

Exponential Functions

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1 Exponent Laws

- $x^0 = 1$
- $x^{-1} = \frac{1}{x}$
- $\left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$
- $x^1 = x$
- $x^{-a} = \frac{1}{x^a}$
- $\left(\frac{x}{y}\right)^{-a} = \frac{y^a}{x^a}$
- $x^a x^b = x^{a+b}$
- $(x^a)^b = x^{ab}$
- $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
- $(ax^b)^c = a^c x^{bc}$

2 Radical Powerlaws

- $x^{\frac{1}{a}} = \sqrt[a]{x}$
- $x^{\frac{b}{a}} = \sqrt[a]{x^b}$
- $x^{-\frac{b}{a}} = \frac{1}{\sqrt[a]{x^b}}$

Steps to Solve:

1. Convert all radicals to fractional exponents
2. Use powerlaws to simplify
3. Convert back to root form

3 Rational Exponent Equations

When solving rational exponent equations, isolate the variable first then flip the exponent

In general:

$$\begin{aligned}a &= b + cx^{\frac{e}{f}} \\ \frac{a-b}{c} &= x^{\frac{e}{f}} \\ \left(\frac{a-b}{c}\right)^{\frac{f}{e}} &= \left(x^{\frac{e}{f}}\right)^{\frac{f}{e}} \\ \left(\frac{a-b}{c}\right)^{\frac{f}{e}} &= x\end{aligned}$$

4 Exponential Equations

When solving exponential equations:

1. Get a single term on each side
2. Make the bases match
3. "Drop" the bases, set exponents equal

Types can include:

- normal easy types
- divide first
- apples - $x^{a+b} = x^a x^b$
- hidden quadratic (trinomial) - $x^{ab} = (x^a)^b$

5 Properties of Exponential Functions

Exponential Function - A function in the form $y = b^x$ where b can change your key points. The greater the b, the steeper. Any values make the function go through (0, 1) always.

Transformations - $y = ab^{k(x-d)} + c$