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Team reference

Challenge Accepted!

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Tabelas, fórmulas e teoremas

242		242		1	21	2
2^2	4	3^2	9		2!	2
2^3	8	3^3	27		3!	6
2^4	16	3^4	81		4!	24
2^5	32	3^5	243		5!	120
2^6	64	3^6	729		6!	720
2^7	128	3^7	2.187]	7!	5.040
2^8	256	3^8	6.561		8!	40.320
2^9	512	3^9	19.683	1	9!	362.880
2^10	1.024	3^10	59.049]	10!	3.628.800
2^11	2.048	3^11	177.147]	11!	39.916.800
2^12	4.096	3^12	531.441	1	12!	[limite int] 479.001.600
2^13	8.192	3^13	1.594.323]	13!	6.227.020.800
2^14	16.384	3^14	4.782.969		14!	87.178.291.200
2^15	32.768	3^15	14.348.907]	15!	1.307.674.368.000
2^16	65.536	3^16	43.046.721	1	16!	20.922.789.888.000
2^17	131.072	3^17	129.140.163	1	17!	355.687.428.096.000
2^18	262.144	3^18	387.420.489]	18!	6.402.373.705.728.000
2^19	524.288	3^19	1.162.261.467	1	19!	121.645.100.408.832.000
2^20	1.048.576	3^20	3.486.784.401	1	20!	2.432.902.008.176.640.000
			1	•		[limite unsigned long long]

Catalan numbers. Catalan numbers are defined by the recurrence:

$$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$$

A closed formula for Catalan numbers is:

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1}$$

Stirling numbers of the first kind. These are the number of permutations of I_n with exactly k disjoint cycles. They obey the recurrence:

$$\begin{bmatrix} n \\ k \end{bmatrix} = (n-1) \begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$$

Stirling numbers of the second kind. These are the number of ways to partition I_n into exactly k sets. They obey the recurrence:

$$\binom{n}{k} = k \binom{n-1}{k} + \binom{n-1}{k-1}$$

Rational Expansion Theorem for Distinct Roots.

If R(z)=P(z)/Q(z), where $Q(z)=q_0(1-\rho_1z)\dots(1-\rho_lz)$ and the numbers (ρ_1,\dots,ρ_l) are distinct, and if P(z) is a polynomial of degree less than l, then

$$[z^n] R(z) = a_1 \rho_1^n + \dots + a_l \rho_l^n, \quad \text{where } a_k = \frac{-\rho_k P(1/\rho_k)}{Q'(1/\rho_k)}.$$
 (7.29)

Proof: Let a_1, \ldots, a_l be the stated constants. Formula (7.29) holds if R(z) = P(z)/Q(z) is equal to

$$S(z) = \frac{a_1}{1 - \rho_1 z} + \dots + \frac{a_1}{1 - \rho_1 z}.$$

$$Q(z) = q_0 + q_1 z + \dots + q_m z^m, \quad \text{where } q_0 \neq 0 \text{ and } q_m \neq 0.$$

The "reflected" polynomial

$$Q^{R}(z) = q_0 z^m + q_1 z^{m-1} + \cdots + q_m$$

has an important relation to Q(z):

$$Q^{R}(z) = q_0(z - \rho_1) \dots (z - \rho_m)$$

$$\iff Q(z) = q_0(1 - \rho_1 z) \dots (1 - \rho_m z).$$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$$\binom{n}{k} = \frac{n}{n-k} \binom{n-1}{k}$$

$$\binom{n}{k} = \frac{n-k+1}{k} \binom{n}{k-1}$$

$$\binom{n+1}{k} = \frac{n+1}{n-k+1} \binom{n}{k}$$

$$\binom{n}{k+1} = \frac{n-k}{k+1} \binom{n}{k}$$

$$x^m = \sum_{k=0}^m \binom{m}{k} x^{\underline{k}},$$

$$\binom{n}{k+1} = \frac{n-k}{k+1} \binom{n}{k}$$

 $\cot 2x = \frac{\cot^2 x - 1}{2\cot x},$

 $\sin \theta$

 $\cos \theta \quad \tan \theta$

$$\begin{array}{ll} \sum_{k=0}^n k = n(n+1)/2 & \sum_{k=0}^b k = (a+b)(b-a+1)/2 \\ \sum_{k=0}^n k^2 = n(n+1)(2n+1)/6 & \sum_{k=0}^n k^3 = n^2(n+1)^2/4 \\ \sum_{k=0}^n k^4 = (6n^5+15n^4+10n^3-n)/30 & \sum_{k=0}^n k^5 = (2n^6+6n^5+5n^4-n^2)/12 \\ \sum_{k=0}^n x^k = (x^{n+1}-1)/(x-1) & \sum_{k=0}^n kx^k = (x-(n+1)x^{n+1}+nx^{n+2})/(x-1)^2 \end{array}$$

$$\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}$$

$$\sum_{k=1}^{n} k^{2} \binom{n}{k} = (n+n^{2})2^{n-2}$$

$$\sum_{k=1}^{n} i^{2} = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^{3} = \frac{n^{2}(n+1)^{2}}{4}$$

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{k} \binom{n}{r-k}$$

$$\binom{n}{k} = \prod_{i=1}^{k} \frac{n-k+i}{i}$$

Table 55 What's the difference?				
$f = \Sigma g$	$\Delta f = g$	$f=\Sigma g$	$\Delta f = g$	
$x^{0} = 1$	0	2 ^x	2 ^x	
$x^{\underline{1}} = x$	1	c^{x}	$(c-1)c^{x}$	
$x^2 = x(x-1)$	2x	$c^{x}/(c-1)$	c^{x}	
$\chi \frac{m}{}$	mx^{m-1}	cf	$\mathrm{c}\Deltaf$	
$x^{\underline{m+1}}/(m+1)$	χ m	f + g	$\Delta f + \Delta g$	
H_x	$x^{-1} = 1/(x+1)$	f g	$f\Delta g + Eg\Delta f$	

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \\ \sin(\alpha-\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \\ \tan(\alpha+\beta)=\frac{\tan\alpha\cos\beta}{1-\tan\alpha\tan\beta} \\ \cos^2\alpha=\frac{1}{2}(1+\cos2\alpha) \\ \sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2} \\ \sin\alpha-\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2} \\ \sin\alpha-\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2} \\ \sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2} \\ \sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2} \\ \cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2} \\ \cos\alpha-\cos\beta=2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha+\beta}{2} \\ \sin\alpha\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \\ \sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-beta)] \\ \tan\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \\ \cos\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-beta)] \\ \tan\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \\ \cos\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\ \sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-beta)] \\ \tan\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\cos\beta=\frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha+\beta)] \\ \sin\alpha+\beta=\frac{1}{2}\cos\alpha$$

 $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$,

 $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y,$

 $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y.$

 $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y},$

 $\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot x + \cot y},$

Derivatives:

1.
$$\frac{d(cu)}{dx} = c\frac{du}{dx},$$

$$2. \ \frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

1.
$$\frac{d(cu)}{dx} = c\frac{du}{dx}$$
, 2. $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$, 3. $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$,

4.
$$\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$$

$$4. \ \frac{d(u^n)}{dx}=nu^{n-1}\frac{du}{dx}, \qquad 5. \ \frac{d(u/v)}{dx}=\frac{v\left(\frac{du}{dx}\right)-u\left(\frac{dv}{dx}\right)}{v^2}, \qquad 6. \ \frac{d(e^{cu})}{dx}=ce^{cu}\frac{du}{dx},$$

$$6. \ \frac{d(e^{cu})}{dx} = ce^{cu}\frac{du}{dx},$$

7.
$$\frac{d(c^u)}{dx} = (\ln c)c^u \frac{du}{dx},$$

8.
$$\frac{d(\ln u)}{dx} = \frac{1}{u} \frac{du}{dx},$$

$$9. \ \frac{d(\sin u)}{dx} = \cos u \frac{du}{dx},$$

$$10. \ \frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx},$$

11.
$$\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx},$$

12.
$$\frac{d(\cot u)}{dx} = \csc^2 u \frac{du}{dx},$$

13.
$$\frac{d(\sec u)}{dx} = \tan u \sec u \frac{du}{dx}$$
,

14.
$$\frac{d(\csc u)}{dx} = -\cot u \csc u \frac{du}{dx}$$

15.
$$\frac{d(\arcsin u)}{dx} = \frac{1}{\sqrt{1 - u^2}} \frac{du}{dx},$$

16.
$$\frac{d(\arccos u)}{dx} = \frac{-1}{\sqrt{1 - u^2}} \frac{du}{dx},$$

17.
$$\frac{d(\arctan u)}{dx} = \frac{1}{1+u^2} \frac{du}{dx},$$

18.
$$\frac{d(\operatorname{arccot} u)}{dx} = \frac{-1}{1+u^2} \frac{du}{dx},$$

19.
$$\frac{d(\operatorname{arcsec} u)}{dx} = \frac{1}{u\sqrt{1-u^2}}\frac{du}{dx},$$

20.
$$\frac{d(\arccos u)}{dx} = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx},$$

21.
$$\frac{d(\sinh u)}{dx} = \cosh u \frac{du}{dx},$$

22.
$$\frac{d(\cosh u)}{dx} = \sinh u \frac{du}{dx},$$

23.
$$\frac{d(\tanh u)}{dx} = \operatorname{sech}^2 u \frac{du}{dx},$$

24.
$$\frac{d(\coth u)}{dx} = -\operatorname{csch}^2 u \frac{du}{dx},$$

$$\mathbf{25.}\ \frac{d(\operatorname{sech} u)}{dx} = -\operatorname{sech} u\ \tanh u \frac{du}{dx},$$

26.
$$\frac{d(\operatorname{csch} u)}{dx} = -\operatorname{csch} u \operatorname{coth} u \frac{du}{dx}$$

27.
$$\frac{d(\operatorname{arcsinh} u)}{dx} = \frac{1}{\sqrt{1+u^2}} \frac{du}{dx},$$

28.
$$\frac{d(\operatorname{arccosh} u)}{dx} = \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx},$$

$$29. \ \frac{d(\operatorname{arctanh} u)}{dx} = \frac{1}{1 - u^2} \frac{du}{dx},$$

30.
$$\frac{d(\operatorname{arccoth} u)}{dx} = \frac{1}{u^2 - 1} \frac{du}{dx},$$

$$\mathbf{31.}\ \frac{d(\operatorname{arcsech} u)}{dx} = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx},$$

32.
$$\frac{d(\operatorname{arccsch} u)}{dx} = \frac{-1}{|u|\sqrt{1+u^2}} \frac{du}{dx}.$$

Integrals:

1.
$$\int cu \, dx = c \int u \, dx,$$

$$2. \int (u+v) dx = \int u dx + \int v dx,$$

3.
$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1,$$

3.
$$\int x^n dx = \frac{1}{n+1}x^{n+1}$$
, $n \neq -1$, 4. $\int \frac{1}{x} dx = \ln x$, 5. $\int e^x dx = e^x$,

$$6. \int \frac{dx}{1+x^2} = \arctan x,$$

7.
$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx,$$

8.
$$\int \sin x \, dx = -\cos x,$$

$$9. \int \cos x \, dx = \sin x,$$

10.
$$\int \tan x \, dx = -\ln|\cos x|,$$

11.
$$\int \cot x \, dx = \ln|\cos x|,$$

12.
$$\int \sec x \, dx = \ln|\sec x + \tan x|,$$
 13.
$$\int \csc x \, dx = \ln|\csc x + \cot x|,$$

13.
$$\int \csc x \, dx = \ln|\csc x + \cot x|,$$

14.
$$\int \arcsin \frac{x}{a} dx = \arcsin \frac{x}{a} + \sqrt{a^2 - x^2}, \quad a > 0,$$

15.
$$\int \arccos \frac{x}{a} dx = \arccos \frac{x}{a} - \sqrt{a^2 - x^2}, \quad a > 0,$$

17.
$$\int \sin^2(ax)dx = \frac{1}{2a} (ax - \sin(ax)\cos(ax)),$$

16.
$$\int_{0}^{\infty} \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2), \quad a > 0,$$

18.
$$\int \cos^2(ax)dx = \frac{1}{2a}(ax + \sin(ax)\cos(ax)),$$

19.
$$\int \sec^2 x \, dx = \tan x$$
, 20. $\int \csc^2 x \, dx = -\cot x$,

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Dilworth's theorem

In any partially ordered set, the maximum number of elements in any antichain equals the minimum number of chains in any partition of the set into chains.

Matrix-tree theorem (Kirchhoff's theorem)

Let matrix T = [tij], where tij is the number of multiedges between i and j, for i!=j, and tii = -degi. Number of spanning trees of a graph is equal to the determinant of a matrix obtained by deleting any k-th row and k-th column from T.

Gale-Shapley algorithm (SMP)

While there is a free man m: let w be the most-preferred woman to whom he has not yet proposed, and propose m to w. If w is free, or is engaged to someone whom she prefers less than m, match m with w, else deny proposal.

Pick's theorem

A of this polygon in terms of the number i of lattice points in the interior located in the polygon and the number b of lattice points on the boundary placed on the polygon's perimeter. A=i+b/2-1

Catalan number, Narayana number

{ 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, ...}
$$C(n) = {2n \choose n} - {2n \choose n+1}$$

narayana is the number of paths from (0, 0) to (2n, 0), with steps only northeast and southeast, not straying below the x-axis, with k peaks.

$$N(n,k) = 1/n * \binom{n}{k} * \binom{n}{k-1}$$

$$N(n,1) + N(n,2) + ... + N(n,n) = C(n)$$

Lucas's theorem

For non-negative integers m and n and a prime p

$$\binom{n}{m} = \prod \binom{ni}{mi} \pmod{p}$$

Wilson's theorem

$$n > 1 \text{ primo } \Leftrightarrow (n-1)! = -1 \pmod{n}$$

Derangements

Number of permutations of the elements of a set such that none of the elements appear in their original position. 1, 0, 1, 2, 9, 44, 265, 1854, 14833,...

$$D(n) = (n-1) * (D(n-1) + D(n-2)) = n * D(n-1) + (-1)^{n}.$$

Fermat primes

A Fermat prime is a prime of form $2^{2^{n}} + 1$. The only known Fermat primes are 3, 5, 17, 257, 65537. A number of form 2n + 1 is prime only if it is a Fermat prime.

Perfect numbers

n > 1 is called perfect if it equals sum of its proper divisors and 1. Even n is perfect i $n = 2^{p^{-1}}(2^p - 1)$ and $2^p - 1$ is prime (Mersenne's). No odd perfect numbers are yet found.

Carmichael numbers

A positive composite n is a Carmichael number $(a^{n^{-1}} \equiv 1 \pmod{n})$ for all a: gcd(a, n) = 1, i n is square-free, and for all prime divisors p of n, p - 1 divides n - 1.

Precedência

Lv	Operator	Description	Grouping
1	::	scope	Left-to-right
2	() []> ++ casts typeid	postfix	Left-to-right
	++ ~ ! sizeof new delete	unary (prefix)	
3	* &	indirection and ref	Right-to-left
	+ -	unary sign operator	
4	(type)	type casting	Right-to-left
5	.* ->*	pointer-to-member	Left-to-right
6	* / %	multiplicative	Left-to-right
7	+ -	additive	Left-to-right
8	<< >>	shift	Left-to-right
9	< > <= >=	relational	Left-to-right
10	== !=	equality	Left-to-right
11	&	bitwise AND	Left-to-right
12	^	bitwise XOR	Left-to-right
13		bitwise OR	Left-to-right
14	&&	logical AND	Left-to-right
15		logical OR	Left-to-right
16	?:	conditional	Right-to-left
17	= *= /= %= += -= >>= <<= &= ^= =	assignment	Right-to-left
18	,	comma	Left-to-right

Heavy-Light Tree

```
1 int to[maxe], ant[maxe], m;
2 int adj[maxv], n;
3 vector< vector<int> > path;
4 int parent[maxv];
5 int path_id[maxv], path_index[maxv];
6 int subtree[maxv];
   int niv[maxv];
8 int in[maxv], out[maxv], tempo;
   int top sort[maxv], pos;
10
11 inline void init() {
     memset(adj,-1,sizeof adj), n = m = 0; 
12
13
14 inline void add(int u, int v) {
      to[m] = v, ant[m] = adi[u], adi[u] = m++;
15
     to[m] = u, ant[m] = adj[v], adj[v] = m++;
16
     n = \max(n, \max(++u, ++v)); 
17
18
19 void dfs(int no = 0, int pai = -1, int nivel = 0) {
20
     in[no] = tempo++;
21
     subtree[no] = 1;
22
     niv[no] = nivel;
23
     parent[no] = pai;
     for(int i = adi[no]; i >= 0; i = ant[i]) {
24
25
        int prox = to[i];
26
        if(prox == pai) continue;
27
        dfs(prox,no,nivel+1);
28
        subtree[no] += subtree[prox]; }
29
      out[no] = tempo++;
      top sort[pos++] = no; }
30
31
32 void heavy_light(int root = 0) {
33
     pos = 0, dfs(root);
34
     path.clear();
35
     vector<bool> parent edge processed(n, false);
36
     parent edge processed[root] = true;
37
     for(int i = 0; i < pos; i++) {
38
        int x = top sort[i];
39
        if(parent edge processed[x])
40
          continue;
41
        vector<int> this path;
```

```
42
        this path.push back(x);
        for(;;) {
43
           int is heavy = (2*subtree[x] >= subtree[parent[x]]);
44
45
          parent edge processed[x] = true;
          x = parent[x];
46
47
           this path.push back(x);
48
           if(!is heavy || parent edge processed[x]) break; }
49
        path.push back(this path); }
50
      for(int i = path.size()-1; i >= 0; i--)
51
        for(int j = path[i].size()-2; j >= 0; j--)
52
          path id[ path[i][i] ] = i,
          path index[ path[i][j] ] = j;
53
54 }
55
56 inline bool is ancestor(int x, int y) {
      return in[v] <= in[x] && out[x] <= out[v]; }
57
58
59 int lca(int u, int v) {
60
      for(;;) {
        if(is ancestor(v,u))
61
62
          return u;
63
        else if(is ancestor(u,v))
64
          return v;
65
        int fu = path[path id[u]].back();
66
        int fv = path[path id[v]].back();
67
        if(niv[fu] >= niv[fv])
68
          u = fu;
69
        else
          v = fv;
70
71
72 }
LIS
```

```
int dolis(int z = 0) {
   rep(i,n) { // se puder repetir, trocar por upper_bound
   int x = lower_bound(lis,lis+z,i,cmp)-lis;
   if( x == z ) z++;
   lis[x] = i;
   pai[i] = x ? lis[x-1] : -1; }
   return z;
}
```

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KD-Tree

```
1 const int maxn = (1 << 20);
   struct P {
3
      int x, v, id;
      ll dist(P p) { return sq(x - p.x) + sq(y - p.y); }
   } p[maxn];
   11 resp[maxn];
   int N;
8
   bool cmpX(P p1, P p2) {
10
      return (p1.x != p2.x ? p1.x < p2.x : p1.y < p2.y); }
11 bool cmpY(P p1, P p2) {
12
      return (p1.y != p2.y ? p1.y < p2.y : p1.x < p2.x); }
13
14 void doit(int ini = 0, int fim = N-1, char flip = 0) {
15
      if(ini < fim) {</pre>
16
        int raiz = (ini + fim) / 2;
17
        nth element(p+ini, p+raiz, p+fim+1, flip ? cmpY : cmpX);
18
        doit(ini, raiz - 1, flip^1);
        doit(raiz + 1, fim, flip^1); }
19
20 }
21
22 void guery(P ponto, ll& best, int ini = 0,
23
               int fim = N-1, char flip = 0) {
24
      if(ini > fim) return;
25
26
      int raiz = (ini + fim) / 2;
27
      11 dist = ponto.dist(p[raiz]);
28
      if(dist > 0) { best = min(best, dist); }
29
      11 D = flip? sq(ponto.y-p[raiz].y) :sq(ponto.x-p[raiz].x);
30
31
      if((flip ? ponto.y <= p[raiz].y : ponto.x <= p[raiz].x)) {</pre>
32
        query(ponto, best, ini, raiz - 1,flip^1);
33
        if(D <= best) query(ponto, best, raiz+1, fim,flip^1);</pre>
34
      } else {
35
        query(ponto, best, raiz + 1, fim,flip^1);
36
        if(D <= best) query(ponto, best, ini, raiz - 1,flip^1);</pre>
37
38
```

Treap

```
1 struct TNode {
      int x, y, z;
3
      TNode *L, *R;
4
      TNode() {}
5
      TNode(int x, TNode *L, TNode *R) {
6
        this->x = xi
7
        this->y = rand();
8
        this->z = 0;
9
        this -> L = L_i
10
        this->R = R_i
11
   } *nil = new TNode(0,NULL,NULL), node[maxn];
13 typedef TNode* Node; Node arv[maxn];
14
15 void fix(Node &root) {
      if(root != nil) root->z = root->L->z + root->R->z + 1; }
16
17
18 void split(Node root, Node &L, Node &R, int x) {
19
      if(root == nil)
20
        return L = nil, R = nil, void();
21
      if(root->x <= x)
22
        return L = root, split(root->R,L->R,R,x), fix(L);
23
      return R = root, split(root->L, L,R->L,x), fix(R);
24
25 Node merge(Node L, Node R) {
26
      if(L == nil) return R; else if(R == nil) return L;
27
      if(L->y >= R->y) return L->R = merge(L->R,R), fix(L), L_i
28
      else return R->L = merge(L,R->L), fix(R), R;
29
30 void insert(Node &root, Node &add) {
31
      if(root == nil || add->y >= root->y)
32
        split(root, add->L, add->R, add->x), root = add;
33
      else if(add->x < root->x) insert(root->L, add);
34
      else insert(root->R, add);
35
      fix(root); }
36
37 void remove(Node &root, int x) {
38
      if(root->x == x)
39
        return root = merge(root->L, root->R), fix(root);
40
      if(x < root -> x) remove(root -> L, x), fix(root);
41
      else remove(root->R,x), fix(root); }
```

```
42
43 int kth(Node root, int x) {
44    int myx = root->L->z + 1;
45    if(x < myx) return kth(root->L, x);
46    if(x > myx) return kth(root->R, x - myx);
47    return root->x;
48 }
```

Pontos de Articulação, Componentes Biconexos e Pontes

```
void gerar(int no) {
2
      vector<int> vet;
3
      while (q--) {
        int no2 = from[pilha[q]];
4
5
        vet.push back(no2);
6
        if(from[pilha[q]] == no) break;
7
8
      // vet.size() == 2 => ponte
9
10
11 void dfs(int no, int dad) {
12
      pai[no] = dad, ord[no] = low[no] = tempo++;
13
      markv[no] = 1;
14
15
      for(int i = adj[no]; i >= 0; i = ant[i]) {
16
        int prox = to[i];
17
        if(!marke[i]) {
           marke[i] = marke[i^1] = 1;
18
           pilha[q++] = i; }
19
20
        if(!markv[prox]) {
21
           dfs(prox,no);
22
           low[no] = min(low[no],low[prox]); }
23
        else if(prox != dad) {
           low[no] = min(low[no],ord[prox]); }
24
        if(q && pai[prox] == no && low[prox] >= ord[no]) {
25
26
           gerar(no); }
27
28
29
30 bool articulation(int no) {
31
      if(pai[no] == -1) {
        int ct = 0;
32
```

```
33
        for(int i = adi[no]; i >= 0; i = ant[i]) {
           int prox = to[i];
34
35
           ct += (pai[prox] == no);
36
37
        return ct >= 2;
38
39
      else {
40
        for(int i = adj[no]; i >= 0; i = ant[i]) {
41
           int prox = to[i];
42
           if(pai[prox] == no && low[prox] >= ord[no]) {
43
             return 1;
44
45
46
        return 0;
47
48
49
50 bool bridge(int x, int y) {
51
      if(ord[x] > ord[y]) swap(x,y);
52
      return pai[y] == x \&\& low[y] > ord[x]; // freq[x][y] == 1?
53 }
```

KonigtheoremHopcroftkarpDilworththeorem

```
public class KonigtheoremHopcroftkarpDilworththeorem {
2.
3
      public static void main(String[] args) throws Exception {
4
        new KonigtheoremHopcroftkarpDilworththeorem().solve();
5
6
7
      int a[][]; // adjacent matrix
8
      int n;
9
10
      int match[], pai[], vis[], passo = 0;
11
12
      void path(int no) {
13
        while(no >= 0) {
14
          match[ pai[no] ] = no;
15
          match[ no ] = pai[no];
16
          no = pai[pai[no]];
17
18
```

++passo;

```
19
                                                                       63
                                                                                for(int i = 0; i < n; i++)
20
                                                                                       if(match[i] == -1 && vis[i] != passo) { // left
      int fila[], pos, q;
                                                                       64
21
                                                                       65
                                                                                  pos = q = 0;
22
      int hopcroftKarp() {
                                                                       66
                                                                                  fila[q++] = i; vis[i] = passo;
        fila = new int[2*n];
23
                                                                       67
                                                                                  while(pos < q) {</pre>
24
        pai = new int[2*n];
                                                                       68
                                                                                     int no = fila[pos++]; cover[no] = false;
25
                                                                       69
        vis = new int[2*n];
                                                                                     for(int j = 0; j < n; j + +) if(a[no][j] != 0) {//right
26
        match = new int[2*n]; Arrays.fill(match,-1);
                                                                       70
                                                                                       int prox = i + n;
27
        for(int i = 0; i < n; i++) { // left
                                                                                       if(match[no] == prox) continue;
                                                                       71
28
                                                                       72
                                                                                       else cover[prox] = true;
           pos = q = 0;
29
           fila[q++] = i; vis[i] = ++passo; pai[i] = -1;
                                                                       73
                                                                                       if(match[prox]!= -1 &&vis[match[prox]] != passo){
30
           boolean found = false;
                                                                       74
                                                                                         vis[match[prox]] = passo;
           while(pos < q && !found) {</pre>
                                                                       75
                                                                                         fila[q++] = match[prox];
31
32
             int no = fila[pos++];
                                                                       76
33
             for(int j = 0; j < n; j + +) if(a[no][j] != 0) {//right
                                                                       77
34
                int prox = i + n;
                                                                       78
                                                                       79
35
                if(match[prox] == -1) {
36
                  pai[prox] = no;
                                                                       80
                                                                                for(int i = 0; i < n; i++) if(cover[n + i]) {
37
                  path(prox);
                                                                       81
                                                                                  cover[i] = true;
38
                  found = true;
                                                                       82
39
                  break;
                                                                       83
40
                                                                       84
41
                else if(vis[ match[prox] ] != passo) {
                                                                       85
                                                                             void solve() throws Exception {
42
                  vis[ match[prox] ] = passo;
                                                                       86
                                                                                int h = hopcroftKarp(); int foi = 0;
43
                  pai[prox] = no;
                                                                       87
                                                                                System.out.printf("%d\n",n - h);
44
                  pai[ match[prox] ] = prox;
                                                                       88
                                                                               minVertexCover();
                  fila[q++] = match[prox];
45
                                                                       89
                                                                                for(int i = 0; i < n; i++) if(!cover[i] && !cover[n+i]){
46
                                                                       90
                                                                                  if(foi++ != 0) System.out.printf(" ");
47
                                                                       91
                                                                                  System.out.printf("%d",i+1);
48
                                                                       92
49
                                                                       93
                                                                               System.out.printf("\n");
50
        int mf = 0;
                                                                       94
        for(int i = 0; i < n; i++) mf += (match[i] != -1 ?1:0);
51
                                                                       95 }
52
        return mf;
53
54
55
      boolean cover[];
56
57
      void minVertexCover() {
58
        cover = new boolean[2*n];
59
        for(int i = 0; i < n; i++) if(match[i] != -1) {
60
           cover[i] = true;
61
```

MaxFlow e Gomory-Hu

```
int level[maxv], fila[maxv], copy adj[maxv];
2
   int bfs(int source, int sink) {
      memset(level,-1,sizeof level);
5
      level[source] = 0;
6
      int pos = 0, tam = 0;
      fila[tam++] = source;
8
      while(pos < tam) {</pre>
9
        int now = fila[pos++];
10
        for(int i = adj[now]; i >= 0; i = ant[i])
11
                if(cap[i] && level[to[i]] == -1) {
12
           level[to[i]] = level[now] + 1;
13
           fila[tam++] = to[i];
14
15
16
      return level[sink] != -1;
17
18
19 int dfs(int no, int sink, int flow) {
20
      if(no == sink) return flow;
21
      for(int &i = copy adj[no]; i \ge 0; i = ant[i])
22
             if(cap[i] && level[no] + 1 == level[to[i]]) {
23
        int nflow = dfs(to[i],sink,min(flow,cap[i]));
24
        if(nflow) {
           cap[i] -= nflow, cap[i^1] += nflow;
25
           return nflow;
26
27
28
29
      return 0;
30
31
32 long long maxflow(int source, int sink) {
33
      long long mf = 0;
34
      while(true) {
35
        if(bfs(source,sink)) {
36
           memcpy(copy_adj,adj,sizeof adj);
37
           while(true) {
38
             int add = dfs(source,sink,1<<30);</pre>
             if(add) mf += add; else break;
39
40
41
        } else break;
```

```
42
43
      return mf;
44 }
45
46 int parent[maxv];
   int answer[maxv][maxv];
48
49 void gomory hu() {
50
      static int cap2[maxe]; rep(i,m) cap2[i] = cap[i];
51
      memset(parent,0,sizeof parent);
52
      memset(answer,0x3f,sizeof answer);
53
      for(int i = 1; i < n; i++)
54
        rep(k,m) cap[k] = cap2[k];
55
        int f = maxflow(i, parent[i]);
56
        bfs(i,parent[i]);
57
        for (int j = i+1; j < n; j++)
58
           if (level[j] != -1 && parent[j]==parent[i]) {
59
             parent[j] = i;
60
61
62
        answer[i][parent[i]]=answer[parent[i]][i] = f;
63
        for (int j=0; j<i;++j) {
64
           answer[i][i] = answer[j][i] =
65
               min(f,answer[parent[i]][i]);
66
67
68
```

return p; }

Mincost MaxFlow Pape int dist[maxv], pot[maxv], pai[maxv]; set< pair<int,int> > heap; 1 int mate[maxn], qf[maxn], n; void update(int no, int ndist, int p) { char inner[maxn]; if(ndist >= dist[no]) return; vector<int> adi[maxn]; // bidirectional 5 if(dist[no] < inf) heap.erase(pair<int,int>(dist[no],no)); 6 dist[no] = ndist, pai[no] = p; int func() { 7 heap.insert(pair<int,int>(dist[no],no)); } 6 memset(mate,-1,sizeof(mate)); 8 7 for(int $i = 0; i < n; i++) {$ 9 pair<int,int> top() { 8 if(mate[i] != -1) continue; 10 pair<int,int> ret = *heap.begin(); 9 for(int $j = adj[i].size()-1; j >= 0; j--) {$ 11 heap.erase(heap.begin()); 10 int prox = adi[i][i]; 12 return ret; } 11 if(mate[prox] != -1) continue; 13 12 mate[i] = prox, mate[prox] = i; 14 int djikstra(int source, int sink) { 13 break; } 15 heap.clear(), memset(dist,inf,sizeof dist); 14 16 update(source, 0, -1); 15 for(int i = 0; i < n; i++) { 17 while(heap.size()) { if(mate[i] != -1) continue; 16 18 pair<int,int> p = top(); memset(inner,0,sizeof(inner)); 17 19 for(int i = adi[p.second]; i >= 0; i = ant[i]) if(cap[i])18 queue<int> q; 20 update(to[i],p.first+w[i]+pot[p.second]-pot[to[i]],i); q.push(i), inner[i] = 1, qf[i] = -1; 19 21 20 while(!q.empty()) { 22 return dist[sink] < inf; } 21 int u = q.front(), v; q.pop(); 23 22 for(int $j = adj[u].size()-1; j >= 0; j--) {$ 24 pair<int,int> mcmf(int source, int sink) { 23 int prox = adj[u][j]; 25 //need bellman-ford? 2.4 if(inner[prox]) continue; //memset(pot,0x3f,sizeof pot), pot[source] = 0; 26 25 if(mate[prox] == -1) { 27 //for(int k = 0; k < n; k++) for(int i = 0; i < n; i++)26 while($u \ge 0$) { // aumente 28 // for(int j = adj[i]; j >= 0; j = ant[j]) if(cap[j])int old = mate[u]; 27 29 // pot[to[j]] = min(pot[to[j]], pot[i] + w[j]);mate[u] = prox, mate[prox] = u; 28 30 memset(pot,0,sizeof pot); 29 prox = old, u = gf[u];31 pair<int, int> p(0,0); // cost, flow 30 qoto next; } 32 while(djikstra(source,sink)) { 31 $for(v = u; v \ge 0; v = qf[v]) if(v == prox) break;$ 33 int cost = 0, flow = inf; 32 $if(v != prox) {$ 34 for(int x = sink; x != source; x = from[pai[x]]) 33 inner[prox] = 1;if(cap[pai[x]] < flow) flow = cap[pai[x]];</pre> 35 34 g.push(mate[prox]), gf[mate[prox]] = u; } 36 for(int x = sink; x != source; x = from[pai[x]]) 35 37 $cap[pai[x]] = flow, cap[pai[x]^1] += flow,$ 36 38 cost += w[pai[x]]*flow; 37 next:; 39 for(int x = 0; x < n; x++) pot[x] += dist[x]; 38 40 p.first += cost, p.second += flow; int tot = 0;39 41 40 for(int i = 0; i < n; i++) tot += (mate[i] != -1);

41

return tot / 2; }

Stable Marriage

```
struct S {
2
      // sorted list of indices by
      int lista[maxn], pref[maxn]; preference
   } h[maxn], m[maxn];
   int prox[maxn], quem[maxn], n;
   void find(int H, int M) {
8
      if(quem[M] == -1) {
9
        quem[M] = H;
10
11
      else if( m[M].pref[H] < m[M].pref[quem[M]] ) {</pre>
12
        int now = quem[M];
13
        quem[M] = H;
        find(now,h[now].lista[prox[now]++]);
14
15
16
      else {
17
        find(H,h[H].lista[prox[H]++]);
18
19
20
21 void process() {
22
      memset(prox, 0, sizeof(prox));
23
     memset(quem,-1,sizeof(quem));
     rep(i,n) if(i) find(i,h[i].lista[prox[i]++]);
24
25
      rep(i,n) if(i) printf("%d %d\n",quem[i],i);
26 }
```

Corte Mínimo - Stoer-Wagner

```
1 int adj[110][110];
2 int n,m, inf = 1<<30;
3 bool foi[110], mark[110];
4 int cap[110];</pre>
```

```
int corte() {
      int ret, S, T;
      memcpy(mark,foi,sizeof foi);
9
      fr(i,0,n) if( !foi[i] ) {
10
        fr(i,0,n) cap[i] = adi[i][i];
11
        mark[i] = true;
12
        S = i;
13
        break;
14
15
      while( true ) {
16
        int x, y = 0;
17
        fr(i,0,n) if (!mark[i] && cap[i] > y ) x = i,y = cap[i];
18
        if(!y) break;
19
        ret = y;
20
        T = S_i
2.1
        S = x_i
2.2
        mark[S] = true;
2.3
        fr(i,0,n) if( !mark[i] && adj[S][i] ) {
24
           cap[i] += adj[S][i];
25
26
27
      foi[S] = true;
28
      fr(i,0,n) {
29
        adj[i][T] += adj[i][S];
30
        adj[T][i] += adj[S][i];
31
32
      return ret;
33 }
34 bool read() {
35
      scanf("%d%d", &n, &m);
      memset(foi,false,sizeof foi);
37
      memset(adj,0,sizeof adj);
38
      fr(i,0,m) {
39
        int a,b,c;
40
        scanf("%d%d%d", &a, &b, &c); a--,b--;
        adj[a][b] = adj[b][a] = c;
41
42
43
      int res = inf;
44
      fr(k,1,n) {
45
        res = min(res, corte());
46
47
      printf("%d\n",res);
48
      return true;
49 }
```

```
14
                                                                                    for(int i = 0; i < N; i++) {
 Componente Fortemente Conexo
                                                                                       swap(A[linha][i],A[pos][i]);
                                                                      15
                                                                      16
  void dfs(int no, int p) {
                                                                      17
                                                                                    swap(B[linha],B[pos]);
2
      d[no] = low[no] = tempo++;
                                                                      18
      pai[no] = p, mark[no] = 1;
3
                                                                      19
                                                                                  double F = A[linha][pivo]; // norm
4
      pilha[q++] = no;
                                                                      2.0
                                                                                  for(int i = 0; i < N; i++) {
5
      rep(i,adj[no].size()) {
                                                                      21
                                                                                    A[linha][i] /= F;
6
        int prox = adj[no][i];
                                                                       22
7
        if(!mark[prox]) {
                                                                       23
                                                                                 B[linha] /= F;
8
           dfs(prox,no);
                                                                       24
9
           low[no] = min(low[no],low[prox]);
                                                                       25
                                                                                 for(int i = linha+1; i < M; i++) {
10
                                                                                    F = A[i][pivo];
                                                                       26
11
        else if(comp[prox] == -1) {
                                                                       27
                                                                                    if( cmp(F) ) {
12
           low[no] = min(low[no], d[prox]);
                                                                      28
                                                                                      for(int j = pivo; j < N; j++) {
13
                                                                       29
                                                                                         A[i][j] = (A[linha][j] * F) -
14
                                                                      30
                                                                                                        (A[i][j] * A[linha][pivo]);
15
      if(low[no] == d[no]) {
                                                                      31
16
        while(true) {
                                                                      32
                                                                                      B[i] = (B[linha] * F) - (B[i] * A[linha][pivo]);
17
           int x = pilha[--q];
                                                                      33
18
           comp[x] = ncomp;
                                                                       34
19
           if(x == no) break;
                                                                      35
                                                                                  linha++;
20
                                                                       36
21
        ncomp++;
                                                                      37
22
                                                                      38
23 }
                                                                      39
                                                                             for(int i = linha; i < M; i++) {
                                                                      40
                                                                               if( cmp(B[i]) ) {
                                                                      41
                                                                                 printf("IMPOSSIBLE\n");
 Gauss
                                                                      42
                                                                                  assert(0);
                                                                      43
                                                                                  return; }
                                                                      44
   void gauss() {
                                                                      45
2
      int linha = 0;
                                                                            if(linha < N) {</pre>
                                                                      46
      for(int pivo = 0; pivo < N; pivo++) {</pre>
3
                                                                      47
                                                                               printf("VERY SOLUTIONS\n");
4
        int pos = -1;
                                                                      48
                                                                               assert(0);
5
        double vmax = 0;
                                                                      49
                                                                               return; }
6
        for(int i = linha; i < M; i++) {</pre>
                                                                      50
7
           if(cmp(A[i][pivo]) && cmp(fabs(A[i][pivo]), vmax) >0 ){
                                                                      51
                                                                             for(int i = linha-1; i >= 0; i--) {
8
             pos = i;
                                                                      52
                                                                               assert( cmp(A[i][i]) );
9
             vmax = fabs(A[i][pivo]);
                                                                      53
                                                                               B[i] /= (A[i][i]);
10
                                                                      54
                                                                               for(int j = 0; j < i; j++) {
11
                                                                       55
                                                                                 B[j] -= (A[j][i] * B[i]); 
12
        if(pos != -1) {
                                                                      56
13
           if(pos != linha) {
```

57 }

Teroema Chinês do Resto

```
pair<ll,ll> euclides(ll x, ll y) {
2
      if(!y) return pair<ll, ll>(1,0);
3
      if(!x) return pair<11,11>(0,1);
4
      pair<ll.ll> p = euclides(v%x,x);
5
      return pair<11,11>(p.second - p.first*(y/x), p.first);
6
7
   11 inverse(ll x.ll mod) {
      pair<11,11> p = euclides(x,mod);
8
9
      return ((p.first % mod) + mod) % mod;
10
11
12 ll chinese(ll *A, ll *B, int n) {
      11 \mod = 1, res = 0;
13
14
      rep(i,n) mod *= B[i];
15
      rep(i,n) {
16
        ll add = A[i] * (mod / B[i]) %
                       mod * inverse(mod / B[i], B[i]) % mod;
17
18
        res = (res + add) % mod;
19
20
      return res;
21 }
22
23 ll gcd(ll x, ll y) {
24
      while(x) y %= x, swap(x,y);
25
      return y;
26 }
27
28 vector<ll> solve(ll a, ll b, ll n) { // a*x=b
29
      vector<ll> v;
30
      pair<ll.ll> s = euclides(a, n);
31
      ll q = qcd(a,b);
32
      if (b % q == 0) {
33
        11 \times 0 = ((s.first*(b/q)) % n + n) % n;
34
        for(int i = 0; i < q; i++) {
35
           11 \text{ nx} = x0 + i*(n/q);
36
           v.push\_back((nx % n + n) % n);
37
38
        sort(v.begin(),v.end());
39
40
      return v;
41 }
```

ou

```
int extended euclid(int a, int b, int &x, int &y) {
     int xx = y = 0;
3
     int yy = x = 1;
4
     while (b) {
5
       int q = a/b;
6
       int t = b; b = a%b; a = t;
7
       t = xx; xx = x-q*xx; x = t;
8
        t = yy; yy = y-q*yy; y = t;
9
10
     return a;
11 }
12
13 PII chinese remainder theorem(int x, int a, int y, int b) {
14
     int s, t;
15
     int d = \text{extended euclid}(x, y, s, t);
     if (a\%d != b\%d) return make pair(0, -1);
16
     return make pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
17
18
19
20
21 PII chinese remainder theorem(const VI &x, const VI &a) {
22
     PII ret = make pair(a[0], x[0]);
23
     for (int i = 1; i < x.size(); i++) {
       ret = chinese remainder theorem(
24
25
                       ret.first, ret.second, x[i], a[i]);
26
       if (ret.second == -1) break;
27
28
     return ret;
29 }
```

Pollard Rho

```
ll mulMod(ll a, ll b, ll n) {
      if(a == 0 \mid | b <= (1LL << 62) / a) return a * b % n;
3
      11 \text{ result} = 0;
4
      if(a < b) swap(a,b);
5
      for(; b != 0; b >>= 1) {
6
        if(b & 1) {
7
           result += a; if(result >= n) result -= n; }
8
        a <<= 1; if(a >= n) a -= n; }
9
      return result; }
10
11 ll powMod(ll a, ll exp, ll n) {
12
      11 \text{ result} = 1;
13
      for(; exp != 0; exp >>= 1) {
14
        if(exp & 1) {
15
           result = mulMod(result, a, n); }
16
        a = mulMod(a, a, n); }
17
      return result;
18 }
19
20 ll f(ll x, ll c, ll n) {
21
      11 result = mulMod(x, x, n);
22
      result += c; if(result >= n) result -= n;
23
      return result; }
24
25 ll gcd(ll u, ll v) {
26
      while(u) v = u, swap(u,v); return v; }
27
28 bool primeTestMillerRabin(ll n, ll a) {
29
     11 d = n-1, x;
30
     int s = 0, r;
31
      while ((d \& 1) == 0 \&\& d != 0) s++, d >>= 1;
32
      while (a \ge n) a \ge 1;
33
      x = powMod(a, d, n);
34
      if (x != 1 \&\& x != n-1) {
35
       r = 1;
36
        while (r \le s-1 \&\& x != n-1) {
37
         x = mulMod(x, x, n);
38
          if (x == 1) return false; else r++; }
39
        if (x != n-1) return false; }
40
      return true; }
41
```

```
42 bool multipleMillerRabin(long long n) {
43
      if (n <= 1) return false;
      if (n <= 3) return true;
45
      if ((n \& 1) == 0) return false;
46
      return
47
        primeTestMillerRabin(n, 2) &&
48
        primeTestMillerRabin(n, 3) &&
49
        primeTestMillerRabin(n, 5) &&
50
        primeTestMillerRabin(n, 7) &&
51
        (n < 3215031751LL ||
52
        (primeTestMillerRabin(n, 11) &&
53
           (n < 2152302898747LL \mid )
54
           (primeTestMillerRabin(n, 13) &&
55
             (n < 3474749660383LL ||
56
             (primeTestMillerRabin(n, 17) &&
57
               (n < 341550071728321LL ||
58
                (primeTestMillerRabin(n, 23))))));
59
60
61 ll pollardsRho(ll n, bool testPrime) {
62
      for (int count = 0, c = 1;;) {
63
        11 x = 2, y = 2, pot = 1, lam = 1, d;
64
        do {
65
           ++count;
66
           if (count == 12) {
67
             if (testPrime) return multipleMillerRabin(n) ?n:1;
68
             else if (multipleMillerRabin(n)) return n;
69
           if(pot == lam) x = y, pot <<= 1, lam = 0;
70
           y = f(y,c,n), lam++;
71
72
           d = gcd(x >= y ? x - y : y - x, n);
73
        } while (d == 1);
74
        if (d != n) return d; else c++; }
75 }
76
77 bool tripleMillerRabin(int x) {
78
      return
        primeTestMillerRabin(x, 2) && primeTestMillerRabin(x, 7)
79
80
        && primeTestMillerRabin(x, 61); }
81
82 bool isPrime(ll n) {
83
      if (n <= 0x7fffffff) return tripleMillerRabin((int)n);</pre>
84
      return pollardsRho(n, true) == n;
85 }
```

Fast Fourier Transform

```
typedef complex<double> T;
   typedef vector<T> VT;
   const int maxn = 1 << 20;
   T A[maxn];
   void fft(T* A, int n, bool inv = false) {
7
     if(n==1) return;
8
     T *A0=A, *A1=A+n/2; rep(i,n) A[i]=A[i];
9
     rep(i,n/2) A0[i]=A[2*i], A1[i]=A[2*i+1];
10
     fft(&A0[0],n/2,inv), fft(&A1[0],n/2,inv);
11
     T = \exp(T(0,(inv?-2:2)*M PI/n)), wnow=T(1,0);
12
     rep(i,n/2)
13
          A[i]=A0[i]+wnow*A1[i],
14
          A[i+n/2]=A0[i]-wnow*A1[i],wnow*=w;
15
     rep(i,n) A[i] = A[i];
16 }
17
18 void multiply(VT& A, VT B) {
19
     int n = A.size() + B.size();
20
     while(n \& (n-1)) n++;
21
     A.resize(n), B.resize(n);
22
     fft(&A[0],n,false), fft(&B[0],n,false);
23
     rep(i,n) A[i] *= B[i];
24
     fft(&A[0],n,true);
25
     rep(i,n) A[i] /= n;
26 }
```

Josephus

```
1 ll pilha[1<<21];
   ll josephus(ll n, ll k, int st = 0) {
3
      pilha[st++] = n;
4
      while( n = pilha[st-1] ) {
5
        if( n == 1 || k == 1 ) break;
6
        if (k > n) pilha[st++] = n-1;
7
        else pilha[st++] = n-n/k;
8
9
     11 \text{ res} = 0;
10
      if(k == 1) res = n-1;
```

```
11
      while( st ) {
12
        n = pilha[--st];
13
        if (k > n) res = (res + k)%n;
14
        else res = res - n%k, res += (res < 0 ? n : res/(k-1));
15
16
     return res;
17
18
     // recursivo
     if( n == 1 ) return 0;
19
2.0
     if( k == 1 ) return n-1;
21
     if (k > n) return (josephus(n-1, k) + k);
22
     res = josephus(n-n/k, k) - n%k;
23
     if( res < 0 ) res += n;
24
     else res += res/(k-1);
25
     return res;
26 }
```

Polinômios

```
typedef complex<double> cdouble;
   int cmp(cdouble x, cdouble y = 0) {
3
      return cmp(abs(x), abs(y));
4
   const int TAM = 200;
   struct poly {
      cdouble poly[TAM]; int n;
8
      poly(int n = 0): n(n) \{ memset(p, 0, sizeof(p)); \}
9
      cdouble& operator [](int i) { return p[i]; }
10
     poly operator ~() {
11
        poly r(n-1);
        for(int i = 1; i \le n; i++) r[i-1] = p[i] * cdouble(i);
12
13
        return r;
14
      pair<poly, cdouble> ruffini(cdouble z) {
15
        if (n == 0) return make pair(poly(), 0);
16
17
        poly r(n-1);
        for (int i = n; i > 0; i--) r[i-1] = r[i] * z + p[i];
18
        return make pair(r, r[0] * z + p[0]);
19
20
21
      cdouble operator ()(cdouble z) {
22
        return ruffini(z).second;
23
```

```
24
      cdouble find one root(cdouble x) {
                                                                     11
                                                                             rep(j,k) if (N[j]){
25
        poly p0 = *this, p1 = ~p0, p2 = ~p1;
                                                                     12
                                                                                 c[i]-=c[e]*A[l][i];
26
        int m = 1000;
                                                                     13
                                                                                 rep(i,m) if (i!=1) A[i][i]-=A[i][e]*A[1][i];
27
        while (m--) {
                                                                     14
           cdouble y0 = p0(x);
                                                                             kt[1]=e; N[x]=1; c[x]=c[e]*-A[1][x];
28
                                                                     15
           if (cmp(v0) == 0) break;
29
                                                                     16 }
30
           cdouble G = p1(x) / y0;
                                                                     17 VT doit(int k){
31
           cdouble H = G * G - p2(x) - v0;
                                                                     18
                                                                             VT res; T best;
           cdouble R = sqrt(cdouble(n-1)*(H * cdouble(n) - G*G));
32
                                                                             while (1){
                                                                     19
33
           cdouble D1 = G + R, D2 = G - R;
                                                                                 int e=-1, l=-1;
                                                                     2.0
34
           cdouble a = cdouble(n) / (cmp(D1, D2) > 0 ? D1 : D2);
                                                                     21
                                                                                 rep(i,k) if (N[i] && c[i]>eps) {e=i; break;}
35
             x -= a;
                                                                     22
                                                                                 if (e==-1) break;
                                                                     23
36
             if (cmp(a) == 0) break;
                                                                                 rep(i,m)
                                                                                     if (A[i][e]>eps && (l==-1 || best>b[i]/A[i][e]))
37
                                                                     24
                                                                                         best=b[ l=i ]/A[i][e];
38
           return x;
                                                                     25
39
                                                                     26
                                                                                 if (l==-1) /*ilimitado*/ return VT();
40
      vector<cdouble> roots() {
                                                                     2.7
                                                                                 pivot(k,l,e);
41
        poly q = *this;
                                                                     2.8
42
        vector<cdouble> r;
                                                                     29
                                                                             res.resize(k,0); rep(i,m) res[kt[i]]=b[i];
43
        while (q.n > 1) {
                                                                     30
                                                                             return res;
44
           cdouble z(rand()/double(RAND MAX),
                                                                     31 }
45
                    rand()/double(RAND MAX));
                                                                     32
46
           z = g.find one root(z); z = find one root(z);
                                                                     33 VT simplex(vector<VT> &AA,VT &bb,VT &cc){
47
           g = g.ruffini(z).first;
                                                                     34
                                                                             int n=AA[0].size(),k;
48
           r.push back(z);
                                                                     35
                                                                             m=AA.size(); k=n+m+1; kt.resize(m);
49
                                                                     36
                                                                             b=bb; c=cc; c.resize(n+m); A=AA;
50
                                                                     37
        return r;
                                                                             rep(i,m){
51
                                                                     38
                                                                                A[i].resize(k); A[i][n+i]=1; A[i][k-1]=-1; kt[i]=n+i;}
52 };
                                                                     39
                                                                             N=VI(k,1); rep(i,m) N[kt[i]]=0;
                                                                             int pos=min_element(b.begin(),b.end())-b.begin();
                                                                     40
                                                                     41
                                                                             if (b[pos]<-eps){
                                                                     42
                                                                                 c=VT(k,0); c[k-1]=-1; pivot(k,pos,k-1); res=doit(k);
 Simplex
                                                                     43
                                                                                 if (res[k-1]>eps) /*impossivel*/ return VT();
                                                                     44
                                                                                 rep(i,m) if (kt[i]==k-1)
1 const double eps = 1e-9;
                                                                     45
                                                                                     rep(j,k-1)
   typedef long double T;
                                                                     46
                                                                                         if (N[j] && (A[i][j]<-eps | eps<A[i][j])){
   typedef vector<T> VT; typedef vector<int> VI;
                                                                     47
                                                                                             pivot(k,i,j); break;
   vector<VT> A; VT b,c,res; VI kt,N; int m;
                                                                     48
5
                                                                     49
                                                                                 c=cc; c.resize(k,0); rep(i,m) rep(j,k) if (N[j])
б
   inline void pivot(int k,int l,int e){
                                                                         c[j]-=c[kt[i]]*A[i][j];
7
       int x=kt[l]; T p=A[l][e];
                                                                     50
8
       rep(i,k) A[l][i]/=p; b[l]/=p; N[e]=0;
                                                                             res=doit(k-1); if (!res.empty()) res.resize(n);
                                                                     51
9
       rep(i,m)
                                                                     52
                                                                             return res;
```

53 }

if(i!=1) b[i]-=A[i][e]*b[1],A[i][x]=A[i][e]*-A[1][x];

Aho-Corasick

```
struct Trie {
2
      int adj[52], pai, fail, c;
3
      bitset<1000> b;
4
      void init(char ch, int dad) {
5
        c = ch, pai = dad, fail = 0;
6
        memset(adj,-1,sizeof(adj)), b.reset(); }
      int& operator[](int i) { return adj[i]; }
8
      int func(int);
9
    } no[maxn]; int nno;
10
11 int Trie::func(int x) {
12
      int % res = adj[x];
13
     if(res == -1) {
14
        if(fail == 0 \&\& no[0][x] == -1) res = 0;
15
        else res = no[fail].func(x);
16
17
      return res; }
18
19 int func(char c) {
20
      if('a' <= c && c <= 'z') return c - 'a';
21
      else return c - 'A' + 26; }
22
23 void add(char *s, int indice) {
24
      int atual = 0;
      for(int i = 0; s[i]; i++) {
25
26
        int& prox = no[atual][func(s[i])];
27
        if(prox == -1) {
28
           prox = nno++, no[prox].init(s[i],atual);
29
        } atual = prox;
30
31
      no[atual].b[indice] = 1; }
32
33 #define err(w,c,a) (no[w][c]==-1 | | no[w][c]==a)
34 void ativar() {
35
      queue<int> 0;
36
      for(int i = 0; i < 52; i++) {
37
        if(no[0][i] != -1) {
38
           Q.push(no[0][i]);
39
40
41
      while(!Q.empty()) {
```

```
42
        int atual = 0.front(); 0.pop();
43
        int c = func(no[atual].c);
        bitset<1000> &b = atual[nol.b;
44
45
        int& w = no[atual].fail = no[atual].pai;
46
        while(w && err(w,c,atual)) {
47
          w = no[w].fail, b = no[w].b;
48
        if(!err(w,c,atual)) {
          w = no[w][c], b = no[w].b;
49
50
        for(int i = 0; i < 52; i++) {
51
          if(no[atual][i] != -1) {
52
             O.push(no[atual][i]);
53
54
55
56
57
   #undef err
58
59 bitset<1000> func(char texto[]) {
60
     bitset<1000> b;
61
     int atual = 0;
62
      for(int i = 0; texto[i]; i++) {
63
        atual = no[atual].func( func(texto[i]) );
64
        b |= no[atual].b; }
65
     return b; }
```

KMP

```
1 void kmp(char * p, char * s) {
     static int f[1<<20];
3
     f[0] = f[1] = 0;
4
     for( int i = 1, j = 0; p[i]; )
5
        if(p[i] == p[j]) f[++i] = ++j;
6
        else if(j) j = f[j];
        else f[++i] = 0;
     for( int i = 1, j = *s = *p ; s[i-1] ; ) {
        if( !p[j] ) printf("%d ", i); //match
9
10
        if(s[i] == p[j]) ++i, ++j;
11
        else if(j) j = f[j];
12
        else ++i;
13
14 }
```

Manacher

```
int manacher() {
2.
      n = strlen(s);
3
      for(int i = 0, l = 0, r = -1; i < n; i++) { // even
        int k = (i > r ? 0 : min(even[r + 1 - i - 1], r - i));
4
5
        while (0 \le i-k \&\& i+k+1 \le n \&\& s[i-k] == s[i+k+1]) k++;
6
        even[i] = k;
        if(i + k > r) l = i - k + 1, r = i + k;
        // 1 <= x <= even[i] : string(s+i-x+1, s+i+x+1)
9
        // for(int x = 1; x \le even[i]; x++)
10
        // cout << string(s + i - x + 1, s + i + x + 1) << endl;
11
12
      for(int i = 0, l = 0, r = -1; i < n; i++) { // odd
        int k = (i > r ? 0 : min(odd[r + l - i], r - i));
13
        while(0 \le i-k-1 && i+k+1 \le n && s[i-k-1] == s[i+k+1]) k++i
14
        odd[i] = k; if(i + k > r) l = i - k, r = i + k;
15
        // 0 \le x \le odd[i] : string(s + i - x, s + i + x + 1);
16
17
        // for(int x = 0; x \le odd[i]; x++)
        // cout << string(s + i - x, s + i + x + 1) << endl;
18
19
20
      int palindromes = 0;
21
      for(int i = 0; i < n; i++)
22
        palindromes += even[i], palindromes += (odd[i] + 1);
23
      return palindromes;
24 }
```

Array de Sufixo

```
int cmp(int i, int j) {
      return dp[(i += skip) <= n ?i:n][h]
3
              < dp[(j+=skip) <= n ?j:n][h];
4
6
   void suffixArray(int len = 1<<30) {</pre>
7
      n = strlen(s);
8
      for(int i = 0; i \le n; i++)
9
        suffix[i] = i, dp[i][0] = s[i];
      skip = h = 0, sort(suffix.suffix+n+1.cmp);
10
11
      for(skip=1;skip;skip<<=1) {</pre>
12
        int q = -1;
```

```
13
        for(int i = 0, i = 0; i \le n;) {
           if(j==n+1 or dp[suffix[i]][h] != dp[suffix[j]][h]) {
14
15
             sort(suffix+i,suffix+j,cmp);
16
             dp[suffix[i]][h+1] = ++q;
17
             while(++i < j) 
18
                if(cmp(suffix[i-1],suffix[i])) q++;
19
                dp[suffix[i]][h+1] = q;
20
21
           } else j++;
2.2
23
        if(++h && q >= n) break; else if(skip >= len) break;
24
25
26
27 int lcp(int i, int j) {
28
      if(i == i) return n - i;
29
      int len = 0;
30
      for(int k = h; k \ge 0; k--) if(dp[i][k] == dp[j][k])
31
        len += (1 << k), i += (1 << k), i += (1 << k);
32
      return len;
33 }
```

Regex

```
public class RegularExpression {
3
      /*
                 :: Matches any sign
       * [ab]
                  :: Set definition, can match the letter a or b
6
       * [^ab] :: This can match any character except a or b
       * [a-d1-7] :: Ranges, letter between a and d and
                      figures from 1 to 7, will not match d1
9
       * X | Z
                  :: Finds X or Z
10
       * XZ
                  :: Finds X directly followed by Z
       * X*
                  :: X occurs zero or more times
11
12
      * X+
                  :: X occurs one or more times
                  :: X occurs between Y and Z times
13
       * X{Y,Z}
       * $
                  :: Checks if a line end follows
14
       * /
15
16
17
      public static void main(String[] args) throws Exception {
18
        Pattern pattern = Pattern.compile("a{1,500}b"); // regex
```

```
19
        Matcher matcher = pattern.matcher("aaaaab"); // input
        boolean found = false;
20
21
        while(matcher.find()) {
22
           System.out.println(
23
              "at ("+ matcher.start() +","+ matcher.end() + ")");
24
           found = true;
25
26
        if(!found) {
27
           System.out.println("null");
2.8
29
        System.out.println(matcher.matches() ? "yes" : "no");
30
31 }
```

Árvore de Sufixo

```
const int K = 26, maxn = 500000+10;
   struct state {
3
      int pos, length, link, v, w;
4
      int next[K];
5
   };
   state st[maxn];
   int cnt, last;
   void init() {
      cnt = last = 0;
10
     st[0].link = -1;
11
     st[0].pos = -1;
12
      st[0].w = INT MAX;
13
      memset (st[0].next, -1, sizeof st[0].next);
14
      ++cnt; }
15
16 void sa_extend (int position, char c) {
17
      int nlast = cnt++;
      st[nlast].length = st[last].length + 1;
18
      st[nlast].pos = position;
19
20
      st[nlast].w = INT MAX;
      memset (st[nlast].next, -1, sizeof st[0].next);
21
22
      int p;
23
      for (p=last; p!=-1 && st[p].next[c]==-1; p=st[p].link)
24
        st[p].next[c] = nlast;
25
      if (p == -1)
        st[nlast].link = 0;
26
```

```
else {
27
        int q = st[p].next[c];
28
29
        if (st[p].length + 1 == st[q].length)
30
           st[nlast].link = q;
31
        else {
32
           st[cnt].length = st[p].length + 1;
33
           memcpy (st[cnt].next, st[q].next, sizeof st[0].next);
34
           st[cnt].link = st[q].link;
35
           st[cnt].pos = -1;
36
           st[cnt].w = INT MAX;
37
           for (; p!=-1 && st[p].next[c]==q; p=st[p].link)
38
             st[p].next[c] = cnt;
           st[q].link = st[nlast].link = cnt;
39
40
           ++cnt;
41
42
43
      last = nlast;
44 }
45
46 void build dfa(char *s, int n){
47
      init();
48
      for (size t i=0; i<n; ++i) {
49
        sa extend (i, s[i]-'a');
50
51 }
```

Algoritmo Z

```
1 int main() {
      scanf("%s",str);
3
     n = strlen(str);
      fz[0] = n;
      for(int i = 1, l = 0, r = 0; i < n; i++) {
6
        if(1 \&\& i + fz[i-1] < r)
          fz[i] = fz[i-1];
8
        else {
9
           int j = min(1 ? fz[i-1] : 0, i > r ? 0 : r-i);
10
          while(str[i+j] == str[j] \&\& ++j);
          fz[i] = j, l = i, r = i + j;
11
12
13
14
```

Poker

```
const char* rank names = "**23456789TJOKA";
   const char* suit names = "CDHS";
3
4
   struct card {
5
      int rank, suit;
6
      int read() {
        char ch[2];
8
        if (scanf(" %c%c", &ch[0], &ch[1]) == EOF) return 0;
9
        for (rank = 0; rank names[rank] != ch[0]; rank++);
10
        for (suit = 0; suit names[suit] != ch[1]; suit++);
11
        return 1;
12
13
      void print() {
14
        printf("%c%c", rank names[rank], suit names[suit]); }
15 };
16
17 struct freq lt {
18
     int* freq;
19
      freq lt(int* freq): freq(freq) {}
20
      bool operator ()(const card A, const card B) const {
2.1
        if (int t = freq[A.rank] - freq[B.rank]) return t > 0;
2.2
        else return A.rank > B.rank;
23
24
   };
25
26 struct hand {
27
      card C[5];
28
      int type() {
29
        int freq[15]; memset(freq, 0, sizeof(freq));
30
        sort(C, C+5, freq lt(freq));
31
        bool flush = true, straight = true;
32
        for (int i = 0; i < 5; i++) {
33
          if (i && C[i].suit != C[i-1].suit) flush = false;
34
           if (i && !(C[i].rank == 5 && C[i-1].rank == 14)
35
             && C[i].rank != C[i-1].rank - 1) straight = false;
36
          freq[C[i].rank]++;
37
38
        sort(C, C+5, freq_lt(freq));
39
40
        int kind[5]; memset(kind, 0, sizeof(kind));
41
        for (int i = 2; i <= 14; i++) kind[freq[i]]++;
```

```
42
        if (straight && flush) return 8;
43
        else if (kind[4]) return 7;
        else if (kind[3] && kind[2]) return 6;
44
45
        else if (flush) return 5;
46
        else if (straight) return 4;
47
        else if (kind[3]) return 3;
48
        else return kind[2];
49
50
      bool operator <(hand H) {
51
        if (int t = type() - H.type()) return t < 0;</pre>
52
        for (int i = 0; i < 5; i++)
53
        if (int t = C[i].rank - H.C[i].rank) return t < 0;
54
        return false;
55
56 };
```

Minimum Enclosing Circle

```
typedef pair<Ponto, double> circle;
   Ponto circumcenter(Ponto p, Ponto q, Ponto r) {
3
      Ponto a = p - r, b = q - r,
4
        c = Ponto(a * (p + r) / 2, b * (q + r) / 2);
5
      return Ponto(c%Ponto(a.y, b.y), Ponto(a.x, b.x)%c)/(a%b);
6
   bool in circle(circle C, Ponto p){
      return cmpEPS(C.first.dist(p) , C.second) <= 0;</pre>
9
10 circle spanning circle(Ponto *T, int n) {
      random_shuffle(T, T + n);
11
12
      circle C(Ponto(), 0);
13
      for (int i = 0; i < n; i++) if (!in circle(C, T[i])) {
14
        C = circle(T[i], 0);
15
        for (int j = 0; j < i; j++) if (!in circle(C, T[j])) {
          C = circle((T[i] + T[i]) / 2, T[i].dist(T[i]) / 2);
16
17
          for (int k = 0; k < j; k++) if (!in circle(C, T[k])) {
18
             Ponto o = circumcenter(T[i], T[j], T[k]);
19
             C = circle(o, T[k].dist(o));
20
21
22
23
      return C;
24 }
```

Circulo circulo

```
vector<PT> CircleCInters(PT a, PT b, double r. double R) {
2.
     vector<PT> ret;
3
     double d = sgrt(dist2(a, b));
     if (d > r+R \mid d+min(r, R) < max(r, R)) return ret;
4
5
     double x = (d*d-R*R+r*r)/(2*d);
6
     double y = sqrt(r*r-x*x);
7
     PT v = (b-a)/di
8
     ret.push back(a+v*x + RotateCCW90(v)*y);
9
     if (y > 0)
10
       ret.push back(a+v*x - RotateCCW90(v)*y);
11
     return ret;
12
```

Distancia 3D

```
double distSP(P a, P b, P c) {
2
     P pp = a + (c-a)/(b-a);
3
     if(!comp(!(a-pp) + !(pp-b), !(a-b))) return !( c-pp );
4
     return min(!(a-c), !(b-c));
5
   double distSS(Pa, Pb, Pc, Pd) {
8
     P ba = b-a;
     P cd = c-di
10
     P ca = c-a;
11
     P w = ba cd;
12
     if(!comp(w*w,0))
13
        return min(min(distSP(a,b,c), distSP(a,b,d)),
14
                    min(distSP(c,d,a), distSP(c,d,b)));
15
16
     double dd = ba%cd*w;
17
     double x = (ca cd*w)/dd;
18
     double y = (ba ca*w)/dd;
19
     double z = (ba cd*ca)/dd;
20
     if (x >= 0 \& x <= 1 \& y >= 0 \& y <= 1) return !(w*z);
21
     return min(min(distSP(a,b,c), distSP(a,b,d)),
22
                 min(distSP(c,d,a), distSP(c,d,b)));
23 }
24
```

```
25 double distPP(Pa. Pb. Pc. Pd) {
      P ba = b-ai
26
      P ca = c-a;
27
28
     P da = d-a;
29
     P w = ba ca;
30
     P q = d - da/w;
31
      double x=(b-a)%(q-a)*w, y=(c-b)%(q-b)*w, z=(a-c)%(q-c)*w;
32
     if(x \le 0 \&\& v \le 0 \&\& z \le 0 | |x > = 0 \&\& v > = 0 \&\& z > = 0)
33
        return !(da/w);
34
     return min( min(distSP(a,b,d), distSP(b,c,d)),
35
                  distSP(c,a,d));
36
37
38 int read() {
39
      fr(i,0,4) scanf("%lf%lf%lf", &p[i].x, &p[i].y, &p[i].z);
40
      fr(i,0,4) scanf("%lf%lf%lf", &q[i].x, &q[i].y, &q[i].z);
41
42
      double dist = 1./0.;
43
      fr(i,0,4) fr(j,i+1,4) fr(k,j+1,4) fr(1,0,4) {
44
        double d = distPP(p[i], p[j], p[k], q[l]);
45
        if( d < dist ) dist = d;
46
        d = distPP(q[i], q[j], q[k], p[l]);
47
        if( d < dist ) dist = d;
48
49
      fr(i,0,4) fr(j,i+1,4) fr(k,0,4) fr(l,k+1,4) {
50
        double d = distSS(p[i], p[j], q[k], q[l]);
51
        if(d < dist) dist = d;
52
53
54
     printf("%.2lf\n", dist);
55
56
     return 1;
57 }
```

Convex Hull 3D

```
1 P p[M], co;
2 vector<int> face[M<<3];</pre>
   int aresta[M][M], f, n;
4
   bool comp2(P a, P b) {
6
      double d = a.x*b.y - a.y*b.x;
      return d > 0 || d == 0 &&
        a.x*a.x+a.y*a.y < b.x*b.x+b.y*b.y; }
8
   bool comp3(P a, P b) {
      double d = a.x*b.v - a.v*b.x;
10
11
      return d \ge 0;
12 void ch2d(vector<int>& w, P X, P Y) {
13
      int n = w.size(); vector<P> q;
14
      fr(i,0,n) q.push back(P(p[w[i]]*X, p[w[i]]*Y, w[i]));
15
      P o = *min element(q.begin(), q.end());
      0.z = 0;
16
17
      fr(i,0,n) q[i] = q[i] - o;
18
      sort(q.begin(), q.end(), comp2);
19
      int m = 0;
      for( int i = 1 ; i < n ; ++i ) {
20
21
        while ( m && comp3(q[i]-q[m-1], q[m]-q[m-1]) ) m--;
22
        a[++m] = a[i];
23
24
      w.resize(++m);
      fr(i,0,m) w[i] = q[i].z+0.5;
25
26 }
27
28 void go(int a, int b) {
29
      if( ~aresta[a][b] ) return;
30
      P A = p[a], v = p[b]-A, w = co-A;
31
      vector<int> plano;
32
      fr(i,0,n) if( i != a && i != b ) {
33
        P u = v%(p[i]-A);
34
        if( u^*w > 0 ) plano = vector<int>(1,i), w = p[i]-A;
35
        else if( u*w == 0 ) plano.push_back(i);
36
37
      plano.push back(a);
38
      plano.push_back(b);
39
      ch2d(plano, v, (co-A)%v);
40
      face[f++] = plano;
41
      int m = plano.size();
```

```
42
      fr(i.0.m) aresta[plano[i]][plano[(i+1)%m]] = f;
      fr(i,0,m)  go(plano[(i+1)%m], plano[i]);
43
44 }
45
46 void convex() {
47
      sort(p,p+n); n = unique(p,p+n) - p;
      int a = min element(p,p+n)-p, b = (a+1)%n;
48
49
      co = p[a];
50
      fr(i,0,n) if( i != a ) {
51
        co = co + p[i];
52
        P v = (p[i]-p[a])%(p[i]-p[b]);
53
        #define meo(a, b) ({ \
54
          P v = (a)%(b); \
55
          v.z != 0 ? v.z < 0 : \
56
          v.y != 0 ? v.y < 0 : \
57
          v.x != 0 ? v.x > 0 : !(a) > !(b); 
58
        })
59
        if(meo(p[i]-p[a], p[i]-p[b]))b = i;
60
61
      co = co*(1./n);
62
      fr(i,0,n) fr(i,0,n) aresta[i][i] = -1; f = 0;
63
      //memset(aresta,-1,sizeof aresta); f = 0;
64
      qo(a,b); qo(b,a);
65 }
66
67 int main() {
68
      int t = -1, caso = 1; cin >> t;
69
      while( t-- && cin >> n && n ) {
        fr(i,0,n) cin >> p[i].x >> p[i].y >> p[i].z;
70
71
        convex();
        double area = 0, vol = 0;
72
73
        fr(i,0,f) {
74
          int m = face[i].size();
75
          Pa = p[face[i][0]];
76
          fr(j,2,m) {
77
             P b = p[face[i][j-1]];
78
             Pc = p[face[i][j]];
79
             area += !((b-a)%(c-a))/2;
80
             vol += c%b*a/6;
81
82
83
        printf("%.6lf %.6lf\n", area, vol);
84
85 }
```

LIS 2D

```
int n, m;
   map<int,int> coast[MAX];
3
4
   bool comp(pii pt, const map<int,int>& ct) { // ct > pt
5
      map<int,int>::const iterator it = ct.lower bound(pt.F);
6
      //if( it != ct.end() \&\& it->F == pt.F ) se x1<=x2
      // \text{return it} -> S >= \text{pt} -> S; // > \text{se y1} <= \text{y2}
8
      if( it == ct.begin() ) return true;
9
      return (--it)->S >= pt.S; // > se y1<=y2
10 }
11 void add(map<int,int>& ct, int x, int y) {
      map<int,int>::iterator it = ct.lower bound(x), jt = it;
12
      if( it != ct.end() && it->F == x &  it->S < v )
13
14
        return; // sem se x1<=x2
15
      if( it != ct.begin() && (--it)->S == y )
16
        return; it = jt; // sem se y1<=y2
17
      while( it != ct.end() && it->S >= y ) ++it;
18
      ct.erase(it,jt); ct[x] = y;
19 }
20 int main() {
21
      while( cin >> n ) {
22
        m = 0; // x1 < x2 && v1 < v2
23
        fr(i,0,n) {
2.4
           int x,y; cin >> x >> y;
           int t = upper_bound(coast, coast+m, pii(x,y), comp)-
25
26
                  coast;
           if( t == m ) coast[m++].clear();
27
           add(coast[t], x,y); }
28
        printf("%d\n", m);
29
30
31 }
```

Convex Hull 2D

```
bool comp1(P a, P b) {
   return a.x < b.x || a.x == b.x && a.y < b.y; }

bool comp2(P a, P b) {
   return a%b > 0 || a%b == 0 && a*a < b*b; }
</pre>
```

```
vector<P> hull(vector<P> p) {
      int n = p.size();
      P o = *min element(p.begin(), p.end(), compl);
9
      fr(i,0,n) p[i] = p[i] - o;
10
      sort(p.begin(), p.end(), comp2);
      int w = n-1; while (w \& p[w] p[w-1] == 0) w--;
11
12
      reverse(p.begin()+w, p.end());// remover se sem colineares
13
14
      int m = 0;
15
      for( int i = 1; i < n; ++i ) { // >= elimina colineares
16
        while (m \& \& (p[i]-p[m-1]) \% (p[m]-p[m-1]) > 0) m--;
17
        p[++m] = p[i]; 
      p.resize(++m);
18
19
      fr(i,0,m) p[i] = p[i] + o;
20
      return p; }
```

DrawPolygon

```
import java.awt.*;
   import java.util.*;
   import javax.swing.*;
   public class DPP extends JPanel {
      static Polygon pol = new Polygon();
6
      static int M = 25, E = 10, W = 800, H = 600;
      public void paint(Graphics q) {
8
        for (int i = 0; i < 50; i++) {
9
           g.drawLine(i*M+E, 0, i*M+E, H);
10
           g.drawLine(0, i*M+E, W, i*M+E); }
11
        q.setColor(Color.blue);
12
        ((Graphics2D)g).setStroke(new BasicStroke(3f));
        q.drawPolygon(pol); }
13
14
      public static void main(String[] args) {
15
        Scanner scan = new Scanner(System.in);
16
        while( scan.hasNextInt() )
           pol.addPoint(E+scan.nextInt()*M,H-E-scan.nextInt()*M);
17
18
        JFrame frame = new JFrame();
19
        frame.setSize(W, H);
20
        frame.getContentPane().add(new DPP());
21
        frame.setDefaultCloseOperation(JFrame.EXIT ON CLOSE);
22
        frame.setVisible(true);
23
24 }
```

Problem	D	F	L	Assunto	Descrição	Idéia
A						
В						
С						
D						
E						
F						
G						
Н						
I						
J						
K						
L						