Chapter 2 Test

1. Expand and simplify the following (3K marks)

$(2x-y-2)^2$ $= (2x-y-2)(2x-y-2)$

$= 4x^2+y^2+4-4xy+4y-8x$

2. Factor each of the following expressions ***completely***: (no expanding) (2K marks each)

a. $8x^4-50y^6$

$= 2(4x^4-25y^6)$

$= 2(2x^2-5y^3)(2x^2+5y^3)$

b. $12x^2-10x-8$

$= 2(6x^2-5x-4)$

$= 2(3x-4)(2x+1)$

c. $8(2c-b)-c(b-2c)$

$= 8(2c-b)+c(2c-b)$

$= (2c-b)(8+c)$

d. $x^2-14xy+48y^2$

$= (x-8y)(x-6y)$

e. $(y-5)^2-(4-x)^2$

$= (y-5-4+x)(y-5+4-x)$

$= (y-9+x)(y-1-x)$

3. Simplify each of the following. State and classify all restrictions (3K marks)

$\frac{2m^2-mn-n^2}{4m^2-4mn-3n^2} = \frac{(2m+n)(m-n)}{(2m+n)(2m-3n)}$

$= \frac{m-n}{2m-3n}$

$m \neq -n/2$ (H)

$m \neq 3n/2$ (A)

4. Simplify the following. (State and classify restrictions) (a/b: 4K marks each)(c: 6A marks)

a. $\frac{x^4-1}{x^2-2x+1} * \frac{x-1}{x^2+2x+1}$

$= \frac{(x^2+1)(x+1)(x-1)}{(x-1)^2} \cdot \frac{(x-1)}{(x+1)^2}$

$= \frac{x^2+1}{x+1}$ $x \neq 1$ (H) $x \neq -1$ (As.)

b. $\frac{ab^2+2}{2ab^2} - \frac{b+2}{2b}$

$= \frac{(ab^2+2)(2b)-(b+2)(2ab^2)}{(2ab^2)(2b)}$

$= \frac{2ab^3+4b-2ab^3-4ab^2}{(2ab^2)(2b)}$

$= \frac{4b-4ab^2}{(2ab^2)(2b)} = \frac{4b(1-ab)}{2b(2ab^2)}$

$= \frac{1-ab}{ab^2}$ $a \neq 0$, $b \neq 0$ As.

c. $\left(\frac{x^2+5x+6}{x^2-3x+2} / \frac{x+3}{x-1} - \frac{6}{x+3}\right)$

$= \left[\frac{(x+3)(x+2)}{(x-2)(x-1)} \div \frac{x+3}{x-1} - \frac{6}{x+3}\right]$

$= \frac{(x+3)(x+2)(x-1)}{(x-1)(x-2)(x+3)} - \frac{6}{x+3}$

$= \frac{(x+2)(x+3)-6(x-2)}{(x-2)(x+3)}$

$= \frac{x^2+5x+6-6x+12}{(x-2)(x+3)}$

$= \frac{x^2-x+18}{(x-2)(x+3)}$

$x \neq 2$ (As.)

$x \neq -3$ (As.)

$x \neq 1$ (Hole)

5. $y = \frac{2x^2+5x-12}{2x+8}$

a) Simplify the expression, state and classify restrictions. (3K marks)

$y = \frac{(2x-3)(x+4)}{2(x+4)}$

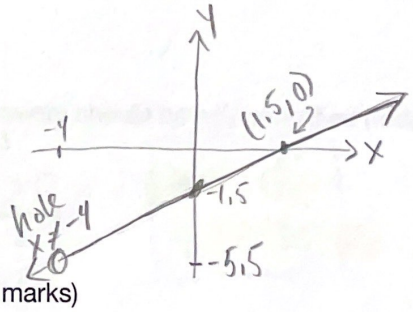
$= \frac{2x-3}{2}$

$= x - 3/2$

$= x - 1.5$, $x \neq -4$ (Hole)

b) Draw the graph of the function (2A marks)

Sketch



c) Clearly label asymptote(s) and/or hole(s) (1C marks)

ok

$A = \frac{1}{2}bh$

6. Determine the area of a triangle with base: $\frac{9y^2-4}{4y-12}$ and height: $\frac{18-6y}{9y^2+12y+4}$. State and classify any restrictions.

$A = \frac{1}{2}\left(\frac{9y^2-4}{4y-12}\right)\left(\frac{18-6y}{9y^2+12y+4}\right)$

$= \frac{6(3y+2)(3y-2)(3-y)}{-12(y-3)(3y+2)(3y+2)}$

$= \frac{3y-2}{-2(3y+2)}$

$y \neq 3$ (h)

$y \neq -2/3$ (as.)

HARD

7. Rewrite $\frac{3}{x^2-x-2}$ as a sum or difference of two rational expressions with DIFFERENT bases and solve it show it equals the expression given (2T marks)

$\frac{3}{x^2-x-2} = \frac{3}{(x-2)(x+1)}$

$= \frac{A}{(x-2)} + \frac{B}{(x+1)}$

$= \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$

$= A(x+1)+B(x+2) = 3$

let $x=-1$: $B(-3) = 3$, $B = -1$

let $x=2$: $A(3) = 3$, $A = 1$

$\frac{1}{(x-2)} - \frac{1}{(x+1)}$

PROOF

$LS = \frac{3}{x^2-x-2}$

$RS = \frac{1}{x-2} - \frac{1}{x+1}$

$= \frac{x+1-x+2}{(x+2)(x+1)}$

$= 3 / (x^2-x-2)$

$LS = RS$

Communication (4C marks) - marks are awarded for proper units and form throughout the test.

Chapter 3 Test

1. Fill in the blank (12C marks)

a. Given the following quadratic $y = x^2 + 6x - 2$

a. Domain: D=ℝ

b. Range: R={y∈ℝ | y≥-11}

c. Vertex: (-3, -11)

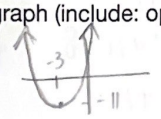
d. Direction of opening: up

e. Axis of symmetry: x=-3

f. Current form the quadratic is given in is called: standard form

g. Number of zeros: D=b²-4ac=36-4(1)(-2)
D>0 ∴ 2 ZEROS

h. Sketch the graph (include: opening, approx.vertex)



b. Given the following quadratic
$g(x) = (2x - 1)(x - 5)$

a. X-intercepts: (½,0) & (5,0)
x=(½+5)/2

b. Axis of symmetry: x=11/4

c. Number of zeros: 2

d. Current form the quadratic is given in is called: factored

2. Simplify the following (3K marks)

$\sqrt{2}(2\sqrt{8} - 3\sqrt{32} + 4\sqrt{50})$ $= \sqrt{2}(4\sqrt{2} - 12\sqrt{2} + 20\sqrt{2})$
$= \sqrt{2}(12\sqrt{2})$
$= 12(2)$
$= 24$

3. Solve by expanding and factoring (4K marks) $(x + 3)(x - 5) = 3(x + 7)$

$x^2 - 2x - 15 - 3x - 21 = 0$
$x^2 - 5x - 36 = 0$
$(x - 9)(x + 4) = 0$
x = 9, -4

4. Find the x-intercepts using the quadratic formula. Answers should be fully simplified (reduced radical form if applicable) (4K marks) $y = x^2 - 4x + 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$
$x = \frac{4 \pm \sqrt{12}}{2}$
$x = \frac{4 \pm 2\sqrt{3}}{2}$
$x = 2 \pm \sqrt{3}$

$x_1 = 2 + \sqrt{3}$
$x_2 = 2 - \sqrt{3}$

5. Determine the point(s) of intersection of the parabola $y = 2x^2 + 12x + 10$ and the line $y = 4x + 2$ (5T marks)

① $y = 2x^2 + 12x + 10$
② $y = 4x + 2$

Sub y into ①
$2x^2 + 12x + 10 = 4x + 2$
$2x^2 + 8x + 8 = 0$
$2(x^2 + 4x + 4) = 0$
$2(x+2)^2 = 0$
$\boxed{x = -2}$

Sub x into ②
$y = 4(-2) + 2$
$y = -8 + 2$
$\boxed{y = -6}$

∴ The POI is $(-2, -6)$.

6. A rectangle has an area of 60m². One side is 7m longer than the other. What are the dimensions of the rectangle? You must set up and solve the equation for full marks. (5A marks)

Let s represent a side (m)

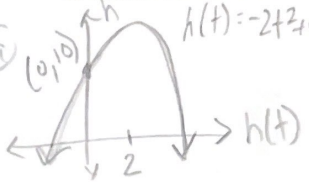
$0 = (s+7)s$
$0 = s^2 + 7s - 60$
$0 = (s+12)(s-5)$
$s = -12, 5$

∵ -12 is invalid
∴ The dims are 5m x 12m

7. A ball is thrown off the roof of the school. Its path can be described by the equation $h(t) = -2t^2 + 8t + 10$ where h is the height in meters after t seconds.

a. Sketch (1C mark)
b. What height was the ball thrown from? Show your work. (1A mark)
c. What time did it take for the ball to reach max height? Show your work. (3A marks)

ⓐ



$h(t) = -2t^2 + 8t + 10$

ⓑ $h(t) = -2t^2 + 8t + 10$
y-int: $(0, 10)$
∴ The ball was thrown from 10m.

ⓒ $h(t) = -2t^2 + 8t + 10$
$= -2(t^2 - 4t) + 10$
$= -2(t-2)^2 + 10 + 8$
$= -2(t-2)^2 + 18$
$V(2, 18)$
↑ t ↑ h

∴ It took 2 seconds to reach max height

Communication (4C marks) - marks are awarded for proper form, units, etc. from throughout the test.

BONUS: (1) Determine the value(s) of k such that the following quadratic has one x-int. $(x^2 - 1)k = (x - 1)^2$

$\boxed{k = 0}$

PROOF

$x^2 - 2x + 1 - kx^2 + k = 0$
$x^2(1-k) - 2x + 1 + k = 0$
a b c

$D = b^2 - 4ac$
$0 = b^2 - 4ac$
$0 = (-2)^2 - 4(1-k)(1+k)$
$0 = 4 - 4 + 4k^2$
$0 = 4k^2$
$0 = k^2$
$\boxed{k = 0}$

SKIPPED

Chapter 4 Test

Fill in the blank (8 K marks)

A. What is the domain of the function $y = -2^{x-1} + 3$ _____

B. What is the range of the function $y = -3 * 2^{2x} - 1$ _____

C. Evaluate $\sqrt[3]{-8y^9x^6y^0}$

D. Evaluate $-27^{\frac{-2}{3}}$

E. Evaluate $32^{\frac{2}{5}}$

F. Convert to simplified radical form: $(24yx^3)^{\frac{-1}{2}}$

G. Given $f(x) = 3^{-x+1} - 2$ state the value of a = _____, k = _____, c = _____, d = _____

1. Power Laws. Express with NO NEGATIVE OR FRACTIONAL exponents (3K marks each)

a. $\sqrt[3]{\frac{x\sqrt{x^6y}\sqrt{x}}{x^{\frac{3}{2}}}}$

2. For the function: $f(x) = -3^{-2x+1} - 2$, given the parent function, state all of the transformations using proper terminology from class. (4C marks)

3. Solve for x. (2K marks each)

a. $26 = -1 + (-9x)^{\frac{3}{4}}$

b. $x = \sqrt{7 + \sqrt[3]{6 + \sqrt{3 + \sqrt[3]{1}}}}$

4. Solve for x for each of the following. (3T mark each)

a. $\left(\frac{1}{6}\right)^{3x+2} * 216^{3x} = \frac{1}{216}$

b. $40(2^{x+2}) = 5$

c. $3^{2x} - 4(3^x) + 3 = 0$

5. Graph the following function. Show your work. Simplify first to make it easier! Correctly draw all transformed graphs (5A marks)

$$f(x) = -2^{x-1} + 3$$

6. Evaluate the following exactly: (3T marks)

a. $0.04^{\frac{1}{2}}$

b. $\sqrt{\sqrt[3]{729}}$

c. $81^{0.75}$

(4C marks) - marks are awarded for proper form, units etc from throughout the test.

Bonus (1 bonus mark) Determine the value of x such that $12^{2x+1} = (2^{3x+7})(3^{3x-4})$

Chapter 5 Trig Test 1

1. Fill in the blank (10K marks)

i. Given 3 sides on an acute triangle use ___cosine law___ to solve the triangle

ii. What is the reciprocal ratio of sine? ___$\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$___

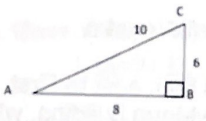
iii. What is the range of the y = sinx graph? ___$R = \{y \in \mathbb{R} \mid -1 \le y \le 1\}$___

iv. The angle from your sight line to an object above you is called ___angle of elevation___

v. How many possible triangles can be made if you are given side m = 9.3, n = 7.2 and <N= 35°?
___two___

vi. How many possible triangles can be made if you are given side c = 3, d = 4 and <D = 70°?
___one___

vii.



a) What is the length of the side adjacent C? ___6___

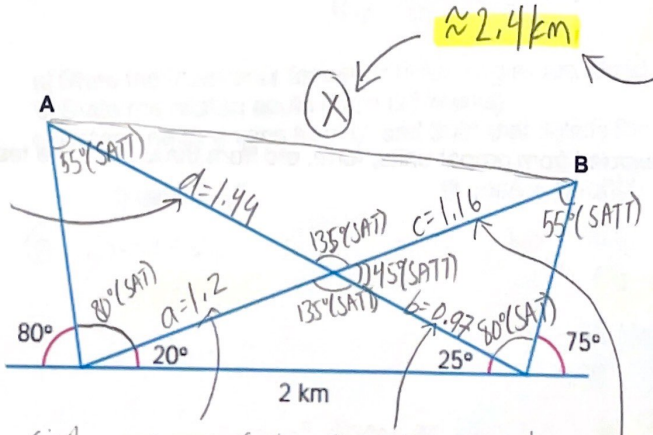
b) Evaluate the ratio of cscC = ___$\frac{1}{\sin C} = \frac{1}{4/5} = \frac{5}{4}$___

c) Evaluate the ratio of secA = ___$\frac{1}{\cos A} = \frac{1}{4/5} = \frac{5}{4}$___

$\sin C = 4/5$

d) What is the value of angle C? ___$C \approx 53°$___

2. Find x. Show all steps and reasoning for full marks (8T mars)



$x^2 = d^2 + c^2 - 2dc\cos X$

$x^2 = (1.44)^2 + (1.16)^2 - 2(1.44)(1.16)\cos 135°$

$x \approx 2.4045$

$\frac{d}{\sin 80°} = \frac{1.2}{\sin 55°}$

$d \approx 1.437$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$; $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 75°} = \frac{2.0}{\sin 135°}$

$a \approx 1.195$

$\frac{b}{\sin 20°} = \frac{2.0}{\sin 135°}$

$b \approx 0.967$

$\frac{c}{\sin 80°} \approx \frac{0.97}{\sin 55°}$

$c \approx 1.163$

3. Given triangle ABC if a = 5m, c = 7.8 and <A = 36°:

a) Prove that 2 possible triangles can be drawn. Show work to support your answer (2T marks)

$LS = a$ $\quad RS = c\sin A$

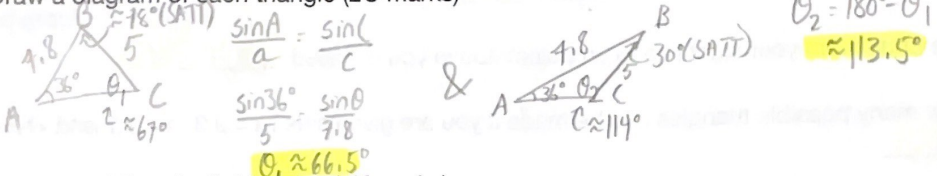
$= 5$ $\quad = 7.8\sin 36°$

≈ 4.58

$\therefore LS > RS$ & $a < c$

$\therefore$ 2 solutions.

b) Draw a diagram of each triangle (2C marks)



$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 36°}{5} = \frac{\sin \theta}{7.8}$

$\theta_1 \approx 66.5°$

& $\quad \theta_2 = 180° - \theta_1$

$\approx 113.5°$

c) Find sides AC on both triangles (7K marks)

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin(77.52°)} = \frac{5}{\sin 36°}$ Δ_1

$b \approx 8.305$

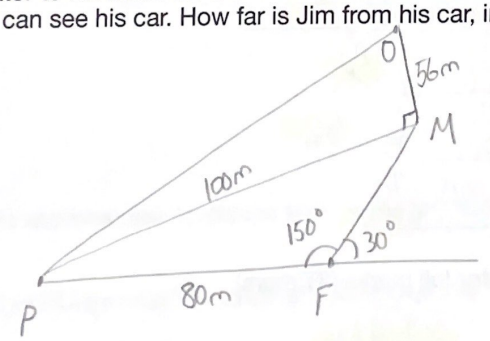
$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin(30.48)} = \frac{5}{\sin 36°}$ Δ_2

$b \approx 4.315$

4. DRAW AND LABEL DIAGRAM ONLY for the following problem. Do not solve (3A marks).

Jim parks his car in a lot on the corner of Park Lane and Main Street. He walks 80m east to First Avenue, turns 30 degrees to the left and follows First Avenue for 100m into the Metro building, where he takes the elevator to his office on the 15h floor. Each floor in the building is 4m in height. From his office window, he can see his car. How far is Jim from his car, in a direct line?



Communication (5C marks) marks are awarded from proper units, form, etc from throughout the test.

NO CALCULATORS

Chapter 5 Trig Test 2

1. Fill in the blank (16T marks)

a. cotX = -2 so X is located in quadrants Q2 or Q4

b. An angle with a clockwise rotation is called negative

c. Find a co-terminal angle for 140°. 500°

d. The principal angle of 620° is 260°

e. Given tanx = 0 and x is from 0° to 360° the possible values for x are: 0°, 360°, 180°

f. Given tanx = $\frac{-1}{\sqrt{3}}$ then x = $\sqrt{3}$, y = -1 and r = 2

$\tan\theta = \frac{-1}{\sqrt{3}}$ ∵ $\tan\theta = y/x$ ∴ $y=-1$, $x=\sqrt{3}$ $y^2+x^2=r^2$ $1+3=r^2$ $r=2$

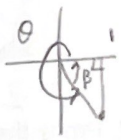
g. Given sinx = 1/5 and cosx = 2/5 then is tanx positive or negative? positive

h. State the related acute angle of 150° 30° $\beta = 180° - \theta$ $\beta = 30°$

i. Given x = 700° then β = 20° $\theta = 700°$, $\theta = 340°$, $\beta = 20°$

j. Given P(1, -1) is a point on a terminal arm: (4T marks)

P(1, -1) $x=1$ $y=-1$ $r=2$ (special Δ)



i. $\tan\theta = \frac{y}{x} = -1$ ii. Tanβ = $|\tan\theta| = 1$ iii. θ is in quadrant 4 iv. β = 45° (special Δ)

2. Evaluate the following: (6K marks)

i. sec 330° = $\frac{1}{\cos 330°} = \frac{2\sqrt{3}}{3}$

ii. csc210° = $\frac{1}{-\sin 30°} = -2$

iii. tan135° = $-\tan 45° = -1$

iv. cos720° = 1

v. csc180° = $\frac{1}{\sin 180°} = \infty$

vi. sin(-30°) = $\sin 330° = -\sin 30° = -1/2$

3. State the exact value and simplify fully into a single fraction: (4K marks)

cos210° + $\sqrt{3}$sec135° $= -\cos 30° + \sqrt{3}\left(\frac{1}{-\cos 45°}\right) = -\sqrt{3}/2 - 2\sqrt{6}/2$

$= -\sqrt{3}/2 + \sqrt{3}(-\sqrt{2})$

$= -\sqrt{3}/2 - \sqrt{6}$

$= \frac{-\sqrt{3} - 2\sqrt{6}}{2}$

4.

a) State the quadrants for which those angles are found in (2A marks)
b) State the related acute angle (2T marks)
c) Determine all angles from 0° and 360° that satisfy the following. (2K marks)

I) tanx = $-\sqrt{3}$

ⓐ ∵ tanx < 0
∴ Q2 & Q4

ⓑ $\tan\beta = |\tan x|$
$\tan\beta = \sqrt{3}$
$\beta = 60°$ (special triangle)

ⓒ In Q2: θ_1, 60°
$\theta_1 = 120°$

In Q4: θ_2, 60°
$\theta_2 = 300°$

II) cosA = 1/sqrt(2)

ⓐ ∵ cosA > 0
∴ Q1 or Q4

ⓑ $\cos\beta = |\cos A|$
$\cos\beta = \frac{1}{\sqrt{2}}$
$\beta = 45°$ (special Δ)

ⓒ In Q1: 45°, θ_1
$\theta_1 = 45°$

In Q4: θ_2, 45°
$\theta_2 = 315°$

5. One of the primary trig rations is given, as well as the quadrant in which the angle is located. Determine an exact expression for the other two primary trig rations cosA and tanA.

$\sin A = \frac{-2}{\sqrt{6}}$, third quadrant (4T marks)

$\because \sin A = y/r$ $\quad x^2+y^2=r^2$

$\therefore y=-2,\ r=\sqrt{6}$ $\quad x^2+4=6$

$x^2=2$

$\because A \in Q3$ $\quad x=\pm\sqrt{2}$

$\therefore x=-\sqrt{2}$

$\cos A = x/r = -\sqrt{2}/\sqrt{6} = -\frac{\sqrt{12}}{6} = -\frac{2\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}$

$\tan A = y/r = -2/\sqrt{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

6. Prove that $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$ (4T marks)

$LS = (\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta$

$= 2\sin^2\theta + 2(1-\sin^2\theta)$

$= 2\sin^2\theta + 2 - 2\sin^2\theta$

$= 2$

$RS = 2$

$\therefore LS = RS$

$\therefore (\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2 = 2$

7. Prove that $\frac{\cos X}{\sin X} - \tan X = \frac{2\cos^2 X - 1}{\sin X \cos X}$. (4T marks)

$LS = \frac{\cos X}{\sin X} - \tan X$

$= \frac{\cos X}{\sin X} - \frac{\sin X}{\cos X}$

$= \frac{\cos^2 X - \sin^2 X}{\sin X \cos X}$

$= \frac{(1-\sin^2 X) - \sin^2 X}{\sin X \cos X}$

$= (1-2\sin^2 X)/(\sin X \cos X)$

$RS = \frac{2\cos^2 X - 1}{\sin X \cos X}$

$= \frac{2(1-\sin^2 X) - 1}{\sin X \cos X}$

$= \frac{2 - 2\sin^2 X - 1}{\sin X \cos X}$

$= \frac{1 - 2\sin^2 X}{\sin X \cos X}$

$\therefore LS = RS$

$\therefore \frac{\cos X}{\sin X} - \tan X = \frac{2\cos^2 X - 1}{\sin X \cos X}$

Communication (4C marks) - marks are awarded for proper communication from throughout the test.

DROPPED

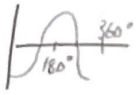
Chapter 6 Test

Fill in the blank: (2C marks each) (20 marks)

1. If the axis is 3 and the amplitude is 3 of a sine graph then the equation of the max is 6 and the min is 0

2. A periodic function f(x) has a period of 4 and f(14) = -5. Given that p is not 14, give 2 examples of p such that f(p) = -5.
p = 18 or p = 10 period=4, f(14)=-5

3. Given a negative cosine function is the interval from 0° to 360°, the x-intercept(s) is (are) (90°,0) & (270°,0) (points) and the maximum is at (180°,1) (points).



4. Given y = sin4x the amplitude of the function is 1 and the axis is y=0

5. Given y = 2cos(2/3)x + 2 the period of the function is 180° and the axis is y=2

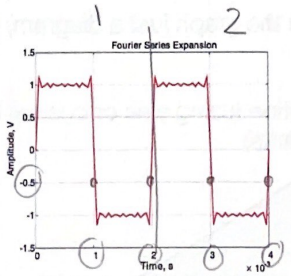
6. A cosine function has a minimum at (3,0), a period of 360° and axis of y = 5. What is a possible equation for this function?
$-5\cos(x-3^\circ)+5$?

7. Given the function y = 2cos(x-90) a sine graph of the same function is $y=2\sin(x-45^\circ)$?

8. Given a graph starts at a min (0,0), has a period of 90° and has a max of 2. Write a possible equation $y=-2\cos 4x$

9. Given f(x) = 3sin90(x-75°)+2 the domain is $\mathbb{R}$ and the range is $\{y\in\mathbb{R} \mid -1\le y\le 5\}$
axis: y=2, amp=3 ⟹ max = axis+amp = 5, min = axis-amp = -1

10. Given the periodic function on the right:



Name 4 x-values (approx) that have a y-value of -0.5 1, 2, 3, 4
What is the period? 2

-cos

11. Given the following function, f(x) = -2cos(2x-180°) $= -2\cos(2(x-90°))$

a. Fill in the following: (4K marks)

Amp = $|a| = 2$

Axis = $c = 0$

Max = 2

Min = -2

Period = $360°/2 = 180°$

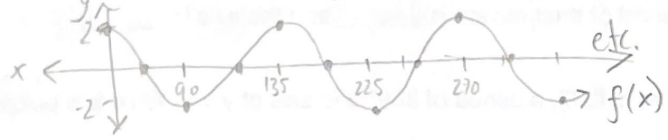
Shift = $d = 90°$ (right)

Key points: _____

$x_1 = 90°$ $y_1 = \text{min} = -2$
$x_2 = 135°$ $y_2 = \text{axis} = 0$
$x_3 = 180°$ $y_3 = \text{max} = 2$
$x_4 = 225°$ $y_4 = \text{axis} = 0$
$x_5 = 270°$ $y_5 = \text{min} = -2$

b. Draw TWO CYCLES of the graph. Include proper labels (3C Marks)

ONE



12. Write a function to represent the motion of a point on a water wheel. The shell has a radius of 2m and the wheel rotates every 30 secs and the bottom of the wheel is 0.6 below water level. Given the water surface is the x-axis and the function starts at a min, determine an equation of the function:

+ sine

using sine: $y = 2\sin(12(x-7.5)) + 1.4$ (2A marks)

- cosine

using cosine: $y = -2\cos 12x + 1.4$ (2A marks)

amp = r = 2
axis: $y = \frac{3.4-0.6}{2} = 1.4$
max = 3.4
min = -0.6
period = 30s, $k = 360/p = 360/30 = 12$

$a = 2$
$c = 1.4$
$d = 0$
$k = 12$

13. A Ferris wheel is 4 feet off the ground. It has a diameter of 26 feet, and rotates once every 32 seconds. If you begin the ride sitting in a chair at the bottom of the ride.....

No

a. Draw a labeled sketch (not the graph just a diagram) (2C marks)

b. Write a trigonometric equation (using sine or cosine) for your height above the ground at t seconds after the ride starts. (3A marks)

c. How high will you be 10 seconds into the ride? Show your work for full marks and write your answer in reduced radical form. (3A marks)

d. During the first rotation, when will you be 10.5 feet high? (Hint: this happens twice) (4A marks)

Communication (3C marks) - marks are awarded for proper use of units, form, etc. from throughout the test.

BONUS (1 bonus mark)
A sine function has a maximum value of 11, a minimum value of -1, a phase shift of 2x/3 rad to the left, and a period of pi. Write an equation for the function.

NO

$I = PRT$

$A = P(1+i)^n$

	Arithmetic	Geometric
Seq.	$t_n = a+(n-1)d$	$t_n = ar^{n-1}$
Series	$S_n = \frac{n}{2}[2a+(n-1)d]$	$S_n = \frac{a(r^n-1)}{r-1}$

Sequences and Series Quiz

All solutions must have proper mathematical support. A grade of zero will be awarded for trial and error solutions.

NO CALCULATORS

1. How many terms are in the sequence 2, 7, 12, 17, ..., 102? (3K marks)

$a=2$, $d=5$

2, 7, 12, 17, ..., 102 (+5)

Arithmetic

$t_n = a+(n-1)d$

$102 = 2+(n-1)5$

$100 = (n-1)5$

$21 = n$

∴ There are 21 terms.

2. The first three terms of the sequence defined by $t_1 = 4$, $t_n = 2t_{n-1}+3$, are (3K marks)

$t_1 = 4$

$t_2 = 2(4)+3 = 11$

$t_3 = 2(11)+3 = 25$

$t_n = 2t_{n-1}+3$

∴ The first 3 terms are 4, 11, 25.

3. Find the sum of the series 3 + 4 + 5 + ... + 97 (4T marks)

$a=3$, $d=1$

3+4+5+...+97 (+1)

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{95} = \frac{95}{2}[2(3)+94(1)]$

$S_{95} = \frac{95}{2}[100]^{50} = 4750$

$t_n = a+(n-1)d$

$97 = 3+(n-1)1$

$94 = (n-1)$

$n = 95$

∴ The sum of the series is 4750

4. For an arithmetic sequence $t_7 = 53$ and $t_{11} = 97$. Determine a, d, t_n and t_{50} for this sequence. (5T marks)

arithmetic

$\begin{cases} t_7 = 53 \\ t_{11} = 97 \end{cases}$

$t_n = a+(n-1)d$

$t_n = -13+(n-1)11$

$t_n = -24+11n$

$t_{50} = -24+11(50)$

$t_{50} = -24+550$

$t_{50} = 526$

$t_n = a+(n-1)d$

① $53 = a+6d$

② $97 = a+10d$

$44 = 4d$

$d = 11$

② − ①

sub d into ①

$53 = a+6(11)$

$a = -13$

5. Given the following series: 2 + 4 + 6 + ... and that the sum of the terms is 110. Find the number of terms in the series. (5T marks)

2+4+6+... (+2, +2)

$S_n = 110$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$110 = \frac{n}{2}[2(2)+(n-1)2]$

$220 = n[4-2+2n]$

$220 = n(2+2n)$

$0 = 2n^2+2n-220$

$0 = n^2+n-110$

$0 = (n+11)(n-10)$

$n_1 = -11$, $n_2 = 10$

∴ n_1 is neg

∴ $n = 10$

6. a) Complete the next 3 lines of Pascal's Triangle. (3K marks)

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
1   5  10  10   5   1
```

b) Determine an expanded expression for: $(x^2+2)^4$. (4K marks)

$(x^2+2)^4 = (x^2)^4+4(x^2)^3(2)+6(x^2)^2(2)^2+4(x^2)(2)^3+(2)^4$

$= x^8+8x^6+24x^4+32x^2+16$

Communication (3C marks) - form, units, formulas, etc. throughout the quiz.

BONUS: Put the following sequence into summation notation

1-1/3+1/5-1/7+...

$$\sum_{i=1}^{\infty}\left(\frac{1}{2i-1}\times(-1)^{i+1}\right)$$

$I = PRT$

$A = P(1+i)^n$

$FV = \frac{R[(1+i)^n - 1]}{i}$

$PV = \frac{R[1-(1+i)^{-n}]}{i}$

Financial Applications Test

1. Joey borrows $1400 to buy a road racing bicycle. He repays the loan with an amount of $2000. Given a rate of 5% simple interest, how long did it take him to pay the loan? (3K marks)

$I = PRT$ $\quad I = A - P$

$600 = 1400(0.05)T \quad I = 2000 - 1400$

$T = 8.571 \quad I = 600$

$\therefore$ 8.571 years

2. Jim's parents want to put some money in account so that in 10 years, Jim will have $10000. The rate of the bank account was 6% per year compounded semi-annually. How much money should Jim's parents invest into the account? (3K marks)

$A = P(1+i)^n$

$10\,000 = P\left(1 + \frac{0.06}{2}\right)^{2(10)}$

$P = \$5536.76$

$\therefore$ They should put in $5536.76

3. Lucy wants to have $20000 in her account in 4 years to buy a new car.

a. How much must she invest per month if her account earns 3.6% annual interest, compounded monthly? (4K marks)

$FV = \frac{R[(1+i)^n - 1]}{i}$

$20000 = \frac{R[(1+\frac{0.036}{12})^{12(4)} - 1]}{0.003}$

$60 = R[(1.003)^{48} - 1]$

$R = 388.01$

$\therefore$ she should invest $388.01 per month

b. How much did Lucy make in interest? (1K mark)

$I = FV - Rn$

$I = 20000 - (388.01)(12)(4)$

$I = 1375.52$

$\therefore$ $1375.52 in interest.

4. Manual invests $1000 in a GIC that earns 4% interest per year, compounded monthly. How much interest is earned after 10 months? (3K marks)

$A = P(1+i)^n$

$A = 1000\left(1 + \frac{0.04}{12}\right)^{10}$

$A = \$1033.84$

$I = A - P$

$I = 1033.84 - 1000$

$I = 33.84$

$\therefore$ $33.84 was earned in interest

5. Lauren plans to withdraw $650 at the end of every 3 months, for 5 years, from an account that earns 6.4% interest, compounded quarterly. How much was there originally? (3K marks)

$PV = \frac{R[1-(1+i)^{-n}]}{i}$

$PV = \frac{650[1-(1+\frac{0.064}{4})^{-4(5)}]}{0.016}$

$PV = 176.806 / 0.016$

$PV = 11050.38$

$\therefore$ There was $11050.38 in the account originally

6. Ken wants to receive $200 every three months at 5.2%/a compounded quarterly. He currently has 6500 to invest. How long will he receive regular payments for? (5K marks)

$PV = R[1-(1+i)^{-n}]$

$6500 = \frac{200[1-(1+\frac{0.052}{4})^{-n}]}{0.013}$

$84.5 = 200[1-(1+\frac{0.052}{4})^{-n}]$

$\frac{84.5}{200} = 1 - (1+0.052/4)^{-n}$

$\frac{115.5}{200} = (1+\frac{0.052}{4})^{-n}$

$\log\left(\frac{115.5}{200}\right) = \log\left(1+\frac{0.052}{4}\right)^{-n}$

$\log\left(\frac{115.5}{200}\right) = -n\log\left(1+\frac{0.052}{4}\right)$

$-n = \log\left(\frac{115.5}{200}\right) / \log(1.013)$

$n = 42.5083$

$n = 4\text{yrs} \times \text{compounding period}$

$\text{years} = n / \text{comp. p.}$

$\text{years} = 10.627$

$\therefore$ He will receive them for 10.627 years.

7. Paul borrows $250 for 4 years at an interest rate that is compounded quarterly. At the end of the 4 years he repays $500. What annual interest rate, compounded quarterly, was Paul charged? (4T marks)

$A = P(1+i)^n$

$500 = 250(1+i)^{4(4)}$

$2 = (1+i)^{16}$

$\sqrt[16]{2} = 1+i$

$i = \sqrt[16]{2} - 1$

$i = 0.04427$

$i = \frac{\text{rate}}{\text{comp. period}}$

BONUS

$\text{rate} = (i)(\text{comp. period})$

$\text{rate} = 0.04427(4)$

$\text{rate} = 0.177$

∴ He was charged 17.7%

8. Jill invests #300 a month at a rate of 5%/a compounded monthly for 2 years. She then takes the money in that account and moves it in an account with a better rate for 5 more years at 6%/a compounded quarterly. How much does he have at the end of the 7 years? (5A marks)

Communication (5C marks) - marks are awarded for proper form, use of formulas, units, sentences, etc.