Exponent Laws

① $x^a x^b = x^{a+b}$ ⑤ $x^1 = x$ ⑨ $\frac{1}{x^{-a}} = x^a$

② $\frac{x^a}{x^b} = x^{a-b}$ ⑥ $x^{-1} = \frac{1}{x}$ ⑩ $\left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$

③ $(x^a)^b = x^{ab}$ ⑦ $\frac{1}{x^{-1}} = x$ ⑪ $\left(\frac{x}{y}\right)^{-a} = \frac{y^a}{x^a}$

④ $x^0 = 1$ ⑧ $x^{-a} = \frac{1}{x^a}$ ⑫ $(ax^b)^c = a^c x^{bc}$

EXAMPLES

ⓐ $\frac{2^4 \cdot 2^{-4} \cdot 4^2}{3^3 \cdot 3 \cdot 9^{-3}} = \frac{2^2 \cdot 2^{-4} \cdot 2^4}{3^3 \cdot 3 \cdot 3^{-6}} = \frac{2^2}{3^{-2}} = 4 \cdot 9 = 36$

ⓑ $\frac{2x^{-2}y^3 z^{-4}}{-3x^5 m^3 n^0} = \frac{2y^3 w m^3}{-3x^2 z^4}$

ⓒ $\left(\frac{3}{2}\right)^{-1}\left(\frac{2x}{3^2 y}\right)^{-2}\left(\frac{x^{-3}}{y^4}\right) = \frac{2}{3}\left(\frac{9y^2}{4x^2}\right)\left(\frac{1}{x^3 y^4}\right) = \frac{27}{2x^5 y^2}$

ⓓ $-(-xy^2)^3(-3x^{-1}y^{-2})^2 = -(x^3 y^6)(9x^{-2}y^{-4}) = 9xy^2$

Rational Exponents

① $x^{1/b} = \sqrt[b]{x}$ $x^{1/2} = \sqrt{x}$

② $x^{a/b} = \sqrt[b]{x^a}$ $x^{1/3} = \sqrt[3]{x}$

③ $x^{-a/b} = \frac{1}{\sqrt[b]{x^a}}$ $x^{1/4} = \sqrt[4]{x}$

* When solving, convert all radicals to exponents, simplify, then convert back.

Examples

ⓐ $\sqrt[3]{x}\sqrt[5]{x^2} = x^{1/3} x^{2/5} = x^{11/15} = \sqrt[15]{x^{11}}$

ⓑ $\sqrt{\sqrt[2]{\sqrt{x}}} = \left(\left((x^{1/2})^{1/2}\right)^{1/2}\right)^{1/2} = x^{1/16} = \sqrt[16]{x}$
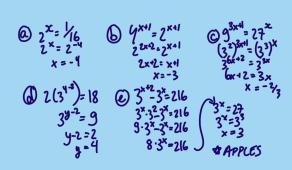
Solving Exponential Equations

↳ The variable is the exponent

① Get a single term on each side
② Get the bases the same
③ "Drop the bases" & set the exp. equal
④ Solve

ⓐ $2^x = \frac{1}{16}$
$2^x = 2^{-4}$
$x = -4$

ⓑ $4^{x+1} = 2^{x+1}$
$2^{2x+2} = 2^{x+1}$
$2x+2 = x+1$
$x = -3$

ⓒ $9^{3x+1} = 27^x$
$(3^2)^{3x+1} = (3^3)^x$
$3^{6x+2} = 3^{3x}$
$6x+2 = 3x$
$x = -\frac{2}{3}$

ⓓ $2(3^{y-2}) = 18$
$3^{y-2} = 9$
$y-2 = 2$
$y = 4$

ⓔ $3^{x+2} - 3^x = 216$
$3^x \cdot 3^2 - 3^x = 216$
$9 \cdot 3^x - 3^x = 216$
$8 \cdot 3^x = 216$
$\begin{cases} 3^x = 27 \\ 3^x = 3^3 \\ x = 3 \end{cases}$
✿ APPLES

Radicals

① $\sqrt{a}\sqrt{b} = \sqrt{ab}$ ④ $(a\sqrt{b})(c\sqrt{d}) = ac\sqrt{bd}$

② $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ ⑤ $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$

③ $\sqrt{a}\sqrt{a} = a$

① $\sqrt{2}\sqrt{3} = \sqrt{6}$ ④ $(2\sqrt{3})(3\sqrt{2}) = 6\sqrt{6}$

② $\sqrt{\frac{4}{9}} = \frac{2}{3}$ ⑤ $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

③ $\sqrt{2}\sqrt{2} = 2$

Simplify
↳ pullout largest perf. square

Multiply
↳ use laws

Add
↳ Like terms

ⓐ $\sqrt{40} = 2\sqrt{10}$

ⓑ $-3\sqrt{50} = -15\sqrt{2}$

ⓐ $(2\sqrt{3})(-3\sqrt{6})$
$= -6\sqrt{18} = -18\sqrt{2}$

ⓑ $(2\sqrt{2})^5$
$= 32(4\sqrt{2}) = 128\sqrt{2}$

ⓐ $-4\sqrt{12} + 7\sqrt{3} - \sqrt{27}$
$= -8\sqrt{3} + 7\sqrt{3} - 3\sqrt{3}$
$= -4\sqrt{3}$

ⓑ $2\sqrt[3]{6} - 4\sqrt[3]{6} + 3\sqrt[3]{6}$
$= 6\sqrt[3]{6} - 4\sqrt[3]{6}$

Complex Numbers   ℂ (imaginary)

↳ treat 'i' like a var $\boxed{i = \sqrt{-1}}$ $\boxed{i^2 = -1}$

<u>Powers of i</u>: Express as i^2 to x pow.
If x is even, mult by 1, if odd, mult by -1

<u>Writing Roots in terms of i</u>: $\sqrt{-1} = i$ so, "pullout" an 'i'!

ⓐ $i^5 = i(i^2)^2 = i(1) = i$ ⓑ $i^{132} = (i^2)^{66} = 1$

ⓒ $i^{602} = (i^2)^{301} = -1$ ⓓ $i^{2002} = -1$

ⓐ $\sqrt{-4} = 2i$ ⓑ $\sqrt{-7} = i\sqrt{7}$

ⓒ $\sqrt{-8} = 2i\sqrt{2}$ ⓓ $\sqrt{-1}(-1)(\sqrt{-1})^2 i\sqrt{-1} = -i(-1)i^2 = -i$

Rationalize Roots & Complex Numbers

↳ we remove all radicals from bottoms of fractions by multiplying by 'one'

1 Term

@ $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

ⓑ $\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

ⓒ $\frac{x\sqrt{xy}}{\sqrt{x^3y}} \cdot \frac{\sqrt{x^3y}}{\sqrt{x^3y}} = \frac{x\sqrt{x^4y^2}}{x^3y} = \frac{x^3y}{x^3y} = 1$

> term – when bottom is a binomial, you must multiply by its __conjugate__.

$a + b\sqrt{c} \rightarrow a - b\sqrt{c}$
$2\sqrt{3} - 4 \rightarrow 2\sqrt{3} + 4$

@ $\frac{3+\sqrt{6}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{3+3\sqrt{3}+\sqrt{6}+\sqrt{18}}{1-3}$

$= \frac{3+3\sqrt{3}+\sqrt{6}+3\sqrt{2}}{-2}$

Imaginary Nums – $i^2 = -1$

@ $\frac{5}{2i} \cdot \frac{i}{i} = \frac{5i}{-2}$

ⓑ $\frac{4+2i}{3i} \cdot \frac{i}{i} = \frac{4i-2}{-3}$

ⓒ $\frac{2+3i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{2+4i+3i-6}{1+4}$

$= \frac{-4+7i}{5}$

Polynomials

Terms – Separated by addition or subtraction $3x+4y+1 \leftarrow 3$ terms $(3xy^6) \div 14 \leftarrow 1$ term

Polynomial – Expression/equation with a single var & pos. int exponents
↳ Monomial (1 term), binomial (2), trinomial (3), ..., n-term polynomial (n)
→ Linear (deg = 1), quadratic (2), cubic (3), quartic (4), quintic (5), ...

Degree – Highest power exponent in a polynomial

Standard Form – Write the terms from highest to lowest power

Leading Coefficient – First coefficient of a polynomial in standard form

Eq^n	Poly?	strd form?	# terms	deg	name (terms)	name (deg)
@ $5x^4$	✓	✓	1	4	monomial	quartic
ⓑ $x^2 - x^{-3}$	✗	—	—	—	—	—
ⓒ $x^3 + 2x^5 + 7$	✓	✗	3	5	trinomial	quintic

Multiplying Polynomials

Distributive Law

$a(b+c) = ab + ac$
$a(b+c+d) = ab + ac + ad$
$(a+b)(c+d) = ac + ad + bc + bd$

Shortcuts

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$

EXAMPLES

@ $(x+3)(x-2) = x^2 + x - 6$

ⓑ $(a^2-2b)(3a^2-b^2) = 3a^4 - a^2b^2 - 6a^2b + 2b^3$

ⓒ $(2x-3)^3 = (2x-3)(2x-3)^2 = (2x-3)(4x^2-12x+9)....$
$= 8x^3 - 36x^2 + 54x - 27$

ⓓ $(2x^3y-4xy^2)^2 = 4x^6y^2 - 16x^4y^3 + 16x^2y^4$