## 02a.eigen\_face\_steps

## April 21, 2018

```
In [1]: import matplotlib.pyplot as plt
        import numpy as np
        from eigenface.yalefaces import YaleFaceDb
        np.set_printoptions(precision=2, suppress=True, formatter={'float': '{: 0.2f}'.format}
In [2]: db = YaleFaceDb()
        def gen_images():
            images = np.empty(shape=(3,4,4))
            images[0,:,:] = np.array([[1,2,3,4],[5,6,7,8],[5,6,7,8],[5,6,7,8]])
            images[1,:,:] = np.array([[2,3,4,5],[6,7,8,9],[5,6,7,8],[5,6,7,8]])
            images[2,:,:] = np.array([[0,2,4,6],[3,5,7,9],[5,6,7,8],[5,6,7,8]])
            return images
        # images = gen_images().astype(dtype=np.float)
        images = db.get_list().astype(dtype=np.float)
        labels = db.get_label()
        print(images[:,:,:,0])
        images.shape
[[[ 255.00 255.00
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                                 253.00
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                                         254.00
                                                 252.00]]
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                                  247.00
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  [ 255.00 255.00
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                                                  255.00]]]
Out[2]: (165, 100, 100, 1)
In [3]: # M training images with width NxN
        # ==> np.array with N^2 row (number of pixels), M column (number of images)
        def convert_matrix_presentation(images):
            vector2d = []
            for image in images:
                vector = image.flatten()
                vector2d.append(vector)
            return np.array(vector2d)
        def test_matrix_presentation():
            return convert matrix presentation(images)
        vector_matrix = test_matrix_presentation()
        print(vector_matrix)
        print(vector_matrix.shape)
```

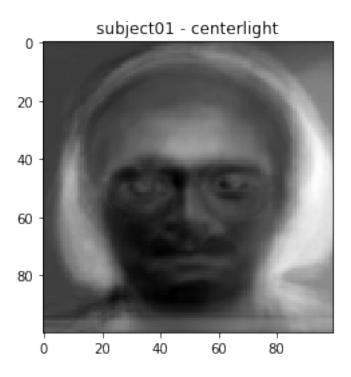
```
[[ 255.00 255.00 255.00 ..., 255.00 255.00 255.00]
 [ 255.00 255.00 255.00 ..., 255.00 254.00 252.00]
 [ 255.00 255.00 255.00 ..., 255.00 255.00 252.00]
 [ 255.00 255.00 255.00 ..., 27.00 40.00 101.00]
 [ 255.00 255.00 255.00 ..., 139.00 222.00 223.00]
 [ 255.00 255.00 255.00 ..., 255.00 255.00 255.00]]
(165, 10000)
In [4]: mean_vector = vector_matrix.mean(axis=0)
       print(mean_vector)
       print(mean_vector.shape)
[ 248.67 248.90 248.86 ..., 209.83 212.42 214.73]
(10000,)
In [5]: vector_mean_matrix = vector_matrix[:,:] - mean_vector
       print(vector_mean_matrix)
[[ 6.33  6.10  6.14 ..., 45.17  42.58  40.27]
[ 6.33 6.10 6.14 ..., 45.17 41.58 37.27]
 [ 6.33 6.10 6.14 ..., 45.17 42.58 37.27]
 [ 6.33 6.10 6.14 ..., -182.83 -172.42 -113.73]
 [ 6.33 6.10 6.14 ..., -70.83 9.58 8.27]
 [ 6.33 6.10 6.14 ..., 45.17 42.58 40.27]]
In [6]: covariance_matrix = np.matmul(vector_mean_matrix, vector_mean_matrix.T) # vector_matrix
       print(covariance_matrix.shape)
       print(covariance_matrix)
(165, 165)
[[ 29355065.79 8846414.05 9457439.14 ..., 581126.85 2089275.27 566094.10]
 [ 8846414.05 25373990.32 20047641.41 ..., -2797640.88 -3135223.47 -2588681.63]
[ 9457439.14 20047641.41 25803591.49 ..., -847127.80 72573.62 -918147.55]
 [ 581126.85 -2797640.88 -847127.80 ..., 42143862.91 32331982.33 21220256.16]
 [ 2089275.27 -3135223.47 72573.62 ..., 32331982.33 37888239.75 16034989.58]
 [ 566094.10 -2588681.63 -918147.55 ..., 21220256.16 16034989.58 32993349.42]]
In [7]: u, eigen_value, eigen_vector_vi = np.linalg.svd(covariance_matrix)
       print("u:", u.shape, "\n", u)
       print("eigen_value:\n", eigen_value)
       print("eigen_vector_vi: ", eigen_vector_vi.shape, "\n", eigen_vector_vi)
```

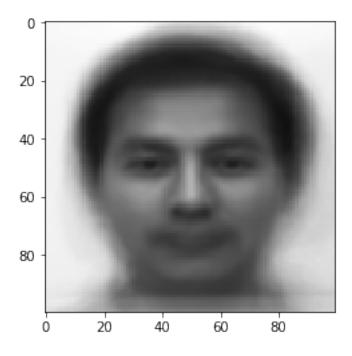
```
u: (165, 165)
 [[-0.06 -0.04 0.02 ..., 0.00 0.00 -0.08]
 [ 0.03 0.05 0.02 ..., 0.00 0.00 -0.08]
 [ 0.03 0.03 0.00 ..., 0.00 0.00 -0.08]
 . . . ,
 [ 0.04 -0.07 0.08 ..., 0.00 0.00 -0.08]
 [0.04 - 0.07 \ 0.04 \dots, \ 0.00 \ 0.00 - 0.08]
 [ 0.05 -0.09 0.11 ..., 0.00 0.00 -0.08]]
eigen value:
 [ 1074499909.05 973989965.73 693086965.82 399710027.64 386363173.15 302872688.11 209279
 5159310.30 5084672.34 4940174.67 4776120.35 4691205.32 4616503.28 4518791.45 4455352.0
  0.00]
eigen_vector_vi: (165, 165)
 [[-0.06 0.03 0.03 ..., 0.04 0.04 0.05]
 [-0.04 0.05 0.03 ..., -0.07 -0.07 -0.09]
 [ 0.02 0.02 0.00 ..., 0.08 0.04 0.11]
 [0.00 - 0.00 \ 0.00 \ \dots, -0.00 - 0.00 - 0.00]
 [ 0.00 -0.00 -0.00 ..., -0.00 -0.00 -0.00]
 [-0.08 -0.08 -0.08 ..., -0.08 -0.08 -0.08]]
In [8]: # Check Covar.x = r.x
       print('Check Eigen Properties Wx = rx:')
       print('Wx:', np.matmul(covariance matrix, eigen vector vi[0,:]))
       print('rx:', eigen_value[0] * eigen_vector_vi[0,:])
Check Eigen Properties Wx = rx:
Wx: [-62514077.58 29641461.92 33867343.71 -195682964.92 43701962.63 27569130.16 -49524581.9
  64446837.28 28256621.50 64620596.47 -141553928.33 46116786.13 22975904.66 -23609906.41
  14573830.44 43016134.71 56745566.52 70993865.57 -176335605.10 47028808.85 76074210.49
64446837.28 28256621.50 64620596.47 -141553928.33 46116786.13 22975904.66 -23609906.41
  14573830.44 43016134.71 56745566.52 70993865.57 -176335605.10 47028808.85 76074210.49
In [9]: eigen_vector_u0 = np.matmul(vector_mean_matrix.T, eigen_vector_vi[0,:])
       print(eigen_vector_u0)
[ 2.67 -1.35 -1.73 ..., 415.28 397.83 415.44]
In [10]: eigen_vector_ui = np.matmul(vector_mean_matrix.T, eigen_vector_vi[:,:].T).T
        print(eigen_vector_ui)
[[ 2.67 -1.35 -1.73 ..., 415.28 397.83 415.44]
 [ 53.18 50.94 51.45 ..., -276.83 -274.72 -262.91]
 [ 108.12  106.08  102.10  ..., -71.50  -81.29  -65.64]
```

```
[0.00 \ 0.00 \ 0.00 \ \dots, -0.00 \ -0.00 \ 0.00]
 [ 0.00 0.00 0.00 ..., -0.00 -0.00 -0.00]
 [-0.00 -0.00 -0.00 ..., -0.00 -0.00 -0.00]]
In [11]: print('Check Eigen Properties Wx = rx:')
         print('Wx:', np.matmul(np.matmul(vector_mean_matrix.T,vector_mean_matrix), eigen_vector_mean_matrix)
         print('rx:', eigen_value[0] * eigen_vector_ui[0,:])
Check Eigen Properties Wx = rx:
Wx: [ 2869581037.27 -1448751383.92 -1861773809.08 ..., 446218945567.74 427472261589.23 44638
rx: [ 2869581037.27 -1448751383.92 -1861773809.08 ..., 446218945567.74 427472261589.22 4463
In [12]: norms = np.linalg.norm(eigen_vector_ui, axis=1)
         print(norms)
[ 32779.57 31208.81 26326.54 19992.75 19656.12 17403.24 14466.50 13263.08 12870.51 12
  1675.16 1647.79 1635.35 1630.89 1618.45 1607.96 1582.94 1559.10 1541.22 1527.69 15
In [13]: norm_ui = np.divide(eigen_vector_ui.T, norms).T
         print(norm_ui)
         (norm_ui[2]*norm_ui[2]).sum()
[[ 0.00 -0.00 -0.00 ..., 0.01 0.01 0.01]
 [0.00 \ 0.00 \ 0.00 \ \dots, -0.01 \ -0.01 \ -0.01]
 [ 0.00 0.00 0.00 ..., -0.00 -0.00 -0.00]
 [0.01 \ 0.01 \ 0.01 \dots, -0.00 \ -0.00 \ 0.00]
 [0.00 \ 0.00 \ 0.00 \ \dots, -0.01 \ -0.00 \ -0.01]
 [-0.02 -0.01 -0.01 ..., -0.00 -0.01 -0.01]]
Out[13]: 1.0000000000000036
In [14]: eigen_faces = norm_ui.reshape(images.shape)
         print("Eigen Faces:\n", eigen_faces[:,:,:,0])
Eigen Faces:
 [[[ 0.00 -0.00 -0.00 ..., 0.00 0.00 0.00]
  [ 0.00 0.00 0.00 ..., 0.00 0.00 0.00]
  [0.00 \ 0.00 \ 0.00 \ \dots, \ 0.00 \ 0.00 \ 0.00]
  [ 0.00 0.00 0.00 ..., 0.02 0.02 0.02]
  [ 0.00 0.00 0.00 ..., 0.01 0.01 0.01]
  [ 0.01 0.01 0.01 ..., 0.01 0.01 0.01]]
```

[[ 0.00 0.00 0.00 ..., 0.00 0.00 0.00]

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[0.00 \ 0.00 \ 0.00 \ \dots, \ 0.00 \ 0.00 \ 0.00]
  [ 0.00 0.00 0.00 ..., 0.00 0.00 0.00]
  [ 0.00 0.00 0.00 ..., -0.01 -0.01 -0.00]
  [ 0.00
          0.00 0.00 ..., -0.01 -0.01 -0.01]
          0.00 0.00 ..., -0.01 -0.01 -0.01]]
  [ 0.00
 [[0.00 \ 0.00 \ 0.00 \ ..., -0.00 \ -0.00 \ -0.00]
  [0.01 \ 0.00 \ 0.00 \ \dots, -0.00 \ -0.00 \ -0.00]
  [0.01 \ 0.01 \ 0.01 \ \dots, -0.00 \ -0.00 \ -0.00]
  . . . ,
  [ 0.01 0.01 0.01 ..., -0.01 -0.01 -0.00]
  [0.01 \ 0.01 \ 0.01 \ \dots, -0.00 \ -0.00 \ -0.00]
  [ 0.01 0.01 0.01 ..., -0.00 -0.00 -0.00]]
 [[ 0.01  0.01  0.01 ..., -0.01 -0.00 -0.00]
  [0.00 \ 0.00 \ 0.00 \ \dots, -0.00 \ -0.00 \ -0.00]
  [0.01 \ 0.01 \ 0.01 \ \dots, -0.00 \ -0.00 \ -0.00]
  [0.01 \ 0.01 \ 0.01 \ \dots, -0.02 \ -0.01 \ -0.00]
  [0.01 \ 0.00 \ -0.00 \ \dots, \ -0.00 \ -0.00 \ -0.01]
  [ 0.01 0.01 0.01 ..., -0.00 -0.00 0.00]]
 [[ 0.00 0.00 0.00 ..., 0.02 0.01 0.01]
  [ 0.00 0.00 0.00 ..., 0.01 0.01 0.01]
  [ 0.01 0.01 0.01 ...,
                            0.01 0.01 0.01]
  [-0.01 -0.01 -0.01 ..., -0.00 0.00 0.00]
  [0.01 \ 0.01 \ 0.00 \dots, -0.00 \ 0.01 \ 0.00]
  [ 0.00 0.00 0.01 ..., -0.01 -0.00 -0.01]]
 [[-0.02 -0.01 -0.01 ..., 0.00 -0.00 0.00]
  [0.00 \ 0.01 \ 0.00 \dots, \ 0.01 \ 0.00 \ 0.00]
  [ 0.00 0.00 0.00 ..., 0.01 0.00 0.00]
  [-0.01 -0.01 -0.00 ..., 0.01 0.02 -0.00]
  [-0.00 -0.00  0.00  ..., -0.01 -0.00 -0.00]
  [ 0.02 0.01 0.01 ..., -0.00 -0.01 -0.01]]]
In [15]: plt.imshow(eigen_faces[0,:,:,0].astype(dtype=np.float), cmap='gray')
         plt.title(\frac{1}{8} - \frac{1}{8}\frac{1}{(labels[0,0], labels[0,1])}
         plt.show()
```





In [ ]: