

Intro to Functions

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1 Function Definitions

2 Function Notation

3 Parent Functions

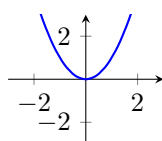
Quadratic Function

$$y = x^2$$

Key Points = $\{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}$

Domain = $\{\mathbb{R}\}$

Range = $\{y \in \mathbb{R} | y \geq 0\}$



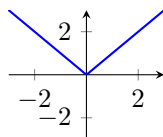
Absolute Value Function

$$y = |x|$$

Key Points = $\{(-2,2), (-1,1), (0,0), (1,1), (2,2)\}$

Domain = $\{\mathbb{R}\}$

Range = $\{y \in \mathbb{R} | y \geq 0\}$



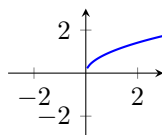
Root Function

$$y = \sqrt{x}$$

Key Points = $\{(0,0), (1,1), (4,2), (9,3)\}$

Domain = $\{x \in \mathbb{R} | x \geq 0\}$

Range = $\{y \in \mathbb{R} | y \geq 0\}$



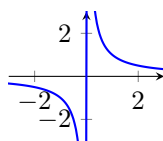
Reciprocal Function

$$y = \frac{1}{x}$$

Key Points = $\{(-2, -\frac{1}{2}), (-1, -1), (-\frac{1}{2}, -2), (\frac{1}{2}, 2), (1, 1), (2, \frac{1}{2})\}$

Domain = $\{x \in \mathbb{R} | x \neq 0\}$

Range = $\{y \in \mathbb{R} | y \neq 0\}$



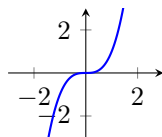
Cubic Function

$$y = x^3$$

Key Points = $\{(-2,-8), (-1,-1), (0,0), (1,1), (2,8)\}$

Domain = $\{\mathbb{R}\}$

Range = $\{\mathbb{R}\}$



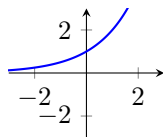
Exponential Function

$$y = 2^x$$

Key Points = $\{(-2, 1/4), (-1, 1/2), (0,1), (1,2), (2,4)\}$

Domain = $\{\mathbb{R}\}$

Range = $\{y \in \mathbb{R} | y > 0\}$



4 Domain and Range

5 The Inverse Function

6 Transformations

$$y = af(k(x - d)) + c$$

Where:

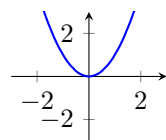
- a and c affect the y-coordinate
- k and d affect the x-coordinate

Functions

Quadratic Function

$$y = x^2$$

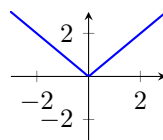
$$y = a(k(x - d))^2 + c$$



Absolute Value Function

$$y = |x|$$

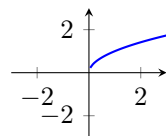
$$y = a|k(x - d)| + c$$



Root Function

$$y = \sqrt{x}$$

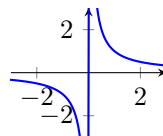
$$y = a\sqrt{k(x - d)} + c$$



Reciprocal Function

$$y = \frac{1}{x}$$

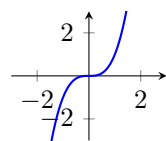
$$y = \frac{a}{k(x - d)} + c$$



Cubic Function

$$y = x^3$$

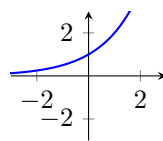
$$y = a(k(x - d))^3 + c$$



Exponential Function

$$y = 2^x$$

$$y = a2^{k(x - d)} + c$$



Effects of Letters

a:

$$0 < |a| < 1$$

Vertical compression by a factor of $|a|$
Multiply y values by $|a|$

$$|a| > 1$$

Vertical stretch by a factor of $|a|$
Multiply y values by $|a|$

$$a < 0$$

Reflection over the x-axis
Multiply y values by -1

c:

$$c > 0$$

Shift up c units
Add c to y values

$$c < 0$$

Shift down c units
Add c to y values

k:

$$0 < |k| < 1$$

Horizontal stretch by a factor of $\frac{1}{|k|}$
Multiply x values by $\frac{1}{|k|}$

$$|k| > 1$$

Horizontal compression by $\frac{1}{|k|}$
Multiply x values by $\frac{1}{|k|}$

$$k < 0$$

Reflection over the y-axis
Multiply x values by -1

d:

$$d > 0$$

Shift right d units
Add d to x values

$$d < 0$$

Shift left d units
Add d to x values

Notes:

- k must be factored out in order to determine the value of d
- The order to complete transformations is:

- 1) Stretch/Compress
- 2) Reflect
- 3) Shift