Chapter 3: Geometric Properties

Properties of Quadrilaterals (How to prove ↓)

Name	Shape	Desc.	Find	Show
Square		4 equal sides, 90° angles	4 lengths, 4 slopes	① lengths equal ② opp. slopes equal ③ adj. slopes - neg rec.
Rectangle		2 sets equal side lengths & 90°	4 lengths, 4 slopes	① 2 pairs matching len ② opp. slopes equal ③ adj. slopes - neg rec.
Parallelogram		opposite sides are parallel	4 slopes	① opp. slopes equal
Trapezoid		ONE pair of parallel sides	4 slopes	① only 2 slopes equal
Rhombus		all sides equal len, opp. sides //	4 lengths	① lengths equal
Kite		2 pairs of equal length adj. sides	4 lengths	① show adj. len equal

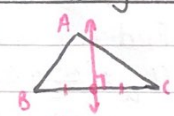
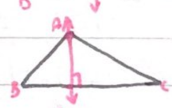
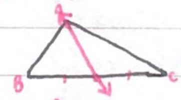
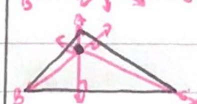
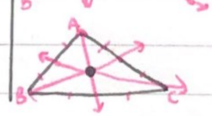
ⓐ A square is also a rectangle, rhombus, and parallelogram
ⓑ A rectangle is also a parallelogram
ⓒ A rhombus is also a parallelogram.

Triangles

Right triangle - 3 sided shape with 90° angle.

	Method 1	Method 2
Prove:	① calculate 3 slopes	① Calculate 3 lengths (LS/RS)
	② show 2 are neg. rec.	② Show pythag is satisfied

Verify Properties of Triangles.

Name	Desc.	Drawing	How to find:
Right Bisector ($y=mx+b$)	line that is ⊥ to a side		① Calculate slope BC ② use neg. reciprocal as slope(m) ③ calculate midpoint BC ④ sub in midpoint & slope into $y=mx+b$
Altitude ($y=mx+b$)	the height of a shape		① calculate slope BC ② use neg. reciprocal as slope(m) ③ use point A ④ plug in ② & ③ (slope & A) into $y=mx+b$
Median ($y=mx+b$)	a line that joins a midpoint to the opposite vertex		① calculate midpoint BC ② use point A & ① to find slope ③ chose any point (A OR midpoint) and use the slope to plug into $y=mx+b$
Circumcentre (POI)	POI of 3 right bisectors		① Find eq^{ns} of all 3 right bisectors ② Do sub/elim with any 2. ③ Check using LS/RS with 3rd line.
Orthocentre (POI)	POI of 3 altitudes		① Find eq^{ns} of all 3 altitudes ② Do sub/elim with any 2 ③ Check using LS/RS with 3rd line.
Centroid (POI)	POI of 3 medians		① Find eq^{ns} of all 3 medians ② Do sub/elim with any 2 ③ Check using LS/RS with 3rd line.

& also:

Length of Altitude:

① Find eq^n of line BC

② Sub/elim with eq^n of line for altitude to find POI

③ Use length formula $l=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance from a Point to a Line

Case I: given a point & the eqⁿ of a line

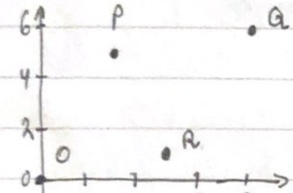
① given eq^n l_1
② find eq^n l_2 (use ⊥m & A)
③ use sub/elim to find POI
④ sub A & POI into length formula

Case II: Given a point & 2 points on the line

① find slope of l_1 using points x & y
② plug in ① & point x //y into $y=mx+b$ (for l_1)
③ Follow steps ①-④ of case I.

Examples

ⓐ



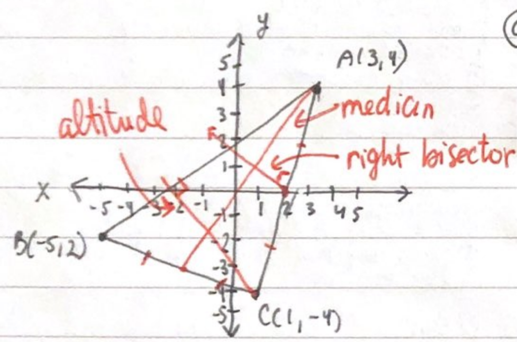
$m = \frac{y_2-y_1}{x_2-x_1}$

$m_{OP} = \frac{5-0}{3-0} = 5/3$ $\quad m_{PQ} = \frac{6-5}{8-3} = 1/5$ $\quad m_{QR} = \frac{6-1}{8-5} = 5/3$ $\quad m_{OR} = \frac{-1-0}{5-0} = 1/5$

∵ $m_{OP} = m_{QR}$ && $m_{QR} = m_{OR}$ ∥ ($l_{OP} // l_{QR}$ && $l_{PQ} // l_{OR}$)

∴ The shape is a parallelogram.

ⓑ A(3,4) B(-5,2) C(1,-4)



ⓐ Find the median from A to BC

$M_{BC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-5+1}{2}, \frac{2-4}{2}\right) = (-2,-1)$

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4+1}{3+2} = 1$

$y = mx+b$
$y = x+b$
$\boxed{y = x+1}$

sub in (-2,-1)
$-1 = -2+b$
$b = 1$

ⓑ Find the right bisector of AC

$M_{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (2,0)$

$m_{AC} = \frac{y_2-y_1}{x_2-x_1} = \frac{4+4}{3-1} = 4$

∵ $l_{AC} \perp l_1$ ∴ $m = -1/4$

$y = mx+b$
$y = -\frac{1}{4}x+b$
sub in (2,0)
$0 = -\frac{1}{4}(2)+b$
$b = \frac{1}{2}$

$\boxed{y = -\frac{1}{4}x + \frac{1}{2}}$

ⓒ Find the altitude from side AB to C

$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{2-4}{-5-3} = 1/4$

∵ they are perpendicular ∴ $m = -4$

$y = mx+b$
$y = -4x+b$
sub in C(1,-4)
$-4 = -4(1)+b$
$b = 0$
$\boxed{y = -4x}$