# Exponent Laws

① $x^a x^b = x^{a+b}$
② $x^a / x^b = x^{a-b}$
③ $(x^a)^b = x^{ab}$
④ $x^0 = 1$
⑤ $x^1 = x$
⑥ $x^{-1} = 1/x$
⑦ $1/x^{-1} = x$
⑧ $x^{-a} = 1/x^a$
⑨ $1/x^{-a} = x^a$
⑩ $(x/y)^{-1} = y/x$
⑪ $(x/y)^{-a} = x^a/y^a$
⑫ $(ax^b)^c = a^c x^{bc}$

## EXAMPLES

ⓐ $\frac{2^4 \cdot 2^{-4} \cdot 4^2}{3^3 \cdot 3 \cdot 9^{-3}} = \frac{2^2 \cdot 2^{-4} \cdot 2^4}{3^3 \cdot 3 \cdot 3^{-6}} = \frac{2^2}{3^{-2}} = 4 \cdot 9 = 36$

ⓑ $\frac{2x^{-2}y^3z^{-4}}{-3w^5m^{-3}n^0} = \frac{2y^3wm^3}{-3x^2z^4}$

ⓒ $\left(\frac{3}{2}\right)^{-1}\left(\frac{12x}{3^5y}\right)^{-2}\left(\frac{9x^3}{y^4}\right) = \frac{2}{3}\left(\frac{81y^2}{4x^2}\right)\left(\frac{1}{x^3y^4}\right) = \frac{27}{2x^5y^2}$

ⓓ $-(-xy^2)^3(-3x^{-1}y^{-2})^2 = -(-x^3y^6)(9x^{-2}y^{-4}) = 9xy^2$

# Rational Exponents

① $x^{1/b} = \sqrt[b]{x}$ $\quad x^{1/2} = \sqrt{x}$
② $x^{a/b} = \sqrt[b]{x^a}$ $\quad x^{1/3} = \sqrt[3]{x}$
③ $x^{-a/b} = \frac{1}{\sqrt[b]{x^a}}$ $\quad x^{1/4} = \sqrt[4]{x}$

* When solving, convert all radicals to exponents, simplify, then convert back.

## Examples

ⓐ $\sqrt[3]{x}\sqrt[5]{x^2} = x^{1/3}x^{2/5} = x^{11/15} = \sqrt[15]{x^{11}}$

ⓑ $\sqrt{\sqrt{\sqrt{\sqrt{x}}}} = ((x^{1/2})^{1/2})^{1/2} = x^{1/16} = \sqrt[16]{x}$

# Solving Exponential Equations

↳ The variable is the exponent
① Get a single term on each side
② Get the bases the same
③ "Drop the bases" & set the exp. equal
④ Solve

ⓐ $2^x = 1/16$
$2^x = 2^{-4}$
$x = -4$

ⓑ $4^{x+1} = 2^{x+1}$
$2^{2x+2} = 2^{x+1}$
$2x+2 = x+1$
$x = -3$

ⓒ $9^{3x+1} = 27^x$
$(3^2)^{3x+1} = (3^3)^x$
$3^{6x+2} = 3^{3x}$
$6x+2 = 3x$
$x = -2/3$

ⓓ $2(3^{y-2}) = 18$
$3^{y-2} = 9$
$y-2 = 2$
$y = 4$

ⓔ $3^{x+2} - 3^x = 216$
$3^x \cdot 3^2 - 3^x = 216$
$9 \cdot 3^x - 3^x = 216$
$8 \cdot 3^x = 216$
$3^x = 27$
$3^x = 3^3$
$x = 3$
* APPLES

# Radicals

① $\sqrt{a}\sqrt{b} = \sqrt{ab}$
② $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$
③ $\sqrt{a}\sqrt{a} = a$
④ $(a\sqrt{b})(c\sqrt{d}) = ac\sqrt{bd}$
⑤ $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$

① $\sqrt{2}\cdot\sqrt{3} = \sqrt{6}$
② $\sqrt{4/9} = 2/3$
③ $\sqrt{2}\cdot\sqrt{2} = 2$
④ $(2\sqrt{3})(3\sqrt{2}) = 6\sqrt{6}$
⑤ $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

Simplify
↳ pullout largest perf. square

Multiply
↳ use laws

Add
↳ Like terms

ⓐ $\sqrt{40} = 2\sqrt{10}$
ⓑ $-3\sqrt{50} = -15\sqrt{2}$

ⓐ $(2\sqrt{3})(-3\sqrt{6})$
$= -6\sqrt{18} = -18\sqrt{2}$

ⓑ $(2\sqrt{2})^5$
$= 32(4\sqrt{2}) = 128\sqrt{2}$

ⓐ $-4\sqrt{12} + 7\sqrt{3} - \sqrt{22}$
$= -8\sqrt{3} + 7\sqrt{3} - 3\sqrt{3}$
$= -4\sqrt{3}$

ⓑ $2\sqrt[3]{6} - 4\sqrt[3]{6} + 3\sqrt[3]{6}$
$= 6\sqrt[3]{6} - 4\sqrt[3]{6}$

# Complex Numbers $\mathbb{C}$ (imaginary)

↳ treat 'i' like a var $\quad i = \sqrt{-1}$ $\quad i^2 = -1$

Powers of i: Express as $i^2$ to x pow.
If x is even, mult by 1, if odd, mult by $-1$
Writing Roots in terms of i: $\sqrt{-1} = i$ so, "pullout" an 'i'.

ⓐ $i^5 = i(i^2)^2 = i(1) = i$
ⓑ $i^{132} = (i^2)^{66} = 1$
ⓒ $i^{602} = (i^2)^{301} = -1$
ⓓ $i^{2002} = -1$
ⓐ $\sqrt{-4} = 2i$
ⓑ $\sqrt{-7} = i\sqrt{7}$
ⓒ $\sqrt{-8} = 2i\sqrt{2}$
ⓓ $\sqrt{-1}(-1)(\sqrt{-1})^2 i\sqrt{-1} = -i(-1)i^2 = -i$

# Rationalize Roots & Complex Numbers

↳ we remove all radicals from bottoms of fractions by multiplying by 'one'

**1 Term**

ⓐ $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

ⓑ $\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

ⓒ $\frac{x\sqrt{xy}}{\sqrt{x^3y}} \cdot \frac{\sqrt{xy^3}}{\sqrt{x^3y}} = \frac{x\sqrt{x^4y^2}}{x^3y} = \frac{x^3y}{x^3y} = 1$

**>term** - when bottom is a binomial, you must multiply by its conjugate.

$a+b\sqrt{c} \rightarrow a-b\sqrt{c}$

$2\sqrt{3}-4 \rightarrow 2\sqrt{3}+4$

ⓐ $\frac{3+\sqrt{6}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{3+3\sqrt{3}+\sqrt{6}+\sqrt{18}}{1-3} = \frac{3+3\sqrt{3}+6+3\sqrt{2}}{-2}$

**Imaginary Nums** - $i^2 = -1$

ⓐ $\frac{5}{2i} \cdot \frac{i}{i} = \frac{5i}{-2}$

ⓑ $\frac{4+2i}{3i} \cdot \frac{i}{i} = \frac{4i-2}{-3}$

ⓒ $\frac{2+3i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{2+4i+3i-6}{1+4} = \frac{-4+7i}{5}$

# Polynomials

Terms - Separated by addition or subtraction $3x+4y+1$ ← 3 terms; $(3+y^6)\div 14$ ← 1 term

Polynomial - Expression/equation with a single var & pos. int exponents

↳ Monomial (1 term), binomial (2), trinomial (3), ..., n-term polynomial (n)

→ Linear (deg = 1), quadratic (2), cubic (3), quartic (4), quintic (5), ...

Degree - Highest power exponent in a polynomial

Standard Form - Write the terms from highest to lowest power

Leading coefficient - First coefficient of a polynomial in standard form

| Eqn | Poly? | strd form? | # terms | deg | name (terms) | name (deg) |
|---|---|---|---|---|---|---|
| ⓐ $5x^4$ | ✓ | ✓ | 1 | 4 | monomial | quartic |
| ⓑ $x^2 - x^{-3}$ | X | — | — | — | — | — |
| ⓒ $x^3+2x^5+7$ | ✓ | X | 3 | 5 | trinomial | quintic |

# Multiplying Polynomials

**Distributive Law**

$a(b+c) = ab+ac$

$a(b+c+d) = ab+ac+ad$

$(a+b)(c+d) = ac+ad+bc+bd$

**Shortcuts**

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

$(a+b)(a-b) = a^2-b^2$

**EXAMPLES**

ⓐ $(x+3)(x-2) = x^2+x-6$

ⓑ $(a^2-2b)(3a^2-b^2) = 3a^4-a^2b^2-6a^2b+2b^3$

ⓒ $(2x-3)^3 = (2x-3)(2x-3)^2 = (2x-3)(4x^2-12x+9) = 8x^3-36x^2+54x-27$

ⓓ $(2x^3y-4xy^2)^2 = 4x^6y^2-16x^4y^3+16x^2y^4$