







Chapter 3: Geometric Properties

Properties of Quadrilaterals

(How to prove ↓)

Name	Shape	Desc.	Find	Show
Square		4 equal sides, 90° angles	4 lengths, 4 slopes	① lengths equal ② opp. slopes equal ③ adj. slopes - neg rec.
Rectangle		2 sets equal side lengths & 90°	4 lengths, 4 slopes	① 2 pairs matching len ② opp. slopes equal ③ adj. slopes - neg rec.
Parallelogram		opposite sides are parallel	4 slopes	① opp. slopes equal
Trapezoid		ONE pair of parallel sides	4 slopes	① only 2 slopes equal
Rhombus		all sides equal len, opp. sides //	4 lengths	① lengths equal
Kite		2 pairs of equal length adj. sides	4 lengths	① show adj. len equal

① A square is also a rectangle, rhombus, and parallelogram.

② A rectangle is also a parallelogram.

③ A rhombus is also a parallelogram.

Triangles

Right triangle - 3 sided shape with 90° angle.



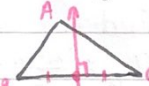
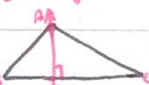




Method 1

Prove: ① calculate 3 slopes
② show 2 are neg. rec.

Method 2

① Calculate 3 lengths (LS/RS)
② show pythag is satisfied

Verify Properties of Triangles.

Name	Desc.	Drawing	How to find:
Right Bisector ($y=mx+b$)	line that is \perp to a side		① Calculate slope BC ② use neg. reciprocal as slope(m) ③ calculate midpoint BC ④ sub in midpoint & slope into $y=mx+b$
Altitude ($y=mx+b$)	the height of a shape		① calculate slope BC ② use neg. reciprocal as slope(m) ③ use point A ④ plug in ② & ③ (slope & A) into $y=mx+b$
Median ($y=mx+b$)	a line that joins a midpoint to the opposite vertex		① calculate midpoint BC ② use point A & ① to find slope ③ chose any point (A or midpoint) and use the slope to plug into $y=mx+b$
Circumcentre (POI)	POI of 3 right bisectors		① Find eq ^{ns} of all 3 right bisectors ② Do sub/elim with any 2. ③ Check using LS/RS with 3rd line.
Orthocentre (POI)	POI of 3 altitudes		① Find eq ^{ns} of all 3 altitudes ② Do sub/elim with any 2. ③ Check using LS/RS with 3rd line.
Centroid (POI)	POI of 3 medians		① Find eq ^{ns} of all 3 medians ② Do sub/elim with any 2. ③ Check using LS/RS with 3rd line.

& also:

Length of Altitude:

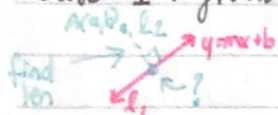
① Find eqⁿ of line BC

② Sub/elim with eqⁿ of line for altitude to find POI

③ Use length formula $l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

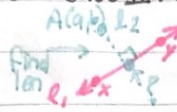
Distance from a Point to a Line

Case I: given a point & the eqⁿ of a line



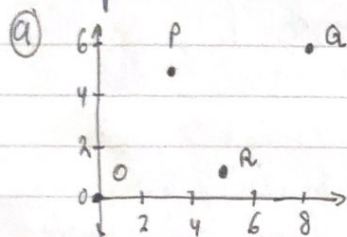
- ① given eqⁿ l_1
- ② find eqⁿ l_2 (use l_1 & A)
- ③ use sub/elim to find P
- ④ sub A & P into length formula

Case II: Given a point & 2 points on the line



- ① find slope of l_1 using points x & y
- ② plug in ① & point x // y into $y = mx + b$ (for l_1)
- ③ Follow steps ①-④ of case I.

Examples



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{OP} = \frac{5-0}{6-0} = \frac{5}{6}$$

$$m_{PQ} = \frac{3-5}{8-6} = \frac{-2}{2} = -1$$

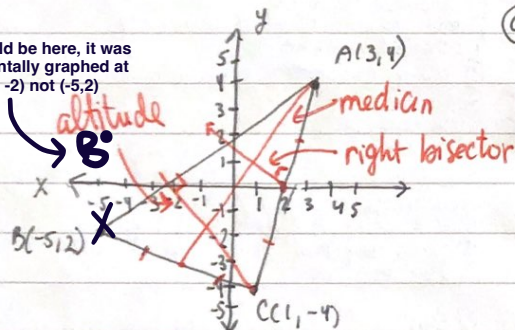
$$m_{QR} = \frac{1-3}{5-8} = \frac{-2}{-3} = \frac{2}{3}$$

$$m_{OR} = \frac{1-0}{5-0} = \frac{1}{5}$$

$\therefore m_{OP} = m_{QR}$ & $m_{PQ} = m_{OR} \parallel (l_{OP} \parallel l_{QR} \text{ & } l_{PQ} \parallel l_{OR})$
 \therefore The shape is a parallelogram.

⑥ A(3,4) B(-5,2) C(1,-4)

B should be here, it was accidentally graphed at (-5, -2) not (-5, 2)



① Find the median from A to BC

$$M_{BC} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-5+1}{2}, \frac{2-4}{2} \right) = \left(-2, -1 \right)$$

$$y = mx + b \quad \text{sub in } (-2, -1)$$

$$y = x + b \quad -1 = -2 + b$$

$$\boxed{y = x + 1} \quad b = 1$$

② Find the right bisector of AC

$$M_{AC} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{3+1}{2}, \frac{4-4}{2} \right) = (2, 0)$$

$$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-4}{1-3} = \frac{-8}{-2} = 4$$

$$\therefore l_{AC} \perp l_1 \therefore m = -\frac{1}{4}$$

$$y = mx + b$$

$$y = -\frac{1}{4}x + b$$

$$\text{sub in } (2, 0)$$

$$0 = -\frac{1}{4}(2) + b$$

$$b = \frac{1}{2}$$

$$\boxed{y = -\frac{1}{4}x + \frac{1}{2}}$$

③ Find the altitude from side AB to C

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{-5-3} = \frac{-2}{-8} = \frac{1}{4}$$

\therefore they are perpendicular
 $\therefore m = -4$

$$y = mx + b$$

$$y = -4x + b$$

$$\text{sub in } C(1, -4)$$

$$-4 = -4(1) + b$$

$$b = 0$$

$$\boxed{y = -4x}$$