Task
Prove that every odd natural number is of one of the forms $4n+1$ or $4n+3$, where n is an integer.

Solution
Task is equavalent to proving the following theorem:

Theorem. $\forall m \in \mathbb{N} : 2 \mid m \Rightarrow \exists n \in \mathbb{Z}\,[m = 4n+1 \vee m = 4n+3]$.

Proof. Let's take arbitrary $m \in \mathbb{N}$. By division theorem, $m \in \mathbb{N}$ can be expressed in one of these forms: $m = 4n, m = 4m+1, m = 4n+2, m = 4m+3$, for some $n \in \mathbb{Z}$.
Of these forms, two express even numbers, $4n = 2 \cdot 2n$ and $4n+2 = 2(2n+1)$, since both are divisible by 2.
Two remaining forms, $4n+1$ and $4n+3$, are not divisible by 2, so they are odd. Since our choice of m was arbitrary, this implies that if m is odd, it can be expressed in one of the forms $4n+1$ or $4n+3$, for some $n \in \mathbb{Z}$.
This concludes the proof. $\square$