A watch company has been selling 1200 watches per week at \$18 each. A survey indicates that for every \$2 increase in price, there will be a drop of 40 sales per week. If it costs \$10 to make a watch, what should the selling price be in order to <u>maximize</u> revenue? let x rep. amount of increase

① $P = (18 + 2x)$

② $q = (1200 - 40x)$

③ $R = pq$

$R = (18+2x)(1200-40x)$
$= 21600 - 720x + 2400x - 80x^2$
$= -80x^2 + 1680x + 21600$
$= -80\left(x - \frac{21}{2}\right)^2 + 30420$

$V\left(21/2,\ 30420\right)$
$\quad x \qquad R$

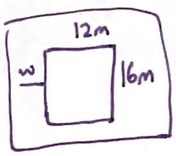
$p = (18 + 21)$
$= 39$

$\therefore \$39$

$8\overline{)168}$ = 21, -16, 08

$21600 + 8820 = 30420$

A garden measuring 12m x 16m is to have a pathway installed around it. The pathway is of equal width all the way around. The <u>new total area</u> will be 285m^2. What is the width of the pathway?



① $285 = (12+2w)(16+2w)$
$0 = -93 + 56w + 4w^2$
$0 = 4w^2 + 56w - 93$
$0 = (2w-3)(2w+31)$
$w = 3/2,\ -31/2$

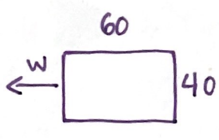
$\because -31/2$ is neg, $w = 3/2$

$\therefore$ The width is 1.5m.

$285 - 192 = 93$

$24 + 32 = 56$

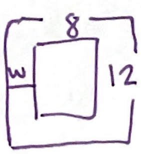
A rectangular playground 40m by 60m is to be doubled in area by extending the length and width by an equal amount. By how much should the length and the width be lengthened?



$2(60)(40) = (60+w)(40+w)$
$2(60)(40) = 60(40) + 100w + w^2$
$0 = -2400 + 100w + w^2$
$0 = w^2 + 100w - 2400$
$0 = (w+120)(w-20)$
$\because$ can't be neg, $w = 20$

$\therefore$ by 20m.

You have a picture 8" by 12" and you want to matt and frame it. The area of the matt must equal the area of the picture. Find the width of the matt correct to one decimal place.



$A_m = A_p$

$(8+w)(12+w) - 12(8) = 12(8)$
$96 + 20w + w^2 = 192$
$w^2 + 20w - 96 = 0$
$(w+24)(w-4) = 0$
$\because$ can't be neg, $\underline{w = 4}$

$\therefore 4"$.

12, 8

96 = 2 × 48, 48 = 2 × 24, 24 = 2 × 12, 12 = 2 × 6, 6 = 2 × 3

8, 12

24, 4

$8 \times 12 = 96$

7 13

The area of a rectangular field is 2275m^2. The field is enclosed by 200m of fencing. What are the dimensions of the field?

let l, w represent length, width (m)

$2275 m^2$

① $2275 = lw$

② $2l + 2w = 200 \Rightarrow l = 100 - w$

sub l into ①

$2275 = w(100-w)$

$0 = -w^2 + 100w - 2275$

$0 = -(w^2 - 100w + 2275)$

$0 = -(w-65)(w-35)$

$w = 65, 35$

$D = b^2 - 4ac$
$= 10000 - 4(2275)$
$= 10000 - 9100$
$= 9000$
$= 30 \cdot 30$

sub in $w = 65$,
$l = 100 - 65$
$= 35$

sub in $w = 35$ in
$l = 100 - 35$
$= 65$

he dims are
5m x 35m

The sum of two numbers is 16. Find the numbers if the sum of the squares is a min.

let a represent a number
let b represent a number

① $a + b = 16 \Rightarrow a = 16 - b$

② $M = a^2 + b^2$

sub a into ②

$M = (16-b)^2 + b^2$

$M = 256 - 32b + b^2 + b^2$

$M = 2b^2 - 32b + 256$

$M = 2(b-8)^2 + 128$

$V(8, 128)$
b M

sub in into (
$a = 16 - 8$
$a = 8$

$\boxed{\therefore 8 \& 8}$

Find two numbers that have a difference of 13 and whose squares when added together yield a minimum.

let a & b represent numbers

① $a - b = 13 \Rightarrow a = 13 + b$

② $M = a^2 + b^2$

sub a into ②

$M = 169 + 26b + b^2 + b^2$

$M = 2b^2 + 26b + 169$

$M = 2(b + 13/2)^2 + 169/2$

$\therefore$ no whole numbers

The sum of two positive numbers is 12. If their product is 35, find the numbers.

let a, b represent numbers

① $a + b = 12 \Rightarrow a = 12 - b$

② $35 = ab$

sub a into ②

$35 = (12-b)b$

$35 = -b^2 + 12b$

$0 = -b^2 + 12b - 35$

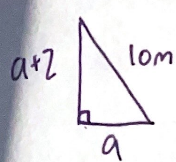
$0 = -(b^2 - 12b + 35)$

$0 = (b-7)(b-5)$

$b_1 = 7$
$b_2 = 5$

$\therefore$ the numbers are 7 & 5.

The hypotenuse of a right angled triangle is 10m. One of the other sides is 2m longer than the third side. Find the lengths of all three sides.



let a represent shortest side

① $a^2+(a+2)^2=100$

$2a^2+4a+4=100$

$2a^2+4a-96=0$

$a^2+2a-48=0$

$\rightarrow (a+8)(a-6)=0$

$a=-8,6$

$\because$ a can't be neg

$\therefore a=6$

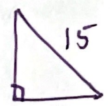
$\therefore$ the sides are 6m, 8m, & 10m

$D=b^2-4ac$

$216-4(108)$

The hypotenuse of a right angled triangle is 15cm. The sum of the other two sides is 21cm. Find the lengths of the other two sides.

let a represent one side, let b represent the second.



① $a^2+b^2=225$

② $a+b=21 \Rightarrow a=21-b$

sub a into ①

$(21-b)^2+b^2=225$

$441-42b+b^2+b^2=225$

$2b^2-42b+216=0$

$b^2-21b+108=0$

$\rightarrow (b-9)(b-12)=0$

$b=9,12$

$\boxed{\therefore 9cm \,\&\, 12cm}$

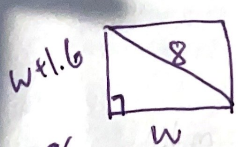
108 — 2, 54 — 2, 26 — 2, 18 — 3, 6 — 2, 3

27, 4 ⟌108, 8, 28, 28, 0

21, ·21, 21, 420, 441, 225, 216

The diagonal of a rectangle is 8cm. The length of the rectangle is 1.6m more than the width. Find the dimensions of the rectangle.

let w represent width



① $w^2+(w+1.6)^2=64$

$2w^2+3.2w+2.56=64$

$200w^2+320w+256=64$

$200w^2+320w+192=0$

$\rightarrow 100w^2+160w+96=0$

$50w^2+80w+48=0$

$25w^2+40w+24=0$

$(5w+\ )(5w+\ )$

$D=b^2-4ac$

$=1600-4(25)(24)$

$=1600-2400$

$1.6 \times 1.6 = 2.56$

96, 2⟌192, 18, 12 — 48, 2⟌96, 8, 16

The price for a new magazine is $\boxed{p(x) = -6x + 40}$ where p(x) represents selling price (in thousands of \$) per magazine and x represents the number of magazines sold (in thousands of \$). The cost function $\boxed{c(x) = 4x+20}$ represents the cost of producing one magazine (in thousands of \$). Calculate the max profit and the number of magazines sold to produce that profit.

$P=x(-6x+40)+x(4x+20)$

$P=-6x^2+40x+4x^2+20x$

$P=-2x^2+60x$

$P=-2(x^2-30x)$

$P=-2(x-15)^2+450$

$V(15, 450)$
$\quad x \quad P$

$\therefore$ max profit is 450,000, # magazines is 15.

C ↑ cost, quantity × cost

A company that builds phones earns a profit on sales according to the quadratic relation P = -10(N-50)^2+15000, where P is the profit earned in dollars, and N is the number of crates of phones sold.

a) How many crates of phones need to be sold to make a maximum profit?
b) What is the maximum profit that can be made?

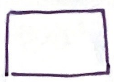
$P = -10(N-50)^2 + 15000$

$V(50, 15000)$

$\uparrow$ crates $\uparrow$ profit

a) 50 crates
b) $15,000

You have 12m of fence. What dimensions will give you a maximum area if the shape fenced is a rectangle? let l, w represent length, width (m)



① $2l + 2w = 12 \Rightarrow l + w = 6 \Rightarrow l = 6 - w$
② $A = lw$

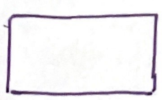
sub ① into ②
$A = (6-w)w$
$A = -w^2 + 6w$
$A = -(w-3)^2 + 9$

$V(3, 9)$
$w \quad A$

sub 3 into ①
$l = 6 - 3$
$l = 3$

$\boxed{\therefore 3m \times 3m}$

A rectangular field is to be enclosed with 600m of fencing. What dimensions will produce a maximum area? let l, w represent length, width (m)



① $2l + 2w = 600 \Rightarrow l = 300 - w$
② $A = lw$

sub l into ②
$A = w(300 - w)$
$A = -w^2 + 300w$
$A = -(w - 150)^2 + 22500$

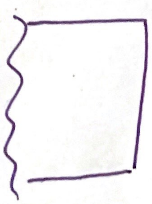
$V(150, 22500)$
$w \quad A$

sub 150 into ①
$l = 300 - 150$
$l = 150$

$\boxed{\therefore 150m \times 150m}$

A rectangular field bounded on one side by a lake is to be fenced on 3 sides by 800m of fence. What dimensions will produce a maximum area?

let l, w represent length, width (m)



① $l + 2w = 800 \Rightarrow l = 800 - 2w$
② $A = lw$

sub l into ②
$A = w(800 - 2w)$
$A = -2w^2 + 800$
$A = -2(w - 200)^2 + 80000$

$V(200, 80000)$
$w \quad A$

sub w into ①
$l = 800 - 2(200)$
$l = 800 - 400$
$l = 400$

$\boxed{\therefore 200m \times 400m}$

$\frac{\cdot 4}{64}$ $\frac{\cdot 4}{256}$

If a show charges $1.60/person, 200 people will buy a ticket. For every 40 cent increase in price, 10 less people will buy a ticket. What is the ticket price that will give the max revenue.

let i represent amount of increase

① $p = 1.60 + 0.4i$
② $q = 200 - 10i$
③ $R = pq$

$R = (1.60 + 0.4i)(200 - 10i)$
$R = 320 - 16i + 80i - 4i^2$
$R = -4i^2 + 64i + 320$
$R = -4(i-8)^2 + 576$

$V(8, 576)$
$i \quad R$

$p = 1.60 + 0.4(8)$
$p = 1.60 + 3.2$
$p = 4.80$

∴ $4.80 will give max revenue.

The profit P(x) of a cosmetics company, in thousands of dollars, is given by P(X) = -5x^2 + 400x - 2550, where x is the amount spent on advertising, in thousands of dollars.

a) Determine the max profit the company can make
b) Determine the amount spent on advertising that will result in the max profit
c) What amount must be spent on advertising to obtain a profit of at least 4 000 000?

$P(x) = -5x^2 + 400x - 2550$

x-ads ($1 k)

a) $= -5(x-40)^2 + 5450$
$V(40, 5450)$
$x \quad P$
∴ $5,450,000

b) $40,000

ⓒ $4{,}000 \geq -5(x-40)^2 + 5450$
$-1450 \geq -5(x-40)^2$
$290 \geq (x-40)^2$
$\pm\sqrt{290} \geq x - 40$
$\boxed{\pm\sqrt{290} + 40 \geq x}$

$5450 - 4000 = 1450$

$1450 \div 5 = 290$
-10
45
-45
00

The cost of a ticket to a hockey arena seating 800 people is $3. At this price every ticket is sold. A survey indicates that if the price is increased, attendance will fall by 100 for every dollar of increase. What price results in the greatest revenue? What is the greatest revenue?

max

let x rep. amount of increase

① $R = (3+x)(800-100x)$
$R = 2400 + 500x - 100x^2$
$R = -100(x - 5/2)^2 + 2625$
$V(5/2, 2625)$
$x \quad R$

25
00

$P = 3 + x$
$= 3 + 5/2$

∴ $P = \$5.5$,
$R = \$2625$