1.- Find the Fourth Taylor polynomial $P_4(x)$ for the function $f(x) = xe^{x^2}$ about $x_0 = 0$.

- Find a upper bound for $f(x) - P_4(x)$, for $0 \leq x \leq 0.4$.
- Approximate $\int_0^{0.4} f(x)dx$ using $\int_0^{0.4} P_4(x)dx$
- Find an upper error bound for the error in the previous point using $\int_0^{0.4} P_4(x)dx$
- Approximate $f'(0.2)$ using $P_4'(0.2)$, and find the error

2.- Suppose two point (x_0, y_0) and $(x_1 y_1)$ are on a straight line. Two formulas are available to find the x-intercept of the line

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \quad and \quad x = x_0 - \frac{(x_1 - x_0)y_0}{y_1 - y_0} \tag{184}$$

- Show that both formulas are algebraically correct
- Use the data $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ and the three digit rounding arithmetic to compute the $x-$intercept both ways. Which method is better and why?

3.- Use the Bisection method to find solutions accurate to within 10^{-5} for the following problems

- $x - 2^{-x} = 0$ for $0 \leq x \leq 1$.
- $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$

4.- Use the fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$

5.- Let $f(x) = x^2 - 6$. With $p_0 = 3$ and $p_1 = 2$, find p_3

- Use the Secant method.
- Use the method of False Position
- Which of the two is closer to $\sqrt{6}$

6.- Use Newton's method to find solutions accurate to within 10^{-5} for the following problems.

$$e^x + 2^{-x} + 2\cos x - 6 = 0, \quad for \quad 1 \leq x \leq 2$$

$$\ln(x-1)+\cos(x-1)=0 \quad for \quad 1.3 \leq 2.$$

7.- Find the approximations to within 10^{-4} to all the real zeros of the following polynomials using Newton's method

$$f(x)=x^3-2x^2-5, \qquad and \qquad f(x)=x^3+4.001x^2+4.002x+1.101$$

Repeat the exercise using Muller's method.

8.- For the given functions $f(x)$, let $x_0=0, x_1=0.6$ and $x_2=0.9$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(0.45)$, and find the actual error.

$$f(x)=\ln(x+1), \qquad f(x)=\tan x.$$

9.- Use the cubic splines for the given value of x to approximate $f(x)$ and $f'(x)$ and calculate the actual error

$$f(x)=x\ln x, \quad approximate \quad f(8.4)\ ,\ f'(8.4)$$

with $x=[8.3,8.6]$ and $f(x)=[17.56492,18.50515]$ and

$$f(x)=x\cos x-2x^2+3x-1 \quad approximate \quad f(0.25)\ ,\ f'(0.25)$$

with $x=[0.1,0.2,0.3,0.4]$ with $f(x)=[-0.62049958,-0.28398668,0.00660095,0.24842440]$

10.- Use the forward-difference formula and the backward-difference formula to determine each missing entry

a.

x	$f(x)$	$f'(x)$
2.1	−1.709847	
2.2	−1.373823	
2.3	−1.119214	
2.4	−0.9160143	
2.5	−0.7470223	
2.6	−0.6015966	

b.

x	$f(x)$	$f'(x)$
−3.0	9.367879	
−2.8	8.233241	
−2.6	7.180350	
−2.4	6.209329	
−2.2	5.320305	
−2.0	4.513417	

The data were taken from $f(x)=\tan x$ and $f(x)=e^{x/3}+x^2$. Compute the actual errors.

11.- Approximate the following integral using the Trapezoidal, Simpson's and Midpoint rule.

$$\int_0^{0.35} \frac{2}{x^2-4}dx, \qquad \int_0^{\pi/2} e^{3x}\sin 2x dx.$$

12.- Given the function f at the following values, approximate $\int_{1.8}^{2.6} f(x)dx$.

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46675

13.- Determine the values of n and h required to approximate (within 10^{-4})

$$\int_0^2 e^{2x}\sin 3x dx$$

Use the Composite Trapezoidal, Simpson's and Midpoint rule.

14.- Use Adaptative quadrature to approximate the following integral to within 10^{-5}

$$\int_0^5 [4x\cos(2x)-(x-2)^2]dx$$

15.- Approximate the following integral using Gaussian quadrature with $n=2$, and compare your results to the exact values of the integral

$$\int_3^{3.5} \frac{x}{\sqrt{x^2-4}}dx, \qquad \int_0^1 x^2e^{-x}dx$$

16.- Approximate the following double integrals, and compare the results to the exact answers

$$\int_{2.1}^{2.5}\int_{1.2}^{1.4} xy^2dydx, \qquad \int_1^{1.5}\int_0^x (x^2+\sqrt{x})dydx.$$

17.- Approximate the following triple integral, and compare the result to the exact answer

$$\int_0^{\pi}\int_0^x\int_0^{xy} \frac{1}{y}\sin(z/y)dzdydx$$

18.- Use the Euler's method to approximate the solutions

$$y' = te^{3t} - 2y, \quad 0 \leq t \leq 1, \quad y(0) = 0, \ with \ h = 0.5. \tag{185}$$

$$y' = \cos 2t + \sin 3t, \quad 0 \leq t \leq 1, \quad y(0) = 1, \ with \ h = 0.25 \tag{186}$$

19.- Use the Taylor method of order two with $h = 0.1$ to approximate the solution to

$$y' = 1 + t\sin(ty) \quad 0 \leq t \leq 2, \quad y(0) = 0$$

20.- Use the Modified Euler, Heun's, Midpoint, Runge Kutta order two and four to approximate the solutions to each of the following initial value problem.

$$y' = y/t - (y/t)^2, \quad 1 \leq t \leq 2, \quad y(1) = 1, \ with \ h = 0.1, \quad (sol: y(t) = t/(1 + \ln t)) \tag{187}$$

$$y' = -5t + 5t^2 + 2t, \quad 0 \leq t \leq 1, \quad y(0) = 1/3, \ with \ h = 0.1, \quad (sol: y(t) = t^2 + \frac{1}{3}e^{-5t}) \tag{188}$$

21.- Use the Adams-Moulton and Adams-Bashforth methods to approximate the solutions to the following initial-value problem. In each case use starting values obtained from the Runge-Kutta method of order four.

$$y' = y/t - (y/t)^2, \quad 1 \leq t \leq 2, \quad y(1) = 1, \ with \ h = 0.1, \quad [sol: y(t) = t/(1 + \ln t)] \tag{189}$$

$$y' = -(y + 1)(y + 3), \quad 0 \leq t \leq 2, \quad y(0) = -2, \ with \ h = 0.1 \quad [sol: y(t) = -3 + 2/(1 + e^{-2t})] \tag{190}$$