We can generally assume that the three pegs are named A, B, and C and have initial coordinates:  
  
A(0, 1), B(0, 0), C(1,0).

X (1, 1)

(0, 1)

A

B

C (1, 0)

(0, 0)

(Given the rules for moves) We must prove that **either**:

1. There is a sequence of moves that ends up in one of the three pegs residing at (1, 1), or
2. No sequence of moves can end in one of the three pegs residing at (1, 1)

**Definition**:

Every point with integer coordinates in the 2-dimensional space has (coordinates-) **oddness**, which is the ordered pair of oddness-values (***odd*** or ***even***) of its **X** and **Y** coordinates.  
  
**Lemma**:  
*An existing peg jumping over another peg preserves its oddness*.

**Proof**:  
There are 6 cases:

|  |  |  |  |
| --- | --- | --- | --- |
| Jumping peg | Peg jumped-over | Operation (jump) | Resulting position |
| B | A | 🡺 | X |
| (even, even) | (even, odd) | 🡺 | (even, even) |
| (0, 0) | (0, 1) | 🡺 | (0, 2) |

|  |  |  |  |
| --- | --- | --- | --- |
| Jumping peg | Peg jumped-over | Operation (jump) | Resulting position |
| B | C | 🡺 | X |
| (even, even) | (odd, even) | 🡺 | (even, even) |
| (0, 0) | (1, 0) | 🡺 | (2, 0) |

|  |  |  |  |
| --- | --- | --- | --- |
| Jumping peg | Peg jumped-over | Operation (jump) | Resulting position |
| A | B | 🡺 | X |
| (even, odd) | (even, even) | 🡺 | (even, odd) |
| (0, 1) | (0, 0) | 🡺 | (0, -1) |

|  |  |  |  |
| --- | --- | --- | --- |
| Jumping peg | Peg jumped-over | Operation (jump) | Resulting position |
| A | C | 🡺 | X |
| (even, odd) | (odd, even) | 🡺 | (even, odd) |
| (0, 1) | (1, 0) | 🡺 | (2, -1) |



|  |  |  |  |
| --- | --- | --- | --- |
| Jumping peg | Peg jumped-over | Operation (jump) | Resulting position |
| C | B | 🡺 | X |
| (odd, even) | (even, even) | 🡺 | (odd, even) |
| (1, 0) | (0, 0) | 🡺 | (-1, 0) |

|  |  |  |  |
| --- | --- | --- | --- |
| Jumping peg | Peg jumped-over | Operation (jump) | Resulting position |
| C | A | 🡺 | X |
| (odd, even) | (even, odd) | 🡺 | (odd, even) |
| (1, 0) | (0, 1) | 🡺 | (-1, 2) |

As can be seen, in all 6 possible cases above, the jumping peg preserves its original oddness after the jump.

Now, **the** **main proof**:  
  
The target location (***1, 1***) has oddness (***odd, odd***).  
None of the three pegs has this oddness, which means that landing at location (***1, 1***) is impossible, as it requires oddness of (***odd, odd***) – as per the Lemma, but the oddness of the three pegs is, and always will be: ***(even, odd)***, ***(even, even)***, ***(odd, even)***.