Factoring

Common Factor Monomial - take out a single term
- ★ if leading term is negative, take the neg.
- ★ if all values are taken out, put a one
- ★ only expand in [ ] (to see if further factorable)

ⓐ $6x^2-10x = 2x(3x-5)$

ⓑ $-4x^2y^2z + 8x^3yz^3 - 16x^2y^4z^4 = -4x^2yz(y - 2xz^2 + 4y^3z^3)$

Common Factor Binomial - remove a binomial from the expression

ⓐ $4x(w+1)+5y(w+1) = (w+1)(4x+5y)$

ⓑ $-8x^2y^4(x+1)^2 - 4xy^4(x+1) = -4xy^4(x+1)[2x(x+1)+1]$
$= -4xy^4(x+1)(2x^2+2+1)$

Bonus - Negative exponents – you have to divide the exponents!
- ★ always take out variable with the smallest exp. out of all terms

Example (no negs.) $x^4+x^3+x^2 = x^2\left(\frac{x^4}{x^2}+\frac{x^3}{x^2}+\frac{x^2}{x^2}\right) = x^2(x^2+x+1)$

With negatives:

ⓐ $x^{-3}y^2(x+1)^{-2} + 2x^{-4}y^{-1}(x+1)^{-2} = x^{-4}y^{-1}(x+1)^{-2}(xy^3+2)$
(x^{-3}/x^{-4}, y^2/y^{-1})

Factor by Grouping
- ↳ 4 terms (or more) ★ sometimes terms must be re-ordered
- → pair up the terms & common factor monomial each pair ★ nothing common → take out '1'
- → common factor binomial whole thing ★ binomials must match

ⓐ $ac+bc+ad+bd = c(a+b)+d(a+b)$
$= (a+b)(c+d)$

ⓒ $2x-3y^2+2y-3xy = 2x+2y-3xy-3y^2$ ★ RE-ORDER
$= 2(x+y)-3y(x+y)$
$= (x+y)(2-3y)$

ⓓ $ma-mb+a^2-ab+2a-2b$
$= m(a-b)+a(a-b)+2(a-b)$
$= (a-b)(m+a+2)$

Simple Trinomials - x^2+bx+c $\qquad \boxed{(x+a)(x+b) = x^2+(a+b)x+ab}$

ⓐ $x^2-8x+12 = (x-6)(x-2)$

ⓑ $x^2-2xy-15y^2 = (x-5y)(x+3y)$

ⓒ $x^4+6x^2+8 = (x^2+2)(x^2+4)$

ⓓ $(x^2+x)^2+(x^2+x)-6 = (x^2+x+3)(x^2+x-2)$

Difference of Squares: $\boxed{a^2-b^2=(a+b)(a-b)}$

ⓐ $x^2-81=(x+9)(x-9)$ ⓑ $8x^2-18y^2=2(4x^2-9y^2)=2(2x+3y)(2x-3y)$

→ Perfect square trinomial: $\boxed{(a+b)^2=a^2+2ab+b^2}$ $\boxed{(a-b)^2=a^2-2ab+b^2}$

ⓐ $4x^2+20x+25=(2x+5)^2$ ⓑ $9a^2-24a+16=(3a-4)^2$

Complex Trinomials

ax^2+bx+c

$(\_+\_)(\_+\_)$ $\quad +=b$

Steps

① choose two numbers that multiply to the first term (order does not matter)
② guess two numbers that multiply to the last term (order does matter)
③ first * last + middle * middle = center term

ⓐ $2x^2+5x-12=(2x-3)(x+4)$ ⓑ $3x^2-14x-24=(3x+4)(x-6)$
ⓒ $6x^2+x-1=(2x+1)(3x-1)$

Factor 3 in 1 - Grouping

Case I $\boxed{x\ x\ x}\ x$ Case II $x\ \boxed{x\ x\ x}$ – trinomial is a perfect square

① $x^2-6x+9-y^2=(x-3)^2-y^2$
$=(x-3+y)(x-3-y)$

② $w^2-a^2-6a-9=w^2-(a^2+6a+9)$ * take out the negative (don't show this step)
$=w^2-(a+3)^2$
$=(w+a+3)(w-a-3)$

Sum & Difference of Cubes

Sum of cubes $\boxed{a^3+b^3=(a+b)(a^2-ab+b^2)}$ * divide by 3
Diff. of cubes $\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$ * divide by 2 with diff. of squares

ⓐ $27x^3+64=(3x+4)(9x^2-12x+16)$
ⓑ $x^9-x^3y^3=x^3(x^6-y^3)=x^3(x^2-y)(x^4+x^2y+y^2)$