

Unit 1 - Intro to Functions

① go to graph

$$② f(x) = 3x^2 - 2, g(x) = -2x$$

$$\checkmark a) f(-2) = 3(-2)^2 - 2 = 10 \quad \checkmark b) g(x+6) = -2(x+6) = -2x - 12$$

$$\checkmark c) g(5) = -2(5) = -10 \quad \checkmark d) f(g(-2)) = 3(-2(-2))^2 - 2 = 46$$

$$\checkmark e) f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4 \quad \checkmark f) f(g(x)) = 3(-2x)^2 - 2 = 12x^2 - 2$$

$$③ f(x) = 7x - 5 \quad \text{sub in } f(x)$$

$$f(x) = 9 \quad 9 = 7x - 5$$

$$\checkmark x = 2$$

$$④ f(x) = 3x - 4 \quad \text{Graph} \quad f(x) \quad f^{-1}(x) \quad \because \text{VLT passes for both}$$

$$\text{let } y = 3x - 4 \quad \text{- on graph} \quad \checkmark D = \mathbb{R} \quad \checkmark D = \mathbb{R} \quad \checkmark \therefore \text{both } f^{-1}s$$

$$x = 3y - 4$$

$$\text{paper}$$

$$\checkmark R = \mathbb{R}$$

$$\checkmark R = \mathbb{R}$$

$$\frac{x+4}{3} = y$$

$$\checkmark f^{-1}(x) = \frac{1}{3}x + \frac{4}{3}$$

$$⑤ f(x) = -2(x-1)^2 + 2 \quad \text{Graph}$$

$$\text{let } y = -2(x-1)^2 + 2 \quad \text{- on graph}$$

$$x = -2(y-2)^2 + 2$$

$$\text{paper}$$

$$f(x)$$

$$f^{-1}(x)$$

$$D = \mathbb{R}$$

$$D = \{x \in \mathbb{R} \mid x \geq 1\}$$

$$R = \mathbb{R}$$

$$\pm \sqrt{\frac{x-2}{2}} = y$$

$$\checkmark f^{-1}(x) = \sqrt{\frac{1}{2}(x-2)}, x \geq 1 \text{ for } f(x)$$

$\because \text{VLT pass} \therefore \text{both } f^{-1}s$

$$⑥ f(x) = \frac{1}{x-2} + 3 \quad \text{Graph}$$

$$\text{let } y = \frac{1}{x-2} + 3 \quad \text{- look at graph paper}$$

$$x = \frac{1}{y-3} + 2$$

$$(y-2)(x-3) = 1$$

$$(y-2) = \frac{1}{x-3}$$

$$y = \frac{1}{x-3} + 2$$

$$\checkmark f^{-1}(x) = \frac{1}{x-3} + 2$$

$$⑦ f(x) = \sqrt{x+3} + 2$$

$$\text{let } y = \sqrt{x+3} + 2$$

$$x = \sqrt{y+3} + 2$$

$$(x-2)^2 = y$$

$$\checkmark f^{-1}(x) = (x-2)^2 - 3$$

$$\text{Graph - look at graph paper}$$

$$\checkmark f^{-1}(x) = (x-2)^2 - 3$$

$$⑧ f(x) = -\sqrt{2(x-3)} + 4$$

$$\checkmark \text{base: } y = \sqrt{x}$$

- reflection over x-axis

- horizontal compression by a factor of $\frac{1}{2}$

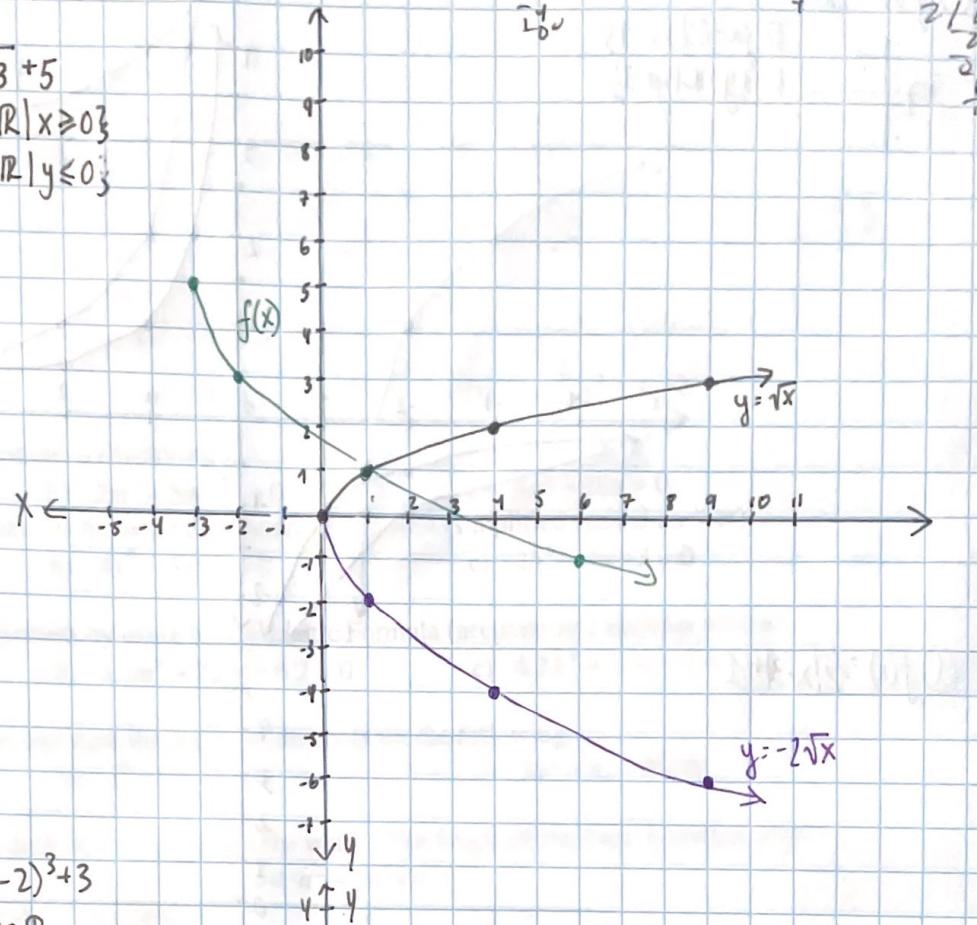
- shift 3 right, 4 up

Unit 1: Functions

Qua

$$\textcircled{1} @ f(x) = -2\sqrt{x+3} + 5$$

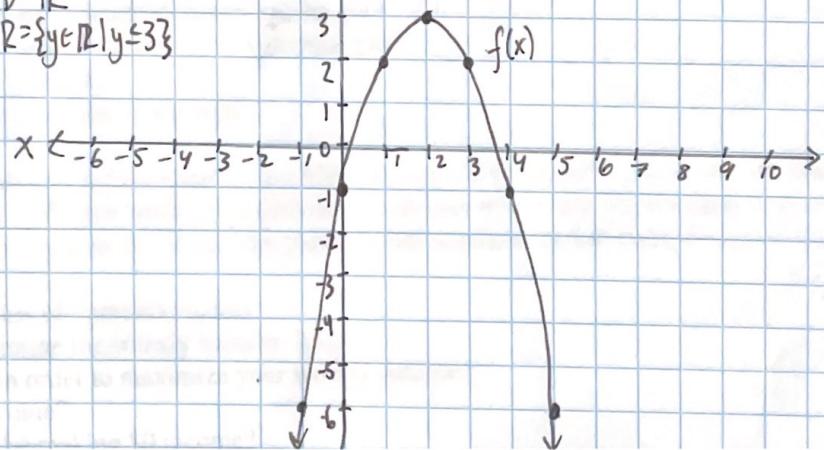
$$\begin{aligned} D &= \{x \in \mathbb{R} \mid x \geq 0\} \\ R &= \{y \in \mathbb{R} \mid y \leq 0\} \end{aligned}$$



flare

$$\textcircled{6} f(x) = -(x-2)^3 + 3$$

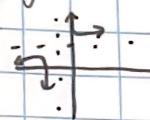
$$\begin{aligned} D &= \mathbb{R} \\ R &= \{y \in \mathbb{R} \mid y \leq 3\} \end{aligned}$$



$$\begin{array}{r} 270 \\ 2540 \\ -4x \\ \hline 49 \\ -40 \\ \hline 9 \end{array}$$

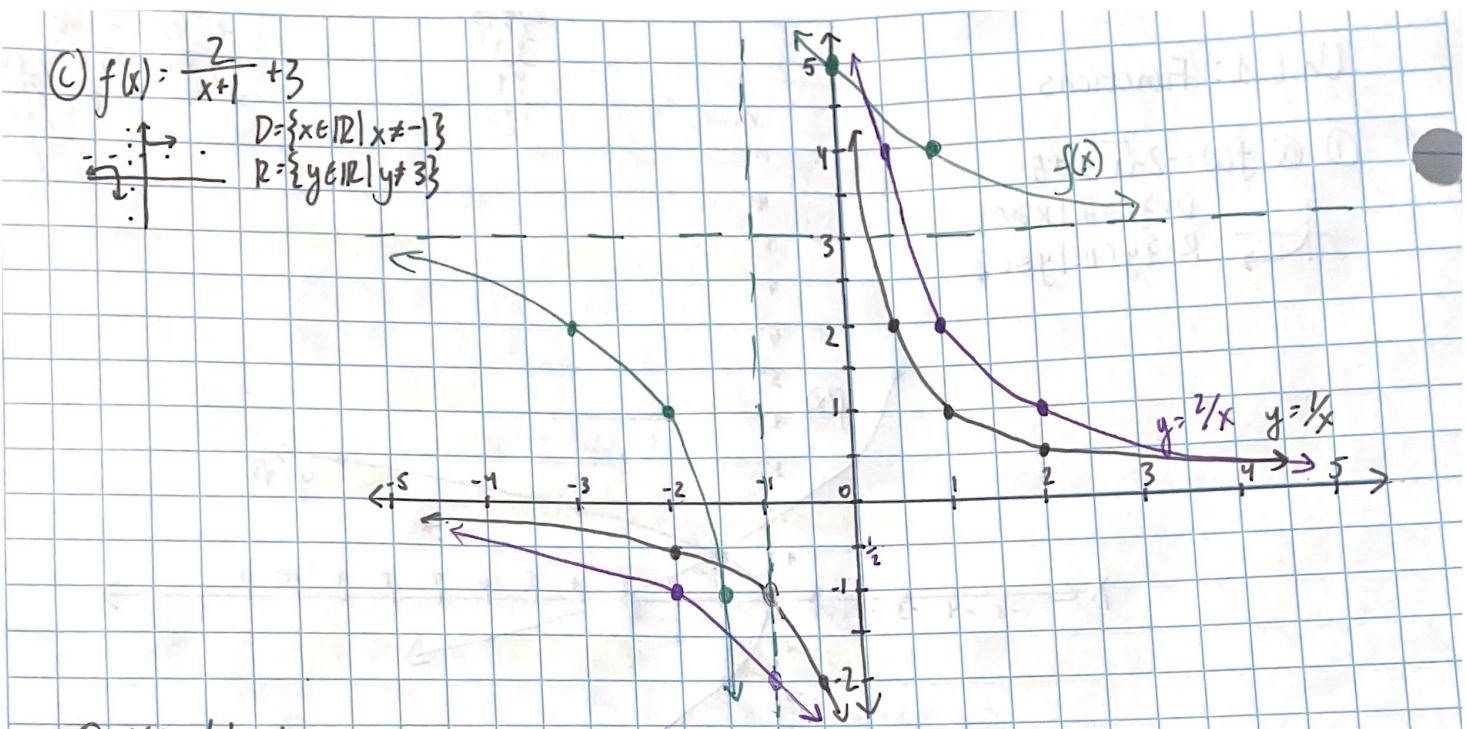
$$\begin{array}{r} 225 \\ 145 \\ -145 \\ \hline 180 \\ 270 \\ -20 \\ \hline 70 \\ -10 \\ \hline 60 \end{array}$$

$$\textcircled{1} \quad f(x) = \frac{2}{x+1} + 3$$



$$D = \{x \in \mathbb{R} \mid x \neq -1\}$$

$$R = \{y \in \mathbb{R} \mid y \neq 3\}$$



Unit 2: Algebraic Expressions

$$\textcircled{1} \quad 2(a-4b) - 3(2a-b) \quad \textcircled{6} \quad 4m(4m^2 - 5mn) - 6n(m^2 - 8n) \quad \textcircled{12} \quad (3x-7)(2x+3) \quad \textcircled{18} \quad (2x+1)(2x-1) - (2x-5)^2$$

$$= 2a - 8b - 6a + 3b = 36m^3 - 20m^2n - 6m^2n + 48n^3 = 6x^2 + x - 12 = 4x^2 - 1 - 4x^2 - 20x + 25$$

$$= -4a - 5b = 36m^2 - 26m^2n + 48n^3 = -20x + 24$$

$$\textcircled{2} \quad 15 + 5(y-20) = 3(y-1) \quad \textcircled{16} \quad \frac{3}{x} - \frac{3}{3} = \frac{1}{2} \quad \textcircled{17} \quad \frac{3}{x-2} - \frac{2}{x-4} = 0$$

$$5y - 85 = 3y - 3 \quad 18 - 4x = 3x \quad 3(x-4) - 2(x-2) = 0$$

$$y = 41 \quad x = 18/7 \quad x = 8$$

$$\textcircled{3} \quad 20x - 15 \quad \textcircled{16} \quad 14x^3 + 6x^2y - 2x \quad \textcircled{17} \quad 2m(a-b) + c(a-b) \quad \textcircled{18} \quad 3x - 9 + 2mx - 6m \quad \textcircled{19} \quad 5xy + 20x - 7x^2y - 28x^2$$

$$= 5(4x-3) \quad = 2x(7x^2 + 3xy - 2) \quad = (a-b)(2m+c) \quad = 3(x-3) + 2m(x-3) \quad = 5x(y+4) - 7x^2(y+4)$$

$$= (x-3)(3+2m) \quad \cancel{x}(y+4)(5x-7x^2)$$

$$\textcircled{20} \quad m^2 - 13m + 42 \quad \textcircled{21} \quad c^2 + 5c - 36 \quad \textcircled{22} \quad 12x^2 - 5x - 72 \quad \textcircled{23} \quad 5x^2 - 17x + 6 \quad \textcircled{24} \quad 6x^2 - 23x + 21$$

$$= (m-7)(m-6) \quad = (c+9)(c-4) \quad = (3x-8)(4x+9) \quad = (5x-2)(x-3) \quad = (3x-7)(2x-3)$$

$$\textcircled{25} \quad 2a^2 - 17a + 21 \quad \textcircled{26} \quad 4x^2 + 36x + 81 \quad \textcircled{27} \quad 25x^2 - 60xk + 36k^2 \quad \textcircled{28} \quad 9x^2 - 144y^2 \quad \textcircled{29} \quad x^4 - 81$$

$$= (2a-3)(a-7) \quad = (2x+9)^2 \quad = (5x+6k)^2 \quad = 9(x+4y)(x-4y) \quad = (x^2+9)(x+3)(x-3)$$

$$\textcircled{30} \quad 16 - (x-3)^2 \quad \textcircled{31} \quad 10x^2 - 25x + 10 \quad \textcircled{32} \quad 8x^2 - 2$$

$$= (4+x-3)(4-x+3) \quad = 5(2x^2 - 5 + 2) \quad = 2(4x^2 - 1)$$

$$= (1+x)(7-x) \quad = 5(2x-1)(x-2) \quad = 2(2x+1)(2x-1)$$

$$\textcircled{33} \quad \frac{6x^2y - 4xy^2 + 10x^2y^2}{2x^2y - 2xy} \quad \textcircled{34} \quad \frac{2-x}{x^2-9} = \frac{-\frac{1}{x-3}}{\frac{(x+3)(x-3)}{(x+3)(x-3)}} \quad \textcircled{35} \quad \frac{x+2}{x^2-3x-10} = \frac{\frac{1}{x+2}}{\frac{(x-5)(x+2)}{(x-5)(x+2)}}$$

$$= \frac{2x(y-2y+5xy)}{-2xy} \quad \cancel{x \neq 0 \text{ (h.)}} \quad \checkmark = \frac{-1}{x+3} \quad \checkmark = \frac{1}{x-5}$$

$$\textcircled{36} \quad -(3x-2y+5xy) \quad \textcircled{37} \quad \cancel{x \neq -3 \text{ (as.)}, x \neq 3 \text{ (h.)}} \quad \textcircled{38} \quad \cancel{x \neq 5 \text{ (as.)}, x \neq -2 \text{ (h.)}}$$

$$\textcircled{39} \quad \frac{2x^2 - 5x + 3}{2x^2 - 2x} \cdot \frac{2x^2 + 2x}{4x - 9} \quad \textcircled{40} \quad \frac{2n^2 - n - 1}{n^2 + 10n + 25} \div \frac{2n^2 - 3n - 2}{n^2 - 3n - 10}$$

$$= \frac{(2x+1)(x-3)}{2x(x-1)} \cdot \frac{2x(x+1)}{(2x+3)(2x-3)} \quad = \frac{(2n+1)(n-1)}{(n+5)(n+5)} \cdot \frac{(n+2)(n-2)}{(2n+1)(2n-1)}$$

$$= \frac{(2x+1)(x-3)(x+1)}{(x-1)(2x+3)(2x-3)} \quad \cancel{x \neq 0 \text{ (h.)}} \quad \cancel{x \neq -5 \text{ (as.)}, n \neq 2 \text{ (h.)}} \quad x \neq 0 \text{ (h.)} \quad = \frac{10}{3} - \frac{12}{5}$$

$$= \frac{(2x+1)(x-3)(x+1)}{(x-1)(2x+3)(2x-3)} \quad \cancel{x \neq 1, x \neq -3, x \neq 2} \quad = \frac{\sqrt{n-1}}{n+5} \quad \cancel{x \neq -5 \text{ (as.)}, n \neq 2 \text{ (h.)}} \quad = 50 - 36 = 14$$

$$\textcircled{41} \quad \frac{4}{x-1} + \frac{3x}{5} = \frac{20 + 3x^2 - 3x}{5(x-1)} \quad \textcircled{42} \quad \frac{2}{x^2 - 2x - 8} - \frac{2}{x^2 - 16} - \frac{3}{(x-4)(x+2)} \quad \textcircled{43} \quad \frac{x+2}{x-1} - \frac{x+3}{x-5} = \frac{(x+2)(x-5) - (x+3)(x-1)}{(x-1)(x-5)}$$

$$\checkmark x \neq 1 \text{ (as.)} \quad \checkmark \frac{3x^2 - 3x + 20}{5(x-1)} \quad = \frac{2(x+4) - 3(x+2)}{(x+2)(x+4)(x-2)} \quad x \neq 1 \text{ (as.)} \quad = \frac{x^2 - 8x - 10 - x^2 + 2x - 3}{(x-1)(x-5)}$$

$$x \neq 2 \text{ (as.)} \quad = \frac{(x+2)}{(x+2)(x+4)(x-2)} \quad x \neq 5 \text{ (as.)} \quad = \frac{-6x - 13}{(x-1)(x-5)}$$

$$x \neq 2 \text{ (h.)} \quad = -\frac{1}{(x+4)(x-2)}$$

$$\textcircled{44} \quad \frac{2x-5y}{xy} \quad \text{for } x=2 \text{ and } y=-4$$

$$2(2) - 5(-4)$$

$$(2)(-4)$$

$$= \frac{24}{-8}$$

$$\checkmark -3$$

Unit 3: Quadratic Functions

$$\begin{array}{llll}
 \textcircled{1} \quad \textcircled{a} -2\sqrt{80} & \textcircled{b} 3\sqrt{9} + \sqrt{27} - 2\sqrt{5} - \sqrt{9} & \textcircled{c} -2\sqrt{3}(3\sqrt{6} - 2\sqrt{2}) & \textcircled{d} (5\sqrt{2} + \sqrt{6})(5\sqrt{2} - \sqrt{6}) \\
 = -8\sqrt{5} & = 9\sqrt{6} + 3\sqrt{3} - 10\sqrt{3} - 4\sqrt{6} & = -6\sqrt{18} + 4\sqrt{6} & = 25(2) - 6 \\
 & = 5\sqrt{6} - 7\sqrt{3} & = -18\sqrt{2} + 4\sqrt{6} & = 44 \\
 \textcircled{e} \frac{-14\sqrt{12}}{2\sqrt{3}} & \textcircled{f} \frac{3\sqrt{6} - 2\sqrt{5}}{\sqrt{2}} & \textcircled{g} \frac{3\sqrt{2}}{5 - \sqrt{3}} & \\
 = \frac{-14\sqrt{3}}{\sqrt{3}} & = 3\sqrt{3} - \sqrt{10} & = \frac{15\sqrt{2} + 3\sqrt{6}}{22} & \\
 = -14 & \text{RATIONALIZE} & &
 \end{array}$$

$$\begin{array}{lll}
 \textcircled{2} \quad \textcircled{a} y = x^2 - 6x + 4 & \textcircled{b} y = 2x^2 - 4x + 3 & \textcircled{c} y = -4x^2 + 6x - 2 \\
 y = (x-3)^2 - 5 & y = 2(x-1)^2 + 1 & y = -4(x-2)^2 + 14 \\
 \textcircled{d} y = -\frac{1}{2}x^2 - 5x + 5 & \textcircled{e} y = 2x^2 - 3x + 1 & \textcircled{f} y = \frac{2}{3}x^2 - x - 2 \\
 y = -\frac{1}{2}(x^2 + 10x) + 5 & y = 2(x^2 - \frac{3}{2}x) + 1 & y = \frac{2}{3}(x^2 - \frac{3}{2}x) - 2 \\
 y = -\frac{1}{2}(x+5)^2 + \frac{35}{2} & y = 2(x - \frac{3}{4})^2 + \frac{1}{8} & y = \frac{2}{3}(x - \frac{3}{4})^2 - \frac{19}{8}
 \end{array}$$

$$\begin{array}{lll}
 \textcircled{3} \quad \textcircled{a} x^2 - 5x - 24 = 0 & \textcircled{b} 2m^2 + 5m - 3 = 0 & \textcircled{c} 4x^2 - 20x = 0 \\
 (x-8)(x+3) = 0 & (2m-1)(m+3) = 0 & 4x(x-5) = 0 \\
 \checkmark x = 8, -3 & \checkmark m = \frac{1}{2}, -3 & \checkmark x = 0, 5
 \end{array}$$

$$\begin{array}{lll}
 \textcircled{4} \quad \textcircled{a} 9x^2 - 12x + 2 = 0 & \textcircled{b} 4x^2 + 4x - 1 = 0 & \textcircled{c} 2b^2 + 4b - 3 = 0 \\
 x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 x = \frac{12 \pm \sqrt{144 - 4 \cdot 2}}{18} & x = \frac{-4 \pm \sqrt{16 + 16}}{8} & x = \frac{-4 \pm \sqrt{16 + 24}}{4} \\
 x = \frac{12 \pm 6\sqrt{2}}{18} & x = \frac{-4 \pm 4\sqrt{2}}{8} & x = \frac{-4 \pm \sqrt{40}}{4} \\
 x = \frac{2 \pm \sqrt{2}}{3} & x = \frac{-2 \pm 2}{2} & x = \frac{-2 \pm \sqrt{10}}{2} \\
 \checkmark x_1 = \frac{2 + \sqrt{2}}{3}, x_2 = \frac{2 - \sqrt{2}}{3} & x_1 = -5, x_2 = \frac{1}{4} & \checkmark x_1 = \frac{-2 + \sqrt{10}}{2}, x_2 = \frac{-2 - \sqrt{10}}{2}
 \end{array}$$

$$\begin{array}{lll}
 \textcircled{5} \quad \textcircled{a} 2x^2 - 7x + 1 = 0 & \textcircled{b} 3.1m^2 + 7.2m - 6.2 = 0 & \textcircled{c} 4.2k^2 + 1.3k - 2.4 \\
 x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 x = \frac{7 \pm \sqrt{49 - 8}}{4} & x = \frac{-2.2 \pm \sqrt{(2.2)^2 + 4(3.1)(-6.2)}}{2(3.1)} & x = \frac{-1.3 \pm \sqrt{1.3^2 + 4(4.2)(2.4)}}{2(4.2)} \\
 x = \frac{7 \pm \sqrt{41}}{4} & \checkmark x_1 = 1.10, x_2 = -1.81 & \checkmark x_1 = 0.62, x_2 = -0.93 \\
 \checkmark x_1 \approx 3.35, x_2 = 0.15 & &
 \end{array}$$



$$(b) @ 2x^2 + 3x + 3 = 0$$

$$D = b^2 - 4ac$$

$$D = (3)^2 - 4(2)(3)$$

$$D = -15$$

$\checkmark \because D < 0 \therefore$ no real roots

$$(b) x^2 + 2x + 1 = 0$$

$$D = b^2 - 4ac$$

$$D = (2)^2 - 4(1)(1)$$

$$D = 0$$

$\checkmark \because D = 0 \therefore$ one real root

$$(c) 3x^2 - 5x - 2 = 0$$

$$D = b^2 - 4ac$$

$$D = (-5)^2 - 4(3)(-2)$$

$$D = 49$$

$\checkmark \because D > 0 \therefore$ 2 real roots

$$(7) h = -4.9t^2 + 34.3t + 2.1$$

$$\textcircled{a} h = -4.9(t^2 - 7t) + 2.1$$

$$h = -4.9(t - \frac{7}{2})^2 + 62.125$$

$$\checkmark t = \frac{7}{2}, h = 62.125$$

$$t = 3.5$$

$\checkmark \textcircled{b}$ 3.5 seconds

$$\textcircled{c} h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$h = \frac{-34.4 \pm \sqrt{34.4^2 - 4(-4.9)(2.1)}}{2(-4.9)}$$

$$h_1 = -0.0605$$

$$h_2 = 7.0809$$

$$\textcircled{d} h = -4.9(1)^2 + 34.3(1) + 2.1$$

$$h = 31.5$$

$$\checkmark 31.5$$

$$\textcircled{e} 2.1 \text{ m}$$

\therefore The max height is 62.125m

$$h_2 = 7.0809$$

$\therefore 7.08 \text{ seconds}$

$$y = 2^x$$

Unit 4: Exponential Functions

(1) (a) $6^2 \cdot 6^{-4} = \frac{1}{36}$ (b) $\frac{5^{-2}}{5^{-4}} = 25$ (c) $\frac{1}{2^{-4}} = 16$ (d) $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$
 (e) $\left(-\frac{2}{3}\right)^0 = 1$ (f) $16^{1/2} = 4$ (g) $8^{2/3} = 4$ (h) $(-3)^{3/5} = -8$
 (i) $125^{-2/3} = \frac{1}{25}$ (j) $\left(\frac{27}{8}\right)^{1/3} = \frac{3}{2}$ (k) $\left(\frac{9}{25}\right)^{-1/2} = \frac{5}{3}$ (l) $2^{-1} + 2^{-2} = \frac{3}{4} = \frac{3}{4}$

(2) (a) $(5x^2y^3)(-2xy^2)$
 $= -10x^3y^5$
 (b) $(-3a^2b^3)^2$
 $= 9a^4b^6$
 (c) $\frac{(-2m^2n)^2}{8mn}$
 $= \frac{m^3n}{2}$
 (d) $\frac{5x^6y^{-1}}{-15x^8y^2} = \frac{x^9}{-3y^5}$
 (e) $\frac{12a^2b^3 - 9ab^2}{3ab^2} = 4ab - 3$

(3) (a) $2^x = 64$
 $x = 6$
 (b) $\frac{1}{4^x} = 32$
 $2^{-2x} = 2^5$
 $x = -5/2$
 (c) $9^{x-2} = 27$
 $3^{2x-4} = 3^3$
 $x = \frac{7}{2}$
 (d) $\frac{1}{3} = 81^x$
 $3^{-1} = 3^{4x}$
 $x = -1/4$
 (e) $7^{5x-1} = 1$
 $7^{5x} = 7$
 $x = 1/5$
 (f) $5^{-x} = \frac{1}{125}$
 $x = 3$
 (g) $2^x = \frac{1}{8}$
 $x = -3$
 (h) $6^{x-3} = 36^{x+1}$
 $x-3 = 2x-2$
 $x = -1$
 (i) $4^{4x+6} = 32^{x-3}$
 $8x+12 = 5x-15$
 $x = -9$
 (j) $2^{x+5} - 5 = 59$
 $x+5 = 6$
 $x = 1$

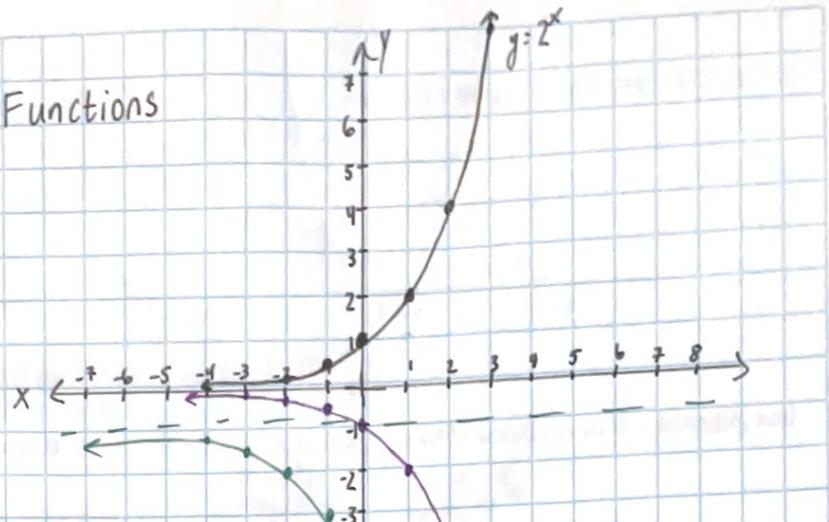
(4) go to graph paper

Unit 4: Exponential Functions

④ ① $f(x) = -2^{x+2} - 1$

$$D = \mathbb{R}$$

$$R = \{y \in \mathbb{R} \mid y < -1\}$$

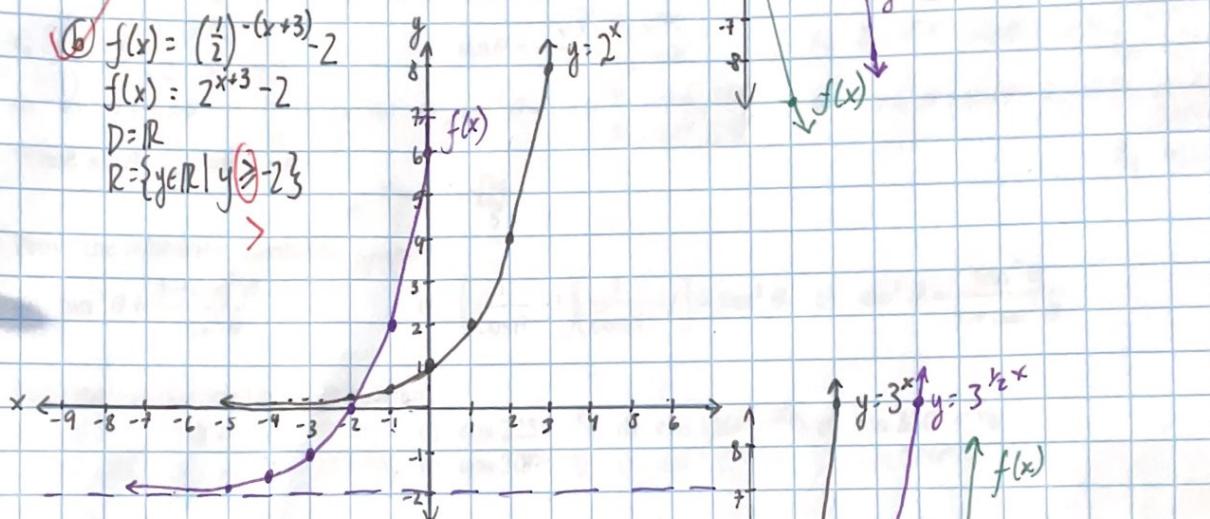


④ ② $f(x) = \left(\frac{1}{2}\right)^{-(x+3)} - 2$

$$f(x) = 2^{x+3} - 2$$

$$D = \mathbb{R}$$

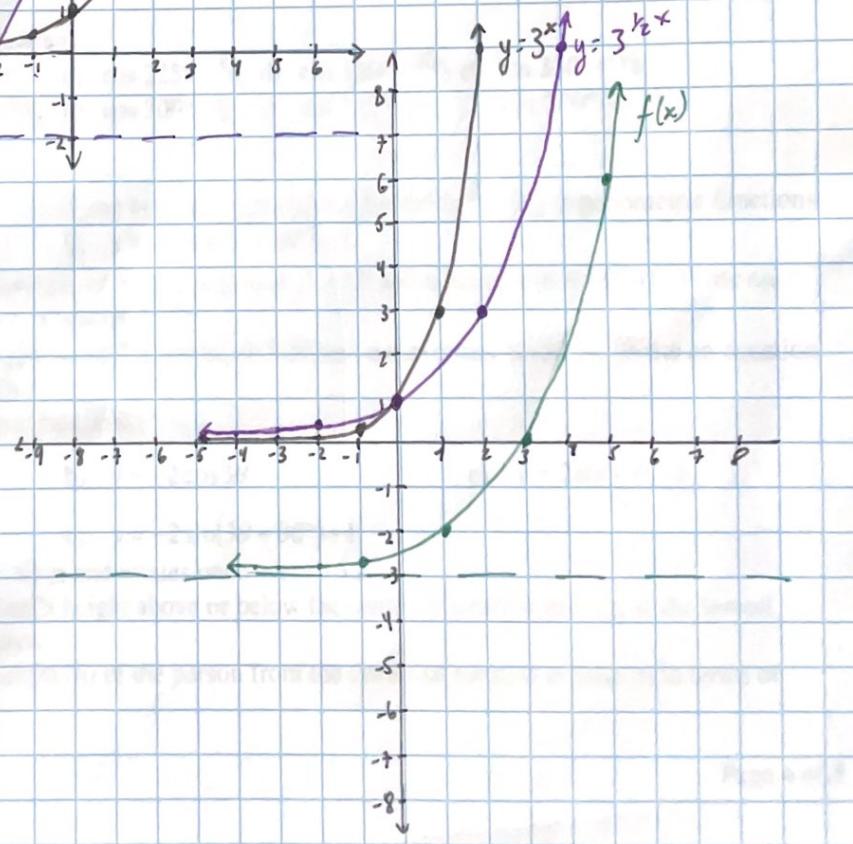
$$R = \{y \in \mathbb{R} \mid y \geq -2\}$$



④ ③ $f(x) = 3^{\frac{1}{2}(x-1)} - 3$

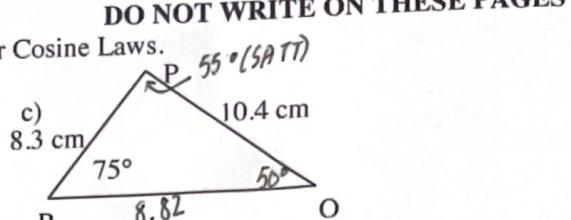
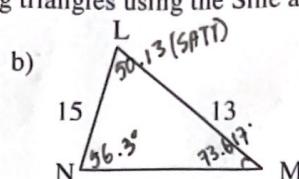
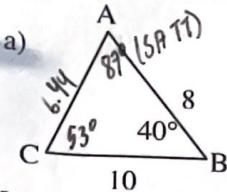
$$D = \mathbb{R}$$

$$R = \{y \in \mathbb{R} \mid y \geq -3\}$$



MCR3U1

2. Solve each of the following triangles using the Sine and/or Cosine Laws.



3. In $\triangle ABC$, $a=62.0$ cm, $\angle B=36^\circ$ and $c=56.0$ cm. Find b . $b=36.908$ cm
4. In $\triangle PQR$, $\angle Q=28^\circ$, $\angle R=116^\circ$ and $p=30.0$ cm. Find r . $r \approx 45.874$ cm
5. In $\triangle ABC$, $\angle A=35^\circ$, $a=30.0$ cm, $c=40.0$ cm. Show why there are two possible solutions to this triangle, and find both of the solutions.
6. Write 2 angles co-terminal with the following.
- a) $156^\circ, 516^\circ, 876^\circ$ b) $-75^\circ, -435^\circ, 285^\circ$
7. Determine two angles between 0° and 360° for the following.
- a) $\sin \theta=-0.573$ $\theta_1: 214.96^\circ$ $\theta_2: 325.04^\circ$ b) $\tan \theta=2.483$ $\theta_1: 68.06^\circ$ $\theta_2: 248.06^\circ$ c) $\cos \theta=-0.53$ $\theta_1: 120.05^\circ$ $\theta_2: 237.996^\circ$
8. Solve the following trigonometric equations for $0^\circ \leq \theta \leq 360^\circ$.
- a) $\cos \theta=\frac{1}{\sqrt{2}}$ $\theta_1: 45^\circ$ $\theta_2: 315^\circ$ b) $\sin \theta=-\frac{\sqrt{3}}{2}$ $\theta_3: 240^\circ$ $\theta_4: 300^\circ$ c) $2\cos^2 \theta-\cos \theta-1=0$ $\theta_1: 120, 240^\circ$ $\theta_2: 0, 360^\circ$
- d) $\sin^2 \theta+\sin \theta=0$ $\theta_1: 0, 180, 360^\circ$ e) $\tan^2 \theta-1=0$ $\theta_1: 135^\circ, 315^\circ$ f) $6\sin^2 \theta+\sin \theta-2=0$ $\theta_1: 221.81^\circ$ $\theta_2: 318.14^\circ$
9. When $\sin \theta=\frac{3}{5}$, what is $\cot \theta$? $\sqrt{\frac{4}{5}}, -\sqrt{\frac{4}{5}}$

10. Prove the following identities. (paper)

$$\text{a) } \tan^2 \theta = \frac{1-\cos^2 \theta}{\cos^2 \theta} \quad \text{b) } \left(\frac{1}{\cos \theta} - 1 \right) \left(\frac{1}{\cos \theta} + 1 \right) = \tan^2 \theta \quad \text{c) } \sin^2 \theta = \frac{\tan^2 \theta}{1+\tan^2 \theta}$$

11. State the exact value for the following.

$$\text{a) } \sin 240^\circ = -\frac{\sqrt{3}}{2} \quad \text{b) } \tan 90^\circ = \infty \quad \text{c) } \cos 225^\circ = -\frac{\sqrt{2}}{2} \quad \text{d) } \cos 150^\circ = -\frac{\sqrt{3}}{2} \quad \text{e) } \sin 330^\circ = -\frac{1}{2}$$

$$\text{f) } \sin 180^\circ = 0 \quad \text{g) } \tan 330^\circ = \frac{\sqrt{3}}{3} \quad \text{h) } \cos 300^\circ = \frac{1}{2} \quad \text{i) } \tan 135^\circ = -1 \quad \text{j) } \cos 270^\circ = 0$$

Unit 5: Trigonometric Ratios

$$\textcircled{1} \quad \textcircled{a} \quad \sin 32^\circ = \frac{4}{x} \quad \textcircled{b} \quad \cos x = \frac{2}{\sqrt{13}} \quad \textcircled{c} \quad \tan 75^\circ = \frac{7}{x} \quad \textcircled{d} \quad \tan x = \frac{10}{7}$$

$\checkmark x \approx 7.548$ $\checkmark x \approx 56.31^\circ$ $\checkmark x = 1.8756$ $\checkmark x \approx 55.008^\circ$

$$\textcircled{2} \quad \textcircled{a} \quad b^2 = a^2 + c^2 - 2ac \cos B \quad \textcircled{b} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \textcircled{c} \quad \frac{\sin B}{r} = \frac{\sin 0}{c}$$

$b^2 = 10^2 + 8^2 - 2(10)(8) \cos 40^\circ$ $15^2 = 13^2 + 12^2 - 2(13)(12) \cos M$ $\frac{\sin 75^\circ}{10.4} = \frac{\sin 0}{8.3}$

$b \approx 6.44$ $M \approx 73.617^\circ$ $\checkmark P \approx 50.433^\circ$

$\frac{\sin B}{b} = \frac{\sin C}{c}$ $13^2 = 15^2 + 12^2 - 2(15)(12) \cos N$ $\frac{P}{\sin P} = \frac{r}{\sin R}$

$\frac{\sin 40^\circ}{6.44} = \frac{\sin C}{8}$ $N \approx 56.251^\circ$ $\frac{P}{\sin P} = \frac{10.4}{\sin 28^\circ}$

$\checkmark C = 53.02^\circ$ $\checkmark P \approx 8.8197$

$$\textcircled{3} \quad \begin{array}{c} B \\ \diagdown \\ \text{triangle} \\ \textcircled{a} \quad 56^\circ \quad b \\ \textcircled{b} \quad \textcircled{c} \end{array} \quad b^2 = a^2 + c^2 - 2ac \cos B$$

$b^2 = 62^2 + 56^2 - 2(62)(56) \cos 36^\circ$

$\checkmark b \approx 36.908 \text{ cm}$

$$\textcircled{4} \quad \begin{array}{c} Q \\ \diagup \\ \text{triangle} \\ \textcircled{r} \quad 30 \text{ cm} \\ \textcircled{P} \quad 35^\circ \quad 116^\circ \quad R \end{array} \quad \frac{r}{\sin R} = \frac{P}{\sin P}$$

$\frac{r}{\sin 116^\circ} = \frac{30}{\sin 36^\circ}$

$\checkmark r \approx 45.874 \text{ cm}$

$$\textcircled{5} \quad \begin{array}{c} B \\ \diagup \\ \text{triangle} \\ \textcircled{A} \quad 40 \text{ cm} \quad 30 \text{ cm} \\ \textcircled{C} \quad 35^\circ \end{array} \quad LS = a \quad RS = c \sin A$$

$= 30 \quad = 40 \sin 35^\circ$

$= 22.943$

$\therefore LS > RS \text{ & } a < c$

$\therefore 2 \text{ sols}$

Sol 1 $\checkmark 50^\circ (\text{SATT})$ Sol 2 $\checkmark 149.886^\circ$

$\begin{array}{c} B \\ \diagup \\ \text{triangle} \\ \textcircled{A} \quad 40 \text{ cm} \quad 30 \text{ cm} \\ \textcircled{C} \quad 35^\circ \quad \textcircled{B} \end{array}$

$\frac{\sin C}{\sin C \cdot \sin A} = \frac{30}{40}$

$\therefore \theta_1 = 49.89^\circ, \theta_2 = 130.11^\circ$

$b^2 = a^2 + c^2 - 2ac \cos B$

$\frac{\sin C}{40} = \frac{\sin 35^\circ}{30}$

$\checkmark b \approx 52.095$

$\checkmark b \approx 13.427$

$\checkmark C \approx 49.89^\circ$

$$\textcircled{6} \quad \textcircled{a} \quad 156^\circ, 516^\circ, 876^\circ \quad \checkmark \textcircled{b} \quad -75^\circ, -435^\circ, 285^\circ$$

$\frac{S}{T}$	$\frac{A}{C}$
---------------	---------------

$$\textcircled{7} \quad \textcircled{a} \quad \sin \theta = -0.573 \quad \textcircled{b} \quad \tan \theta = 2.183 \quad \textcircled{c} \quad \cos \theta = -0.53$$

$\therefore \angle \theta \therefore Q3, Q4$

$\sin \beta = 0.573$

$\beta = 34.96^\circ$

$\tan \beta = 2.183$

$\beta \approx 68.063^\circ$

$\cos \beta = 0.53$

$\beta = 57.995^\circ$

$\checkmark Q_3: \theta_3 = 214.95961^\circ$

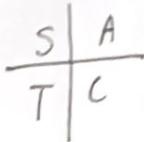
$\checkmark Q_4: \theta_4 = 325.0403^\circ$

$\checkmark \theta_1 = 68.063^\circ$

$\checkmark \theta_2 = 122.005^\circ$

$\checkmark \theta_3 = 248.063^\circ$

$\checkmark \theta_4 = 337.995^\circ$



$$\textcircled{8} \textcircled{a} \cos\theta = \frac{1}{\sqrt{2}}$$

$\because \cos\theta > 0 \therefore Q1, Q4$

Q1:

$\theta = 45^\circ$

(Special)

$\theta = 315^\circ$

(Special Δ)

$\theta = 360^\circ - 45^\circ$

$= 315^\circ$

$$\textcircled{6} \sin\theta = -\frac{\sqrt{3}}{2}$$

$\therefore \sin\theta < 0 \therefore Q3, Q4$

Q3:

$\theta = 240^\circ$

(Special)

$\theta = 300^\circ$

(Special Δ)

$\theta = 240^\circ$

$\theta = 300^\circ$

$$\textcircled{c} 2\cos^2\theta - \cos\theta - 1 = 0$$

$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120^\circ, 240^\circ$$

$$\theta = 0^\circ, 360^\circ$$

$$\theta = 0^\circ, 360^\circ$$

$$\theta = 120^\circ, 240^\circ$$

$$\theta = 0^\circ, 360^\circ$$

$$\textcircled{d} \sin^2\theta + \sin\theta - 1 = 0$$

$$\sin\theta(\sin\theta + 1) = 0$$

$$\sin\theta = 0$$

$$\sin\theta = -1$$

$$\theta = 0^\circ, 180^\circ, 360^\circ$$

$$\theta = 270^\circ$$

$$\textcircled{e} \tan^2\theta - 1 = 0$$

$$\tan^2\theta = 1$$

$$\tan\theta = \pm 1$$

$$\tan\theta = -1$$

$$\tan\theta = 1$$

$$\theta = 135^\circ, 315^\circ$$

$$\theta = 45^\circ, 225^\circ$$

$$\textcircled{f} 6\sin^2\theta + \sin\theta - 2 = 0$$

$$(3\sin\theta + 2)(2\sin\theta - 1) = 0$$

$$\sin\theta = -\frac{2}{3}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 41.81^\circ$$

$$\theta = 30^\circ (\text{I.S. f.t.)}$$

$$\theta = 221.81^\circ$$

$$\theta = 30^\circ, 150^\circ$$

$$318.19^\circ$$

$$\textcircled{g} \sin\theta = \frac{3}{5}$$

$$\therefore \sin\theta = \frac{3}{5} \therefore y = 5, r = 5$$

$$(\theta \text{ is in Q1 only since } r > 0)$$

$$x^2 + y^2 = 1$$

$$x^2 + (5)^2 = 1$$

$$x = \pm \sqrt{24}$$

$$x = \pm 2\sqrt{6}$$

$$\cot\theta = \frac{x}{y}$$

$$\text{in Q1:}$$

$$\cot\theta = \frac{\sqrt{24}}{5}$$

$$\cot\theta = -\frac{\sqrt{24}}{5}$$

$$\text{OK - values bad } -\frac{4}{3}, 8\frac{4}{3}$$

$$\textcircled{h} \textcircled{a} \tan^2\theta = \frac{1 - \cos^2\theta}{\cos^2\theta}$$

$$\text{LS: } \tan^2\theta \quad \text{RS: } \frac{1 - \cos^2\theta}{\cos^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\cos^2\theta}$$

$$= \frac{(1 - \cos^2\theta)}{\cos^2\theta}$$

$$\textcircled{b} \left(\frac{1}{\cos\theta} - 1 \right) \left(\frac{1}{\cos\theta} + 1 \right) = \tan^2\theta$$

$$\text{LS: } \left(\frac{1}{\cos\theta} - 1 \right) \left(\frac{1}{\cos\theta} + 1 \right) = \tan^2\theta$$

$$= \frac{1}{\cos^2\theta} - 1$$

$$= \frac{1 - \cos^2\theta}{\cos^2\theta}$$

$$\therefore \text{LS} = \text{RS} \therefore \dots$$

$$\textcircled{c} \sin^2\theta = \frac{\tan^2\theta}{1 + \tan^2\theta}$$

$$\text{RS: } \frac{\tan^2\theta}{1 + \tan^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \div \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\tan^2\theta}{1 + \tan^2\theta}$$

$$= \sin^2\theta$$

$$\textcircled{i} \textcircled{a} \sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\textcircled{b} \tan 90^\circ = \infty$$

$$\textcircled{c} \cos 225^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\textcircled{d} \cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\textcircled{e} \sin 330^\circ = -\sin 30^\circ = -\frac{1}{2}$$

$$\textcircled{f} \sin 180^\circ = 0$$

$$\textcircled{g} \tan 330^\circ = -\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\textcircled{h} \cos 300^\circ = \cos 60^\circ = \frac{1}{2}$$

$$\textcircled{i} \tan 135^\circ = -\tan 45^\circ = -1$$

$$\textcircled{j} \cos 270^\circ = 0$$

Unit 6: Sinusoidal Functions

① (a) $y = 2\cos(3(\theta - 60^\circ)) - 2$

amp: 2 ✓

h. shift: 60° right ✓

verticals: 2 down ✓

(b) $y = -3\sin(2\theta + 90^\circ) + 1$

$y = -3\sin 2(\theta + 45^\circ) + 1$

amp: 3 ✓ vertical s.: 1 up ✓

h. shift: 45° left ✓

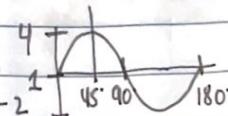
② amp: 3 $\rightarrow [a = \pm 3]$

period: $180^\circ \rightarrow [k=2]$

max: $(45^\circ, 4) \rightarrow [c=1]$

$y = \sin(\theta)$

$y = 3\sin 2\theta + 1$



③ amp: 2 $\rightarrow [a = \pm 2]$

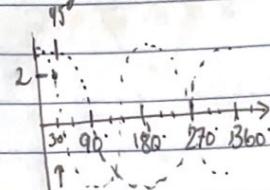
period: $360^\circ \rightarrow [k=1]$

max: $(30^\circ, 1)$

$y = \cos x$

$y = 2\cos(x + 15^\circ)$

$y = 2\cos(\theta - 30^\circ - 1)$



move it back 15° left to y

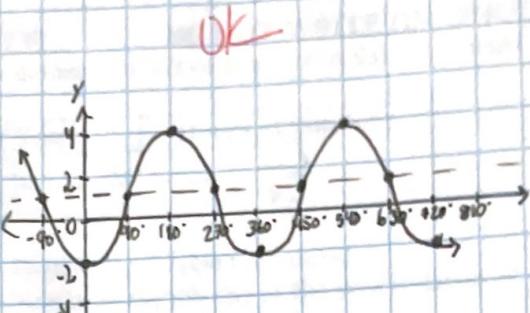
④ go to graph

⑤ go to graph

Unit 6: Sinusodial Functions

④ ② $y = 3 \sin(\theta - 90^\circ) + 1$

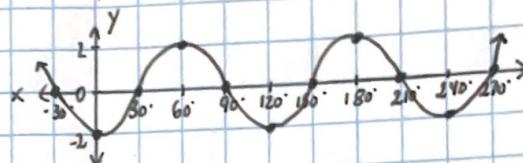
Amp: $|a| = 3$ max: 4
 axis: $y = 1$ min: -2
 period: $360^\circ/k = 360^\circ$
 shift/d: 90°
 $x_1 = 90^\circ, y_1 = 1$
 $x_2 = 180^\circ, y_2 = 4$
 $x_3 = 270^\circ, y_3 = 1$
 $x_4 = 360^\circ, y_4 = -2$
 $x_5 = 450^\circ, y_5 = 1$



OK

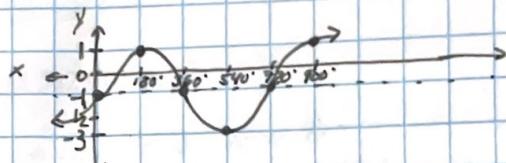
⑥ $y = -2 \cos 3\theta$

A
A Amp: 2 max: 2
 axis: $y = 0$ min: -2
 period: 120°
 d: 0°
 $x_1 = 0^\circ, y_1 = -2$
 $x_2 = 30^\circ, y_2 = 0$
 $x_3 = 60^\circ, y_3 = 2$
 $x_4 = 90^\circ, y_4 = 0$
 $x_5 = 120^\circ, y_5 = -2$



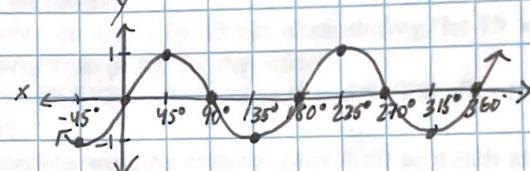
⑦ $y = 2 \sin \frac{1}{2}\theta - 1$

A
A Amp: 2 max: 1
 axis: $y = -1$ min: -3
 period: 720°
 d: 0°
 $x_1 = 0^\circ, y_1 = -1$
 $x_2 = 180^\circ, y_2 = 1$
 $x_3 = 360^\circ, y_3 = -1$
 $x_4 = 540^\circ, y_4 = 3$
 $x_5 = 720^\circ, y_5 = -1$



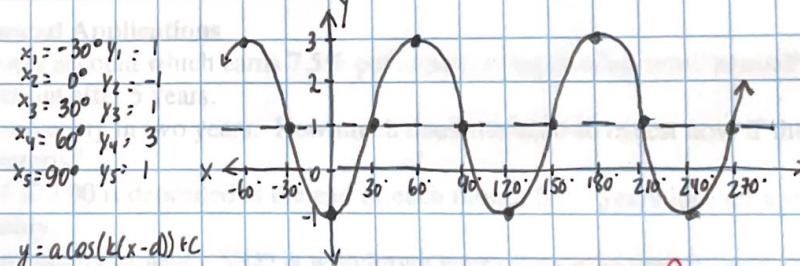
⑧ $y = \cos 2(\theta - 45^\circ)$

A
A Amp: 1 max: 1
 axis: $y = 0$ min: -1
 period: 180°
 d: 45°
 $x_1 = 45^\circ, y_1 = 1$
 $x_2 = 90^\circ, y_2 = 0$
 $x_3 = 135^\circ, y_3 = -1$
 $x_4 = 180^\circ, y_4 = 0$
 $x_5 = 225^\circ, y_5 = 1$



⑨ $y = -2 \sin(3\theta + 90^\circ) + 1$

A
A Amp: 2 max: -3
 axis: $y = 1$ min: -1
 period: 120°
 d: -30°
 $x_1 = -30^\circ, y_1 = 1$
 $x_2 = 0^\circ, y_2 = -1$
 $x_3 = 30^\circ, y_3 = 1$
 $x_4 = 60^\circ, y_4 = 3$
 $x_5 = 90^\circ, y_5 = 1$

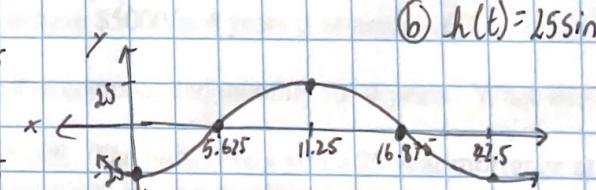


⑩ ① $y = -2 \cos \frac{1}{2}(\theta - 45^\circ)$

Diagram: A circle with radius 25 m and center at (0, 0). The angle theta is measured from the positive x-axis to the line segment connecting the center to a point on the circle in the second quadrant.

$y = a \cos(k(x-d)) + c$
 $y = -2 \cos \left(\frac{1}{2}(\theta - 45^\circ) \right)$

$x_1 = 0^\circ, y_1 = -25$
 $x_2 = 5.625^\circ, y_2 = 0$
 $x_3 = 11.25^\circ, y_3 = 25$
 $x_4 = 16.875^\circ, y_4 = 0$
 $x_5 = 22.5^\circ, y_5 = -25$



?

⑪ $h(t) = 2.5 \sin \left(\frac{\pi}{2}(t - 5.625) \right)$

Unit 7: Sequences & Series

① ② $\overbrace{1, 4, 7, 10, \dots}^{\substack{+3 \\ +3 \\ +3}}$
 $t_n = a + (n-1)d$
 $t_n = 1 + (n-1)3$
 $\checkmark t_n = -2 + 3n$

⑥ $\overbrace{4, 2, 1, \frac{1}{2}, \dots}^{\substack{+1 \\ +\frac{1}{2} \\ +\frac{1}{2}}}$
 $t_n = ar^{n-1}$
 $\checkmark t_n = 4\left(\frac{1}{2}\right)^{n-1}$

② ③ $\overbrace{2, 14, 26, \dots}^{\substack{+12 \\ +12}}$
 $t_n = a + (n-1)d$
 $t_{17} = 2 + (16)12$
 $\checkmark t_{17} = 194$

⑥ $\overbrace{4, -8, 16, \dots}^{\substack{x-2 \\ x-2}}$
 $t_n = ar^{n-1}$
 $t_{17} = 4(-2)^{16}$
 $\checkmark t_{17} = 262144$

③ ④ $\overbrace{5, 13, 21, \dots, 421}^{\substack{+8 \\ +8}}$
 $t_n = a + (n-1)d$
 $421 = 5 + (n-1)8$
 $\checkmark n = 53$

⑥ $\overbrace{3, 12, 48, \dots, 12582912}^{\substack{-4 \\ -4}}$
 $t_n = ar^{n-1}$
 $12582912 = (3)(4)^{n-1}$
 $424 = 8n$
 $\checkmark n = 12$

④ $\overbrace{2, 6, 10, \dots}^{\substack{+4 \\ +4}}$
 $S_n = \frac{n}{2}[2a + (n-1)d]$
 $S_{25} = 12[4 + 24(4)]$
 $S_{25} = 12[100]$
 $\checkmark S_{25} = 1200$
 calc err

⑥ $\overbrace{2, 4, 8, \dots}^{\substack{+2 \\ +2}}$
 $S_n = \frac{a(r^n - 1)}{r-1}$
 $S_{25} = \frac{2(2^{24} - 1)}{1}$
 $\checkmark S_{25} = 67108862$

⑤ $25000 + 200(71) = 1789200 \quad \therefore \$1789,200$

⑥ $a = 30 \quad S_n = \frac{a}{2}[2a + (n-1)d] \quad \therefore \2775
 $d = 5 \quad S_{15} = \frac{15}{2}[2(30) + 14(5)]$
 $n = 15 \quad S_{15} = 2775$

⑦ $\overbrace{800, 1200, 1800}^{\substack{+400 \\ +400}}$
 $t_n = a + (n-1)d$
 $t_{15} = 800 + 14(400)$
 $t_{15} = 325600$

⑧ $100000 \dots$
 $t_n = ar^{n-1}$
 $t_6 = 100000 \left(\frac{2}{5}\right)^5$
 $t_6 = 100000 \left(\frac{32}{3125}\right)$
 $t_6 = 1024$

$S_n = \frac{a(r^n - 1)}{r-1}$
 $S_6 = \frac{100000 \left(\left(\frac{2}{5}\right)^6 - 1\right)}{\frac{2}{5} - 1}$
 $\checkmark S_6 = 165984 \quad \therefore \$165,984$