Task

Say whether the following is true or false and support your answer by a proof.

$$(\exists m \in \mathbb{N})\,(\exists n \in \mathbb{N})\,(3m + 5n = 12)$$

Solution

Let's claim that $(\exists m \in \mathbb{N})\,(\exists n \in \mathbb{N})\,(3m + 5n = 12)$ is false and prove it.

Theorem. $(\forall m \in \mathbb{N})\,(\forall n \in \mathbb{N})\,(3m + 5n \neq 12)$

Proof. Since 12 is even number, then either both $3m$ and $5n$ are even, or both are odd. $3m$ is even only if m is even, similarly $5n$ is even only if n is even. So, either both m and n are event, or both are odd.

In case both m and n are event, here are the possibilities:

$$3 \cdot 2 + 5 \cdot 2 = 16$$

Obviously, all other combinations of even m and n will yield even bigger numbers.

In case both n and m are odd, here are the possibilities:

$$3 \cdot 1 + 5 \cdot 1 = 8$$
$$3 \cdot 1 + 5 \cdot 3 = 18$$
$$3 \cdot 3 + 5 \cdot 1 = 14$$

Obviously, all other combinations of odd m and n will yield even bigger numbers.

Thus we tried all combinations of m and n, and neither of them yielded $3m + 5n = 12$. Hence, $(\forall m \in \mathbb{N})\,(\forall n \in \mathbb{N})\,(3m + 5n \neq 12)$. $\square$