

Task

Prove that for any integer n , at least one of the integers $n, n + 2, n + 4$ is divisible by 3.

Solution

Task is equivalent to proving the following theorem:

Theorem. $\forall n \in \mathbb{N} : 3 \mid n \vee 3 \mid (n + 2) \vee 3 \mid (n + 4)$

Proof. By cases.

By division theorem, all integers can be expressed in one of these forms: $3k, 3k + 1, 3k + 2$, for some $k \in \mathbb{Z}$.

In case $n = 3k$, for some k , it is divisible by 3 (by definition of divisibility).

In case $n = 3k + 1$, for some k , then $n + 2 = 3k + 3 = 3(k + 1)$ is divisible by 3 (by definition of divisibility).

In case $n = 3k + 2$, for some k , then $n + 4 = 3k + 6 = 3(k + 2)$ is divisible by 3 (by definition of divisibility).

Hence, we proved that for any integer n , at least one of the integers $n, n + 2, n + 4$ is divisible by 3. \square