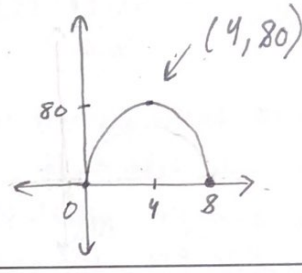
MPM 2D

Quadratic Functions - Worksheet #5
Word Problems with $y = a(x - h)^2 + k$

Name: Anna Denisova
Date: Nov. 10, 2022

1. A golf ball is hit and travels through the air according to the equation $h = -5(t - 4)^2 + 80$, where h is the height of the ball in metres, and t is the time in seconds.

Rough Sketch (label vertex)



x-int
y-int
vertex (max/min)

a) The maximum height of the ball, - (y-val vertex)
The max height is 80m.

b) The time it takes to reach the maximum (vertex) height, - (x-val vertex)
The time is 4 seconds.

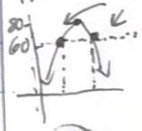
* take furthest one to get where it LANDS, not STARTS (s < e)

c) How long it takes for the ball to land. - (x-int) (the second one)

Let $h = 0$
$0 = -5(t-4)^2 + 80$
$-80 = -5(t-4)^2$
$16 = (t-4)^2$
$\pm 4 = t - 4$
$\pm 4 = t - 4$
$4 = t - 4$ or $-4 = t - 4$
$t = 8$ or $t = 0$
∴ it hit after 8 seconds.

d) When was the ball 60 m (y) above the ground?

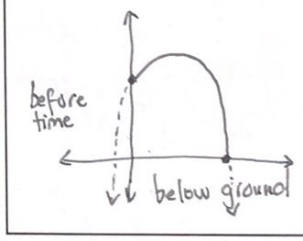
two ans:



Let $h = 60$
$60 = -5(t-4)^2 + 80$
$-20 = -5(t-4)^2$
$4 = (t-4)^2$
$\pm 2 = t - 4$
$2 = t - 4$ $\quad -2 = t - 4$
$t = 6$ $\quad t = 2$
∴ The ball is at 60m after 2 seconds & 6 seconds.

2. A basketball player takes a jump shot. The path of the ball can be described by the equation $h = -5(t - 1)^2 + 7$, where h is the height of the ball in m, and t is the time in seconds. Determine:

Rough Sketch (label vertex)



V(1, 7)

a) The maximum (vertex) height of the ball.
∴ The max height is 7m

b) The time it takes to reach the maximum height,
∴ The max height occurs at 1 second.

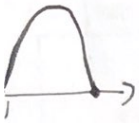
c) The height from which the player shot the ball.
y-int
Let $t = 0$
$h = -5(0-1)^2 + 7$
$h = 2$
∴ The ball was shot 2m high.

$y = a(x-h)^2 + k$

V(50, 15000)

3. A company that builds phones earns a profit on sales according to the quadratic relation $P = -10(N - 50)^2 + 15000$, where P is the profit earned in dollars, and N is the number of crates of phones sold.

don't know y-int



X

a) How many crates of phones need to be sold to make a maximum profit?

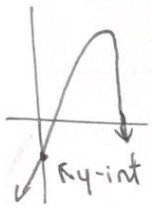
∴ 50 crates need to be sold.

b) What is the maximum profit that can be made?

∴ The max profit is $15,000

now we know! ←

c) If the company only sells 11 crates, will they make a profit or take a loss? How much?



let n = 11

$P = -10(11 - 50)^2 + 15000$

$P = -10(-39)^2 + 15000$

$P = -10(1521) + 15000$

$P = -210$

∴ They have a loss of $210.

y

4. The equation, $h = -0.025(d - 20)^2 + 10$, shows the height of a soccer ball, h metres, as a function of the horizontal distance, d metres, the ball travels until it first hits the ground.

X

V(20, 10)

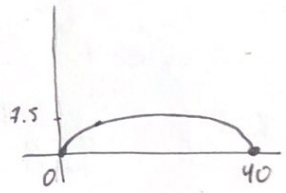
a) What is the maximum height of the ball?

The max height is 10 m.

b) What is the horizontal distance of the ball from the kicker when it reaches its maximum height.

The distance is 20m.

Rough Sketch (label vertex)



c) How far does the ball travel horizontally from when it is kicked until it hits the ground?

let h = 0

$0 = -0.025(d - 20)^2 + 10$

$0 = -\frac{1}{40}(d - 20)^2 + 10$

$-10 = -\frac{1}{40}(d - 20)^2$

$400 = (d - 20)^2$

$\pm 20 = d - 20$

$20 = d - 20$ OR $-20 = d - 20$

$\boxed{d = 40}$ OR $\boxed{d = 0}$

∴ The ball traveled 40 m.

d) What is the height of the ball when it is 10 m horizontally from the kicker?

let d = 10

$h = -\frac{1}{40}(10 - 20)^2 + 10$

$h = -\frac{1}{40}(100)^2 + 10$

$h = -5/2 + 10$

$\boxed{h = 7.5m}$

∴ The height is 7.5 m.

Homework (4.4)
Pg. 186 #12-14

Answers: 1a) 80m b) 4s c) 8sd) 2s and 6s 2.a) 7m b)1s c)2m 3.a) 50 crates b)$15000 c) loses $210 4.a) 10m b) 20m c) 40m d) 7.5m

Anna Denisova Applications of Max/Min Nov. 15, 2022

STEPS
1. let statements
2. set up equations
3. perform substitution to get a single equation
4. complete the square
5. Interpret the vertex
6. ∴ statement

eg. You have 12m of fence. What dimensions will give you a maximum area if the shape fenced is a rectangle.

y is x? confusing Try to NOT use x & y

let x represent the length (m)
let y represent the width (m)

① $2x+2y=12 \rightarrow x+y=6 \rightarrow x=6-y$

② $A=xy$

Sub x into ②

$A=(6-y)y$

$A=6y-y^2$

$A=-y^2+6y$

$A=-(y^2-6y)$

$A=-(y^2-6y+9)+9$

$A=-(y-3)^2+9$

$V(\underset{y}{3},\underset{A}{9})$

factored

standard

$\left(\frac{-6}{2}\right)^2$

vertex

Sub in $y=3$

$x=6-3$

$x=3$

∴ The dimensions are 3m × 3m

sum of squares a^2+b^2

square of sum $(a+b)^2$

eg. The sum of two numbers is 16. Find the numbers if the sum of the squares is a min.

let a represent the first number
let b represent the second number

① $a+b=16 \rightarrow a=16-b$

② $M=a^2+b^2$

→

$(a+b)^2 = a^2 + 2ab + b^2$

① $a = 16 - b$

② $M = a^2 + b^2$

sub a into ②

$M = (16-b)^2 + b^2$

$M = 256 - 32b + b^2 + b^2$

$M = 2b^2 - 32b + 256$

$M = 2(b^2 - 16b) + 256$

$M = 2(b^2 - 16b + 64) + 128$

$M = 2(b-8)^2 + 128$

$V(8, 128)$

$b \quad M$

sub b into ①

$a = 16 - 8$

$\boxed{a = 8}$

$\therefore$ The numbers are 8 & 8.

R Revenue - total income

P Profit - Revenue with costs deducted

p price - how much you sell each item for

q quantity - how many sold/made

C cost - how much it costs to produce each item

* $\boxed{R = pq}$ $\quad P = pq - cq \quad P = R - C$

eg. If a show charges \$1.60/person 200 people will buy a ticket. For every 40¢ increase in price, 10 less people will buy a ticket. What is the ticket price that will give the max revenue.

let x represent the number of increase in price

① $P = 1.6 + 0.4x$

② $q = 200 - 10x$

③ $R = pq$

sub p and q into R

$R = (1.6 + 0.4x)(200 - 10x)$

$R = -4x^2 + 64x + 320$

$R = -4(x-8)^2 + 576$

$V(8, 576)$

$x \quad R$

sub x into P

$P = 1.6 + 0.4(8)$

$P = 4.8$

$\therefore$ \$4.80 will give the max revenue.

Homework:

Worksheet #9, row 78

Ch 1 word problems
↳ each eqⁿ is linear

Ch (before this)
↳ max/min
→ vertex

Ch 6 (now)
↳ quadratics

Anna Denisova — Section 6.5 - Word Problems — Nov 28, 22

STEPS

① let statements
② create 2 equations ☆
③ substitute → goal: eqⁿ with 1 variable
④ Solve by @ Factoring
ⓑ Quadratic formula
⑤ Write a ∴ statement

① The sum of two positive numbers is 12. If their product is 35, find the numbers.

let x represent num 1
let y represent num 2

① $x+y=12 \rightarrow x=12-y$
② $xy=35$

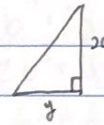
sub x into ②
$y(12-y)=35$
$12y-y^2=35$
$-y^2+12y-35=0$
$y^2-12y+35=0$
$(y-7)(y-5)=0$
$\boxed{y=5,7}$

sub y into ①
$x=7$

check that BOTH answers are VALID

$\therefore$ The numbers are 5 & 7

hypotenuse ② The hyp of a right angled triangle is 10m. One of the other sides is 2m longer than the third side. Find the length of all three sides.



let x represent the first side (m)
let y represent the second side (m)

① $y=x+2$
② $x^2+y^2=100$

$100=10^2$ pythag

sub y into ②
$x^2+(x+2)^2=100$
$x^2+x^2+4x+4=100$
$2x^2+4x-96=0$
$2(x^2+2x-48)=0$
$2(x+8)(x-6)=0$
$\boxed{x=-8,6}$

$\therefore x>0 \therefore$
$x \neq 8$

☆ ONLY 6 is valid, can't be negative

$x=6$
Sub x into ①
$\boxed{y=8}$

$\therefore$ The three sides are 6, 8, 10

③ The area of a rectangular field is $2275\ m^2$. The field is enclosed by 200m of fencing. What are the dimensions of the field?

Let l represent the length
Let w represent the width

① $lw = 2275$

② $2l + 2w = 200 \rightarrow l + w = 100 \xrightarrow{③} l = 100 - w$

Sub ③ into ①

$w(100 - w) = 2275$

$100w - w^2 = 2275$

$w^2 - 100w + 2275 = 0$

factorable but...

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \frac{100 \pm \sqrt{10000 - 4(2275)}}{2}$

$w = \frac{100 \pm \sqrt{10000 - 9100}}{2}$

$w = \frac{100 \pm \sqrt{900}}{2}$

$w = \frac{100 \pm 30}{2}$

$w = 50 \pm 15$

$w_1 = 35$

$w_2 = 65$

sub w into ③ check!

$l = 100 - 35$

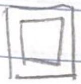
$l = 65$

∴ The dimensions are $35m \times 65m$

Homework: WORD PROBLEMS (row 94)

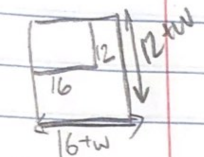
Anna Denisova Word Problems – Part 2 Nov 25, 2022

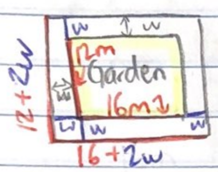
picture frames
↳ matt



eg. A garden measuring 12m × 16m is to have a pathway installed around it. The pathway is of equal width all the way around. The new total area will be 285 m^2. What is the width of the pathway?

eg





A = 285 m^2

let w represent the width of the pathway (m)

① $285 = (16+2w)(12+2w)$ — one eq^n

$285 = 192 + 56w + 4w^2$ — exp.

$0 = 4w^2 + 56w - 93$ — set to zero

$0 = (2w - 3)(2w + 31)$ — factor

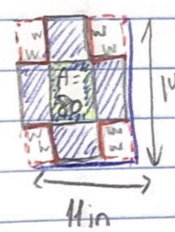
$w = 3/2, -31/2$

∵ $-31/2$ is negative ∴ $w = 3/2$

∴ The width of the pathway is 1.5m.

---

eg. You have to make an open topped box with a base area of 80 in^2. You will be taking a 11 in × 14 in piece of cardboard and cutting equal size squares from each corner, scoring the edges and folding up the box. What should be the dimensions of the squares you cut? Approx your answer.



let w represent the width of the squares (in)

① $80 = (11-2w)(14-2w)$

$80 = 4w^2 - 50w + 154$

$0 = 4w^2 - 50w + 74$

$0 = 2(2w^2 - 25w + 37)$

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \frac{25 \pm \sqrt{625 - 4(2)(37)}}{4}$

$w = \frac{25 \pm \sqrt{625 - 296}}{4}$

$w = \frac{25 \pm \sqrt{329}}{4}$

$w \approx \frac{25 \pm 18.1}{4}$

$w_1 \approx \frac{25 + 18.1}{4}$ ≈ 10.78 (not enough paper) 11-20? NO

CAN'T BE THIS ONE SINCE PAPER IS 11 in × 14

$w_2 \approx \frac{25 - 18.1}{4}$ ≈ 1.74

∵ $2w_1 > 11$ ∴ Impossible

∴ the dim of the squares are 1.74 in × 1.74 in.

26 × 26; 625 − 296 = 329; 18 × 18 = 144 + 180 = 324