# Team notebook

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## Contents

# 1 Algorithms

## 1.1 Mo's algorithm on trees

```
void flat(vector<vector<edge>> &g, vector<int> &a,
    vector<int> &le, vector<int> &ri, vector<int> &cost,
    int node, int pi, int &ts, int w) {

  cost[node] = w;
  le[node] = ts;
  a[ts] = node;
  ts++;
  for (auto e : g[node]) {
    if (e.to == pi) continue;
    flat(g, a, le, ri, cost, e.to, node, ts, e.w);
  }
  ri[node] = ts;
  a[ts] = node;
  ts++;
}

/** Case: cost in nodes: let P = LCA(u, v), le(u) <= le(v)
 *   Case 1:  P = u
 *    In this case, our query range would be [le(u),le(v)].
 *   Case 2:  P != u
 *    In this case, our query range would be [ri(u),le(v)] +
     [le(P),le(P)].*/

// Case when the cost is in the edges.
void compute_queries(vector<vector<edge>> &g) {
  // g is undirected
  int n = g.size();
  lca_tree.init(g, 0);
  vector<int> a(2 * n), le(n), ri(n), cost(n);
  // a: nodes in the flatten array
  // le: left id of the given node
  // ri: right id of the given node
  // cost: cost of the edge from the node to the parent
  int ts = 0; // timestamp
  flat(g, a, le, ri, cost, 0, -1, ts, 0);
  int q; cin >> q;
  vector<query> queries(q);
  for (int i = 0; i < q; i++) {
    int u, v;
    cin >> u >> v;
    u--; v--;
    int lca = lca_tree.query(u, v);
    if (le[u] > le[v])
      swap(u, v);
    queries[i].id = i;
    queries[i].lca = lca;
    queries[i].u = u;
    queries[i].v = v;
    if (lca == u) {
      queries[i].a = le[u] + 1;
      queries[i].b = le[v];
    } else {
      queries[i].a = ri[u];
      queries[i].b = le[v];
    }
  }
  solve_mo(queries, a, le, cost); // this is the usal algorithm
}
```

## 1.2 mo's algorithm

```
const int MN = 5 * 100000 + 100;
const int SN = 708;
struct query {
  int a, b, id;
  query() {}
  query(int x, int y, int i) : a(x), b(y), id(i) {}
  bool operator < (const query &o) const {
    return b < o.b;
  }
};
vector<query> s[SN];
int ans[MN];
struct DS {
  void clear() {}
  void insert(int x) {}
  void erase(int x) {}
  long long query() {}
};
DS data;
int main() {
  int n, q;
```

```
  while (cin >> n >> q) {
    for (int i = 0; i < SN; ++i)
      s[i].clear();
    vector<int> a(n);
    for (auto &i : a) cin >> i;
    for (int i = 0; i < q; ++i) {
      int b, e;
      cin >> b >> e;
      b--; e--;
      s[b / SN].emplace_back(b, e, i);
    }
    for (int i = 0; i < SN; ++i) {
      if (s[i].size()) sort(s[i].begin(), s[i].end());
    }
    for (int b = 0; b < SN; ++b) {
      if (s[b].size() == 0) continue;
      int i = s[b][0].a;
      int j = s[b][0].a - 1;
      data.clear();
      for (int k = 0; k < (int)s[b].size(); ++k) {
        int L = s[b][k].a;
        int R = s[b][k].b;
        while (j < R) { j++; data.insert(a[j]); }
        while (j > R) { data.erase(a[j]); j--; }
        while (i < L) { data.erase(a[i]); i++; }
        while (i > L) { i--; data.insert(a[i]); }
        ans[s[b][k].id] = data.query();
      }
    }
    for (int i = 0; i < q; ++i) {
      cout << ans[i] << endl;
    }
  }
  return 0;
};
```

# 2 Data Structures

## 2.1 BIT

```
int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
  while(x <= N) {
    tree[x] += v;
    x += (x & -x);
  }
}
// get cumulative sum up to and including x
int get(int x) {
  int res = 0;
  while(x) {
    res += tree[x];
    x -= (x & -x);
  }
  return res;
}
// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
  int idx = 0, mask = N;
  while(mask && idx < N) {
    int t = idx + mask;
    if(x >= tree[t]) {
      idx = t;
      x -= tree[t];
    }
    mask >>= 1;
  }
  return idx;
}
```

## 2.2 persistent tree

```
// Persistent binary trie (BST for integers)
const int MD = 31;

struct node_bin {
  node_bin *child[2];
  int val;

  node_bin() : val(0) {
    child[0] = child[1] = NULL;
  }
```

```
};

typedef node_bin* pnode_bin;

pnode_bin copy_node(pnode_bin cur) {
  pnode_bin ans = new node_bin();
  if (cur) *ans = *cur;
  return ans;
}

pnode_bin modify(pnode_bin cur, int key, int inc, int id = MD) {
  pnode_bin ans = copy_node(cur);
  ans->val += inc;
  if (id >= 0) {
    int to = (key >> id) & 1;
    ans->child[to] = modify(ans->child[to], key, inc, id - 1);
  }
  return ans;
}

int sum_smaller(pnode_bin cur, int key, int id = MD) {
  if (cur == NULL) return 0;
  if (id < 0) return 0; // strictly smaller
  // if (id == - 1) return cur->val; // smaller or equal

  int ans = 0;
  int to = (key >> id) & 1;
  if (to) {
    if (cur->child[0]) ans += cur->child[0]->val;
    ans += sum_smaller(cur->child[1], key, id - 1);
  } else {
    ans = sum_smaller(cur->child[0], key, id - 1);
  }
  return ans;
}
```

## 2.3 seg tree

```
/**
 *  Important notes:
 *   - When using lazy propagation remembert to create new versions for
       each push_down operation!!!
 *   - remember to set left and right pointers to NULL
 * */

struct node {
  long long ans, pending;
  node *left, *right;
  node() : ans(0), left(NULL), right(NULL), pending(0) {}
};
```

## 2.4 stl order statistic

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,//key,mapped type, comparator
    rb_tree_tag,tree_order_statistics_node_update> set_t;
//find_by_order(i) devuelve iterador al i-esimo elemento
//order_of_key(k): devuelve la pos del lower bound de k
//Ej: 12, 100, 505, 1000, 10000.
//order_of_key(10) == 0, order_of_key(100) == 1,
//order_of_key(707) == 3, order_of_key(9999999) == 5
```

## 2.5 treap

```
#define null NULL

struct node {
    int x, y, size;
    long long sum;
    node *l, *r;
    node(int k) : x(k), y(rand()), size(1),
                  l(null), r(null), sum(0) { }
};

node* relax(node* p) {
    if (p) {
        p->size = 1;
        p->sum = p->x;
        if (p->l) {
            p->size += p->l->size;
            p->sum += p->l->sum;
        }
```

```
        if (p->r) {
            p->size += p->r->size;
            p->sum += p->r->sum;
        }
    }
    return p;
}

// Puts all elements <= x in l and all elements > x in r.
void split(node* t, int x, node* &l, node* &r) {
    if (t == null) l = r = null; else {
        if (t->x <= x) {
            split(t->r, x, t->r, r);
            l = relax(t);
        } else {
            split(t->l, x, l, t->l);
            r = relax(t);
        }
    }
}

node* merge(node* l, node *r) {
    if (l == null) return relax(r);
    if (r == null) return relax(l);
    if (l->y > r->y) {
        l->r = merge(l->r, r);
        return relax(l);
    } else {
        r->l = merge(l, r->l);
        return relax(r);
    }
}

node* insert(node* t, node* m) {
    if (t == null || m->y > t->y) {
        split(t, m->x, m->l, m->r);
        return relax(m);
    }
    if (m->x < t->x) t->l = insert(t->l, m);
    else t->r = insert(t->r, m);
    return relax(t);
}

node* erase(node* t, int x) {
    if (t == null) return null;
    if (t->x == x) {
        node *q = merge(t->l, t->r);
        delete t;
        return relax(q);
    } else {
        if (x < t->x) t->l = erase(t->l, x);
        else t->r = erase(t->r, x);
        return relax(t);
    }
}

// Returns any node with the given x.
node* find(node* cur, int x) {
    while (cur != null and cur->x != x) {
        if (x < cur->x) cur = cur->l;
        else cur = cur->r;
    }
    return cur;
}

node* find_kth(node* cur, int k) {
  while (cur != null and k >= 0) {
    if (cur->l && cur->l->size > k) {
      cur = cur->l;
      continue;
    }
    if (cur->l)
      k -= cur->l->size;
    if (k == 0) return cur;
    k--;
    cur = cur->r;
  }
  return cur;
}

long long sum(node* p, int x) { // find the sum of elements <= x
    if (p == null) return 0LL;
    if (p->x > x) return sum(p->l, x);
    long long ans = (p->l ? p->l->sum : 0) + p->x + sum(p->r, x);
    assert(ans >= 0);
    return ans;
}
```

## 2.6 wavelet tree

```
struct wavelet {
  vector<int> values, ori;
  vector<int> map_left, map_right;
  int l, r, m;
  wavelet *left, *right;
  wavelet() : left(NULL), right(NULL) {}
  wavelet(int a, int b, int c) : l(a), r(b), m(c), left(NULL),
      right(NULL) {}
};

wavelet *init(vector<int> &data, vector<int> &ind, int lo, int hi) {
  if (lo > hi || (data.size() == 0)) return NULL;
  int mid = ((long long)(lo) + hi) / 2;
  if (lo + 1 == hi) mid = lo; // handle negative values

  wavelet *node = new wavelet(lo, hi, mid);

  vector<int> data_l, data_r, ind_l, ind_r;
  int ls = 0, rs = 0;
  for (int i = 0; i < int(data.size()); i++) {
    int value = data[i];
    if (value <= mid) {
      data_l.emplace_back(value);
      ind_l.emplace_back(ind[i]);
      ls++;
    } else {
      data_r.emplace_back(value);
      ind_r.emplace_back(ind[i]);
      rs++;
    }
    node->map_left.emplace_back(ls);
    node->map_right.emplace_back(rs);
    node->values.emplace_back(value);
    node->ori.emplace_back(ind[i]);
  }

  if (lo < hi) {
    node->left = init(data_l, ind_l, lo, mid);
    node->right = init(data_r, ind_r, mid + 1, hi);
  }
  return node;
}

int kth(wavelet *node, int to, int k) {
  // returns the kth element in the sorted version of (a[0], ..., a[to])
  if (node->l == node->r) return node->m;
  int c = node->map_left[to];
  if (k < c)
    return kth(node->left, c - 1, k);
  return kth(node->right, node->map_right[to] - 1, k - c);
}

int pos_kth_ocurrence(wavelet *node, int val, int k) {
  // returns the position on the original array of the kth ocurrence of
      the value "val"
  if (!node) return -1;

  if (node->l == node->r) {
    if (int(node->ori.size()) <= k)
      return -1;
    return node->ori[k];
  }

  if (val <= node->m)
    return pos_kth_ocurrence(node->left, val, k);
  return pos_kth_ocurrence(node->right, val, k);
}
```

# 3 Dynamic Programming

## 3.1 convex hull trick

```
struct line {
  long long m, b;
  line (long long a, long long c) : m(a), b(c) {}
  long long eval(long long x) {
    return m * x + b;
  }
};

long double inter(line a, line b) {
  long double den = a.m - b.m;
  long double num = b.b - a.b;
  return num / den;
}
```

```
/**
 *  min m_i * x_j + b_i, for all i.
 *     x_j <= x_{j + 1}
 *     m_i >= m_{j + 1}
 * */
struct ordered_cht {
  vector<line> ch;
  int idx; // id of last "best" in query
  ordered_cht() {
    idx = 0;
  }

  void insert_line(long long m, long long b) {
    line cur(m, b);
    // new line's slope is less than all the previous
    while (ch.size() > 1 &&
       (inter(cur, ch[ch.size() - 2]) >= inter(cur, ch[ch.size() - 1]))) {
        // f(x) is better in interval [inter(ch.back(), cur), inf)
        ch.pop_back();
    }

    ch.push_back(cur);
  }

  long long eval(long long x) { // minimum
    // current x is greater than all the previous x,
    // if that is not the case we can make binary search.
    idx = min<int>(idx, ch.size() - 1);
    while (idx + 1 < (int)ch.size() && ch[idx + 1].eval(x) <=
        ch[idx].eval(x))
      idx++;

    return ch[idx].eval(x);
  }
};


/**
 *  Dynammic convex hull trick
 * */

typedef long long int64;
typedef long double float128;

const int64 is_query = -(1LL<<62), inf = 1e18;

struct Line {
  int64 m, b;
  mutable function<const Line*()> succ;
  bool operator<(const Line& rhs) const {
    if (rhs.b != is_query) return m < rhs.m;
    const Line* s = succ();
    if (!s) return 0;
    int64 x = rhs.m;
    return b - s->b < (s->m - m) * x;
  }
};

struct HullDynamic : public multiset<Line> { // will maintain upper hull
    for maximum
  bool bad(iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y->m == z->m && y->b <= z->b;
    }
    auto x = prev(y);
    if (z == end()) return y->m == x->m && y->b <= x->b;
    return (float128)(x->b - y->b)*(z->m - y->m) >= (float128)(y->b -
        z->b)*(y->m - x->m);
  }
  void insert_line(int64 m, int64 b) {
    auto y = insert({ m, b });
    y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
    if (bad(y)) { erase(y); return; }
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }

  int64 eval(int64 x) {
    auto l = *lower_bound((Line) { x, is_query });
    return l.m * x + l.b;
  }
};
```

## 3.2 divide and conquer

```
/**
 * recurrence:
 *   dp[k][i] = min dp[k-1][j] + c[i][j - 1], for all j > i;
 *
 * "comp" computes dp[k][i] for all i in O(n log n) (k is fixed)
 * */

void comp(int l, int r, int le, int re) {
  if (l > r) return;

  int mid = (l + r) >> 1;

  int best = max(mid + 1, le);
  dp[cur][mid] = dp[cur ^ 1][best] + cost(mid, best - 1);
  for (int i = best; i <= re; i++) {
    if (dp[cur][mid] > dp[cur ^ 1][i] + cost(mid, i - 1)) {
      best = i;
      dp[cur][mid] = dp[cur ^ 1][i] + cost(mid, i - 1);
    }
  }

  comp(l, mid - 1, le, best);
  comp(mid + 1, r, best, re);
}
```

## 3.3 dp on trees

```
/**
 *  for any node, save the total answer and the answer of every children.
 *  for the query(node, pi) the answer is ans[node] - partial[node][pi]
 *  cases:
 *    - all children missing
 *    - no child is missing
 *    - missing child is current pi
 * */
void add_edge(int u, int v) {
  int id_u_v = g[u].size();
  int id_v_u = g[v].size();
  g[u].emplace_back(v, id_v_u); // id of the parent in the child's list
      (g[v][id] -> u)
  g[v].emplace_back(u, id_u_v); // id of the parent in the child's list
      (g[u][id] -> v)
}
```

# 4 Geometry

## 4.1 all

```
double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y) {}
  PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)  const { return PT(x*c, y*c ); }
  PT operator / (double c)  const { return PT(x/c, y/c ); }
};

double dot(PT p, PT q)  { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  return os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p)  { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
```

```
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
    double a, double b, double c, double d) {
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
    && fabs(cross(a-b, a-c)) < EPS
    && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c,
     c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y -
        p[i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
      return true;
  return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
```

```
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}
```

## 4.2 center 2 points + radius

```
vector<point> find_center(point a, point b, long double r) {
  point d = (a - b) * 0.5;
  if (d.dot(d) > r * r) {
    return vector<point> ();
  }
  point e = b + d;
  long double fac = sqrt(r * r - d.dot(d));
  vector<point> ans;
  point x = point(-d.y, d.x);
  long double l = sqrt(x.dot(x));
  x = x * (fac / l);
  ans.push_back(e + x);
  x =  point(d.y, -d.x);
  x = x * (fac / l);
  ans.push_back(e + x);
  return ans;
```

```
}
```

## 4.3 closest pair

```
struct point {
  double x, y;
  int id;
  point() {}
  point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
  double a = p.x - o.x, b = p.y - o.y;
  return sqrt(a * a + b * b);
}

double cp(vector<point> &p, vector<point> &x, vector<point> &y) {
  if (p.size() < 4) {
    double best = 1e100;
    for (int i = 0; i < p.size(); ++i)
      for (int j = i + 1; j < p.size(); ++j)
        best = min(best, dist(p[i], p[j]));
    return best;
  }

  int ls = (p.size() + 1) >> 1;
  double l = (p[ls - 1].x + p[ls].x) * 0.5;
  vector<point> xl(ls), xr(p.size() - ls);
  unordered_set<int> left;
  for (int i = 0; i < ls; ++i) {
    xl[i] = x[i];
    left.insert(x[i].id);
  }
  for (int i = ls; i < p.size(); ++i) {
    xr[i - ls] = x[i];
  }

  vector<point> yl, yr;
  vector<point> pl, pr;
  yl.reserve(ls); yr.reserve(p.size() - ls);
  pl.reserve(ls); pr.reserve(p.size() - ls);
  for (int i = 0; i < p.size(); ++i) {
    if (left.count(y[i].id))
      yl.push_back(y[i]);
    else
      yr.push_back(y[i]);

    if (left.count(p[i].id))
      pl.push_back(p[i]);
    else
      pr.push_back(p[i]);
  }

  double dl = cp(pl, xl, yl);
  double dr = cp(pr, xr, yr);
  double d = min(dl, dr);
  vector<point> yp; yp.reserve(p.size());
  for (int i = 0; i < p.size(); ++i) {
    if (fabs(y[i].x - l) < d)
      yp.push_back(y[i]);
  }
  for (int i = 0; i < yp.size(); ++i) {
    for (int j = i + 1; j < yp.size() && j < i + 7; ++j) {
      d = min(d, dist(yp[i], yp[j]));
    }
  }
  return d;
}

double closest_pair(vector<point> &p) {
  vector<point> x(p.begin(), p.end());
  sort(x.begin(), x.end(), [](const point &a, const point &b) {
    return a.x < b.x;
  });
  vector<point> y(p.begin(), p.end());
  sort(y.begin(), y.end(), [](const point &a, const point &b) {
    return a.y < b.y;
  });
  return cp(p, x, y);
}
```

## 4.4 convex hull

```
#define REMOVE_REDUNDANT

typedef double T;
```

```
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) <
      make_pair(rhs.y,rhs.x); }
  bool operator==(const PT &rhs) const { return make_pair(y,x) ==
      make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 &&
      (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >=
        0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <=
        0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
#endif
}
```

### 4.5 triangles

Let a, b, c be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

# 5 Graphs

## 5.1 bridges

```
struct edge{
  int to, id;
  edge(int a, int b) : to(a), id(b) {}
};

struct graph {
  vector<vector<edge> > g;
  vector<int> vi, low, d, pi, is_b;

  int ticks, edges;

  graph(int n, int m) {
    g.assign(n, vector<edge>());
    is_b.assign(m, 0);
    vi.resize(n);
```

```
  low.resize(n);
  d.resize(n);
  pi.resize(n);
  edges = 0;
}

void add_edge(int u, int v) {
  g[u].push_back(edge(v, edges));
  g[v].push_back(edge(u, edges));
  edges++;
}

void dfs(int u) {
  vi[u] = true;
  d[u] = low[u] = ticks++;
  for (int i = 0; i < g[u].size(); ++i) {
    int v = g[u][i].to;
    if (v == pi[u]) continue;
    if (!vi[v]) {
      pi[v] = u;
      dfs(v);
      if (d[u] < low[v])
        is_b[g[u][i].id] = true;


      low[u] = min(low[u], low[v]);
    } else {
      low[u] = min(low[u], d[v]);
    }
  }
}

// Multiple edges from a to b are not allowed.
// (they could be detected as a bridge).
// If you need to handle this, just count
// how many edges there are from a to b.
void comp_bridges() {
  fill(pi.begin(), pi.end(), -1);
  fill(vi.begin(), vi.end(), 0);
  fill(low.begin(), low.end(), 0);
  fill(d.begin(), d.end(), 0);
  ticks = 0;
  for (int i = 0; i < g.size(); ++i)
    if (!vi[i]) dfs(i);
}
};
```

## 5.2 dinic

```
// taken from
    https://github.com/jaehyunp/stanfordacm/blob/master/code/MinCostMaxFlow.cc
typedef long long LL;

struct edge {
  int u, v;
  LL cap, flow;
  edge() {}
  edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};

struct dinic {
  int N;
  vector<edge> E;
  vector<vector<int>> g;
  vector<int> d, pt;

  dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}

  void add_edge(int u, int v, LL cap) {
    if (u != v) {
      E.emplace_back(edge(u, v, cap));
      g[u].emplace_back(E.size() - 1);
      E.emplace_back(edge(v, u, 0));
      g[v].emplace_back(E.size() - 1);
    }
  }

  bool bfs(int S, int T) {
    queue<int> q({S});
    fill(d.begin(), d.end(), N + 1);
    d[S] = 0;
    while(!q.empty()) {
      int u = q.front(); q.pop();
      if (u == T) break;
      for (int k: g[u]) {
        edge &e = E[k];
        if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
          d[e.v] = d[e.u] + 1;
```

```
        q.emplace(e.v);
      }
    }
  }
  return d[T] != N + 1;
}

LL dfs(int u, int T, LL flow = -1) {
  if (u == T || flow == 0) return flow;
  for (int &i = pt[u]; i < int(g[u].size()); ++i) {
    edge &e = E[g[u][i]];
    edge &oe = E[g[u][i]^1];
    if (d[e.v] == d[e.u] + 1) {
      LL amt = e.cap - e.flow;
      if (flow != -1 && amt > flow) amt = flow;
      if (LL pushed = dfs(e.v, T, amt)) {
        e.flow += pushed;
        oe.flow -= pushed;
        return pushed;
      }
    }
  }
  return 0;
}

LL max_flow(int S, int T) {
  LL total = 0;
  while (bfs(S, T)) {
    fill(pt.begin(), pt.end(), 0);
    while (LL flow = dfs(S, T))
      total += flow;
  }
  return total;
}
};
```

## 5.3 euler formula

Euler's formula states that if a finite, connected, planar graph is drawn in the plane without any edge intersections, and $v$ is the number of vertices, $e$ is the number of edges and $f$ is the number of faces (regions bounded by edges, including the outer, infinitely large region), then:

$$f + v = e + 2$$

It can be extended to non connected planar graphs with $c$ connected components:

$$f + v = e + c + 1$$

## 5.4 eurelian

```
struct edge;
typedef list<edge>::iterator iter;

struct edge {
  int next_vertex;
  iter reverse_edge;

  edge(int next_vertex) :next_vertex(next_vertex) {}
};

const int max_vertices = 6666;
int num_vertices;
list<edge> adj[max_vertices]; // adjacency list
vector<int> path;

void find_path(int v) {
  while(adj[v].size() > 0) {
    int vn = adj[v].front().next_vertex;
    adj[vn].erase(adj[v].front().reverse_edge);
    adj[v].pop_front();
    find_path(vn);
  }
  path.push_back(v);
}

void add_edge(int a, int b) {
  adj[a].push_front(edge(b));
  iter ita = adj[a].begin();
  adj[b].push_front(edge(a));
  iter itb = adj[b].begin();
  ita->reverse_edge = itb;
  itb->reverse_edge = ita;
}
```

## 5.5 heavy light decomposition

```
// Heavy-Light Decomposition
struct TreeDecomposition {
  vector<int> g[MAXN], c[MAXN];
  int s[MAXN]; // subtree size
  int p[MAXN]; // parent id
  int r[MAXN]; // chain root id
  int t[MAXN]; // index used in segtree/bit/...
  int d[MAXN]; // depht
  int ts;

  void dfs(int v, int f) {
    p[v] = f;
    s[v] = 1;
    if (f != -1) d[v] = d[f] + 1;
    else d[v] = 0;

    for (int i = 0; i < g[v].size(); ++i) {
      int w = g[v][i];
      if (w != f) {
        dfs(w, v);
        s[v] += s[w];
      }
    }
  }

  void hld(int v, int f, int k) {
    t[v] = ts++;
    c[k].push_back(v);
    r[v] = k;

    int x = 0, y = -1;
    for (int i = 0; i < g[v].size(); ++i) {
      int w = g[v][i];
      if (w != f) {
        if (s[w] > x) {
          x = s[w];
          y = w;
        }
      }
    }
    if (y != -1) {
      hld(y, v, k);
    }
    for (int i = 0; i < g[v].size(); ++i) {
      int w = g[v][i];
      if (w != f && w != y) {
        hld(w, v, w);
      }
    }
  }

  void init(int n) {
    for (int i = 0; i < n; ++i) {
      g[i].clear();
    }
  }

  void add(int a, int b) {
    g[a].push_back(b);
    g[b].push_back(a);
  }

  void build() {
    ts = 0;
    dfs(0, -1);
    hld(0, 0, 0);
  }
};
```

## 5.6 lca

```
void init(vector<vector<edge> > &g, int root) {
  // g is undirected
  dfs(g, root);
  int N = g.size(), i, j;

  for (i = 0; i < N; i++) {
    for (j = 0; 1 << j < N; j++) {
      P[i][j] = -1;
      MI[i][j] = inf;
    }
  }

  for (i = 0; i < N; i++) {
    P[i][0] = T[i];
```

```
    MI[i][0] = W[i];
  }

  for (j = 1; 1 << j < N; j++)
    for (i = 0; i < N; i++)
      if (P[i][j - 1] != -1) {
        P[i][j] = P[P[i][j - 1]][j - 1];
        MI[i][j] = min(MI[i][j-1], MI[ P[i][j - 1] ][j - 1]);
      }
}
```

## 5.7 max flow min cost

```
struct MCMF {
    typedef int ctype;
    enum { MAXN = 1000, INF = INT_MAX };
    struct Edge { int x, y; ctype cap, cost; };
    vector<Edge> E;    vector<int> adj[MAXN];
    int N, prev[MAXN]; ctype dist[MAXN], phi[MAXN];

    MCMF(int NN) : N(NN) {}

    void add(int x, int y, ctype cap, ctype cost) { // cost >= 0
        Edge e1={x,y,cap,cost}, e2={y,x,0,-cost};
        adj[e1.x].push_back(E.size()); E.push_back(e1);
        adj[e2.x].push_back(E.size()); E.push_back(e2);
    }

    void mcmf(int s, int t, ctype &flowVal, ctype &flowCost) {
        int x;
        flowVal = flowCost = 0; memset(phi, 0, sizeof(phi));
        while (true) {
            for (x = 0; x < N; x++) prev[x] = -1;
            for (x = 0; x < N; x++) dist[x] = INF;
            dist[s] = prev[s] = 0;

            set< pair<ctype, int> > Q;
            Q.insert(make_pair(dist[s], s));
            while (!Q.empty()) {
                x = Q.begin()->second; Q.erase(Q.begin());
                FOREACH(it, adj[x]) {
                    const Edge &e = E[*it];
                    if (e.cap <= 0) continue;
                    ctype cc = e.cost + phi[x] - phi[e.y]; // ***
                    if (dist[x] + cc < dist[e.y]) {
                        Q.erase(make_pair(dist[e.y], e.y));
                        dist[e.y] = dist[x] + cc;
                        prev[e.y] = *it;
                        Q.insert(make_pair(dist[e.y], e.y));
                    }
                }
            }
            if (prev[t] == -1) break;

            ctype z = INF;
            for (x = t; x != s; x = E[prev[x]].x)
                { z = min(z, E[prev[x]].cap); }
            for (x = t; x != s; x = E[prev[x]].x)
                { E[prev[x]].cap -= z; E[prev[x]^1].cap += z; }
            flowVal += z;
            flowCost += z * (dist[t] - phi[s] + phi[t]);
            for (x = 0; x < N; x++)
                { if (prev[x] != -1) phi[x] += dist[x]; } // ***
        }
    }
};
```

## 5.8 two sat

```
vector<int> G[MAX];
vector<int> GT[MAX];
vector<int> Ftime;
vector<vector<int> > SCC;
bool visited[MAX];
int n;


void dfs1(int n){
  visited[n] = 1;

  for (int i = 0; i < G[n].size(); ++i) {
    int curr = G[n][i];
    if (visited[curr]) continue;
    dfs1(curr);
  }
```

```
  Ftime.push_back(n);
}

void dfs2(int n, vector<int> &scc) {
  visited[n] = 1;
  scc.push_back(n);

  for (int i = 0;i < GT[n].size(); ++i) {
    int curr = GT[n][i];
    if (visited[curr]) continue;
    dfs2(curr, scc);
  }
}


void kosaraju() {
  memset(visited, 0, sizeof visited);

  for (int i = 0; i < 2 * n ; ++i) {
    if (!visited[i]) dfs1(i);
  }

  memset(visited, 0, sizeof visited);
  for (int i = Ftime.size() - 1; i >= 0; i--) {
    if (visited[Ftime[i]]) continue;
    vector<int> _scc;
    dfs2(Ftime[i],_scc);
    SCC.push_back(_scc);
  }
}

/**
 * After having the SCC, we must traverse each scc, if in one SCC are -b
     y b, there is not a solution.
 * Otherwise we build a solution, making the first "node" that we find
     truth and its complement false.
 **/


bool two_sat(vector<int> &val) {
  kosaraju();
  for (int i = 0; i < SCC.size(); ++i) {
    vector<bool> tmpvisited(2 * n, false);
    for (int j = 0; j < SCC[i].size(); ++j) {
      if (tmpvisited[SCC[i][j] ^ 1]) return 0;
      if (val[SCC[i][j]] != -1) continue;
      else {
        val[SCC[i][j]] = 0;
        val[SCC[i][j] ^ 1] = 1;
      }
      tmpvisited[SCC[i][j]] = 1;
    }
  }
  return 1;
}
```

# 6 Math

## 6.1 FFT

```
typedef long double T;

const T pi = acos(-1);

struct cpx {
   T real, image;
   cpx(T _real, T _image) {
      real = _real;
      image = _image;
   }
   cpx() {}
};

cpx operator + (const cpx &c1, const cpx &c2) {
   return cpx(c1.real + c2.real, c1.image + c2.image);
}
cpx operator - (const cpx &c1, const cpx &c2) {
   return cpx(c1.real - c2.real, c1.image - c2.image);
}
cpx operator * (const cpx &c1, const cpx &c2) {
   return cpx(c1.real * c2.real - c1.image * c2.image , c1.real
       *c2.image + c1.image * c2.real);
}

int rev(int id, int len) {
   int ret = 0;
   for (int i = 0; (1 << i) < len; i++) {
```

```
        ret <<= 1;
        if (id & (1 << i)) ret |= 1;
    }
    return ret;
}

void fft(cpx *a, int len, int dir) {
    for (int i = 0; i < len; i++) {
        A[rev(i, len)] = a[i];
    }

    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);

        cpx wm = cpx(cos(dir * 2 * pi / m), sin(dir * 2 * pi / m ));
        for (int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for (int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
    }
    if (dir == -1) for (int i = 0; i < len; i++) A[i].real /= len,
         A[i].image /= len;
    for (int i = 0; i < len; i++) a[i] = A[i];
}
```

### 6.2 fibonacci

Let A, B and n be integer numbers.

$$k = A - B \tag{1}$$

$$F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1} \tag{2}$$

$$\sum_{i=0}^{n} F_i^2 = F_{n+1} F_n \tag{3}$$

$ev(n)$ = returns 1 if $n$ is even.

$$\sum_{i=0}^{n} F_i F_{i+1} = F_{n+1}^2 - ev(n) \tag{4}$$

$$\sum_{i=0}^{n} F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1} \tag{5}$$

### 6.3 lucas

For non-negative integers $m$ and $n$ and a prime $p$, the following congruence relation holds: :

$$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} \pmod p,$$

where :

$$m = m_k p^k + m_{k-1} p^{k-1} + \cdots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1} p^{k-1} + \cdots + n_1 p + n_0$$

are the base $p$ expansions of $m$ and $n$ respectively. This uses the convention that $\binom{m}{n} = 0$ if $m \leq n$.

## 7 Number theory

### 7.1 NTT

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

/* The following vector of pairs contains pairs (prime, generator)
 * where the prime has an Nth root of unity for N being a power of two.
 * The generator is a number g s.t g^(p-1)=1 (mod p)
 * but is different from 1 for all smaller powers */
vector<PLL> nth_roots_unity {
  {1224736769,330732430},{1711276033,927759239},{167772161,167489322},
   {469762049,343261969},{754974721,643797295},{1107296257,883865065}};

PLL ext_euclid(LL a, LL b) {
  if (b == 0)
    return make_pair(1,0);
  pair<LL,LL> rc = ext_euclid(b, a % b);
  return make_pair(rc.second, rc.first - (a / b) * rc.second);
```

```
}

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
  PLL p = ext_euclid(x, modulo);
  if ( (p.first * x + p.second * modulo) != 1 )
    return -1;
  return (p.first+modulo) % modulo;
}

//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL &root_unity) {
  int n = a.size();
  LL prime = root_unity.first;
  LL basew = mod_pow(root_unity.second, (prime-1) / n, prime);
  if (dir < 0) basew = mod_inv(basew, prime);
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    LL w = 1;
    for (int i = 0; i < mh; i++) {
      for (int j = i; j < n; j += m) {
        int k = j + mh;
        LL x = (a[j] - a[k] + prime) % prime;
        a[j] = (a[j] + a[k]) % prime;
        a[k] = (w * x) % prime;
      }
      w = (w * basew) % prime;
    }
    basew = (basew * basew) % prime;
  }
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  }
}
```

## 7.2 all

```
// Discrete logarithm
// Computes x which a ^ x = b mod n.
long long d_log(long long a, long long b, long long n) {
  long long m = ceil(sqrt(n));
  long long aj = 1;
  map<long long, long long> M;
  for (int i = 0; i < m; ++i) {
    if (!M.count(aj))
      M[aj] = i;
    aj = (aj * a) % n;
  }

  long long coef = mod_pow(a, n - 2, n);
  coef = mod_pow(coef, m, n);
  // coef = a ^ (-m)
  long long gamma = b;
  for (int i = 0; i < m; ++i) {
    if (M.count(gamma)) {
      return i * m + M[gamma];
    } else {
      gamma = (gamma * coef) % n;
    }
  }
  return -1;
}

void ext_euclid(long long a, long long b, long long &x, long long &y,
    long long &g) {
  x = 0, y = 1, g = b;
  long long m, n, q, r;
  for (long long u = 1, v = 0; a != 0; g = a, a = r) {
    q = g / a, r = g % a;
    m = x - u * q, n = y - v * q;
    x = u, y = v, u = m, v = n;
  }
}

/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 * */
long long crt(vector<long long> &a, vector<long long> &x) {
  long long z = 0;
  long long n = 1;
  for (int i = 0; i < x.size(); ++i)
    n *= x[i];

  for (int i = 0; i < a.size(); ++i) {
    long long tmp = (a[i] * (n / x[i])) % n;
```

```
    tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
    z = (z + tmp) % n;
  }

  return (z + n) % n;
}
```

## 7.3 pollard rho

```
const int rounds = 20;
// checks whether a is a witness that n is not prime, 1 < a < n
bool witness(long long a, long long n) {
  // check as in Miller Rabin Primality Test described
  long long u = n - 1;
  int t = 0;
  while (u % 2 == 0) {
    t++;
    u >>= 1;
  }
  long long next = mod_pow(a, u, n);
  if (next == 1) return false;
  long long last;
  for (int i = 0; i < t; ++i) {
    last = next;
    next = mod_mul(last, last, n);
    if (next == 1) {
      return last != n - 1;
    }
  }
  return next != 1;
}
// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
bool miller_rabin(long long n, int it = rounds) {
  if (n <= 1) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  for (int i = 0; i < it; ++i) {
    long long a = rand() % (n - 1) + 1;
    if (witness(a, n)) {
      return false;
    }
  }
  return true;
}

long long pollard_rho(long long n) {
  long long x, y, i = 1, k = 2, d;
  x = y = rand() % n;
  while (1) {
    ++i;
    x = mod_mul(x, x, n);
    x += 2;
    if (x >= n) x -= n;
    if (x == y) return 1;
    d = __gcd(abs(x - y), n);
    if (d != 1) return d;
    if (i == k) {
      y = x;
      k *= 2;
    }
  }
  return 1;
}

// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
  vector<long long> ans;
  if (n == 1)
    return ans;
  if (miller_rabin(n)) {
    ans.push_back(n);
  } else {
    long long d = 1;
    while (d == 1)
      d = pollard_rho(n);
    vector<long long> dd = factorize(d);
    ans = factorize(n / d);
    for (int i = 0; i < dd.size(); ++i)
      ans.push_back(dd[i]);
  }
  return ans;
}
```

## 7.4 totient

```
for (int i = 1; i < MN; i++)
```

```
  phi[i] = i;

for (int i = 1; i < MN; i++)
  if (!sieve[i]) // is prime
    for (int j = i; j < MN; j += i)
      phi[j] -= phi[j] / i;

long long totient(long long n) {
  if (n == 1) return 0;
  long long ans = n;
  for (int i = 0; primes[i] * primes[i] <= n; ++i) {
    if ((n % primes[i]) == 0) {
      while ((n % primes[i]) == 0) n /= primes[i];
      ans -= ans / primes[i];
    }
  }
  if (n > 1) {
    ans -= ans / n;
  }
  return ans;
}
```

# 8 Strings

## 8.1 aho

```
// Max number of states in the matching machine.
// Should be equal to the sum of the length of all keywords.
const int MAXS = 6 * 50 + 10;
const int MAXC = 26;
// Bit i in this mask is on if the keyword with index i
// appears when the machine enters this state.
int out[MAXS];

int f[MAXS]; // Failure function
int g[MAXS][MAXC]; // Goto function, or -1 if fail.

int buildMatchingMachine(const vector<string> &words,
    char lowestChar = 'a', char highestChar = 'z') {
  memset(out, 0, sizeof out);
  memset(f, -1, sizeof f);
  memset(g, -1, sizeof g);

int states = 1; // Initially, we just have the 0 state

for (int i = 0; i < words.size(); ++i) {
  const string &keyword = words[i];
  int currentState = 0;
  for (int j = 0; j < keyword.size(); ++j) {
    int c = keyword[j] - lowestChar;
    if (g[currentState][c] == -1) {
      g[currentState][c] = states++;
    }
    currentState = g[currentState][c];
  }
  // There's a match of keywords[i] at node currentState.
  out[currentState] |= (1 << i);
}

// State 0 should have an outgoing edge for all characters.
for (int c = 0; c < MAXC; ++c) {
  if (g[0][c] == -1) {
    g[0][c] = 0;
  }
}

queue<int> q;
// Iterate over every possible input
for (int c = 0; c <= highestChar - lowestChar; ++c) {
  // All nodes s of depth 1 have f[s] = 0
  if (g[0][c] != -1 and g[0][c] != 0) {
    f[g[0][c]] = 0;
    q.push(g[0][c]);
  }
}
while (q.size()) {
  int state = q.front();
  q.pop();
  for (int c = 0; c <= highestChar - lowestChar; ++c) {
    if (g[state][c] != -1) {
      int failure = f[state];
      while (g[failure][c] == -1) {
        failure = f[failure];
      }
      failure = g[failure][c];
      f[g[state][c]] = failure;
```

```
      // Merge out values
      out[g[state][c]] |= out[failure];
      q.push(g[state][c]);
    }
  }
 }
 return states;
}
int findNextState(int currentState, char nextInput,
    char lowestChar = 'a') {
  int answer = currentState;
  int c = nextInput - lowestChar;
  while (g[answer][c] == -1) answer = f[answer];
  return g[answer][c];
}
```

## 8.2 kmp

```
void kmp(const string &needle, const string &haystack) {
  // Precompute border function
  int m = needle.size();
  vector<int> border(m);
  border[0] = 0;
  for (int i = 1; i < m; ++i) {
    border[i] = border[i - 1];
    while (border[i] > 0 and needle[i] != needle[border[i]]) {
       border[i] = border[border[i] - 1];
    }
    if (needle[i] == needle[border[i]]) border[i]++;
  }

  // Now the actual matching
  int n = haystack.size();
  int seen = 0;
  for (int i = 0; i < n; ++i){
    while (seen > 0 and haystack[i] != needle[seen]) {
      seen = border[seen - 1];
    }
    if (haystack[i] == needle[seen]) seen++;
    if (seen == m) {
      printf("Needle occurs from %d to %d\n", i - m + 1, i);
      seen = border[m - 1];
    }
  }
}
```

## 8.3 suffix array

```
// Complexity: O(n log n)
// * * * IMPORTANT: The last character of s must compare less
//      than any other character (for example, do s = s + '\1';
//      before calling this function).
//Output:
// sa   = The suffix array. Contains the n suffixes of s sorted
//       in lexicographical order. Each suffix is represented
//       as a single integer (the position in the string
//       where it starts).
// rank = The inverse of the suffix array. rank[i] = the index
//       of the suffix s[i..n) in the pos array. (In other
//       words, sa[i] = k <==> rank[k] = i).
//       With this array, you can compare two suffixes in O(1):
//       Suffix s[i..n) is smaller than s[j..n) if and
//       only if rank[i] < rank[j].
namespace SuffixArray {
    int t, rank[MAXN], sa[MAXN], lcp[MAXN];
    bool compare(int i, int j){
       return rank[i + t] < rank[j + t];
    }
    void build(const string &s){
       int n = s.size();
       int bc[256];
       for (int i = 0; i < 256; ++i) bc[i] = 0;
       for (int i = 0; i < n; ++i) ++bc[s[i]];
       for (int i = 1; i < 256; ++i) bc[i] += bc[i-1];
       for (int i = 0; i < n; ++i) sa[--bc[s[i]]] = i;
       for (int i = 0; i < n; ++i) rank[i] = bc[s[i]];
       for (t = 1; t < n; t <<= 1){
          for (int i = 0, j = 1; j < n; i = j++){
             while (j < n && rank[sa[j]] == rank[sa[i]]) j++;
             if (j - i == 1) continue;
             int *start = sa + i, *end = sa + j;
             sort(start, end, compare);
             int first = rank[*start + t], num = i, k;
             for(; start < end; rank[*start++] = num){
                k = rank[*start + t];
                if (k != first and (i > first or k >= j))
```

```
                    first = k, num = start - sa;
                }
            }
        }
        // Remove this part if you don't need the LCP
        int size = 0, i, j;
        for(i = 0; i < n; i++) if (rank[i] > 0) {
            j = sa[rank[i] - 1];
            while(s[i + size] == s[j + size]) ++size;
            lcp[rank[i]] = size;
            if (size > 0) --size;
        }
        lcp[0] = 0;
    }
};
// Applications:
// lcp(x,y) = min(lcp(x,x+1), lcp(x+1, x+2), ... , lcp(y-1, y))
void number_of_different_substrings(){
  // If you have the i-th smaller suffix, Si,
  //   it's length will be |Si| = n - sa[i]
  // Now, lcp[i] stores the number of
  //    common letters between Si and Si-1
  //    (s.substr(sa[i]) and s.substr(sa[i-1]))
  // so, you have |Si| - lcp[i] different strings
  //   from these two suffixes => n - lcp[i] - sa[i]
  for(int i = 0; i < n; ++i) ans += n - sa[i] - lcp[i];
}
void number_of_repeated_substrings(){
  // Number of substrings that appear at least twice in the text.
  // The trick is that all 'spare' substrings that can give us
  // Lcp(i - 1, i) can be obtained by Lcp(i - 2, i - 1)
  // due to the ordered nature of our array.
  // And the overall answer is
  // Lcp(0, 1) +
  //    Sum(max[0, Lcp(i, i - 1) - Lcp(i - 2, i - 1)])
  //    for 2 <= i < n
  // File Recover
  int cnt = lcp[1];
  for(int i=2; i < n; ++i){
    cnt += max(0, lcp[i] - lcp[i-1]);
  }
}
void repeated_m_times(int m){
  // Given a string s and an int m, find the size
  // of the biggest substring repeated m times (find the rightmost pos)
  // if a string is repeated m+1 times, then it's repeated m times too
  // The answer is the maximum, over i, of the longest common prefix
  // between suffix i+m-1 in the sorted array.
  int length = 0, position = -1, t;
  for (int i = 0; i <= n-m; ++i){
    if ((t = getLcp(i, i+m-1, n)) > length){
      length = t;
      position = sa[i];
    } else if (t == length) { position = max(position, sa[i]); }
  }
  // Here you'll get the rightmost position
  // (that means, the last time the substring appears)
  for (int i = 0; i < n; ){
    if (sa[i] + length > n) { ++i; continue; }
    int ps = 0, j = i+1;
    while (j <n && lcp[j] >= length){
      ps = max(ps, sa[j]);
      j++;
    }
    if(j - i >= m) position = max(position, ps);
    i = j;
  }
  if(length != 0)
    printf("%d %d\n", length, position);
  else
    puts("none");
}
void smallest_rotation(){
  // Reads a string of lenght k. Then just double it (s = s+s)
  // and find the suffix array.
  // The answer is the smallest i for which s.size() - sa[i] >= k
  // If you want the first appearence (and not the string)
  // you'll need the second cycle
  int best = 0;
  for (int i=0; i < n; ++i){
    if (n - sa[i] >= k){
      //Find the first appearence of the string
      while (n - sa[i] >= k){
        if(sa[i] < sa[best] && sa[i] != 0) best = i;
        i++;
      }
      break;
    }
  }
  if (sa[best] == k) puts("0");
```

```
  else printf("%d\n", sa[best]);
}
```

## 8.4 z algorithm

```
vector<int> compute_z(const string &s){
  int n = s.size();
  vector<int> z(n,0);
  int l,r;
  r = l = 0;
  for(int i = 1; i < n; ++i){
    if(i > r) {
      l = r = i;
      while(r < n and s[r - l] == s[r])r++;
      z[i] = r - l;r--;
    }else{
      int k = i-l;
      if(z[k] < r - i +1) z[i] = z[k];
      else {
        l = i;
        while(r < n and s[r - l] == s[r])r++;
        z[i] = r - l;r--;
      }
    }
  }
  return z;
}
```

# 9 X - Misc

## 9.1 equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{array}{c} ax + by = e \\ cx + dy = f \end{array} \Rightarrow \begin{array}{c} x = \dfrac{ed - bf}{ad - bc} \\ y = \dfrac{af - ec}{ad - bc} \end{array}$$

In general, given an equation $Ax = b$, the solution to a variable $x_i$ is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where $A'_i$ is $A$ with the $i$'th column replaced by $b$.

## 9.2 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where $V, W$ are lengths of sides opposite angles $v, w$.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

## 9.3 Triangles

Side lengths: $a, b, c$

Semiperimeter: $p = \dfrac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1-\left(\frac{a}{b+c}\right)^2\right]}$

Law of sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

## 9.4 Quadrilaterals

With side lengths $a, b, c, d$, diagonals $e, f$, diagonals angle $\theta$, area $A$ and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

## 9.5 Spherical coordinates

$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

## 9.6 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx}\arcsin x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}\arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}\tan x &= 1 + \tan^2 x & \frac{d}{dx}\arctan x &= \frac{1}{1+x^2} \\ \int \tan ax &= -\frac{\ln|\cos ax|}{a} & \int x\sin ax &= \frac{\sin ax - ax\cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) & \int xe^{ax}dx &= \frac{e^{ax}}{a^2}(ax-1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

## 9.7 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \cdots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \cdots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \cdots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \cdots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \end{aligned}$$

## 9.8 Series

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty) \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1) \\ \sqrt{1+x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty) \end{aligned}$$

## 9.9 Geometric series

$$r \neq 1$$

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k = a\left(\frac{1-r^n}{1-r}\right)$$