# Problem Statement

The National Basketball Association (NBA) desires to achieve a certain level of league parity in order to retain fandom and maintain revenues. The NBA believes that it can establish and maintain league parity by enforcing certain rules in offseason free agency. The NBA believes that designing a multi-agent system that can be used to simulate the dynamics of the offseason free agency will enable them to determine what effects various rules will have on league parity.

NBA parity will be measured by computing a Power Index for each team at the beginning of each iteration (after all offseason free agency activity has finished) and computing each team’s Power Index volatility over time. The specifics of volatility computation are discussed in the Desired Emergent Behavior section. The NBA’s average volatility can then be computed. The NBA wants to maximize this average volatility.

Each agent type will operate under two conditions:

1. An agent will not be able to communicate directly with any other agent of the same type.
2. Agents will strive to maximize their respective utility (see Agent Design section for more information on agent utility).

The agents will make local decisions in an attempt to maximize their individual utility. The NBA would like the rules it introduces to increase the global property of league parity.

# Agent Design

The system will be comprised of two agent types: NBA Players and NBA Teams. Each agent class will be primarily concerned with obtaining its highest individual utility. NBA Players will attempt to maximize their contract size, which will be equivalent to the annual salary multiplied by contract length. NBA Teams will attempt to maximize their overall team Power Index.

**NBA Player**

A NBA Player can either be currently contracted by a Team or available to be signed by Teams. A Player will be available if 1) they were on a Team but their contract expired or 2) they entered the system through the Draft (discussed in Environment Design). Players will also have a finite number of years for which they are available to play in the league. Once they have been in the system for the maximum allotted time, a Team can no longer contract the Player and the Player will be Retired from the system.

A Player who is a free agent will be primarily concerned with joining a Team where they can achieve their maximum contract potential. Contract potential is defined as the following:

Both the salary and the contract length will have upper and lower bounds. A Player cannot accept a contract with a length greater than their remaining time in the league. In addition, a NBA Player will also have a Team Preference Factor (TPF). The TPF will enable a Player to consider the Prestige (a weighted average of the Team’s recent Power Indexes) of a Team along with contract size. The overall contract preference formula will therefore be defined as:

In addition to this, a NBA Player will have a skill value (0-100). This skill value will be generated from a distribution at agent creation. This distribution will be based on the distribution of player efficiency ratings (PER) for real NBA players. The skill attribute will be adjusted each year using a function that takes into account the number of years a Player has been on a Team, their current Team’s Prestige, and the Player’s years left in the league. A Team will consider the Player’s skill when determining whether to offer a contract.

**NBA Team**

A NBA Team will be a collection of Players. Each year, a Team is primarily concerned with maximizing its Power Index relative to the environmental constraints discussed in the Environment Design section below.

The Power Index for a NBA Team will be calculated as a function that takes into account several factors related to a Team. These will include the skills of the individual Players and the Team’s Power Index in previous years. Player Skill will account for 70% of the Team’s Power Index and previous Power Index will account for 30%. These weights were chosen such that a Team performance year-to-year could vary greatly yet there is some carryover from previous years’ performances.

Player Skill will be calculated such that the most skilled Player receives the highest weight with each successive Player receiving less. The weight associated with each rank will be tied to minutes played statistics from the real NBA. For example, each Team’s total minutes will sum to 240 minutes (5 players times 48 minutes). On average, the Player that plays the most minutes might be 35 minutes; this would mean that the top Player on any Team would have their Skill account for 14.5% of the Team’s Player Skill Power Index.

For a Team to maximize its Power Index, there are many factors that must be taken into account. There will be minimum and maximum roster size requirements. The salary cap will also affect the roster for each Team. In any given year, the combination of all salaries paid to a Team’s rostered Players cannot exceed the salary cap for the Team.

A Team will offer contracts to Players. Within the constraints of the current lineup and salary cap, a Team can offer a salary and contract length to a Player not currently signed to a Team. Both the salary and contract length will be based off of Player skill.

# Environment Design

Agents will be Drafted and Retired on a regular schedule. Each year, the Draft will introduce *n* (*n* will be randomly assigned based on a Gaussian distribution) newly created Players. These new Players enter the free agency pool during the offseason in which they are created. Each Player will have a Years Left in the League parameter that will be randomly set based on a Gaussian distribution when the Player is created. Each offseason period, this parameter will be decremented. Once a Player’s Years Left in the League reaches zero, the Player will Retire and leave the simulation.

The league will consist of a fixed number of Teams (30). Each iteration, an offseason period consisting of *w* weeks (where *w* is a fixed simulation parameter) will occur. Each week, a Team will have the ability to offer a contract to *p* Players (where *p* is a fixed simulation parameter). Teams will attempt to sign players such that the Team maximizes its Power Index; however, the Players have the option to accept or reject a contract offered by the Team. Additionally, the Team will be bound by certain constraints including Salary Cap, minimum and maximum Roster Size, and minimum and maximum Yearly Salary for each individual player. These constraints’ values will be based on actual NBA values. As the number of weeks increases, both Teams and Players will become more desperate. A Player’s thresholds for contract acceptance will decrease, as the Player is motivated to be signed by a Team by the end of the offseason period. A Team’s contract offers will become more lucrative in an attempt to fill free space. Once the offseason period ends, each Team’s Power Index and the league average volatility will be computed before another offseason period begins.

# Desired Emergent Behavior

The desired emergent behavior for this multi-agent system is achieving and maintaining a high level of league parity. This is desirable for the NBA because it will keep fans of all Teams interested. We plan on computing volatility in year *i* as follows:

where is the power of Team t in year I, and T is the set of Teams. Note that this can only be calculated after the fifth year in the simulation. The second and third terms are included to allow for some team consistency (2-4 years), while still valuing changes in Power Index over longer spans of time.

# Hypotheses

1. Lowering the salary cap will increase league parity. We believe that this will make it more difficult for Teams to retain skilled Players for long periods of time.
2. Increasing the TPF, which is the weight Players place on Team Prestige (as opposed to contract size), will decrease league parity, since Players with high skill values will be more likely to cluster on the best Teams.
3. Increasing the Max Contract Size will increase parity in the league. This is because a higher Max Contract Size makes it more difficult to retain good Players, since other Teams will be more likely able to offer bigger contracts.

# Experiments

We will run each of these experiments for various numbers of iterations (seasons). Initially, we believe that the number of years will range from a minimum of 5 (so that a reasonable volatility could be computed) to a maximum of 20 (so that any trends or convergence could be reasonably applied to the real NBA).

1. Vary the per-team Salary Cap.
2. Increase the standard deviation for the distribution that determines the value a Player places on team success vs. contract size (success vs. compensation).
3. Vary the Max Contract Size a Team can offer to a Player.

Each of these experiments will be run repeatedly, holding the other parameters constant. For each experiment, we will be observing how changing the parameter influences the average volatility (league parity) of the NBA.