TÍNH CHẤT CỦA TÍCH CHẬP

TÍNH CHẤT PHÉP TÍCH CHẬP

Continuous-time system
$x(t) * h_1(t) + x(t) * h_2(t) =$
$x(t) * \{h_1(t) + h_2(t)\}$
$\{x(t) * h_1(t)\} * h_2(t) = x(t) * \{h_1(t) * h_2(t)\}$
$h_1(t) * h_2(t) = h_2(t) * h_1(t)$

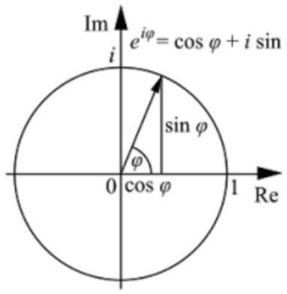
Discrete-time system
$x[n] * h_1[n] + x[n] * h_2[n] =$
$x[n] * \{h_1[n] + h_2[n]\}$
$\{x[n] * h_1[n]\} * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$
$h_1[n] * h_2[n] = h_2[n] * h_1[n]$

CÔNG THỨC EULEUR

CÔNG THỨC EULEUR - LIÊN HỆ GIỮA MŨ PHỨC VÀ LƯỢNG GIÁC

$$\boxed{x(t) = e^{j\omega t}}$$

- **Công thức Euleur:**
  $e^{i\varphi} = cos(\varphi) + i.sin(\varphi)$



- $|e^{i\varphi}| = 1$
- $e^{i0} = 1, e^{i\pi} = -1, e^{i.2\pi} = 1$
- $e^{i\pi/2} = i, e^{i.3\pi/2} = -i$
- $e^{i\varphi} = e^{i(\varphi+2\pi)}$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

MỘT SỐ CHUỖI CƠ BẢN

MỘT SỐ CHUỖI CƠ BẢN

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \dfrac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \\ N & \alpha = 1 \end{cases}$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \qquad |\alpha| < 1$$

$$\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \qquad |\alpha| < 1$$

$$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} \qquad |\alpha| < 1$$

$$\sum_{n=0}^{\infty} n^2\alpha^n = \frac{\alpha^2+\alpha}{(1-\alpha)^3} \qquad |\alpha| < 1$$

TÍNH CHẤT FOURIER LIÊN TỤC

BIỂU DIỄN TÍN HIỆU LIÊN TỤC KHÔNG TUẦN HOÀN

D. Tính chất

Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
	$x_1(t)$	$X_1(\omega)$
	$x_2(t)$	$X_2(\omega)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$
Time shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\pi X(0)\delta(\omega)+\frac{1}{j\omega}X(\omega)$
Convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$
Real signal	$x(t)=x_e(t)+x_o(t)$	$X(\omega)=A(\omega)+jB(\omega)$ $X(-\omega)=X^*(\omega)$
Even component	$x_e(t)$	$\mathrm{Re}\{X(\omega)\}=A(\omega)$
Odd component	$x_o(t)$	$j\,\mathrm{Im}\{X(\omega)\}=jB(\omega)$

Parseval's relations

$$\int_{-\infty}^{\infty} x_1(\lambda)X_2(\lambda)\,d\lambda = \int_{-\infty}^{\infty} X_1(\lambda)x_2(\lambda)\,d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega)X_2(-\omega)\,d\omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$$

FOURIER LIÊN TỤC THƯỜNG GẶP

BIỂU DIỄN TÍN HIỆU LIÊN TỤC KHÔNG TUẦN HOÀN

E. Các cặp biến đổi Fourier thông dụng

$x(t)$	$X(\omega)$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$
$u(-t)$	$\pi\delta(\omega)-\dfrac{1}{j\omega}$
$e^{-at}u(t), a>0$	$\dfrac{1}{j\omega+a}$
$te^{-at}u(t), a>0$	$\dfrac{1}{(j\omega+a)^2}$

$x(t)$	$X(\omega)$
$e^{-a\|t\|}, a>0$	$\dfrac{2a}{a^2+\omega^2}$
$\dfrac{1}{a^2+t^2}$	$e^{-a\|\omega\|}$
$e^{-at^2}, a>0$	$\sqrt{\dfrac{\pi}{a}}\,e^{-\omega^2/4a}$
$p_a(t)=\begin{cases}1 & \|t\|<a\\ 0 & \|t\|>a\end{cases}$	$2a\dfrac{\sin\omega a}{\omega a}$
$\dfrac{\sin at}{\pi t}$	$p_a(\omega)=\begin{cases}1 & \|\omega\|<a\\ 0 & \|\omega\|>a\end{cases}$
$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$
$\sum_{k=-\infty}^{\infty}\delta(t-kT)$	$\omega_0\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0), \omega_0=\dfrac{2\pi}{T}$

TÍNH CHẤT FOURIER RỜI RẠC

BIỂU DIỄN CHUỖI KHÔNG TUẦN HOÀN

D. Tính chất

Property	Sequence	Fourier transform
	$x[n]$	$X(\Omega)$
	$x_1[n]$	$X_1(\Omega)$
	$x_2[n]$	$X_2(\Omega)$
Periodicity	$x[n]$	$X(\Omega+2\pi)=X(\Omega)$
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(\Omega)+a_2X_2(\Omega)$
Time shifting	$x[n-n_0]$	$e^{-j\Omega n_0}X(\Omega)$
Frequency shifting	$e^{j\Omega_0 n}x[n]$	$X(\Omega-\Omega_0)$
Conjugation	$x^*[n]$	$X^*(-\Omega)$
Time reversal	$x[-n]$	$X(-\Omega)$
Time scaling	$x_{(m)}[n]=\begin{cases}x[n/m] & \text{if } n=km\\ 0 & \text{if } n\neq km\end{cases}$	$X(m\Omega)$
Frequency differentiation	$nx[n]$	$j\dfrac{dX(\Omega)}{d\Omega}$
First difference	$x[n]-x[n-1]$	$(1-e^{-j\Omega})X(\Omega)$
Accumulation	$\sum_{k=-\infty}^{n}x[k]$	$\pi X(0)\delta(\Omega)+\dfrac{1}{1-e^{-j\Omega}}X(\Omega)$ $\|\Omega\|\le\pi$
Convolution	$x_1[n]*x_2[n]$	$X_1(\Omega)X_2(\Omega)$
Multiplication	$x_1[n]x_2[n]$	$\dfrac{1}{2\pi}X_1(\Omega)\otimes X_2(\Omega)$
Real sequence	$x[n]=x_e[n]+x_o[n]$	$X(\Omega)=A(\Omega)+jB(\Omega)$ $X(-\Omega)=X^*(\Omega)$
Even component	$x_e[n]$	$\operatorname{Re}\{X(\Omega)\}=A(\Omega)$
Odd component	$x_o[n]$	$j\operatorname{Im}\{X(\Omega)\}=jB(\Omega)$

Parseval's relations

$$\sum_{n=-\infty}^{\infty}x_1[n]x_2[n]=\frac{1}{2\pi}\int_{2\pi}X_1(\Omega)X_2(-\Omega)\,d\Omega$$

$$\sum_{n=-\infty}^{\infty}|x[n]|^2=\frac{1}{2\pi}\int_{2\pi}|X(\Omega)|^2\,d\Omega$$

FOURIER RỜI RẠC THƯỜNG GẶP

BIỂU DIỄN CHUỖI KHÔNG TUẦN HOÀN

E. Các cặp biến đổi Fourier thông dụng

$x[n]$	$X(\Omega)$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\Omega n_0}$
$x[n]=1$	$2\pi\delta(\Omega), \lvert\Omega\rvert \le \pi$
$e^{j\Omega_0 n}$	$2\pi\delta(\Omega-\Omega_0), \lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$\cos\Omega_0 n$	$\pi[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)], \lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$\sin\Omega_0 n$	$-j\pi[\delta(\Omega-\Omega_0)-\delta(\Omega+\Omega_0)], \lvert\Omega\rvert, \lvert\Omega_0\rvert \le \pi$
$u[n]$	$\pi\delta(\Omega)+\dfrac{1}{1-e^{-j\Omega}}, \lvert\Omega\rvert \le \pi$
$-u[-n-1]$	$-\pi\delta(\Omega)+\dfrac{1}{1-e^{-j\Omega}}, \lvert\Omega\rvert \le \pi$
$a^n u[n], \lvert a\rvert < 1$	$\dfrac{1}{1-ae^{-j\Omega}}$
$-a^n u[-n-1], \lvert a\rvert > 1$	$\dfrac{1}{1-ae^{-j\Omega}}$
$(n+1)a^n u[n], \lvert a\rvert < 1$	$\dfrac{1}{(1-ae^{-j\Omega})^2}$
$a^{\lvert n\rvert}, \lvert a\rvert < 1$	$\dfrac{1-a^2}{1-2a\cos\Omega+a^2}$
$x[n]=\begin{cases}1 & \lvert n\rvert \le N_1 \\ 0 & \lvert n\rvert > N_1\end{cases}$	$\dfrac{\sin\left[\Omega\left(N_1+\frac{1}{2}\right)\right]}{\sin(\Omega/2)}$
$\dfrac{\sin Wn}{\pi n}, 0 < W < \pi$	$X(\Omega)=\begin{cases}1 & 0 \le \lvert\Omega\rvert \le W \\ 0 & W < \lvert\Omega\rvert \le \pi\end{cases}$
$\sum_{k=-\infty}^{\infty}\delta[n-kN_0]$	$\Omega_0\sum_{k=-\infty}^{\infty}\delta(\Omega-k\Omega_0), \Omega_0=\dfrac{2\pi}{N_0}$

TÍNH CHẤT LAPLACE

TÍNH CHẤT CỦA BIẾN ĐỔI LAPLACE

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$	$R' \supset R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	$R'=R$
Shifting in s	$e^{s_0t}x(t)$	$X(s-s_0)$	$R'=R+\mathrm{Re}(s_0)$
Time scaling	$x(at)$	$\dfrac{1}{\lvert a\rvert}X(s)$	$R'=aR$
Time reversal	$x(-t)$	$X(-s)$	$R'=-R$
Differentiation in t	$\dfrac{dx(t)}{dt}$	$sX(s)$	$R' \supset R$
Differentiation in s	$-tx(t)$	$\dfrac{dX(s)}{ds}$	$R'=R$
Integration	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\dfrac{1}{s}X(s)$	$R' \supset R \cap \{\mathrm{Re}(s)>0\}$
Convolution	$x_1(t)*x_2(t)$	$X_1(s)X_2(s)$	$R' \supset R_1 \cap R_2$

LAPLACE THƯỜNG GẶP

MỘT SỐ CẶP BIẾN ĐỔI LAPLACE THÔNG DỤNG

$x(t)$	$X(s)$	ROC
$\delta(t)$	1	All s
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}(s) > 0$
$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}(s) < 0$
$tu(t)$	$\frac{1}{s^2}$	$\mathrm{Re}(s) > 0$
$t^k u(t)$	$\frac{k!}{s^{k+1}}$	$\mathrm{Re}(s) > 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}(s) > -\mathrm{Re}(a)$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}(s) < -\mathrm{Re}(a)$

$x(t)$	$X(s)$	ROC
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}(s) > -\mathrm{Re}(a)$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}(s) < -\mathrm{Re}(a)$
$\cos \omega_0 t u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}(s) > 0$
$\sin \omega_0 t u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}(s) > 0$
$e^{-at}\cos \omega_0 t u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}(s) > -\mathrm{Re}(a)$
$e^{-at}\sin \omega_0 t u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}(s) > -\mathrm{Re}(a)$

TÍNH CHẤT LAPLACE 1 PHÍA

TÍNH CHẤT CỦA BIẾN ĐỔI LAPLACE MỘT PHÍA

Tương tự như biến đổi Laplace hai phía, ngoại trừ:

Phép vi phân trong miền thời gian:

$$\frac{dx(t)}{dt} \leftrightarrow sX_I(s) - x(0^-)$$

$$\frac{d^2x(t)}{dt^2} \leftrightarrow s^2X_I(s) - sx(0^-) - x'(0^-)$$

$$\frac{d^nx(t)}{dt^n} \leftrightarrow s^nX_I(s) - s^{n-1}x(0^-) - s^{n-2}x'(0^-) - \cdots - x^{(n-1)}(0^-)$$

Với: $$x^{(r)}(0^-) = \left.\frac{d^rx(t)}{dt^r}\right|_{t=0^-}$$

SO SÁNH LAPLACE 1 PHÍA VÀ 2 PHÍA

SO SÁNH GIỮA BIẾN ĐỔI MỘT PHÍA VÀ HAI PHÍA

Signal	Unilateral Transform $x(t) \xleftrightarrow{\mathcal{L}_u} X(s)$, $y(t) \xleftrightarrow{\mathcal{L}_u} Y(s)$	Bilateral Transform $x(t) \xleftrightarrow{\mathcal{L}} X(s)$, $y(t) \xleftrightarrow{\mathcal{L}} Y(s)$	ROC $s \in R_x$, $s \in R_y$
$ax(t) + by(t)$	$aX(s) + bY(s)$	$aX(s) + bY(s)$	At least $R_x \cap R_y$
$x(t - \tau)$	$e^{-s\tau}X(s)$ if $x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$	$e^{-s\tau}X(s)$	R_x
$e^{s_o t}x(t)$	$X(s - s_o)$	$X(s - s_o)$	$R_x + \text{Re}\{s_o\}$
$x(at)$	$\frac{1}{a}X\left(\frac{s}{a}\right)$, $a > 0$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$
$x(t) * y(t)$	$X(s)Y(s)$ if $x(t) = y(t) = 0$ for $t < 0$	$X(s)Y(s)$	At least $R_x \cap R_y$
$-tx(t)$	$\frac{d}{ds}X(s)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$	$sX(s)$	At least R_x
$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}\int_{-\infty}^{0^-} x(\tau)\,d\tau + \frac{X(s)}{s}$	$\frac{X(s)}{s}$	At least $R_x \cap \{\text{Re}\{s\} > 0\}$

TÍNH CHẤT Z

TÍNH CHẤT CỦA BIẾN ĐỔI Z

Property	Sequence	Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$	$R' \supset R_1 \cap R_2$
Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	$R' \supset R \cap \{0 < \lvert z\rvert < \infty\}$
Multiplication by z_0^n	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$R' = \lvert z_0\rvert R$
Multiplication by $e^{j\Omega_0 n}$	$e^{j\Omega_0 n}x[n]$	$X(e^{-j\Omega_0}z)$	$R' = R$
Time reversal	$x[-n]$	$X\left(\frac{1}{z}\right)$	$R' = \frac{1}{R}$
Multiplication by n	$nx[n]$	$-z\frac{dX(z)}{dz}$	$R' = R$
Accumulation	$\sum_{k=-\infty}^{n} x[n]$	$\frac{1}{1 - z^{-1}}X(z)$	$R' \supset R \cap \{\lvert z\rvert > 1\}$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$R' \supset R_1 \cap R_2$

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Z THƯỜNG GẶP

MỘT SỐ CẶP BIẾN ĐỔI Z THÔNG DỤNG

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	All z
$u[n]$	$\dfrac{1}{1-z^{-1}}, \dfrac{z}{z-1}$	$\|z\| > 1$
$-u[-n-1]$	$\dfrac{1}{1-z^{-1}}, \dfrac{z}{z-1}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 if $(m>0)$ or ∞ if $(m<0)$
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}, \dfrac{z}{z-a}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}, \dfrac{z}{z-a}$	$\|z < \|a\|$
$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}, \dfrac{az}{(z-a)^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}, \dfrac{az}{(z-a)^2}$	$\|z\| < \|a\|$

MỘT SỐ CẶP BIẾN ĐỔI Z THÔNG DỤNG

$x[n]$	$X(z)$	ROC
$(n+1)a^n u[n]$	$\dfrac{1}{(1-az^{-1})^2}, \left[\dfrac{z}{z-a}\right]^2$	$\|z\| > \|a\|$
$(\cos \Omega_0 n)u[n]$	$\dfrac{z^2-(\cos\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\|z\| > 1$
$(\sin \Omega_0 n)u[n]$	$\dfrac{(\sin\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$	$\|z\| > 1$
$(r^n \cos \Omega_0 n)u[n]$	$\dfrac{z^2-(r\cos\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\|z\| > r$
$(r^n \sin \Omega_0 n)u[n]$	$\dfrac{(r\sin\Omega_0)z}{z^2-(2r\cos\Omega_0)z+r^2}$	$\|z\| > r$
$\begin{cases} a^n & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$	$\dfrac{1-a^N z^{-N}}{1-az^{-1}}$	$\|z\| > 0$

SO SÁNH Z 1 PHÍA VÀ 2 PHÍA

SO SÁNH GIỮA BIẾN ĐỔI MỘT PHÍA VÀ HAI PHÍA

Signal	*Unilateral Transform* $x[n] \overset{z_u}{\longleftrightarrow} X(z)$ $y[n] \overset{z_u}{\longleftrightarrow} Y(z)$	*Bilateral Transform* $x[n] \overset{z}{\longleftrightarrow} X(z)$ $y[n] \overset{z}{\longleftrightarrow} Y(z)$	*ROC* $z \in R_x$ $z \in R_y$
$ax[n] + by[n]$	$aX(z) + bY(z)$	$aX(z) + bY(z)$	At least $R_x \cap R_y$
$x[n-k]$	See below	$z^{-k}X(z)$	R_x, except possibly $\|z\| = 0, \infty$
$\alpha^n x[n]$	$X\left(\frac{z}{\alpha}\right)$	$X\left(\frac{z}{\alpha}\right)$	$\|\alpha\| R_x$
$x[-n]$	—	$X\left(\frac{1}{z}\right)$	$\frac{1}{R_x}$
$x[n] * y[n]$	$X(z)Y(z)$ if $x[n] = y[n] = 0$ for $n < 0$	$X(z)Y(z)$	At least $R_x \cap R_y$
$nx[n]$	$-z\frac{d}{dz}X(z)$	$-z\frac{d}{dz}X(z)$	R_x, except possibly addition or deletion of $z = 0$