

Rational Expressions

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1 Adding/Subtracting/Multiplying Polynomials

Polynomial - An algebraic expression containing two or more terms

Adding/Subtracting

When adding or subtracting polynomials, we add/subtract "like" terms

Multiplying

When multiplying polynomials...

- use distributive property (sometimes more than once)
- be careful with signs

Associative Property - the way the factors are grouped in a multiplication problem does not change the product.

2 Factoring Polynomials

Greatest Common Factor (GCF)

Divide out largest common coefficient and variable with max common exponent

Grouping

- Group terms factor GCF from each group
- Factor common brackets if possible

Simple Trinomial

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

Complex Trinomial

$$ax^2 + bx + c, a \neq 1$$

- Find factors of a & place in each bracket
- Find factors of c & place in each bracket
- Check by expanding in your head if it fits, if not, try something else

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Perfect Squares

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Long Division

$$\text{quotient} \times \text{divisor} = \text{dividend}$$

3 Rational Expressions

A rational expression is an expression that is the ratio of two polynomials,

$$f(x) = \frac{R(x)}{H(x)}$$

When working with rational expressions, it is important to state **restrictions**. Restrictions are values of the variable that cause the function to be undefined.

The two types of restrictions are **holes** and **asymptotes**. Restrictions arise from any factors that have ever appeared in the denominator.

A **hole** is a missing value in the graph. It occurs when a restriction is **removed through simplification**.

A **asymptote** is a line that the graph approaches but never crosses. It occurs when a restriction **remains after being simplified**.

Note - 2 rational expressions are equivalent IF they are the same for all possible values of the domain.

4 Multiplying and Dividing Rational Expressions

Steps

1. Factor numerator and denominator

2. Divide out common factors to simplify

3. Identify restrictions

Note - When dividing by fractions we multiply by the reciprocal, restrictions are determined by all factors that have ever appeared in the denominator

5 Adding and Subtracting Rational Expressions

Note: You cannot cross cancel when adding/subtracting, only when multiplying
Goal: Get a common denominator

Steps

1. Factor denominator
2. Identify restrictions
3. Find lowest common denominator
4. Express rational exp. with same LCD
5. Add/subtract tops & keep LCD
6. Expand out and refactor numerator to see if refactorable

6 Graphs of Rational Functions

Rational functions usually come out to be either linear (only holes no asymptotes) or reciprocal (can be both) functions

Steps

1. Simplify expression
2. State and classify restrictions
3. Draw function and add any holes