Intro to Functions

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2023

Contents

1 Function Definitions

Relation - Any relationship (set of ordered pairs)
Function - A relation that maps each input (x) to exactly one output (y)
Finite - Countable (discrete)
Infinite - Uncountable
Mapping Diagram - A visual representation of inputs and outputs (finite only)
Vertical Line Test (VLT) - A relation is not a function if you can draw a vertical line and cross the graph more than once. ($\because$ VLT fails/passes $\therefore$ this is/is not a function.)

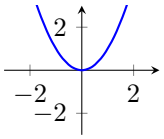
2 Function Notation

$f(x)$

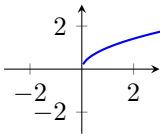
- used in place of y in equations
- helpful for identifying equations and input used

3 Parent Functions

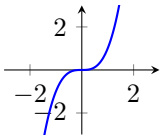
Quadratic Function
$y = x^2$
Key Points = {(-2,4), (-1,1), (0,0), (1,1), (2,4)}
Domain = $\{\mathbb{R}\}$
Range = $\{y \epsilon \mathbb{R} | y \geq 0\}$



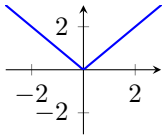
Root Function
$y = \sqrt{x}$
Key Points = {(0,0), (1,1), (4,2), (9,3)}
Domain = $\{x \epsilon \mathbb{R} | x \geq 0\}$
Range = $\{y \epsilon \mathbb{R} | y \geq 0\}$



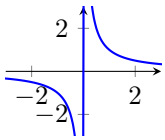
Cubic Function
$y = x^3$
Key Points = {(-2,-8), (-1,-1), (0,0), (1,1), (2,8)}
Domain = $\{\mathbb{R}\}$
Range = $\{\mathbb{R}\}$



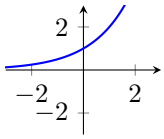
Absolute Value Function
$y = |x|$
Key Points = {(-2,2), (-1,1), (0,0), (1,1), (2,2)}
Domain = $\{\mathbb{R}\}$
Range = $\{y \epsilon \mathbb{R} | y \geq 0\}$



Reciprocal Function
$y = \frac{1}{x}$
Key Points = {(-2,-$\frac{1}{2}$), (-1,-1), (-$\frac{1}{2}$,-2), ($\frac{1}{2}$,2), (1,1), (2,$\frac{1}{2}$)}
Domain = $\{ x \epsilon \mathbb{R} | x \neq 0\}$
Range = $\{ y \epsilon \mathbb{R} | y \neq 0\}$



Exponential Function
$y = 2^x$
Key Points = {(-2, 1/4), (-1, 1/2), (0,1), (1,2), (2,4)}
Domain = $\{\mathbb{R}\}$
Range = $\{ y \epsilon \mathbb{R} | y > 0\}$



4 Domain and Range

Domain - The set of all x values (input)
Range - The set of all y values (output)

Symbols

- $\mathbb{R}$ - the set of real numbers
- $\mathbb{Z}$ - the set of integers
- ϵ - belongs to
- $|$ - such that
- $\leqslant, <, \leqslant, >$

For finite sets there are no duplicates and the values are in order.

Domain & Range From Equations - Common Restrictions
1. Cannot divide by zero
2. Cannot square root negatives
3. Vertex restrictions (use the graph)

5 The Inverse Function

$f^{-1}(x)$

- The inverse function is the "reverse" of the original function
- $f(x)$ operates in a specific way and order, $f^{-1}(x)$ will do the opposite operation in the reverse order
- Referred to as "inverse of f" or "f inverse"

$(x, y) \epsilon f(x), (y, x) \epsilon f^{-1}(x)$

Note:
$f(f^{-1}(x)) = f^{-1}(f(x)) =$ x
$f(x)$ and $f^{-1}(x)$ "cancel" each other

Steps to find the inverse:

1. Given a list of points flip the coordinates
   (x,y) turns to (y,x)
2. Given an equation in function notation
   - convert "y=" to "let f(x) = y"
   - switch x and y variables
   - solve for y
   - if the inverse expression is a function, convert back to $f^{-1}(x)$
3. Given a discrete graph flip all points and replot
4. Given a continuous graph choose select points to flip OR reflect graph in the line y = x

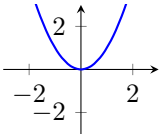
6 Transformations

$$y = af(k(x-d)) + c$$

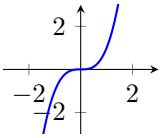
Where:
- a and c affect the y-coordinate
- k and d affect the x-coordinate

Functions

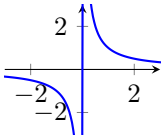
Quadratic Function
$y = x^2$
$y = a(k(x-d))^2 + c$



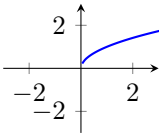
Cubic Function
$y = x^3$
$y = a(k(x-d))^3 + c$



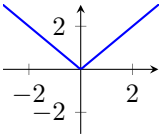
Reciprocal Function
$y = \frac{1}{x}$
$y = \frac{a}{k(x-d)} + c$



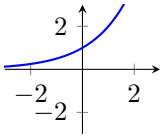
Root Function
$y = \sqrt{x}$
$y = a\sqrt{k(x-d)} + c$



Absolute Value Func
$y = |x|$
$y = a|k(x-d)| + c$



Exponential Func
$y = 2^x$
$y = a2^{k(x-d)} + c$



Effects of Letters

a: $0 < |a| < 1$
Vert. compression by a factor of $|a|$
Multiply y values by $|a|$

$|a| > 1$
Vertical stretch by a factor of $|a|$
Multiply y values by $|a|$

$a < 0$
Reflection over the x-axis
Multiply y values by -1

k: $0 < |k| < 1$
Horizontal stretch by a factor of $\frac{1}{|k|}$
Multiply x values by $\frac{1}{|k|}$

$|k| > 1$
Horizontal compression by $\frac{1}{|k|}$
Multiply x values by $\frac{1}{|k|}$

$k < 0$
Reflection over the y-axis
Multiply x values by -1

c: $c > 0$
Shift up c units
Add c to y values

$c < 0$
Shift down c units
Add c to y values

d: $d > 0$
Shift right d units
Add d to x values

$d < 0$
Shift left d units
Add d to x values

Notes:
- k must be factored out in order to determine the value of d
- The order to complete transformations is:

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1) Stretch/Compress
2) Reflect
3) Shift
```

Piecewise Functions

Piecewise - A **funciton** made up of two or more functions on given intervals.
Notation Examle

$$\begin{cases} function & restrictions \\ x & 0 < x \leq 100 \\ 1 & 100 < x \end{cases}$$