# Problem Statement

The National Basketball Association (NBA) desires to achieve a certain level of league parity in order to retain fandom and maintain revenues. The NBA believes that it can establish and maintain league parity by enforcing certain rules in offseason free agency. The NBA believes that designing a multi-agent system that can be used to simulate the dynamics of the offseason free agency will enable them to determine what effects various rules will have on league parity.

NBA parity will be measured by computing a Power Index for each team at the beginning of each iteration (after all offseason free agency activity has finished) and computing each team’s Power Index volatility over time. The specifics of volatility computation are discussed in the Desired Emergent Behavior section. The NBA’s average volatility can then be computed. The NBA wants to maximize this average volatility.

Each agent type will operate under two conditions:

1. An agent will not be able to communicate directly with any other agent of the same type.
2. Agents will strive to maximize their respective utility (see Agent Design section for more information on agent utility).

The agents will make local decisions in an attempt to maximize their individual utility. The NBA would like the rules it introduces to increase the global property of league parity.

# Agent Design

The system will be comprised of two agent types: NBA Players and NBA Teams. Each agent class will be primarily concerned with obtaining its highest individual utility. NBA Players will attempt to maximize their contract size, which will be equivalent to the annual salary multiplied by contract length. NBA Teams will attempt to maximize their overall team Power Index.

**NBA Player**

A NBA Player can either be currently contracted by a Team or available to be signed by Teams. A Player will be available if 1) they were on a Team but their contract expired or 2) they entered the system through the Draft (discussed in Environment Design). Players will also have a finite number of years for which they are available to play in the league. Once they have been in the system for the maximum allotted time, a Team can no longer contract the Player and the Player will be Retired from the system.

A Player who is a free agent will be primarily concerned with joining a Team where they can achieve their maximum contract potential. Contract potential is defined as the following:

Both the salary and the contract length will have upper and lower bounds. A Player cannot accept a contract with a length greater than their remaining time in the league. In addition, a NBA Player will also have a Team Preference Factor (TPF). The TPF will enable a Player to consider the Prestige (a weighted average of the Team’s recent Power Indexes) of a Team along with contract size. The overall contract preference formula will therefore be defined as:

In addition to this, a NBA Player will have a skill value (0-100). This skill value will be generated from a distribution at agent creation. This distribution will be based on the distribution of player efficiency ratings (PER) for real NBA players. The skill attribute will be adjusted each year using a function that takes into account the number of years a Player has been on a Team, their current Team’s Prestige, and the Player’s years left in the league. A Team will consider the Player’s skill when determining whether to offer a contract.

**NBA Team**

A NBA Team will be a collection of Players. Each year, a Team is primarily concerned with maximizing its Power Index relative to the environmental constraints discussed in the Environment Design section below.

The Power Index for a NBA Team will be calculated as a function that takes into account several factors related to a Team. These will include the skills of the individual Players and the Team’s Power Index in previous years. Player Skill will account for 70% of the Team’s Power Index and previous Power Index will account for 30%. These weights were chosen such that a Team performance year-to-year could vary greatly yet there is some carryover from previous years’ performances.

Player Skill will be calculated such that the most skilled Player receives the highest weight with each successive Player receiving less. The weight associated with each rank will be tied to minutes played statistics from the real NBA. For example, each Team’s total minutes will sum to 240 minutes (5 players times 48 minutes). On average, the Player that plays the most minutes might be 35 minutes; this would mean that the top Player on any Team would have their Skill account for 14.5% of the Team’s Player Skill Power Index.

For a Team to maximize its Power Index, there are many factors that must be taken into account. A Team must have between thirteen and fifteen (inclusive) Players on its roster, which is consistent with NBA regular season regulations. The sum of the annual salaries of all the Players on a Team’s roster is subject to a salary cap. Since the NBA allows teams to exceed this cap and choose to pay luxury tax, we will account for this in our simulation. Teams will be randomly assigned an amount that they are willing to go over the salary cap, ranging from $0 over to no monetary limit.

The offseason simulation will be made up of blocks of time (weeks). Each Team will offer contracts to Players during each week. Within the constraints of the current lineup and salary cap, a Team can offer a salary and contract length to any Player not currently signed to a Team. At the end of each week, Players with contract offers must accept or decline each offer (a Player can accept at most one offer each week). In each week, a team can only send out as many contract offers as it has roster spots available, and the sum of the contract offers must not put them over their salary cap (factoring in luxury tax willingness) if all offers were to be accepted.

The annual contract size offered to Players will depend on the number of roster spots left, the amount of salary cap the Team has available, the current roster of the team, and the Player’s skill. At the beginning of each week, each Team will compute how much each available player would increase its Power Index, based on how the Player’s skill would fit into the current roster. Teams will then make offers to the players that will increase their Power Index the most, where is the number of roster spots available. The size of the contract for player will be computed as follows:

where is the amount that adding Player changes the Team’s Power Index, and is the set of the top Players (in terms of Power Index added).

# Environment Design

Agents will be Drafted and Retired on a regular schedule. Each year, the Draft will introduce *n* (*n* will be randomly assigned based on a Gaussian distribution) newly created Players. These new Players enter the free agency pool during the offseason in which they are created. Each Player will have a Years Left in the League parameter that will be randomly set based on a Gaussian distribution when the Player is created. Each offseason period, this parameter will be decremented. Once a Player’s Years Left in the League reaches zero, the Player will Retire and leave the simulation.

The league will consist of a fixed number of Teams (30). Each iteration, an offseason period consisting of *w* weeks (where *w* is a fixed simulation parameter) will occur, in which Teams make offers, and Players choose to accept or reject them. As the number of weeks increases, both Teams and Players will become more desperate. A Player’s thresholds for contract acceptance will decrease, as the Player is motivated to be signed by a Team by the end of the offseason period. A Team’s contract offers will become more lucrative in an attempt to fill free space. Once the offseason period ends, each Team’s Power Index and the league average volatility will be computed before another offseason period begins.

# Desired Emergent Behavior

The desired emergent behavior for this multi-agent system is achieving and maintaining a high level of league parity. This is desirable for the NBA because it will keep fans of all Teams interested. We plan on computing volatility in year *i* as follows:

where is the power of Team t in year i, T is the set of Teams, and is a weight given to that entry. We will define the weights as follows: . These weights were chosen to value changes in the three to five year span more than recent or distant changes. For calculating parity in years before the eighth year, the unused weights will be dropped, and the remaining ones will be recalculated proportionately. For example, to calculate the parity in year 4, the weights will be: .

# Hypotheses

1. Lowering the salary cap will increase league parity. We believe that this will make it more difficult for Teams to retain skilled Players for long periods of time.
2. Increasing the TPF, which is the weight Players place on Team Prestige (as opposed to contract size), will monotonically decrease league parity, since Players with high skill values will be more likely to cluster on the best Teams.
3. Increasing the Max Contract Size will increase parity in the league. This is because a higher Max Contract Size makes it more difficult to retain good Players, since other Teams will be more likely able to offer bigger contracts.

4. We believe that parity will be minimized when 50% of the Teams are willing to go over the salary cap, and will be maximized when either all or none of the Teams will exceed the salary cap.

5. Teams that sign Players to longer deals will have a higher Power Index, since they will be more likely to keep good players for longer. This will result in lower parity, since the Teams willing to sign longer contracts will remain the same year-to-year.

# Experiments

We will run each of these experiments for various numbers of iterations (seasons). Initially, we believe that the number of years will range from a minimum of 5 (so that a reasonable volatility could be computed) to a maximum of 20 (so that any trends or convergence could be reasonably applied to the real NBA).

1. Vary the per-team Salary Cap.
2. Increase the standard deviation for the distribution that determines the value a Player places on team success vs. contract size (success vs. compensation).
3. Vary the Max Contract Size a Team can offer to a Player.
4. Vary the proportion of teams willing to go over the salary cap, and the amount that each team is willing to go over by.
5. Vary the rate at which Teams decrease their contract offer length each week that passes in the offseason.

Each of these experiments will be run repeatedly, holding the other parameters constant. For each experiment, we will be observing how changing the parameter influences the average volatility (league parity) of the NBA.