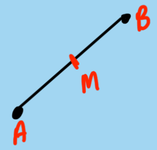
Midpoint



$$M_{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

eg. ⓐ Find the midpoint of segment PQ where $P(2,3)$ & $Q(-1,4)$

$$M_{PQ} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2-1}{2}, \frac{3+4}{2}\right) = \left(\tfrac{1}{2}, \tfrac{7}{2}\right)$$

ⓑ Given the midpoint of PQ is $M(-4,1)$ and $P(-2,-6)$ find Q.

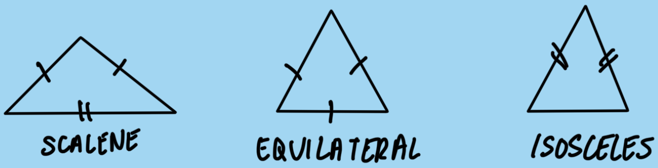
$$M_{PQ} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$(-4, 1) = \left(\frac{-2+x_2}{2}, \frac{-6+y_2}{2}\right)$$

$-4 = \frac{-2+x_2}{2}$ | $1 = \frac{-6+y_2}{2}$

$-6 = x_2$ | $8 = y_2$

$\therefore$ Q is $(-6,8)$

Triangles



Length of a line

$$l_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

eg Determine the length of the line segment joining:

ⓐ $M(2,-4)$ and $N(-3,5)$

$$l_{MN} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-3,-2)^2+(5+4)^2} = \sqrt{(-5)^2+9^2} = \sqrt{106}$$

ⓑ $P(\tfrac{1}{2}, \tfrac{3}{4})$ and $Q(-\tfrac{1}{3}, \tfrac{1}{2})$

$$l_{PQ} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{\left(-\tfrac{2}{6}-\tfrac{3}{6}\right)^2+\left(\tfrac{2}{4}-\tfrac{3}{4}\right)^2} = \sqrt{\left(-\tfrac{5}{6}\right)^2+\left(-\tfrac{1}{4}\right)^2} = \ldots = \frac{\sqrt{109}}{12}$$

Equations of Lines

Slope-intercept form: $\boxed{y = mx+b}$

Standard form: $\boxed{Ax+By+C=0}$

Slope: $\boxed{m = \frac{y_2-y_1}{x_2-x_1}}$

Steps: find the eq^n of a line

① Write the formula $y=mx+b$

② Find slope, find y-int

③ Write equation

eg. Find the eq^n of the line, write ans in standard form.

ⓐ slope is -1 and the line passes through $(1,4)$

$y=mx+b$
$y=-x+b$
$y=-x+5$
$x+y-5=0$

sub in $(1,4)$
$4=-1+b$
$b=5$

ⓑ passing through $(-1,2)$ and $(2,3)$

$y=mx+b$
$y=\tfrac{1}{3}x+b$
$y=\tfrac{1}{3}x+\tfrac{7}{3}$
$3y=x+7$
$0=x-3y+7$

$m=\frac{y_2-y_1}{x_2-x_1}$
$m=\frac{3-2}{2+1}$
$m=\tfrac{1}{3}$

sub in $(-1,2)$
$2=\tfrac{1}{3}(-1)+b$
$b=\tfrac{7}{3}$

ⓒ passing through $(1,4)$ & parallel to line $y+3x-2=0$

$y=mx+b$
$y=-3x+b$
$y=-3x+7$
$3x+y-7=0$

$y=-3x+2$
$\because l_1 \parallel l_2$
$\therefore m=-3$

sub in $(1,4)$
$4=-3(1)+b$
$7=b$

Slope-Point formula

$$y-y_1 = m(x-x_1)$$

m- slope (x_1,y_1) ← point

eg. passing through $(2,-1)$ with slope $\tfrac{3}{4}$

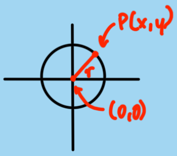
$y-y_1 = m(x-x_1)$

$y-y_1 = \tfrac{3}{4}(x-x_1)$ ← sub in $(2,-1)$

$y+1 = \tfrac{3}{4}(x-2)$

$y = \tfrac{3}{4}x-\tfrac{5}{2}$

Circles



With centre (0,0):

$r^2 = x^2 + y^2$

With centre (a, b):

$r^2 = (x-a)^2 + (y-b)^2$

eg. ⓐ centre (0,0) and radius 4.

$r^2 = x^2 + y^2$

$16 = x^2 + y^2$

ⓑ Centre (-2,3) and radius 2

$r^2 = (x-a)^2 + (y-b)^2$

$4 = (x+2)^2 + (y-3)^2$

Is point (x,y) on the circle $\boxed{x^2+y^2=r^2}$?

↳ LS > RS outside

→ LS = RS on

→ LS < RS in

eg. is the point (3,2) on the circle $x^2+y^2=2$?

$LS = x^2+y^2$ $\quad RS = 2$

$= 2^2 + 3^2$

$= 10$

∵ LS > RS ∴ the point is outside the circle.

Square Rooting Equations - you must use ±

ⓐ $x^2 = 4$

$x \pm \sqrt{4}$

$x \pm 2$

ⓑ $x^2 = 16$

$x \pm 4$

ⓒ $(2x-4)^2 = 81$

$2x-4 = 9$ OR $2x-4 = -9$

$x = 13/2$ $\quad x = -5/2$