# Axtreme example case - problem description

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**Abstract**

This note describes the Axtreme example case.

## 1 Introduction

The aim is to estimate the long-term distribution of individual crest heights in a North-Sea location. The long-term distribution is a combination of the long-term description of the metocean environment in terms of average sea state parameters and the short term variability of individual crest heights in given sea states.

## 2 Long-term distribution of sea states

We let a sea state be defined by the significant wave height $H_s$ and peak wave period $T_p$.

We assume a sea state duration of $T_{ss} = 3$ hours so that the number of sea states in one year is $N_{ss} = 2922$.

For the long-term distribution of sea states we use the joint distribution of $H_s$ and $T_p$ used in [1, 2]. The joint probability density function is in the form $f_{H_s,T_p}(h_s, t_p) = f_{H_s}(h_s) f_{T_p|H_s}(t_p|h_s)$, where $f_{H_s}$ is a three-parameter Weibull distribution with location, scale and shape parameters $\alpha = 0.5026$, $\beta = 1.9536$ and $\gamma = 1.2550$, respectively. The conditional distribution of wave period, given $H_s$ is given by a lognormal distribution in the form:

$$f(t_p|h_s) = \frac{1}{t_p \sigma \sqrt{2\pi}} \exp\left(-\frac{(\ln(t_p) - \mu)^2}{2\sigma^2}\right), \tag{1}$$

where $\mu$ and $\sigma$ depend on $h_s$ through the relations

$$\mu = a_1 + a_2 h_s^{a_3}, \qquad \sigma^2 = b_1 + b_2 e^{-b_3 h_s} \tag{2}$$

The parameters are given in Table 1.

| $a_1$ | $a_2$ | $a_3$ | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|---|---|
| 1.557 | 0.403 | 0.408 | 0.005 | 0.137 | 0.454 |

Table 1: Parameters of model

Samples from this model can be obtained by the provided python function `sample_seastates(n_ss, weib_prms, lognorm_prms)`.

## 3 Short-term distribution of crest heights

In a given sea state ($H_s$, $T_p$), we assume that the distribution of wave crests be described by the Forristall distribution [3]. The Foristall distribution is a Weibull distribution where the distribution parameters are functions of sea state parameters $H_s$, mean wave period $T_{m01}$ and water depth $h$. The number of individual crests during a seastate duration $T_{ss} = 3$ hours is $N = T_{ss}/T_{m02}$, where $T_{m02}$ is the zero-crossing wave period.

For estimating $T_{m01}$ and $T_{m02}$ from $H_s$ and $T_p$ we use the relations in DNV RP-C205 [4] (section 3.5.5.4 and 3.5.5.5). The process of sampling crests for given seastate samples $H_s$ and $T_p$ can be summarized as follows:

1. Calculate JONSWAP $\gamma$ parameters using formula in RP-C205: `gamma_rpc205(Hs, Tp)`
2. Calculate $Tm01$ and $Tm02$ using RP-C205: `Tm01 = Tm01_from_Tp_gamma(Tp, gamma)` and `Tm02 = Tm02_from_Tp_gamma(Tp, gamma)`
3. Calculate the mean wavenumber corresponding to $T_{m01}$: `km01 = omega_to_k_rpc205(2*np.pi/Tm01, h)`
4. Calculate number of individual crests for each seastate: `Nw = 3600*3/Tm02`
5. Sample a maximum crest for each seastate `c_max = ForristallCrest(Hs, Tm01, km01, h).rvs_max(Nw)`

## 4 Estimating return value

After obtaining an array of $N_{ss} = 2922 \cdot N_y$ max crest heights, the $R$-year return value is estimated as the $1 - 1/(2922 \cdot R)$ quantile in the empirical distribution of $c_{max}$. For an accurate estimate of the $R$-year return value $N_y$ should be about 100-times larger than $R$.

## 5 Axtreme approach

I suggest to use a Gumbel or GEV distribution as model for the distribution of $c_{max}$ in a given seastate.

Note that in a more realistic scenario, obtaining samples from the short-term distribution (in this case: `c_max = ForristallCrest(Hs, Tm01, km01, h).rvs_max(Nw)`) will be numerically expensive. I.e. the goal should be to obtain reliable results for relatively moderate number of evaluation of this function.

It probably makes sense to use some kind of importance sampling approach to estimate the return value. The simplest approach is to just sample $T_p$ (and $c_{max}$) for $H_s$ above some limit that is well below the minimum $H_s$ that may give a crest above the return value of interest.

## References


[1] G. Lian, Ø. Hagen, and S. Haver, 'Establishing Unmanning Criteria for a Jacket Structure on the NCS', in Proceedings of the ASME 2021, Stavanger, Norway, 2021, pp. 205–214.

[2] O. Gramstad, T. B. Johannessen, and G. Lian, 'Long-term analysis of wave-induced loads using High Order Spectral Method and direct sampling of extreme wave events', Marine Structures, vol. 91, p. 103473, Sep. 2023.

[3] G. Z. Forristall, 'Wave Crest Distributions: Observations and Second-Order Theory', Journal of Physical Oceanography, vol. 30, no. 8, pp. 1931–1943, 2000.

[4] DNV, Recommended Practice DNV-RP-C205: Environmental Conditions and Environmental Loads. DNV, 2019.