

Exercises: Artificial Intelligence

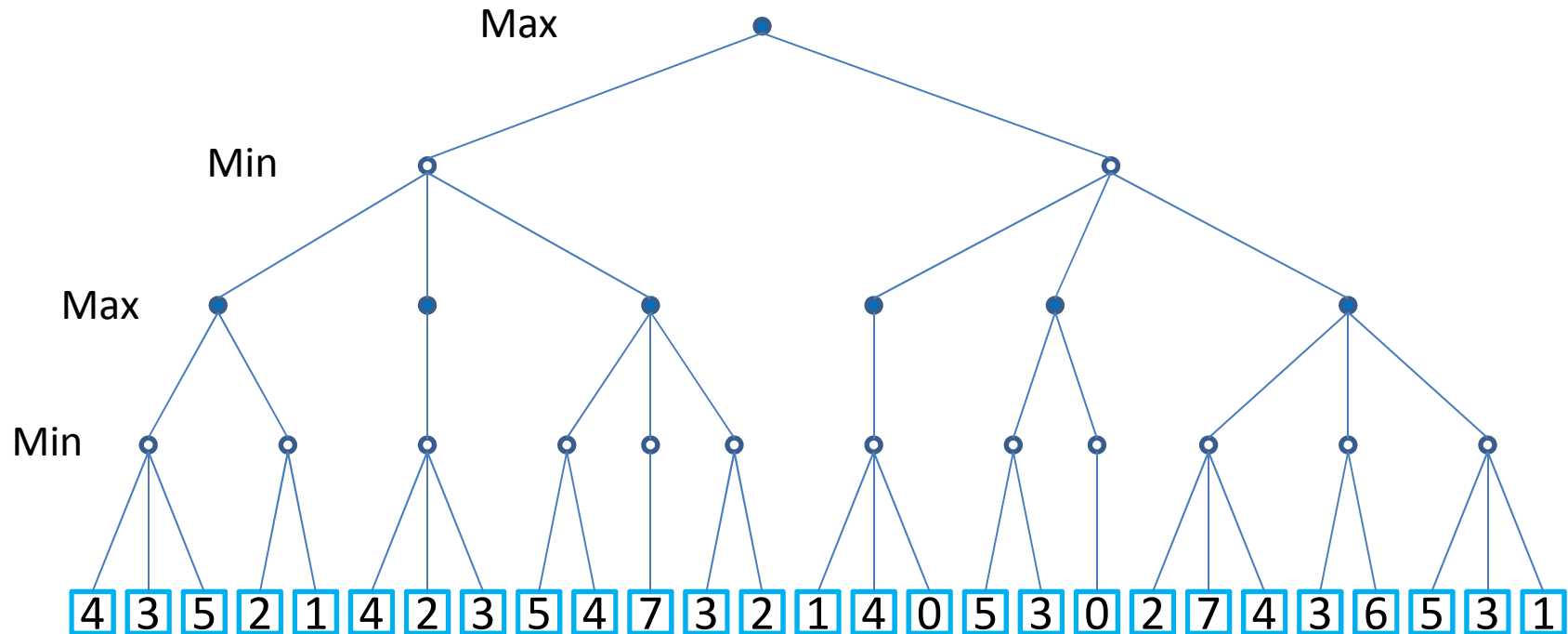
MiniMax & Constraint Processing:
MiniMax Algorithm

MiniMax & Constraint Processing: MiniMax Algorithm

PROBLEM 1

Problem 1

- Perform the minimax algorithm on the figure below. First without, later with $\alpha\beta$ -pruning.

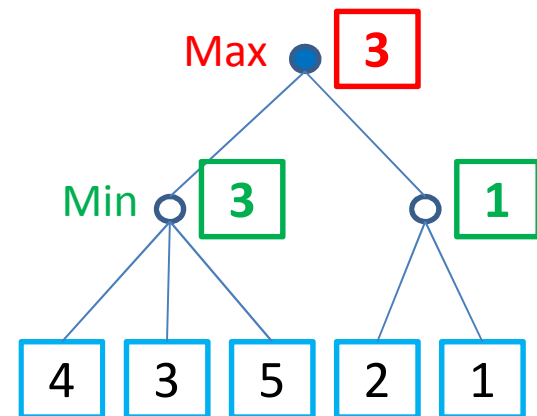


MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX ALGORITHM

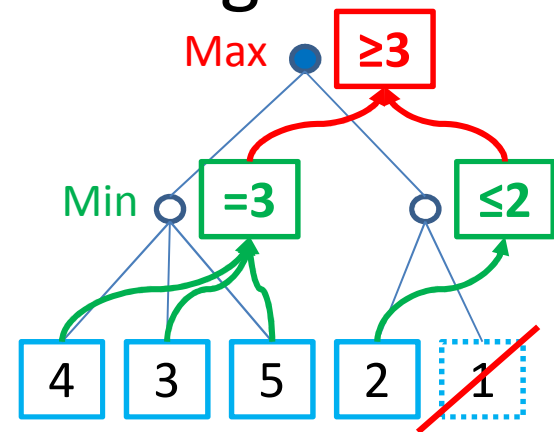
MiniMax Algorithm

- Restrictions
 - 2 players: Max = Computer & Min = Opponent
 - Deterministic, perfect information
- Depth-bound & Evaluation function
 - Construct tree (depth-bound)
 - Compute evaluation leaves
 - Propagate upwards (min/max)



$\alpha\beta$ -Pruning

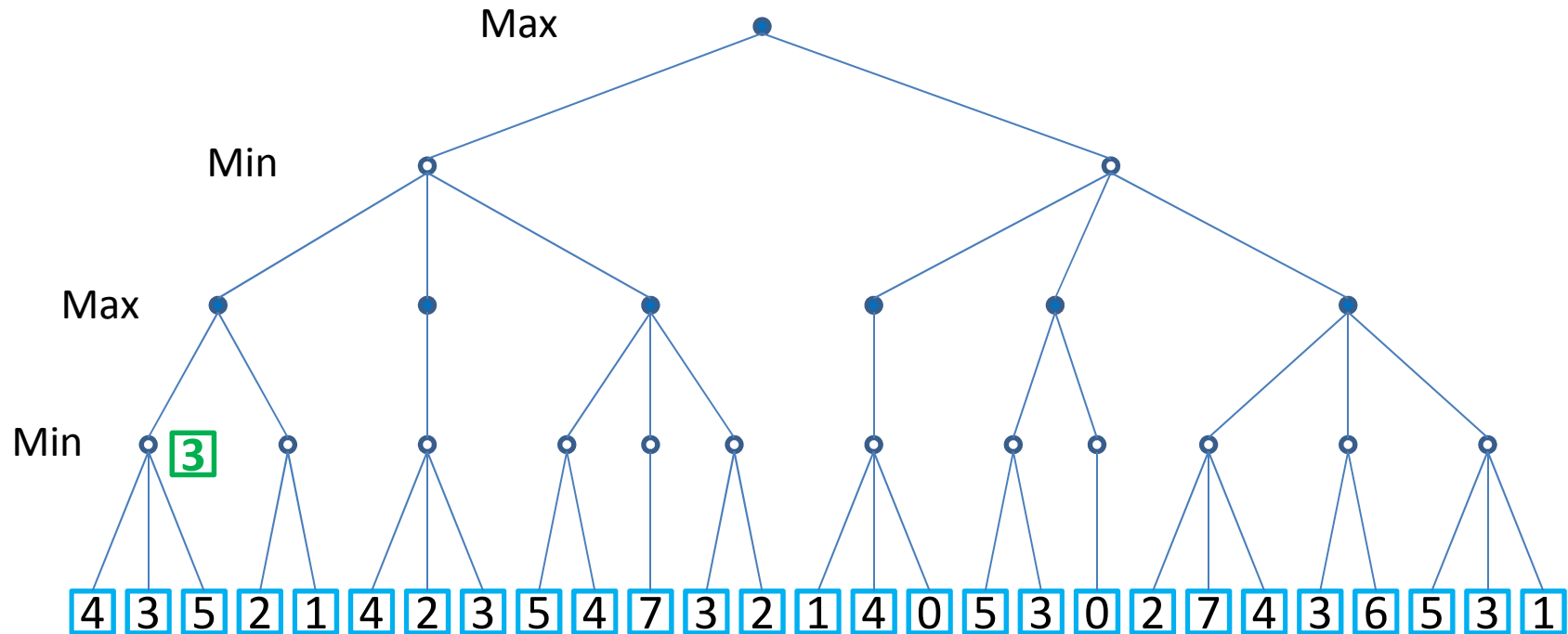
- Generally applied optimization
 - Instead of generating, then propagating
 - Interleave generation and propagation
 - Obtain information on redundant parts
- Generate tree: depth-first & Left-to-right
 - Propagate values of nodes
 - Estimates for parent nodes



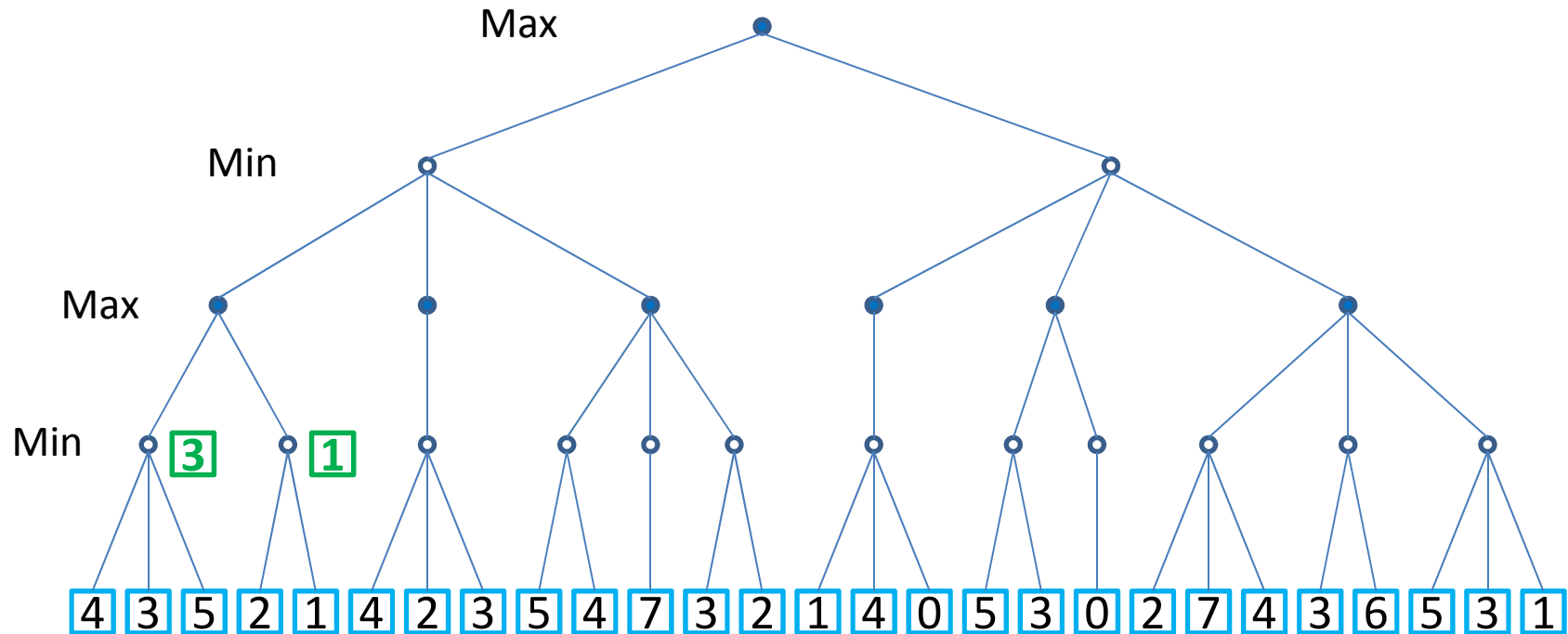
MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX WITHOUT $\alpha\beta$ -PRUNING

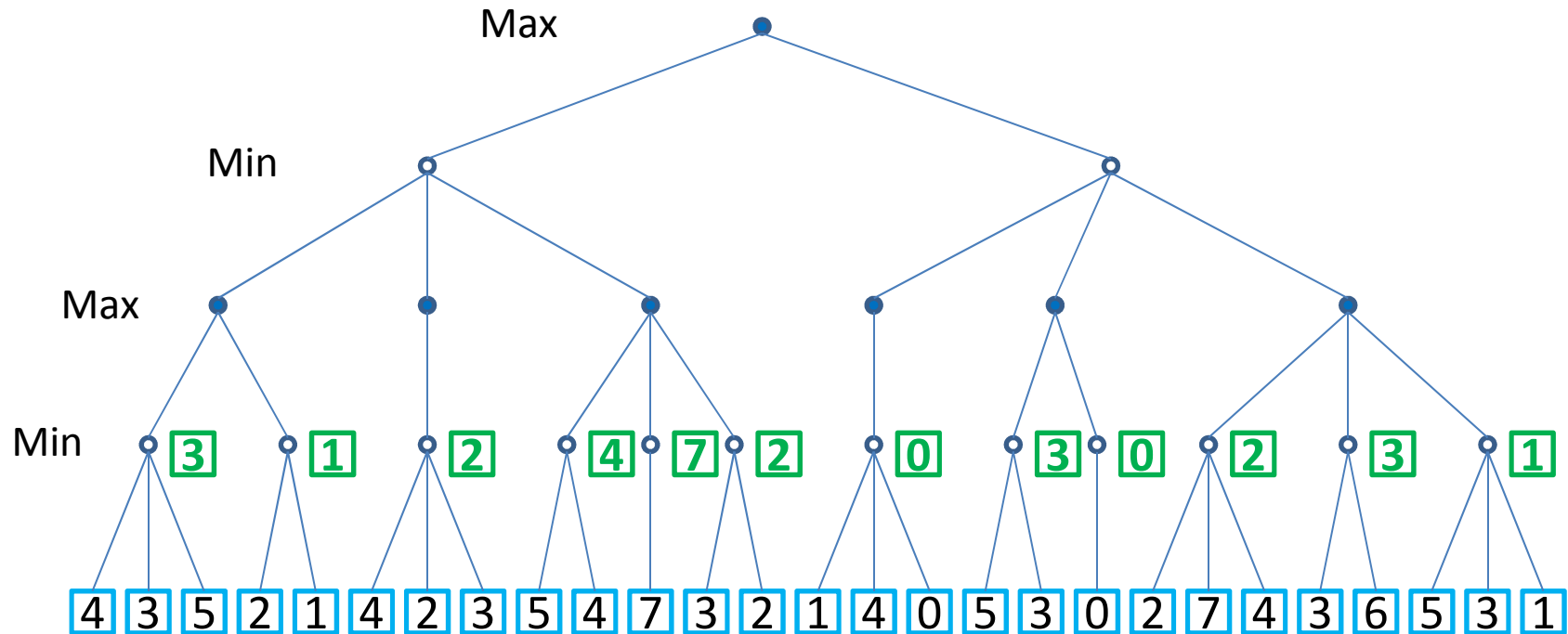
MiniMax without $\alpha\beta$ -pruning



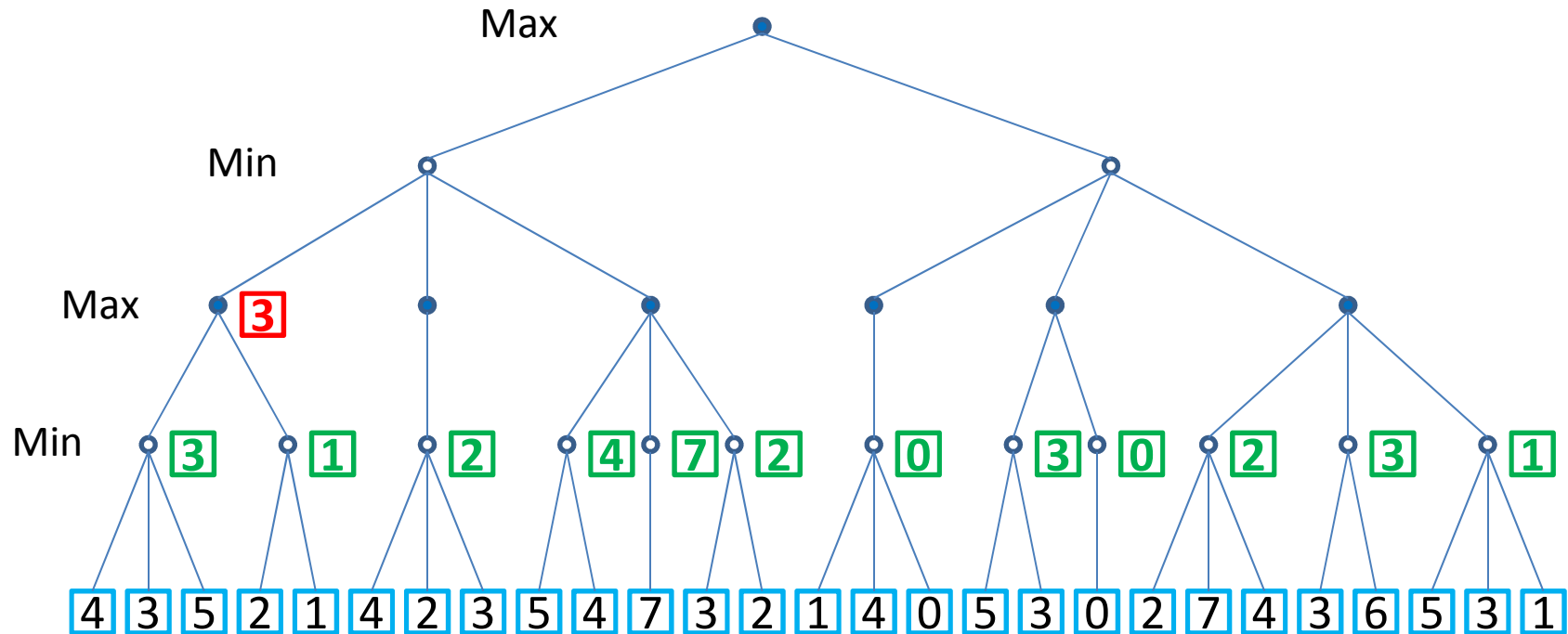
MiniMax without $\alpha\beta$ -pruning



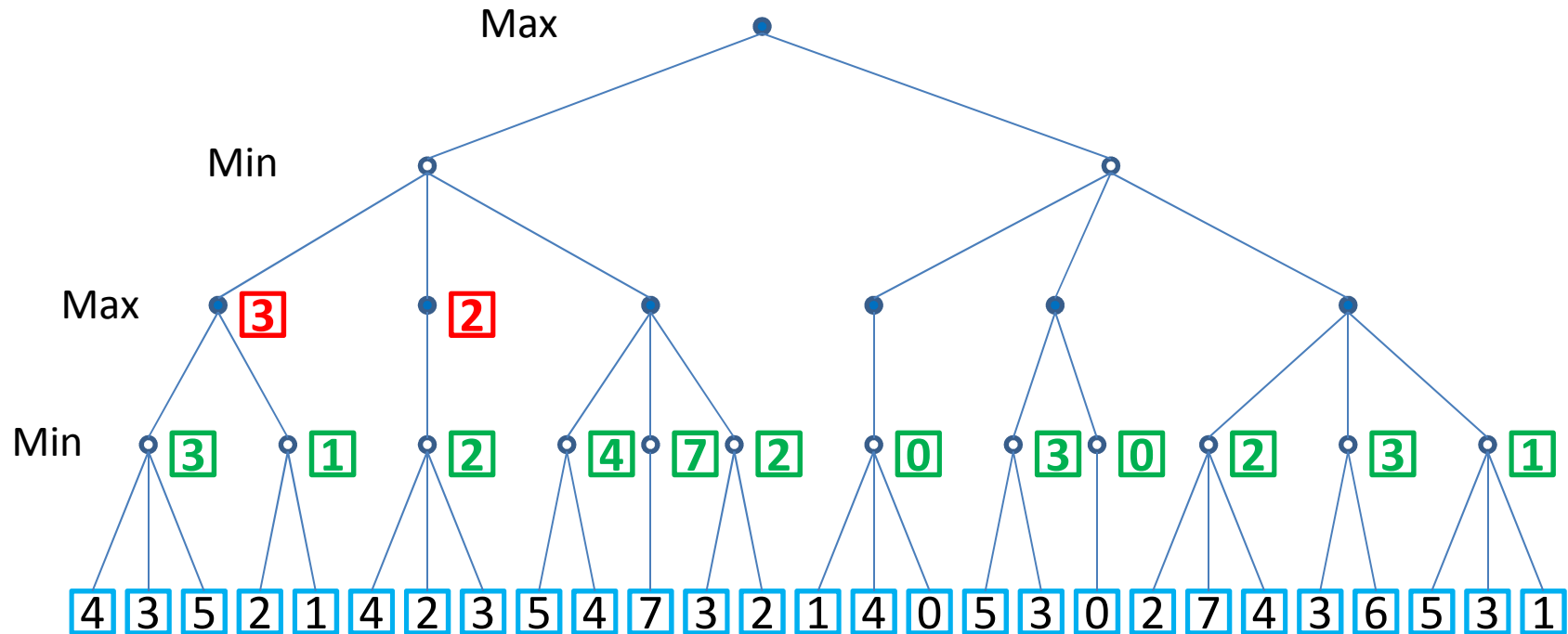
MiniMax without $\alpha\beta$ -pruning



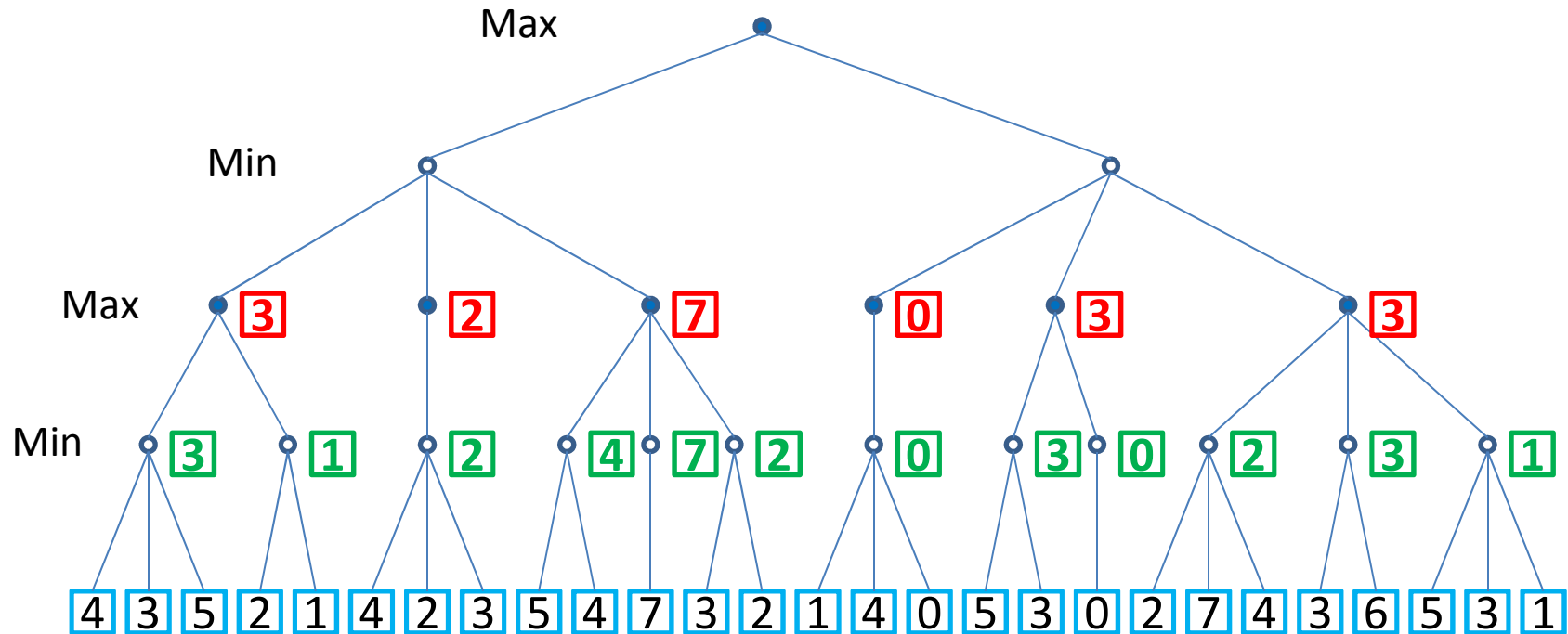
MiniMax without $\alpha\beta$ -pruning



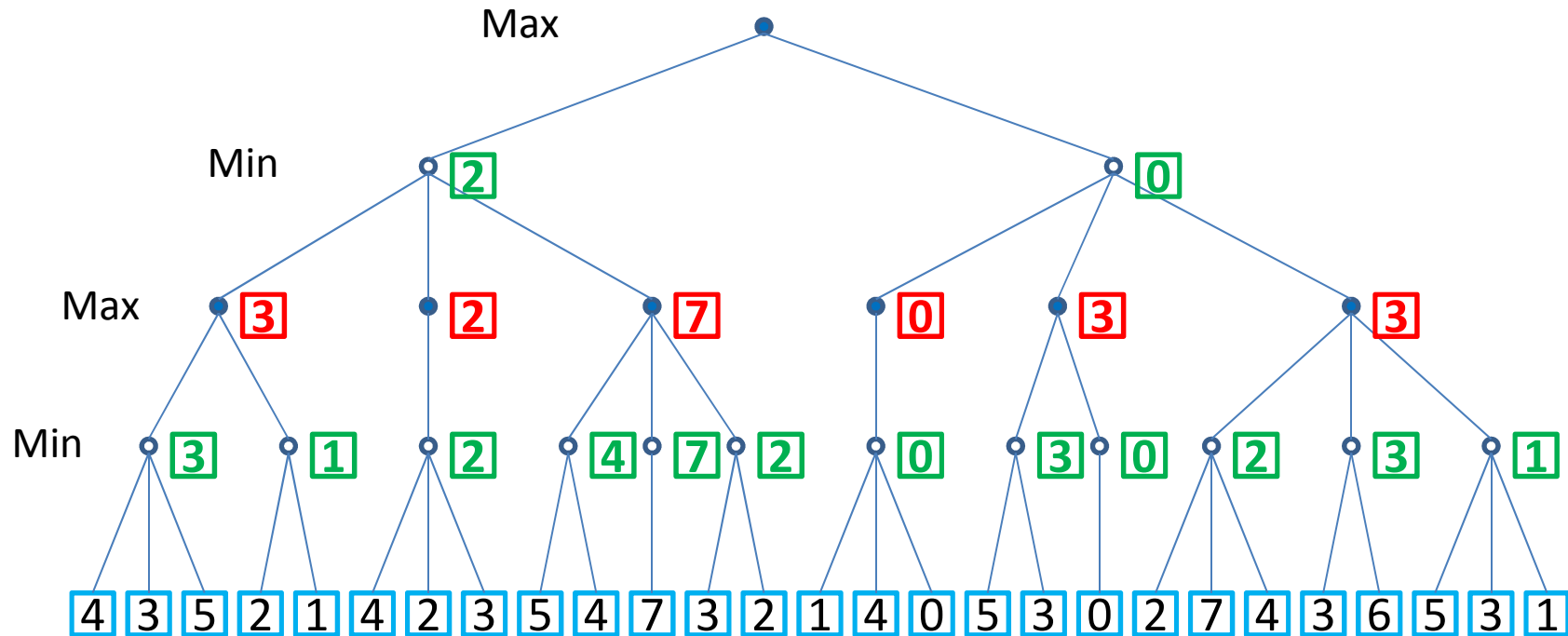
MiniMax without $\alpha\beta$ -pruning



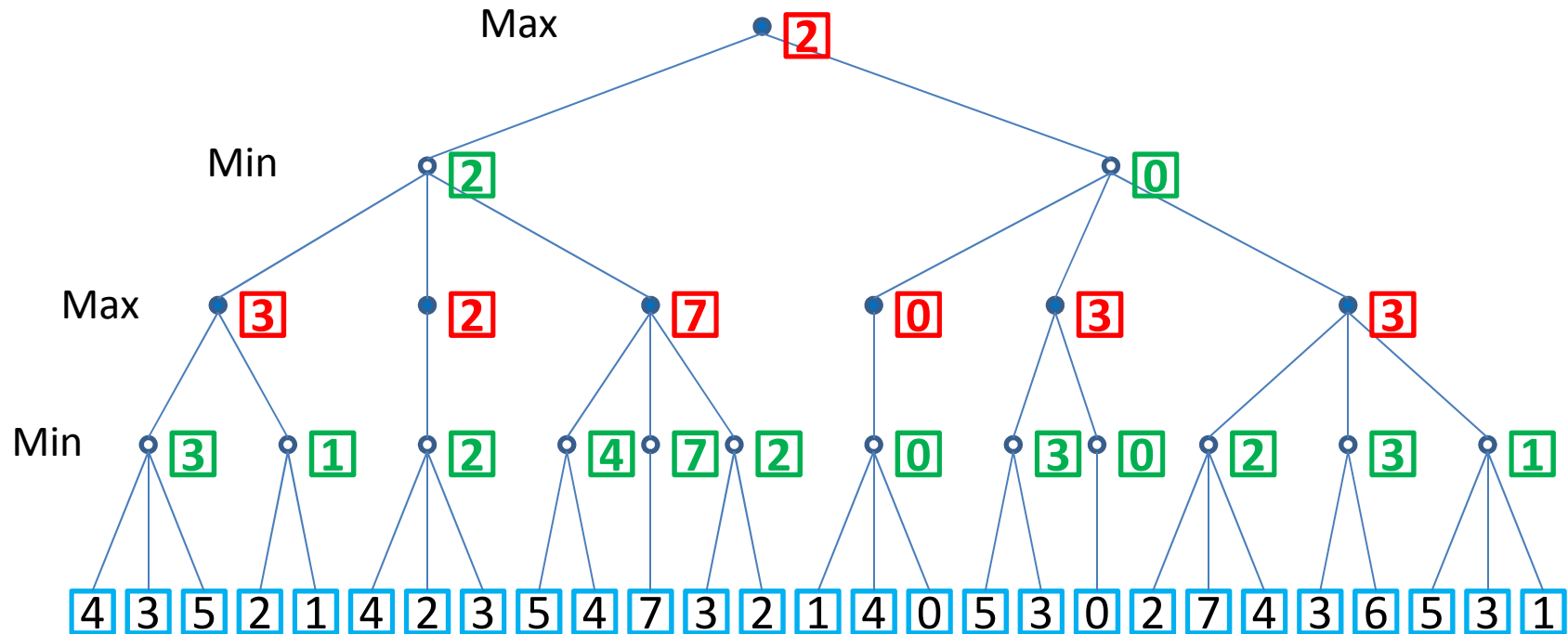
MiniMax without $\alpha\beta$ -pruning



MiniMax without $\alpha\beta$ -pruning



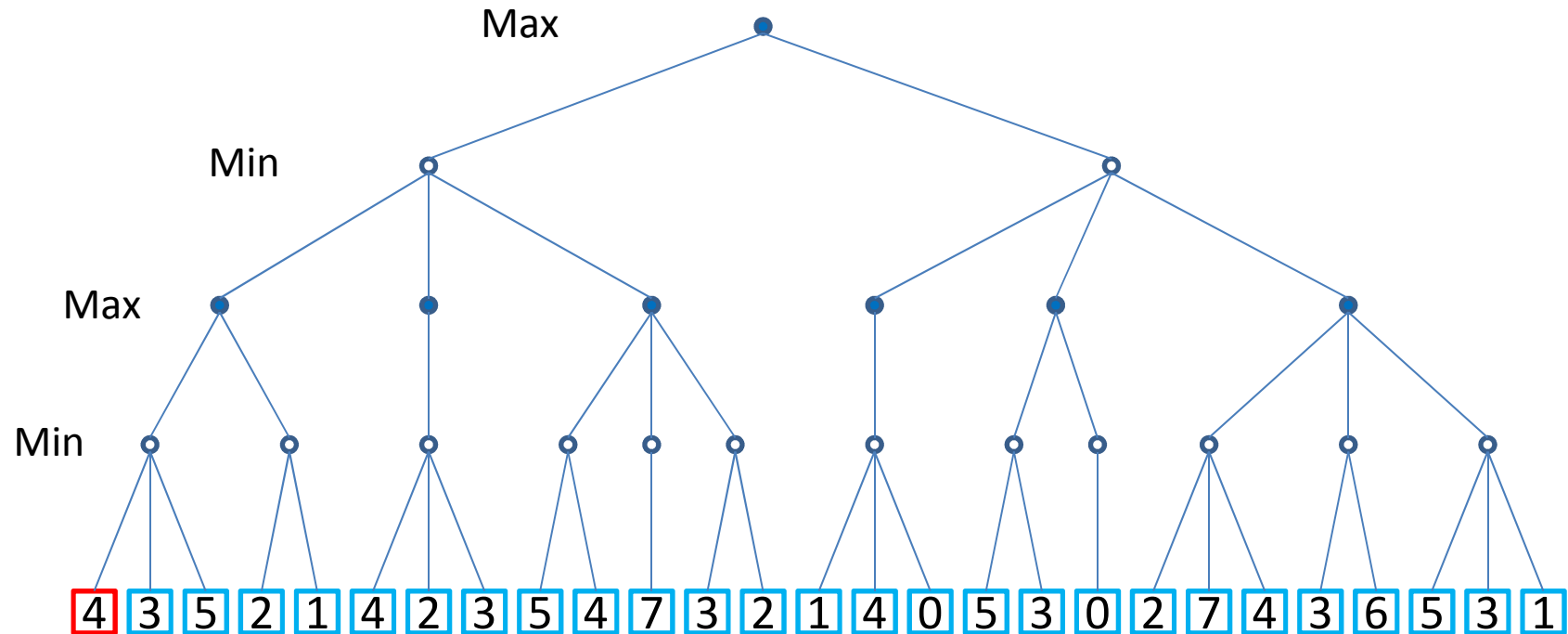
MiniMax without $\alpha\beta$ -pruning



MiniMax & Constraint Processing: MiniMax Algorithm

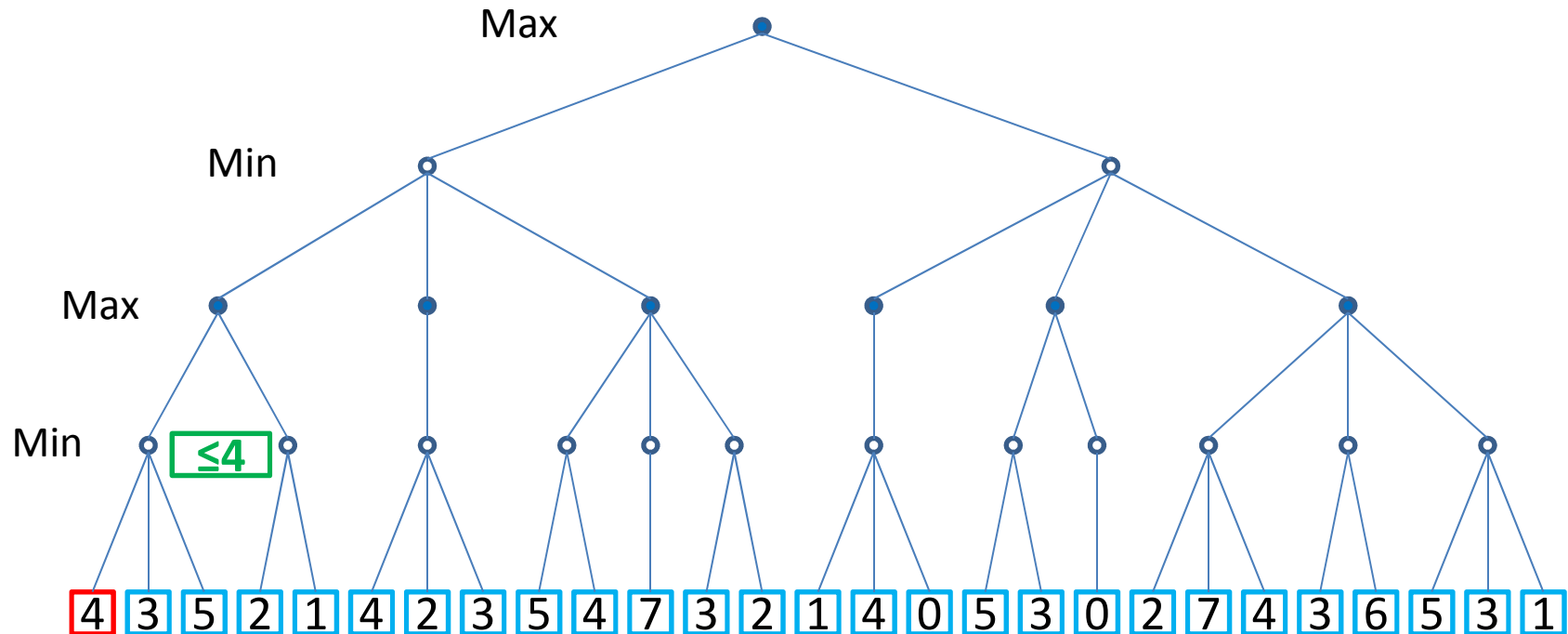
MINIMAX WITH $\alpha\beta$ -PRUNING

MiniMax with $\alpha\beta$ -pruning

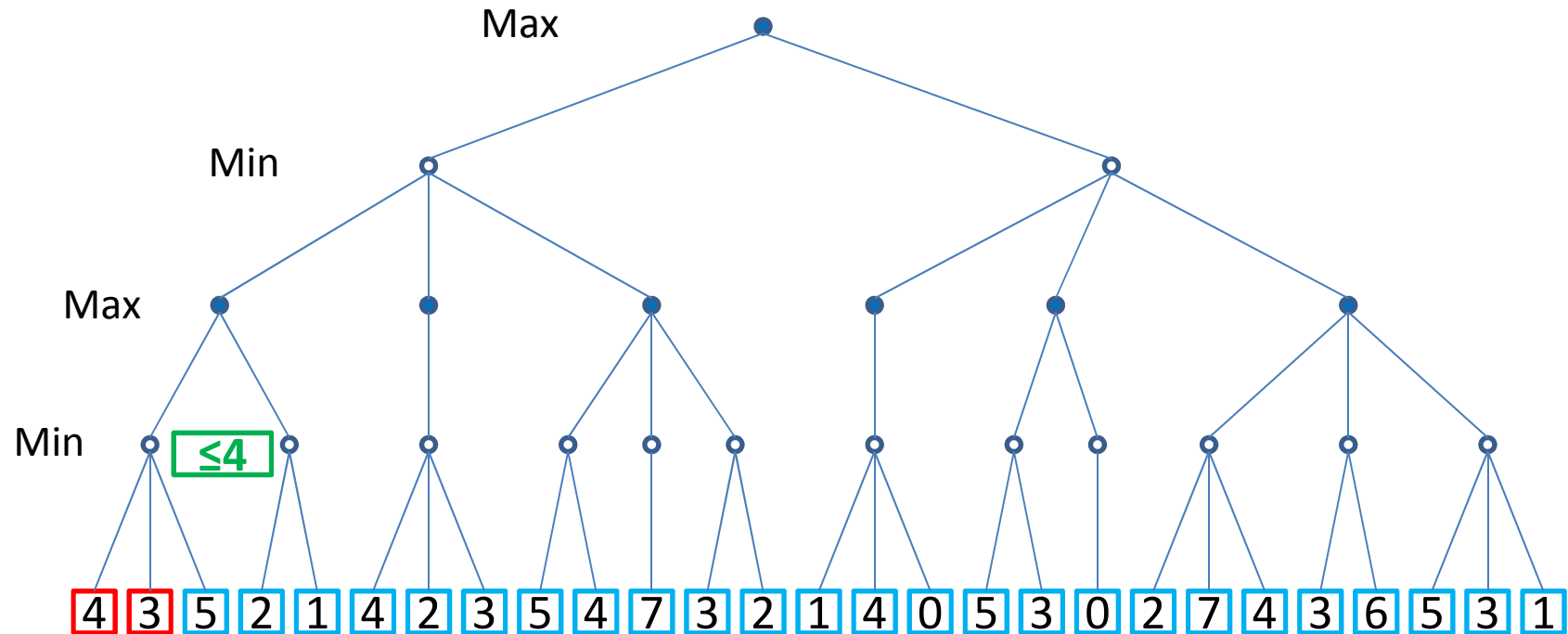


MiniMax with $\alpha\beta$ -pruning

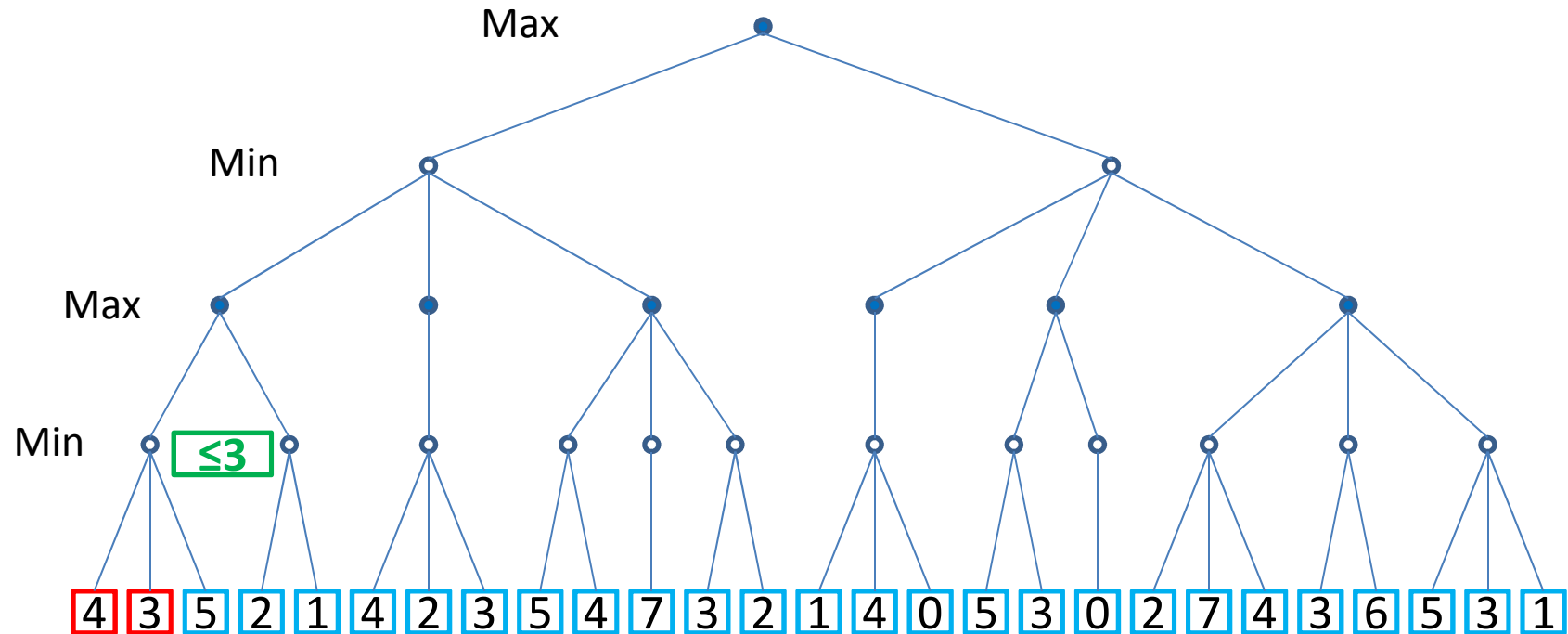
- **α -nodes**: Temporary values at MIN-nodes



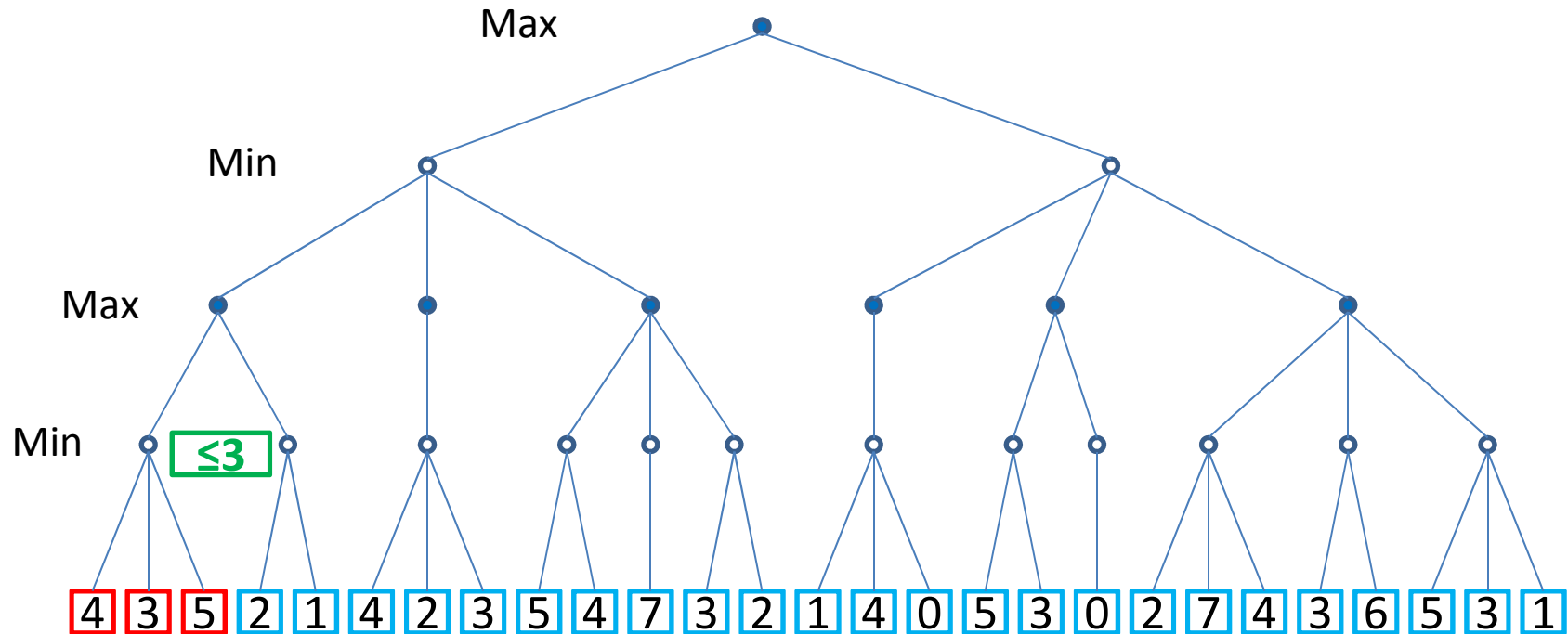
MiniMax with $\alpha\beta$ -pruning



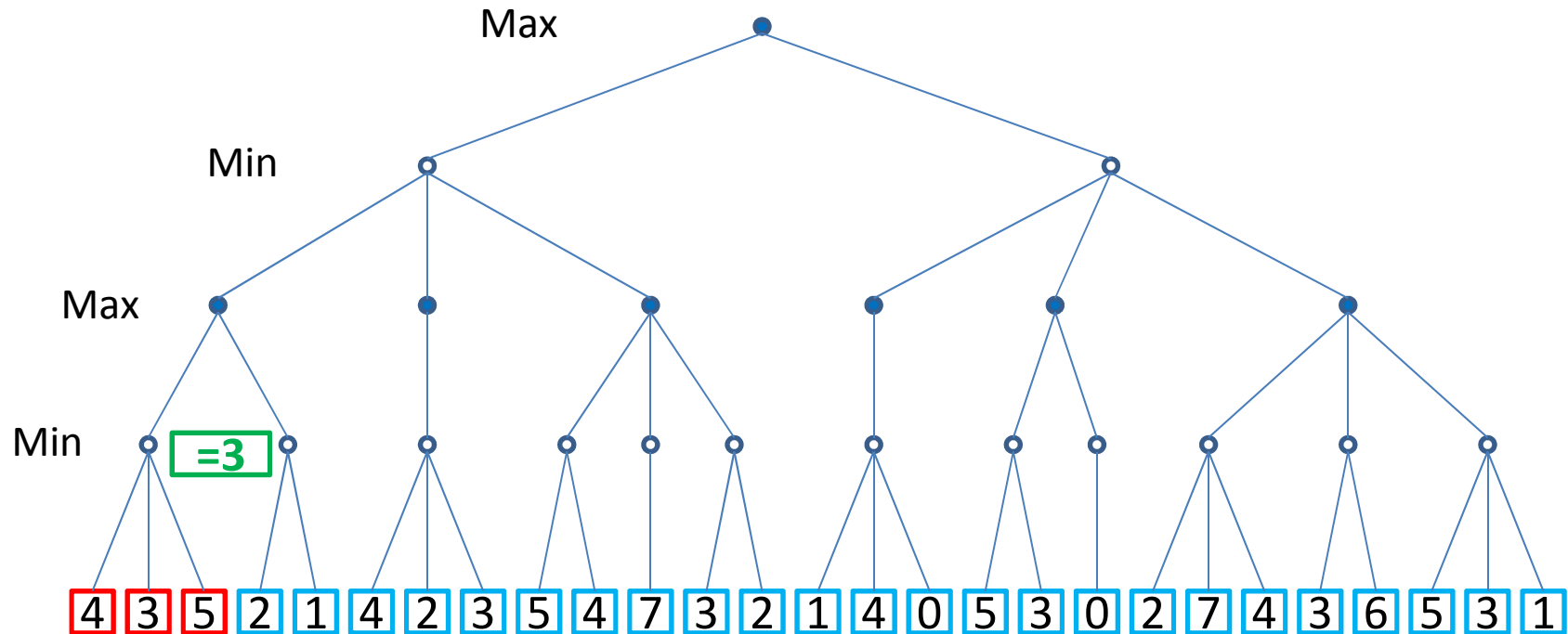
MiniMax with $\alpha\beta$ -pruning



MiniMax with $\alpha\beta$ -pruning

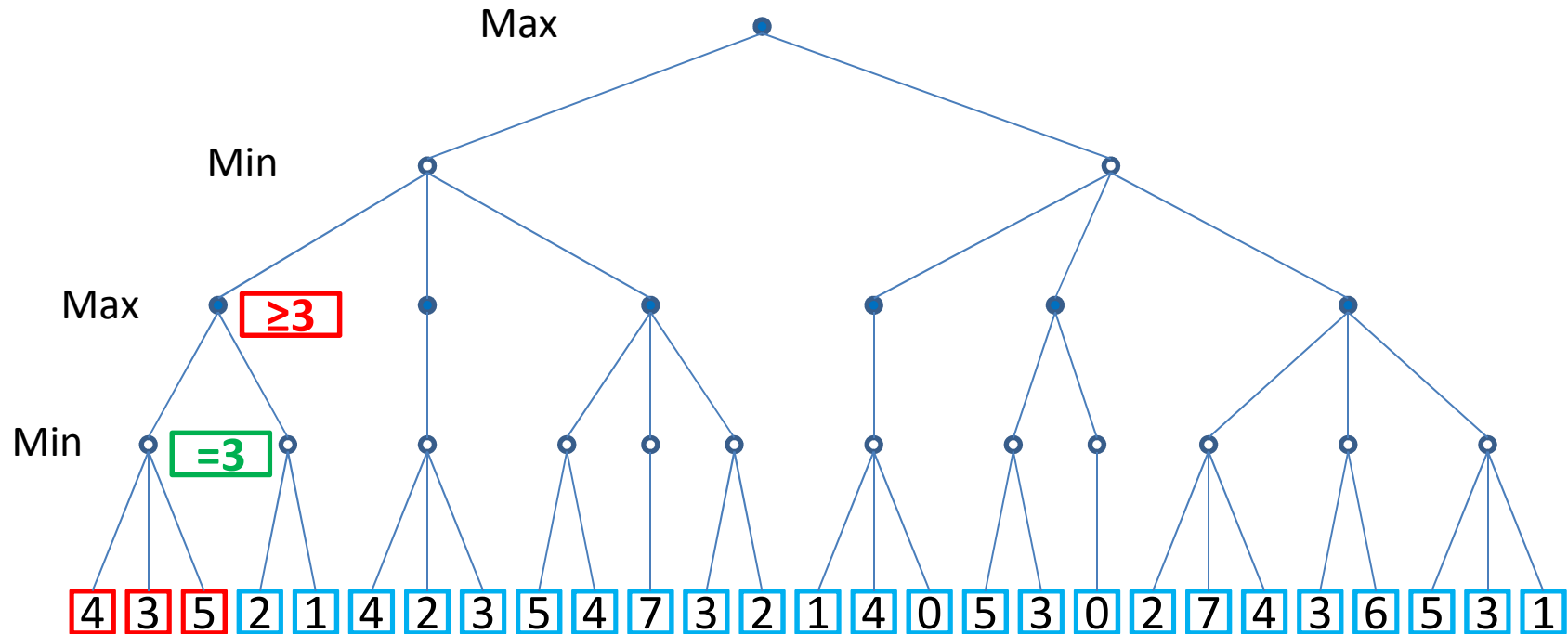


MiniMax with $\alpha\beta$ -pruning

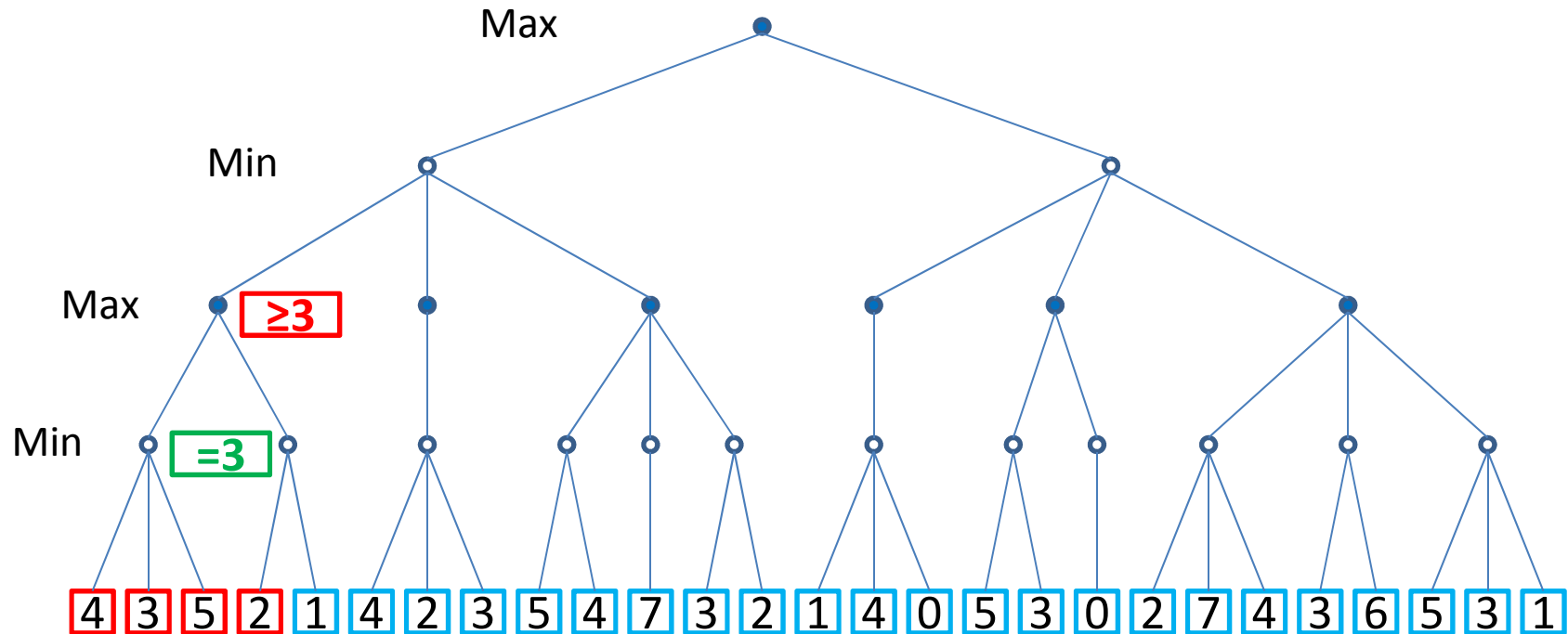


MiniMax with $\alpha\beta$ -pruning

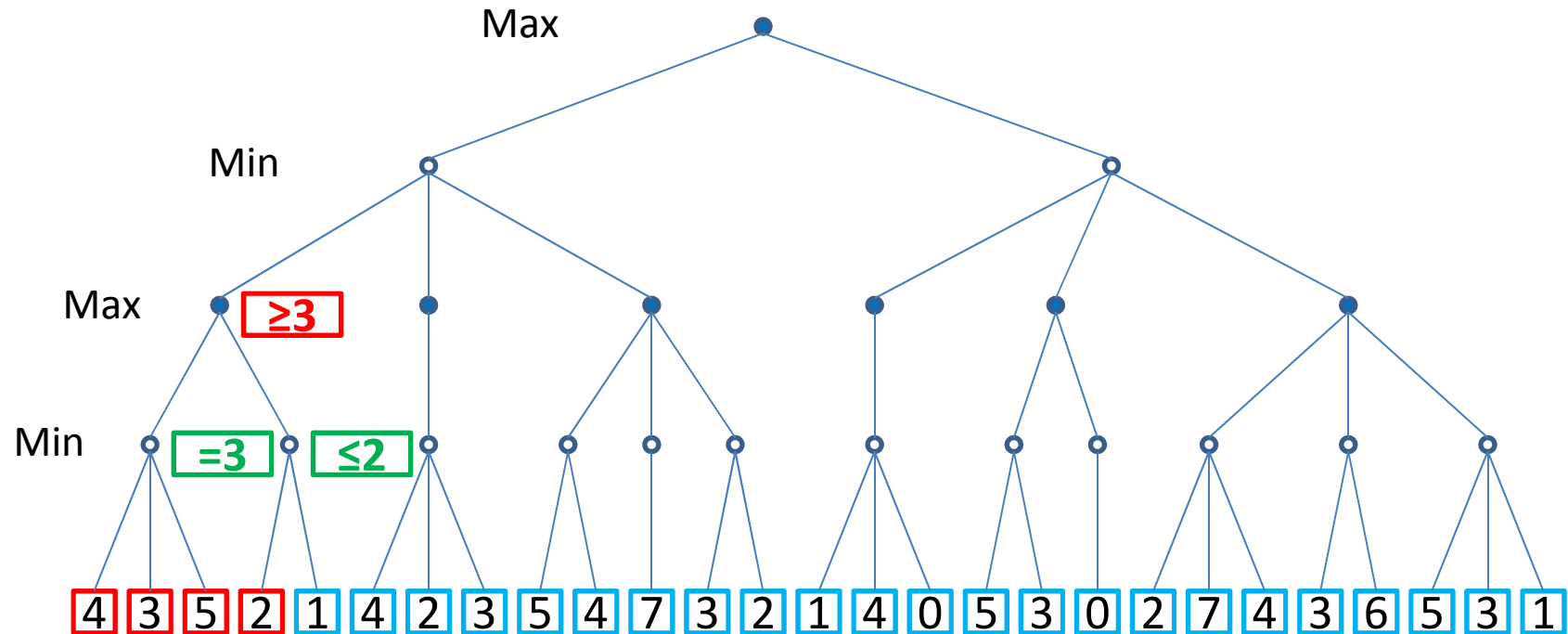
- **β -nodes**: Temporary values at MAX-nodes



MiniMax with $\alpha\beta$ -pruning

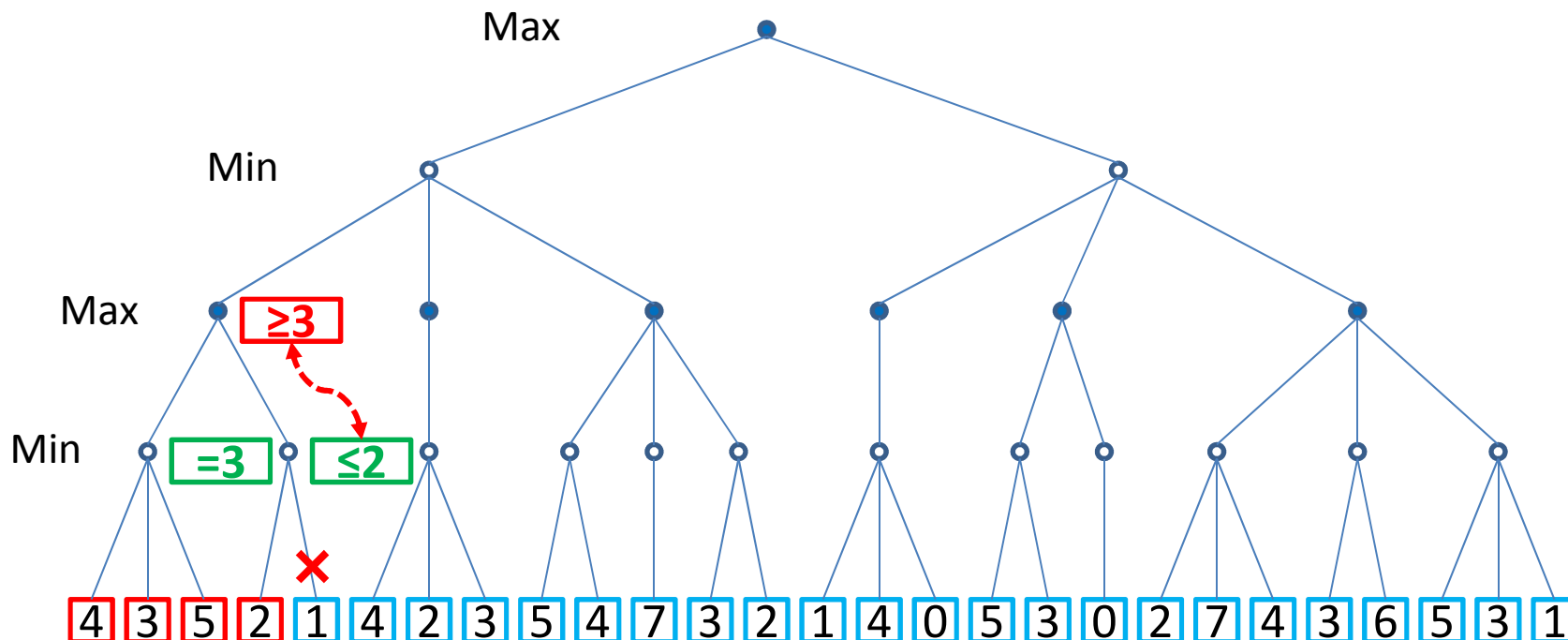


MiniMax with $\alpha\beta$ -pruning

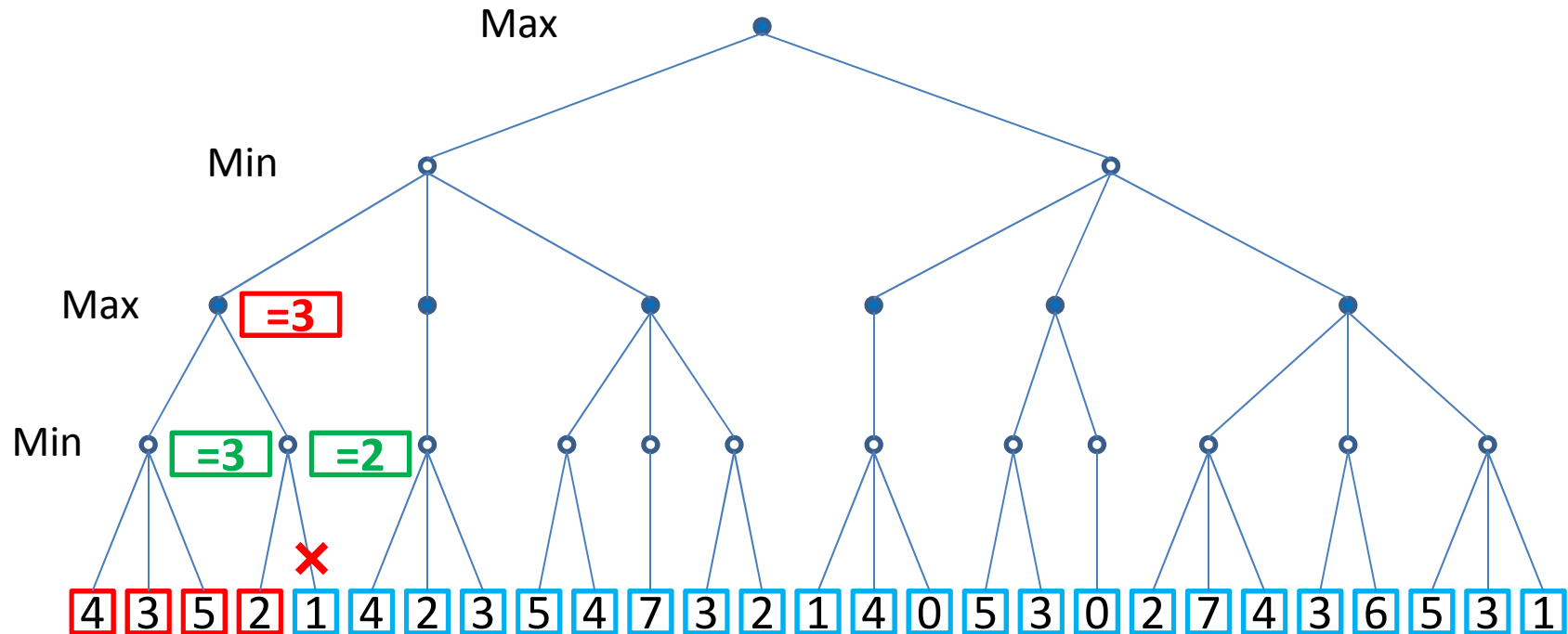


MiniMax with $\alpha\beta$ -pruning

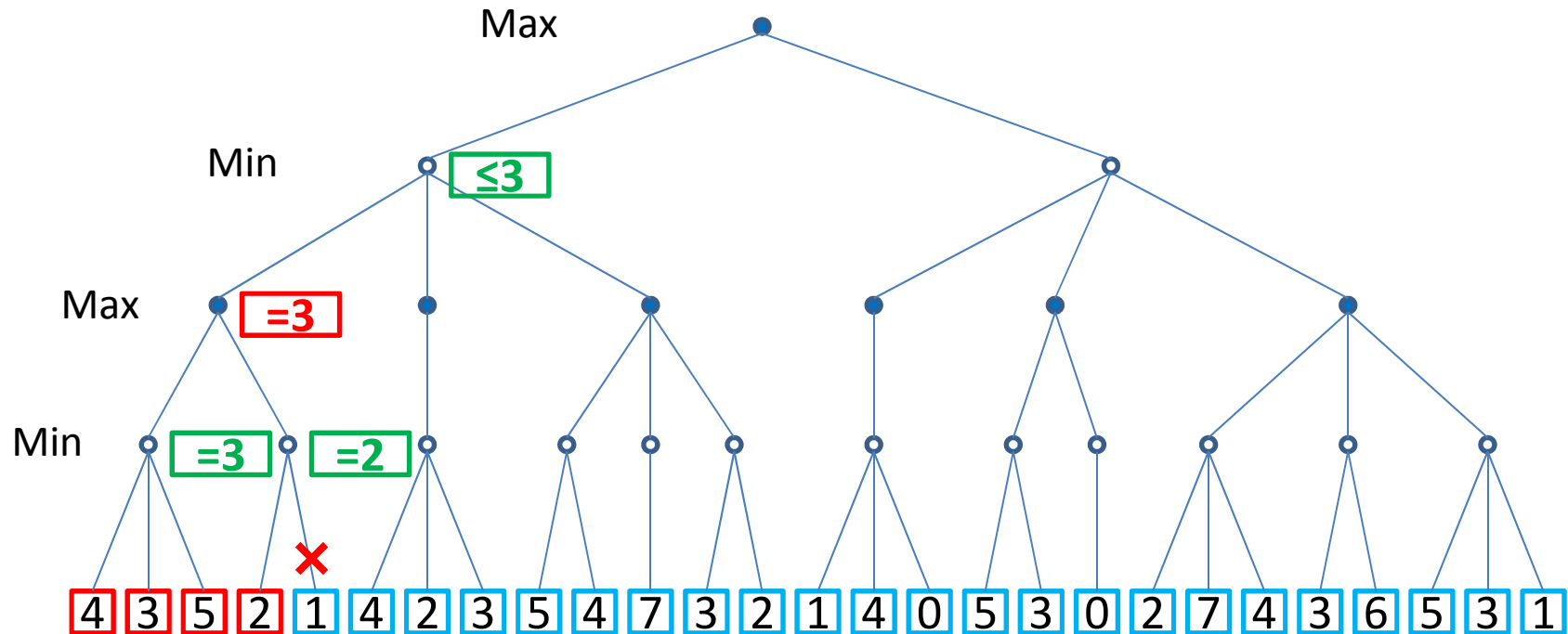
- **Prune: Parent β -node \geq Child α -node**



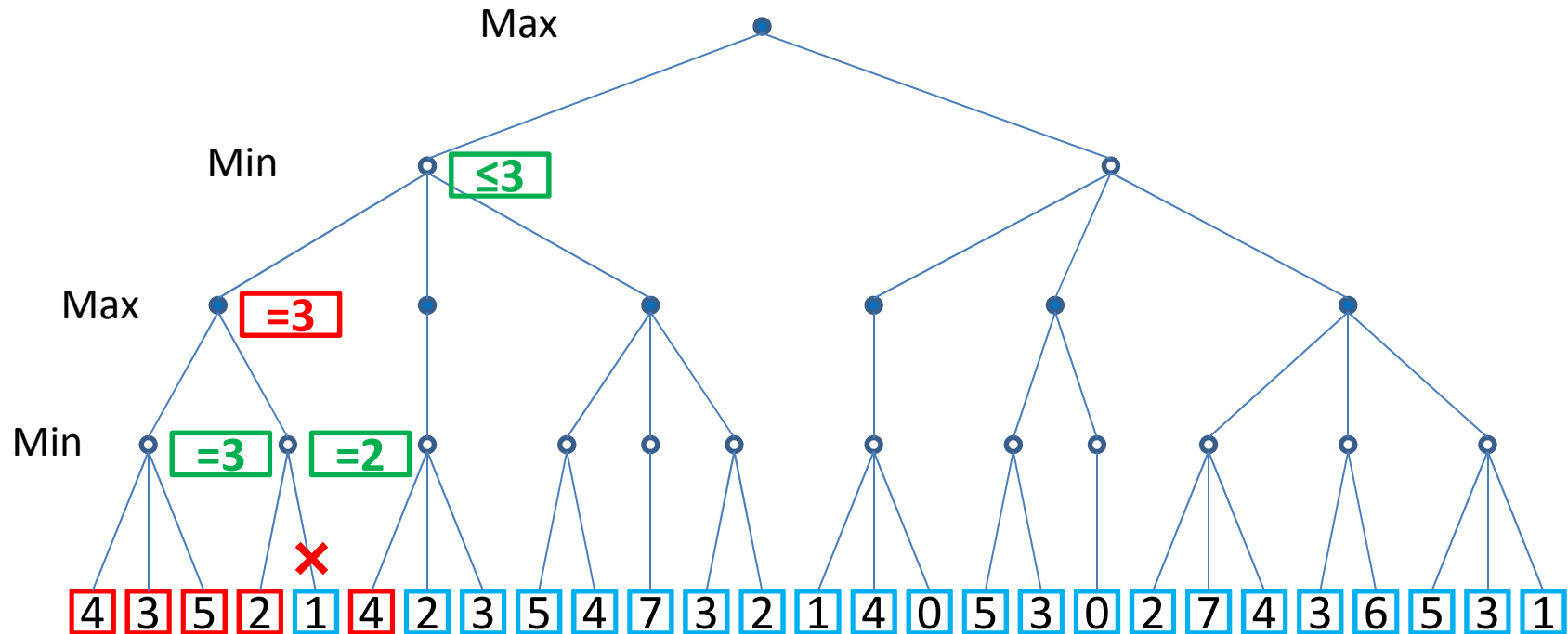
MiniMax with $\alpha\beta$ -pruning



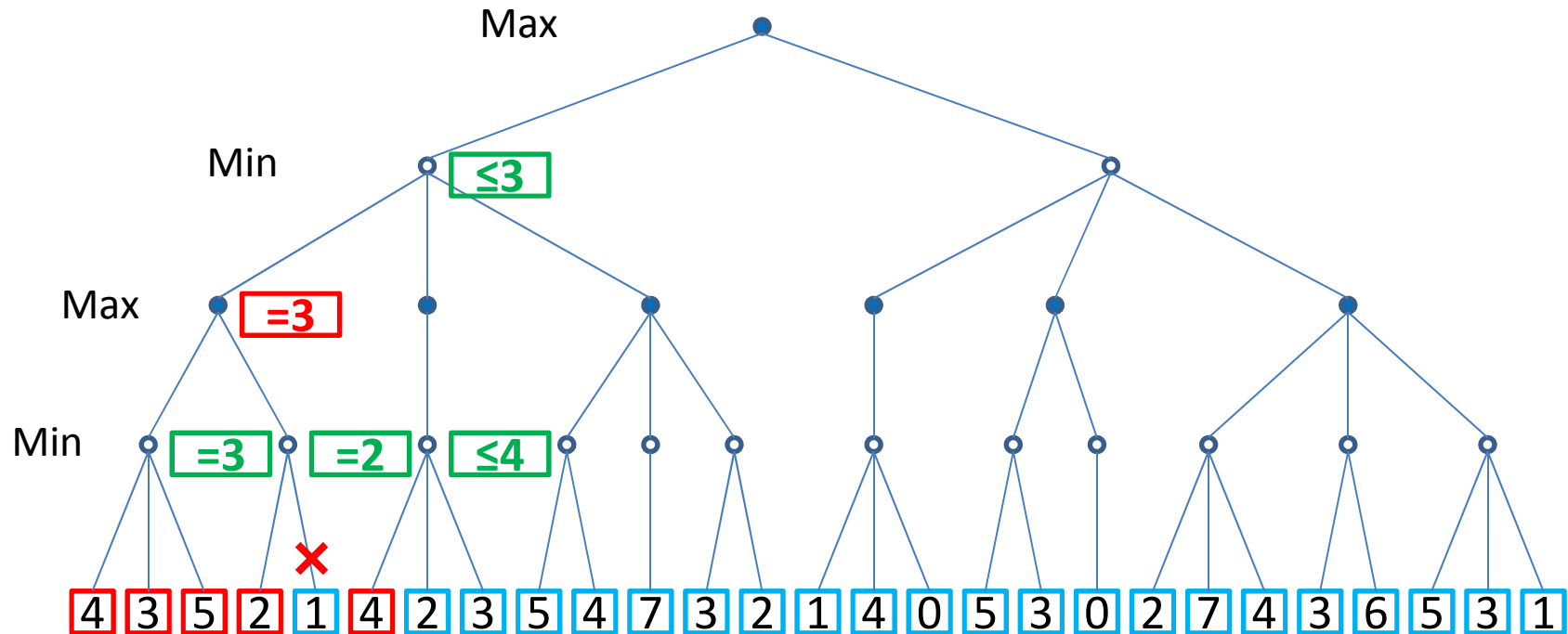
MiniMax with $\alpha\beta$ -pruning



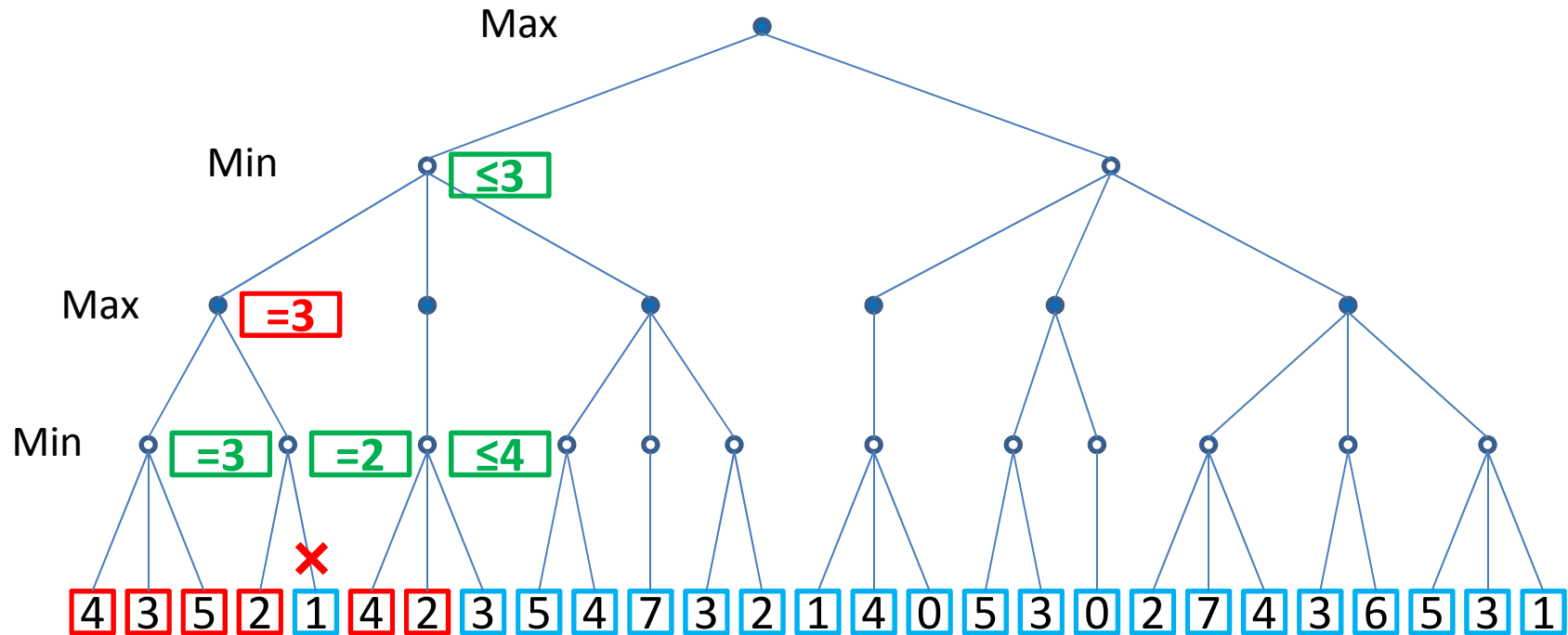
MiniMax with $\alpha\beta$ -pruning



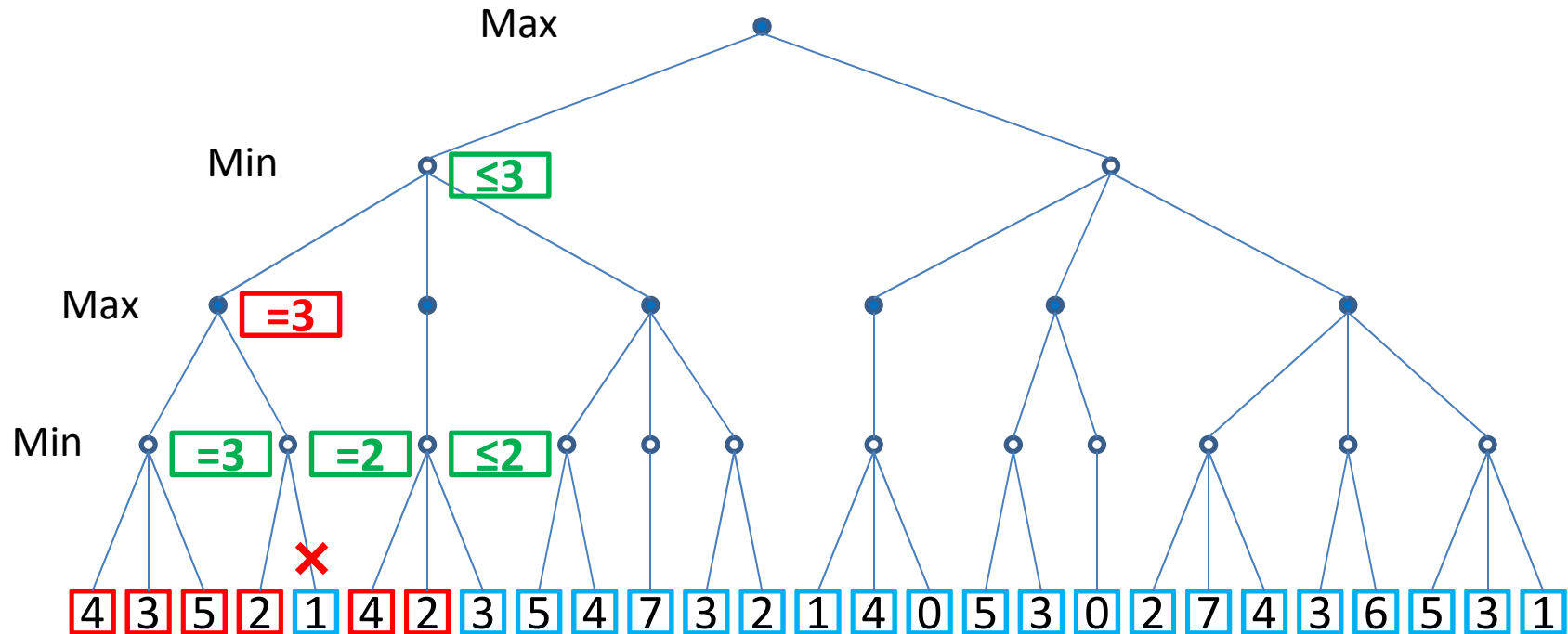
MiniMax with $\alpha\beta$ -pruning



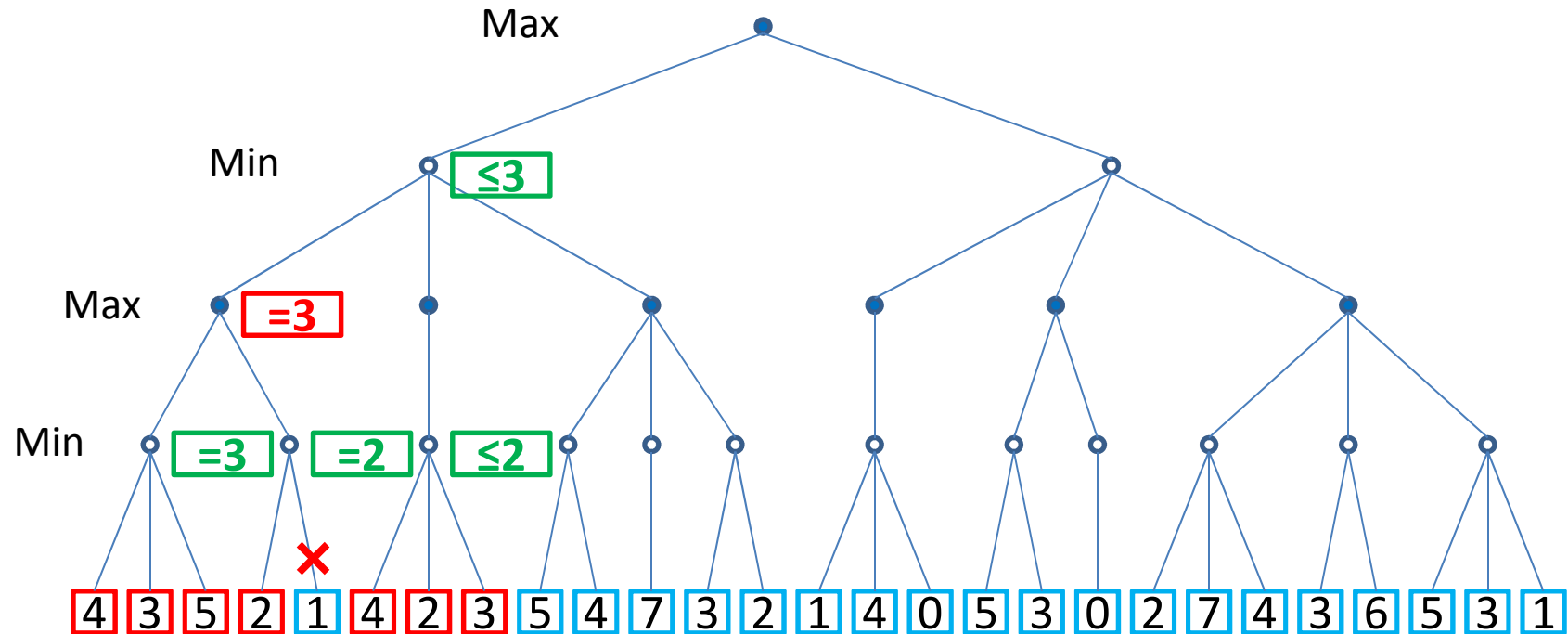
MiniMax with $\alpha\beta$ -pruning



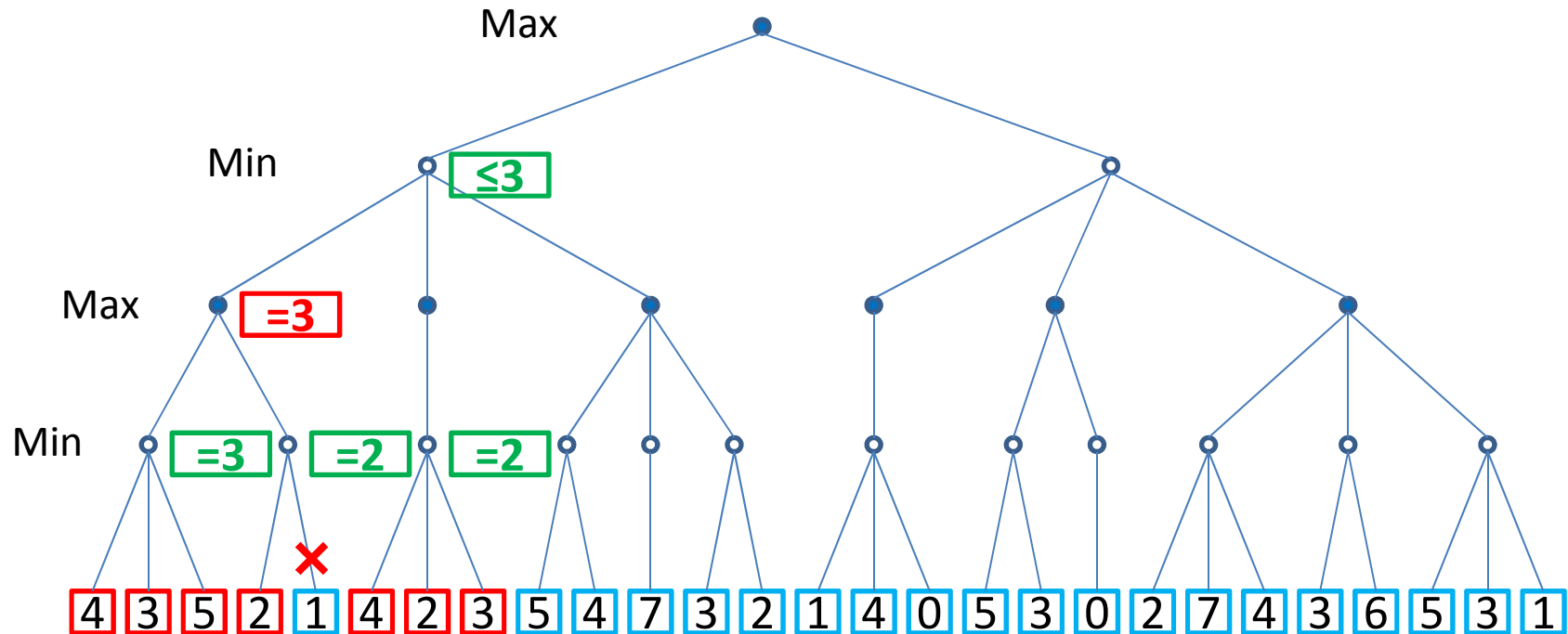
MiniMax with $\alpha\beta$ -pruning



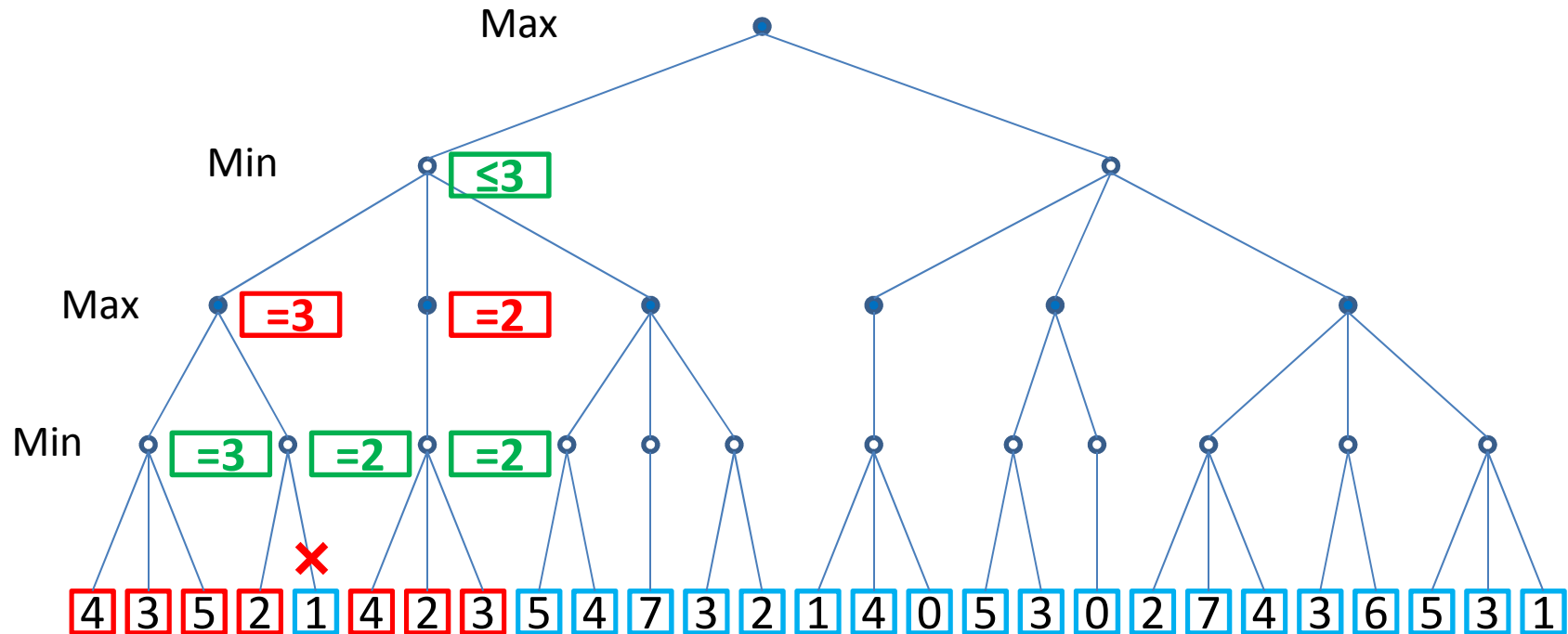
MiniMax with $\alpha\beta$ -pruning



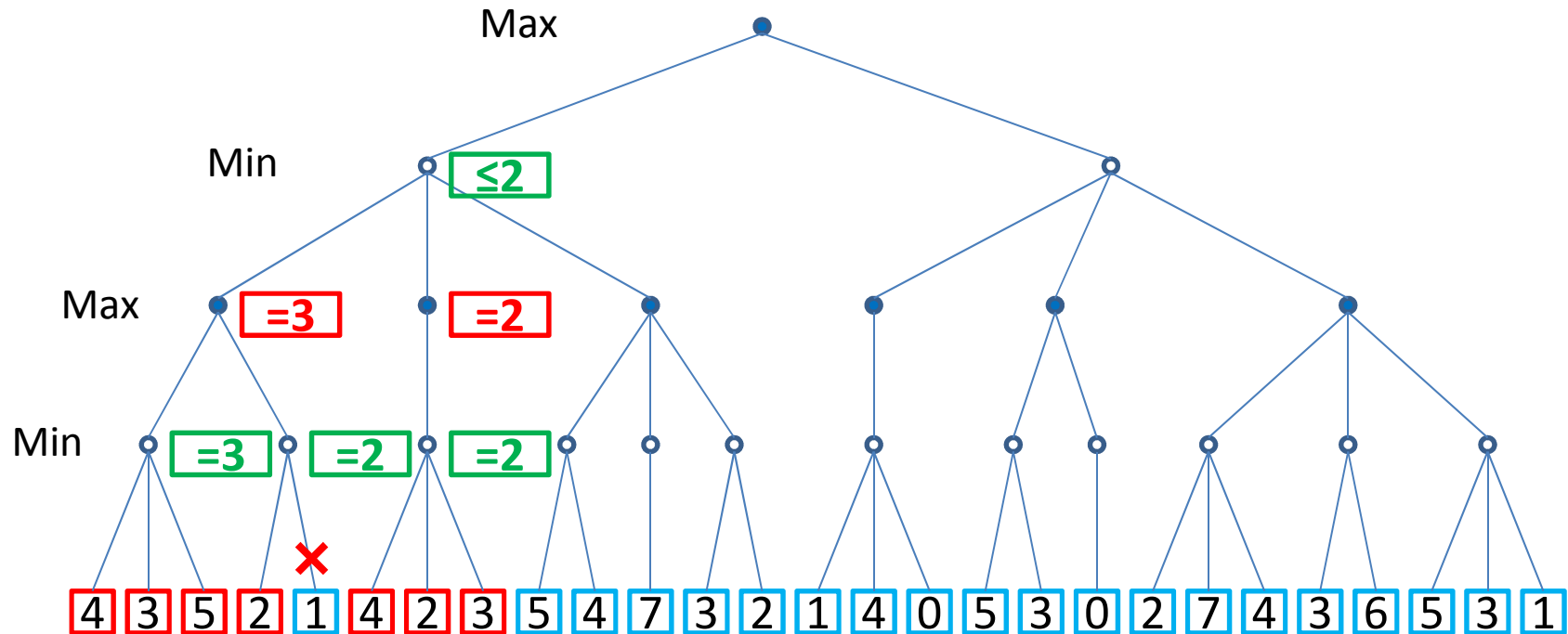
MiniMax with $\alpha\beta$ -pruning



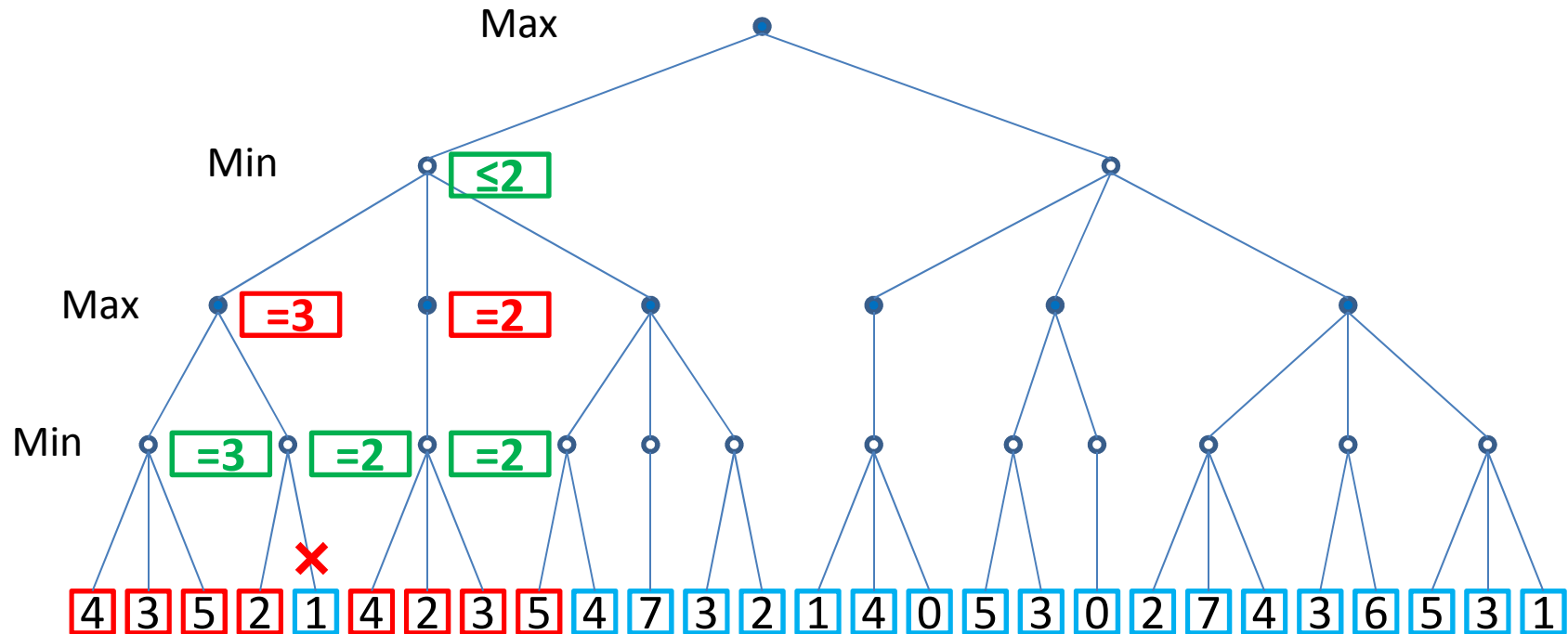
MiniMax with $\alpha\beta$ -pruning



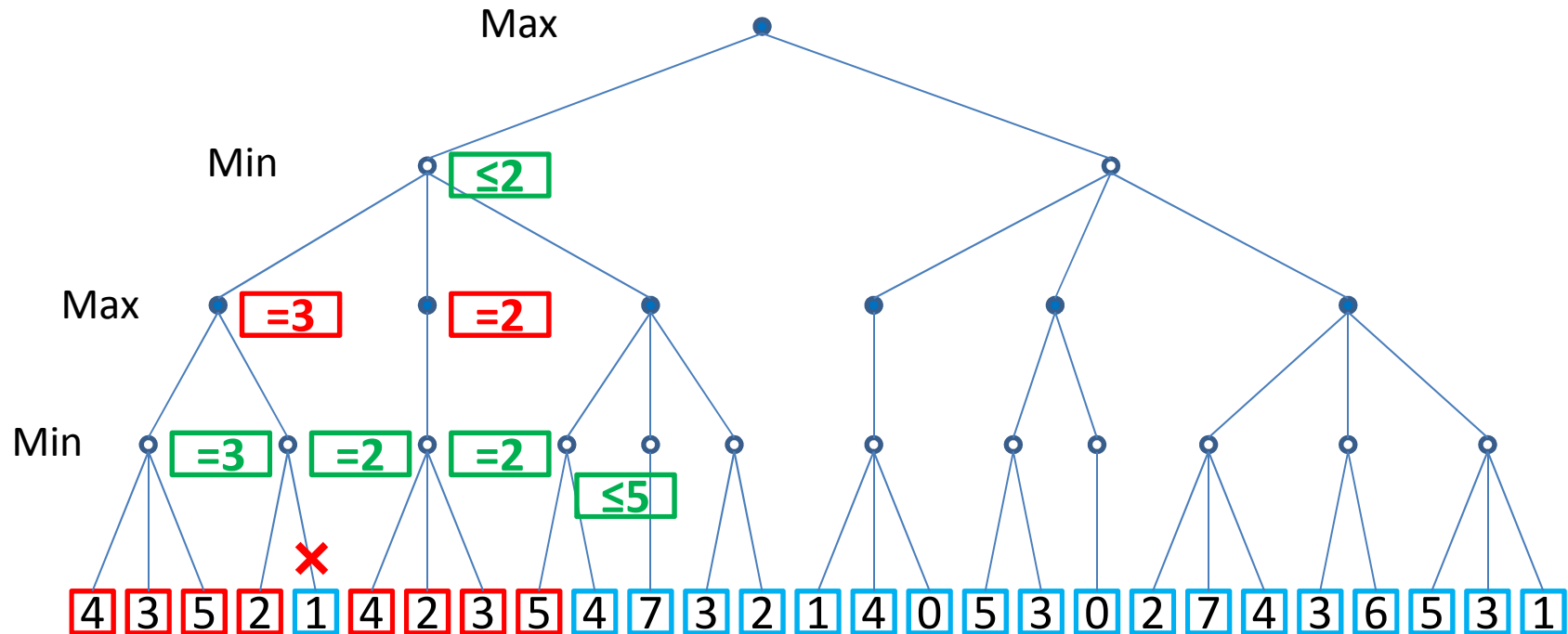
MiniMax with $\alpha\beta$ -pruning



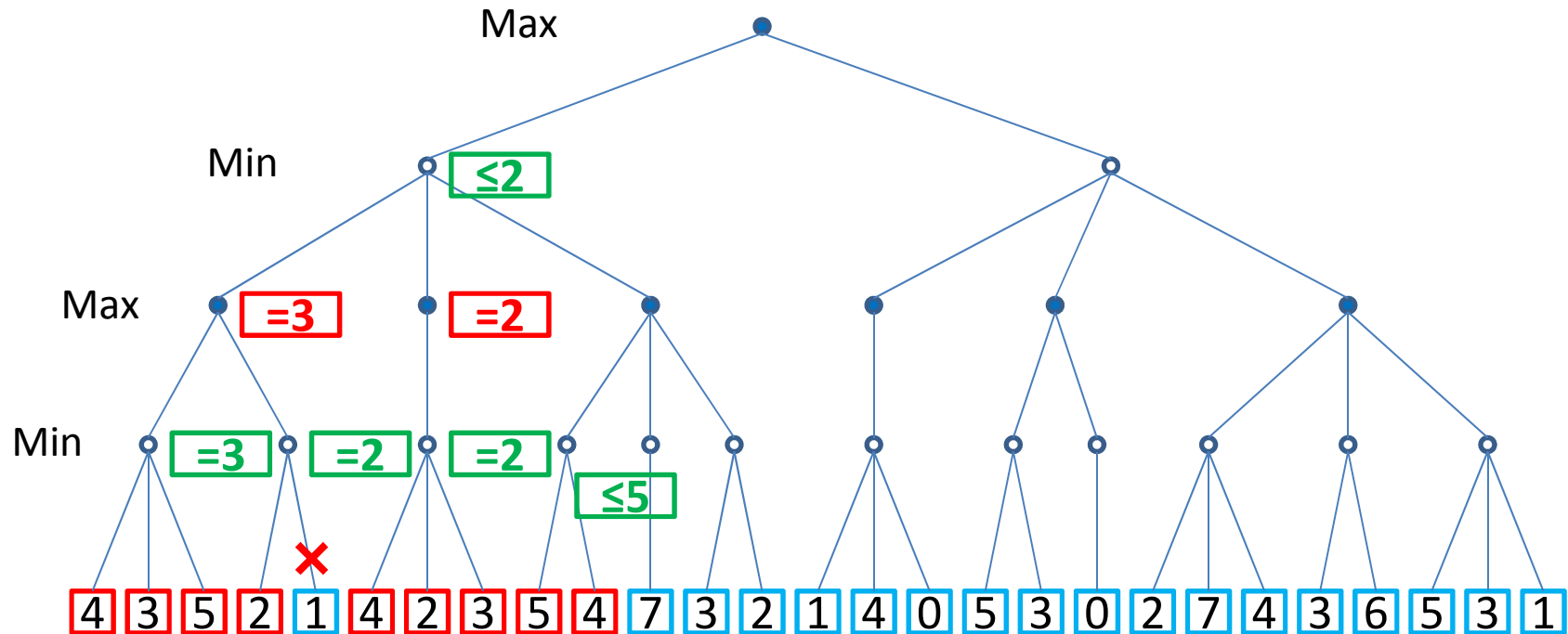
MiniMax with $\alpha\beta$ -pruning



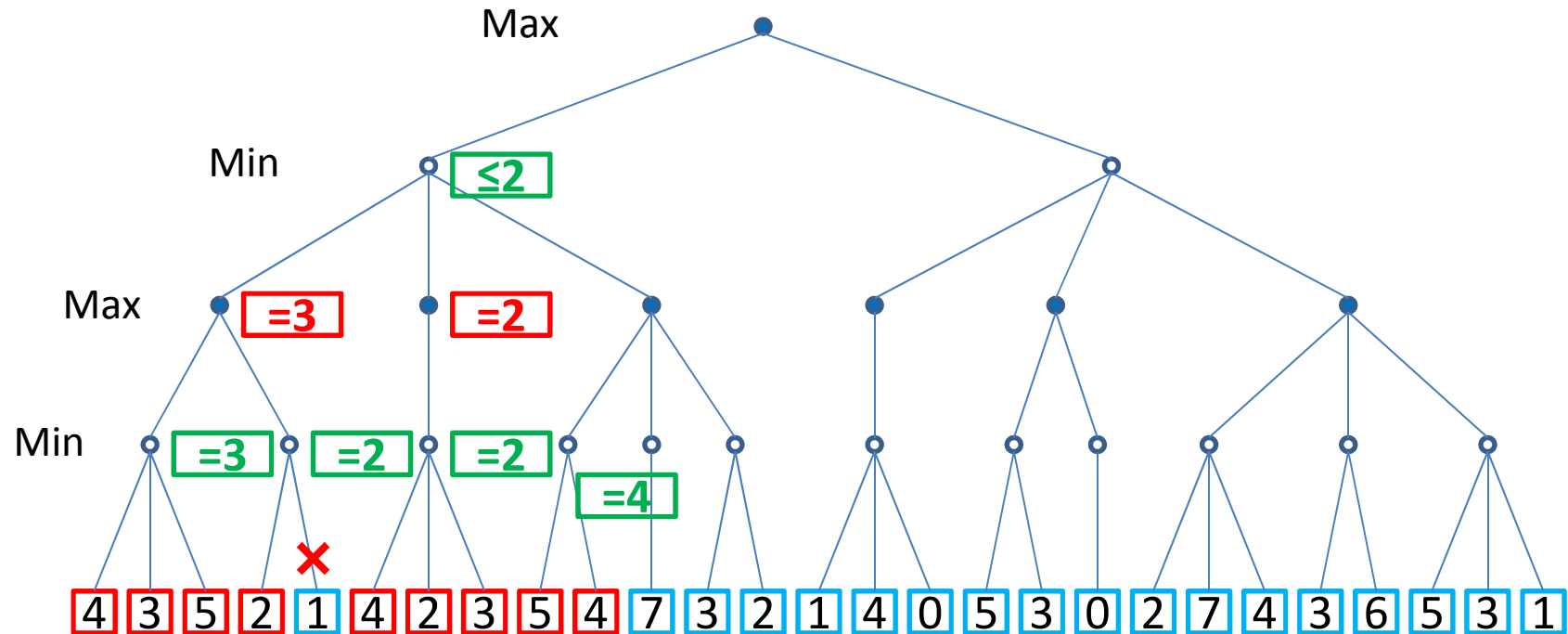
MiniMax with $\alpha\beta$ -pruning



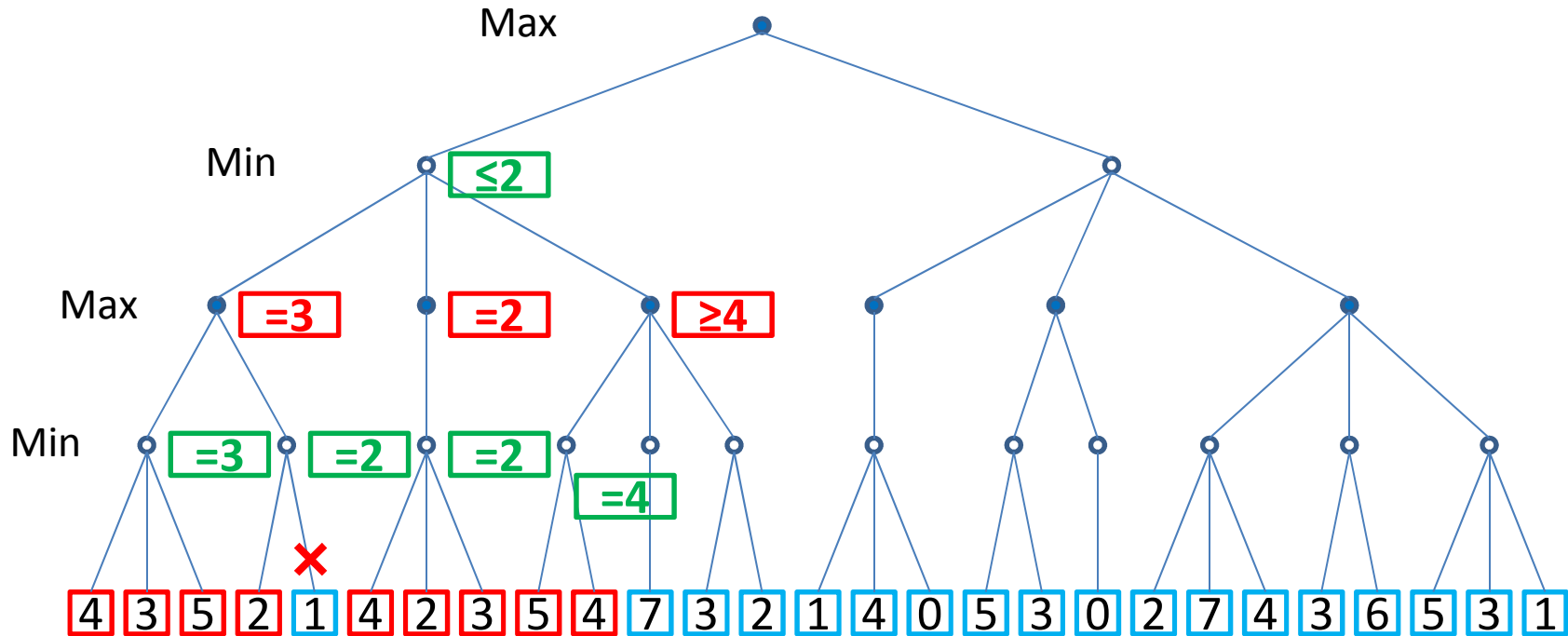
MiniMax with $\alpha\beta$ -pruning



MiniMax with $\alpha\beta$ -pruning

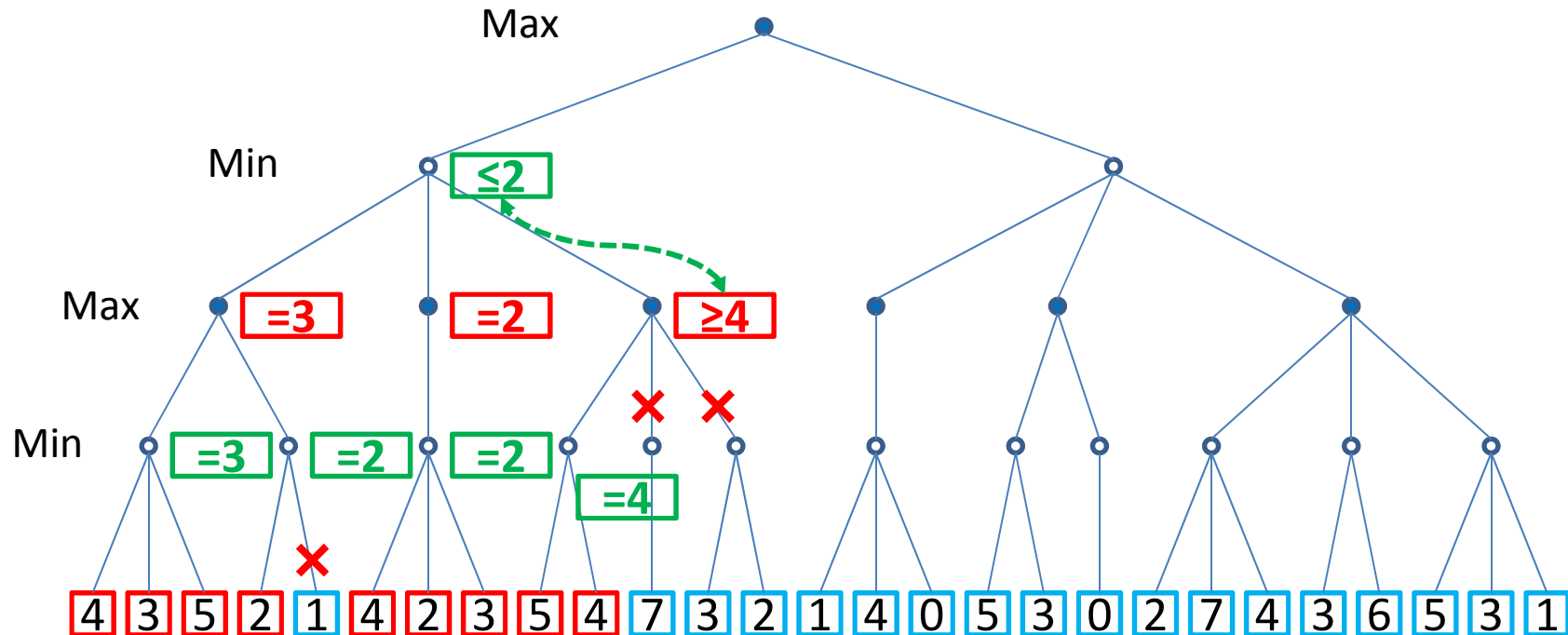


MiniMax with $\alpha\beta$ -pruning

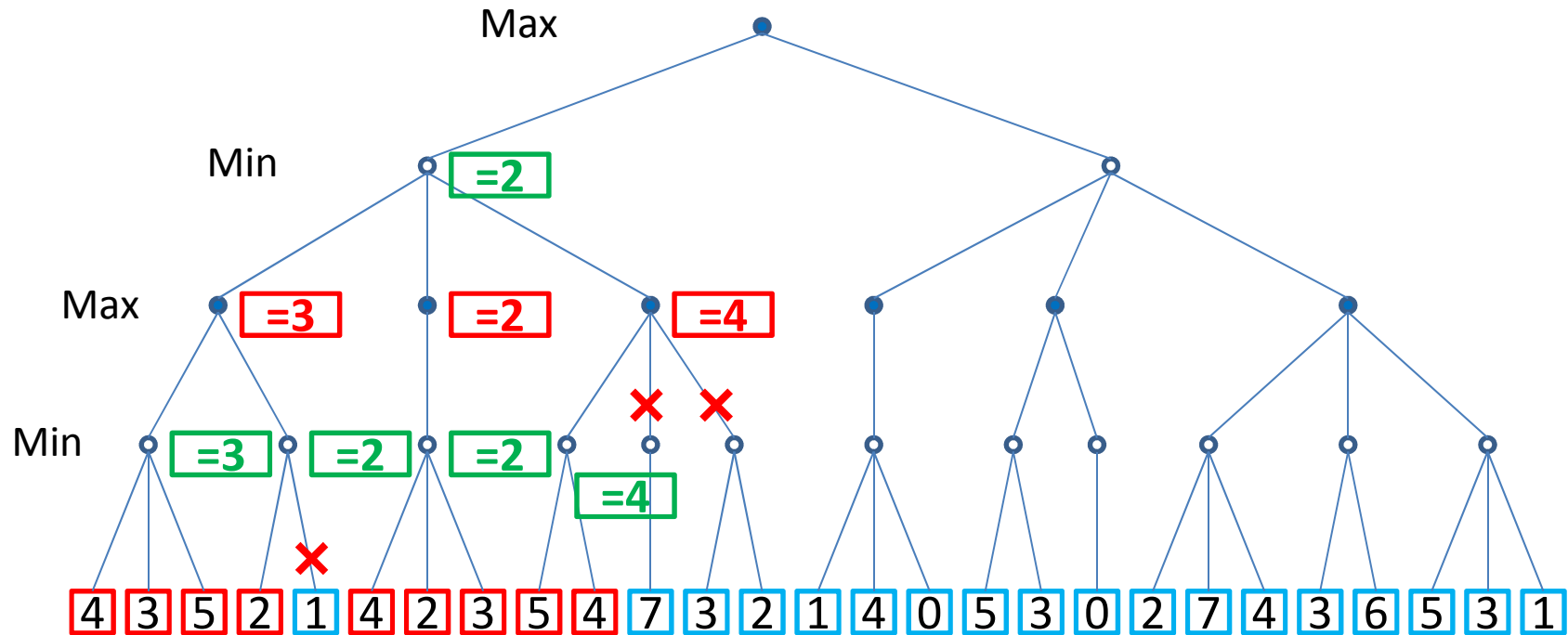


MiniMax with $\alpha\beta$ -pruning

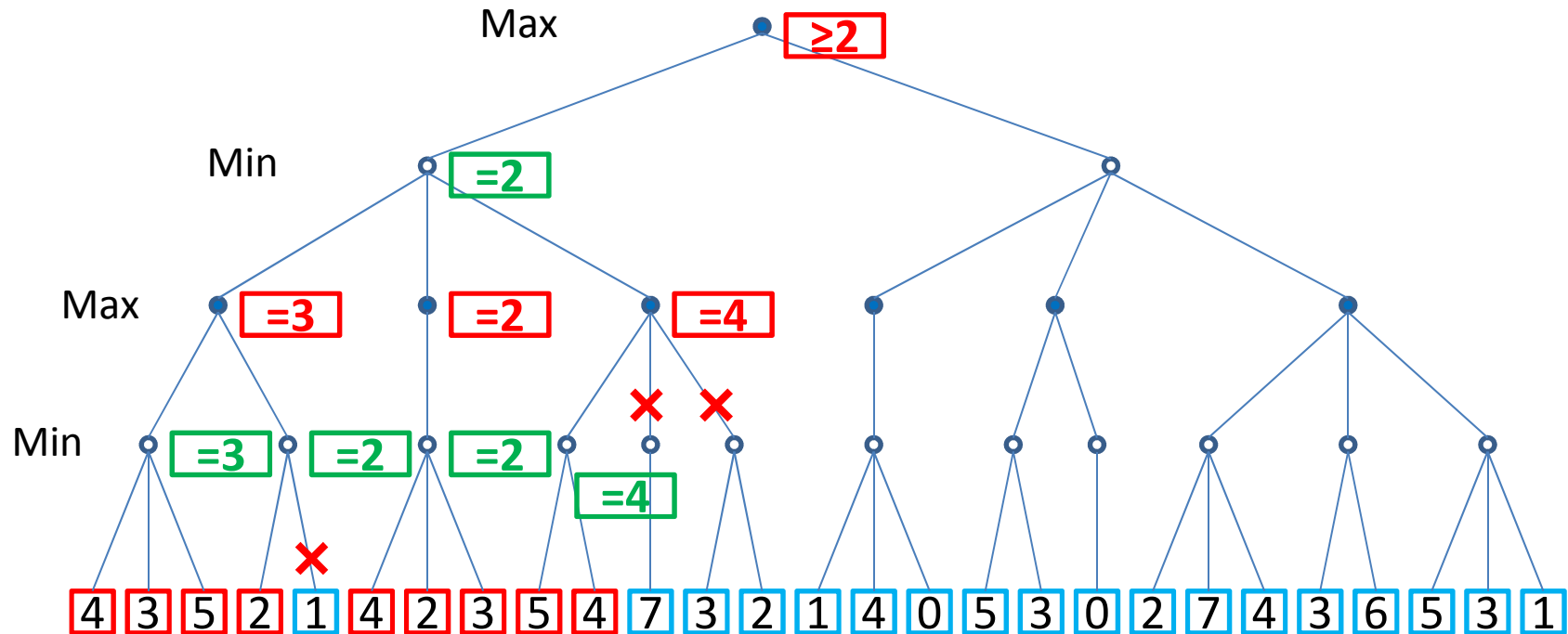
- Prune: **Parent α -node** \leq **Child β -node**



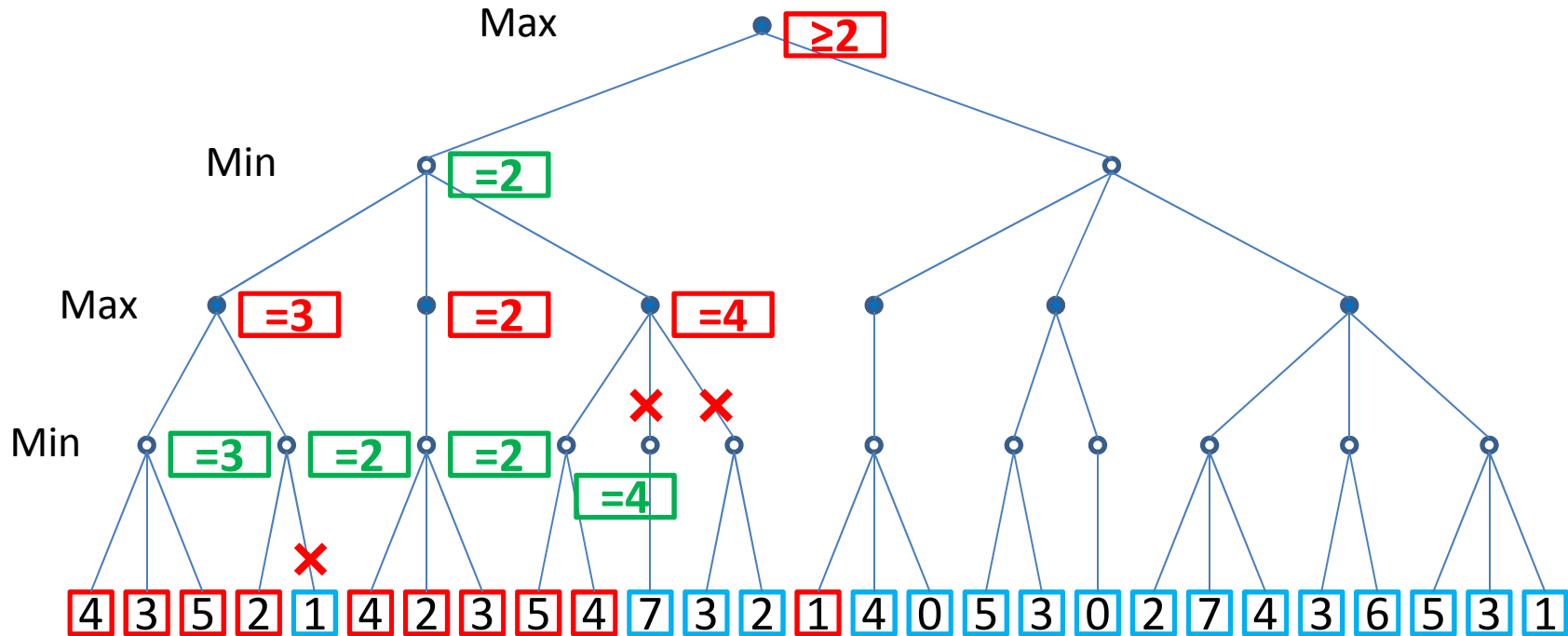
MiniMax with $\alpha\beta$ -pruning



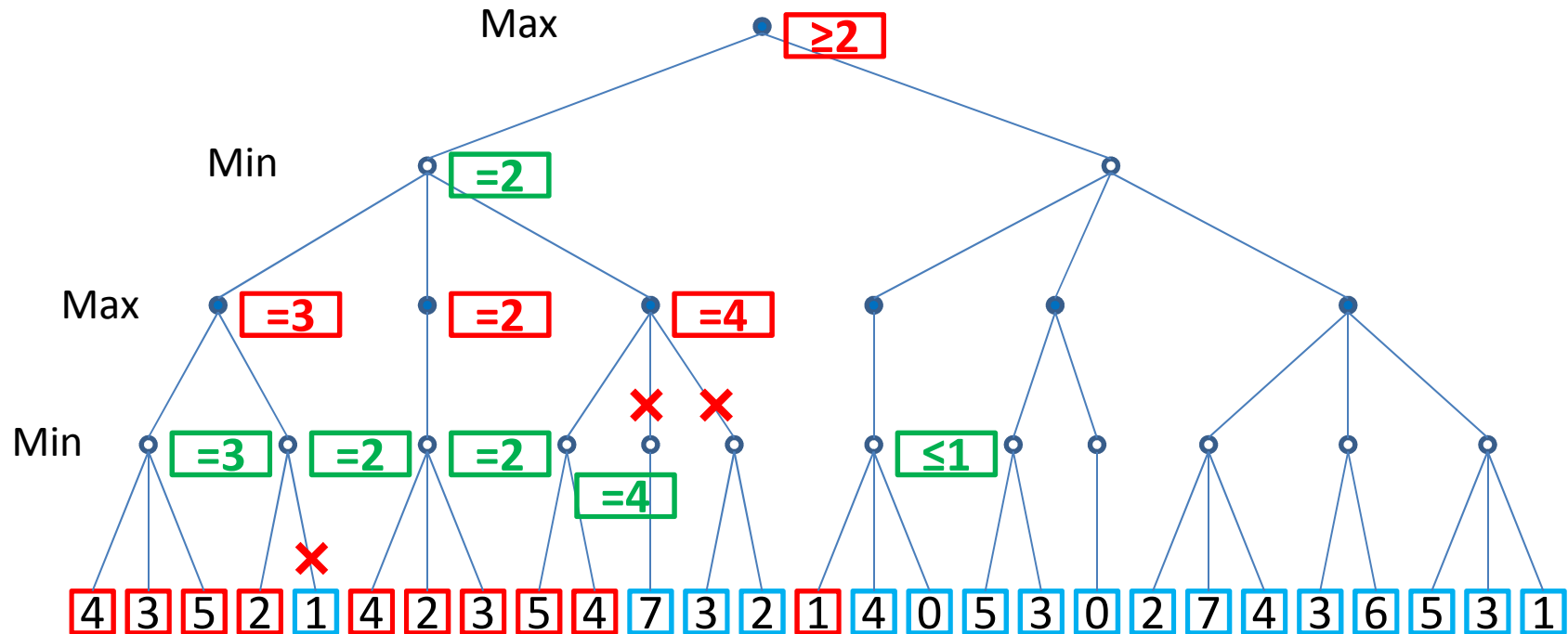
MiniMax with $\alpha\beta$ -pruning



MiniMax with $\alpha\beta$ -pruning

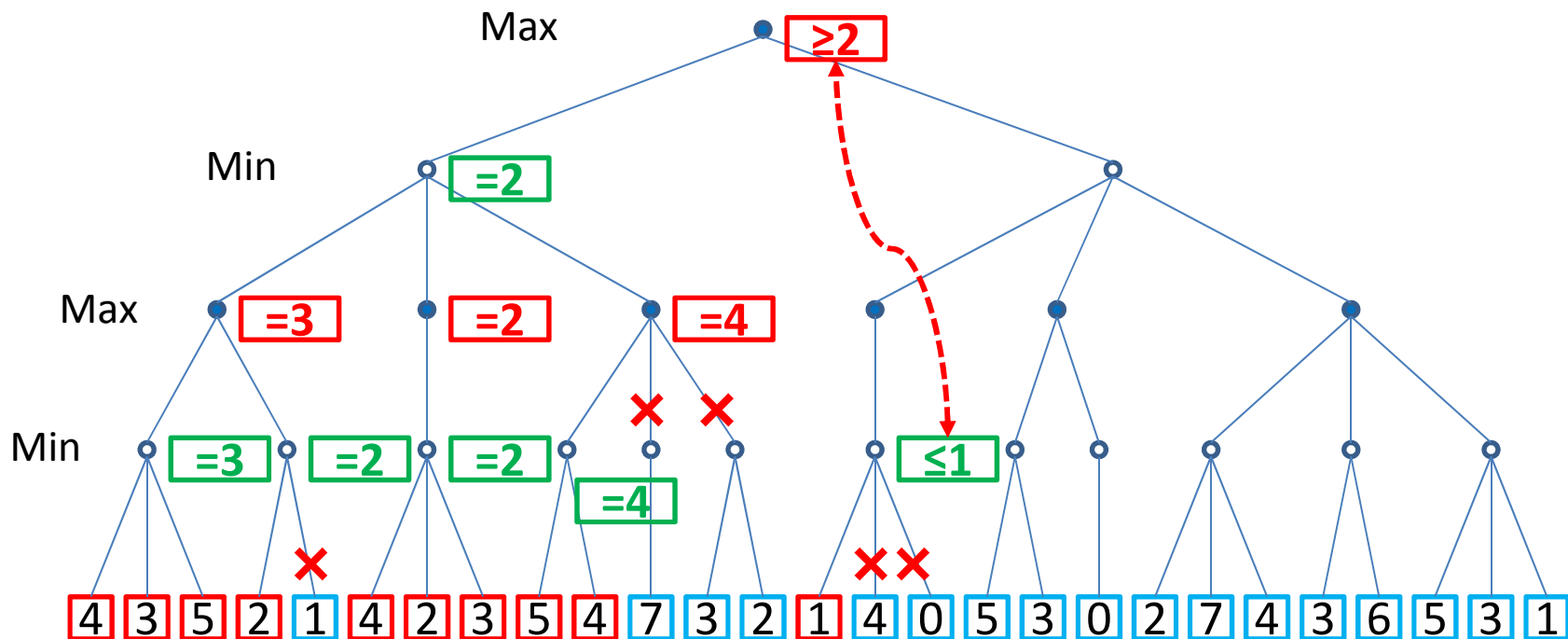


MiniMax with $\alpha\beta$ -pruning

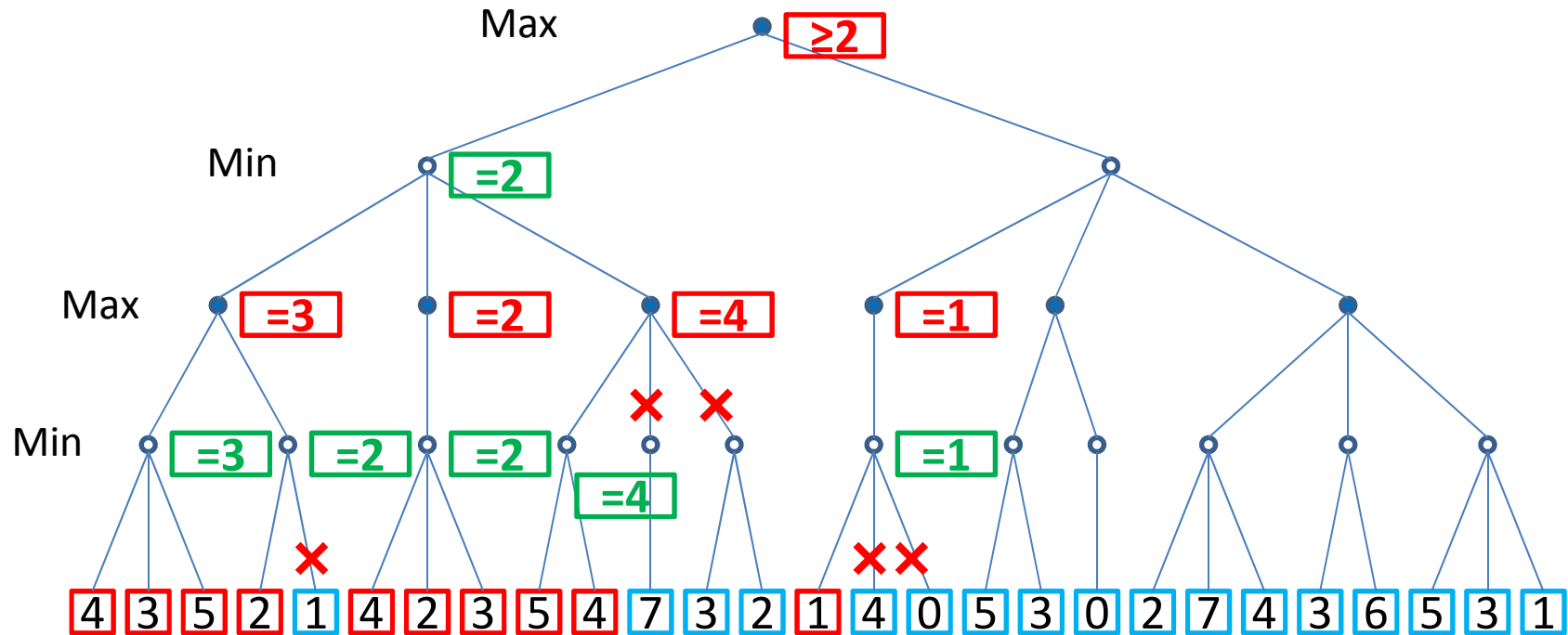


MiniMax with $\alpha\beta$ -pruning

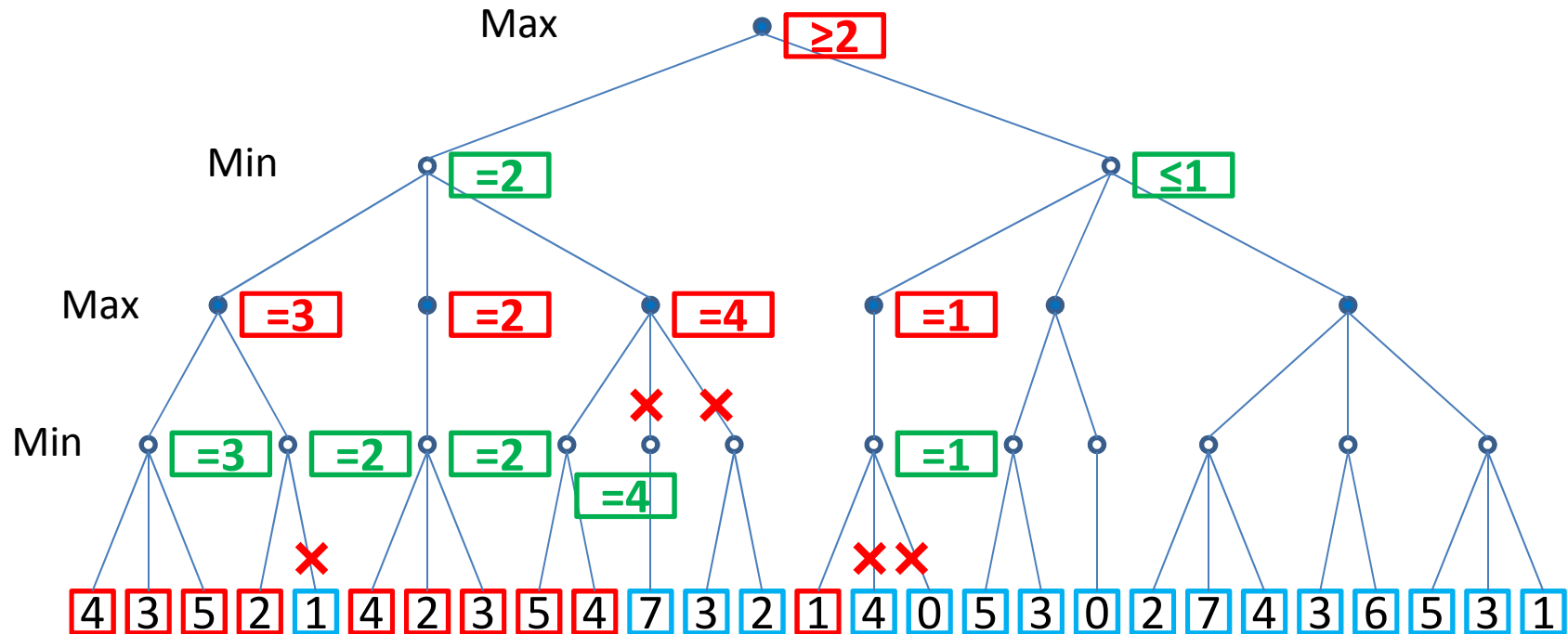
- “Deep” cut-off: **Ancestor β -node $\geq \alpha$ -node**



MiniMax with $\alpha\beta$ -pruning

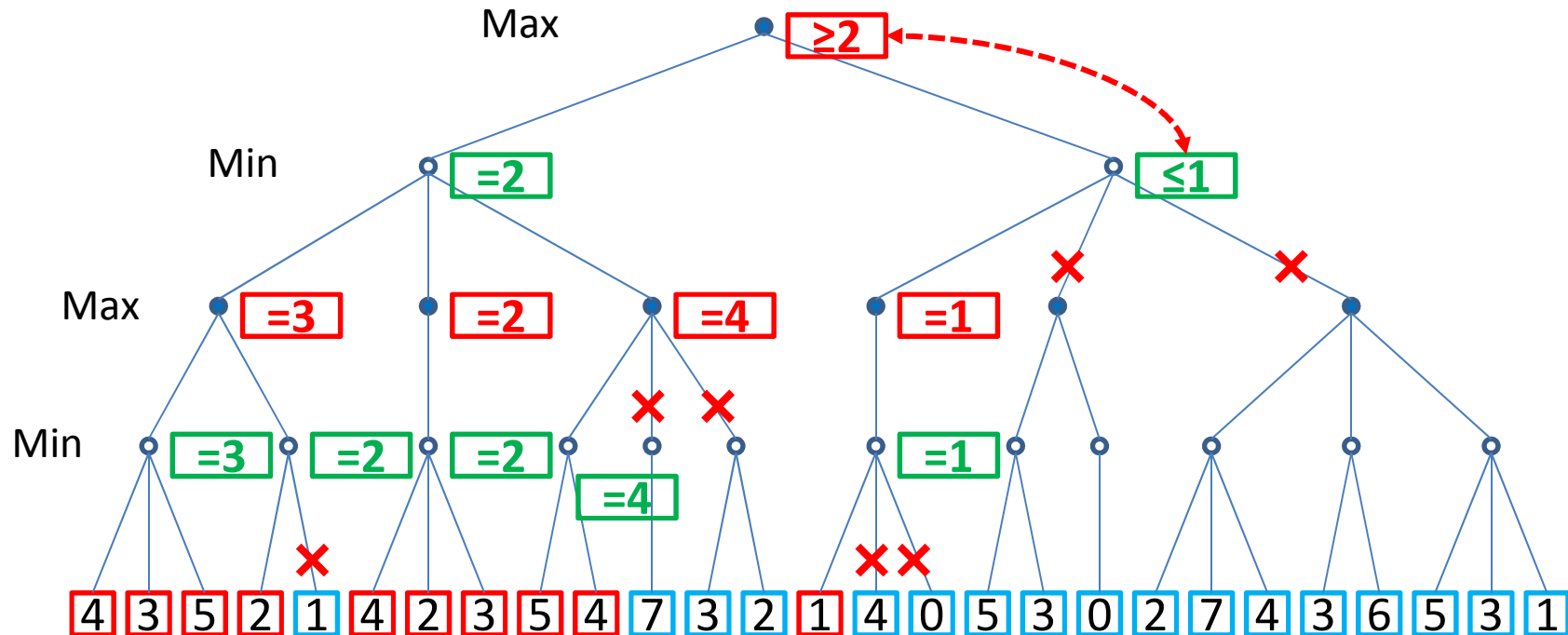


MiniMax with $\alpha\beta$ -pruning



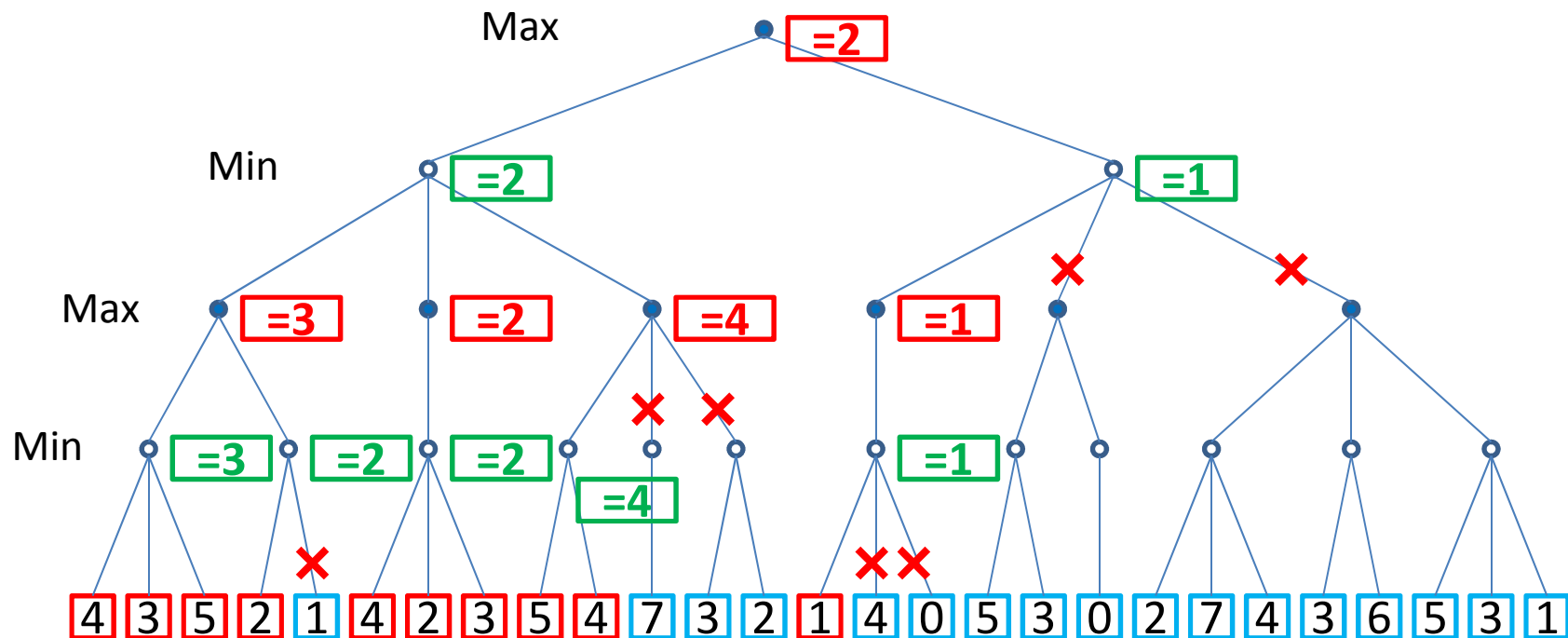
MiniMax with $\alpha\beta$ -pruning

- Prune: **Parent β -node \geq Child α -node**



MiniMax with $\alpha\beta$ -pruning

- **17 static evaluations saved**

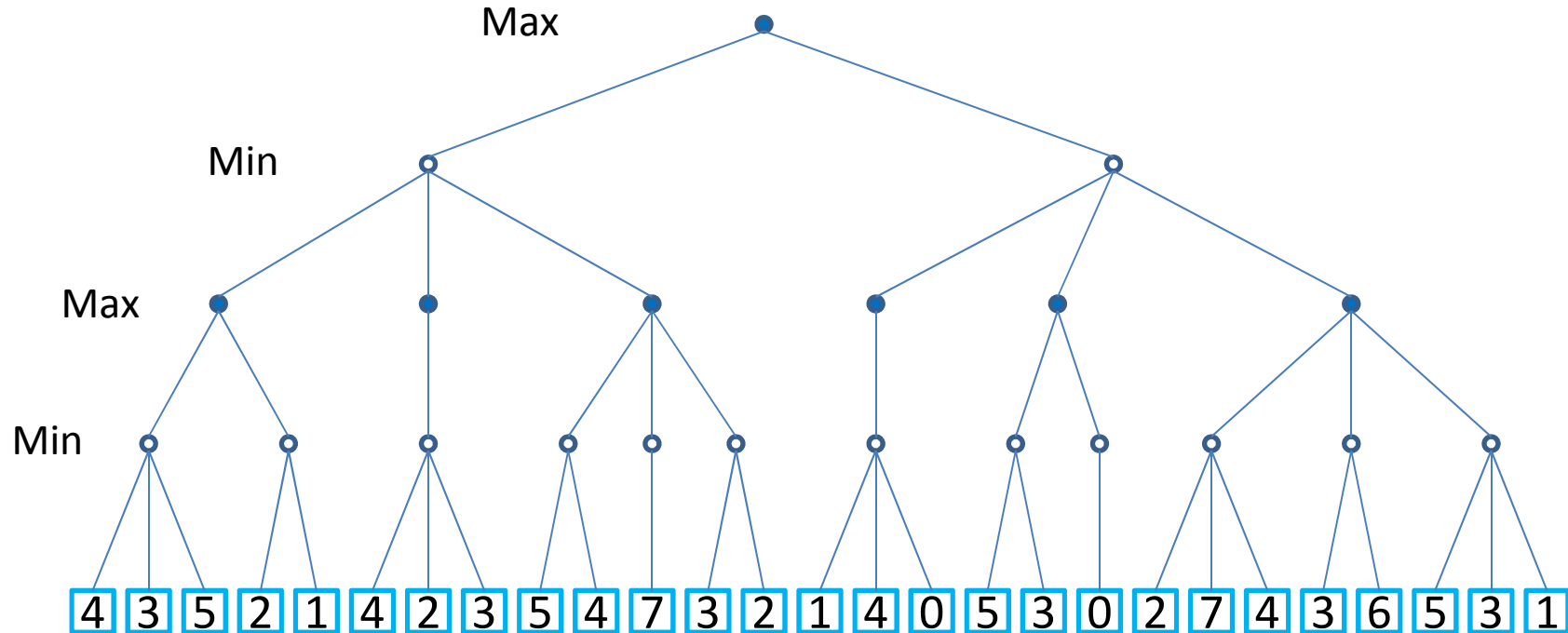


MiniMax & Constraint Processing: MiniMax Algorithm

PROBLEM 2

Problem 2

- Can the nodes be ordered in such a way that $\alpha\beta$ -pruning can cut off more branches?

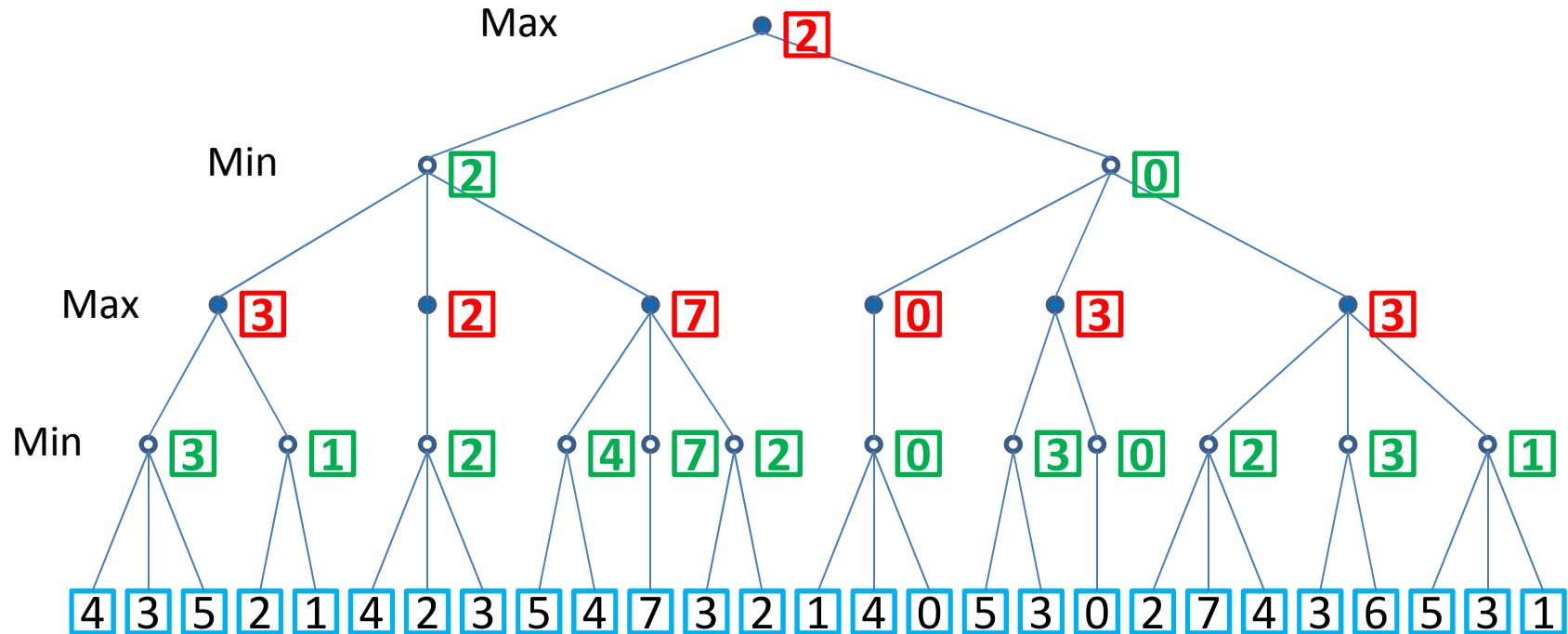


MiniMax & Constraint Processing: MiniMax Algorithm

OPTIMIZING $\alpha\beta$ -PRUNING

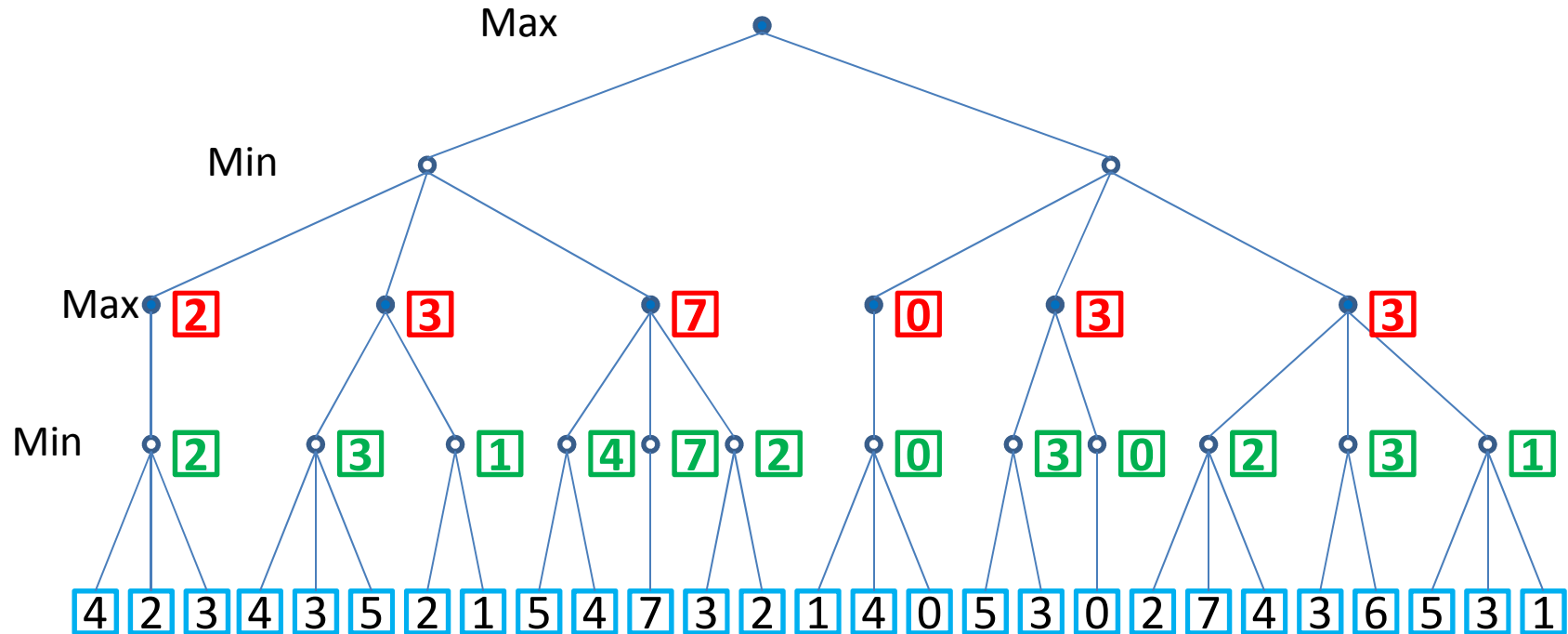
Optimizing $\alpha\beta$ -Pruning

- **Best case:** Each layer best node left-to-right



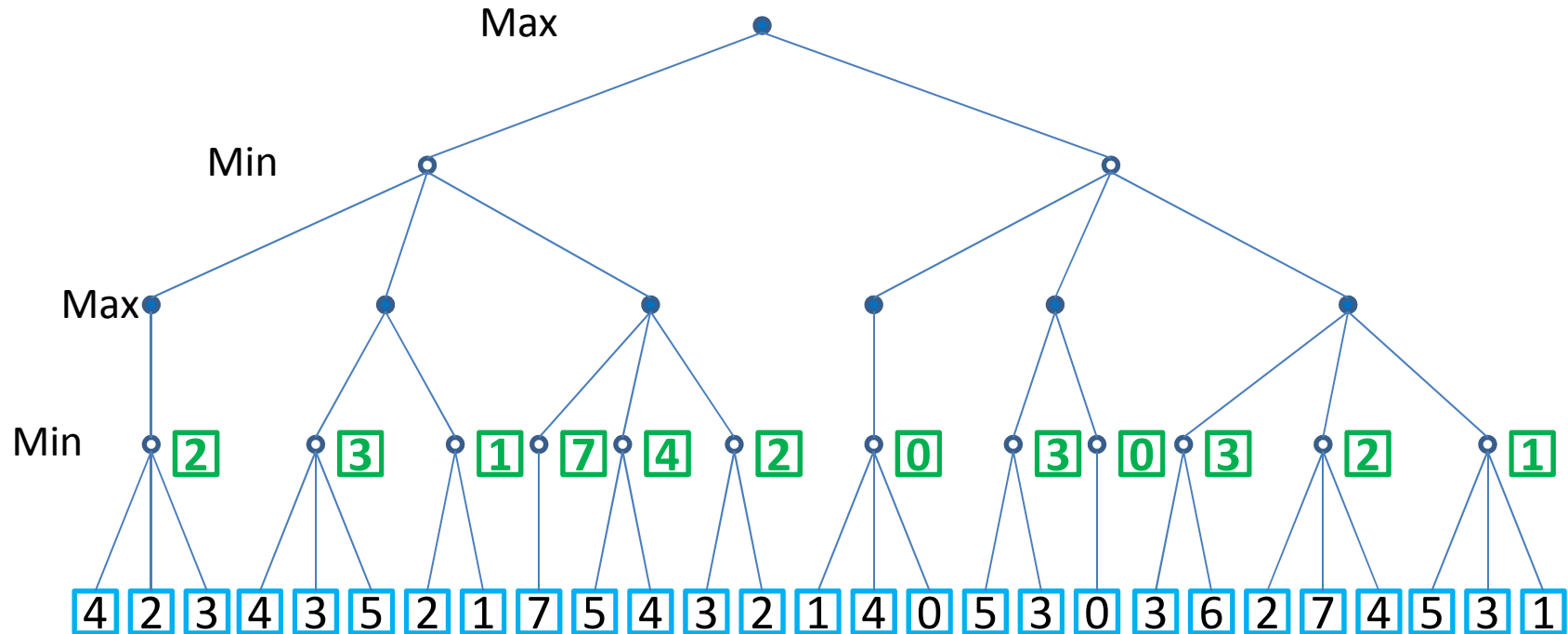
Optimizing $\alpha\beta$ -Pruning

- **Best case:** Each layer best node left-to-right



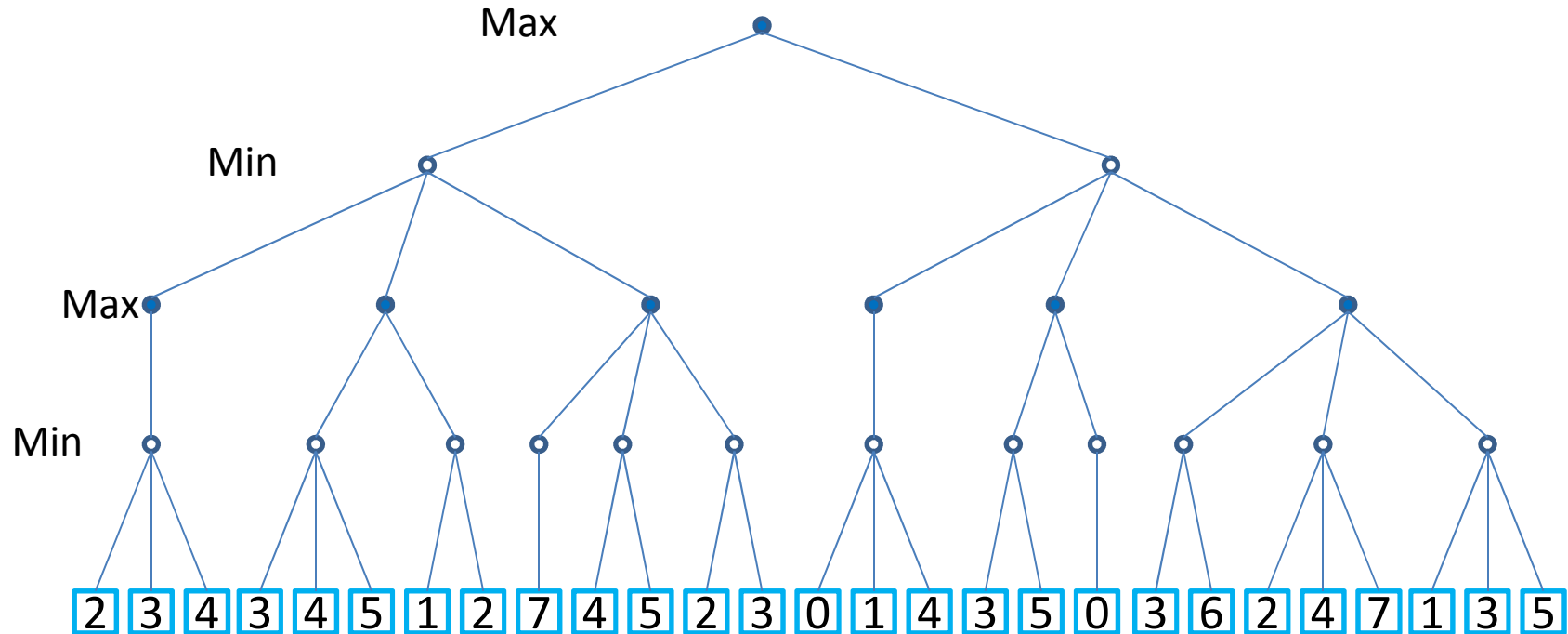
Optimizing $\alpha\beta$ -Pruning

- **Best case:** Each layer best node left-to-right



Optimizing $\alpha\beta$ -Pruning

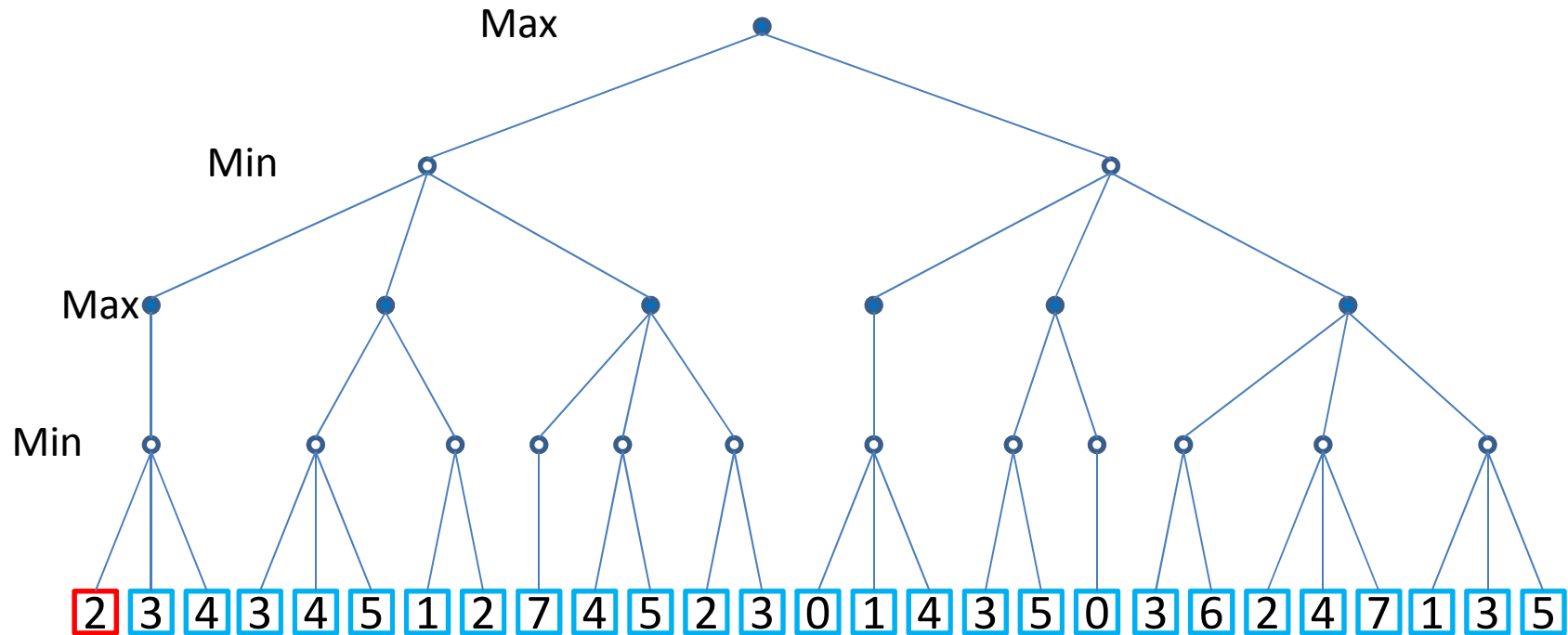
- **Best case:** Each layer best node left-to-right



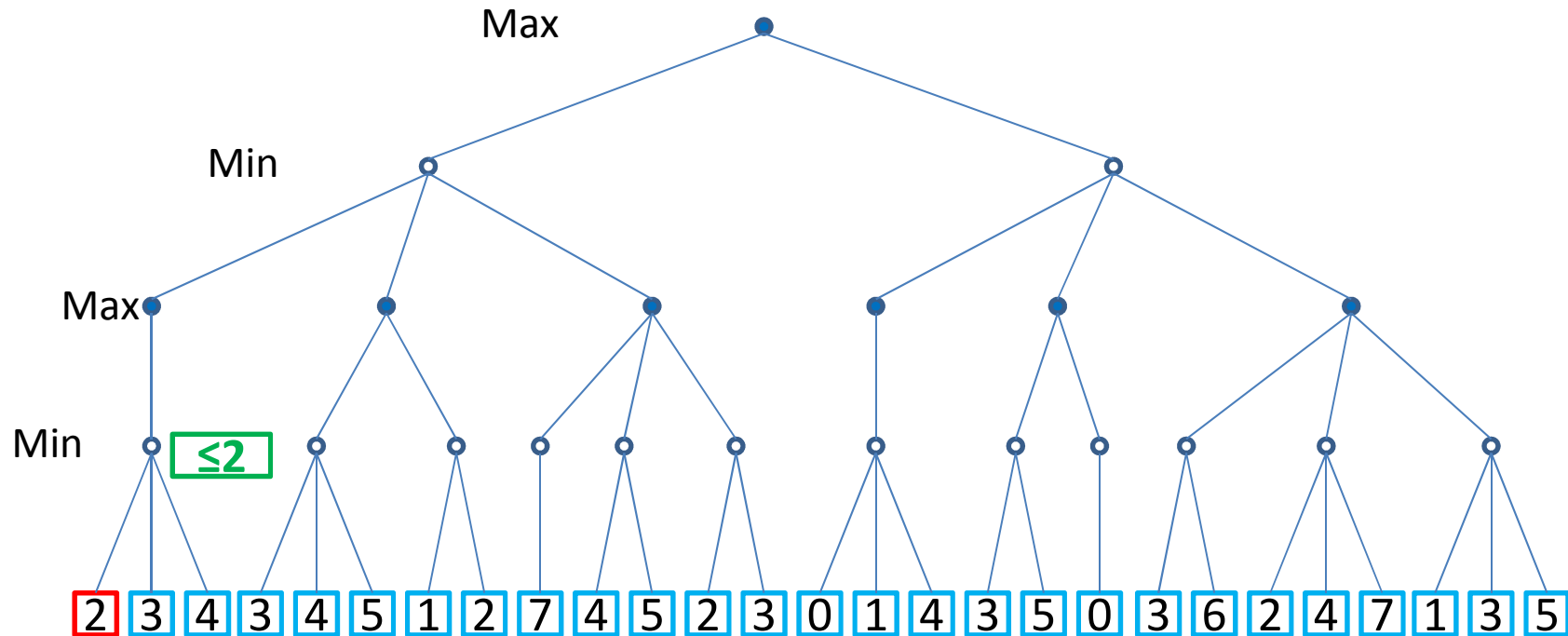
MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX WITH $\alpha\beta$ -PRUNING

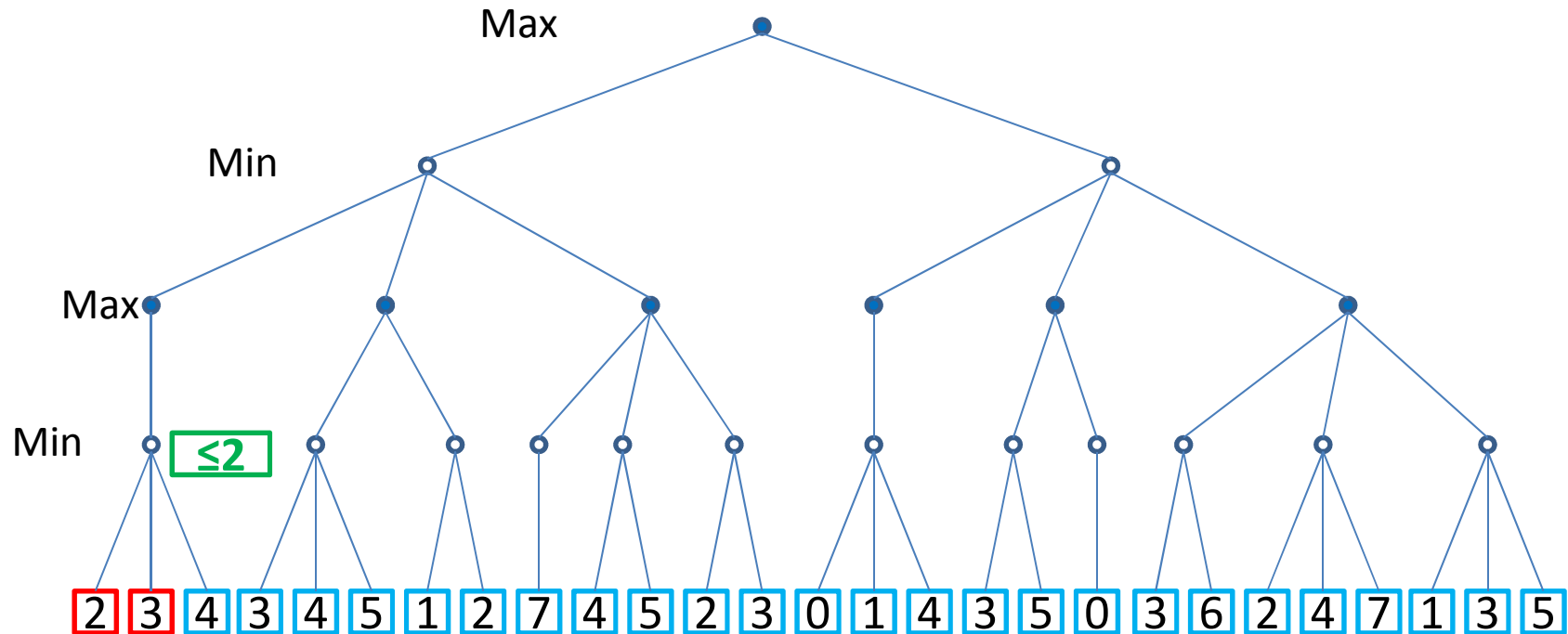
Minimax with $\alpha\beta$ -Pruning



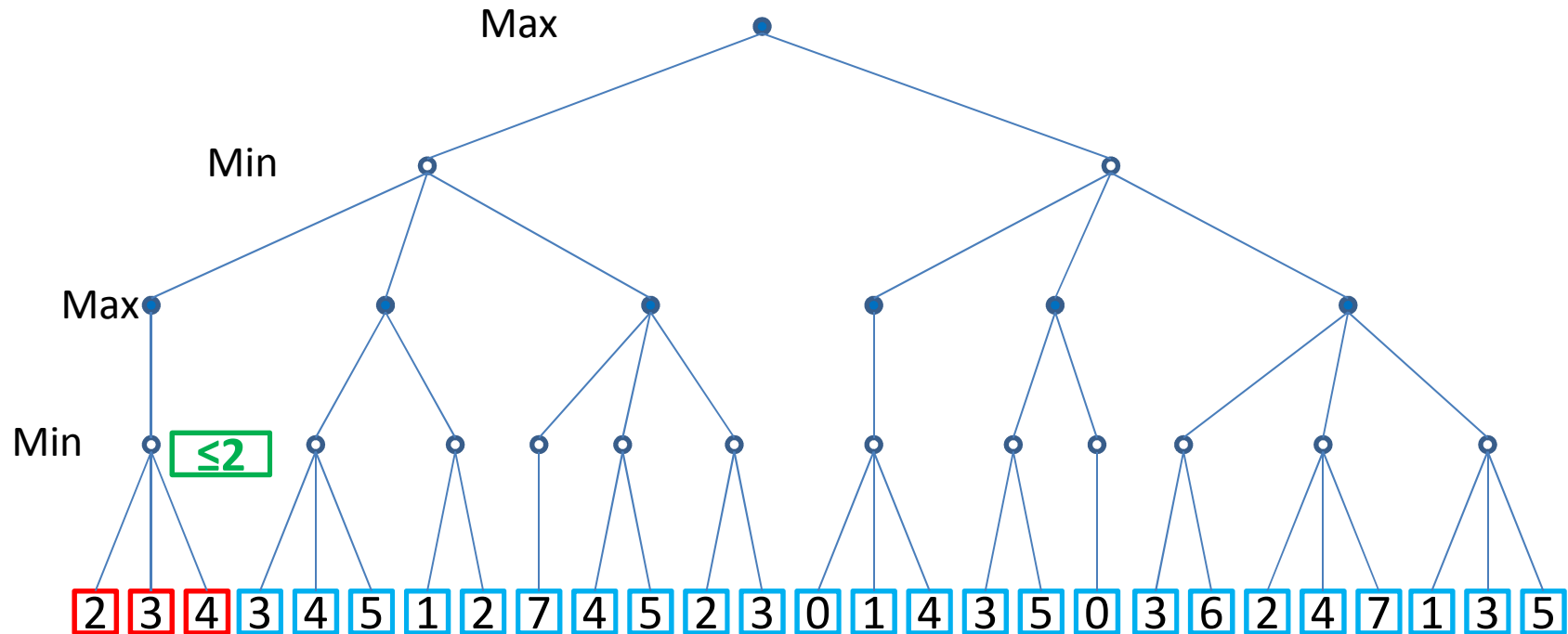
Minimax with $\alpha\beta$ -Pruning



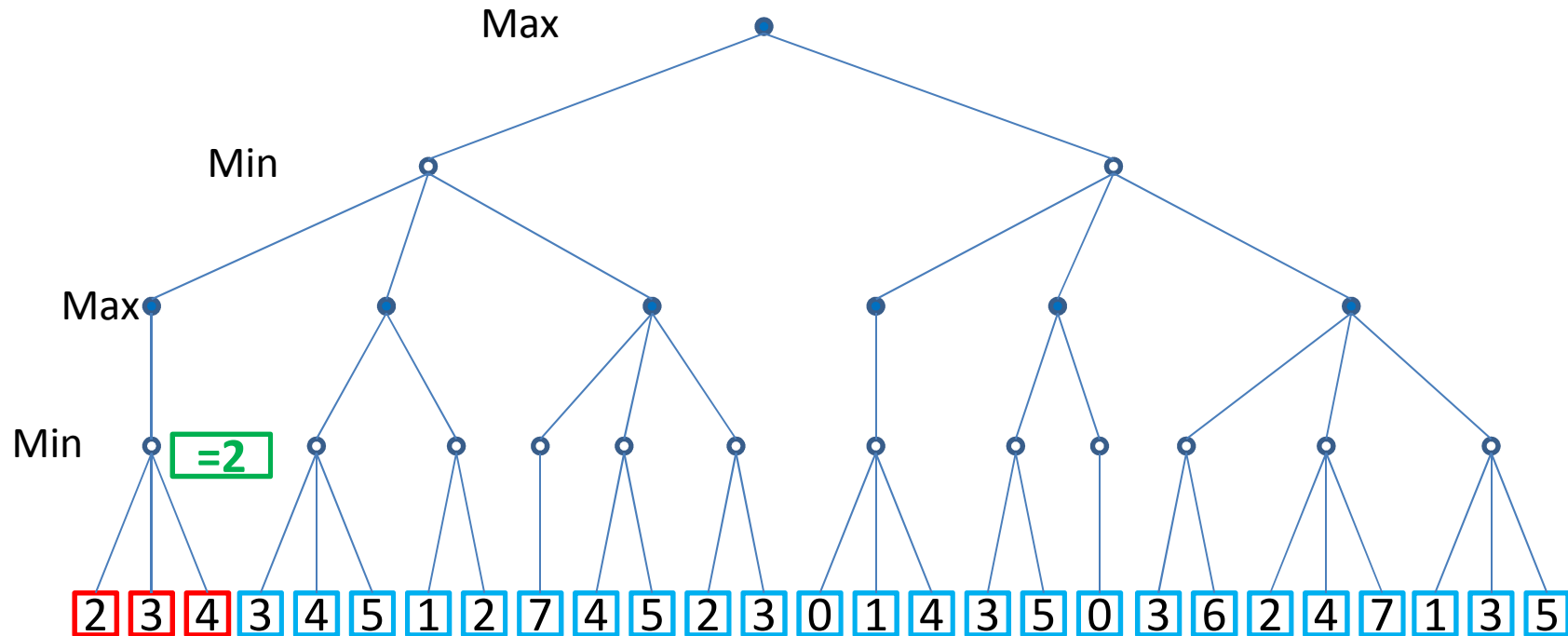
Minimax with $\alpha\beta$ -Pruning



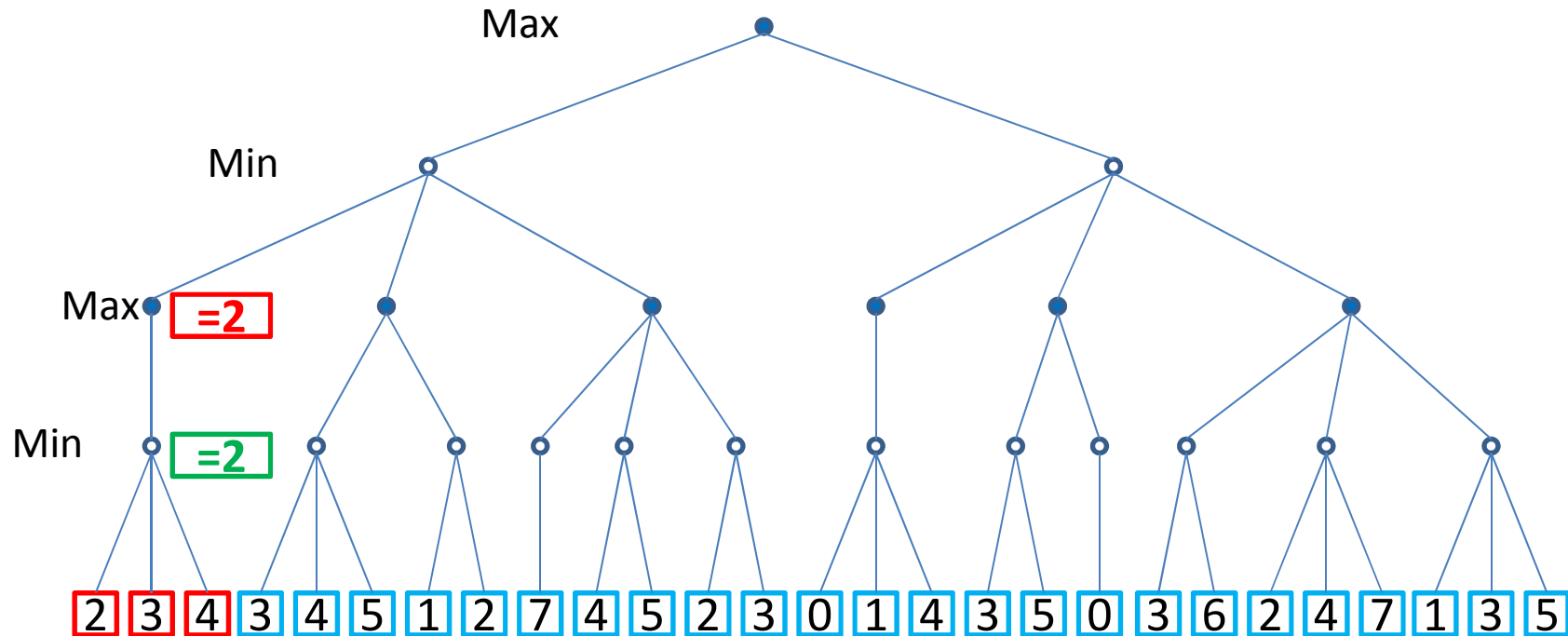
Minimax with $\alpha\beta$ -Pruning



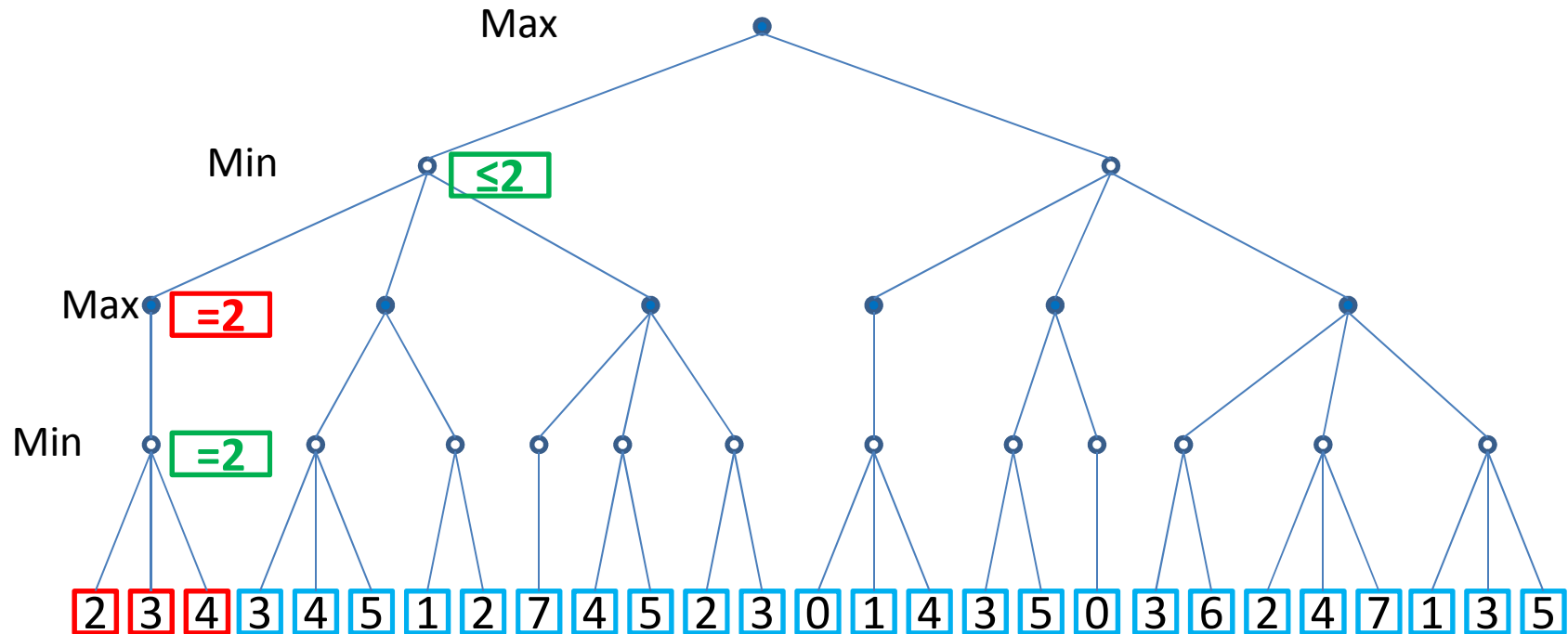
Minimax with $\alpha\beta$ -Pruning



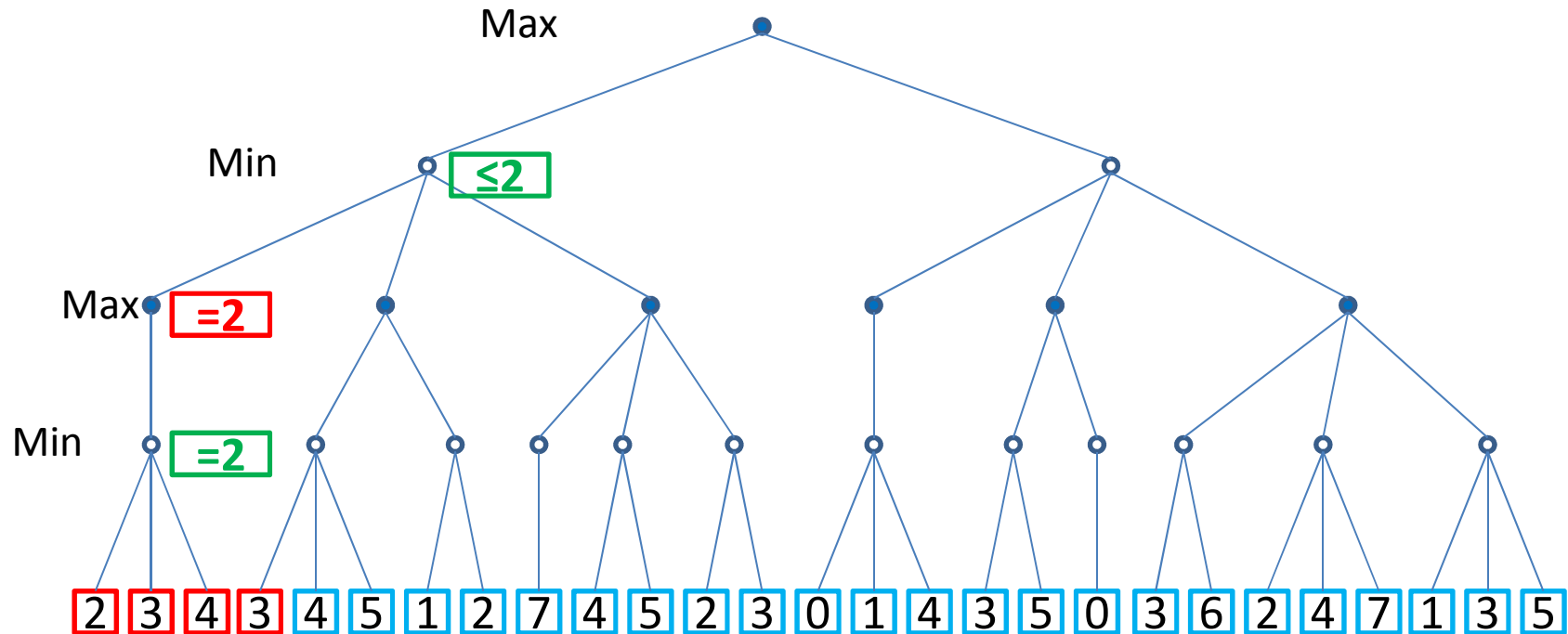
Minimax with $\alpha\beta$ -Pruning



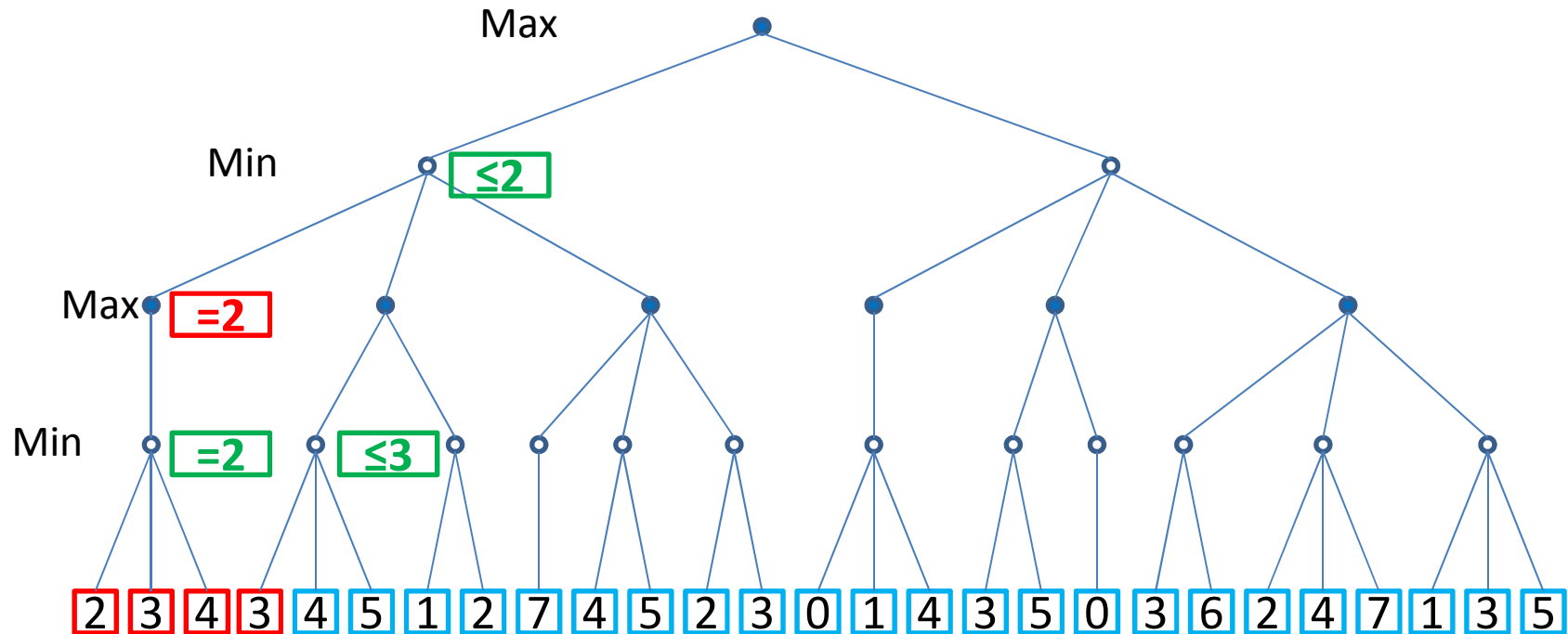
Minimax with $\alpha\beta$ -Pruning



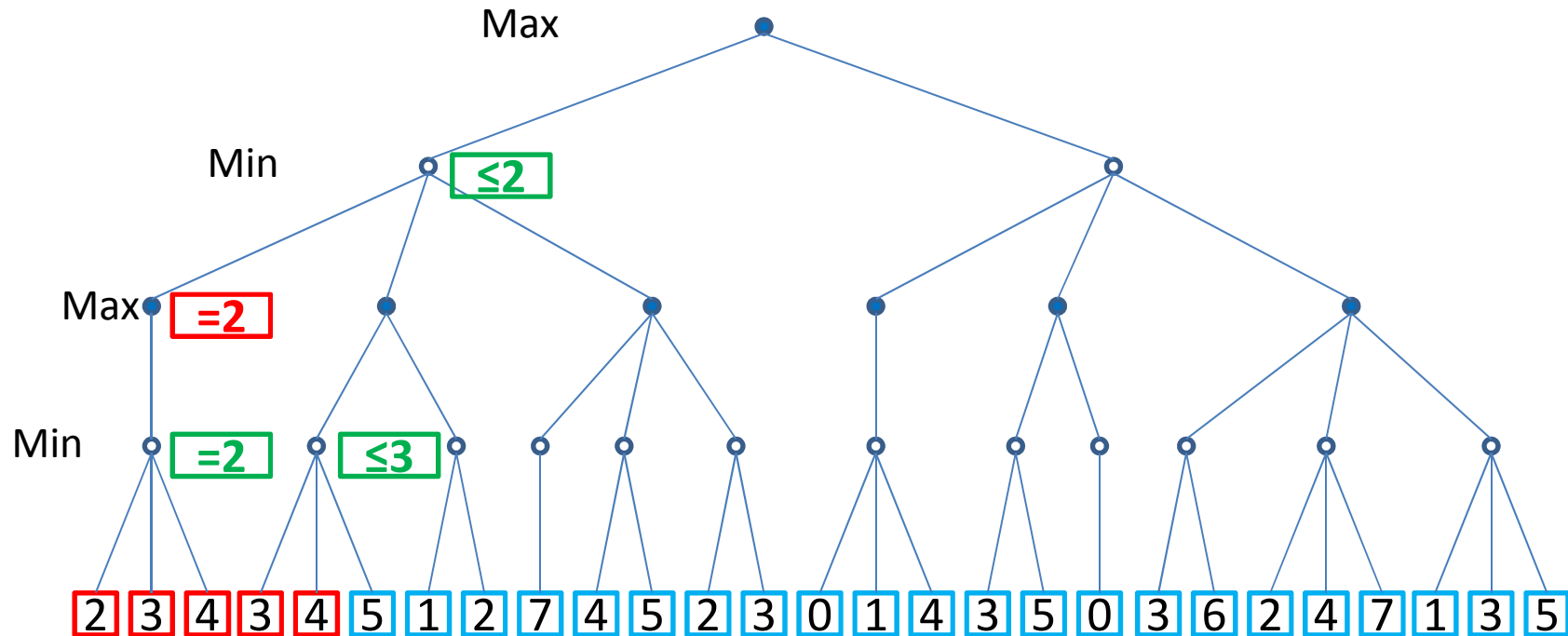
Minimax with $\alpha\beta$ -Pruning



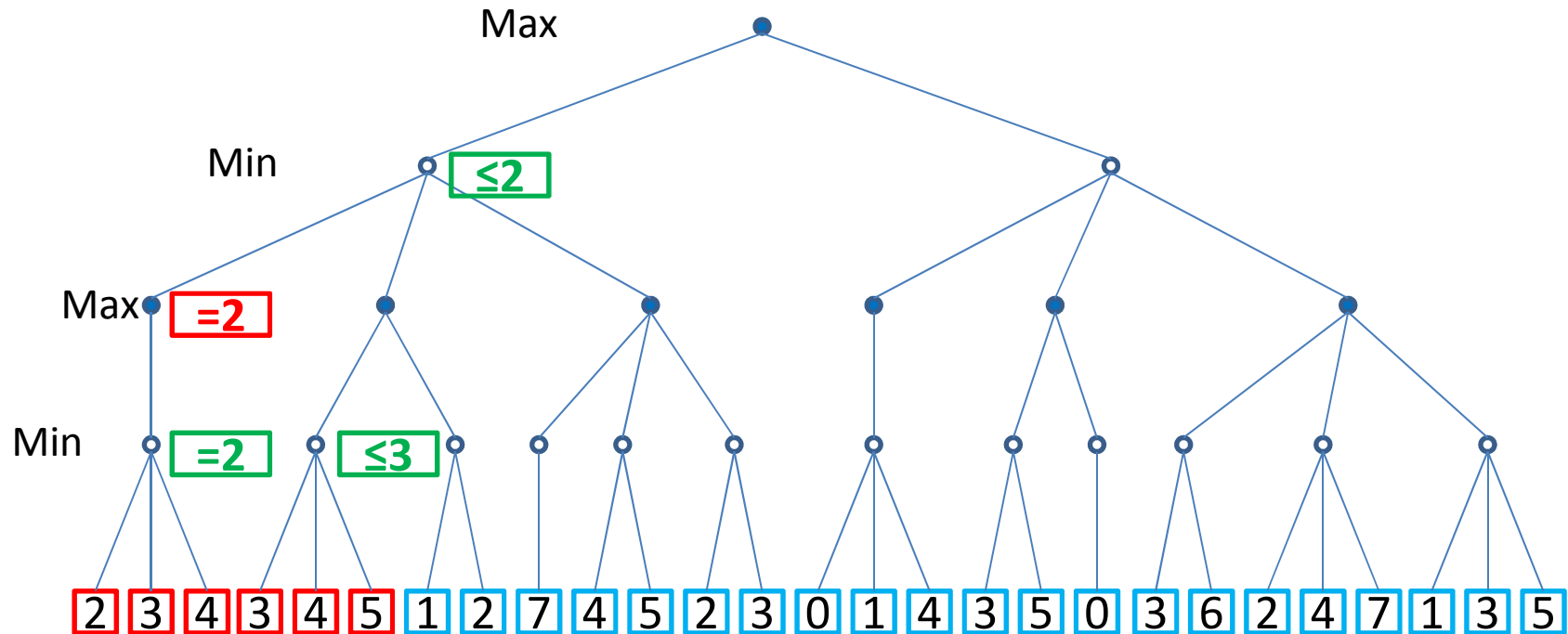
Minimax with $\alpha\beta$ -Pruning



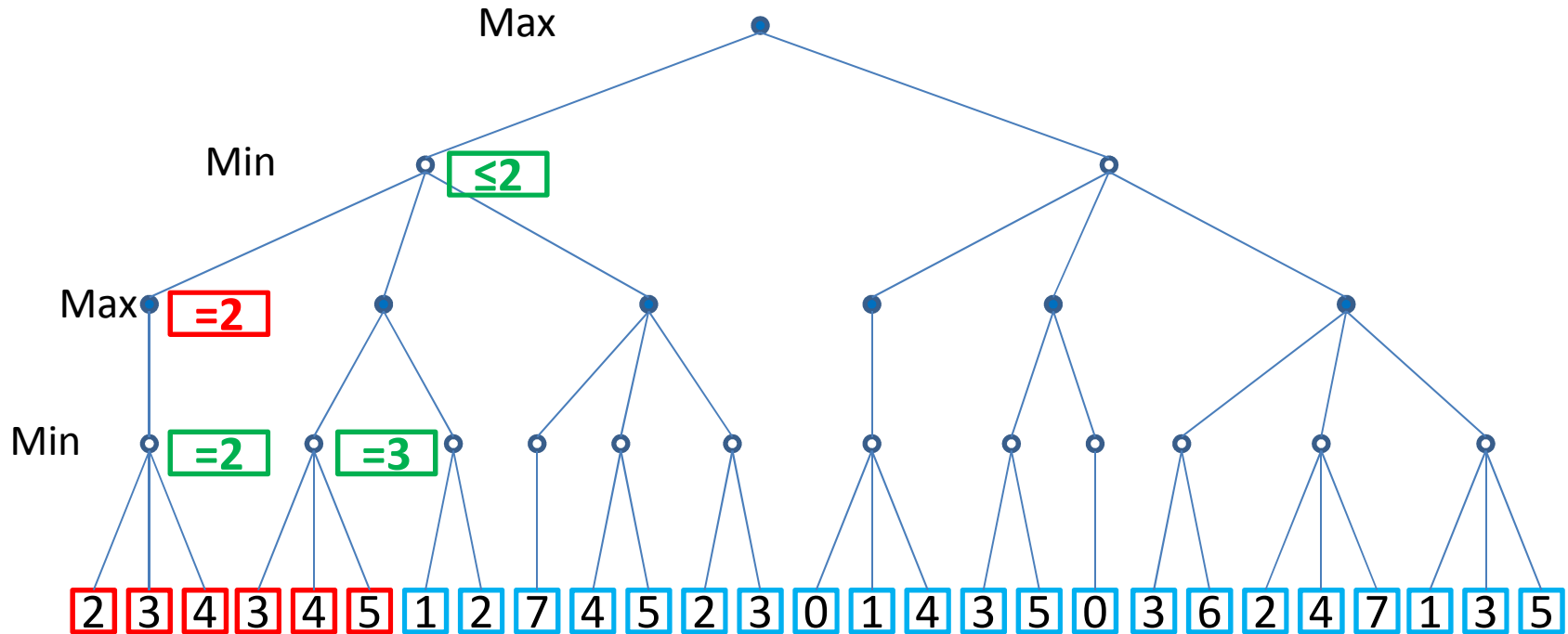
Minimax with $\alpha\beta$ -Pruning



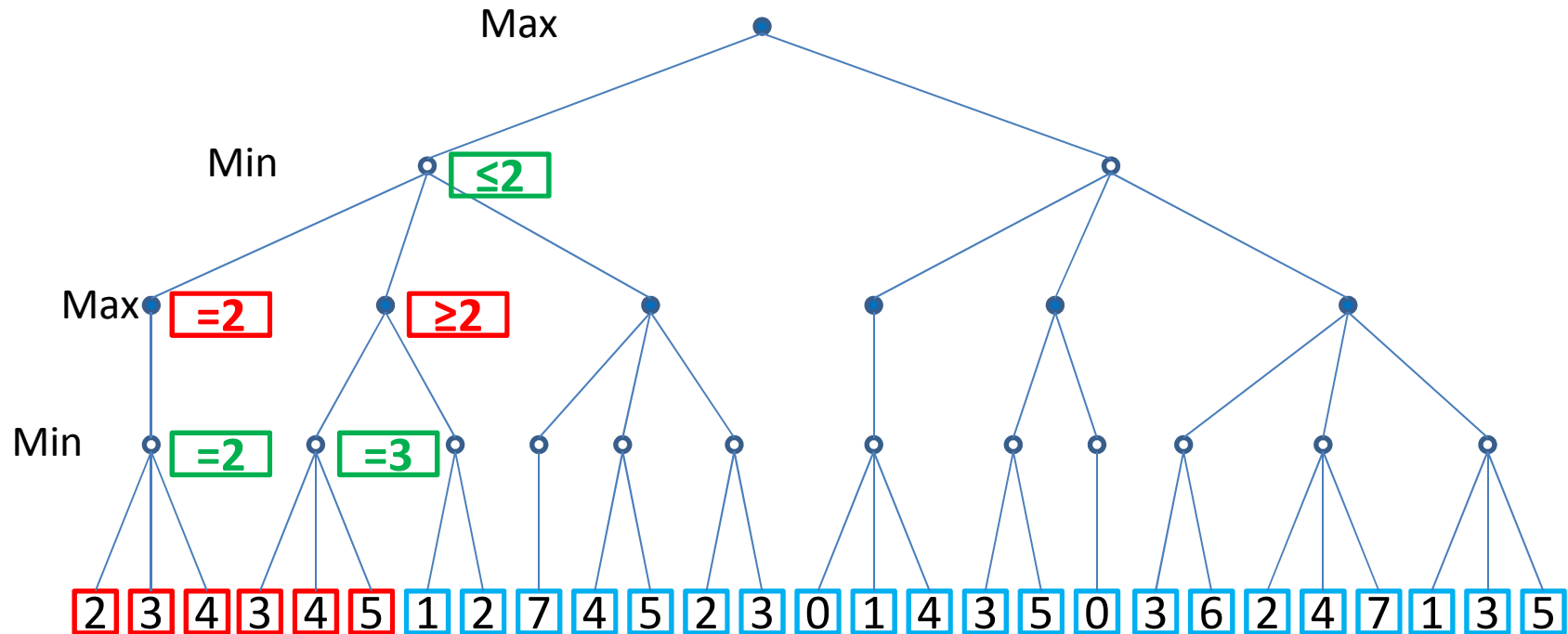
Minimax with $\alpha\beta$ -Pruning



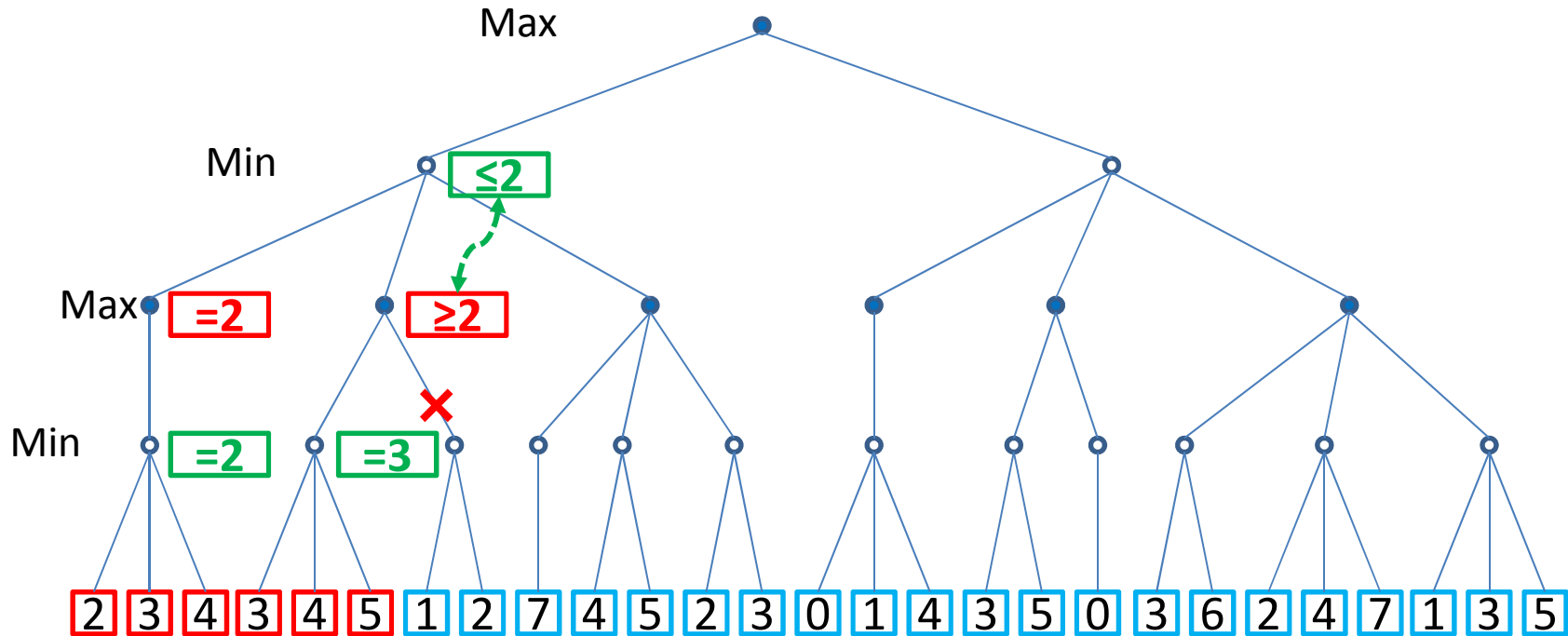
Minimax with $\alpha\beta$ -Pruning



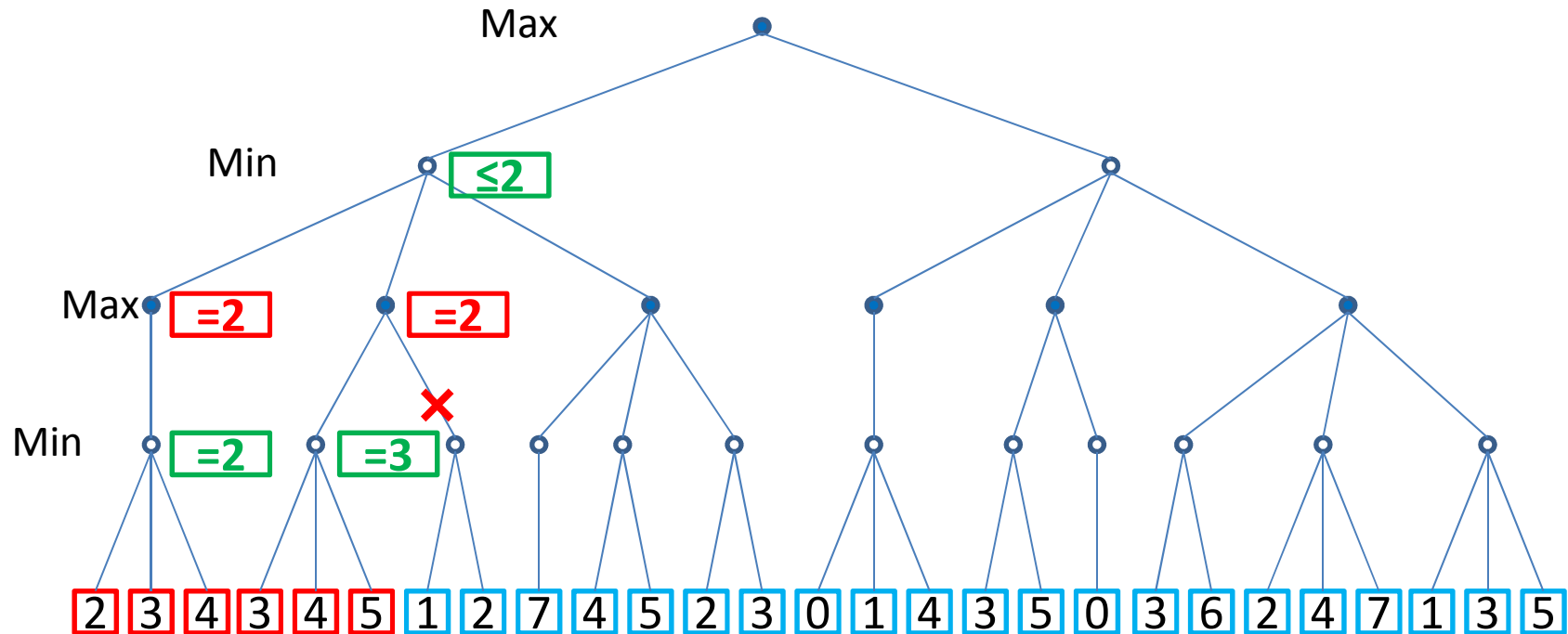
Minimax with $\alpha\beta$ -Pruning



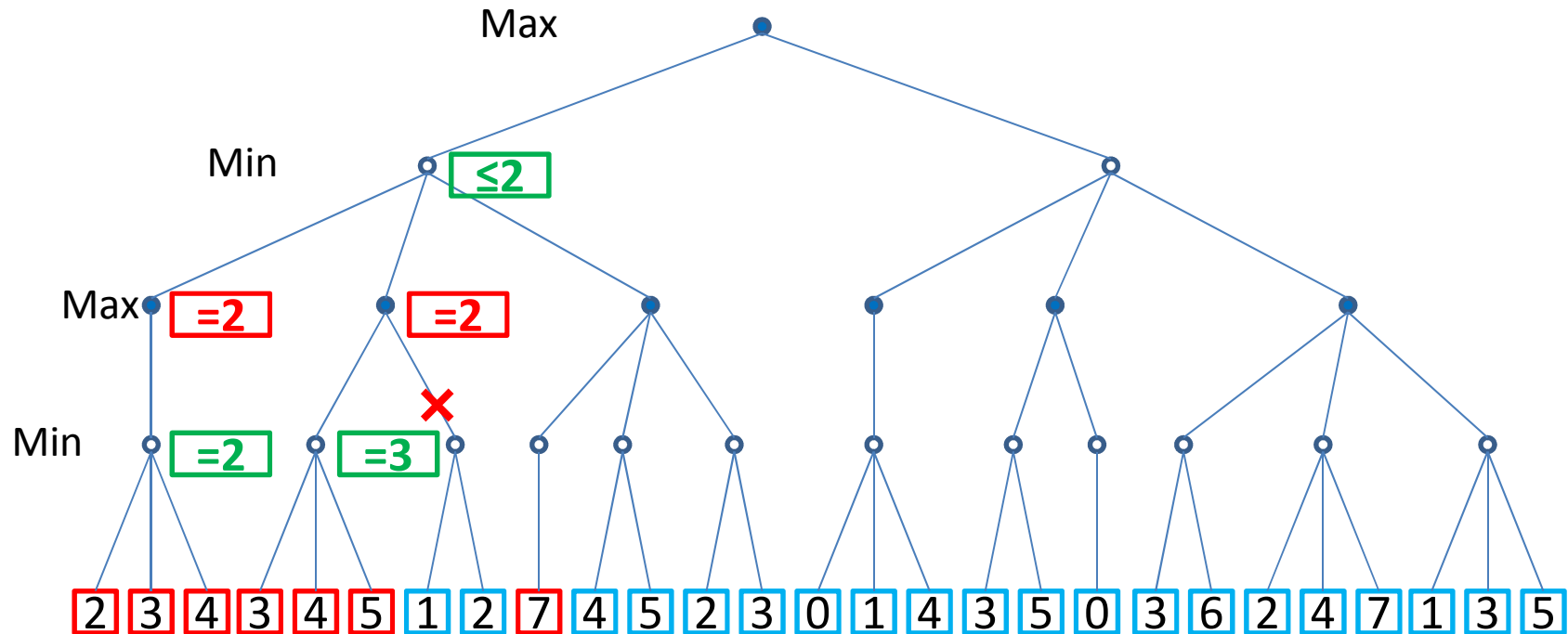
Minimax with $\alpha\beta$ -Pruning



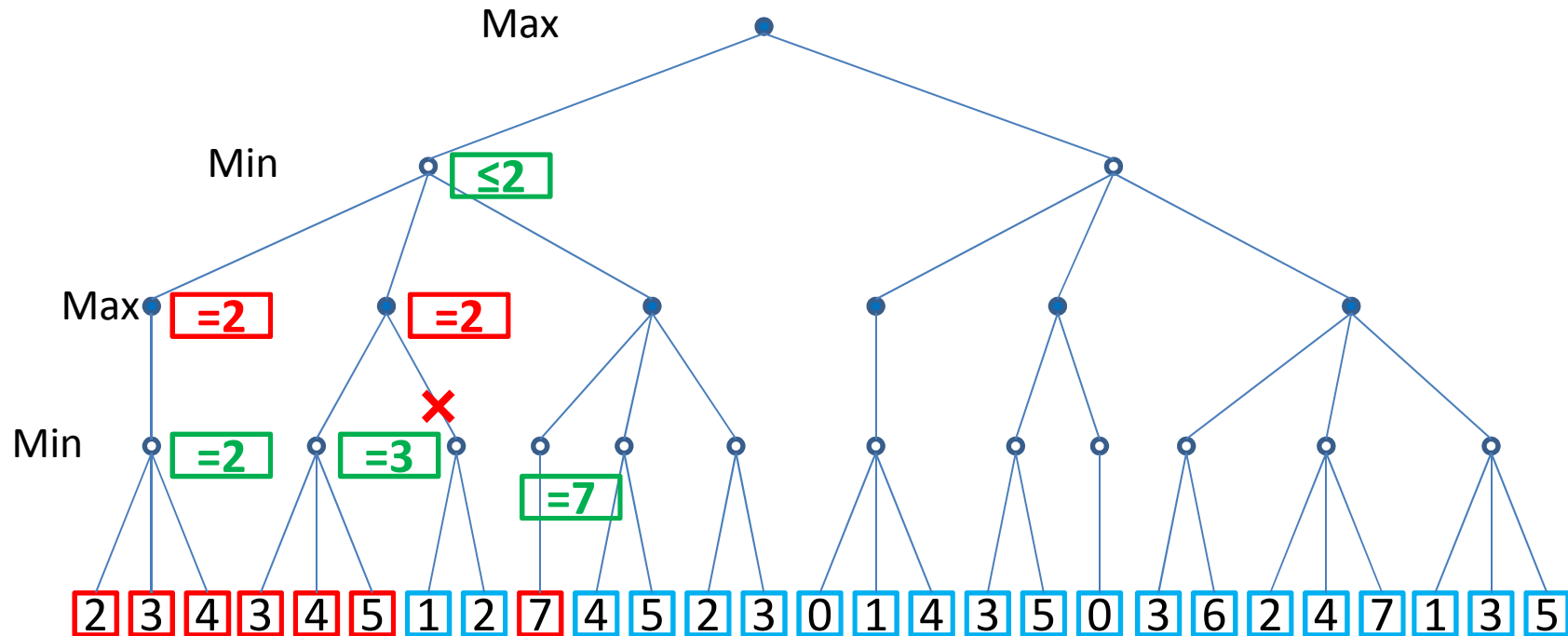
Minimax with $\alpha\beta$ -Pruning



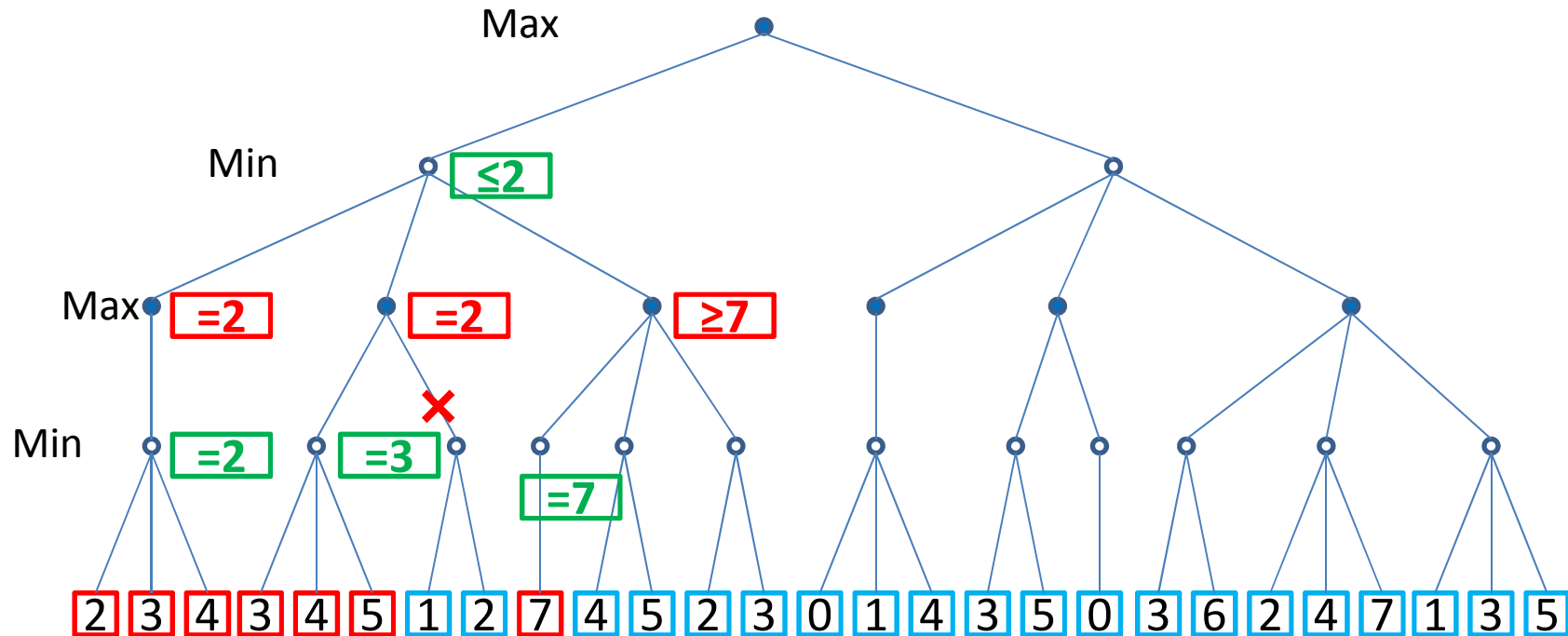
Minimax with $\alpha\beta$ -Pruning



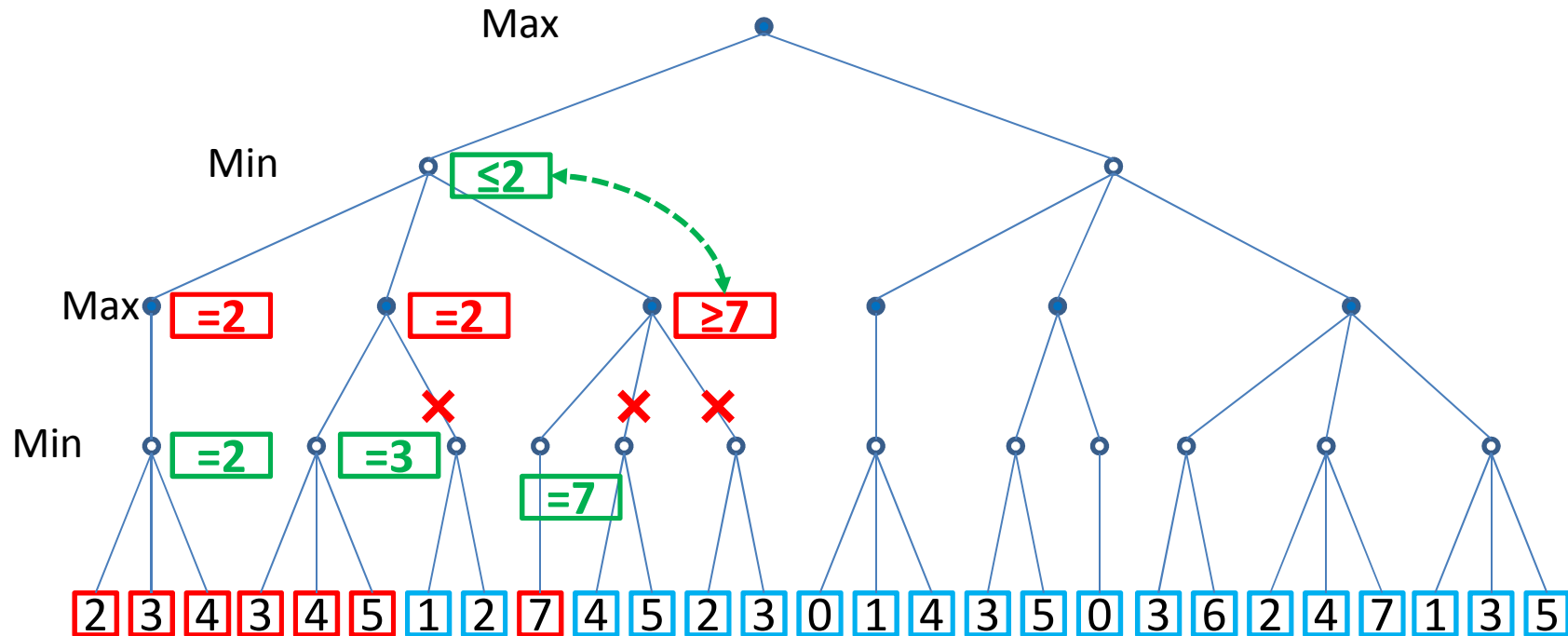
Minimax with $\alpha\beta$ -Pruning



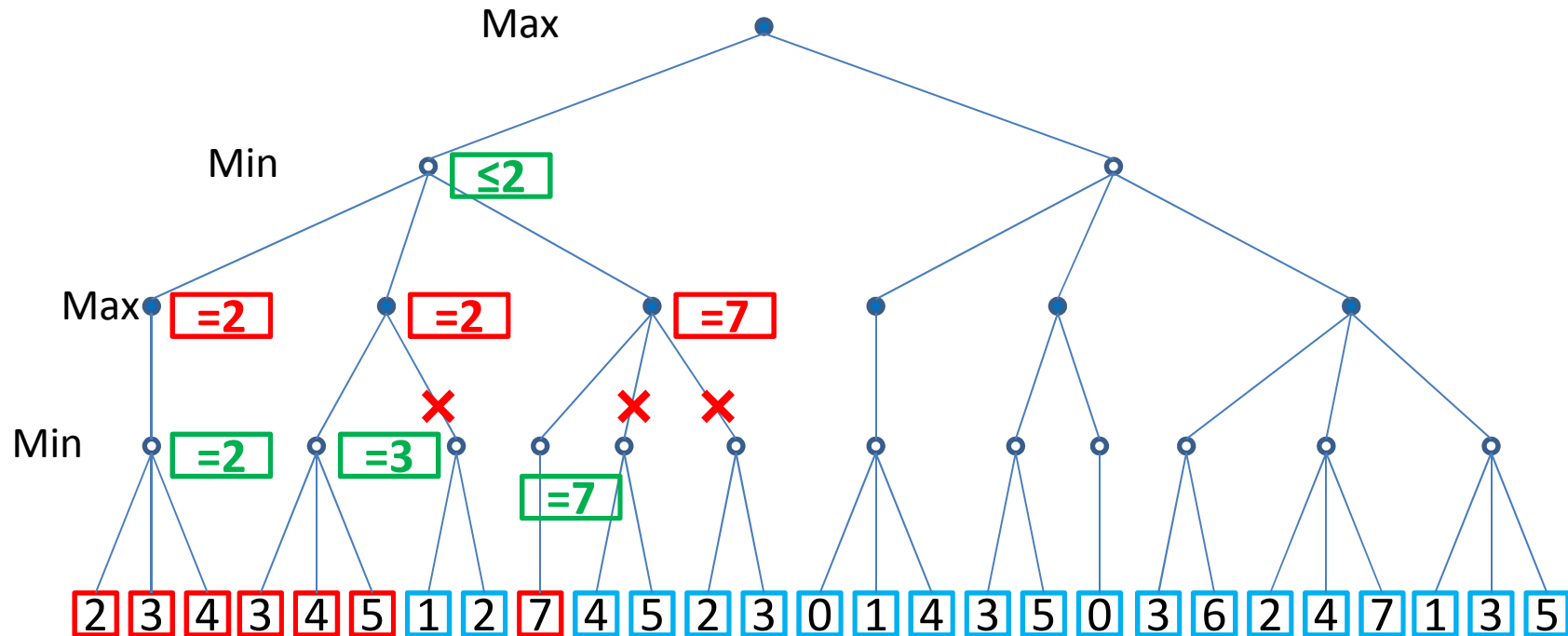
Minimax with $\alpha\beta$ -Pruning



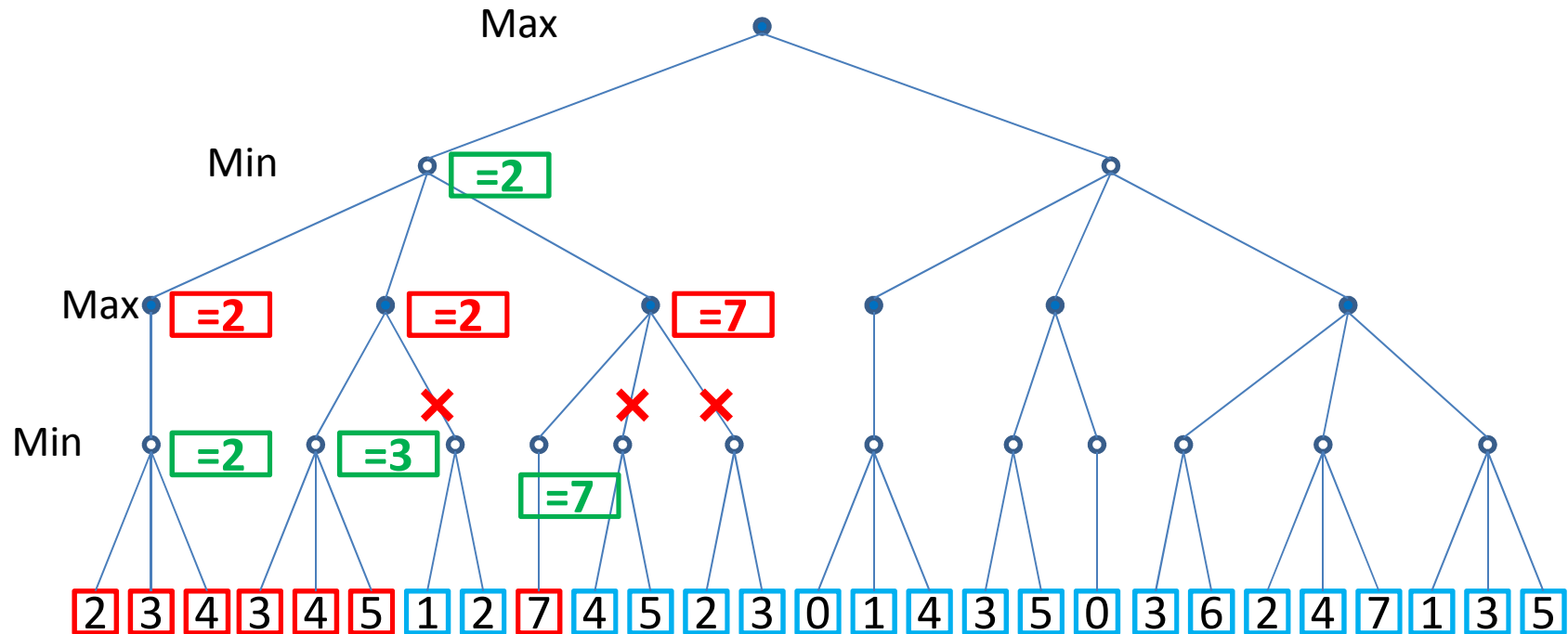
Minimax with $\alpha\beta$ -Pruning



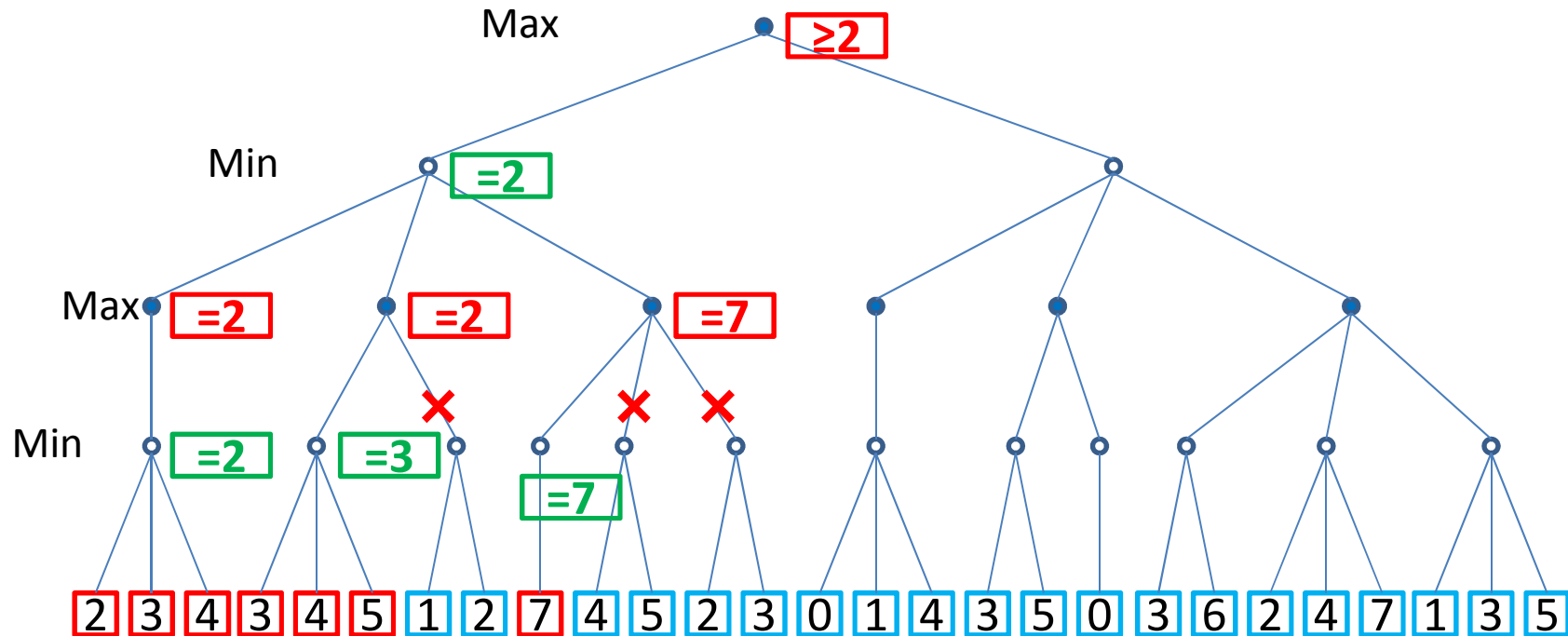
Minimax with $\alpha\beta$ -Pruning



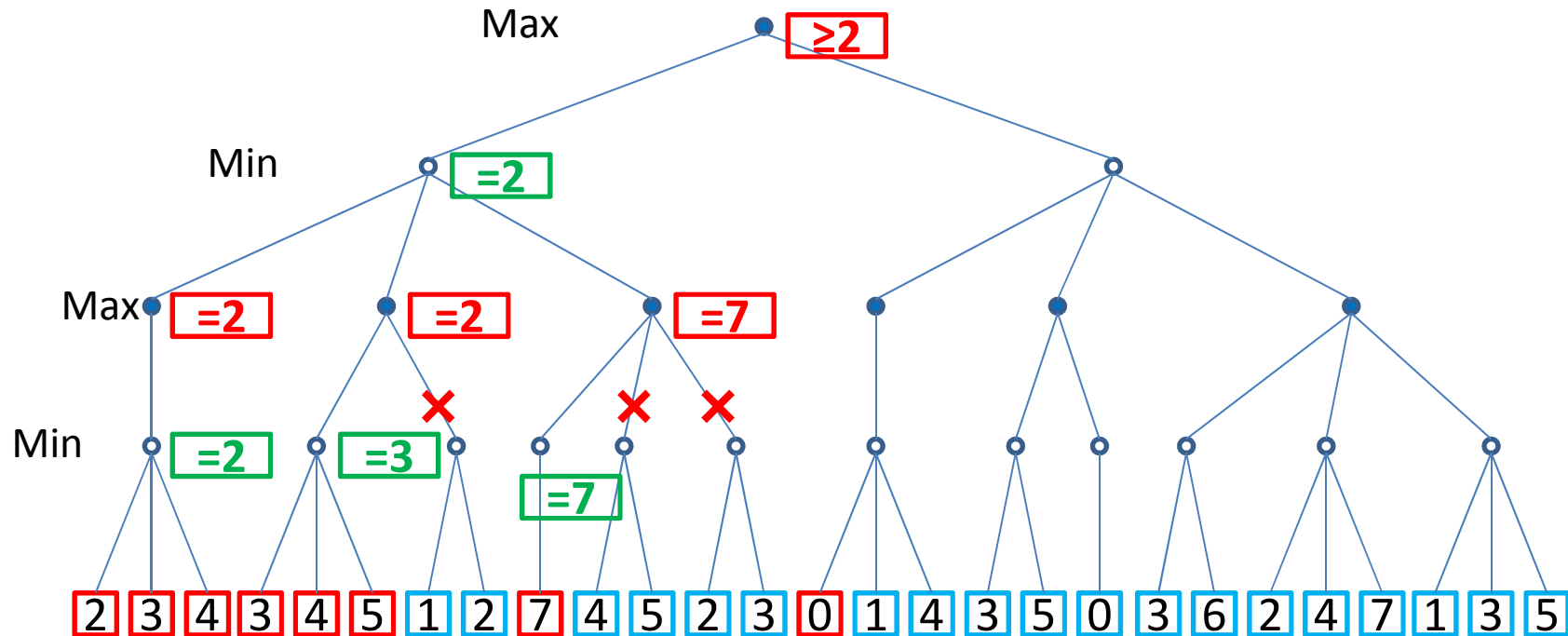
Minimax with $\alpha\beta$ -Pruning



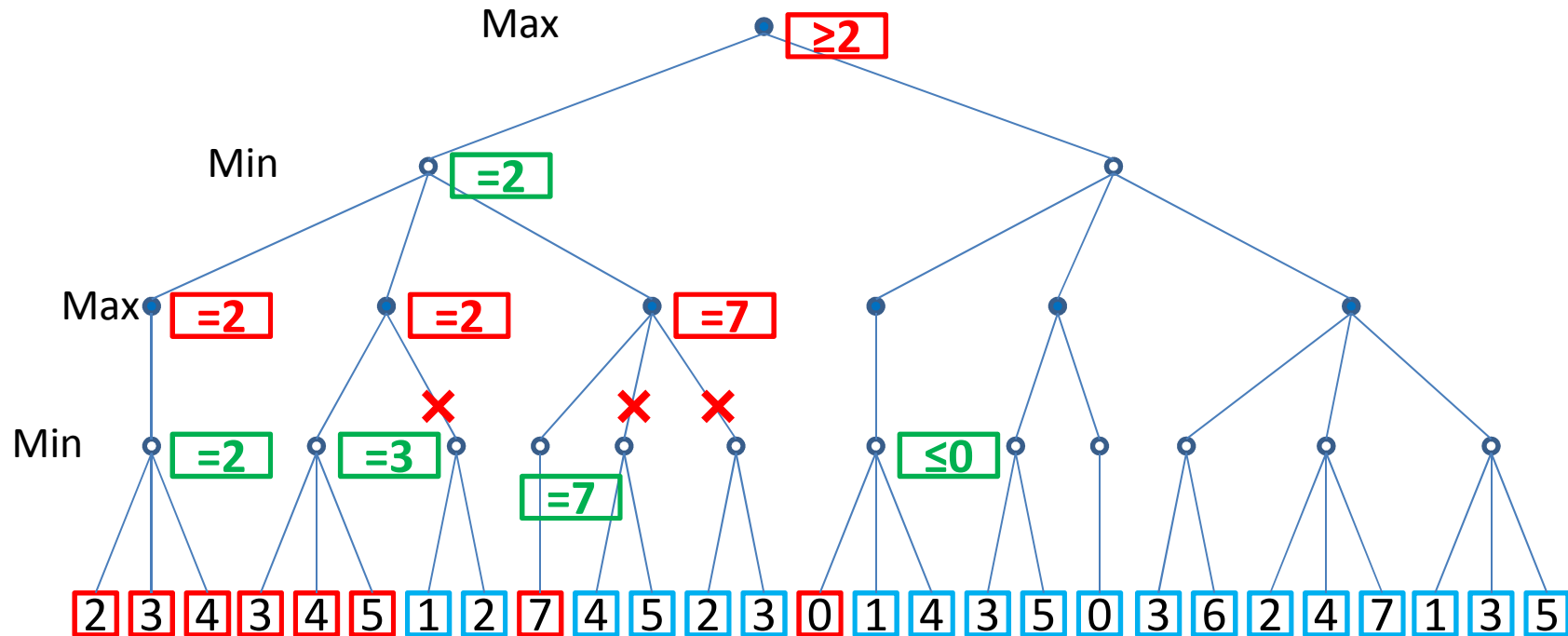
Minimax with $\alpha\beta$ -Pruning



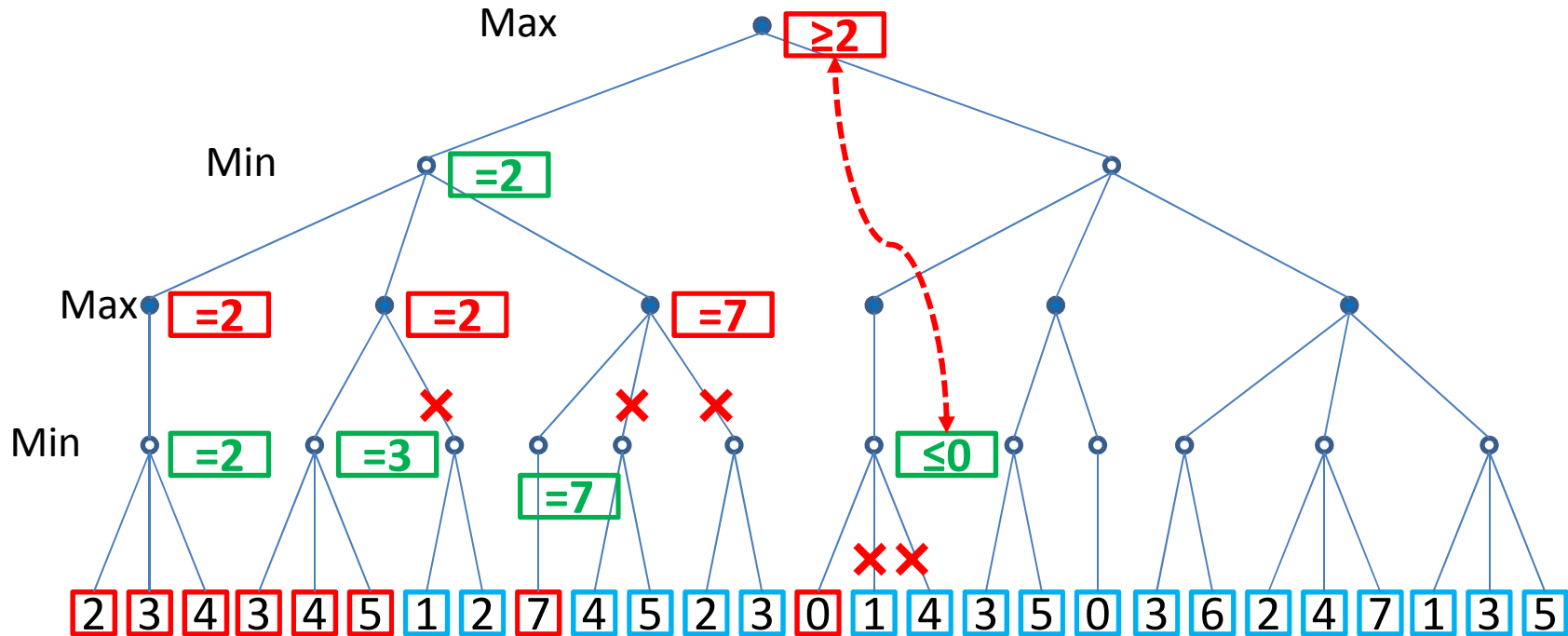
Minimax with $\alpha\beta$ -Pruning



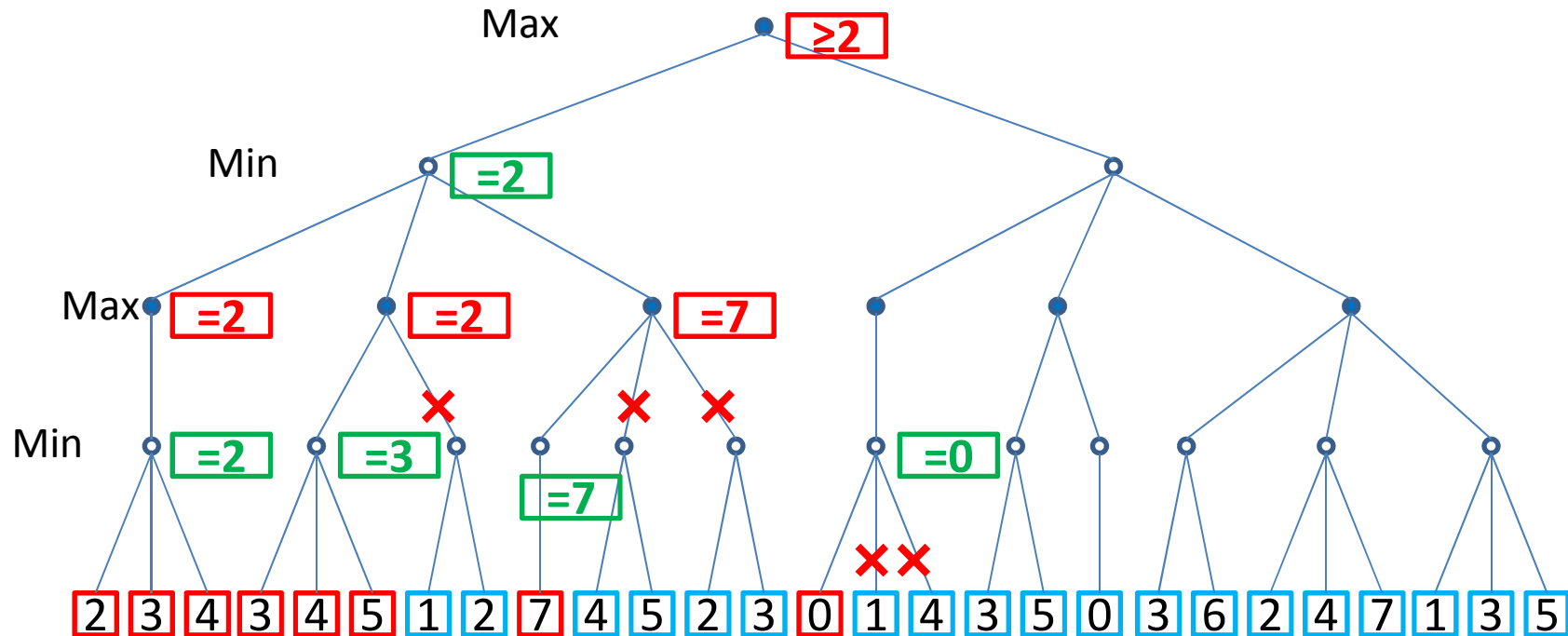
Minimax with $\alpha\beta$ -Pruning



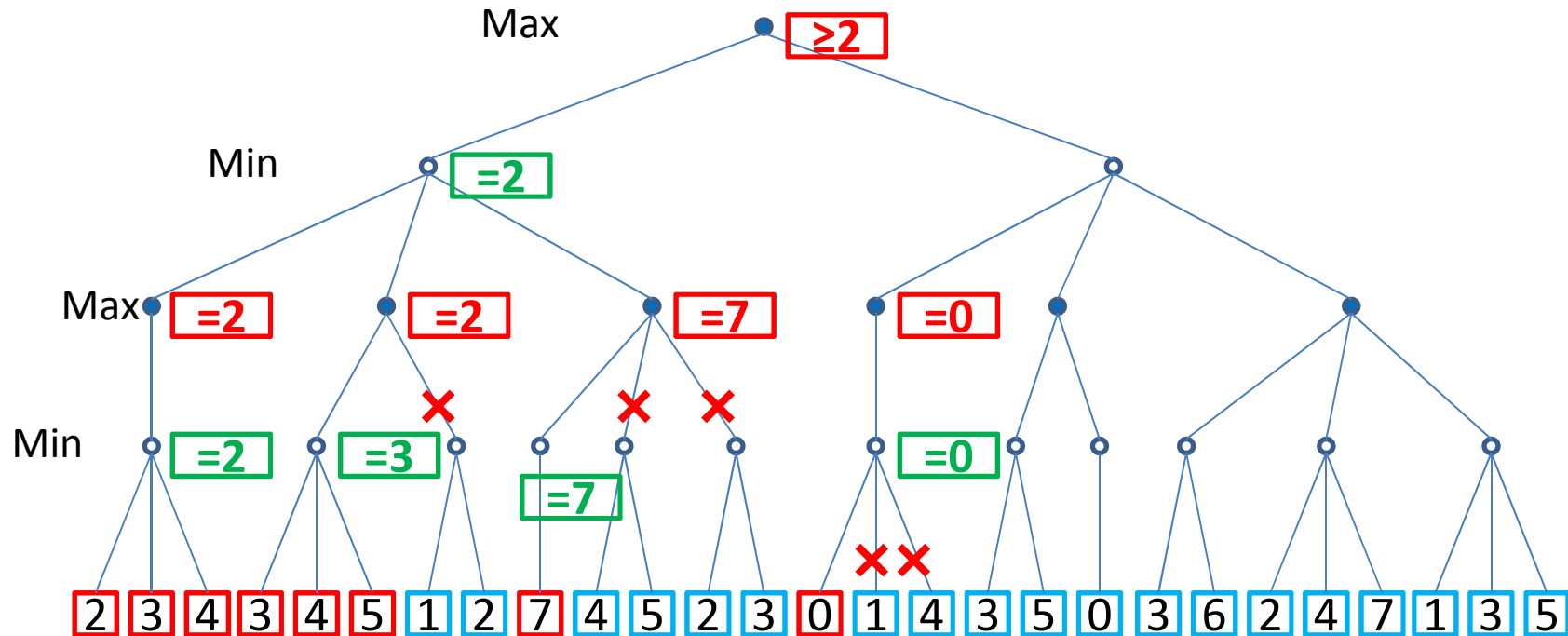
Minimax with $\alpha\beta$ -Pruning



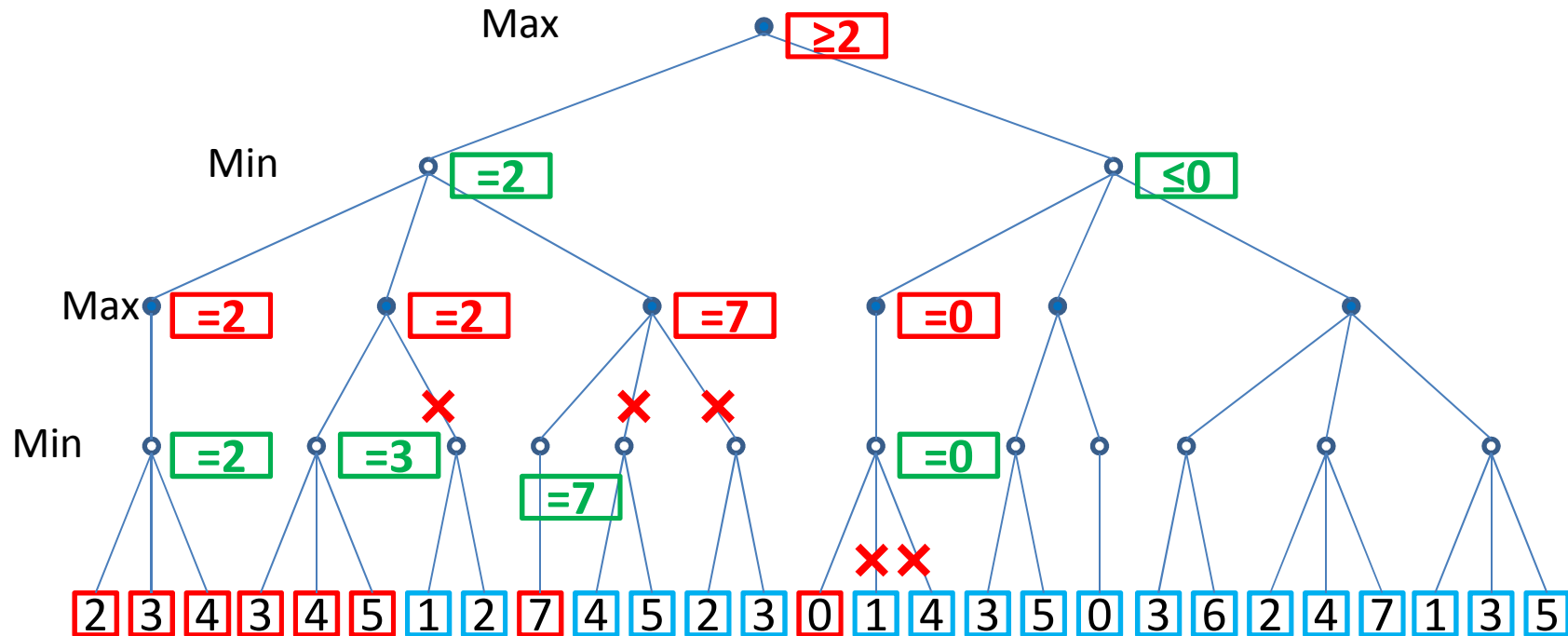
Minimax with $\alpha\beta$ -Pruning



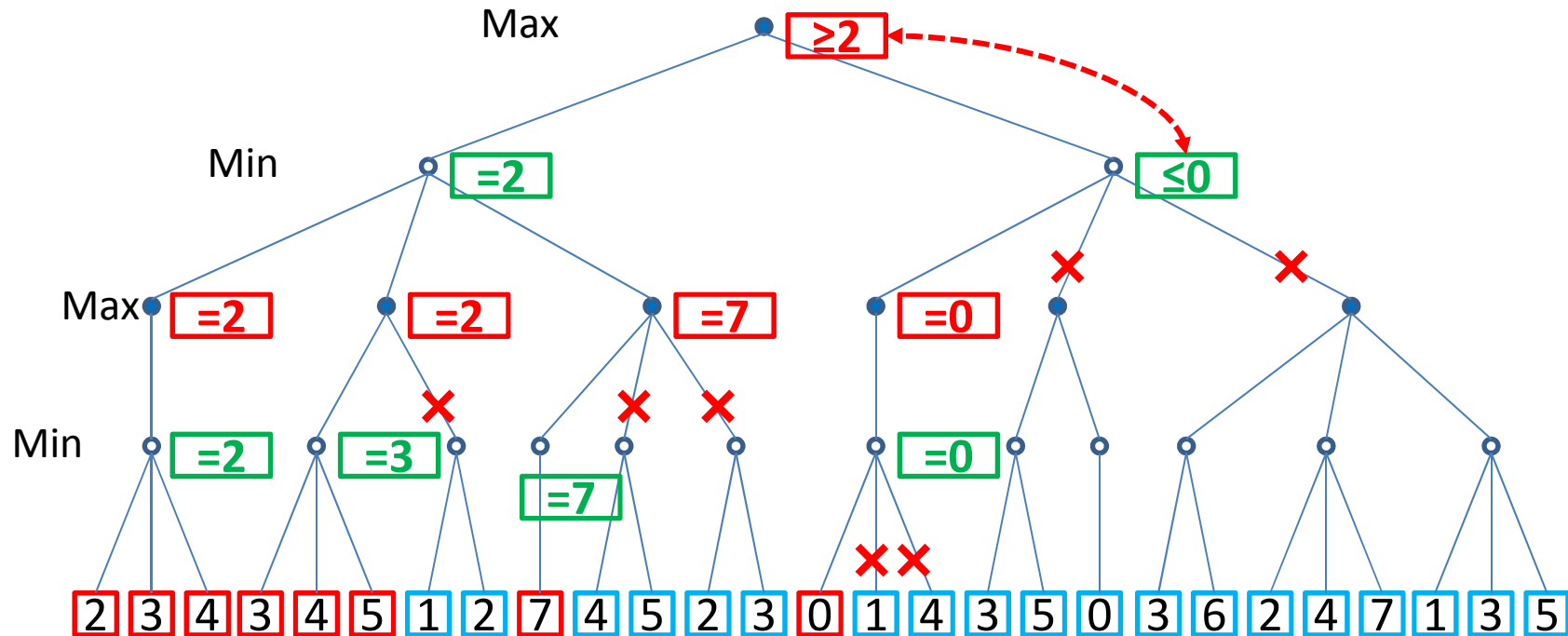
Minimax with $\alpha\beta$ -Pruning



Minimax with $\alpha\beta$ -Pruning

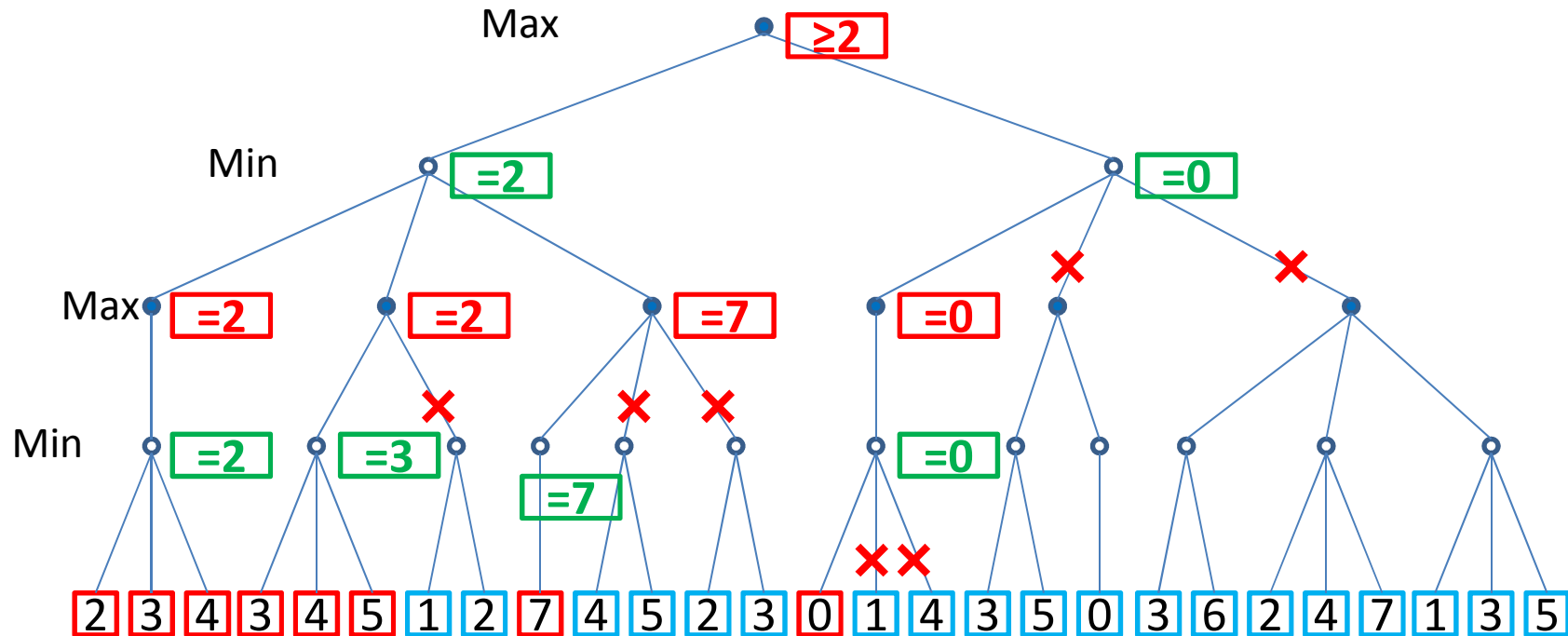


Minimax with $\alpha\beta$ -Pruning



Minimax with $\alpha\beta$ -Pruning

- 19 static evaluations saved



Exercises: Artificial Intelligence

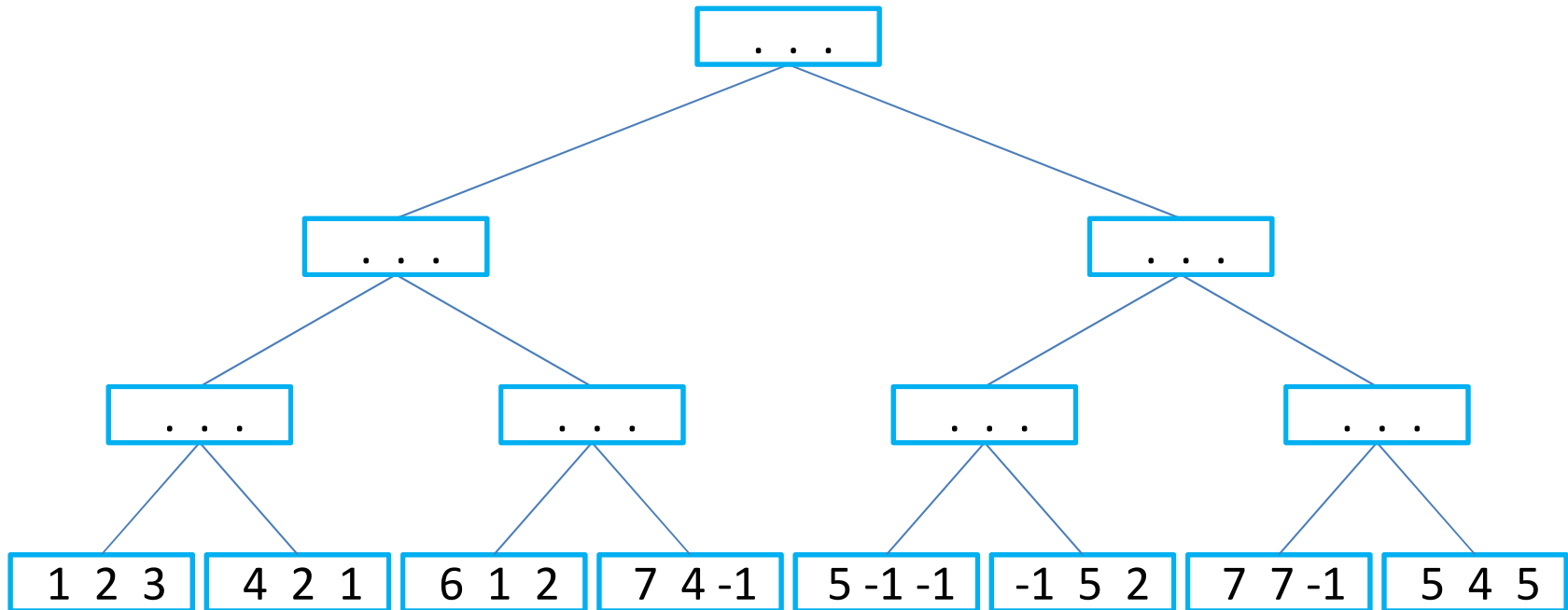
MiniMax & Constraint Processing:
MiniMax Algorithm for 3 Players

MiniMax & Constraint Processing: MiniMax Algorithm for 3 Players

PROBLEM

Problem

- Come up with a MiniMax algorithm for 3 players and apply on the figure below.

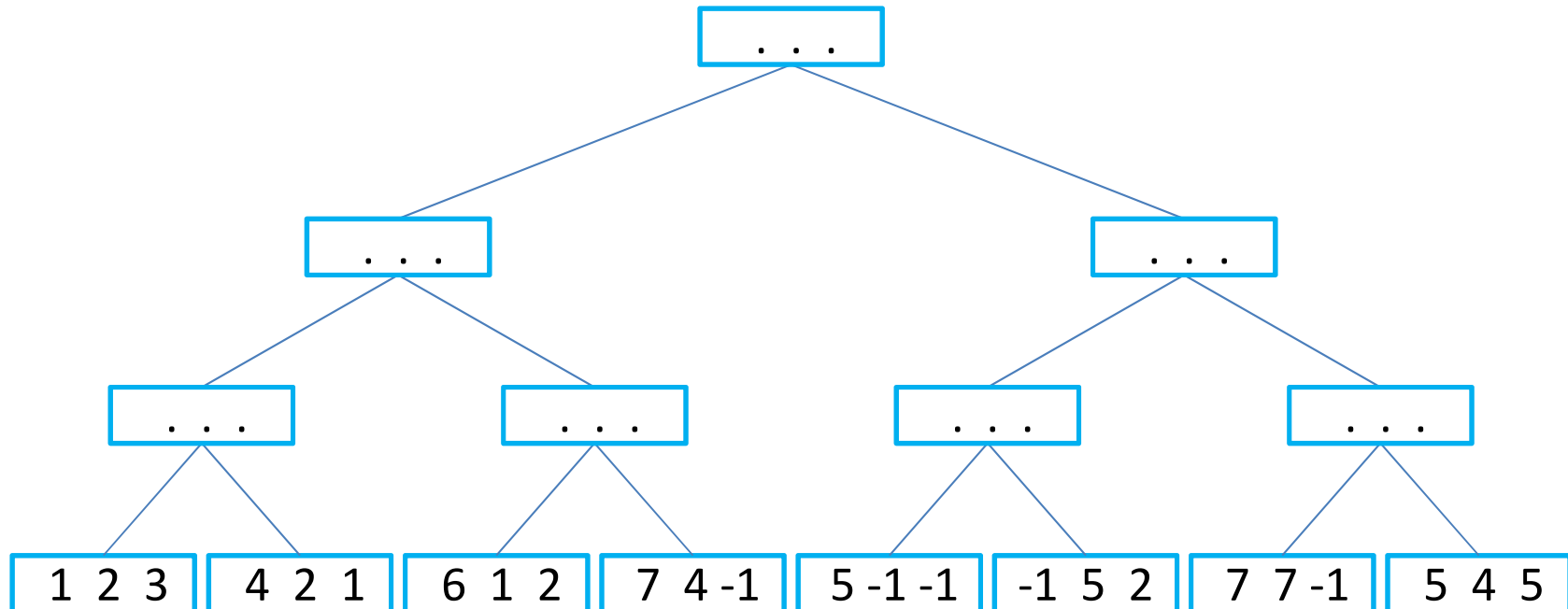


MiniMax & Constraint Processing: MiniMax Algorithm

MINIMAX FOR 3 PLAYERS

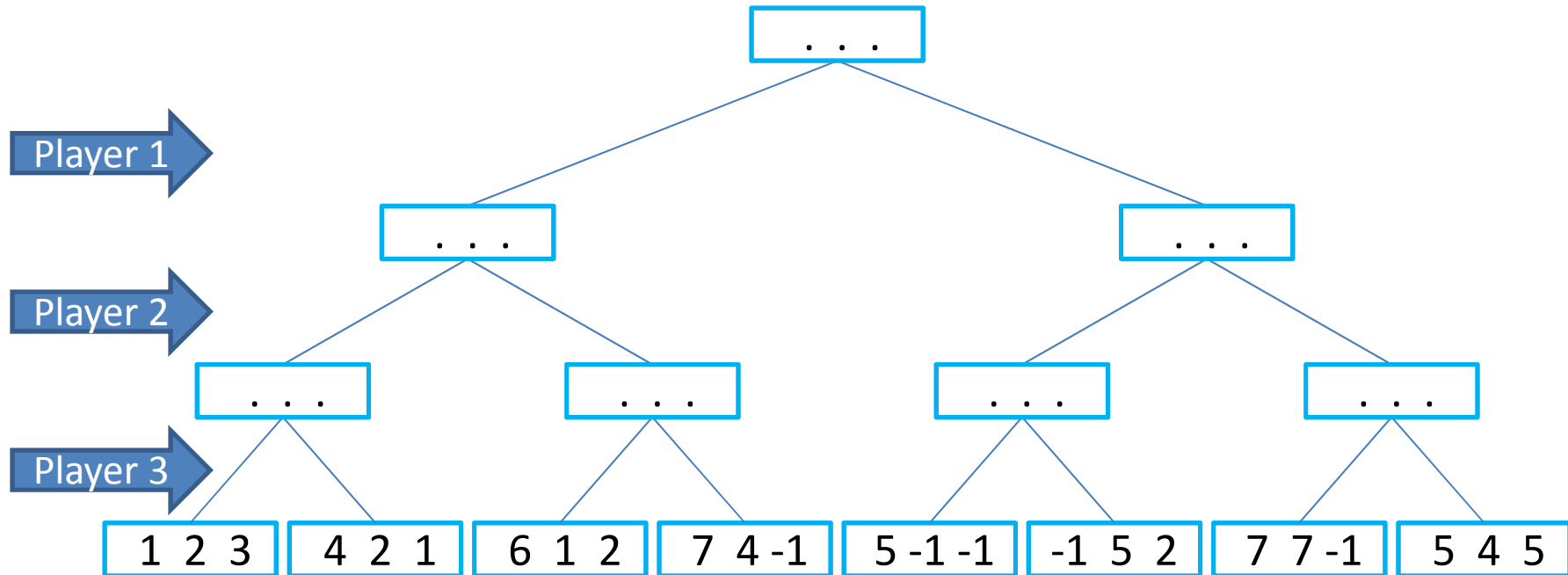
MiniMax For 3 Players

- All players are Max
- Evaluation function given by vector



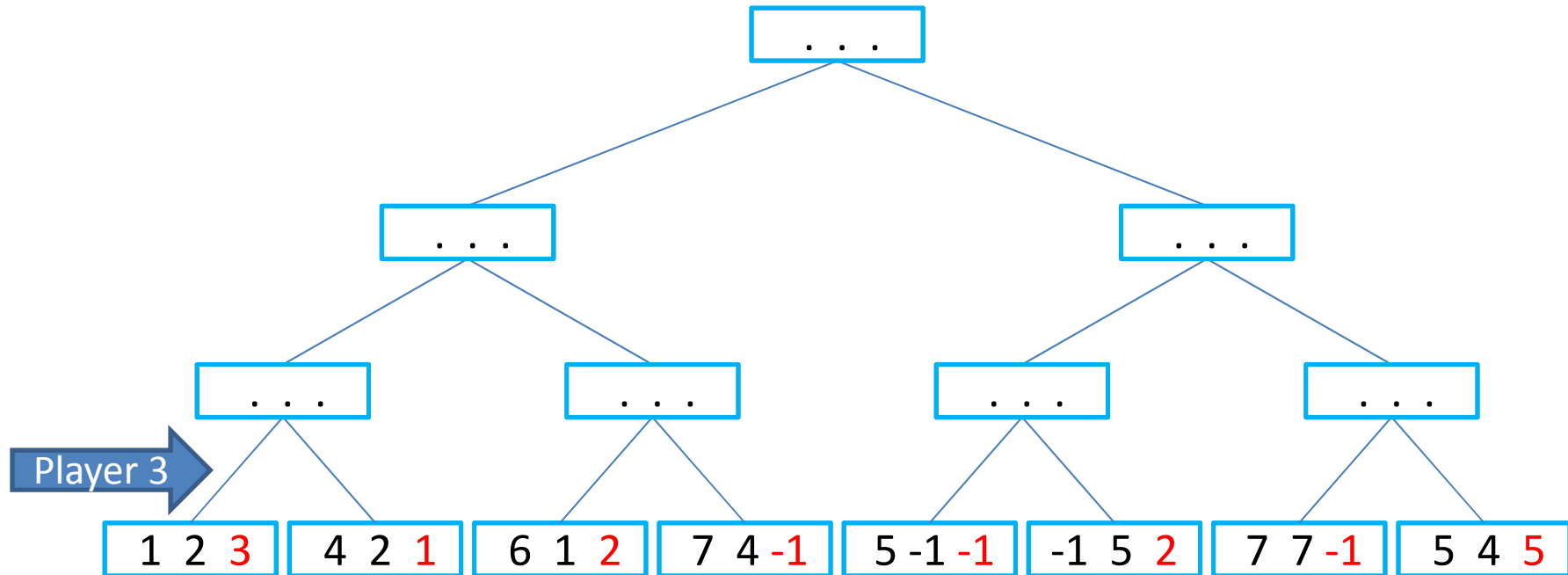
MiniMax For 3 Players

- Each layer assigned to 1 player
- Turn: every 3 layers



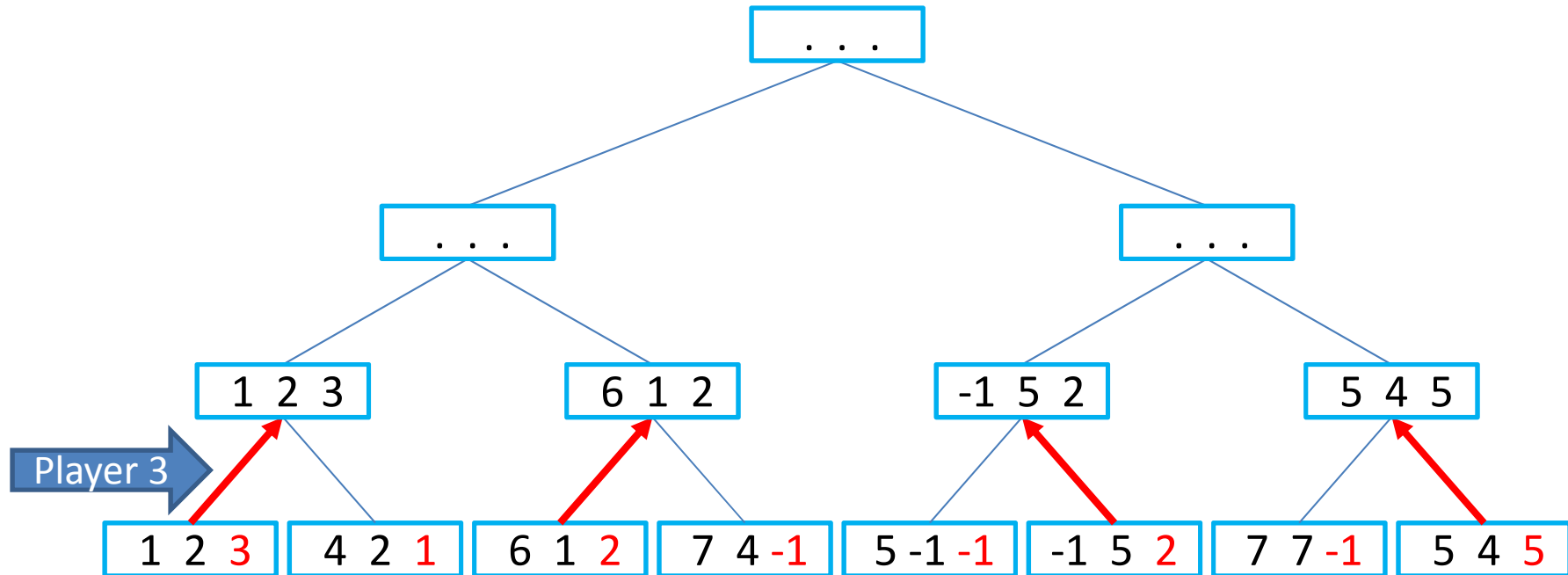
MiniMax For 3 Players

- Max third player: third position of vector



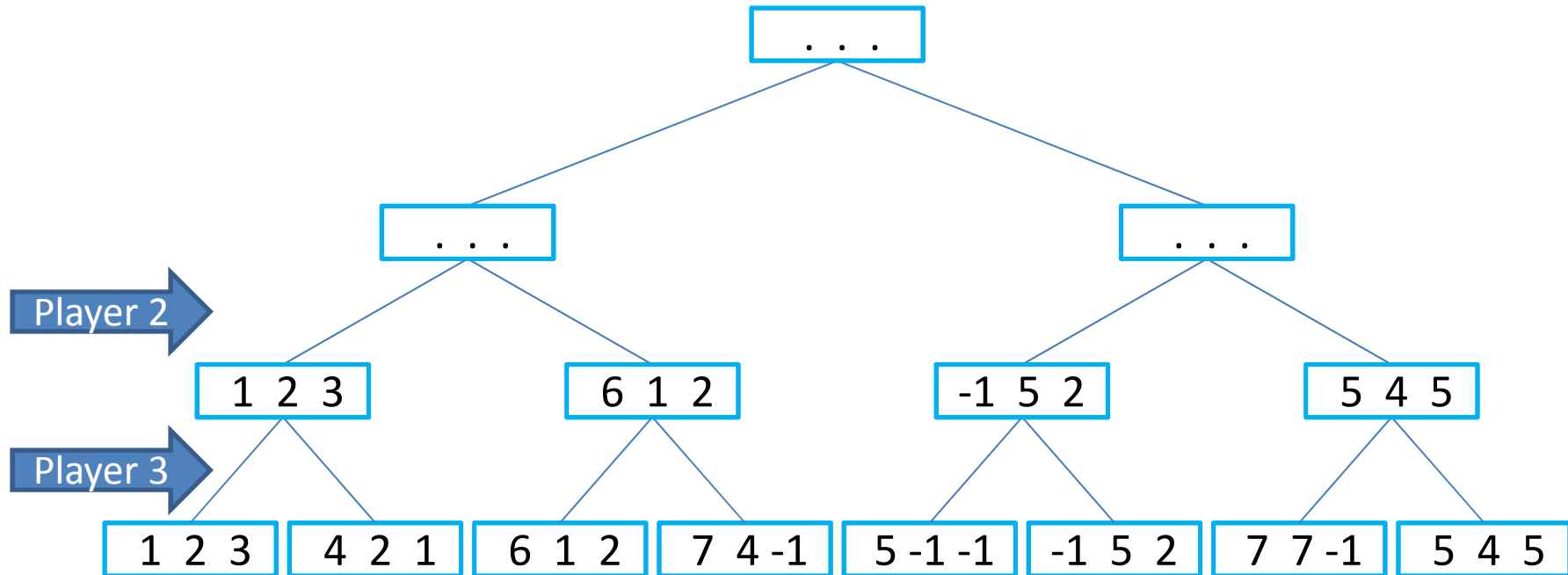
MiniMax For 3 Players

- $\text{MaxThirdPlayer}([1,2,3],[4,2,1]) = [1,2,3]$
- $\text{MaxThirdPlayer}([6,1,2],[7,4,-1]) = [6,1,2]$
- $\text{MaxThirdPlayer}([5,-1,-1],[-1,5,2]) = [-1,5,2]$
- $\text{MaxThirdPlayer}([7,7,-1],[5,4,5]) = [5,4,5]$



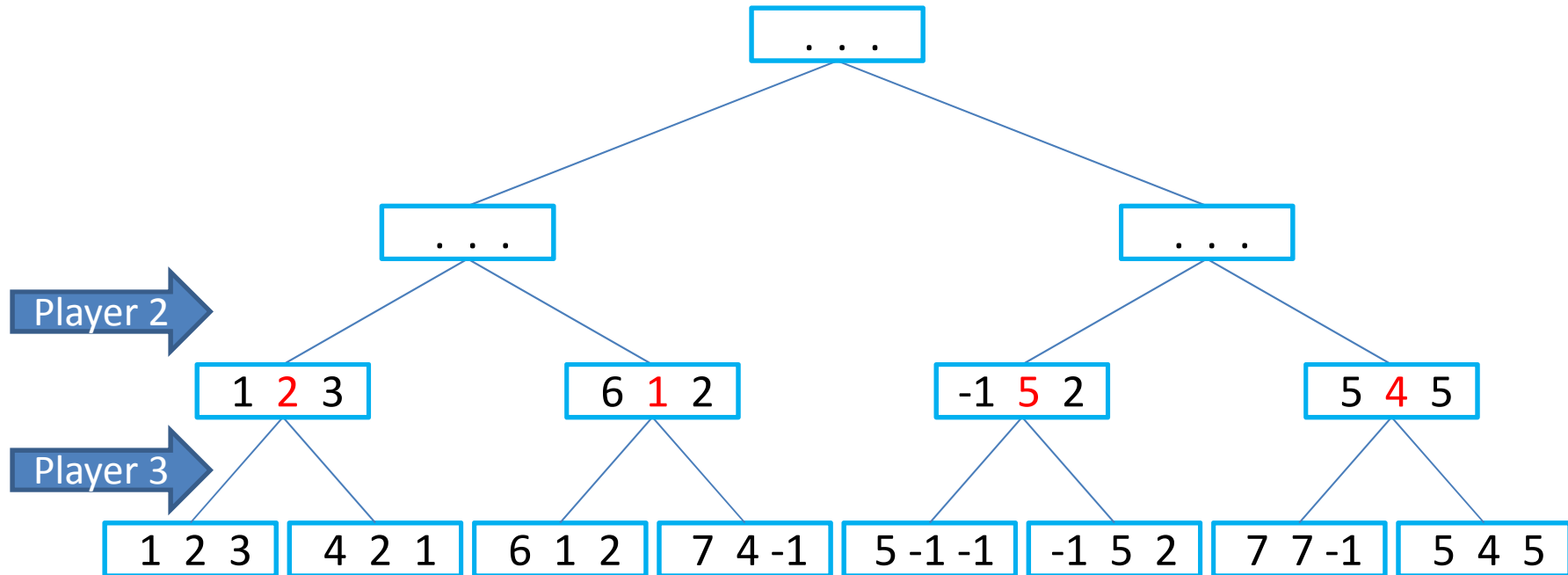
MiniMax For 3 Players

- Second player's move



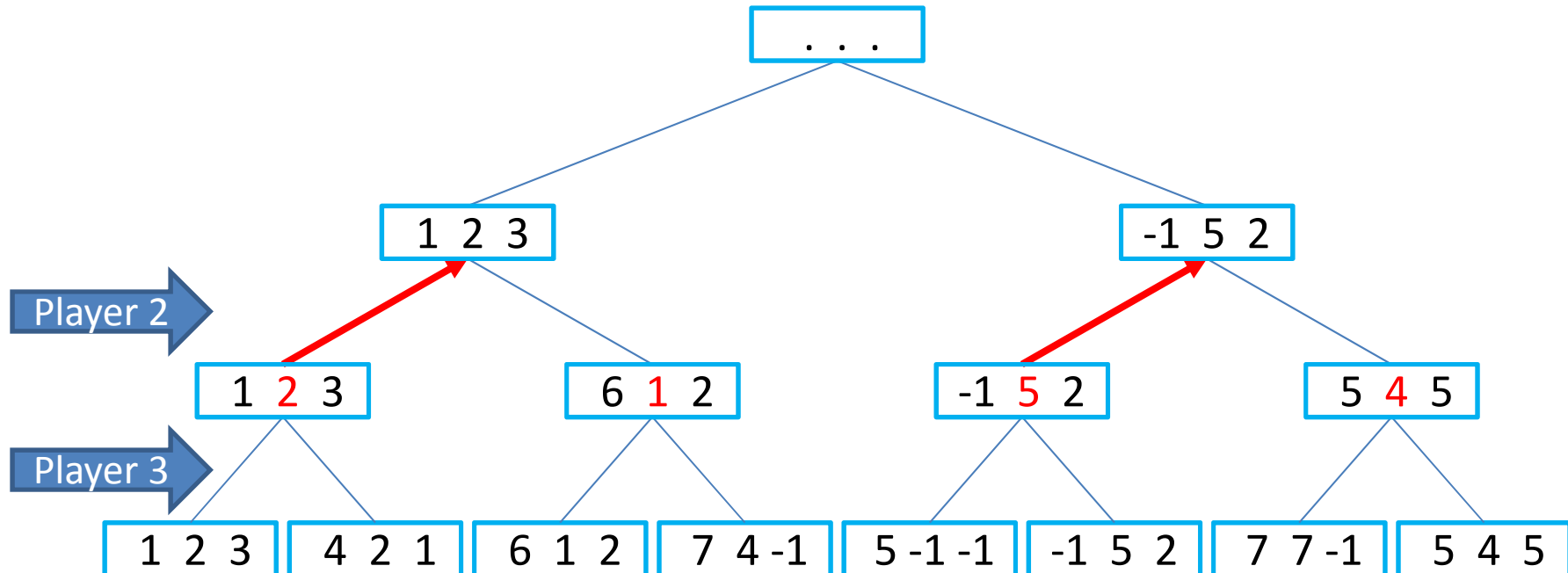
MiniMax For 3 Players

- Max second player: second position of vector



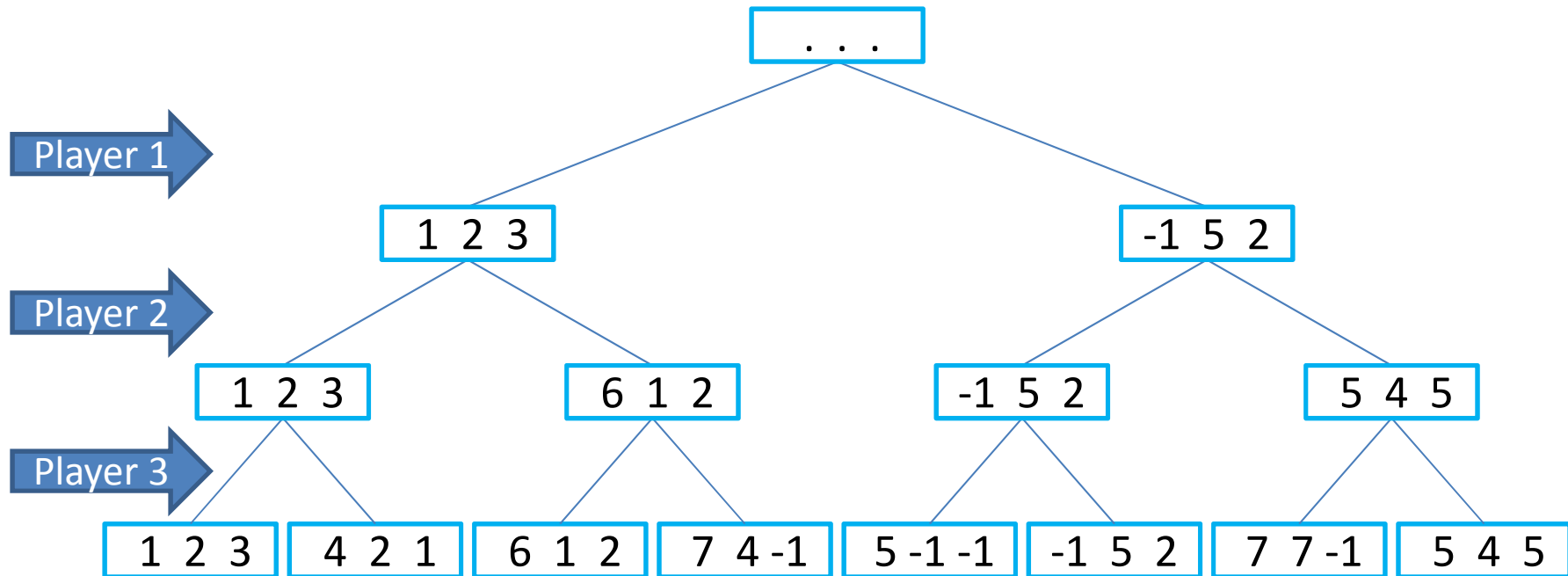
MiniMax For 3 Players

- $\text{MaxSecondPlayer}([1,2,3],[6,1,2]) = [1,2,3]$
- $\text{MaxSecondPlayer}([-1,5,2],[5,4,5]) = [-1,5,2]$



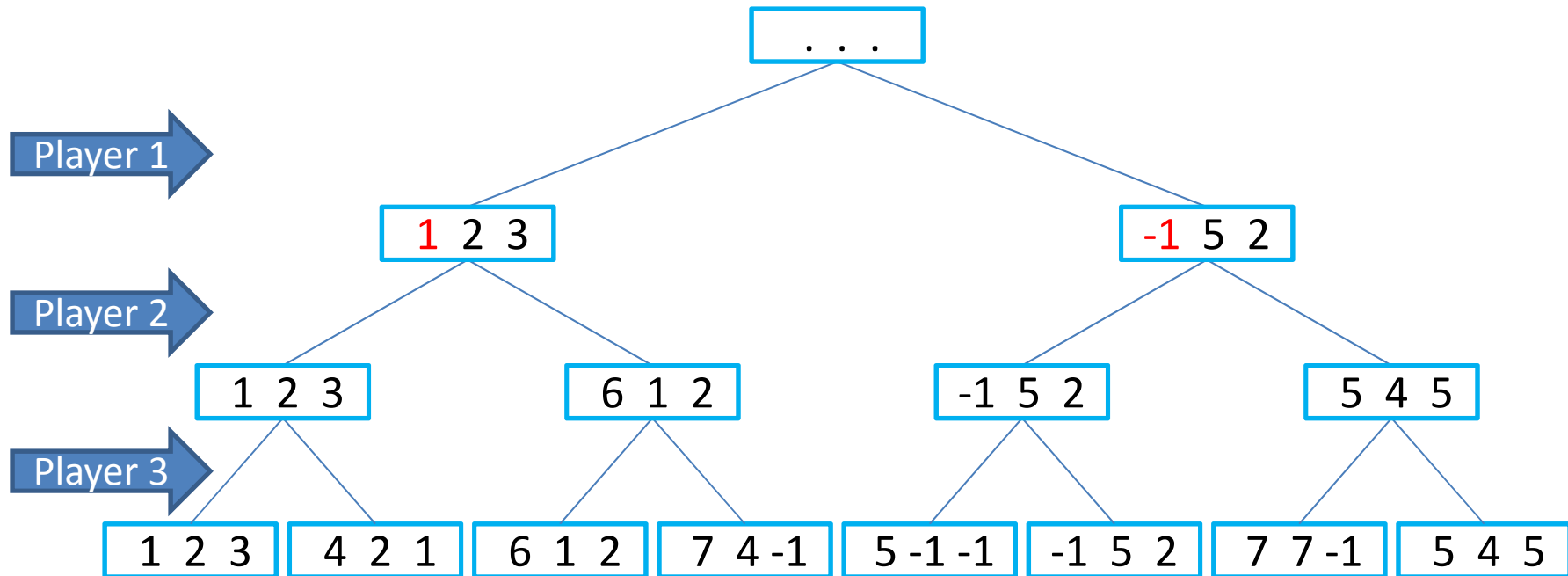
MiniMax For 3 Players

- First player's move



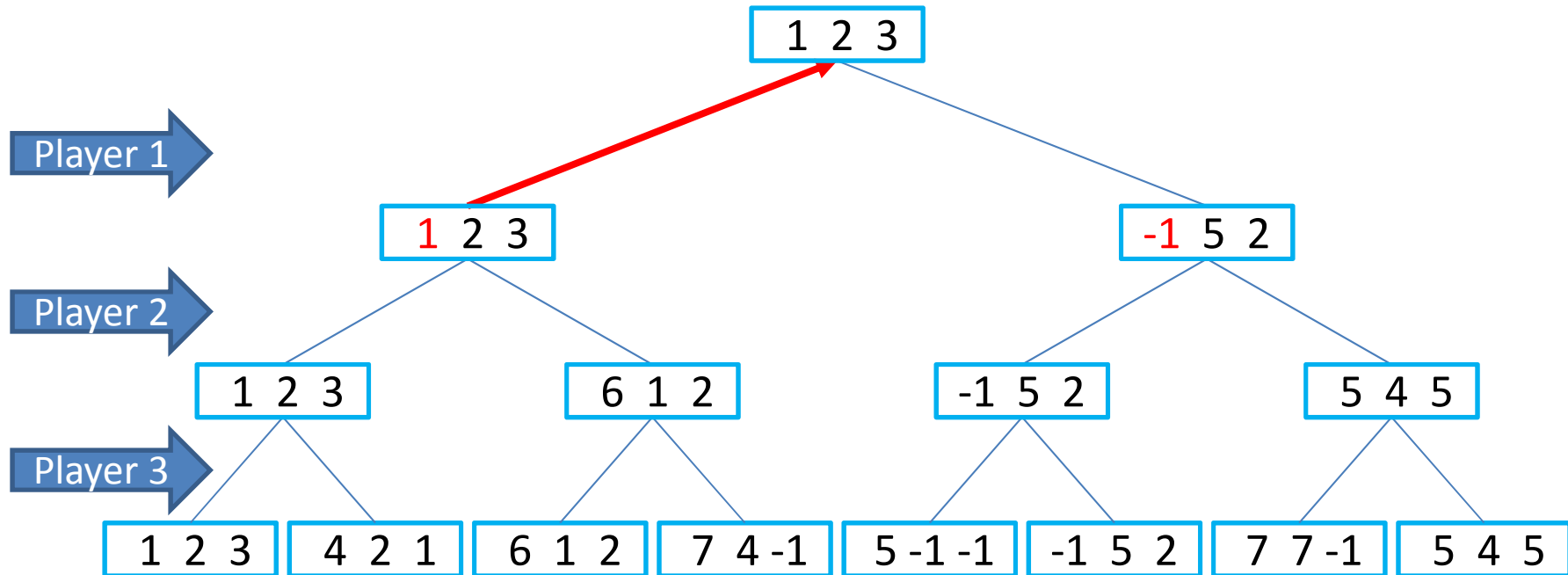
MiniMax For 3 Players

- Max first player: first position of vector



MiniMax For 3 Players

- $\text{MaxFirstPlayer}([1,2,3],[-1,5,4]) = [1,2,3]$



Exercises: Artificial Intelligence

MiniMax & Constraint Processing:
The 4 Houses problem

MiniMax & Constraint Processing: The 4 Houses problem

PROBLEM

Problem

- Variant of the 4 houses problem
 - There are 4 Families: A, B, C & D
 - Living in 4 Houses: 1, 2, 3 & 4
 - C lives in a house with higher number than D
 - D lives next to A in a lower numbered house
 - There is at least one house between D and B
 - C does not live in 3
 - B does not live in 1

Problem

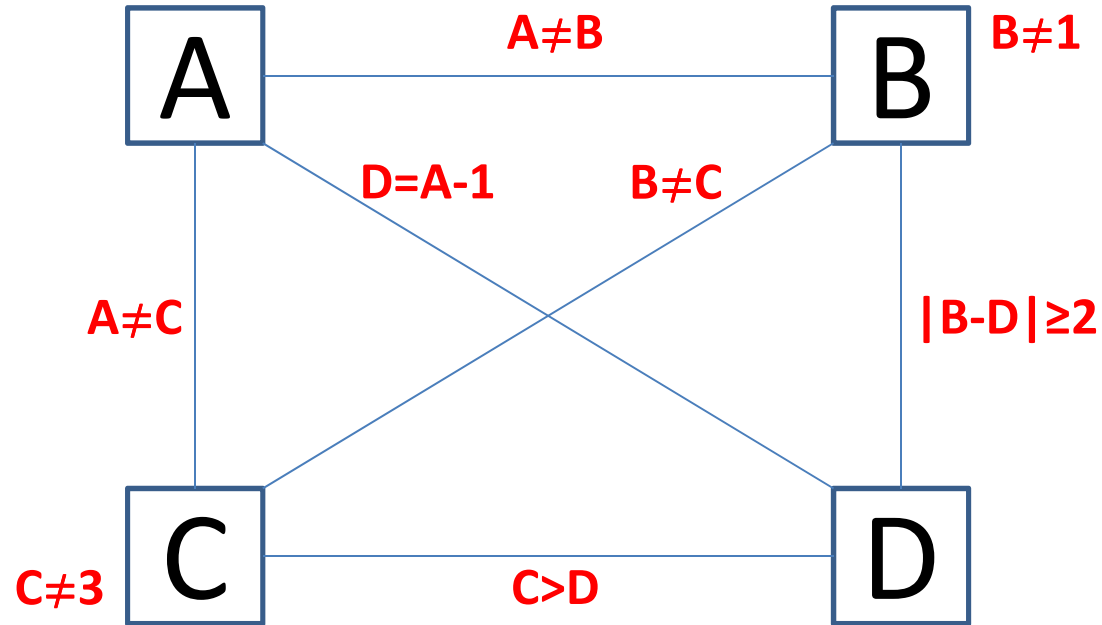
- Which family lives in which house?
 - Solve with backtracking
 - Solve with backjumping
 - Solve with backmarking
- Consider the following sets of assignments:
 - $\{A=1\}, \{A=2, B=2\}, \{A=2, B=3\}, \{A=2, B=3, C=1\}, \{A=2, B=4\}$
 - Which of these are no-goods?
 - You can use arc-consistency arguments to determine the no-goods and the not no-goods

MiniMax & Constraint Processing: The 4 Houses problem

CONSTRAINT PROCESSING: PROBLEM REPRESENTATION

Constraint Processing

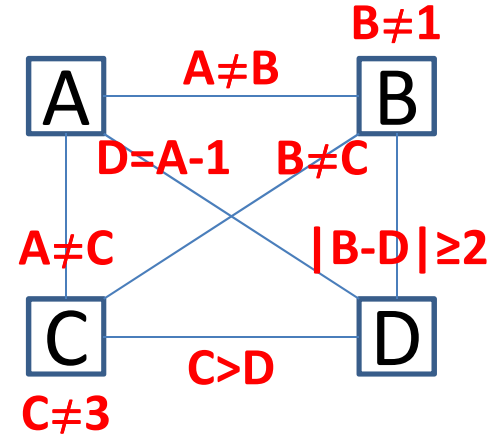
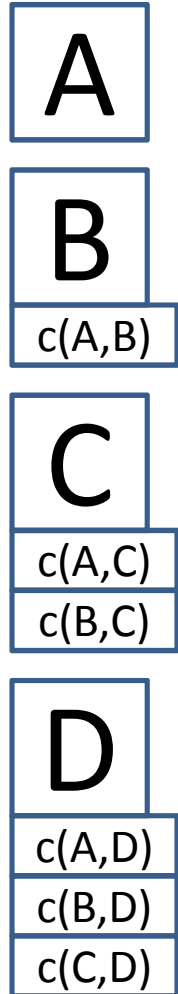
- Problem representation:



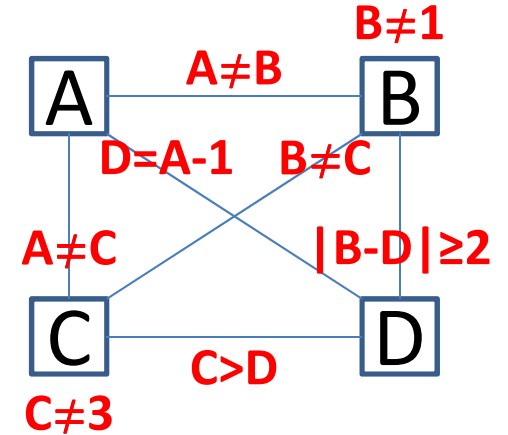
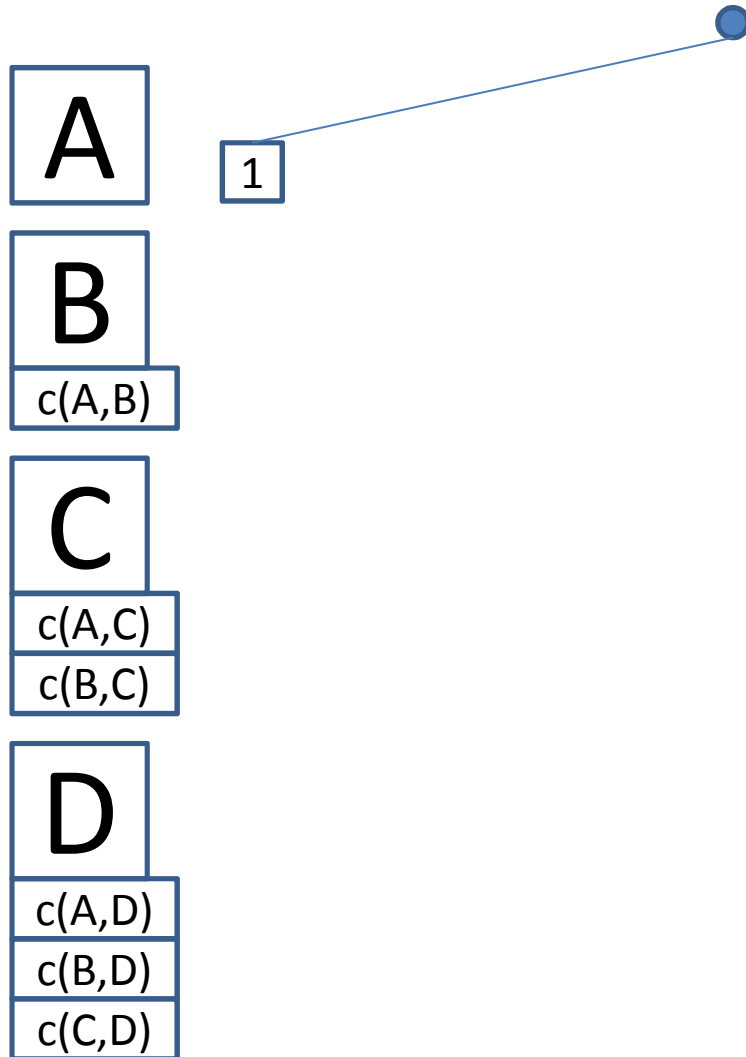
MiniMax & Constraint Processing: The 4 Houses problem

CONSTRAINT PROCESSING: BACKTRACKING

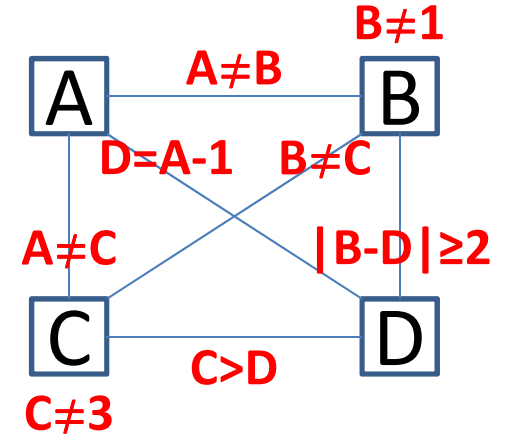
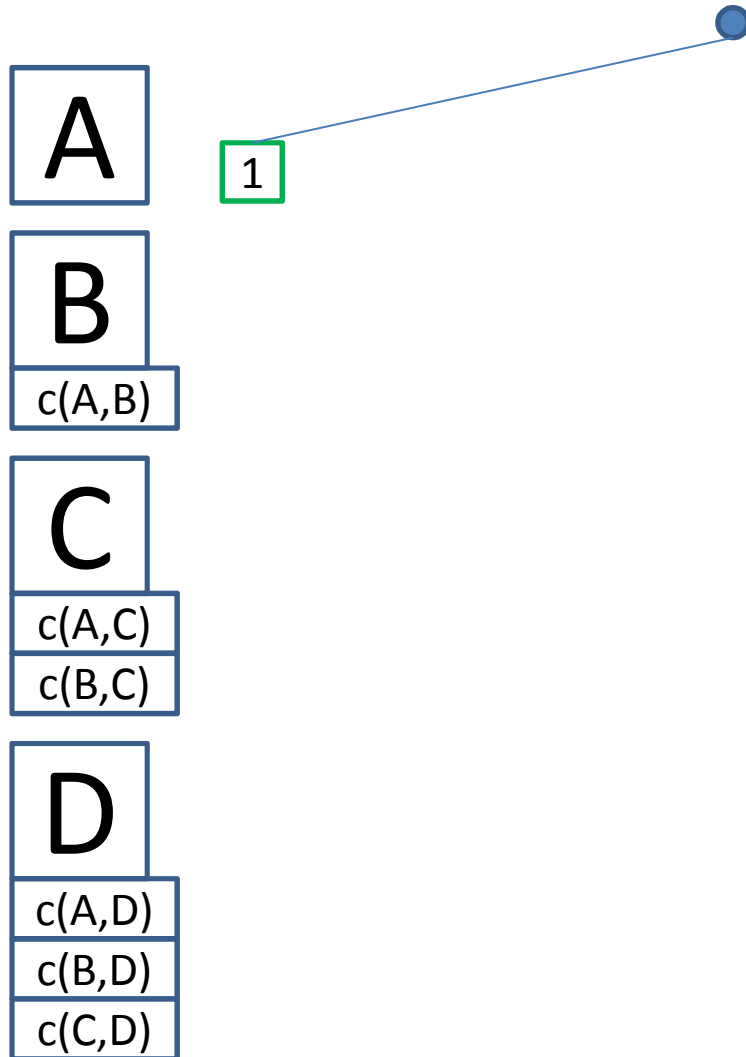
Constraint Processing: Backtracking



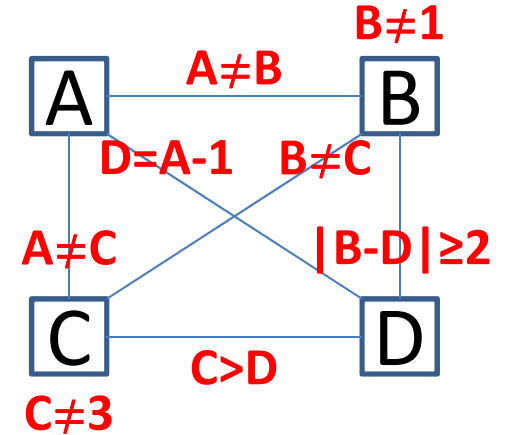
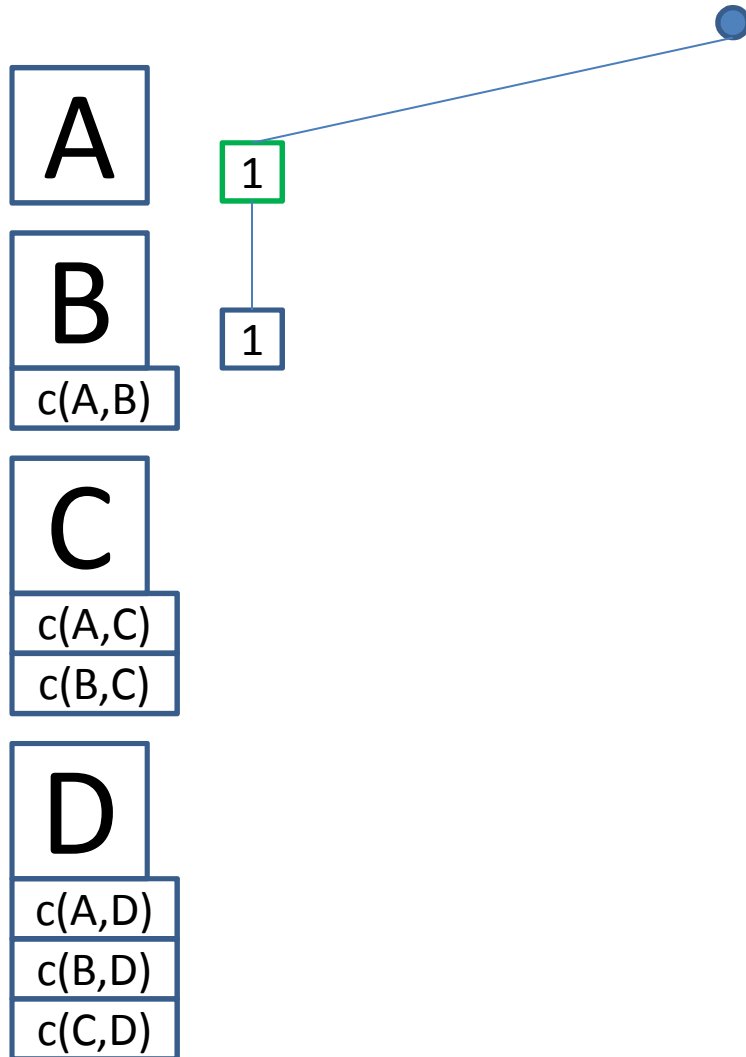
Constraint Processing: Backtracking



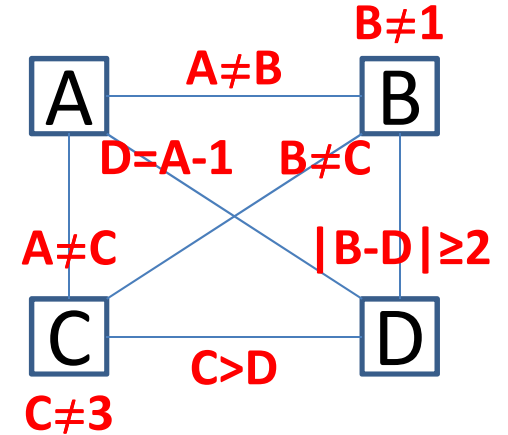
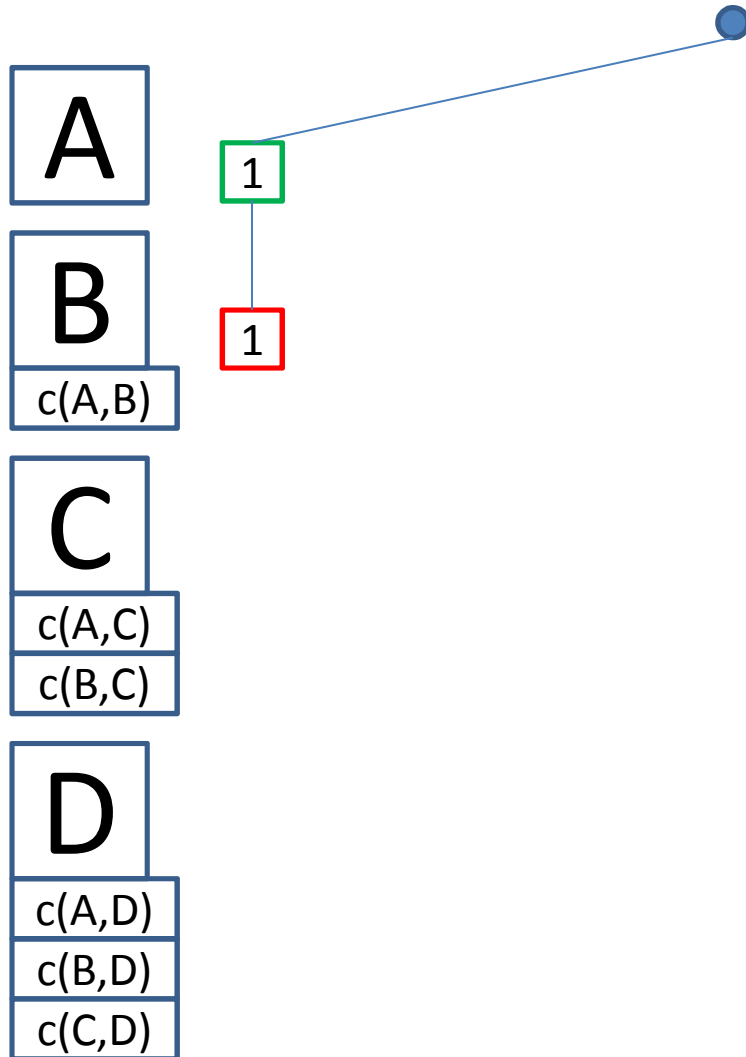
Constraint Processing: Backtracking



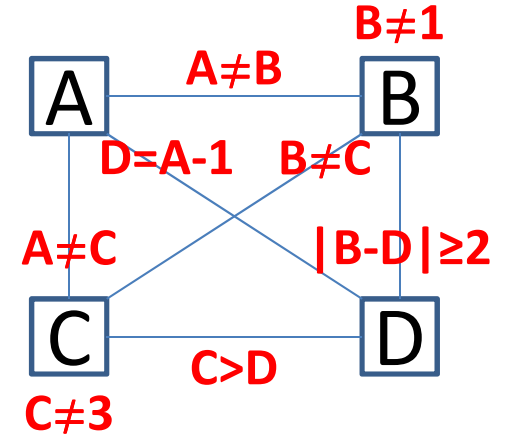
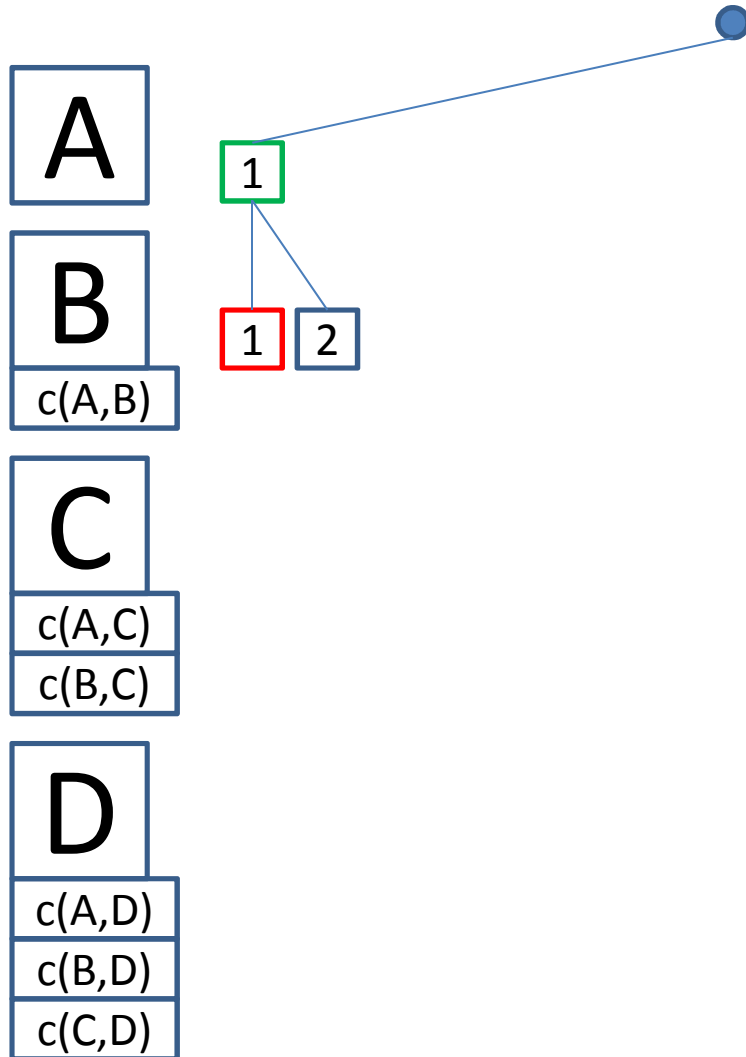
Constraint Processing: Backtracking



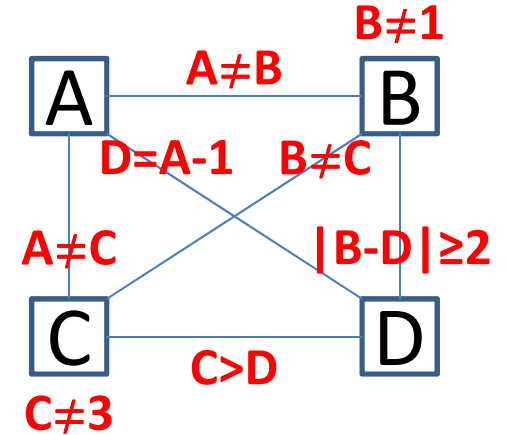
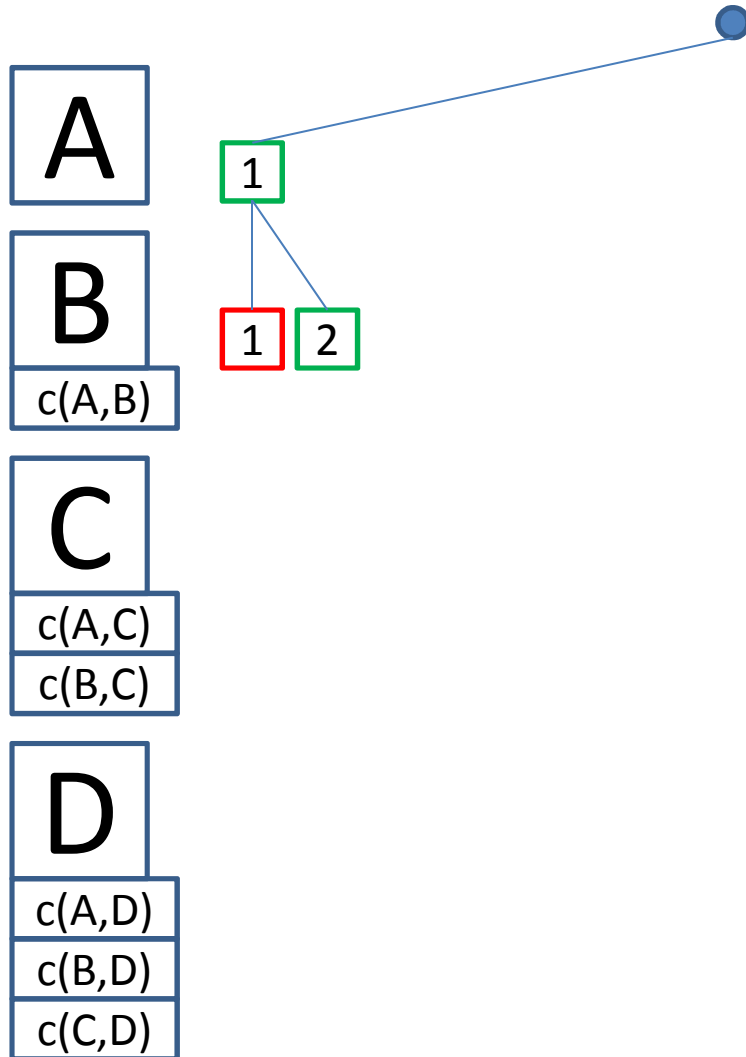
Constraint Processing: Backtracking



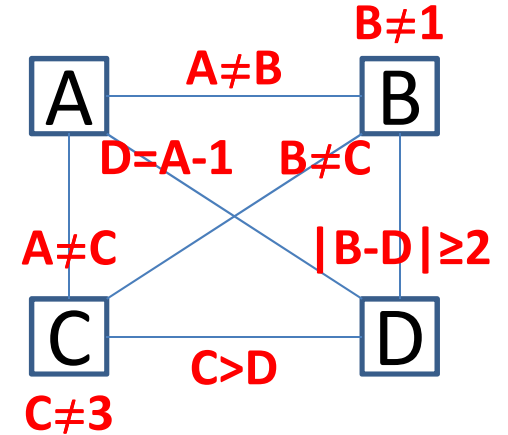
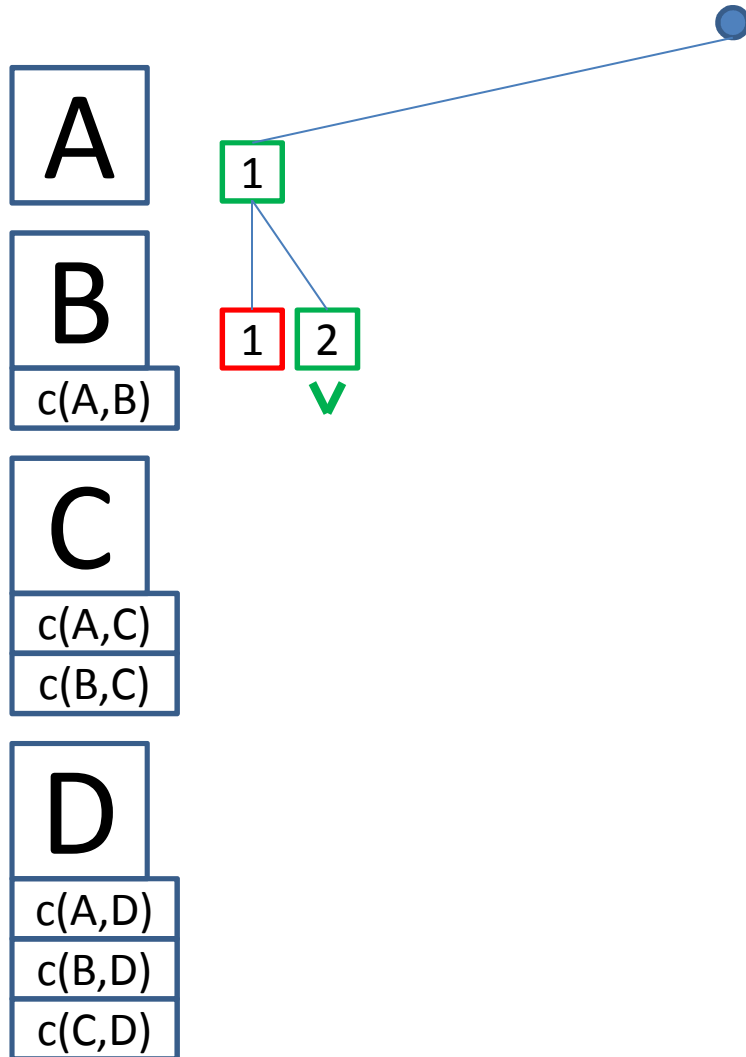
Constraint Processing: Backtracking



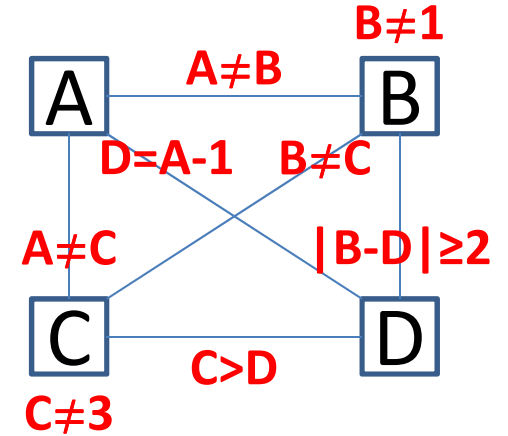
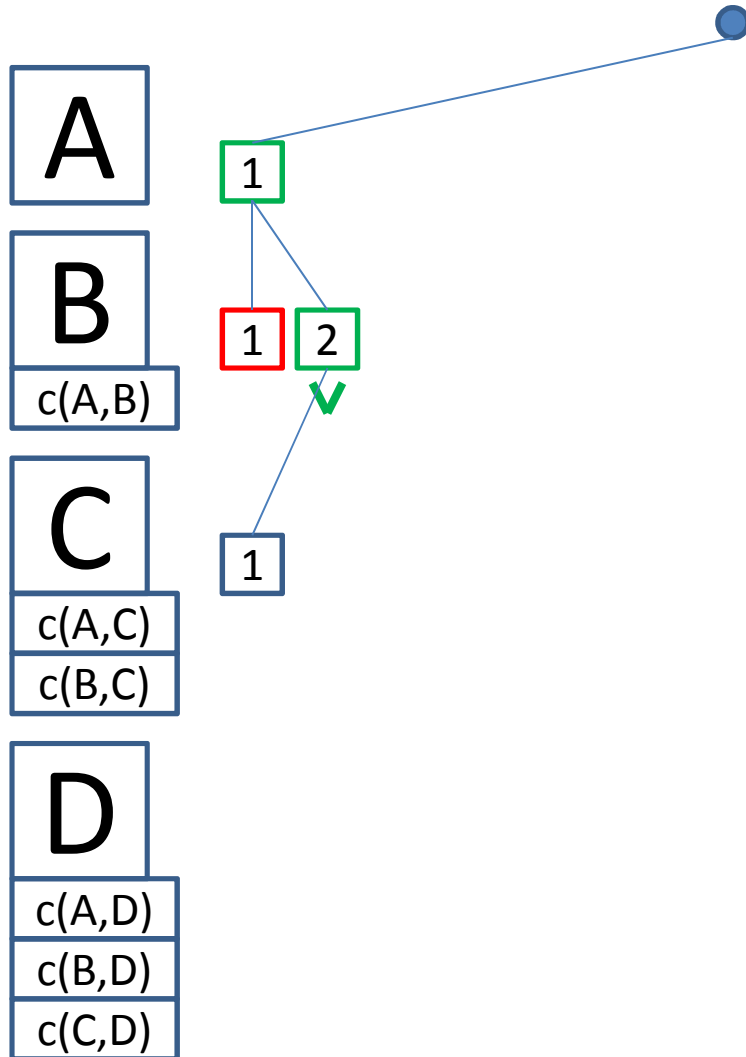
Constraint Processing: Backtracking



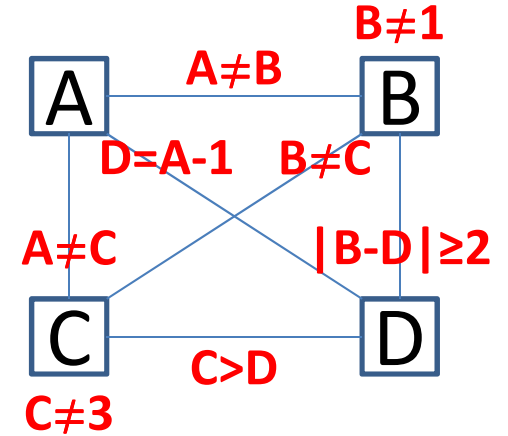
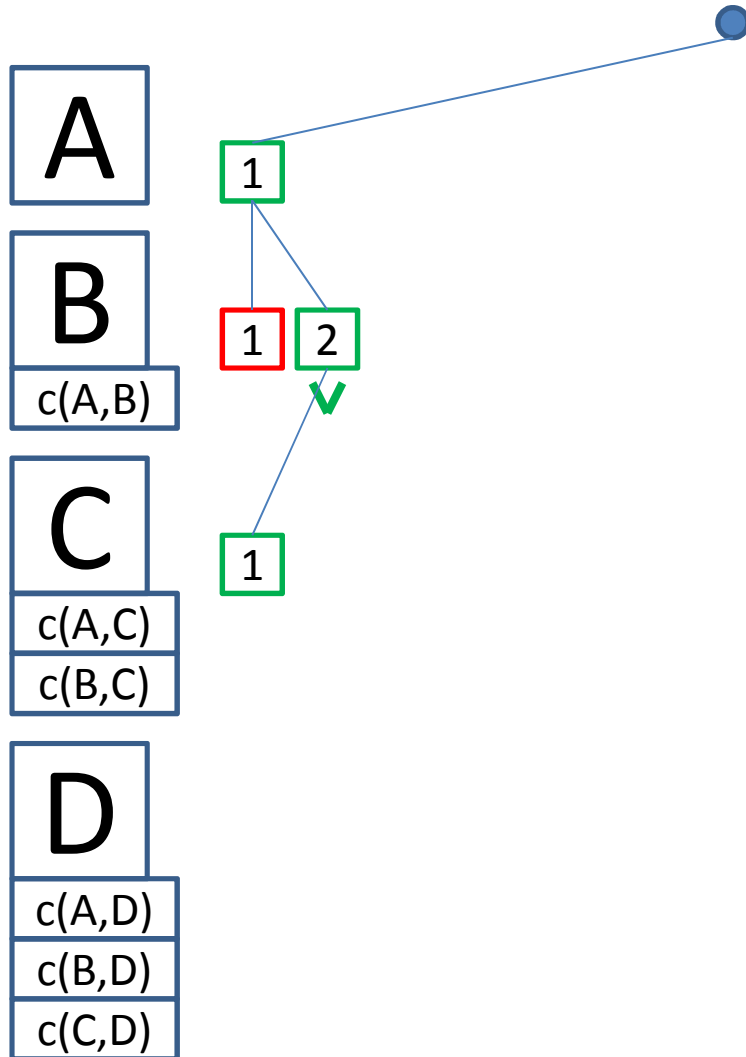
Constraint Processing: Backtracking



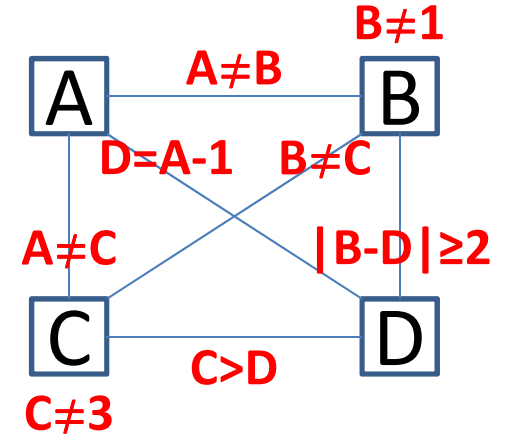
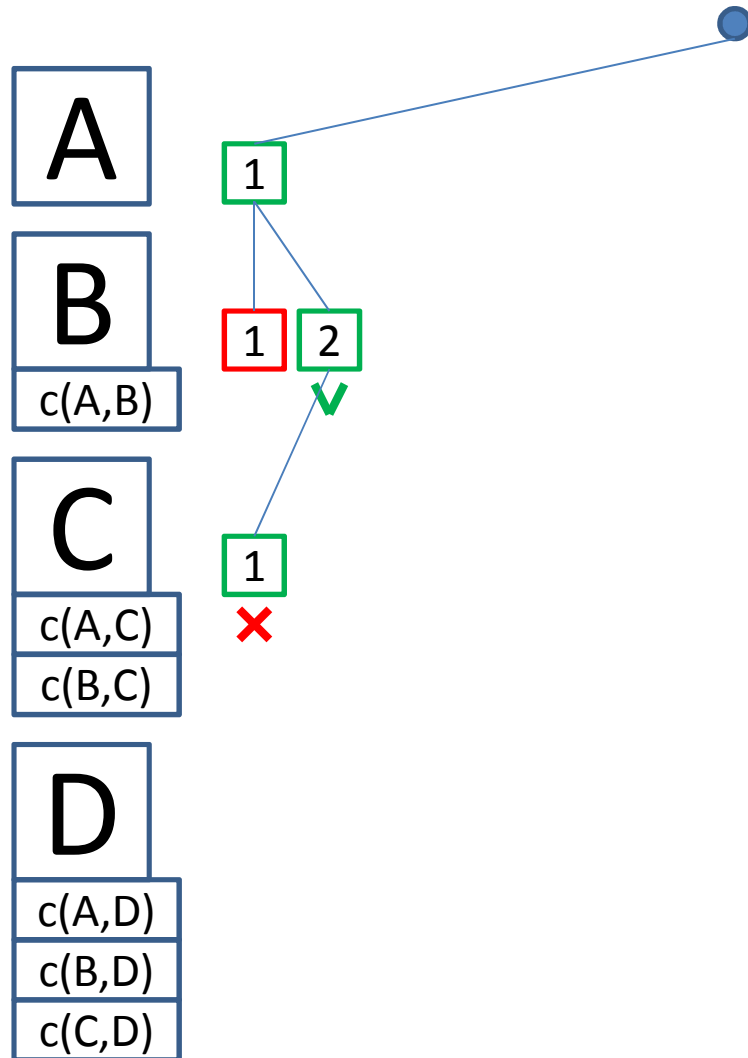
Constraint Processing: Backtracking



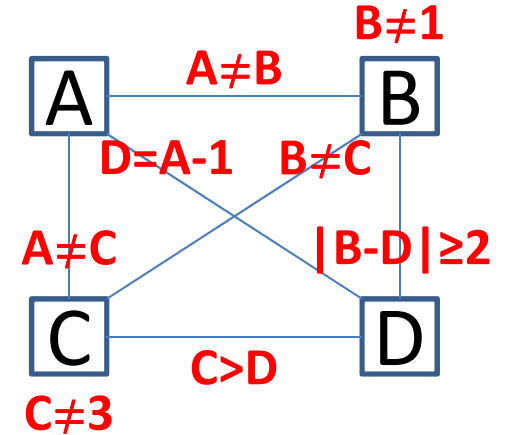
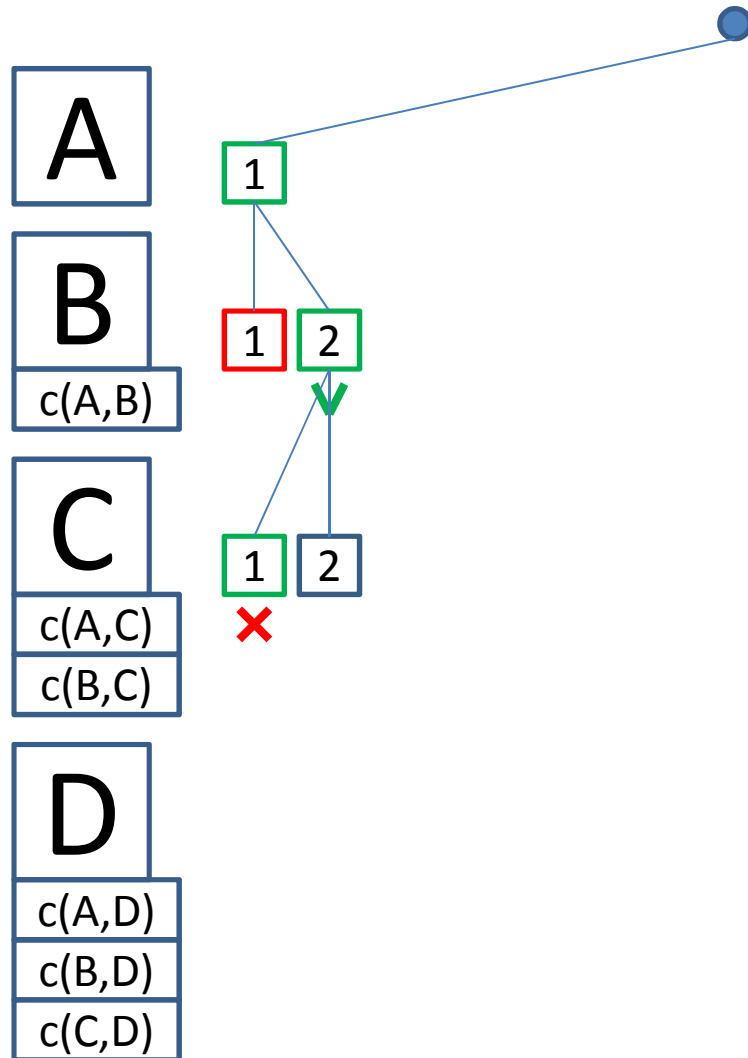
Constraint Processing: Backtracking



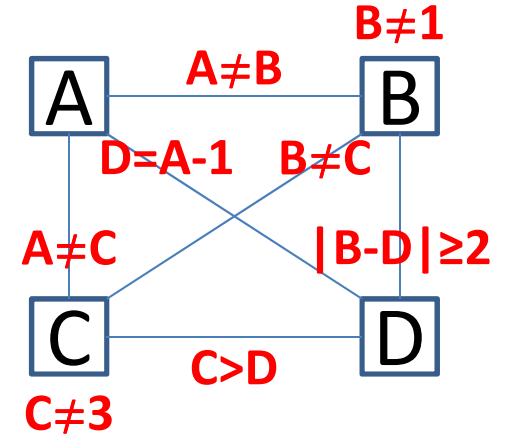
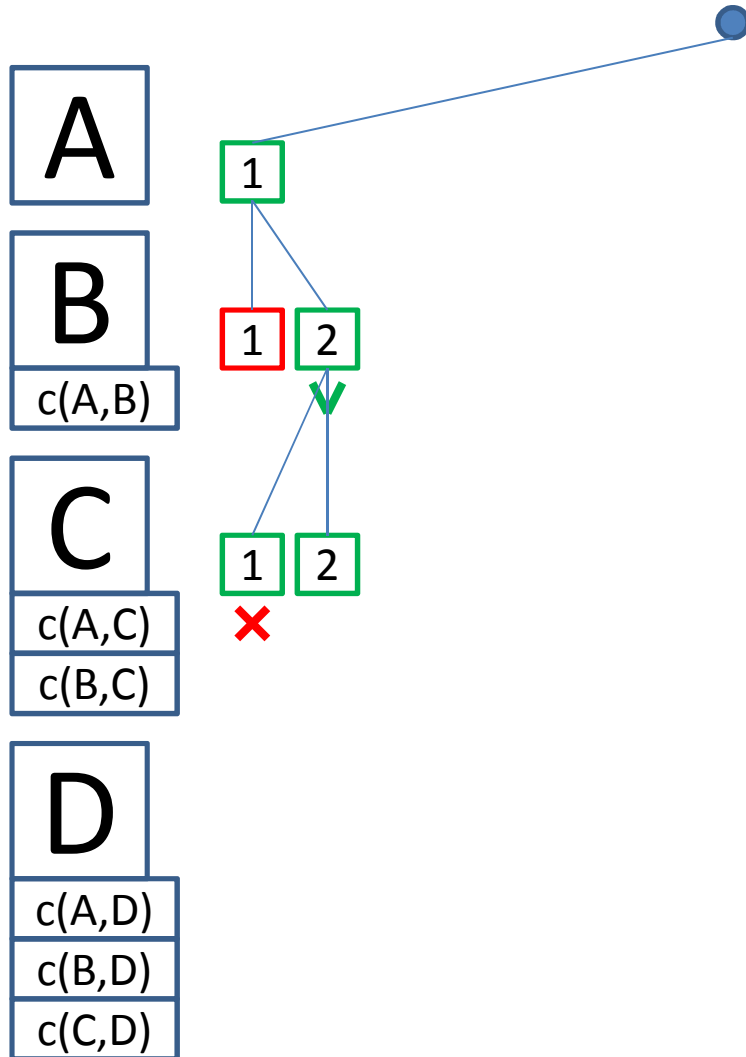
Constraint Processing: Backtracking



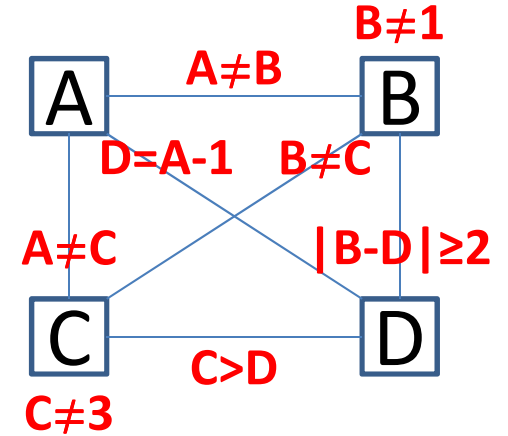
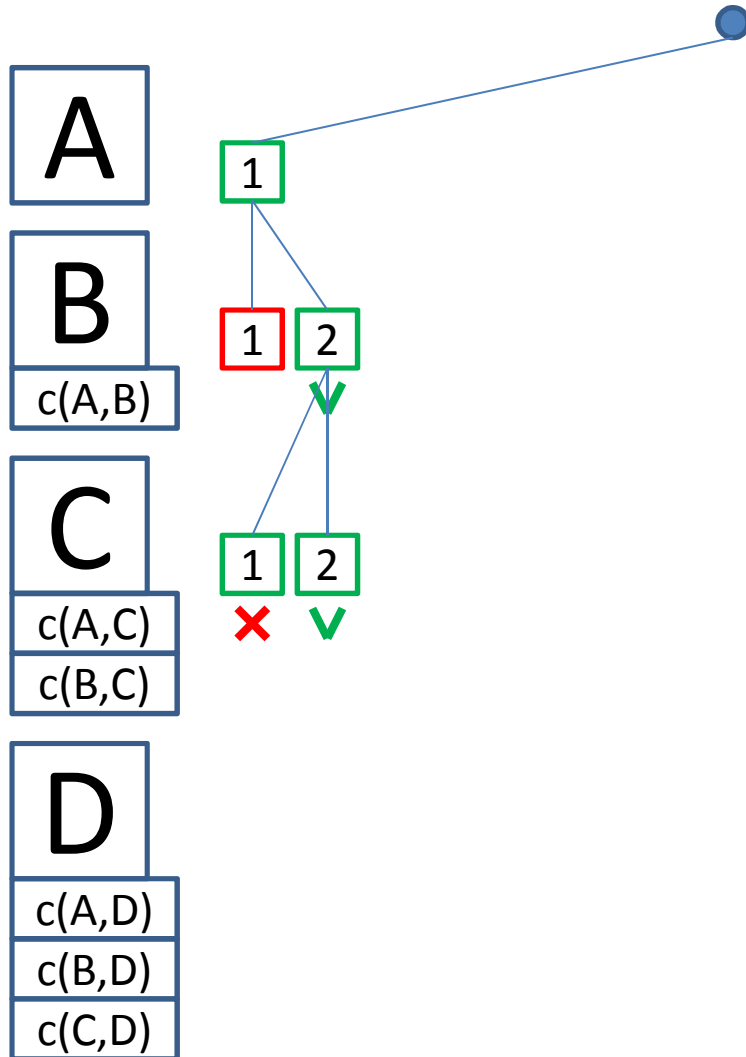
Constraint Processing: Backtracking



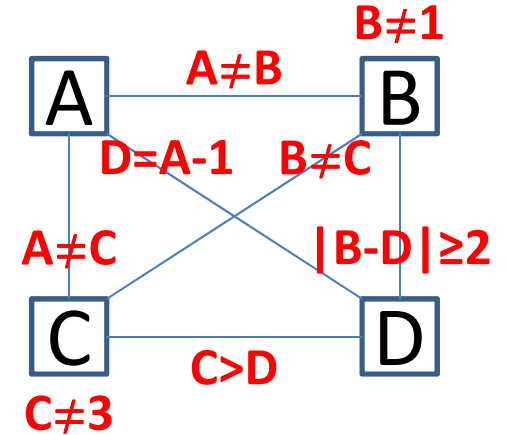
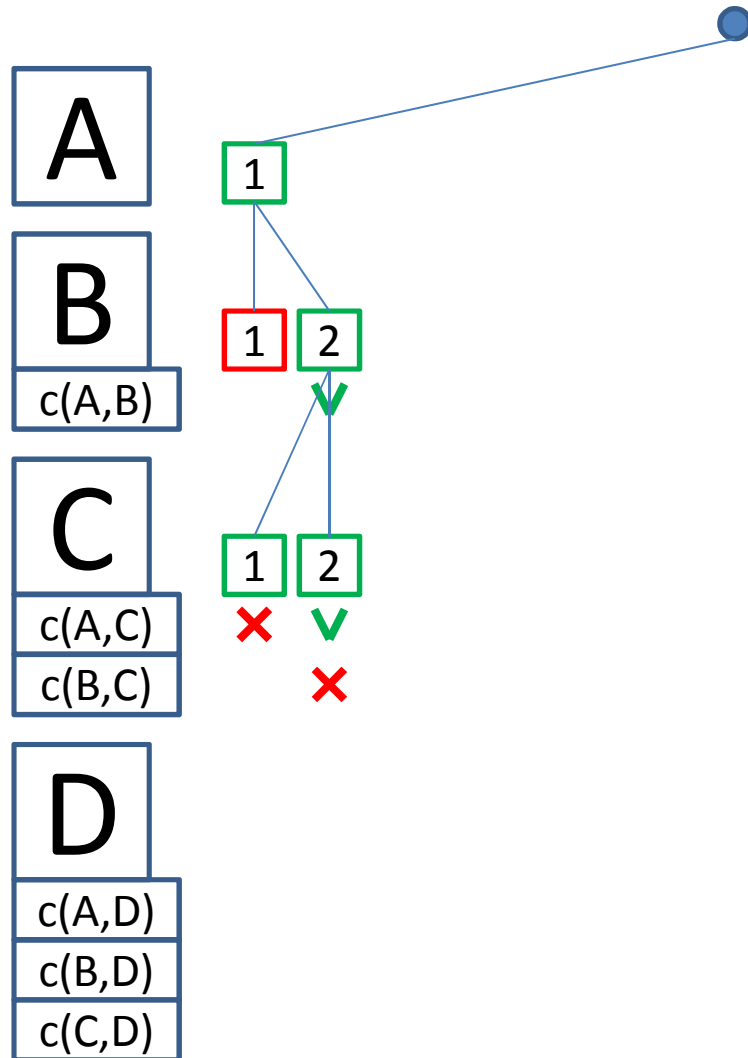
Constraint Processing: Backtracking



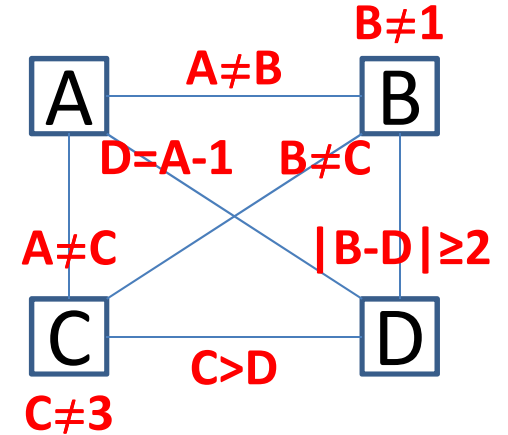
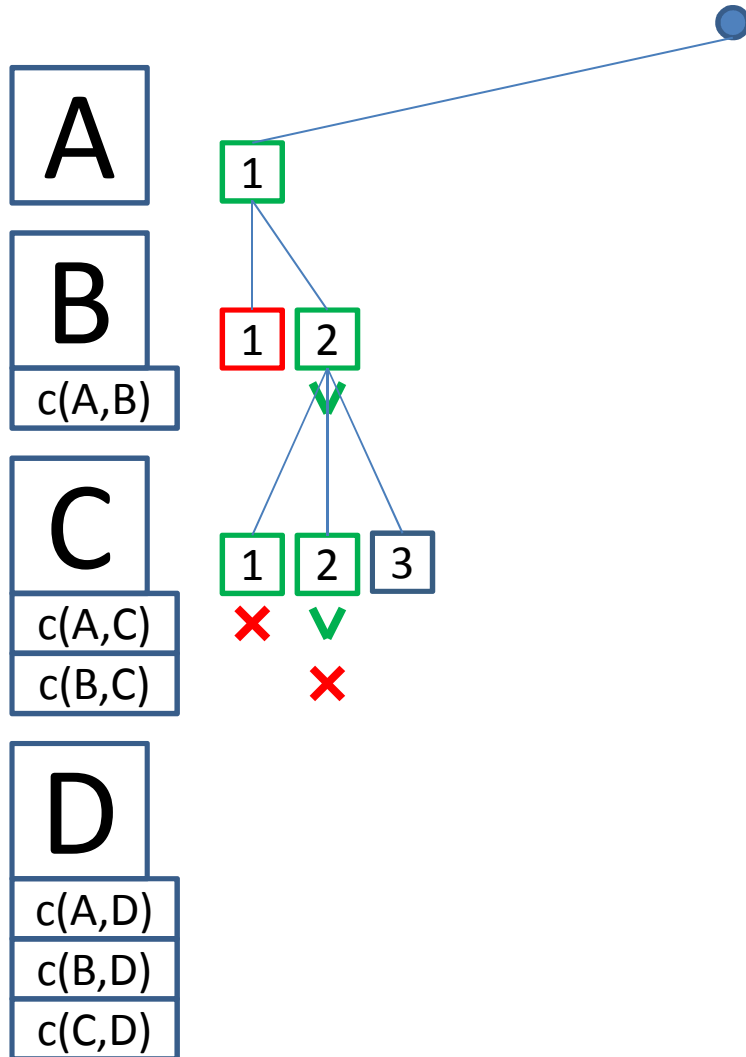
Constraint Processing: Backtracking



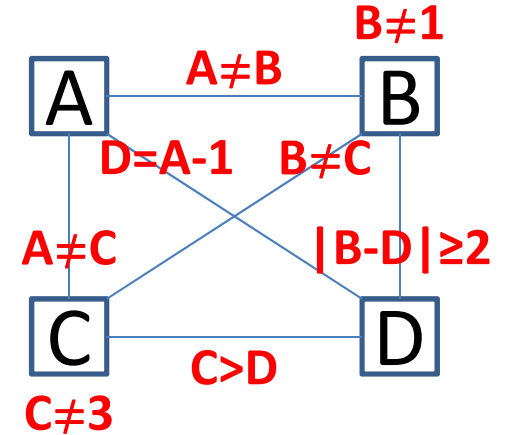
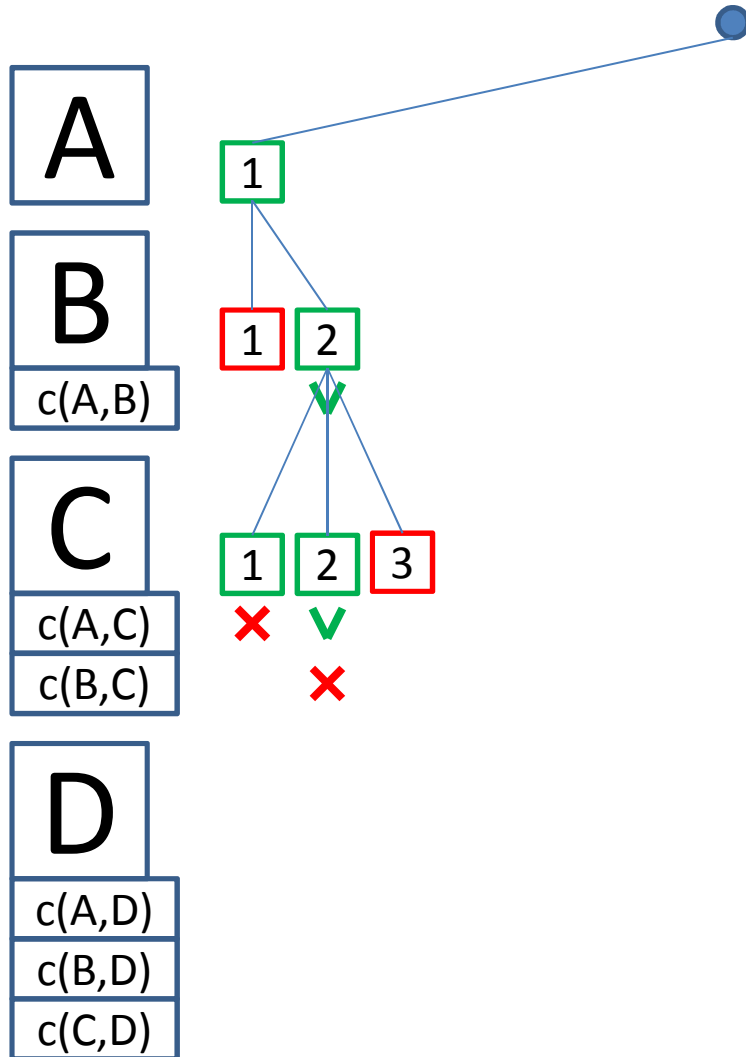
Constraint Processing: Backtracking



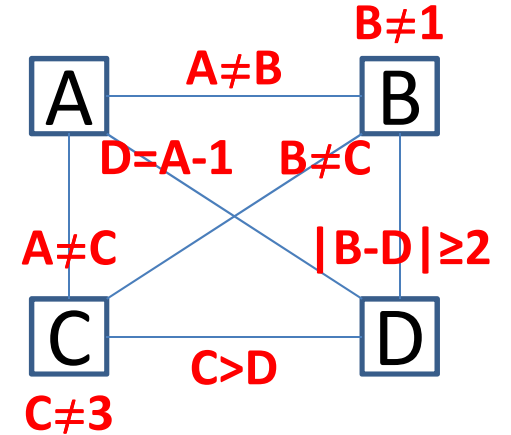
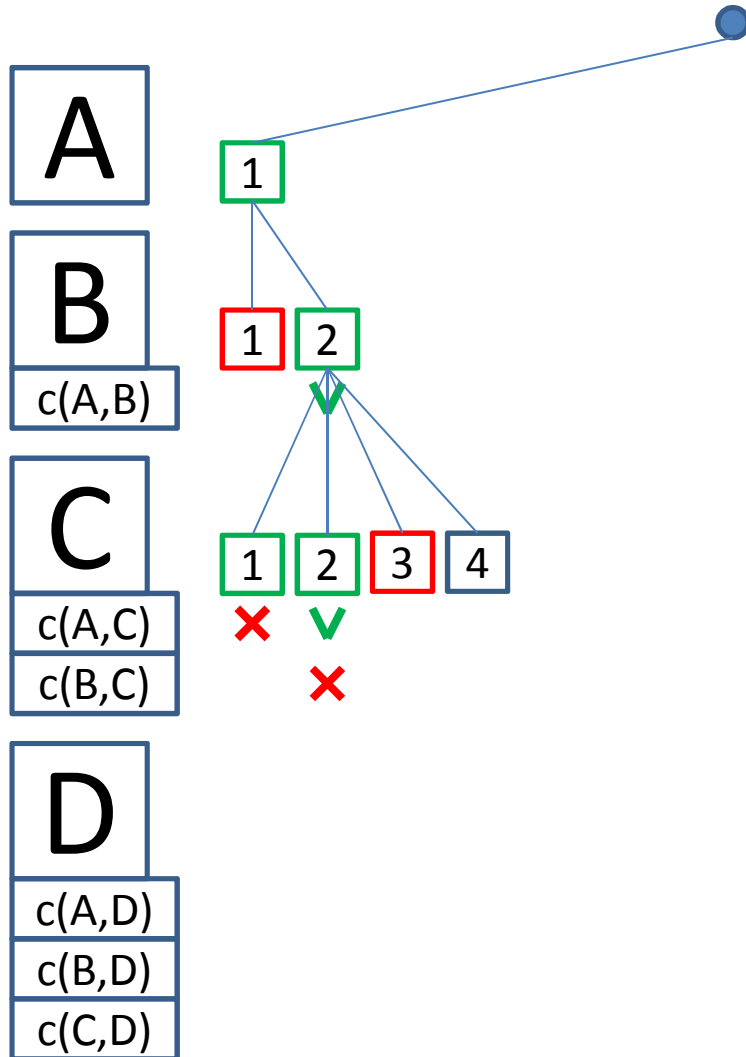
Constraint Processing: Backtracking



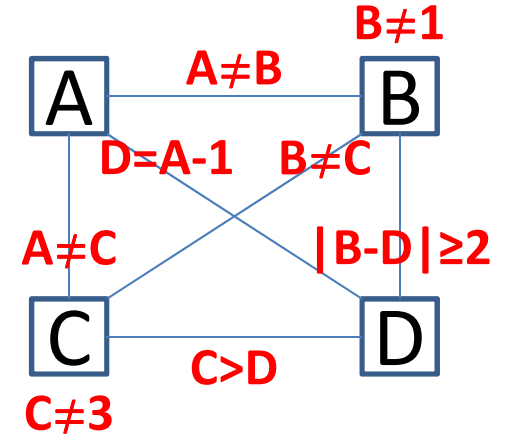
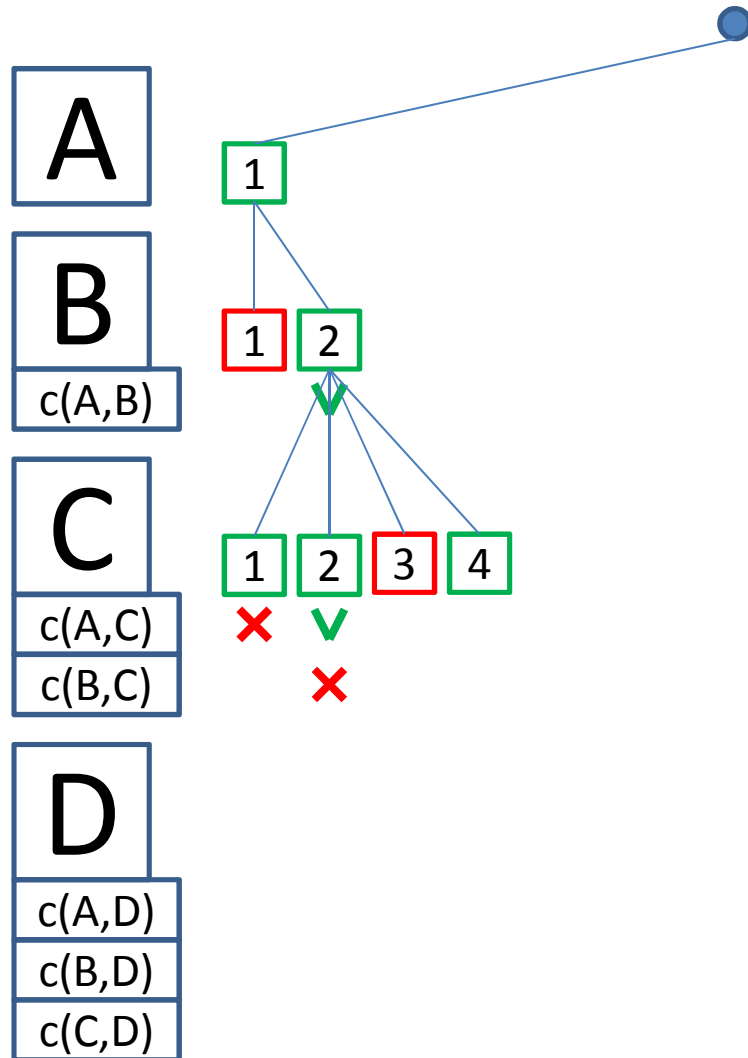
Constraint Processing: Backtracking



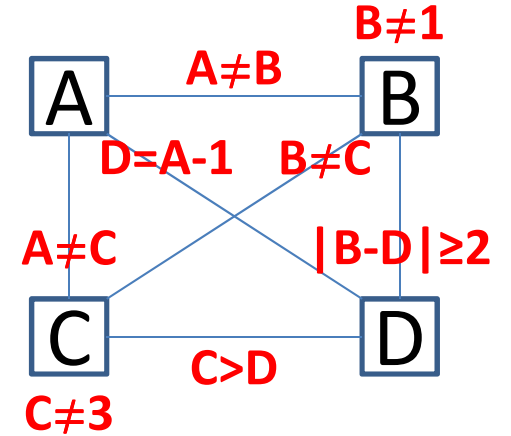
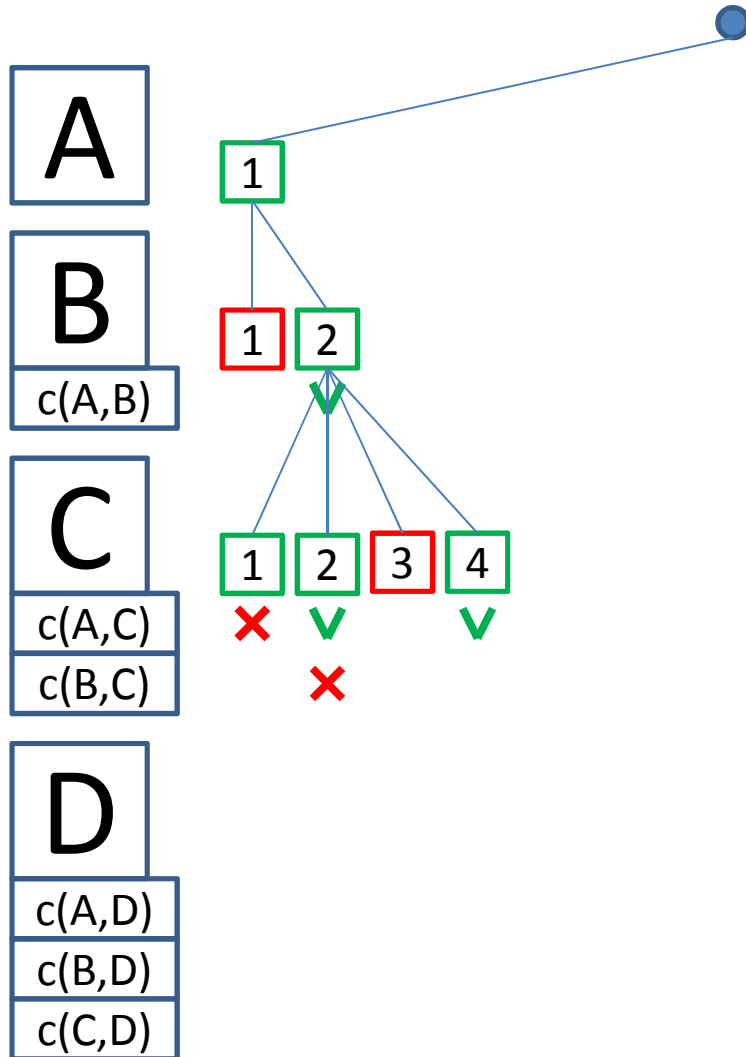
Constraint Processing: Backtracking



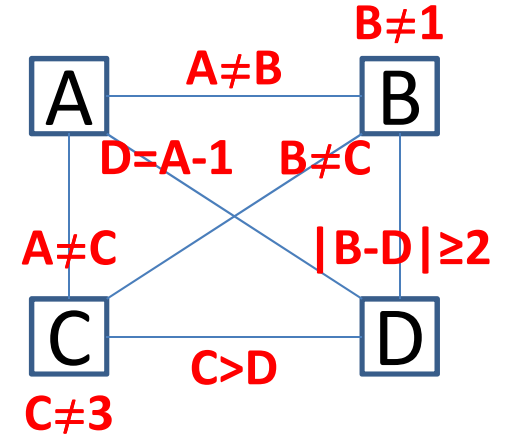
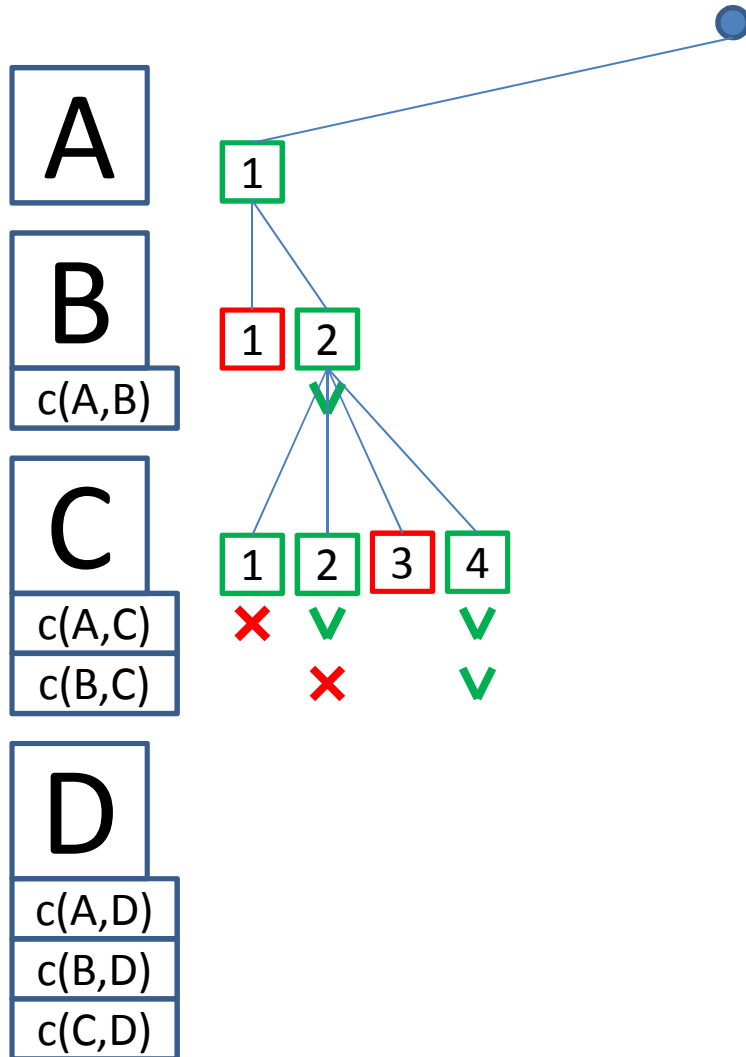
Constraint Processing: Backtracking



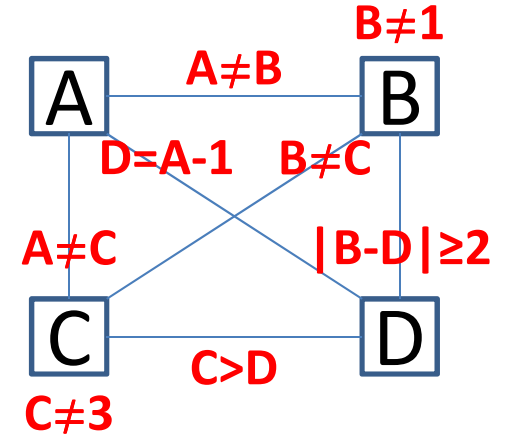
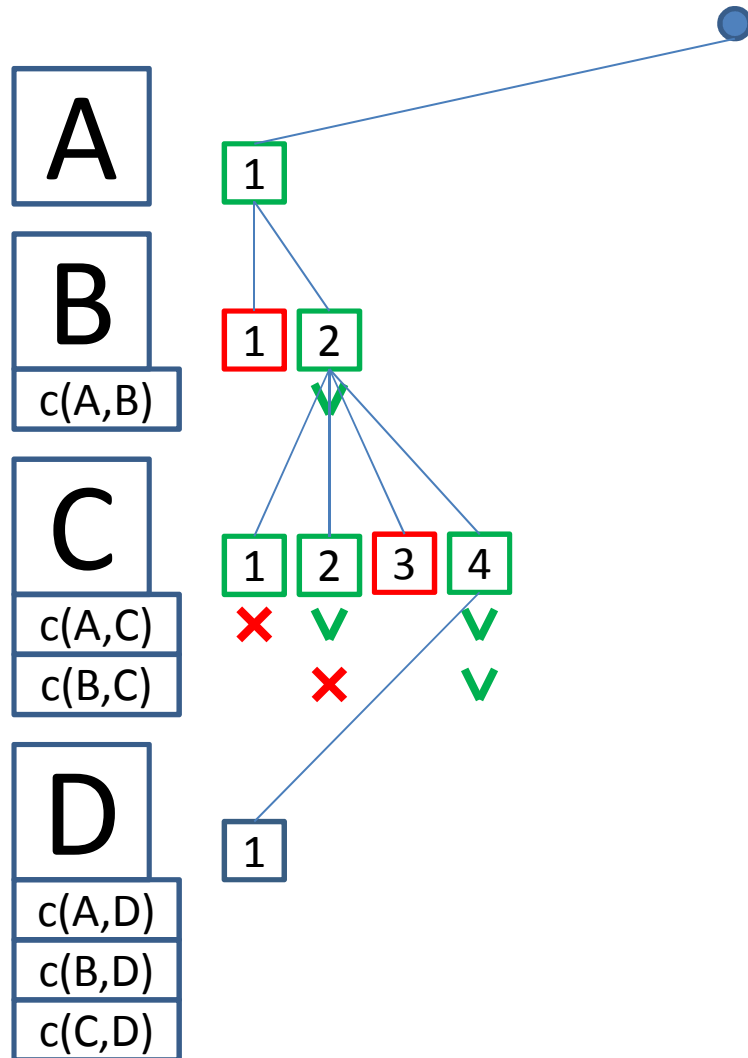
Constraint Processing: Backtracking



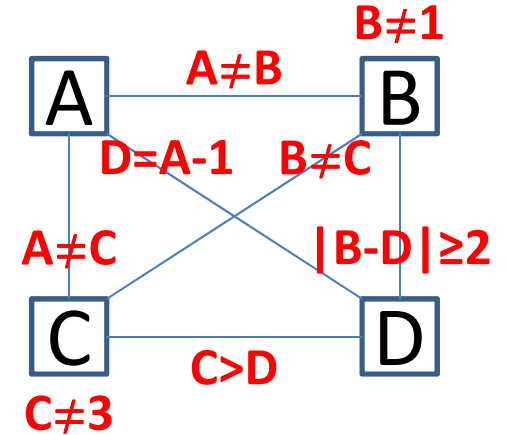
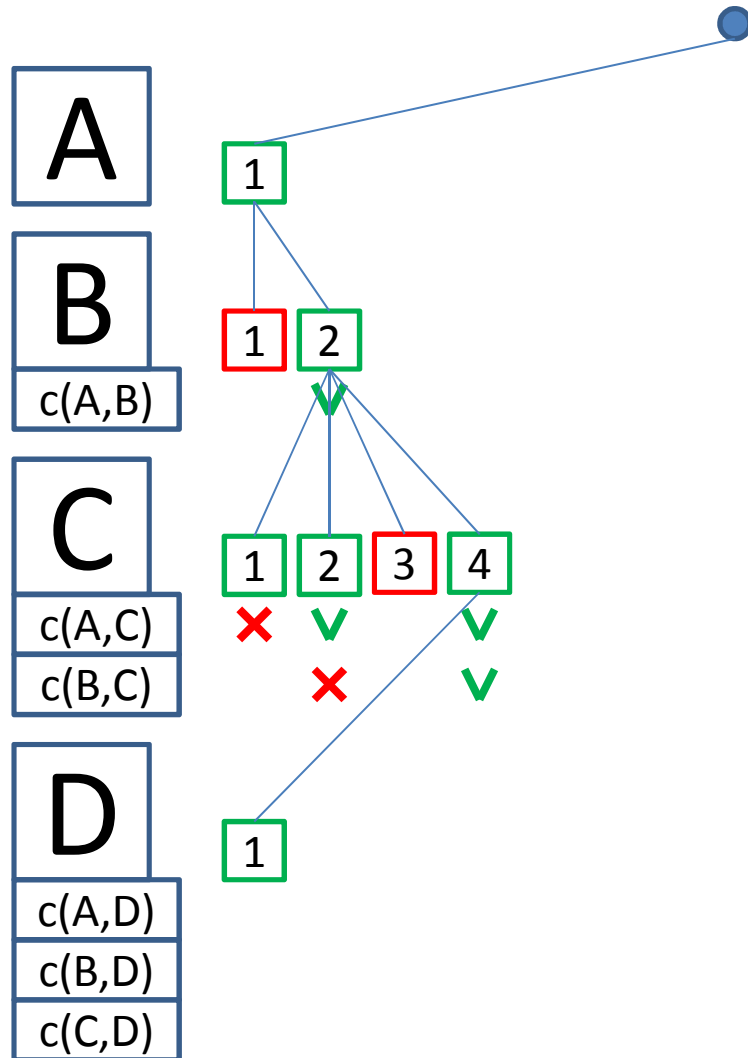
Constraint Processing: Backtracking



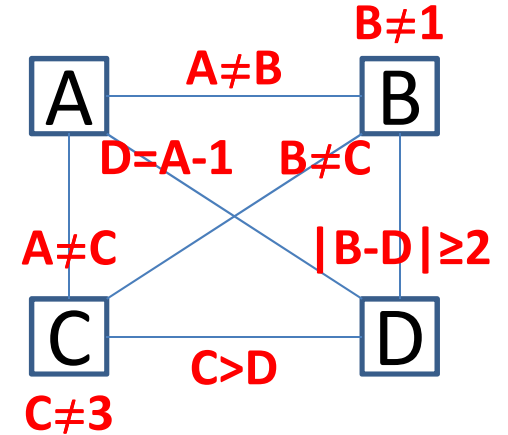
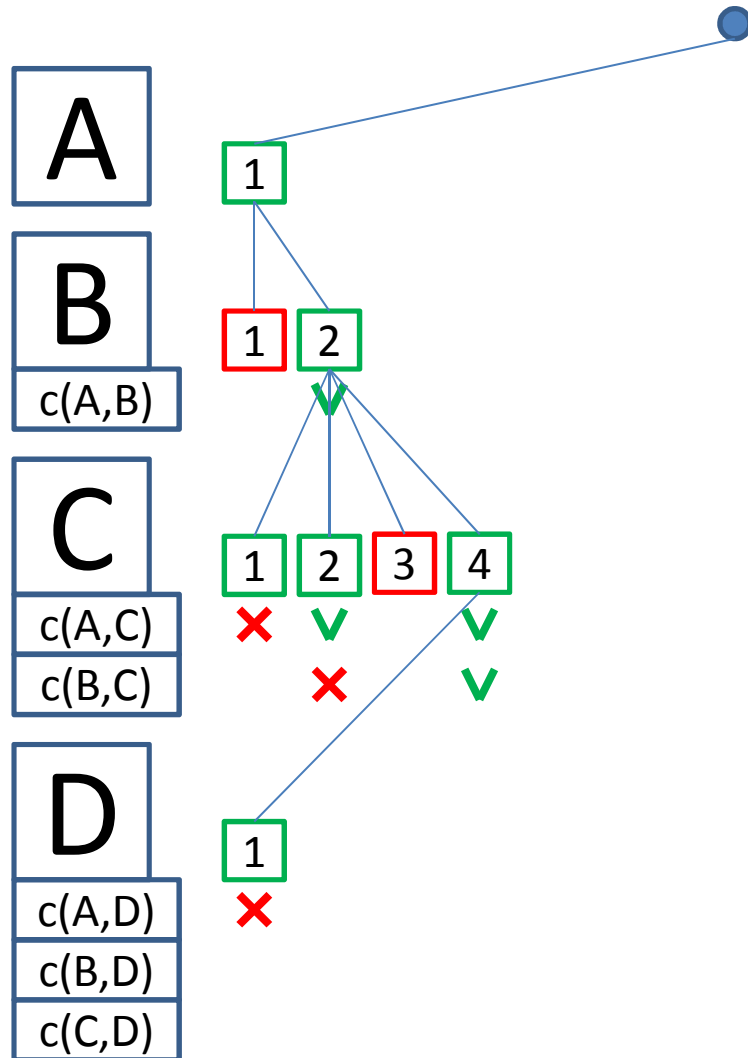
Constraint Processing: Backtracking



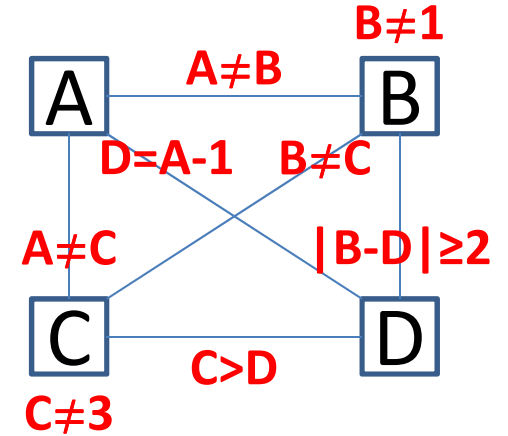
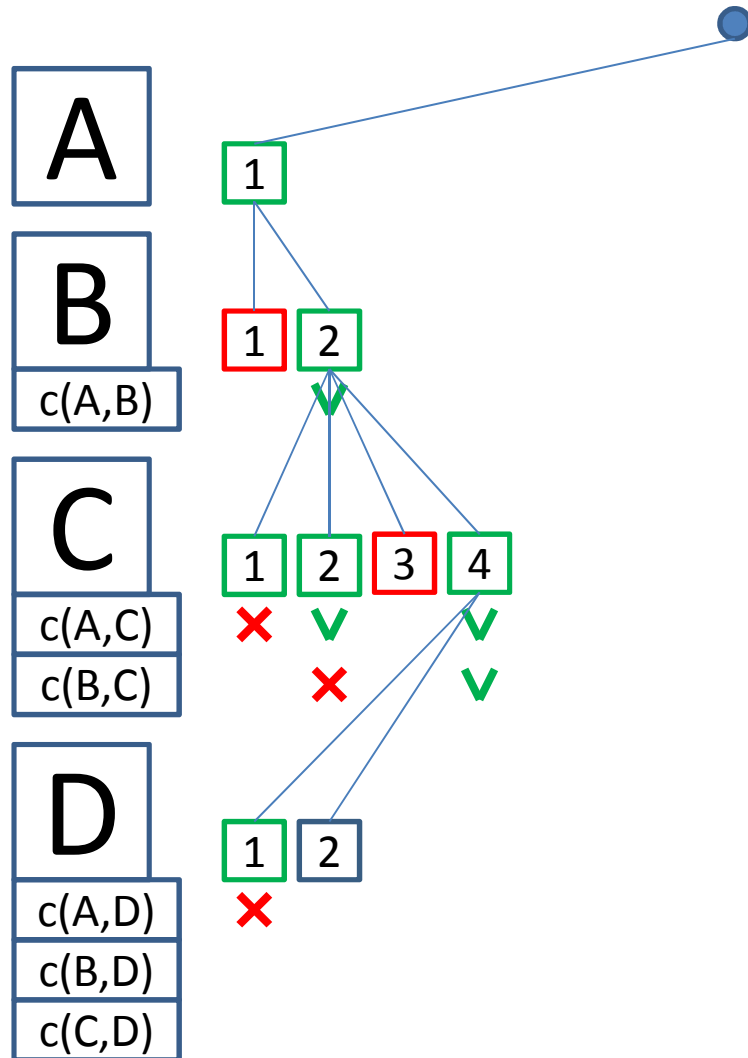
Constraint Processing: Backtracking



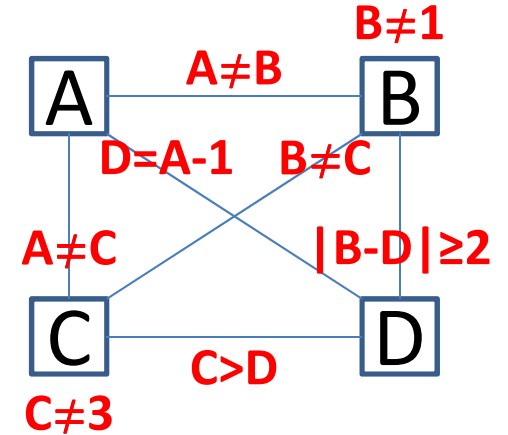
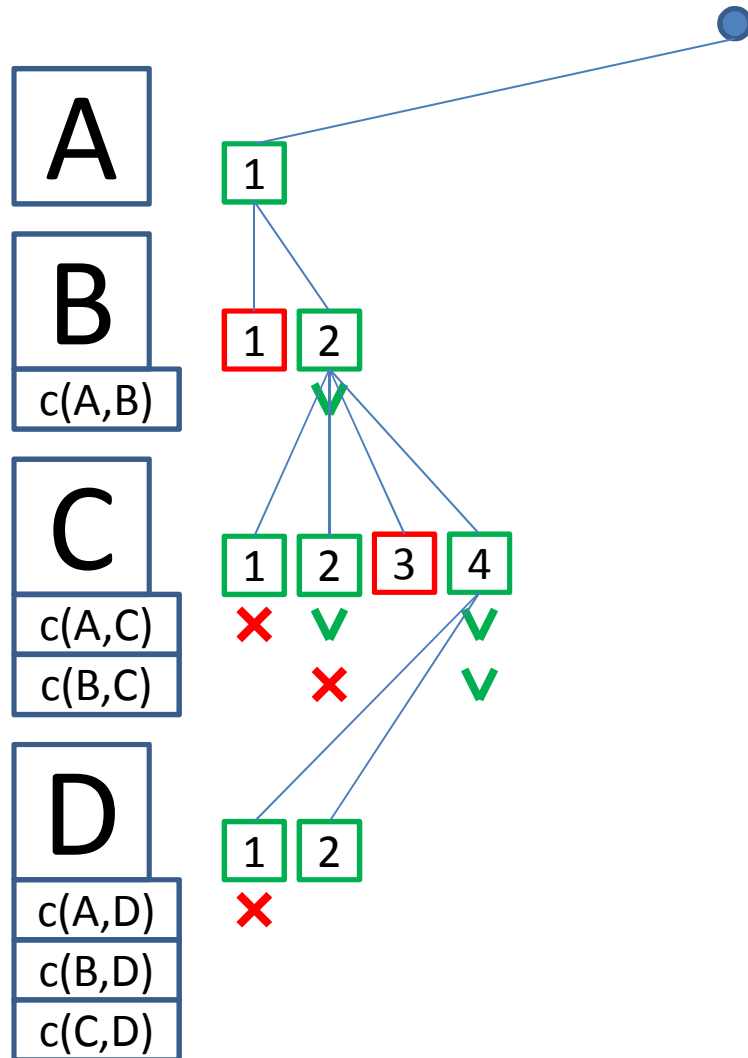
Constraint Processing: Backtracking



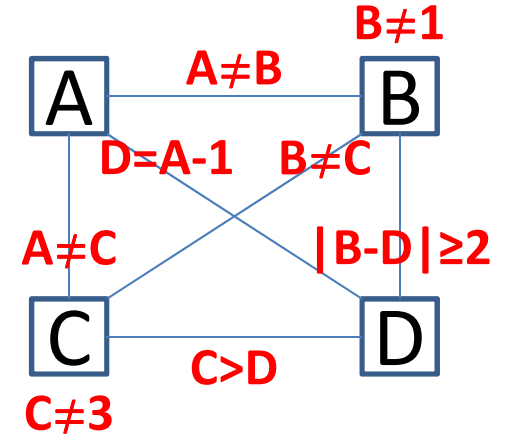
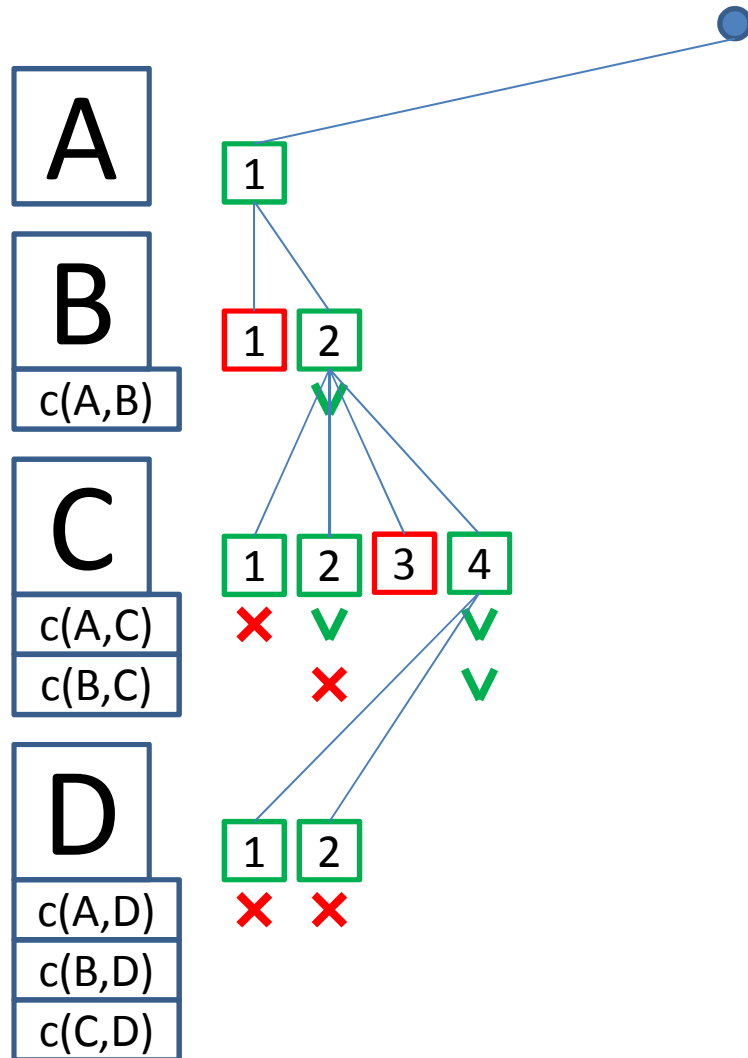
Constraint Processing: Backtracking



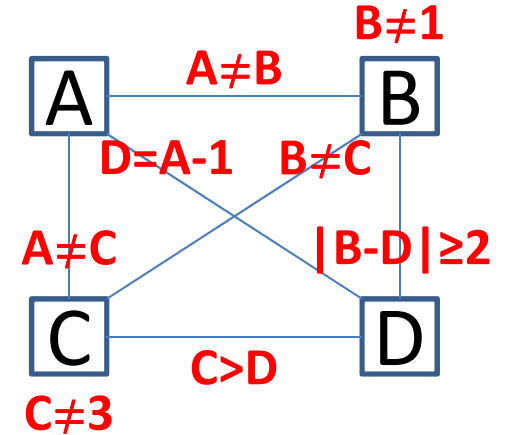
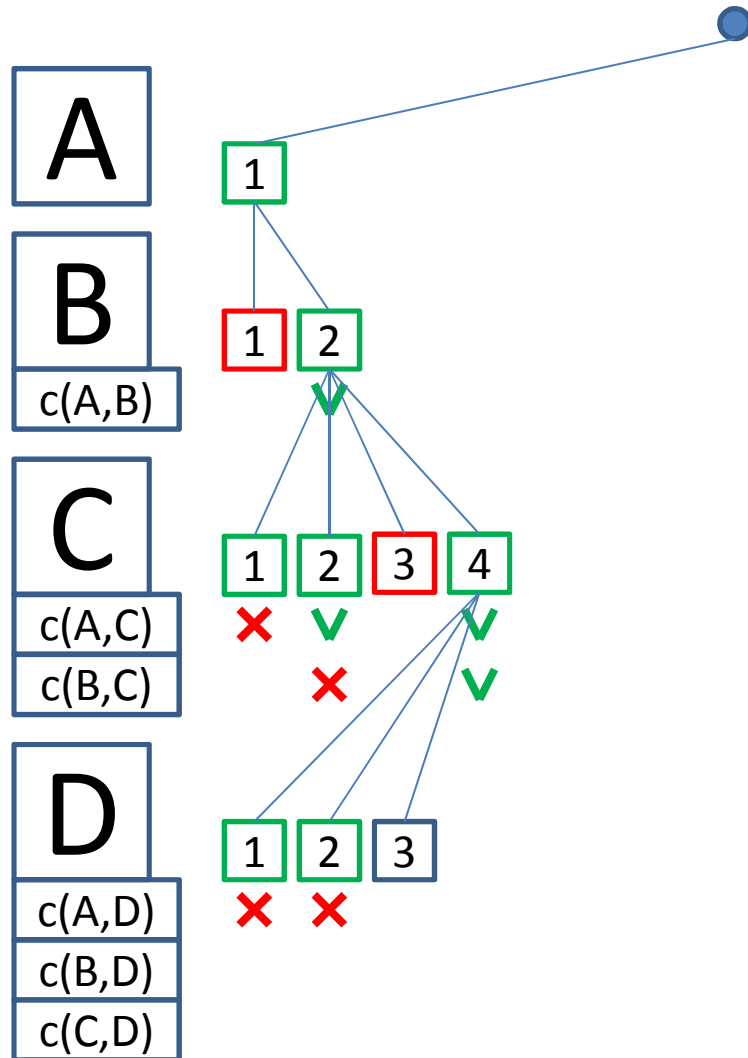
Constraint Processing: Backtracking



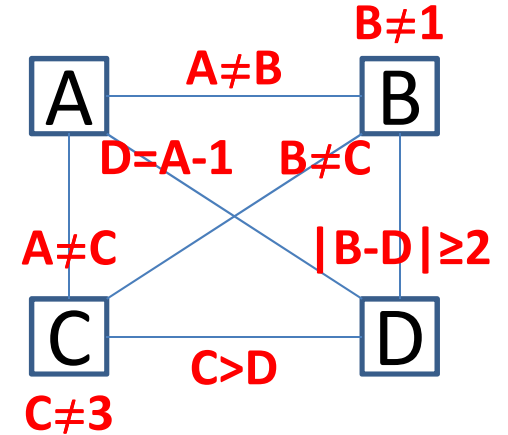
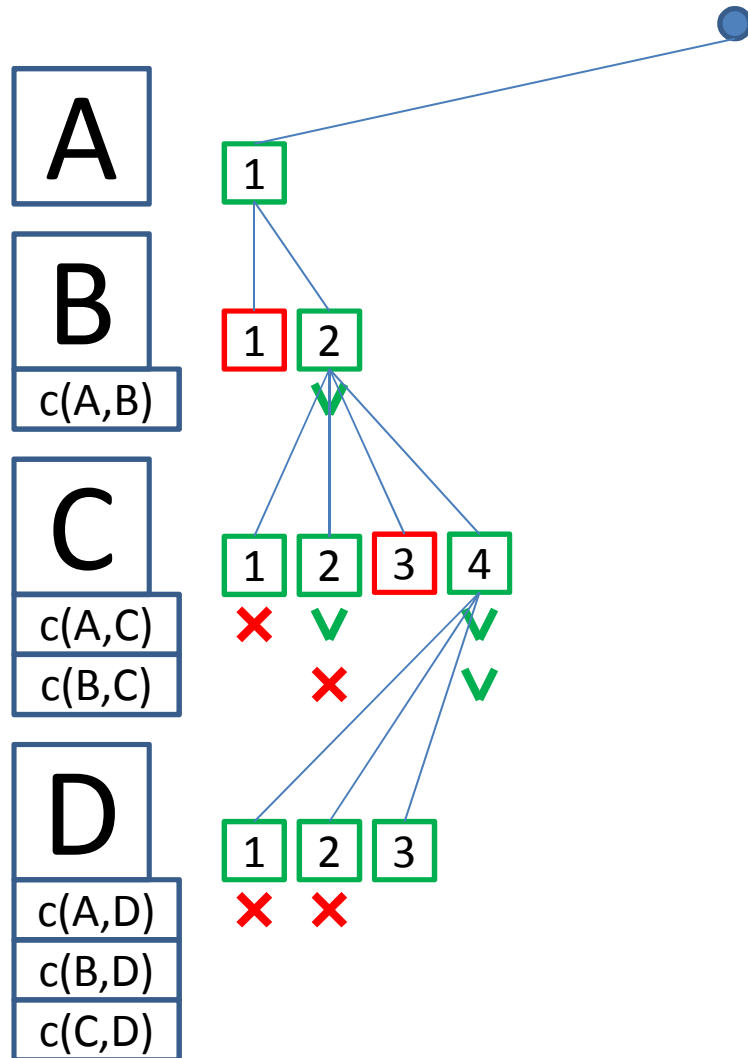
Constraint Processing: Backtracking



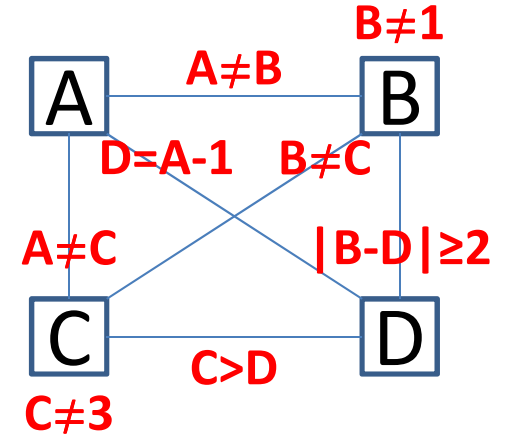
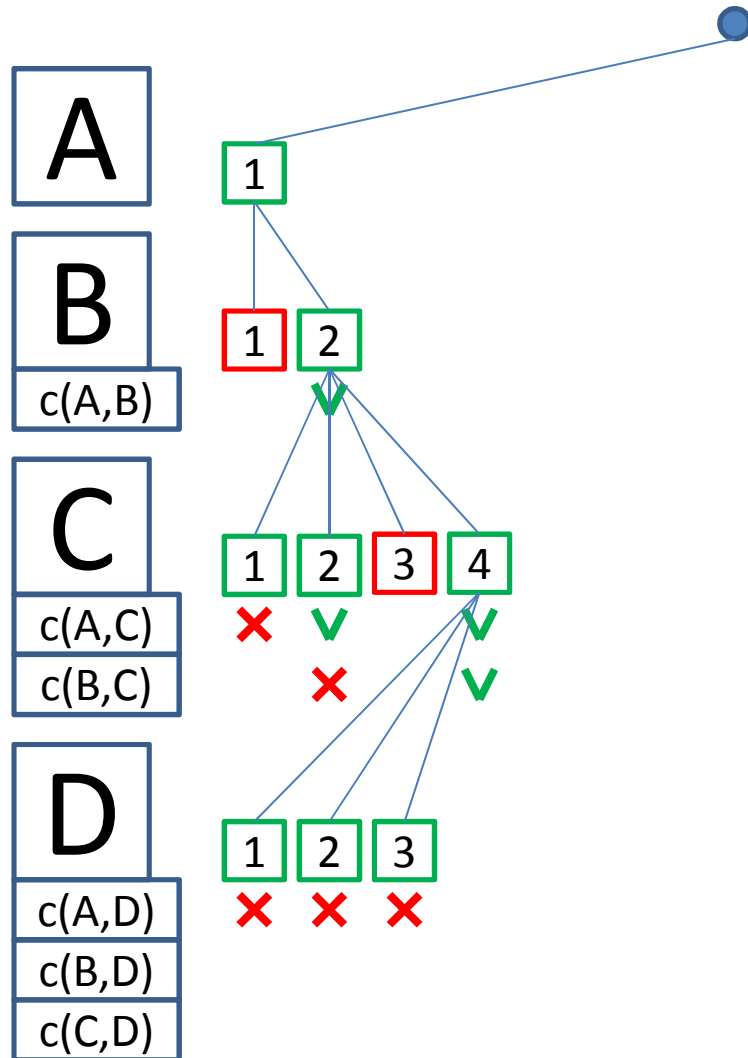
Constraint Processing: Backtracking



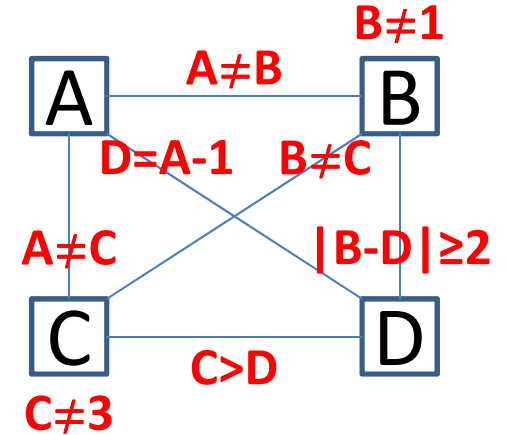
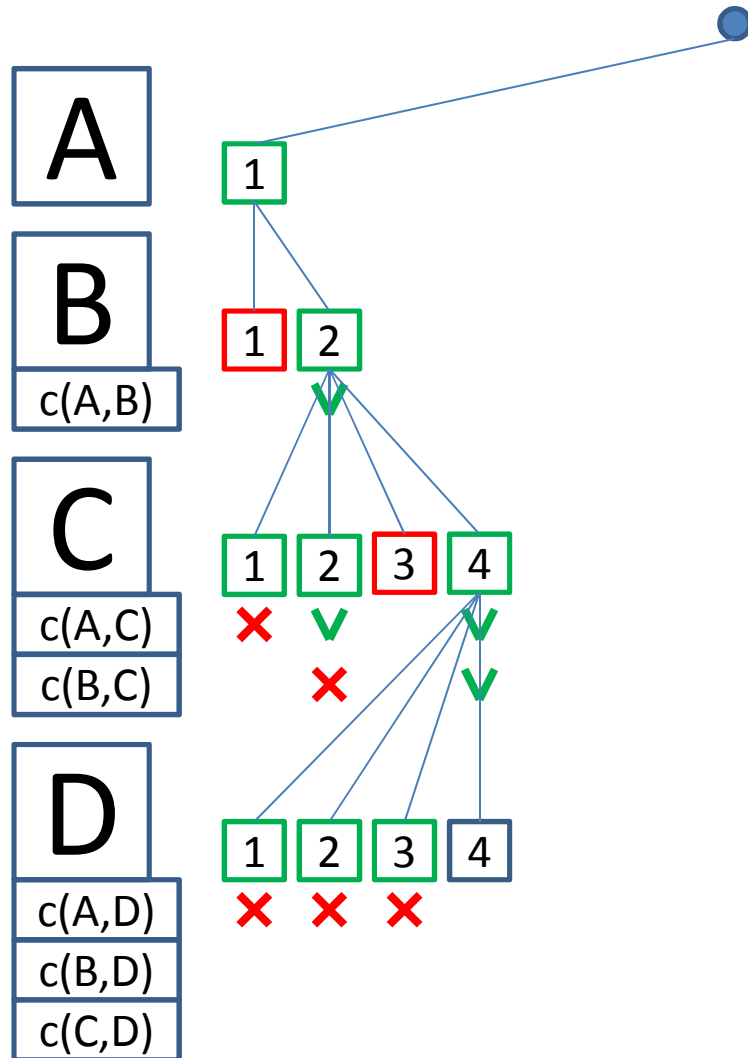
Constraint Processing: Backtracking



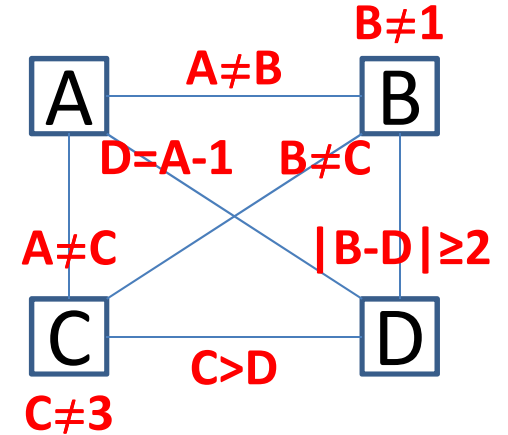
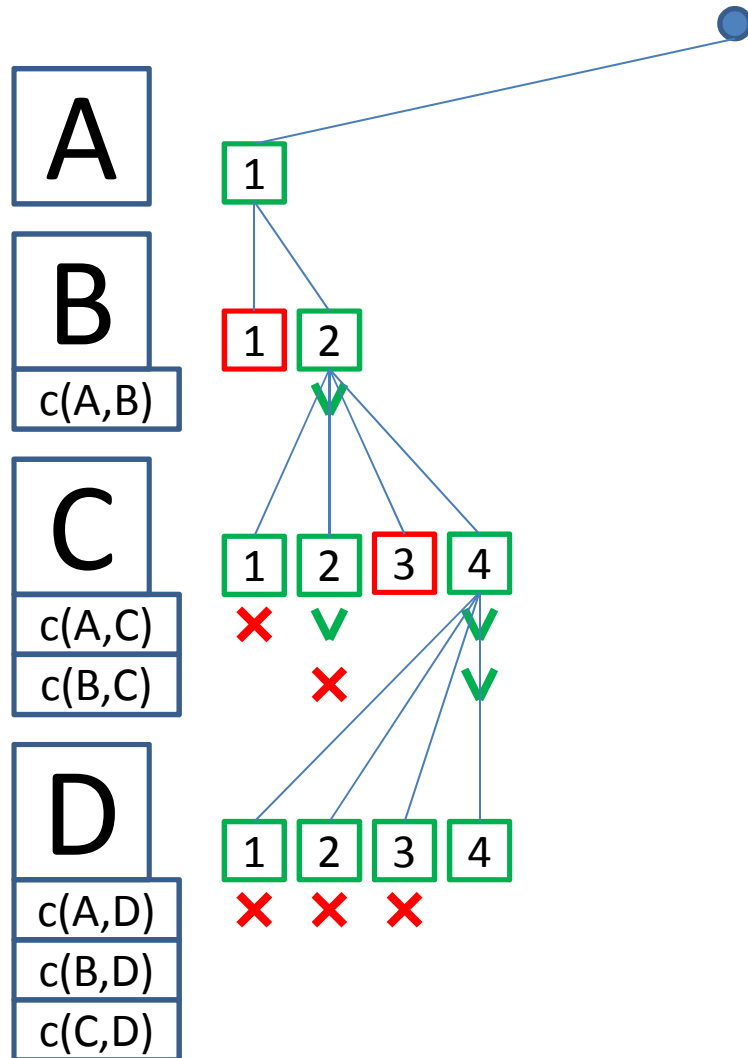
Constraint Processing: Backtracking



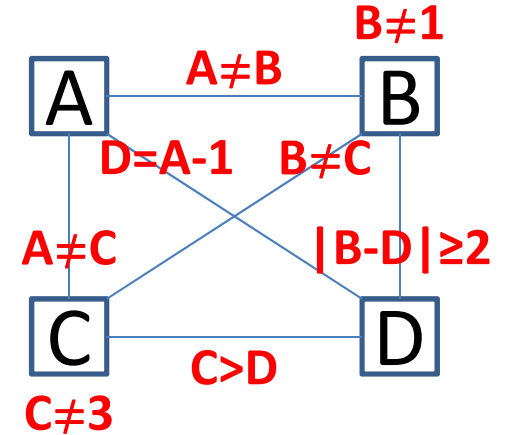
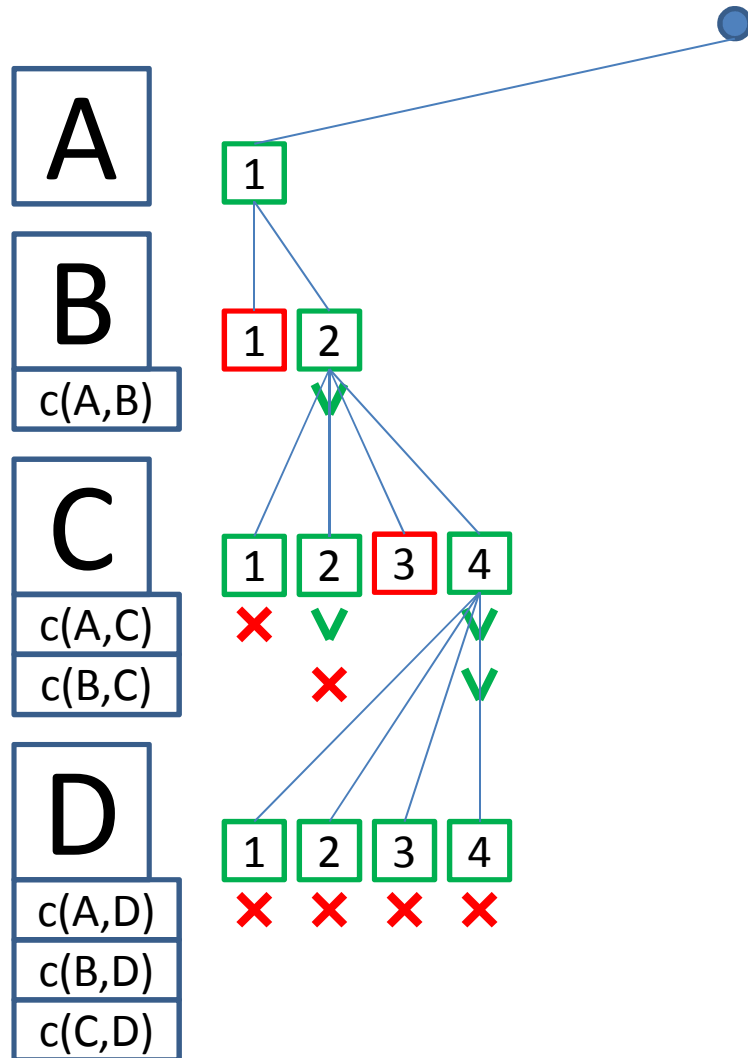
Constraint Processing: Backtracking



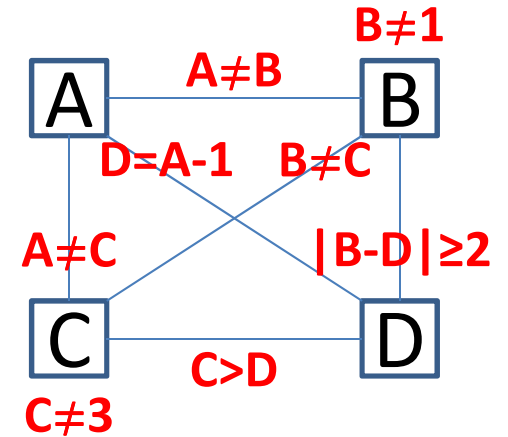
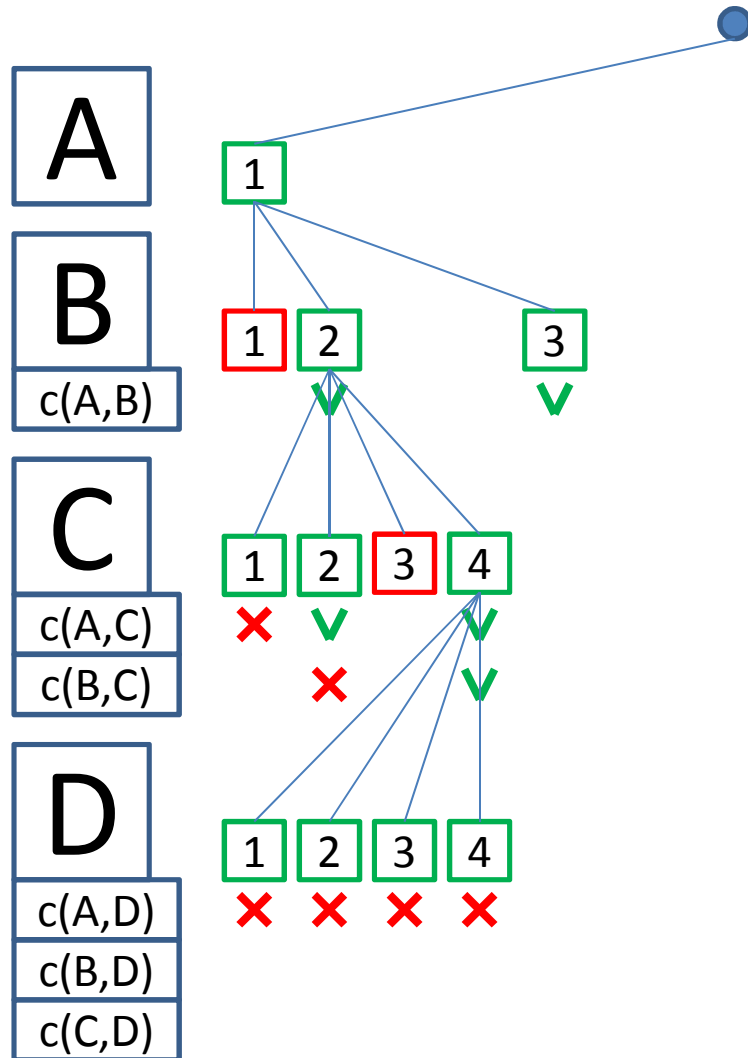
Constraint Processing: Backtracking



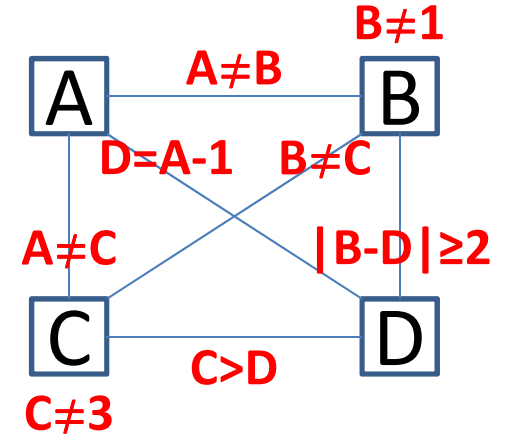
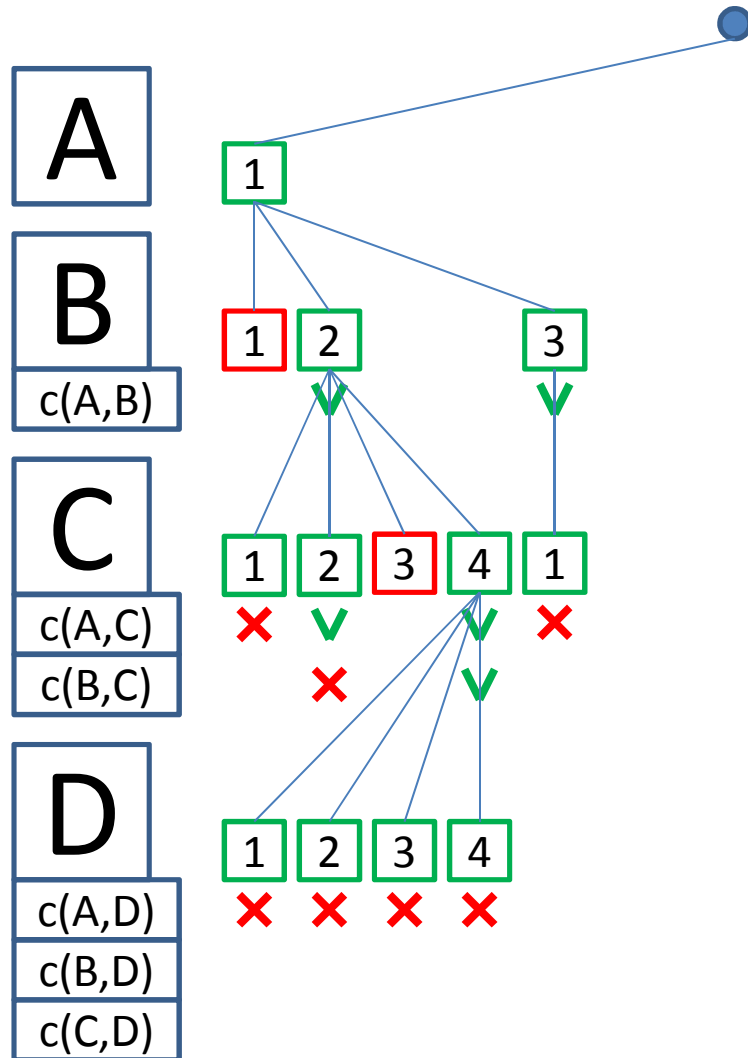
Constraint Processing: Backtracking



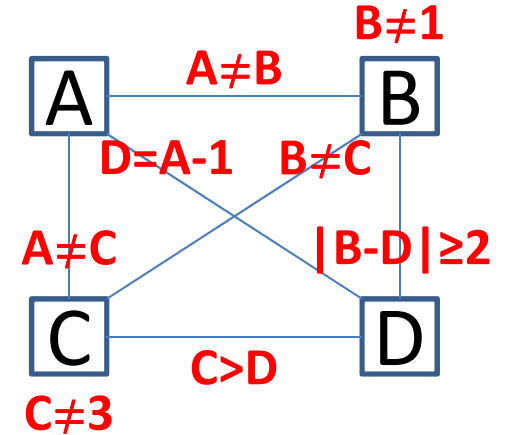
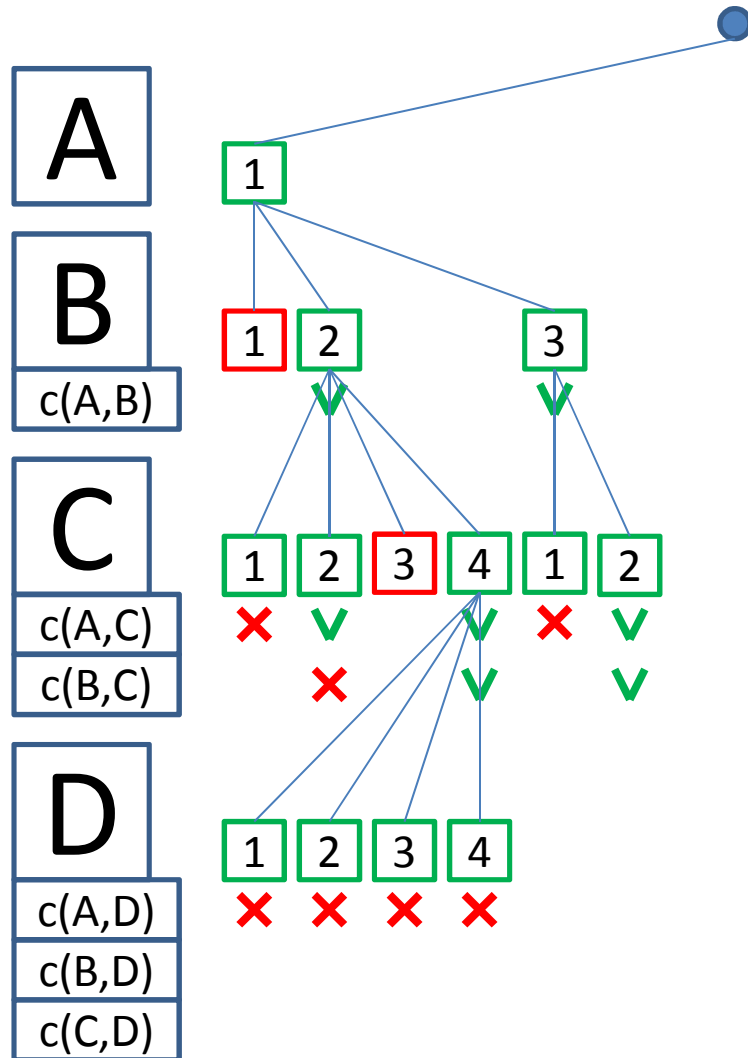
Constraint Processing: Backtracking



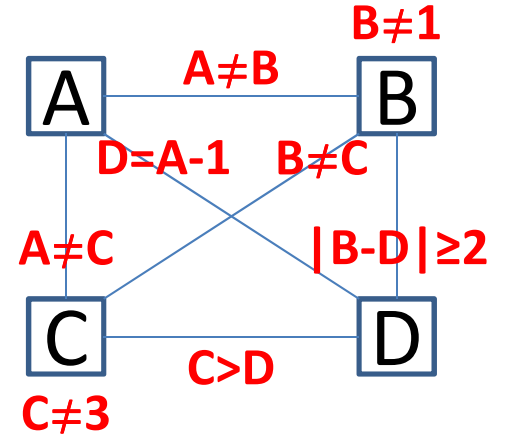
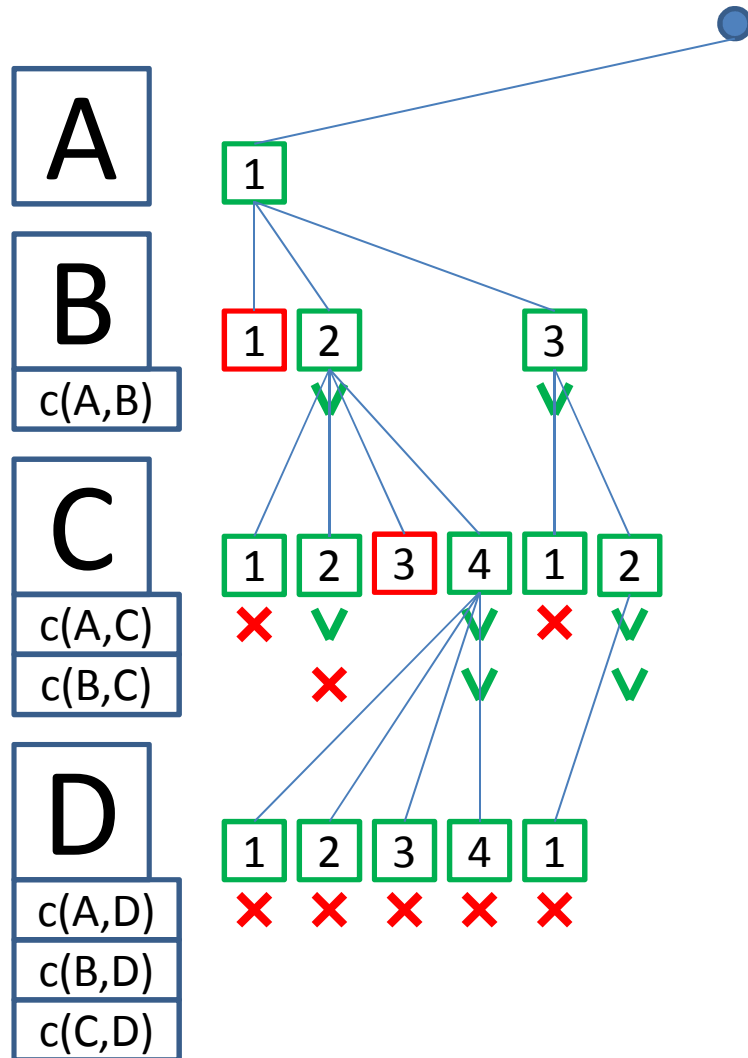
Constraint Processing: Backtracking



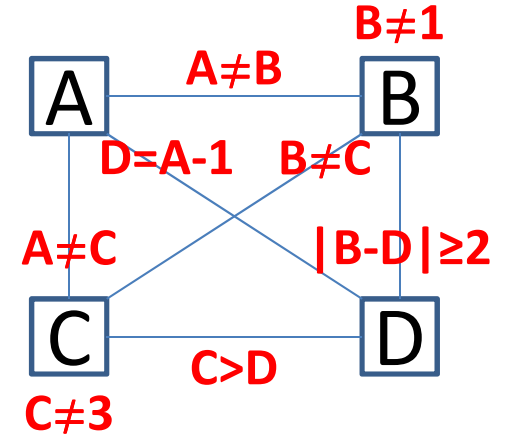
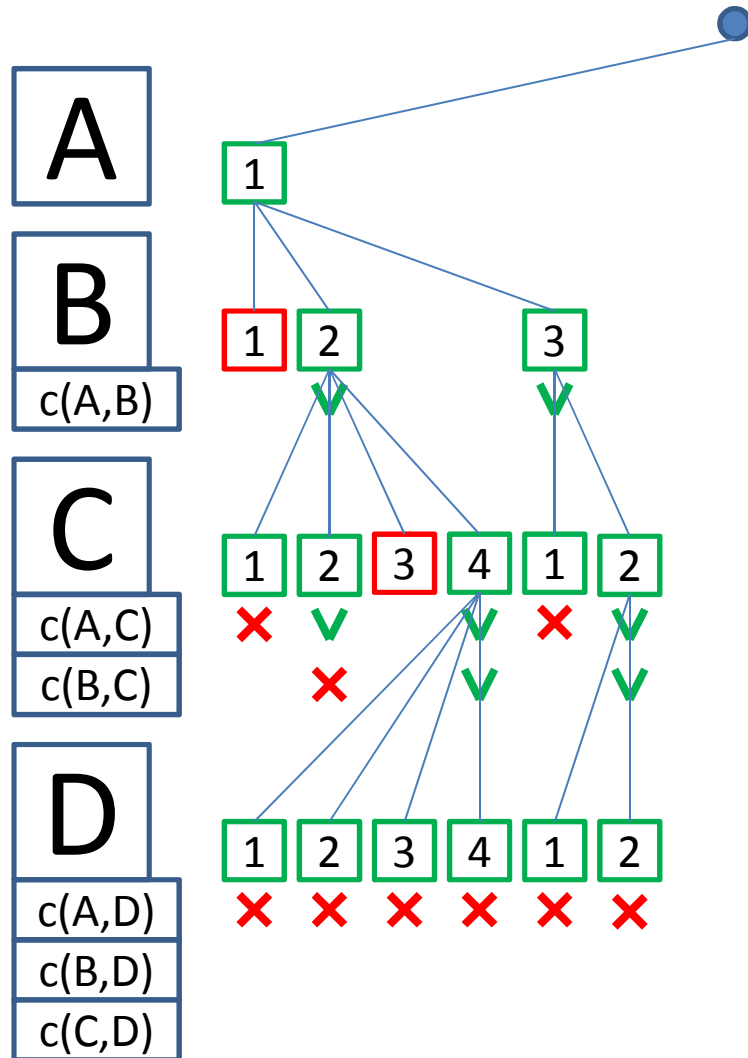
Constraint Processing: Backtracking



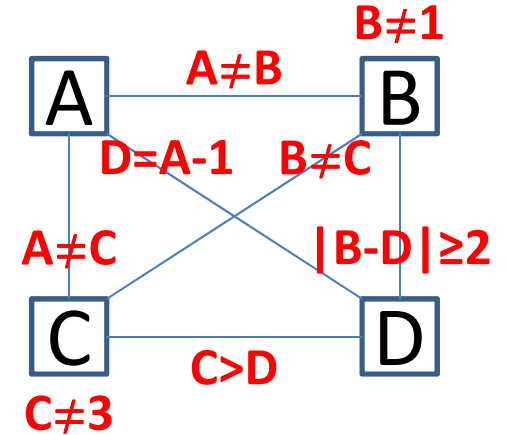
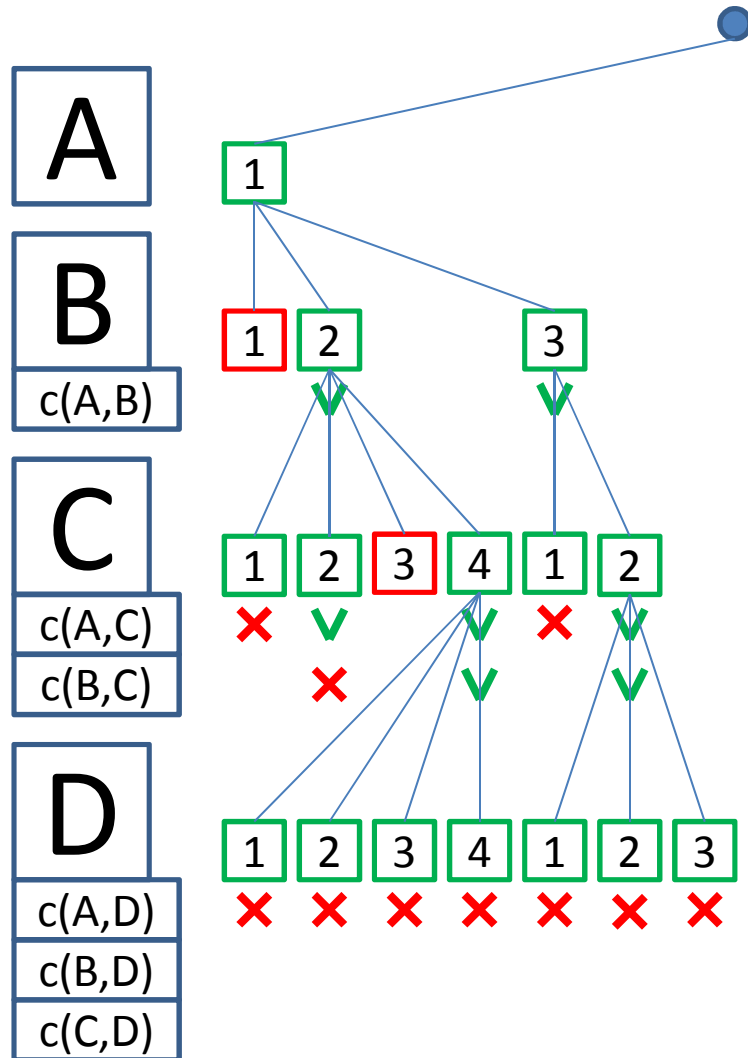
Constraint Processing: Backtracking



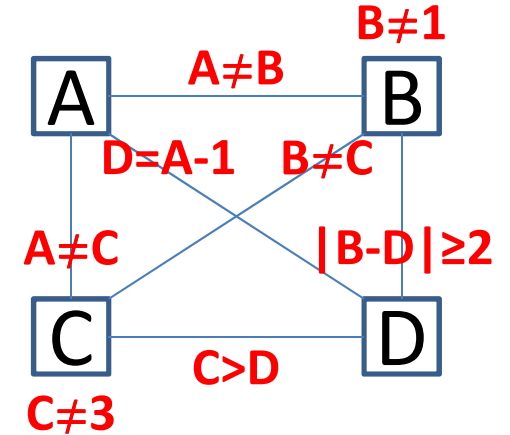
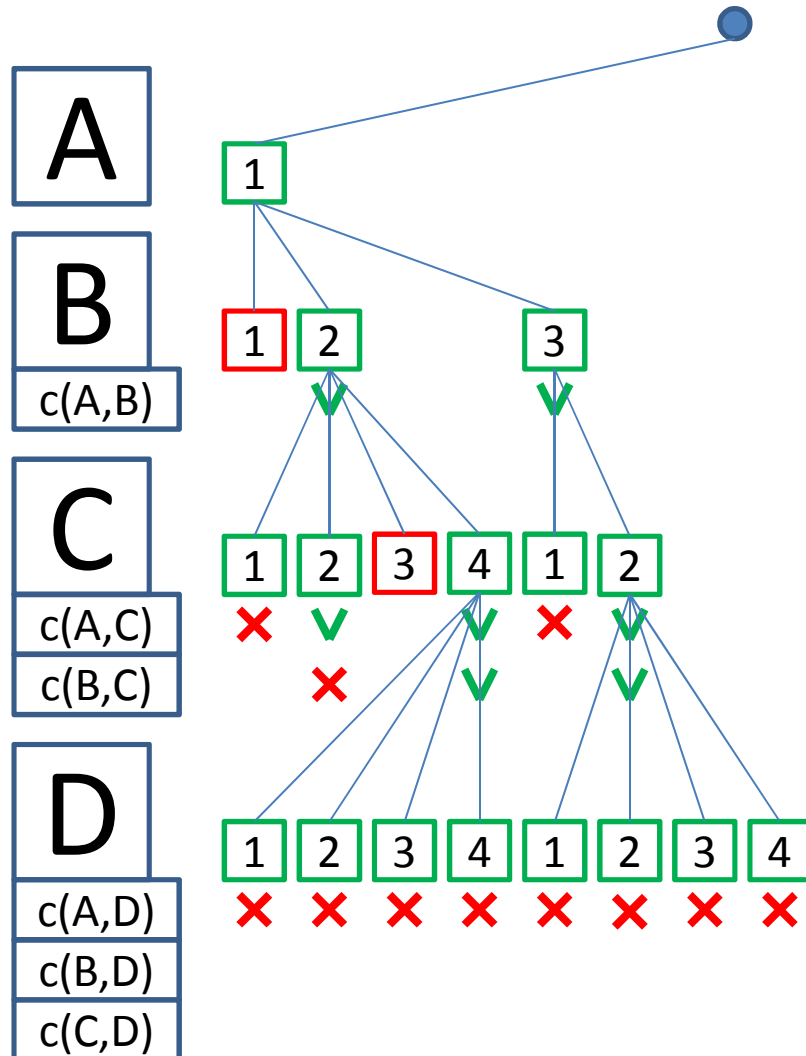
Constraint Processing: Backtracking



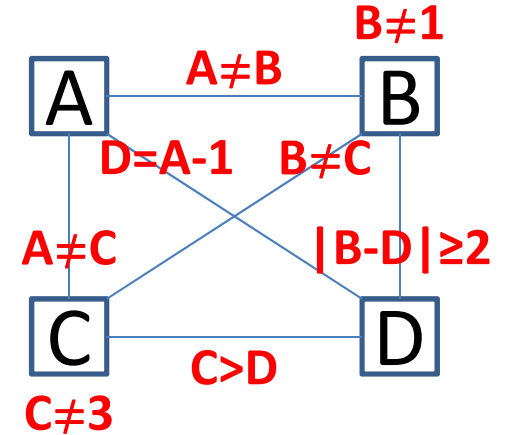
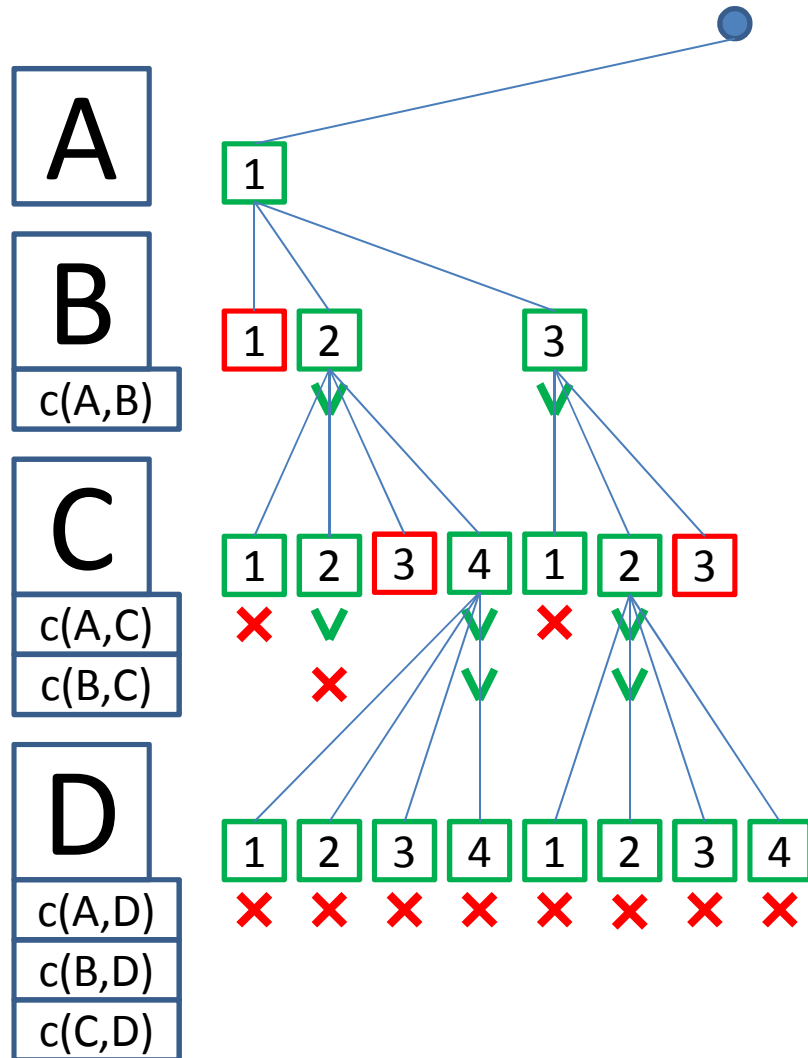
Constraint Processing: Backtracking



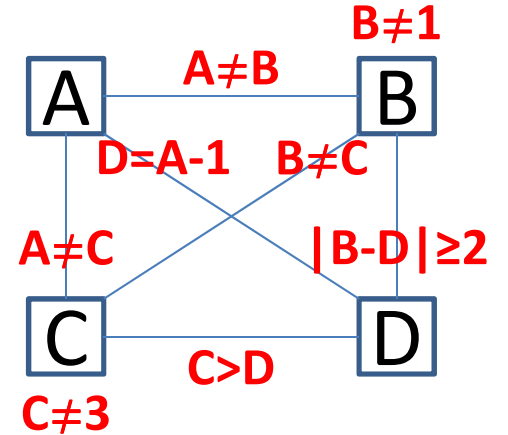
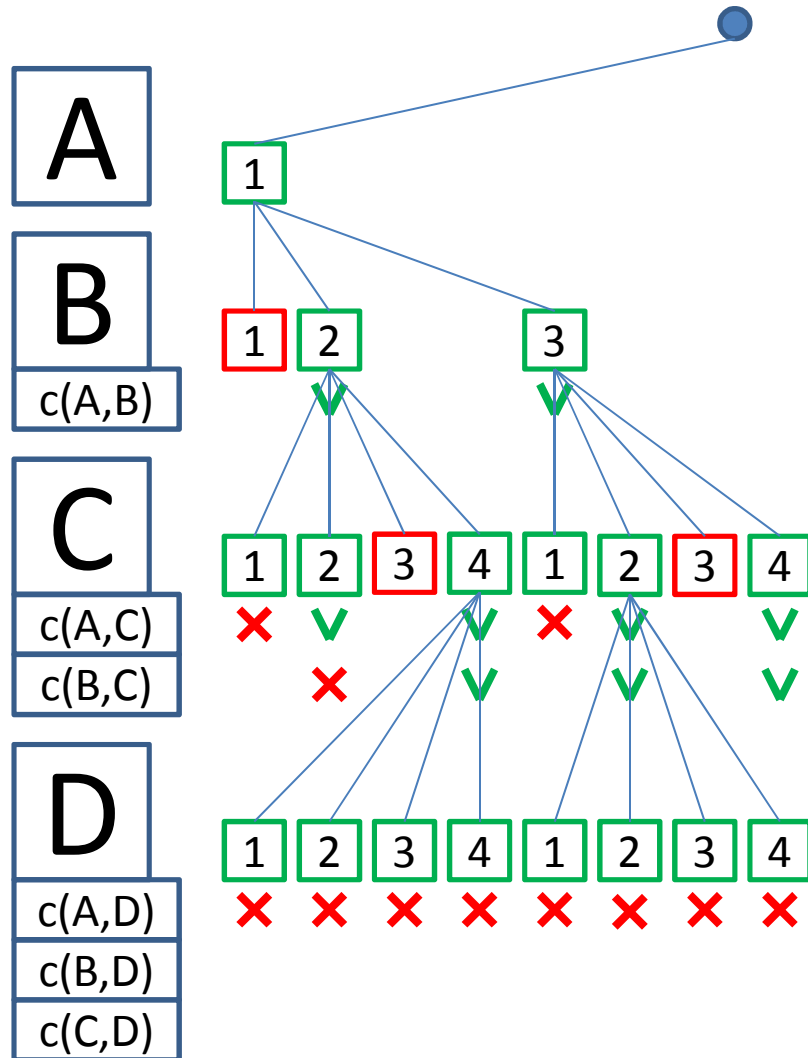
Constraint Processing: Backtracking



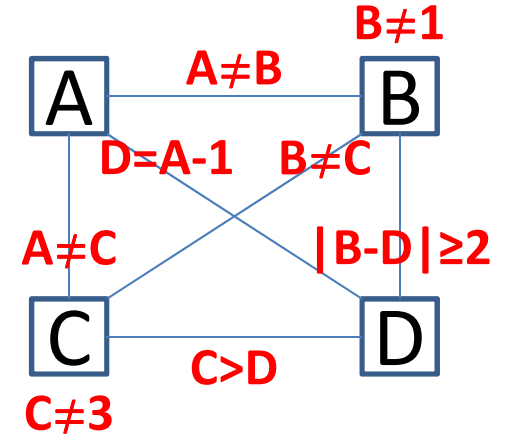
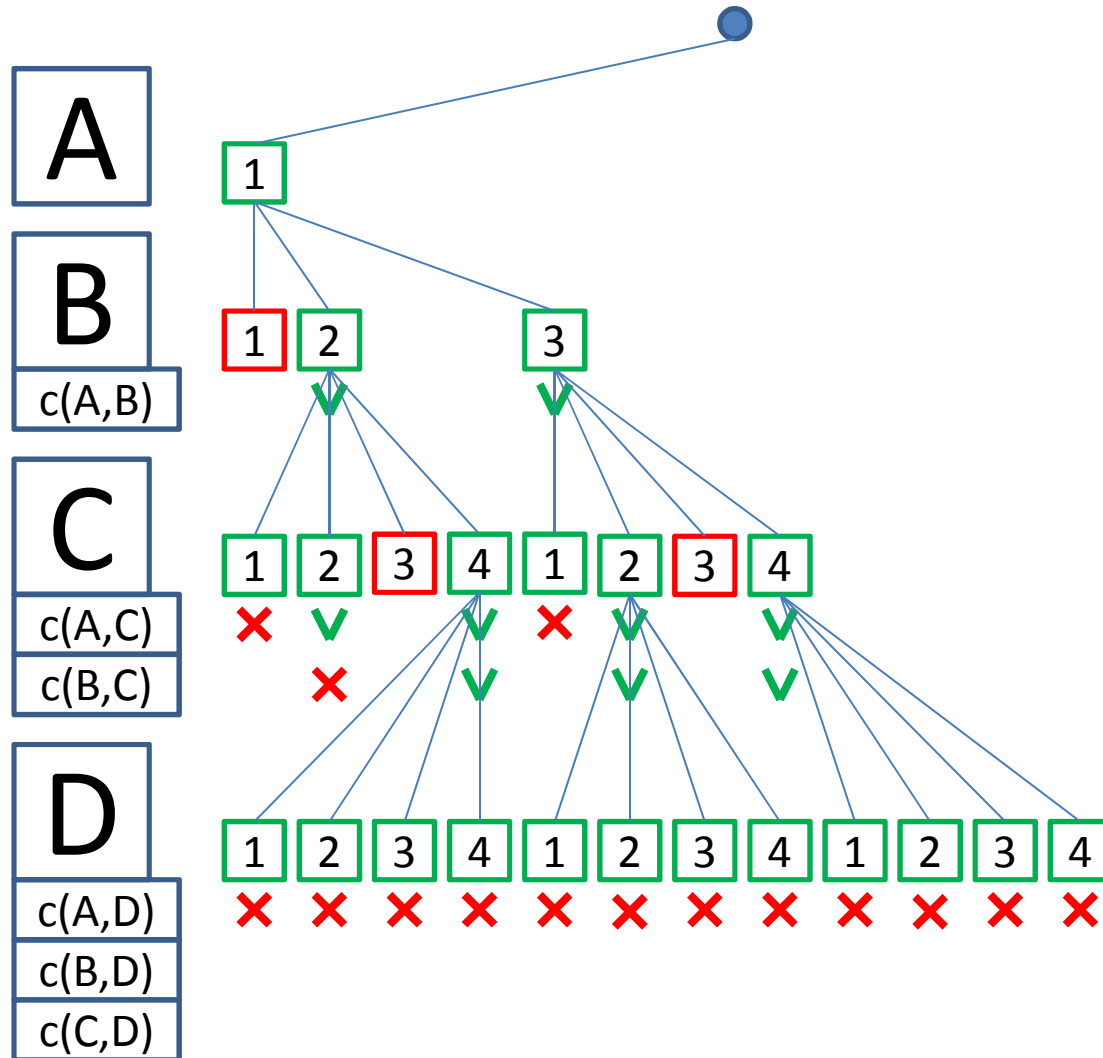
Constraint Processing: Backtracking



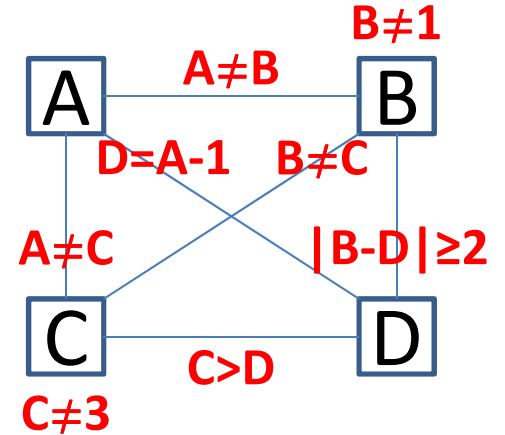
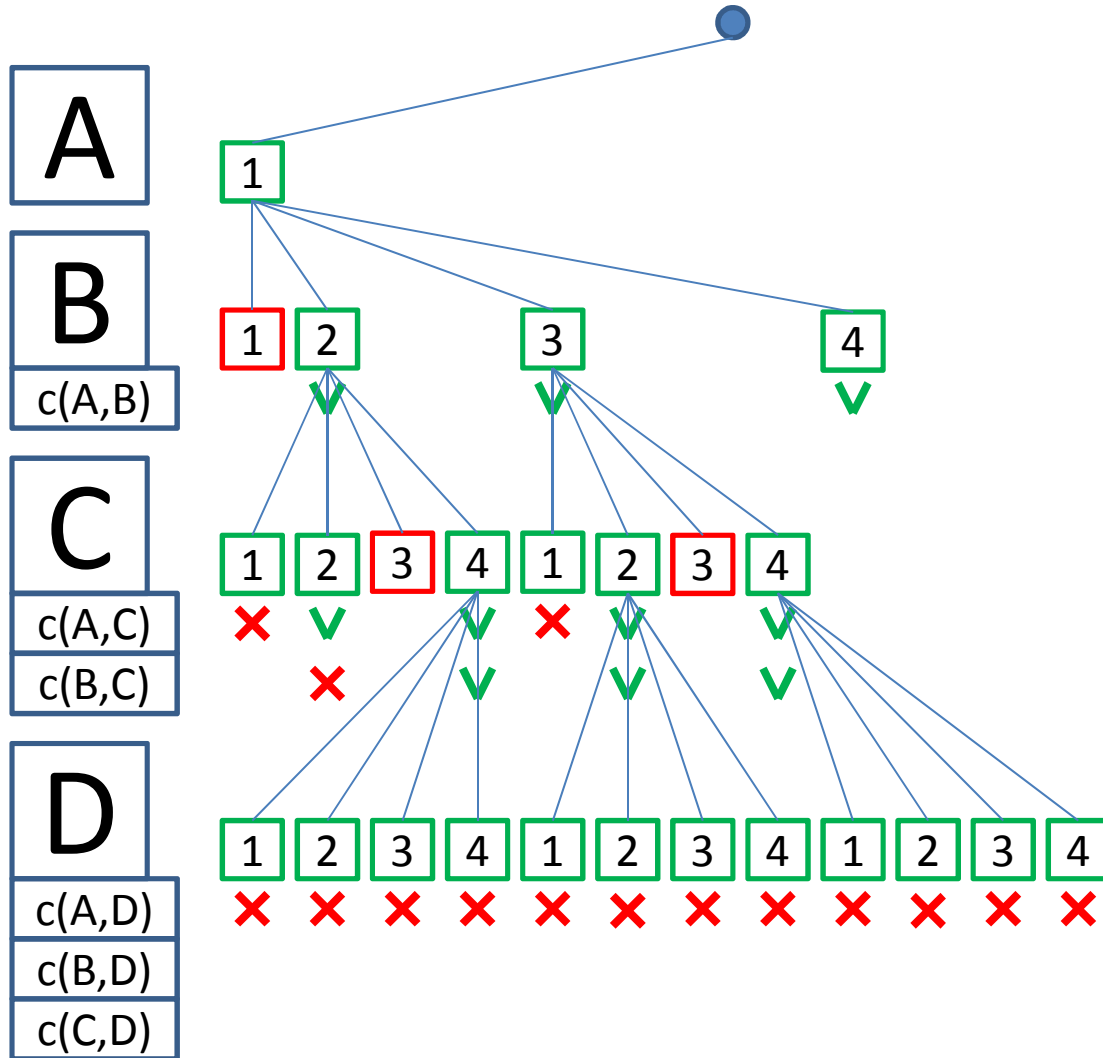
Constraint Processing: Backtracking



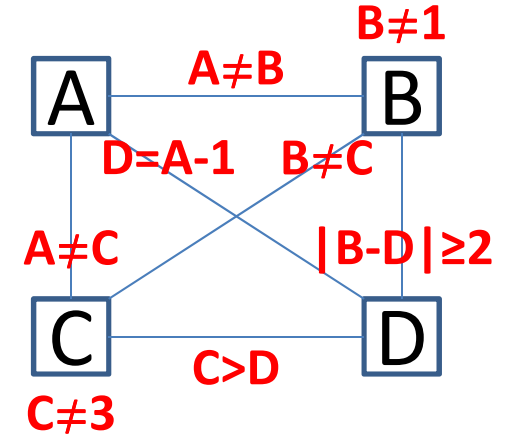
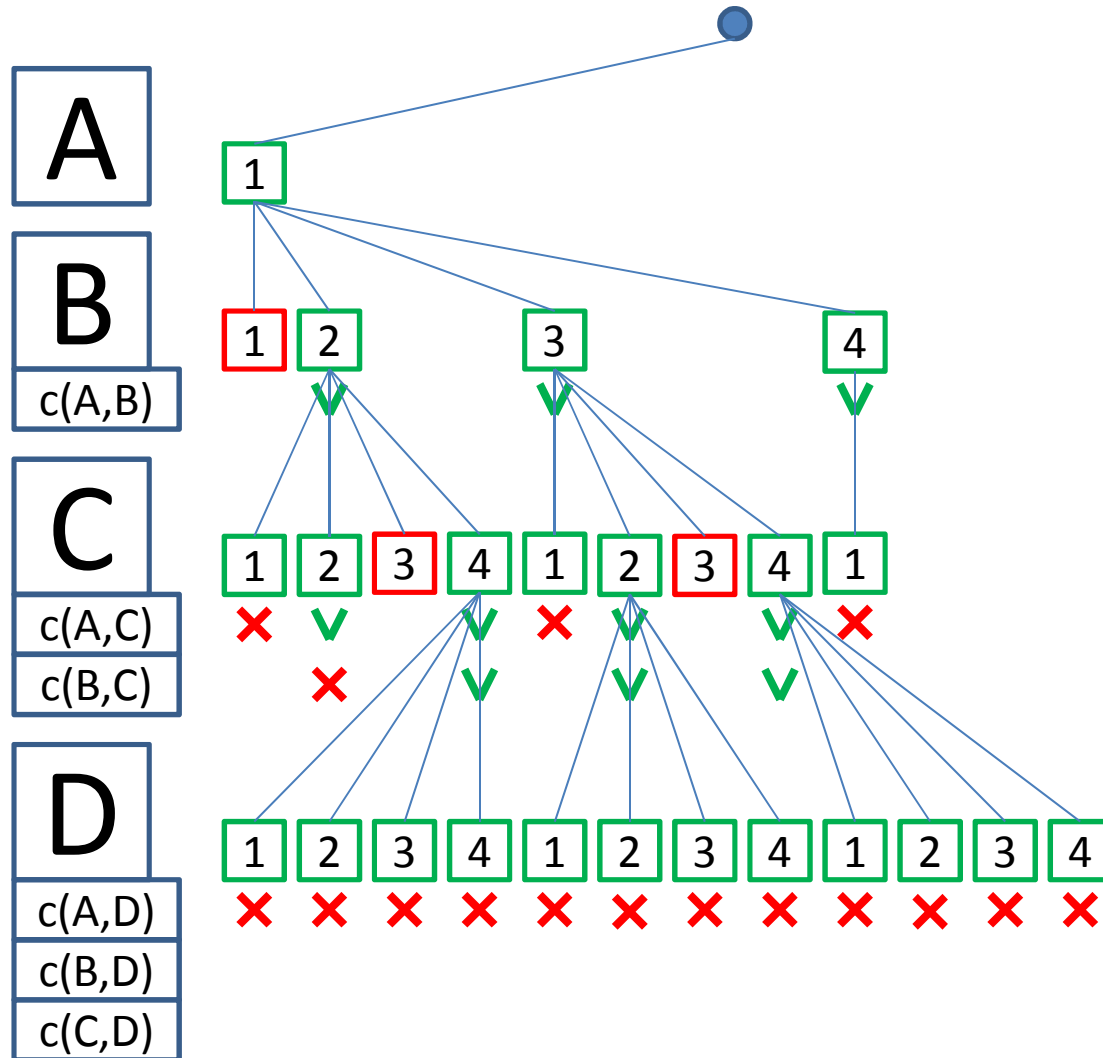
Constraint Processing: Backtracking



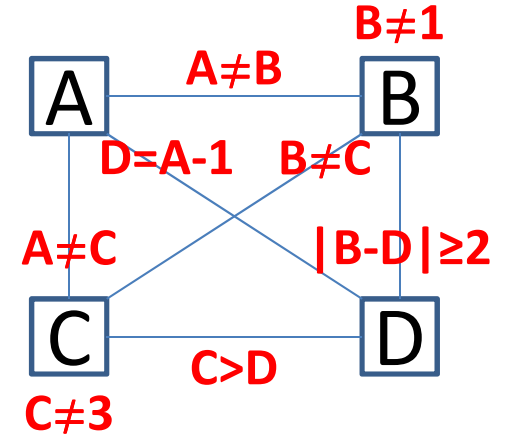
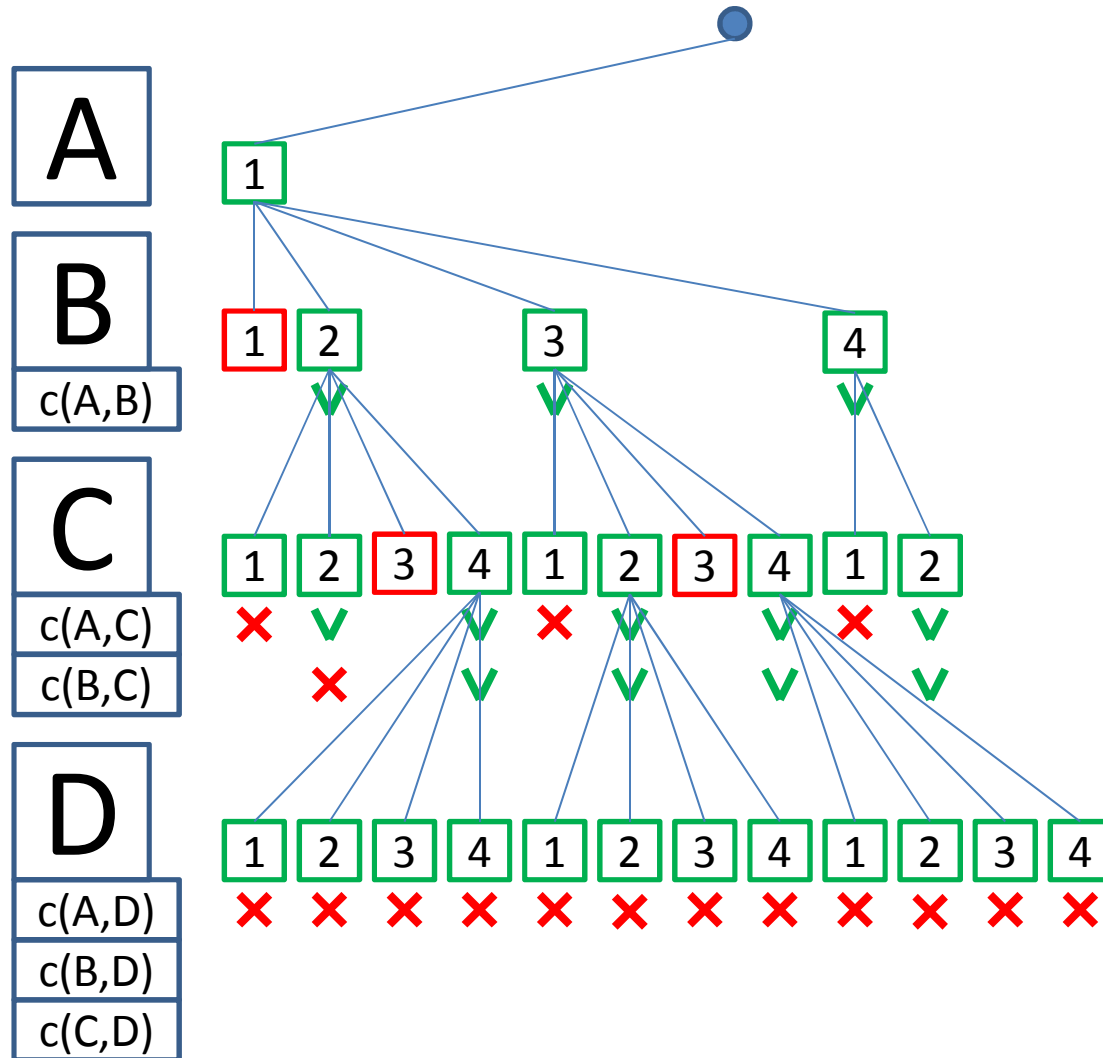
Constraint Processing: Backtracking



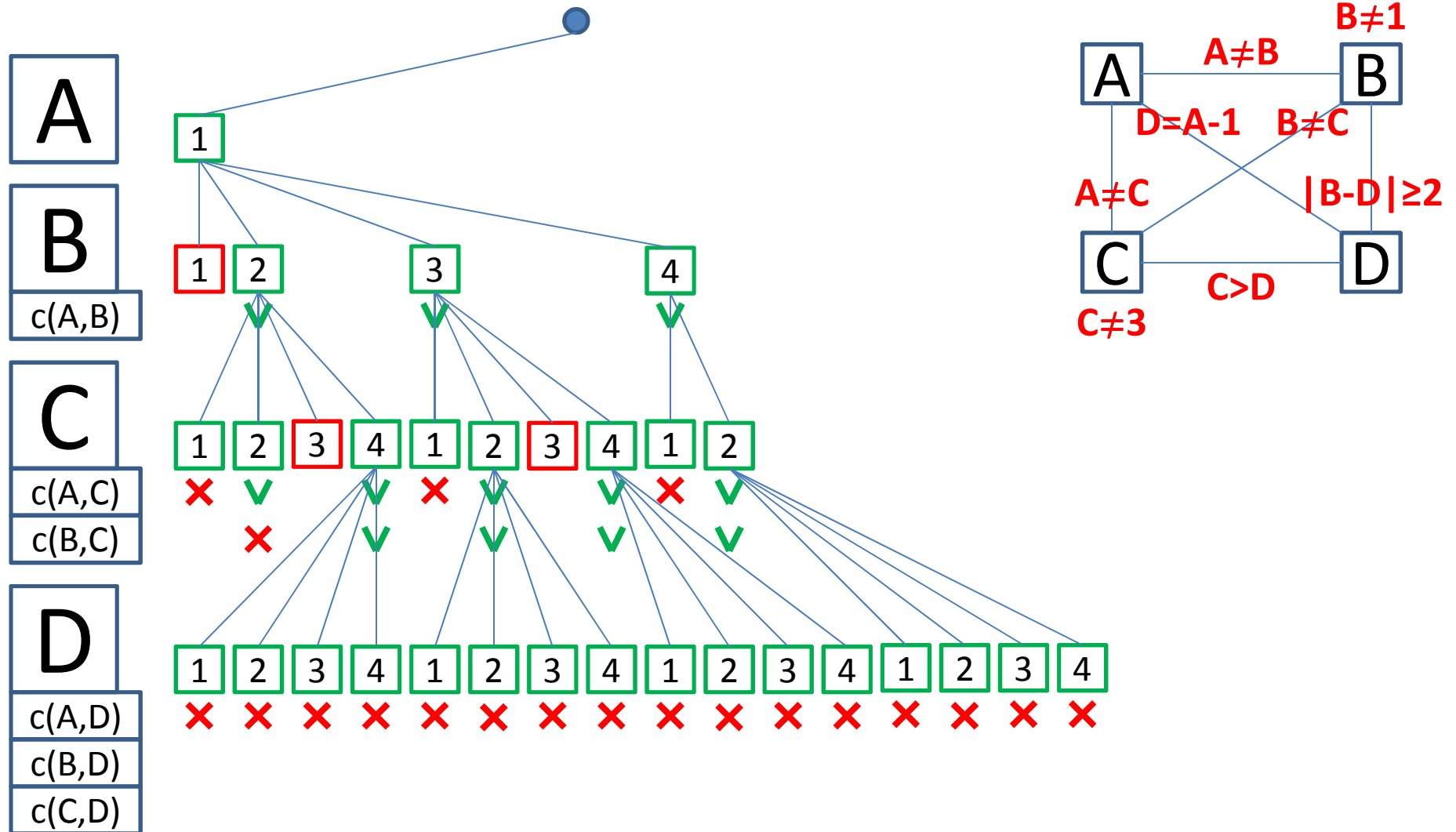
Constraint Processing: Backtracking



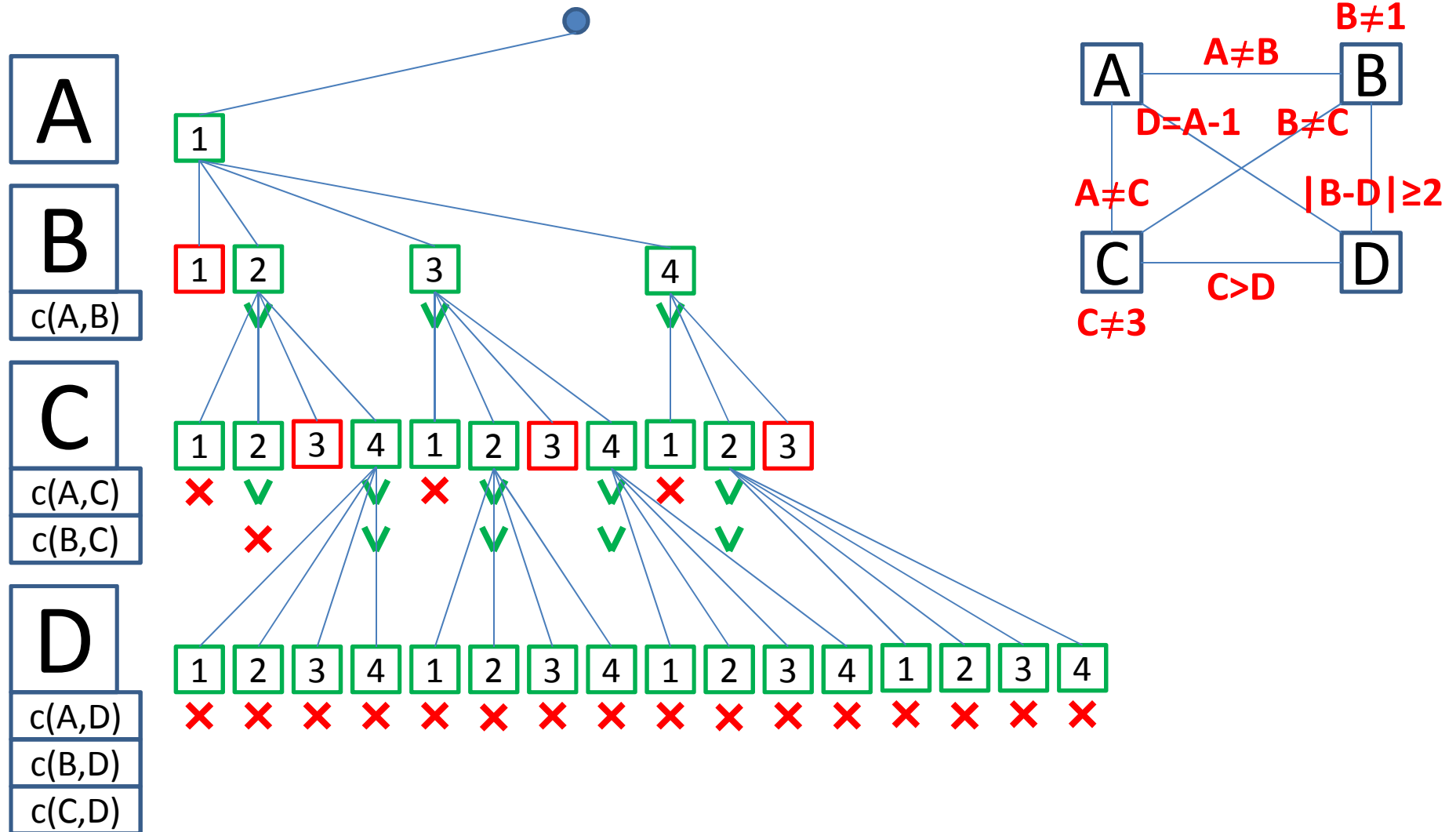
Constraint Processing: Backtracking



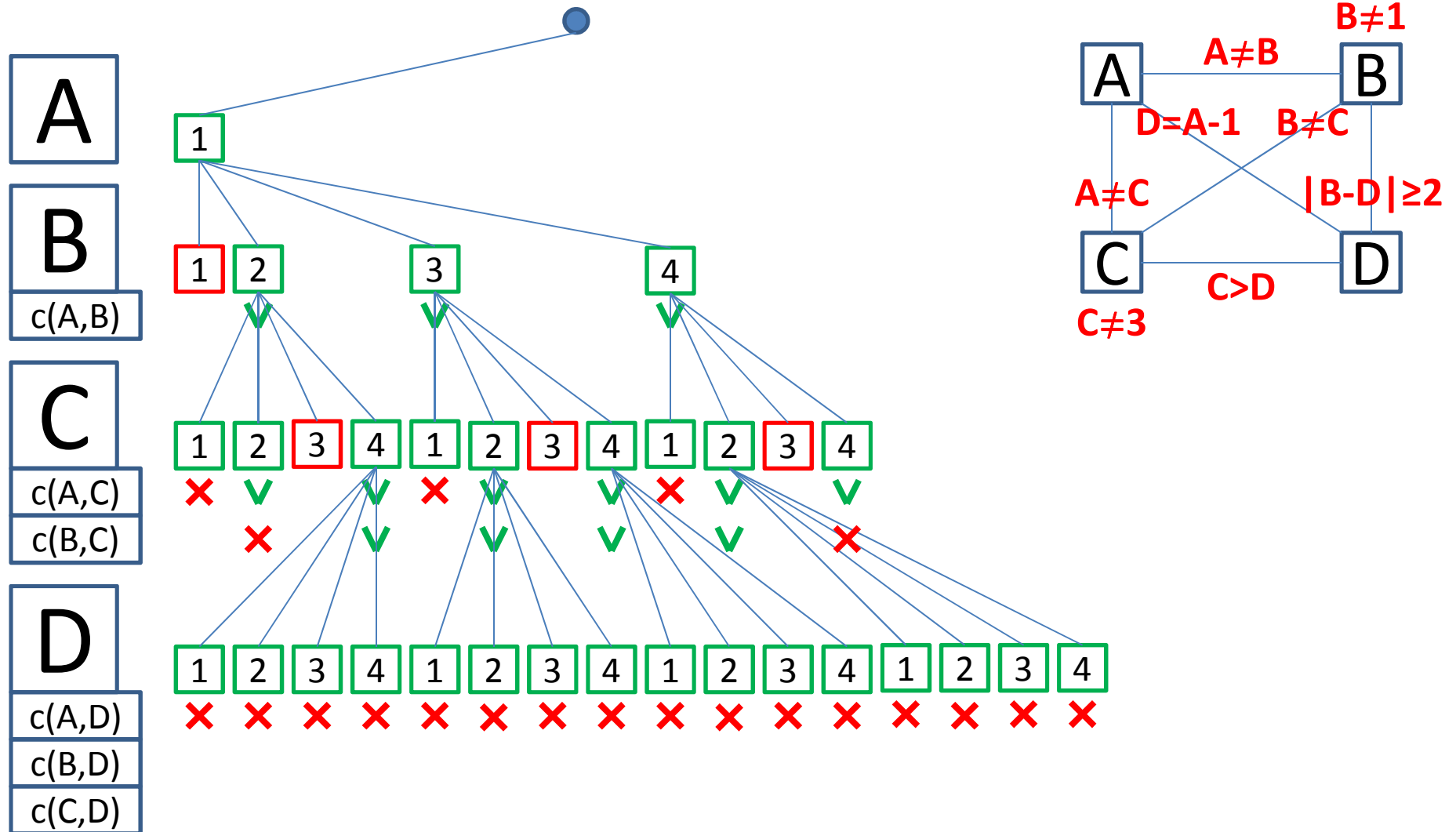
Constraint Processing: Backtracking



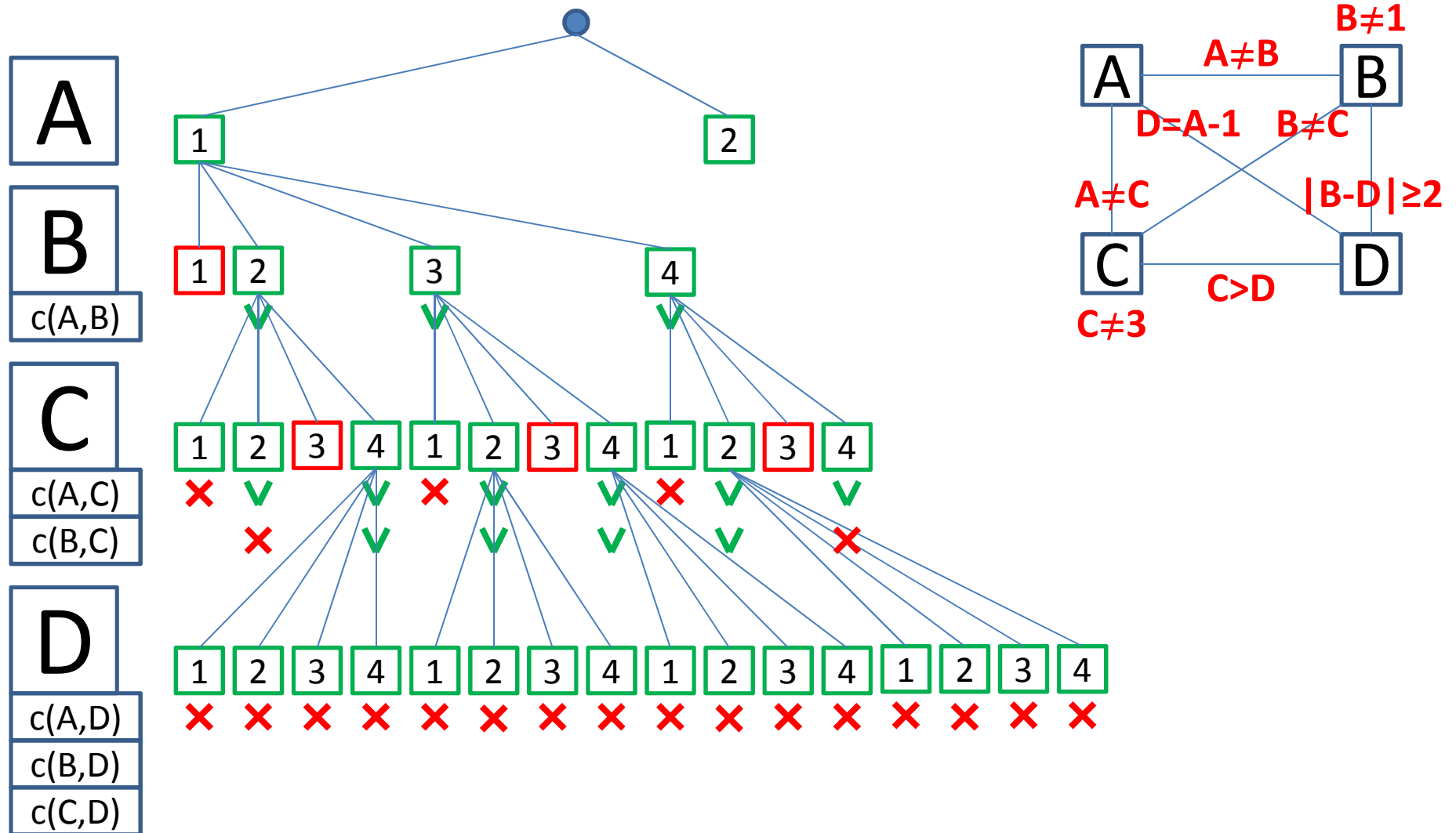
Constraint Processing: Backtracking



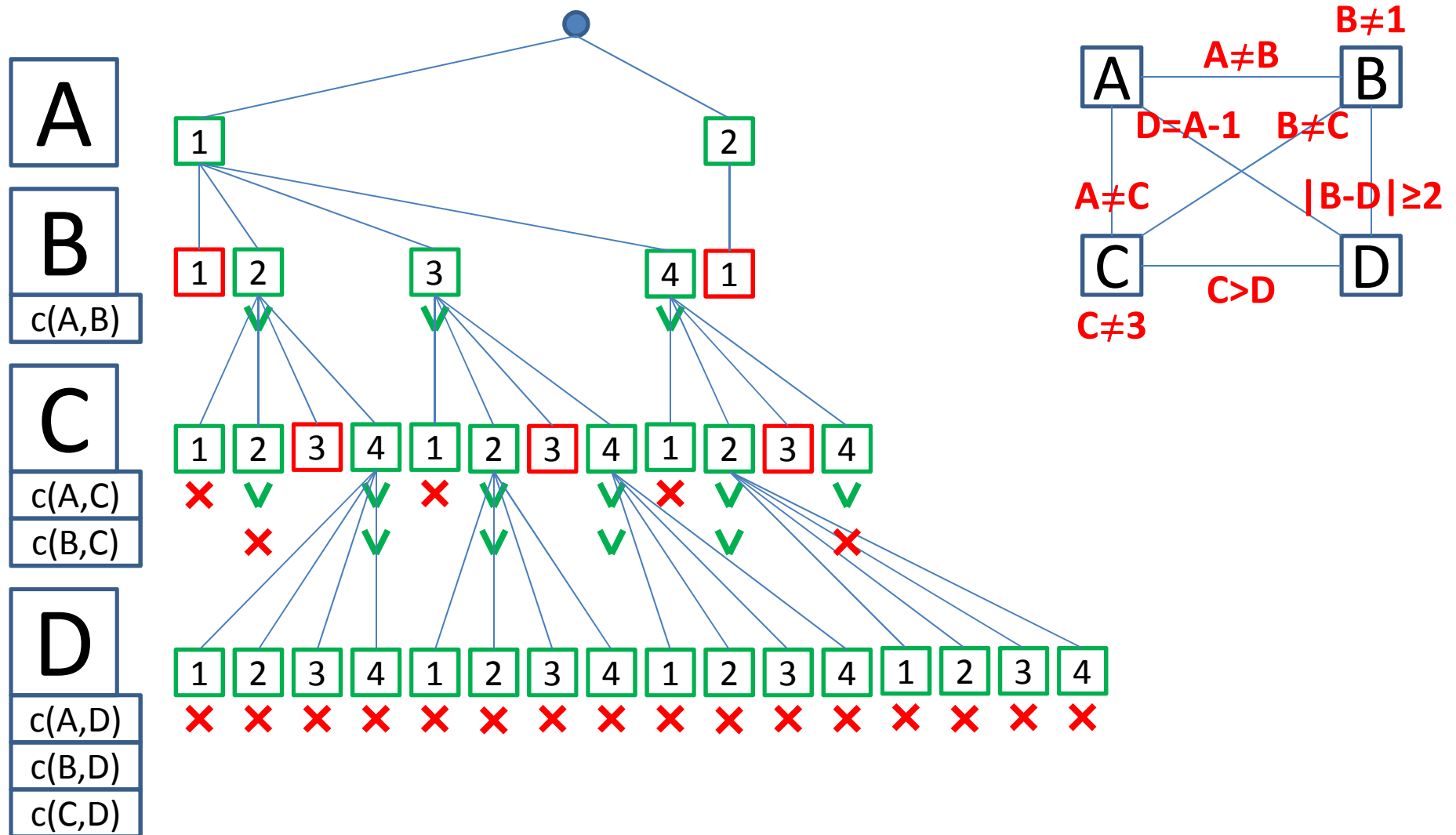
Constraint Processing: Backtracking



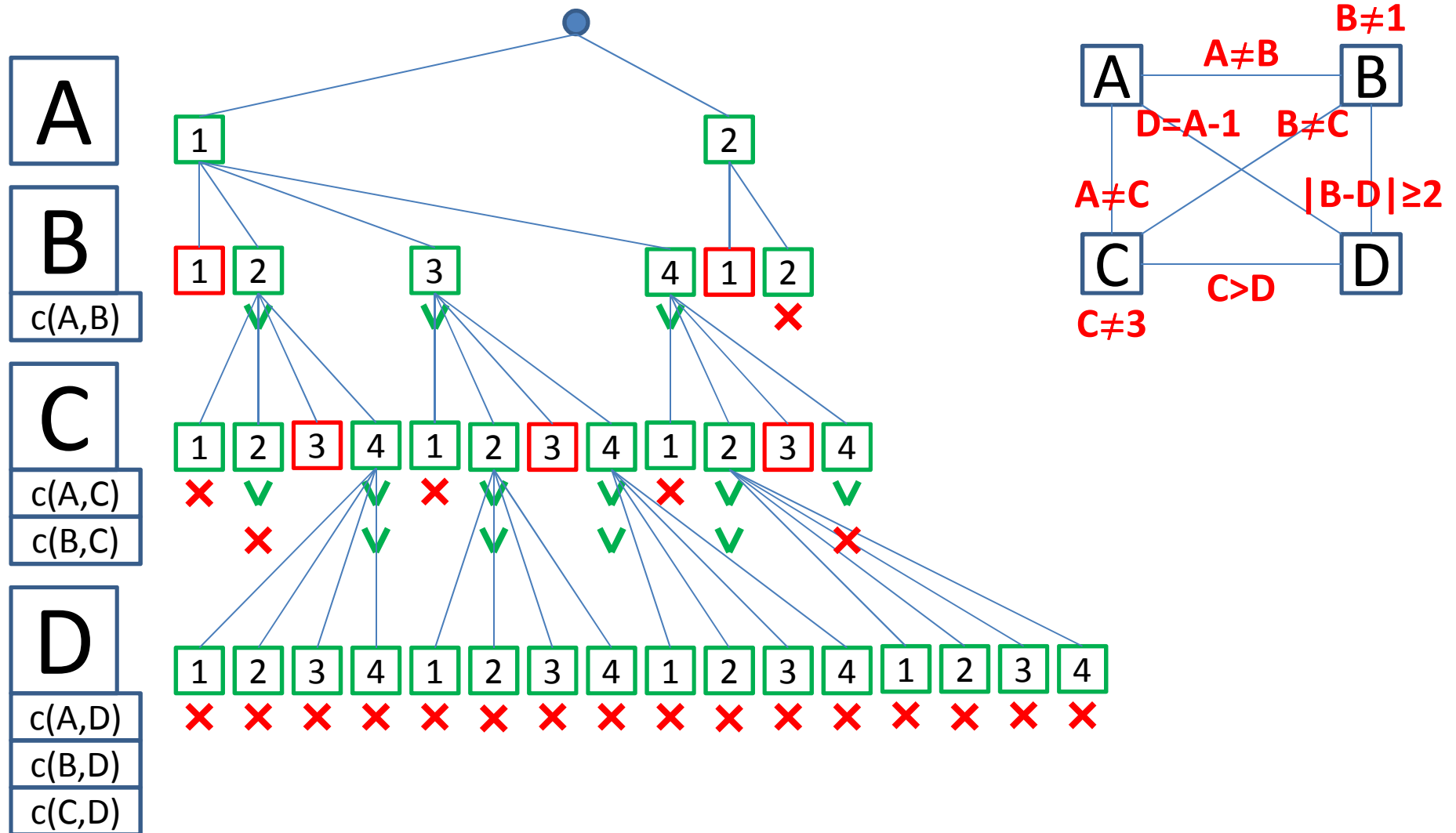
Constraint Processing: Backtracking



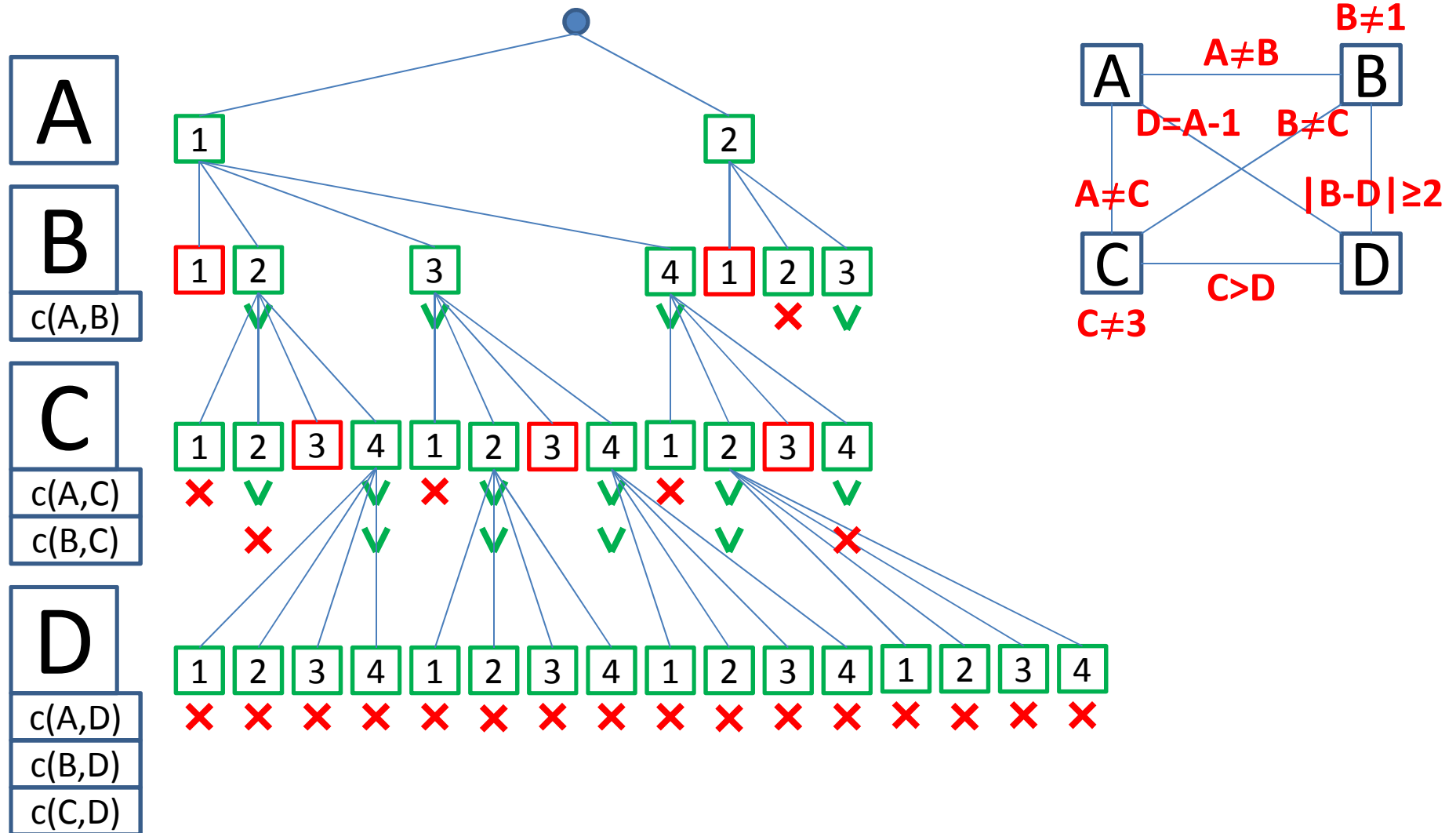
Constraint Processing: Backtracking



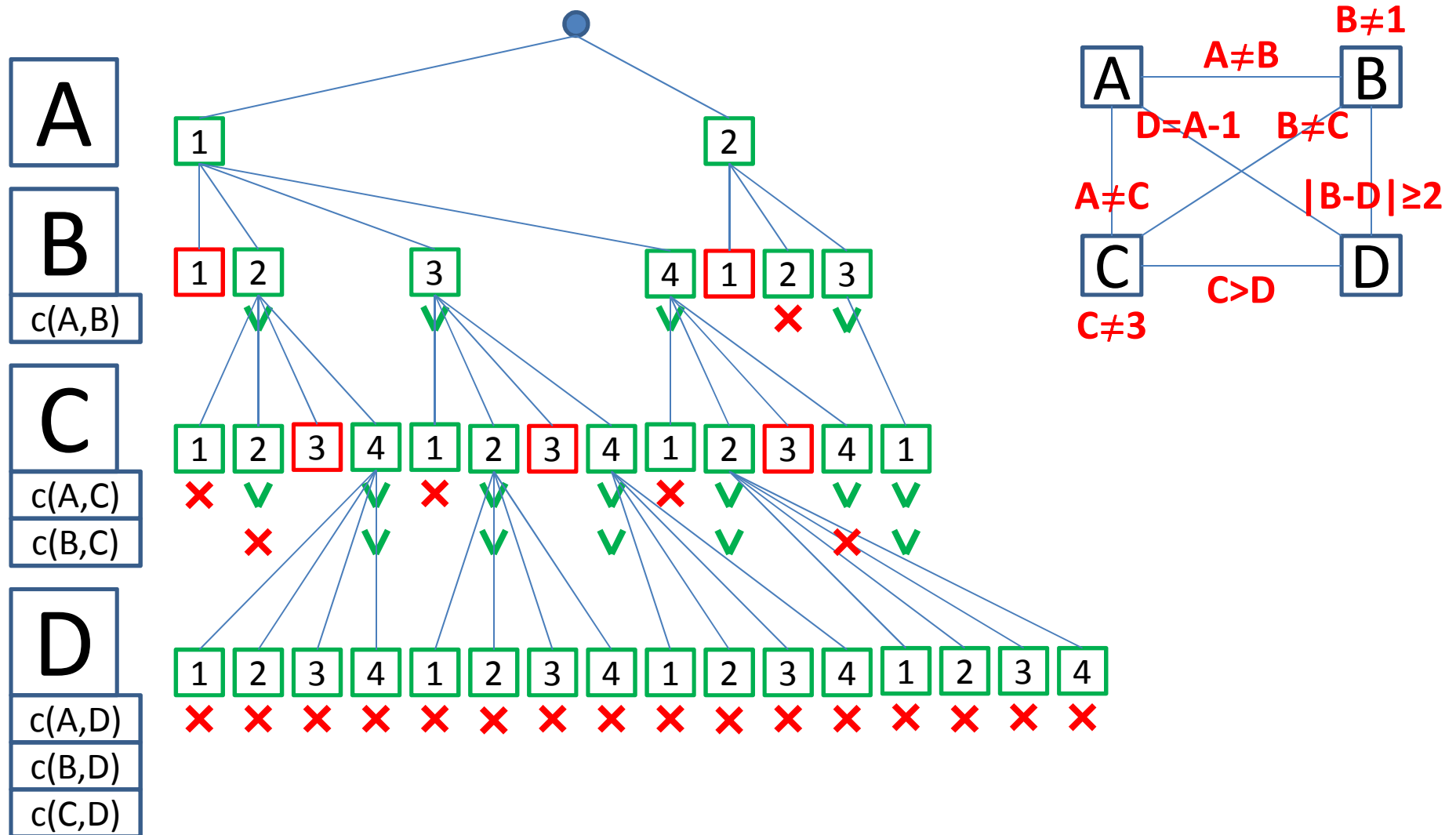
Constraint Processing: Backtracking



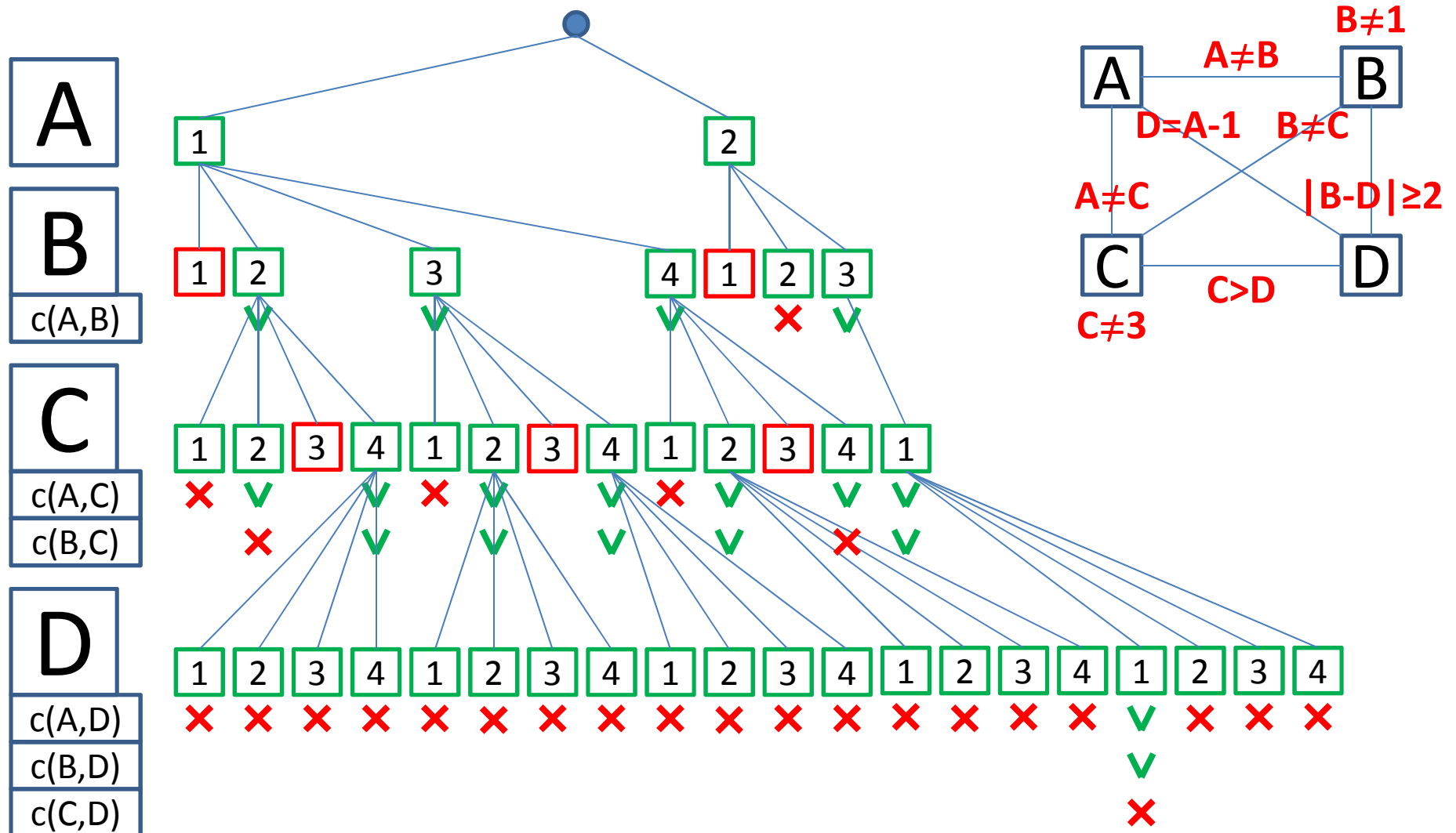
Constraint Processing: Backtracking



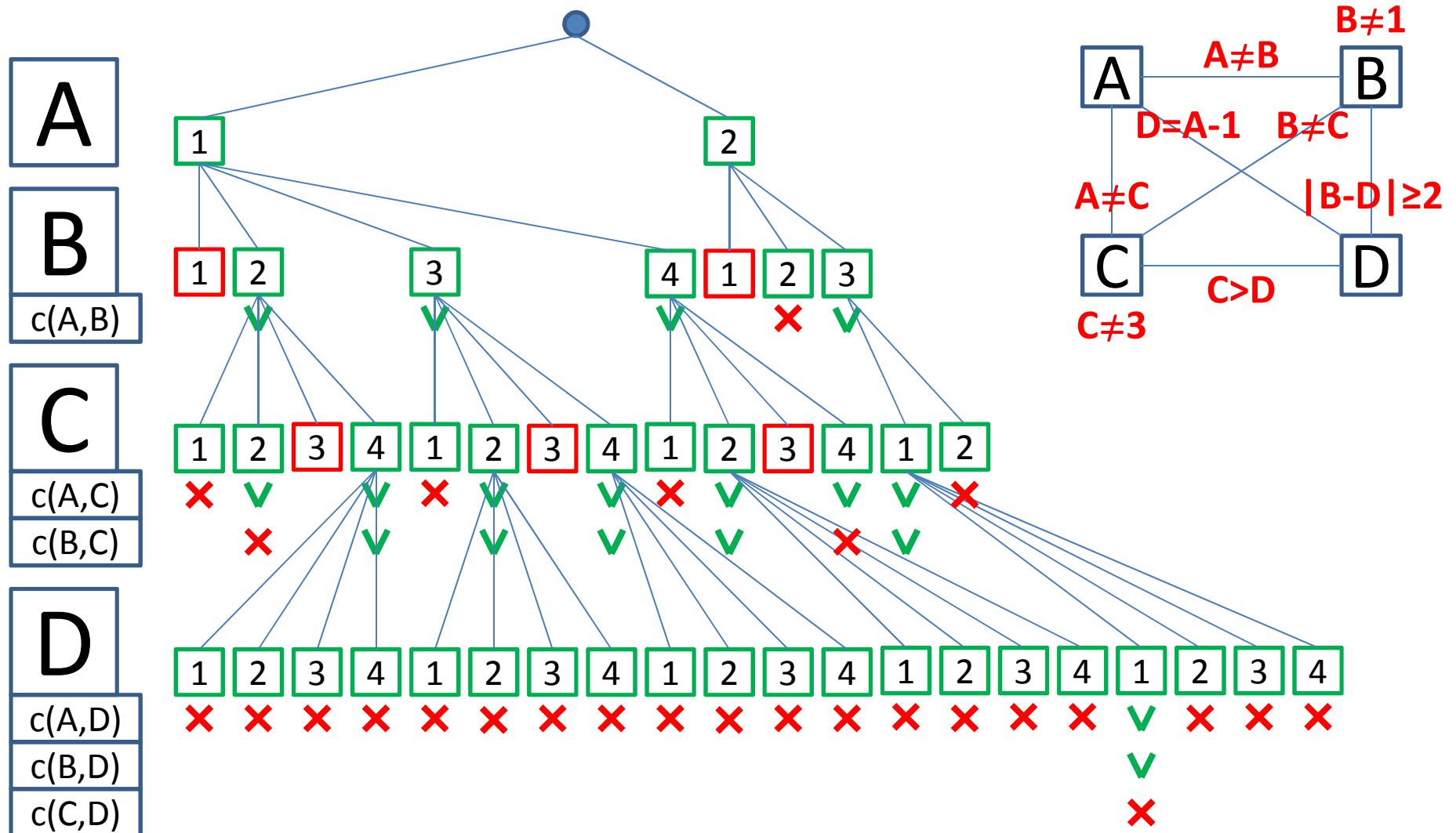
Constraint Processing: Backtracking



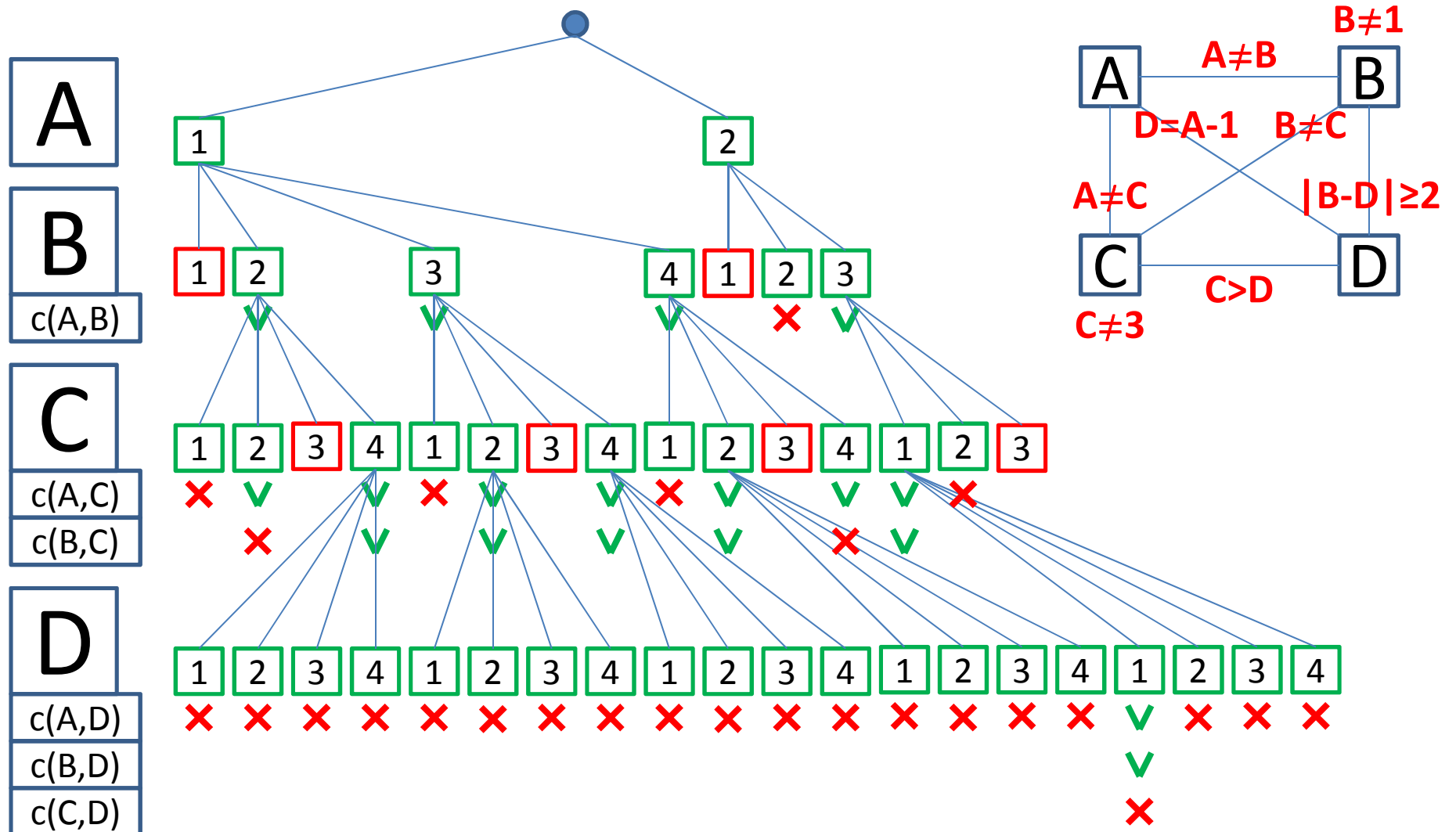
Constraint Processing: Backtracking



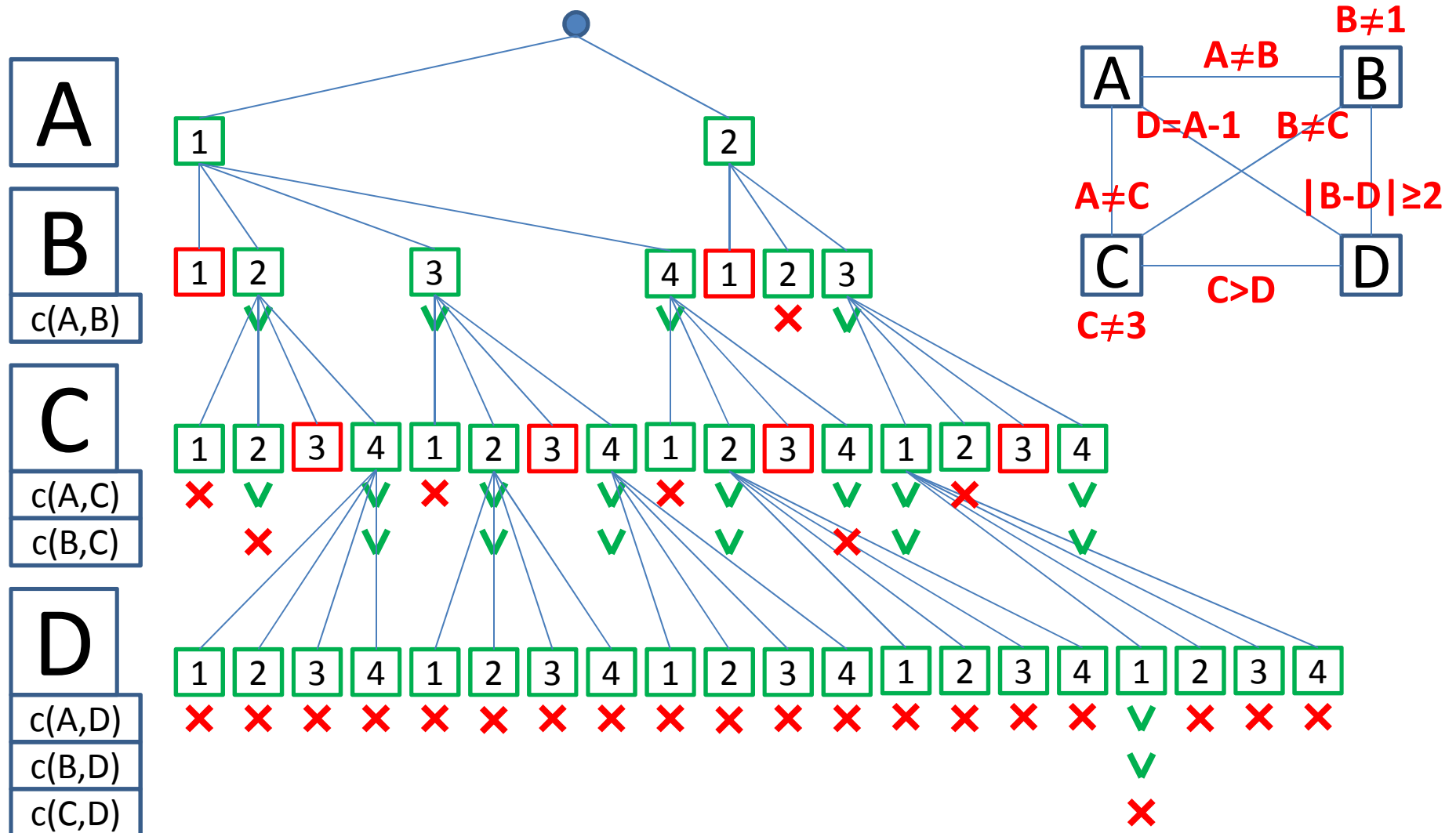
Constraint Processing: Backtracking



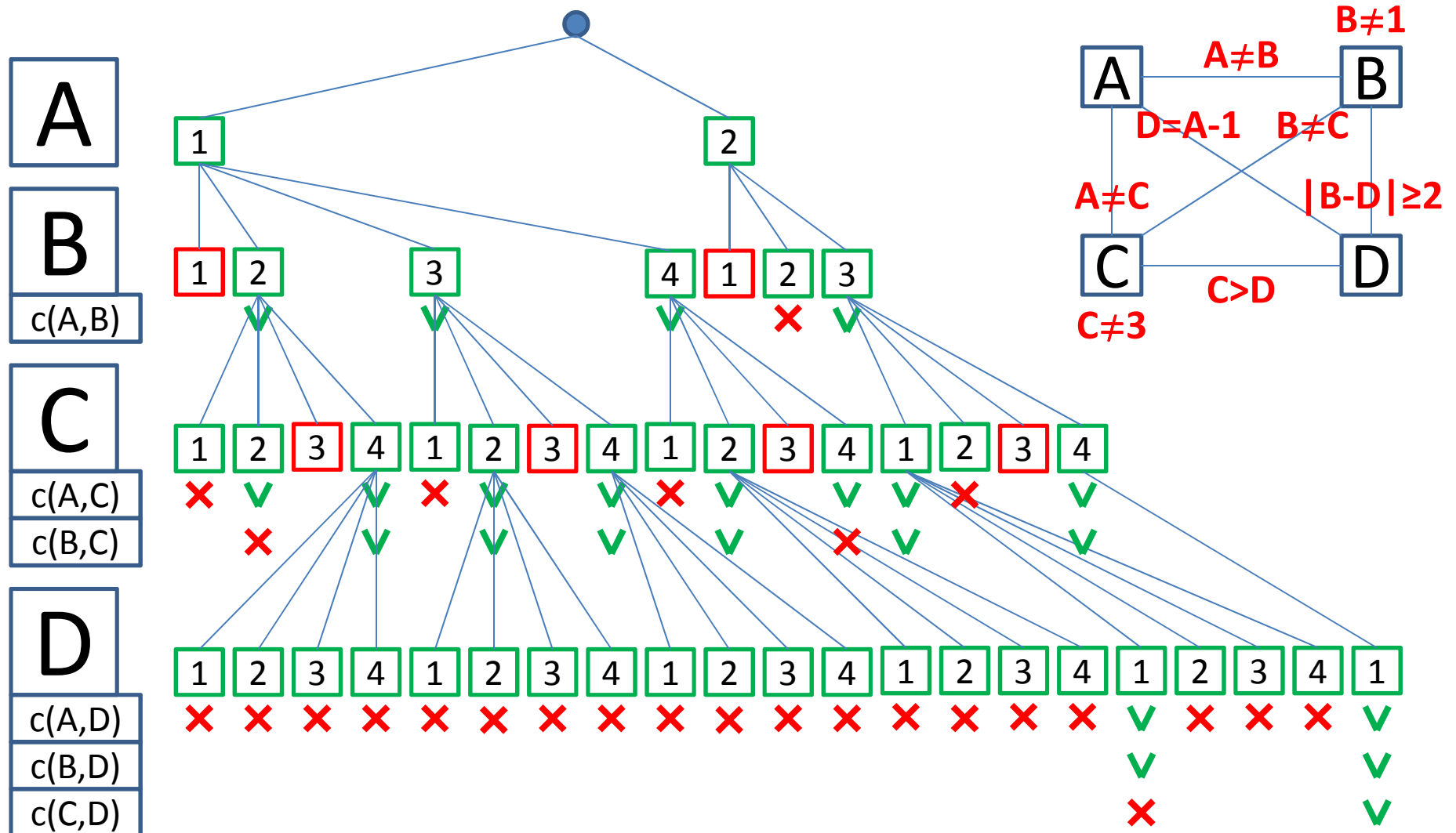
Constraint Processing: Backtracking



Constraint Processing: Backtracking



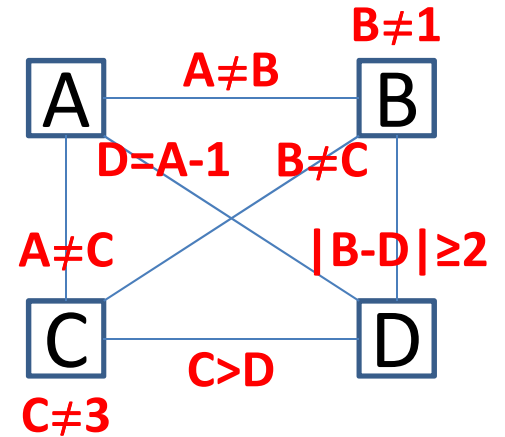
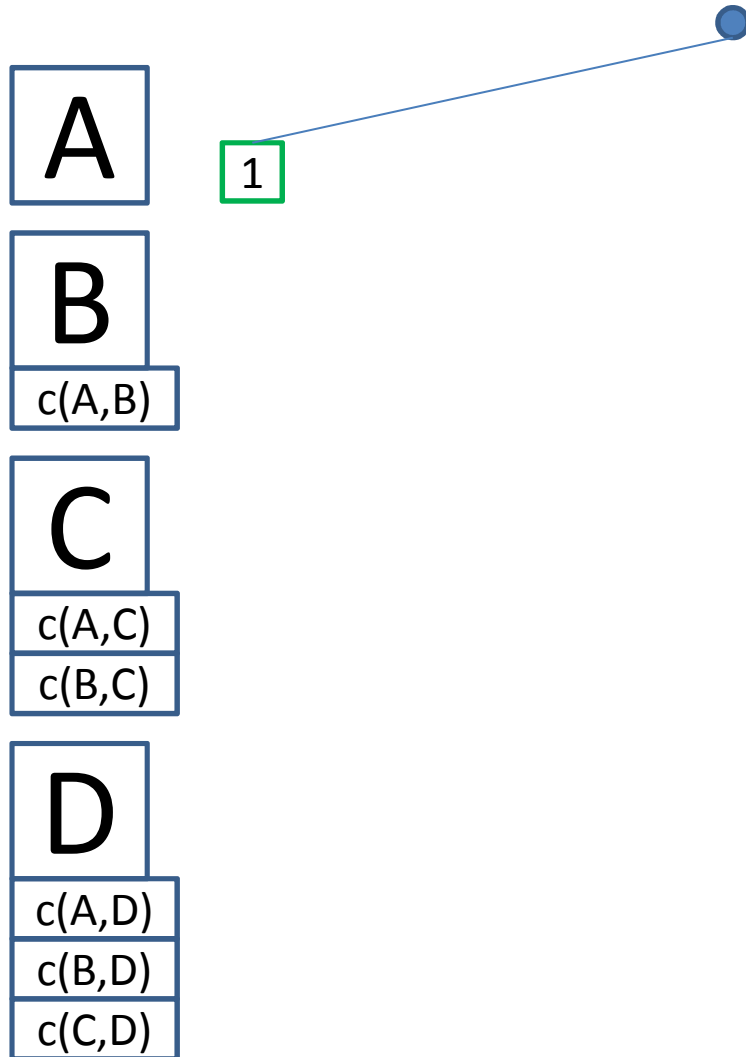
Constraint Processing: Backtracking



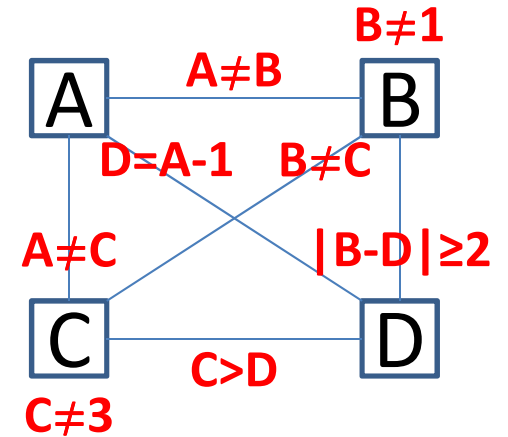
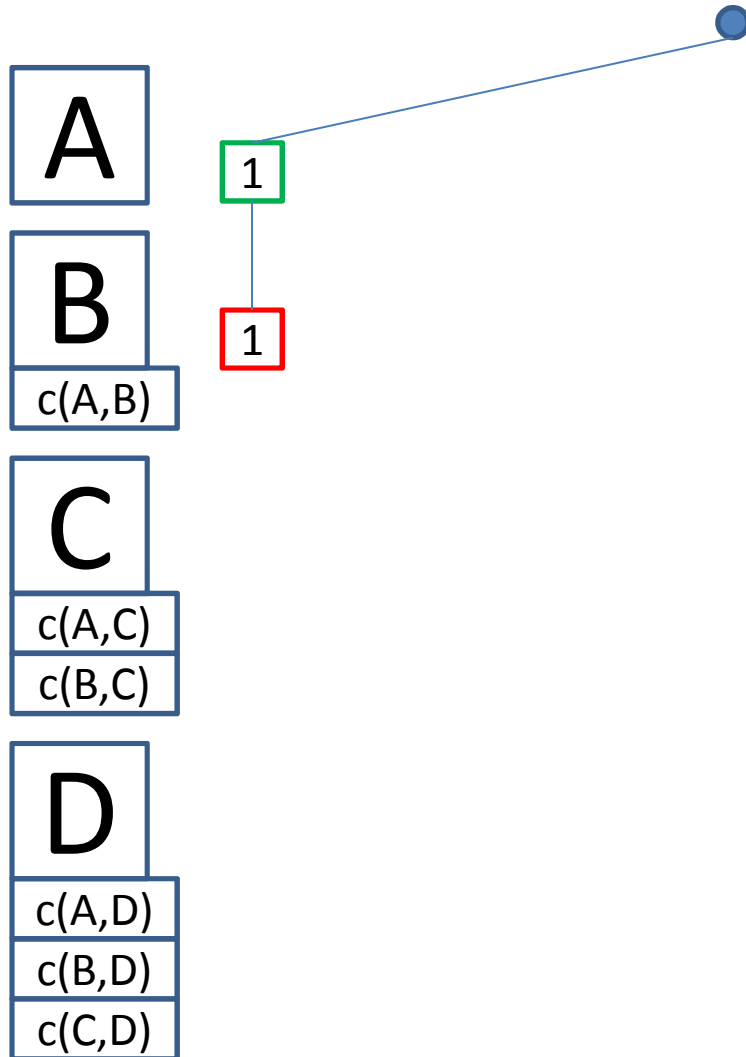
MiniMax & Constraint Processing: The 4 Houses problem

CONSTRAINT PROCESSING: BACKJUMPING

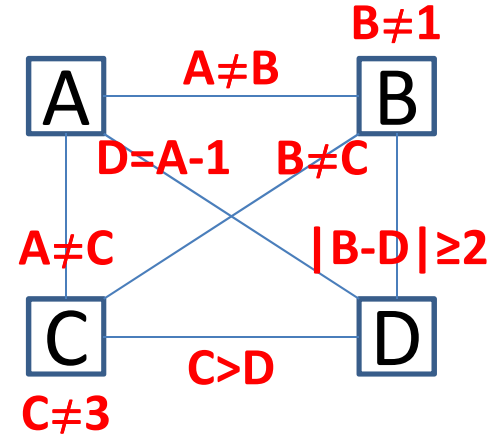
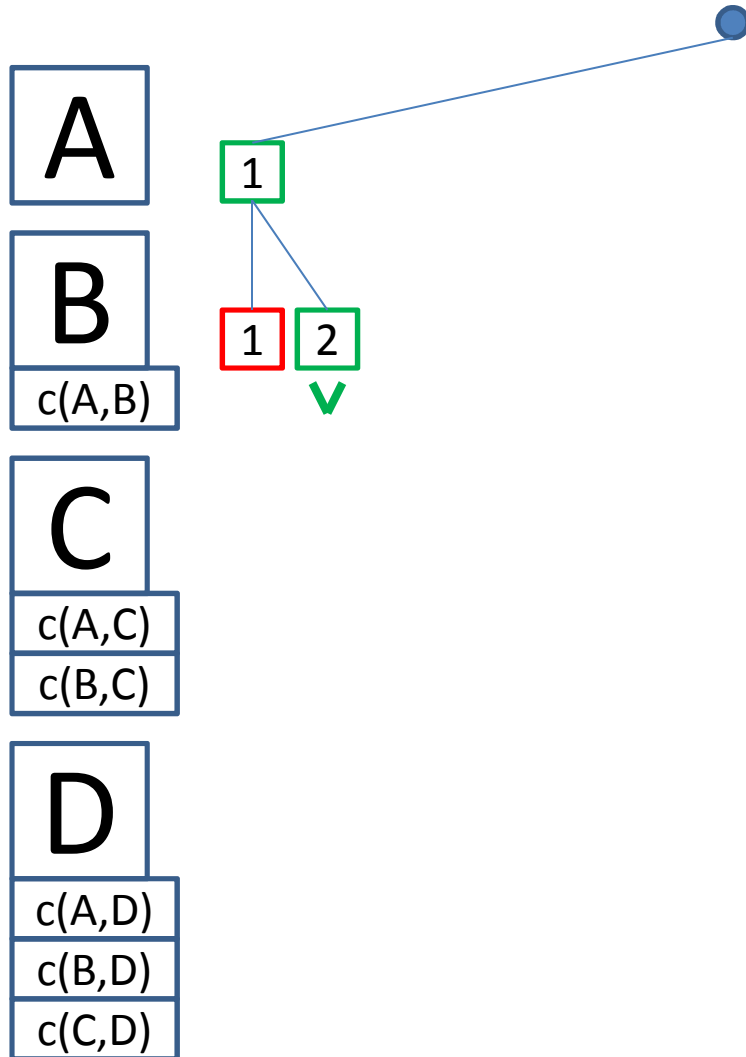
Constraint Processing: Backjumping



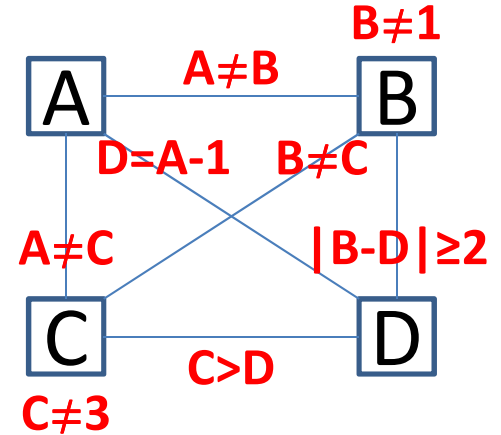
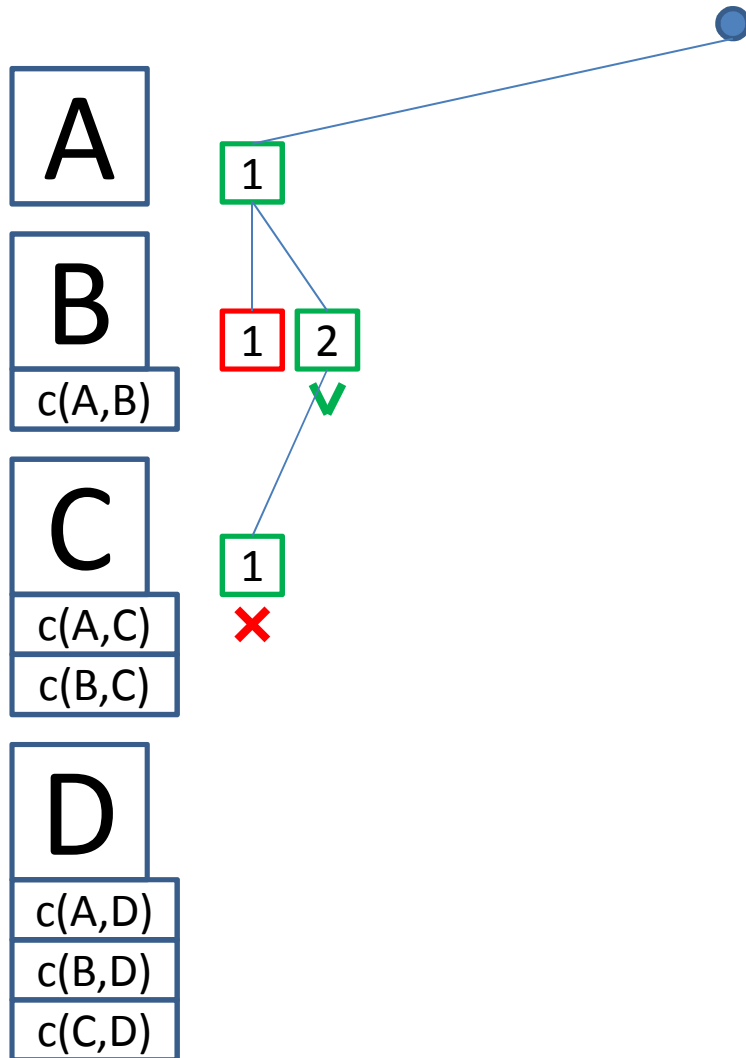
Constraint Processing: Backjumping



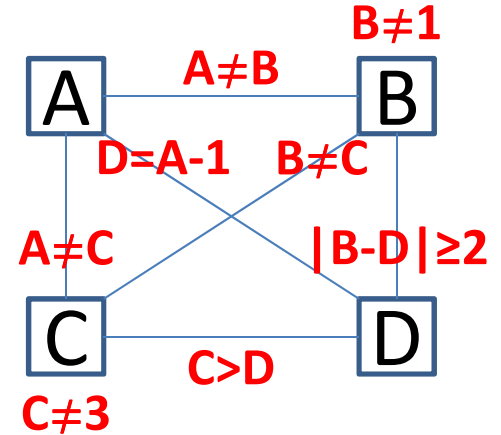
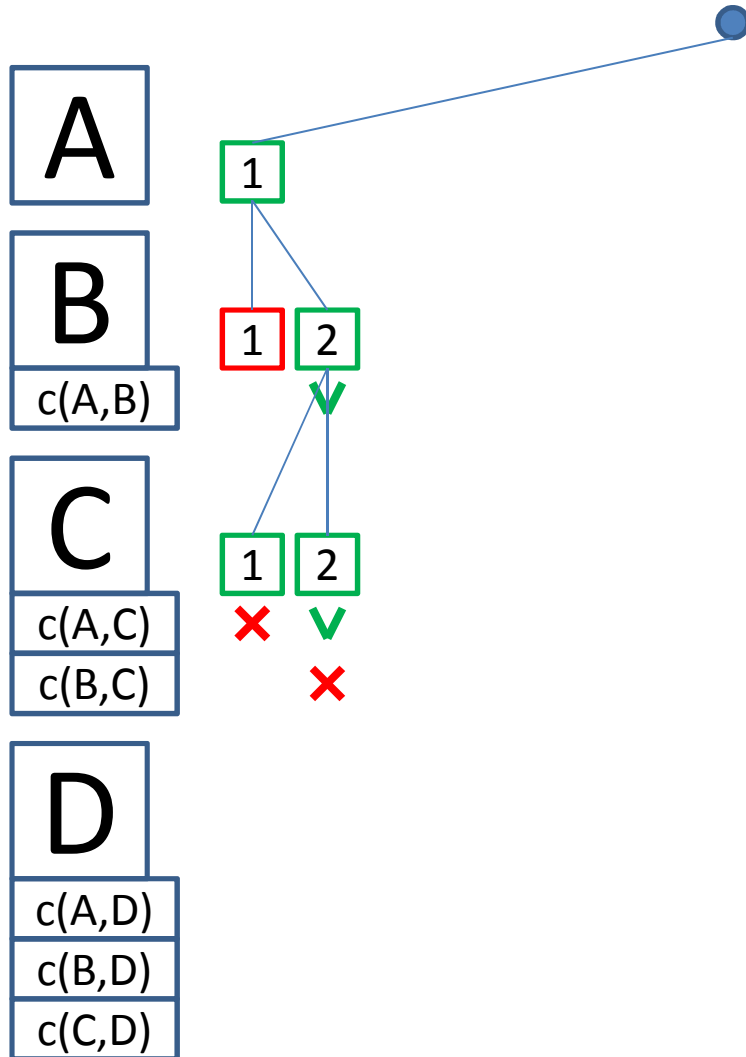
Constraint Processing: Backjumping



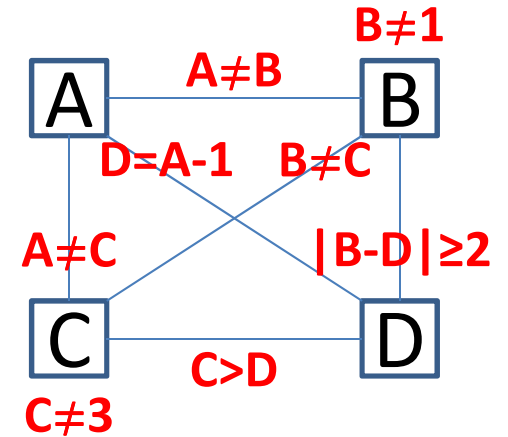
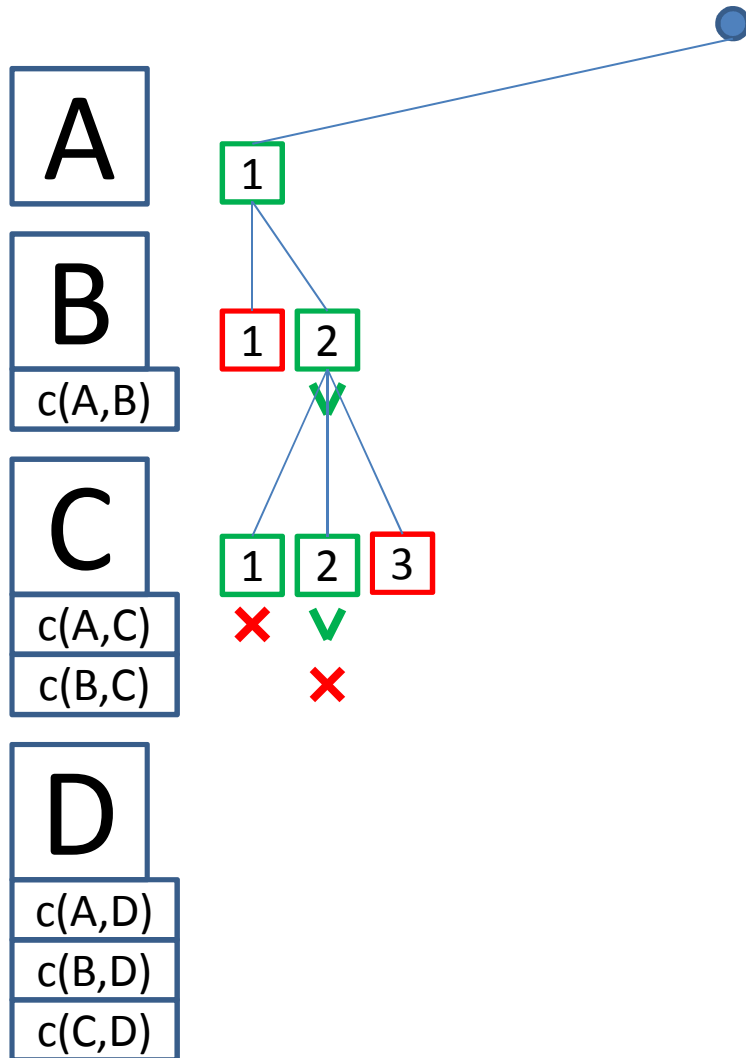
Constraint Processing: Backjumping



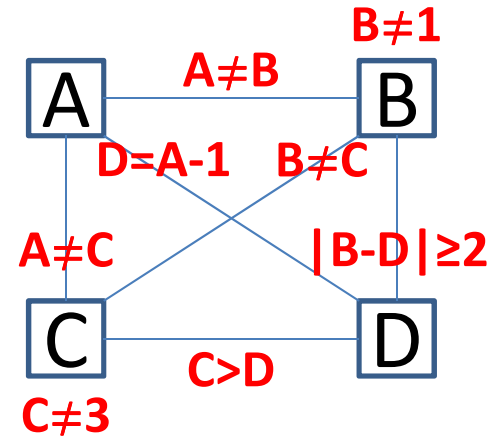
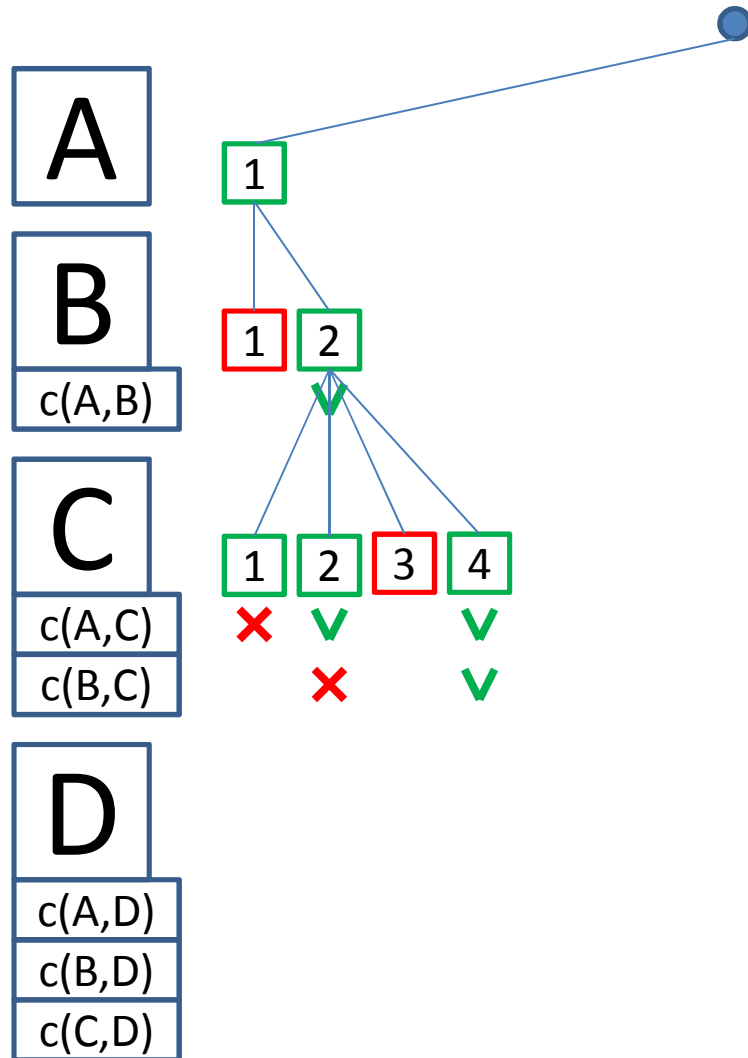
Constraint Processing: Backjumping



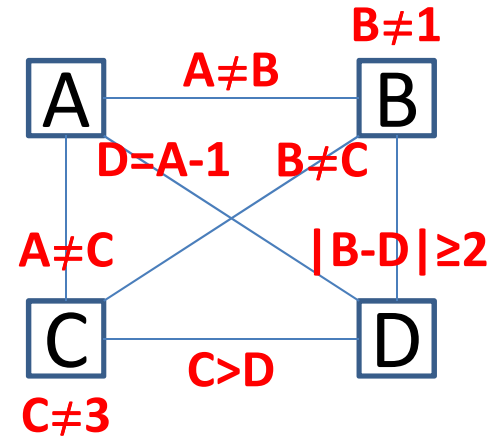
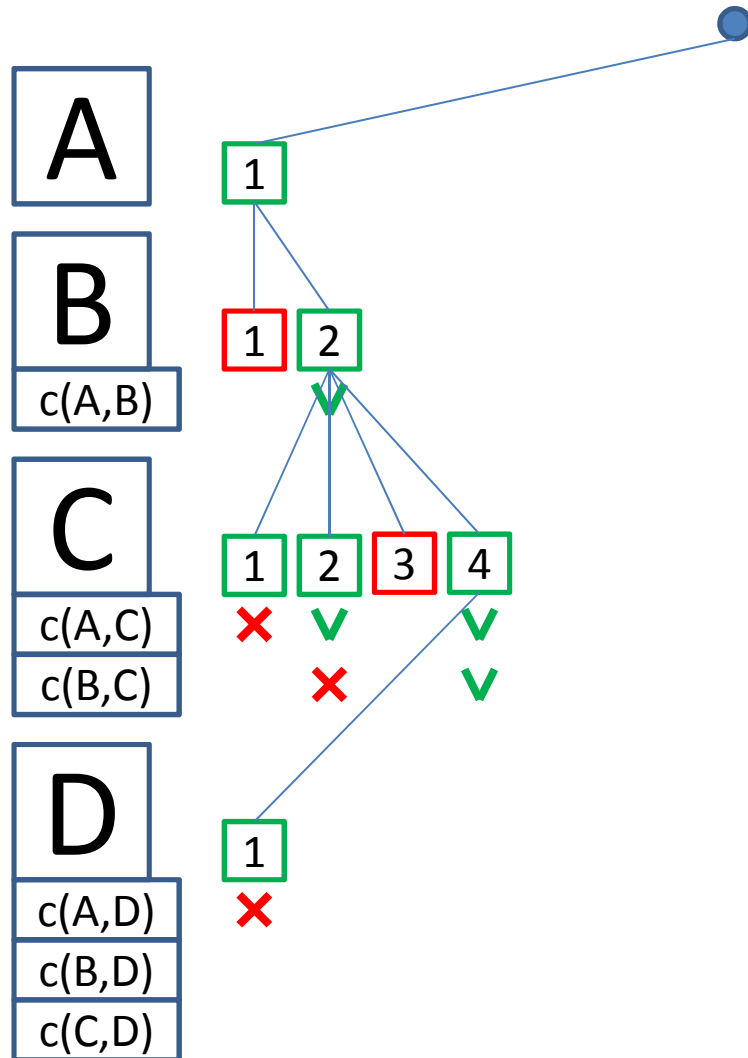
Constraint Processing: Backjumping



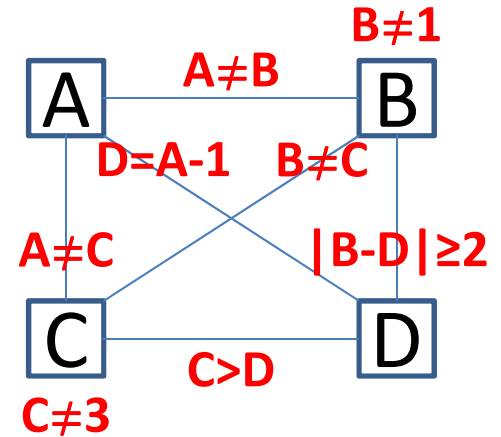
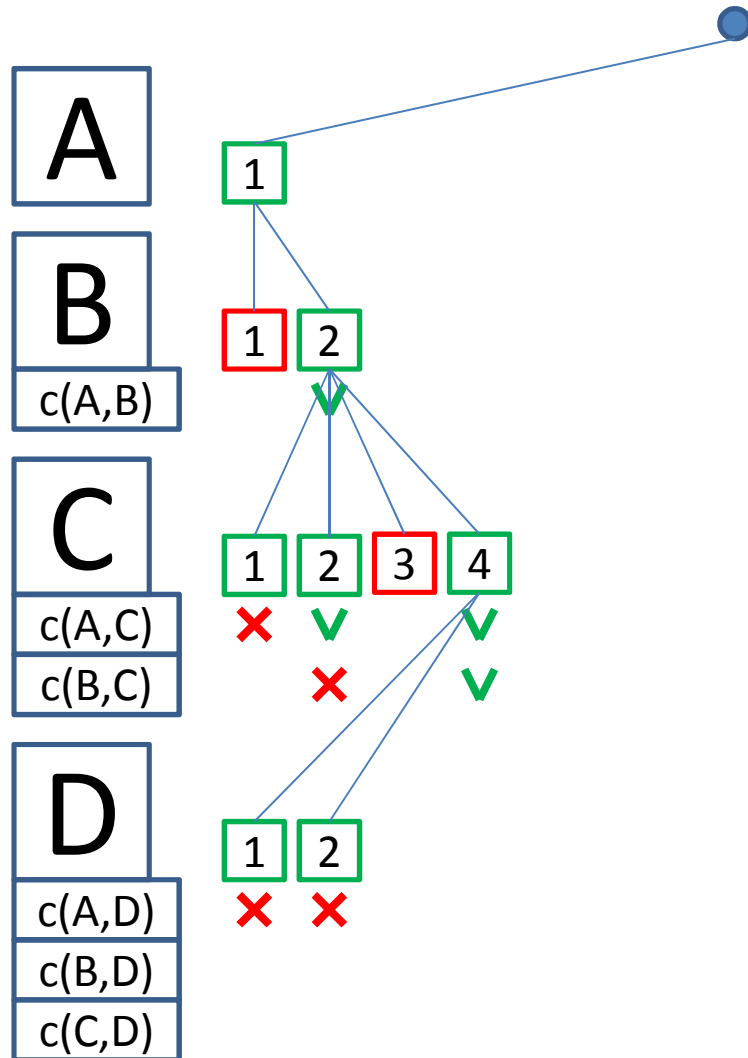
Constraint Processing: Backjumping



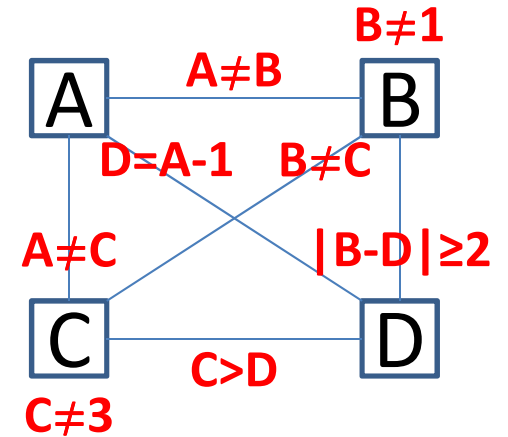
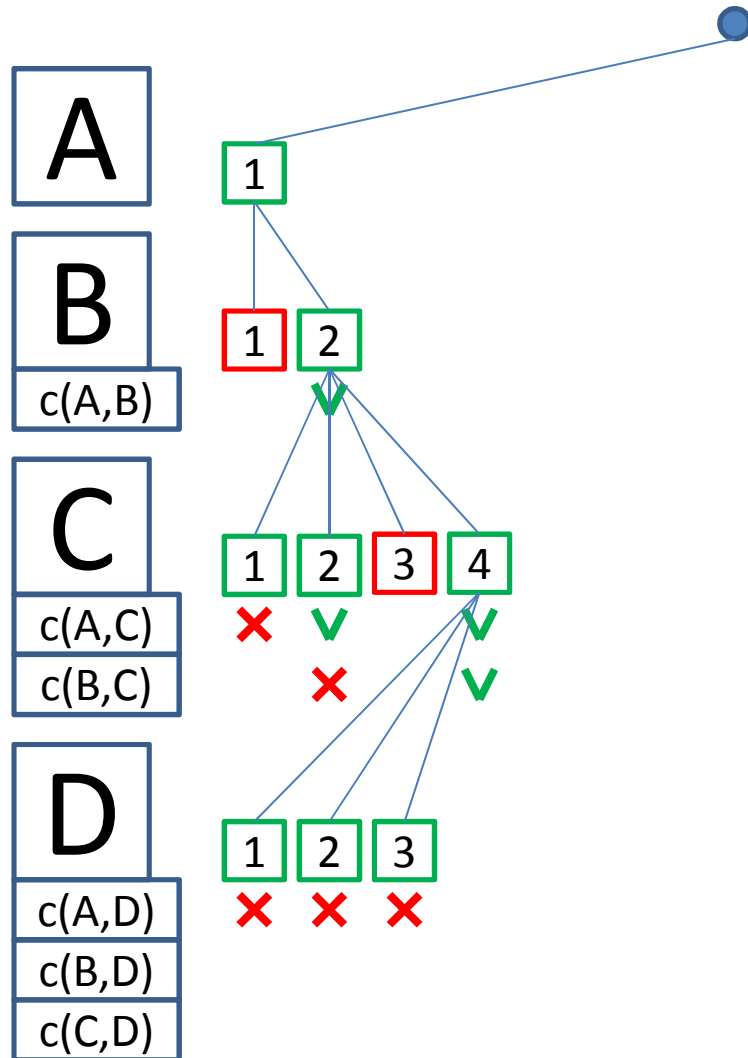
Constraint Processing: Backjumping



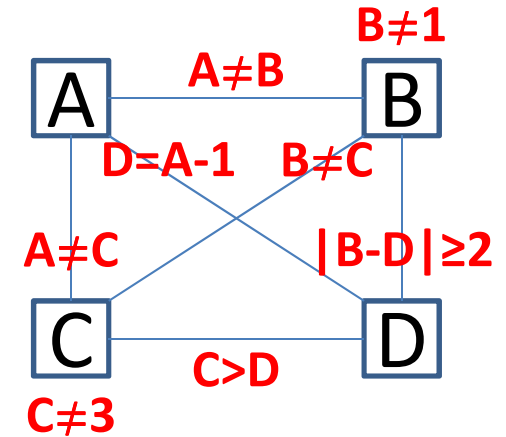
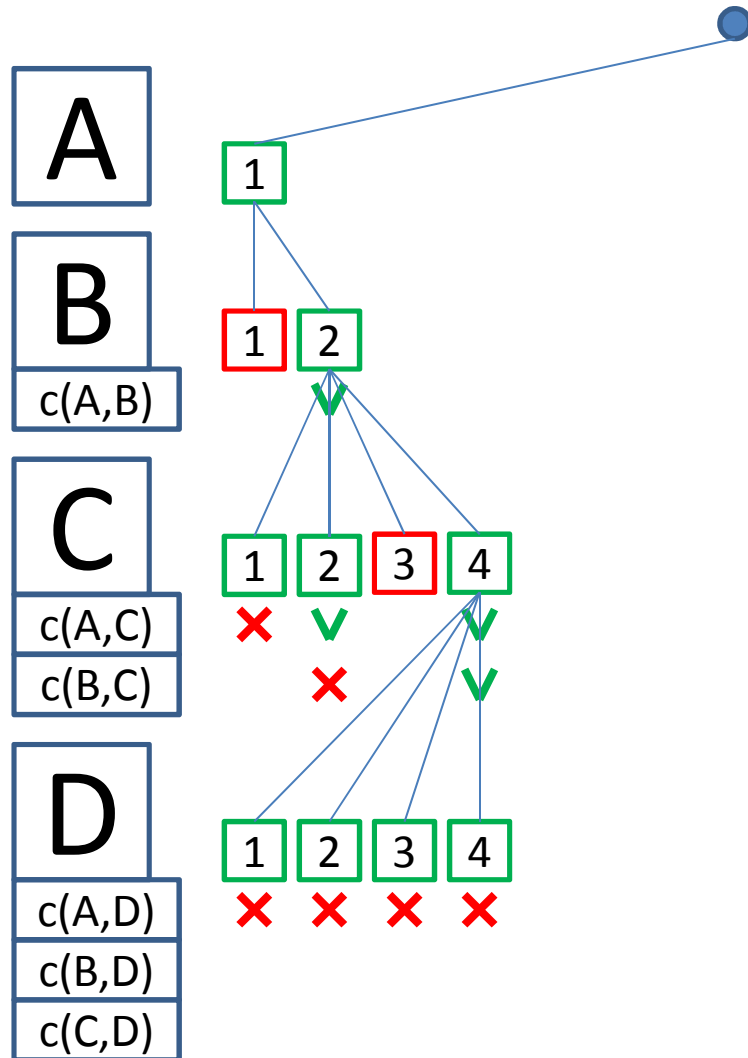
Constraint Processing: Backjumping



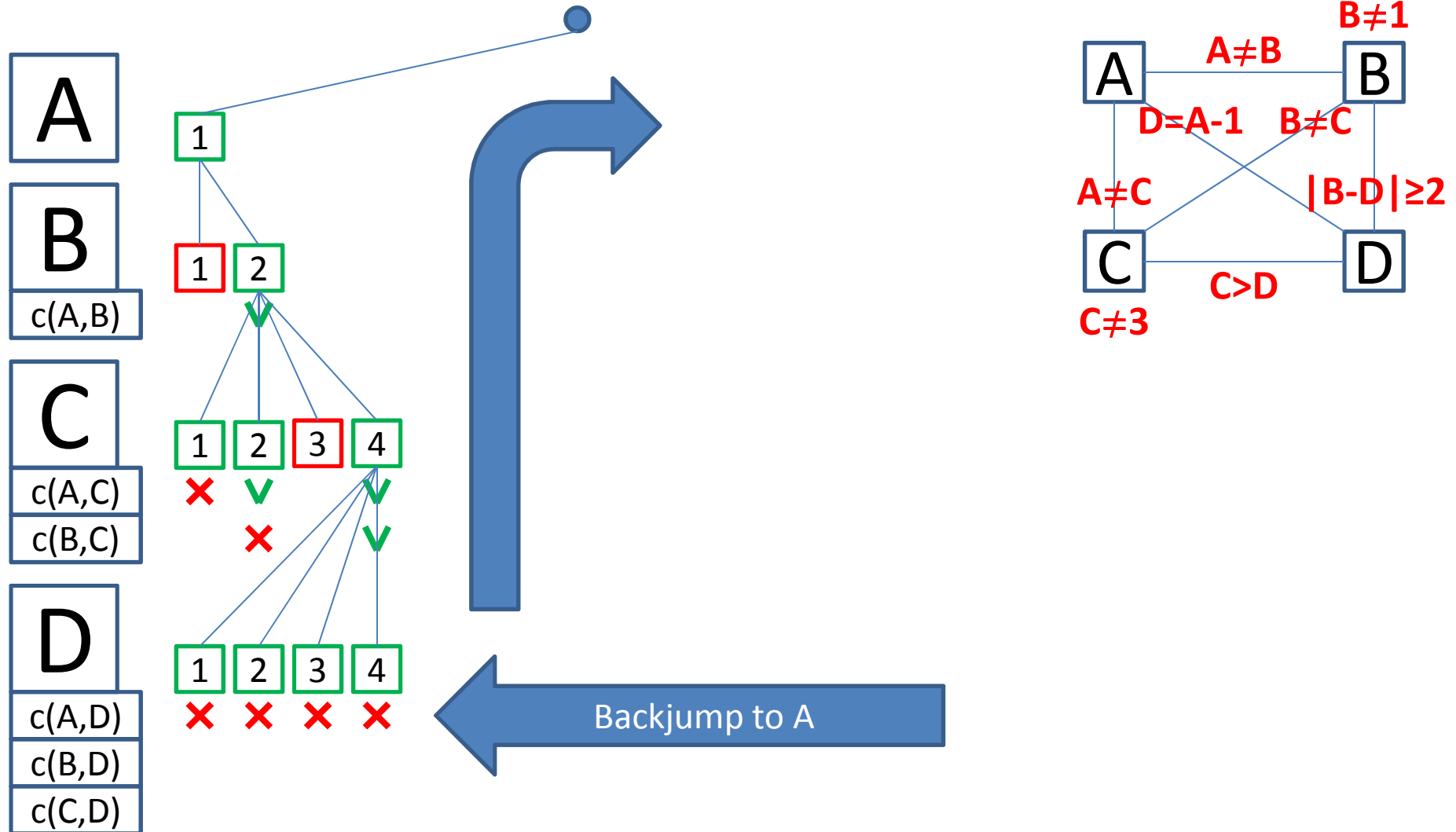
Constraint Processing: Backjumping



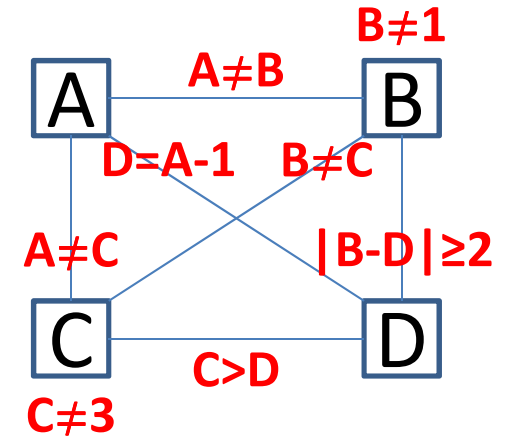
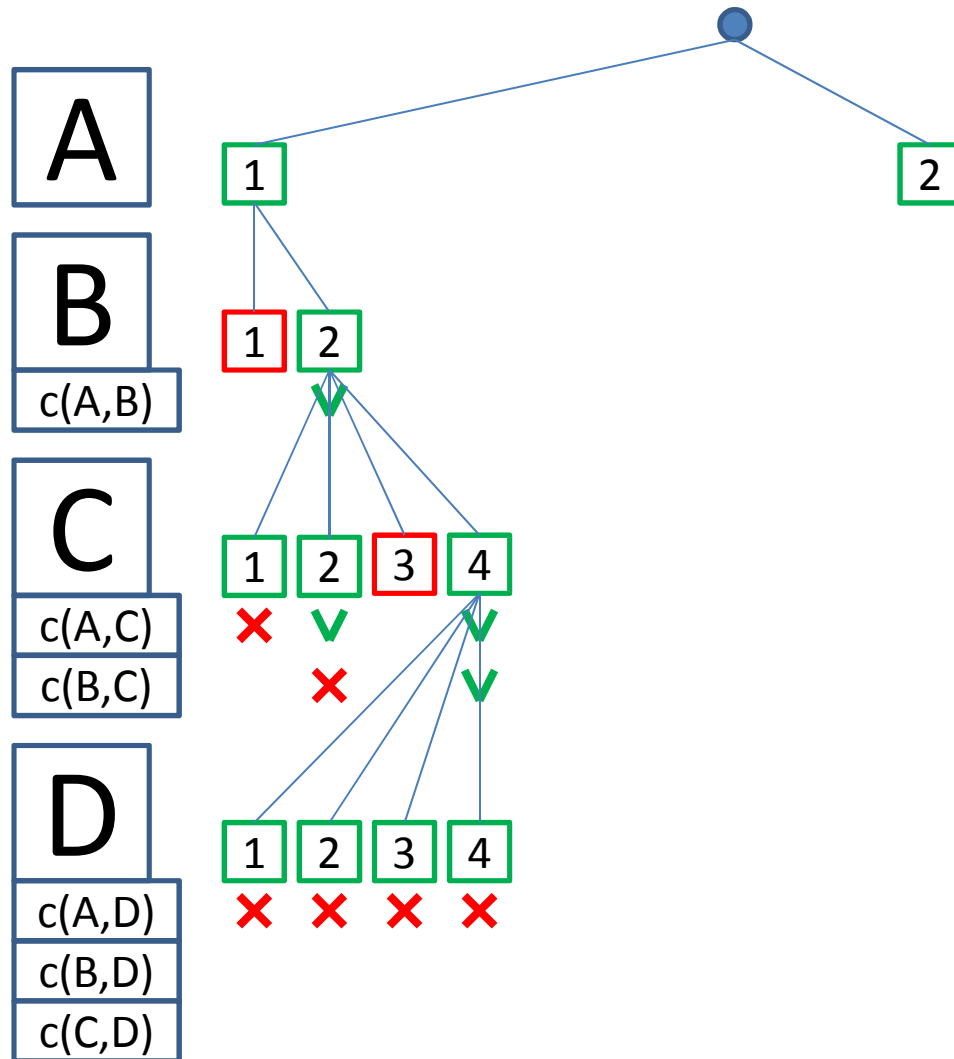
Constraint Processing: Backjumping



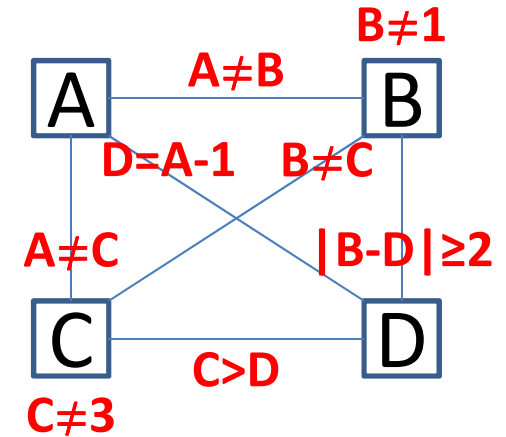
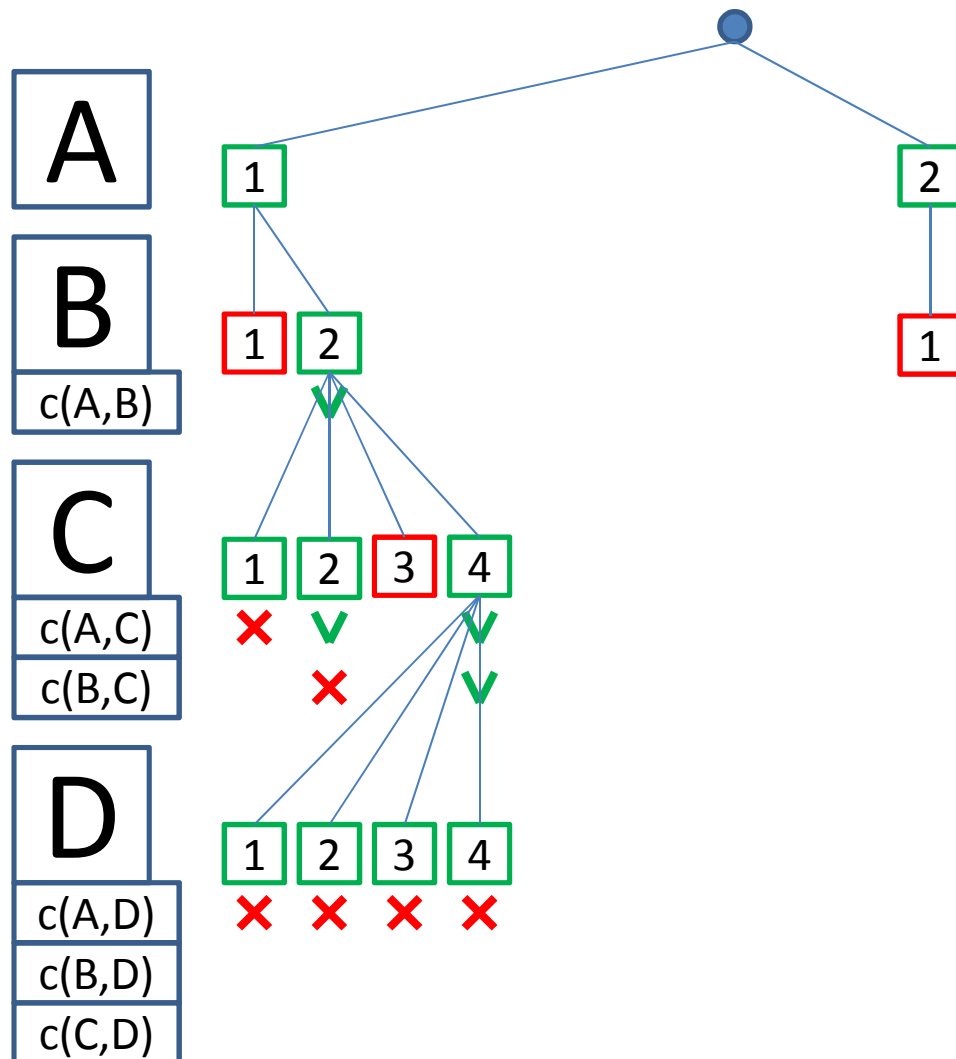
Constraint Processing: Backjumping



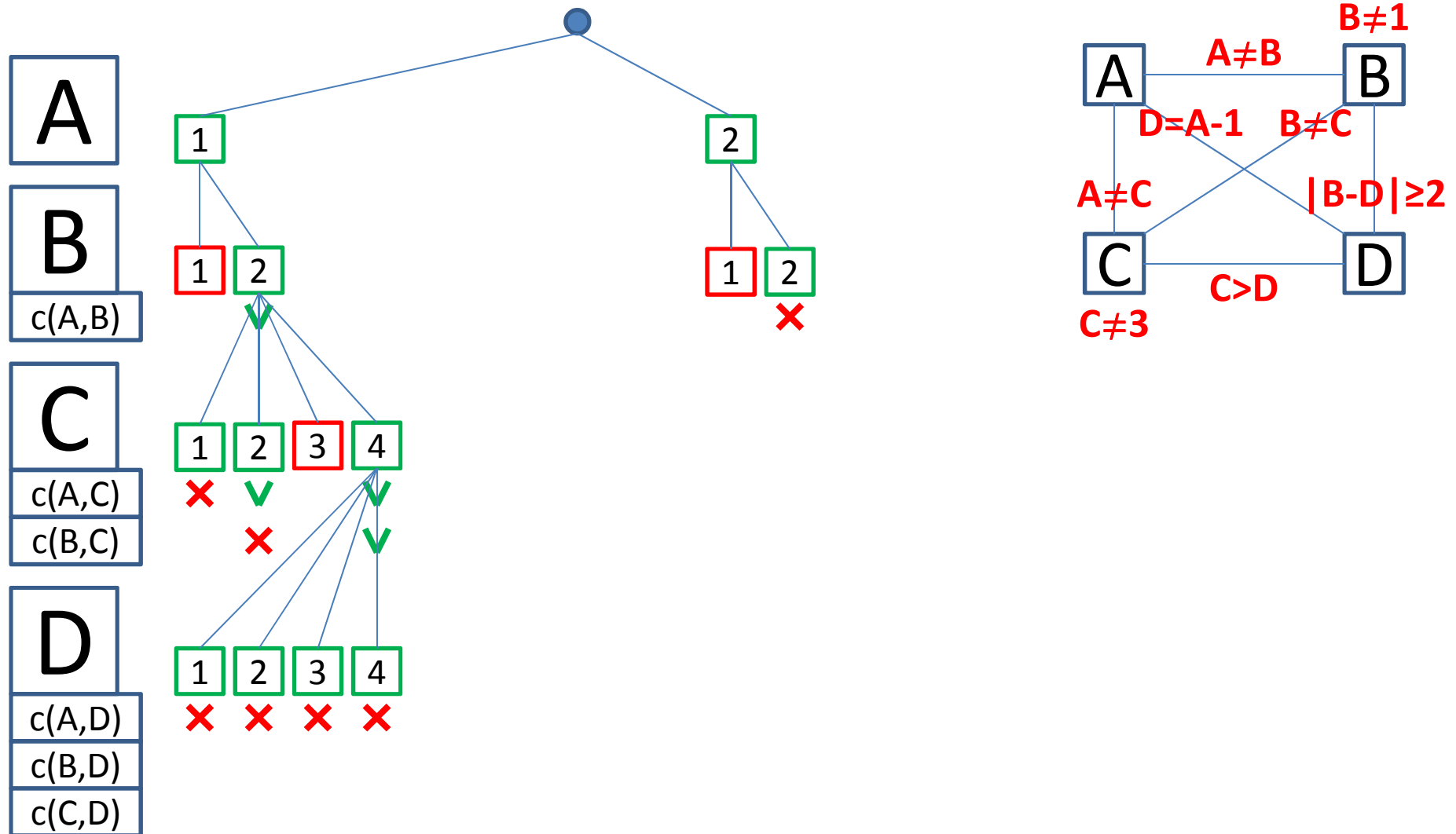
Constraint Processing: Backjumping



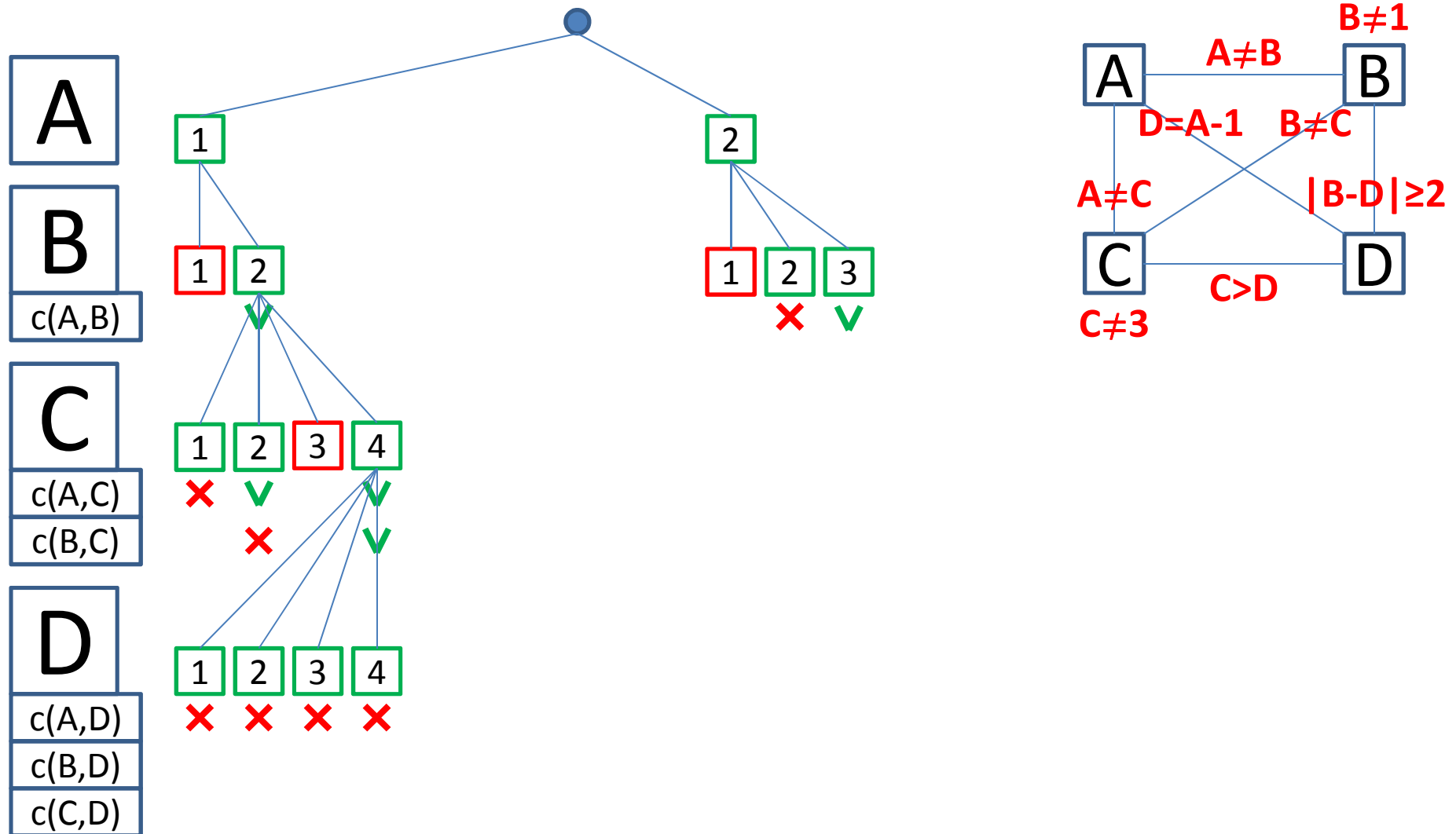
Constraint Processing: Backjumping



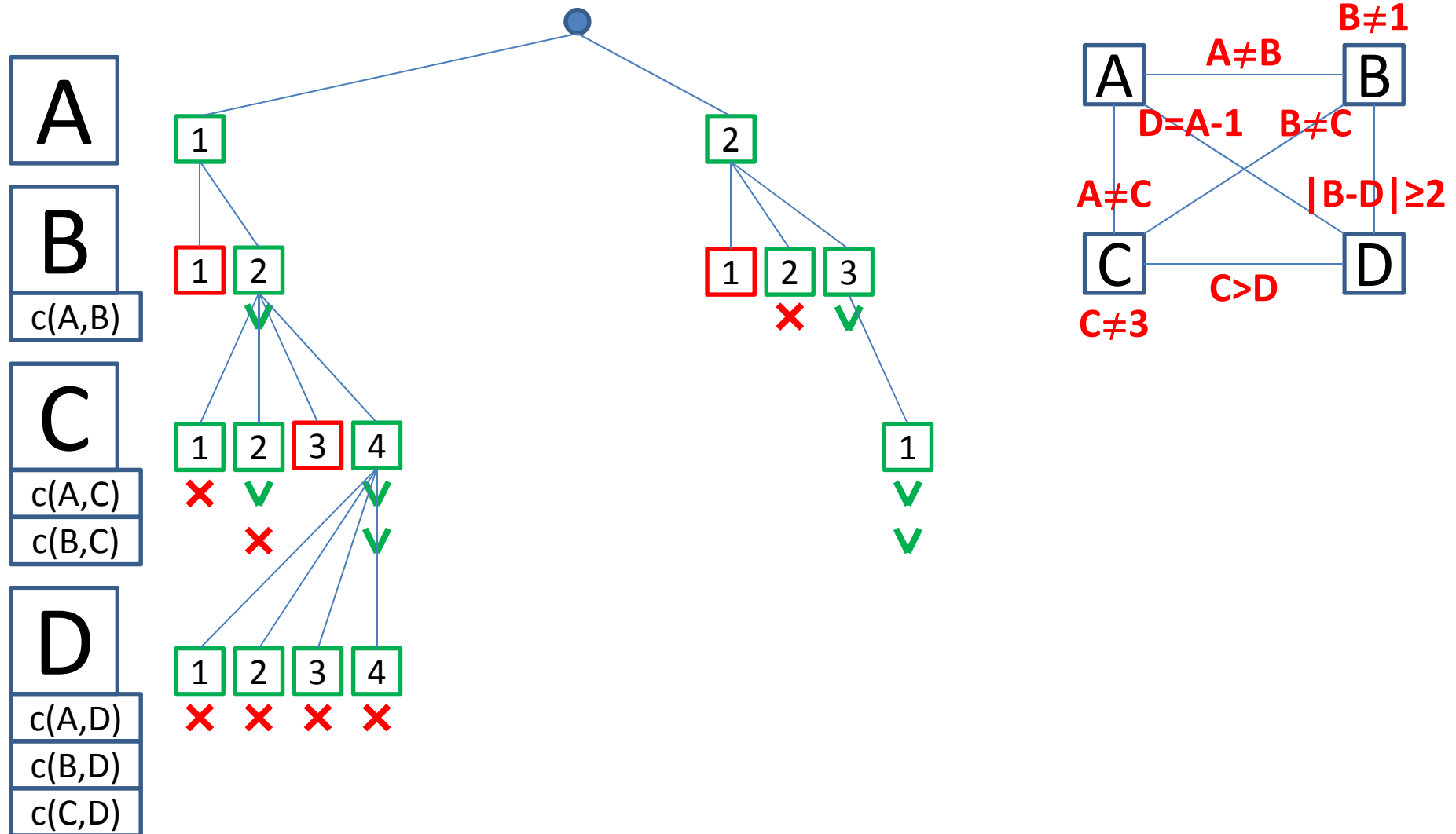
Constraint Processing: Backjumping



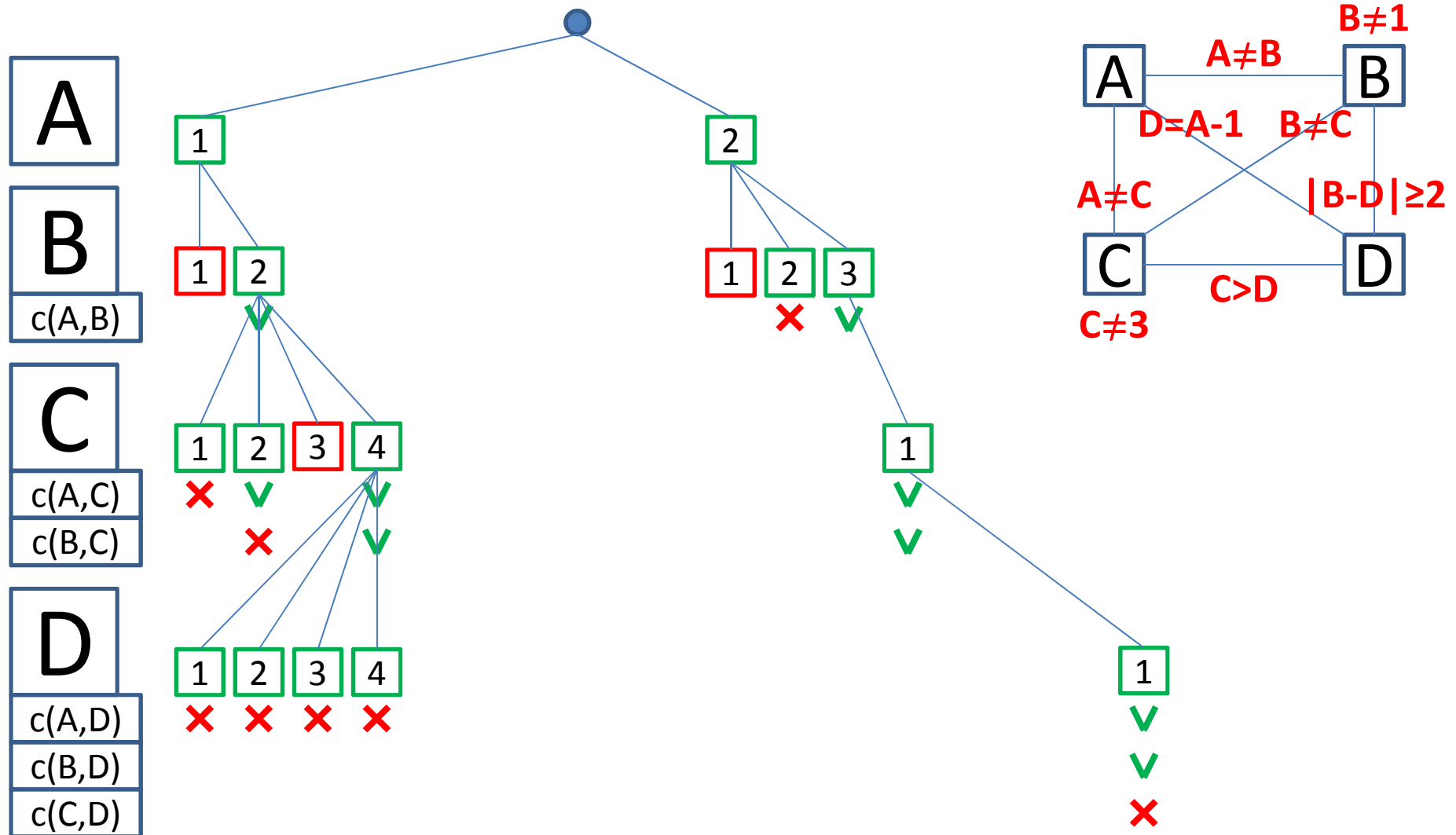
Constraint Processing: Backjumping



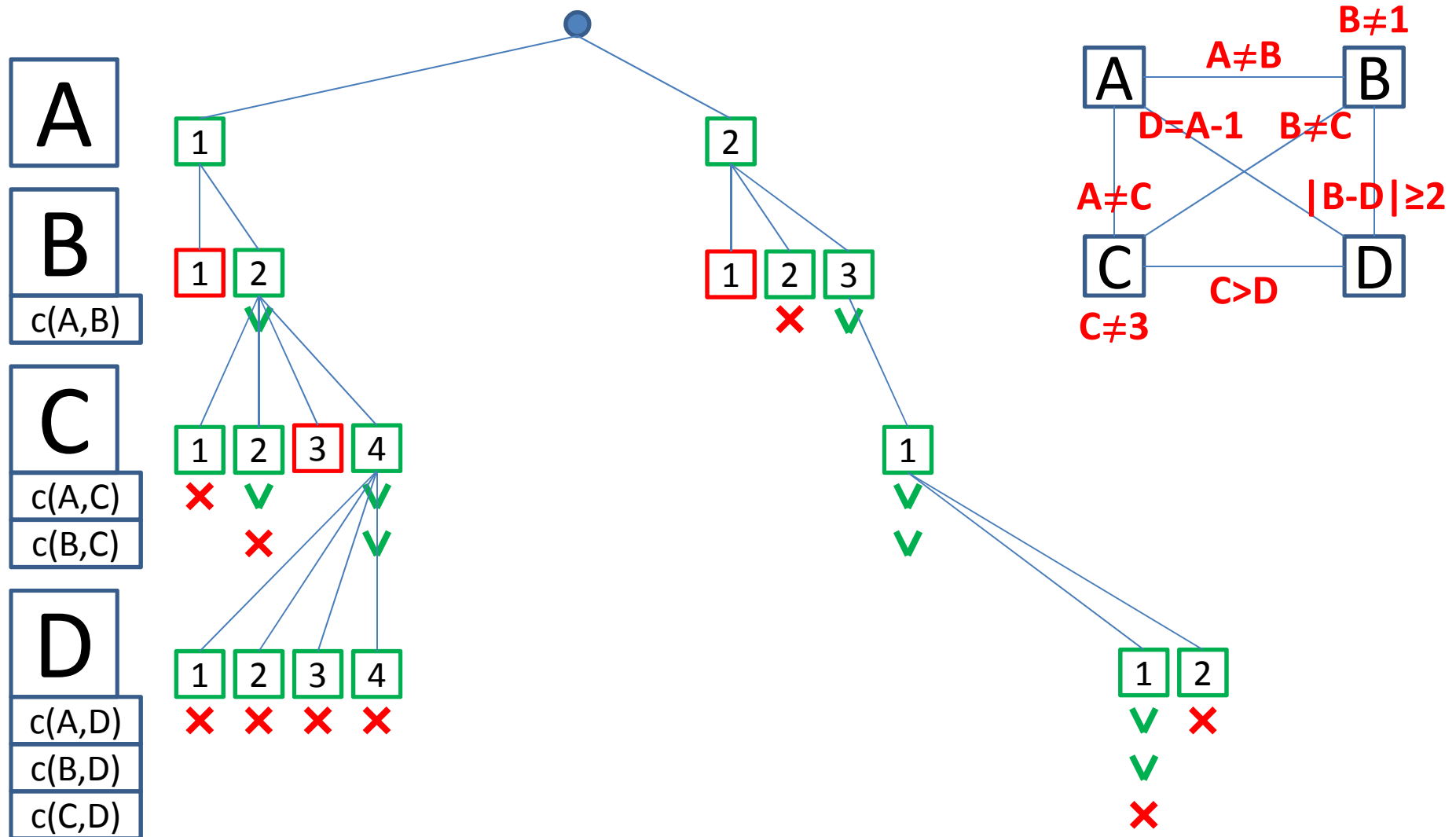
Constraint Processing: Backjumping



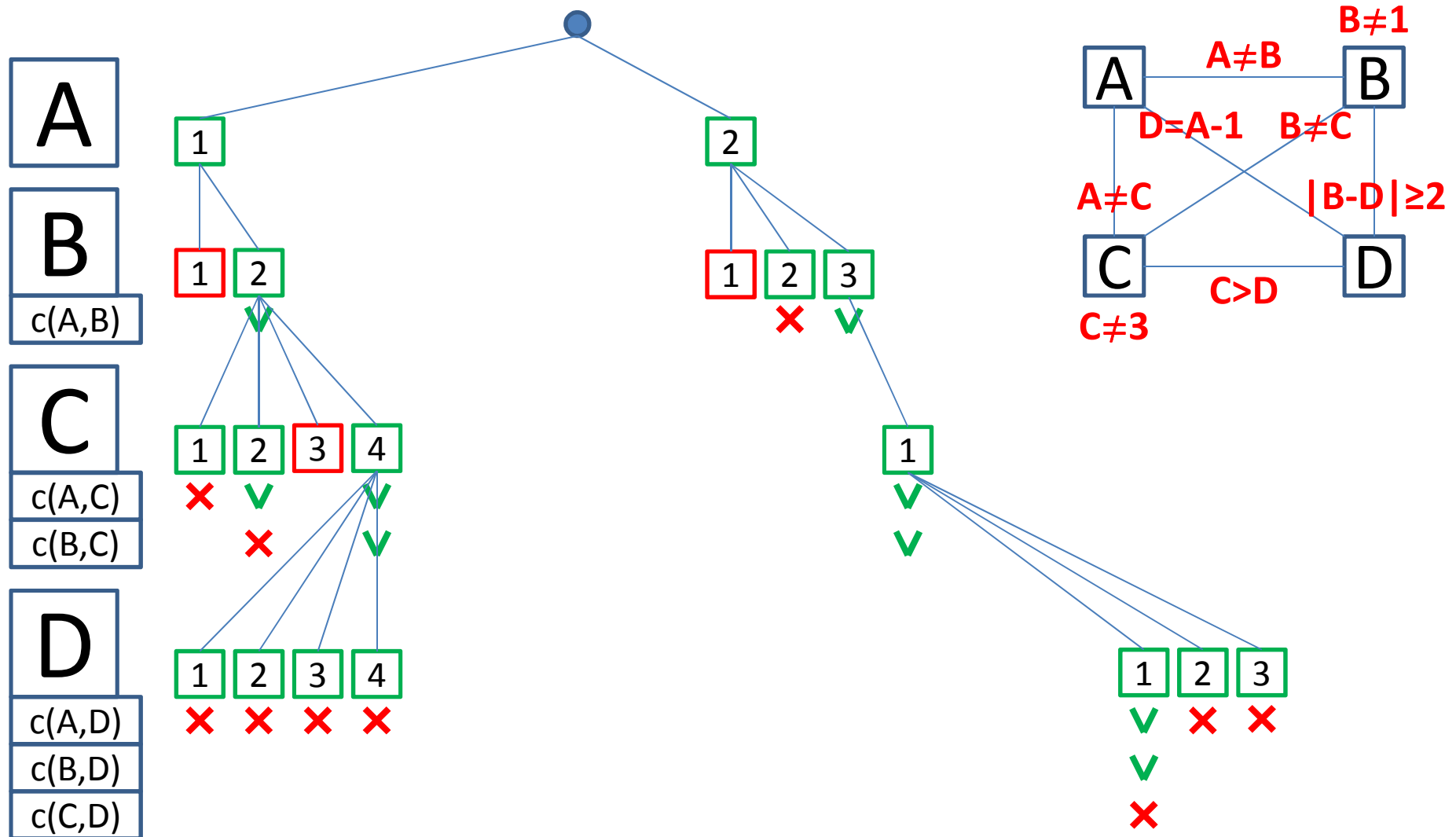
Constraint Processing: Backjumping



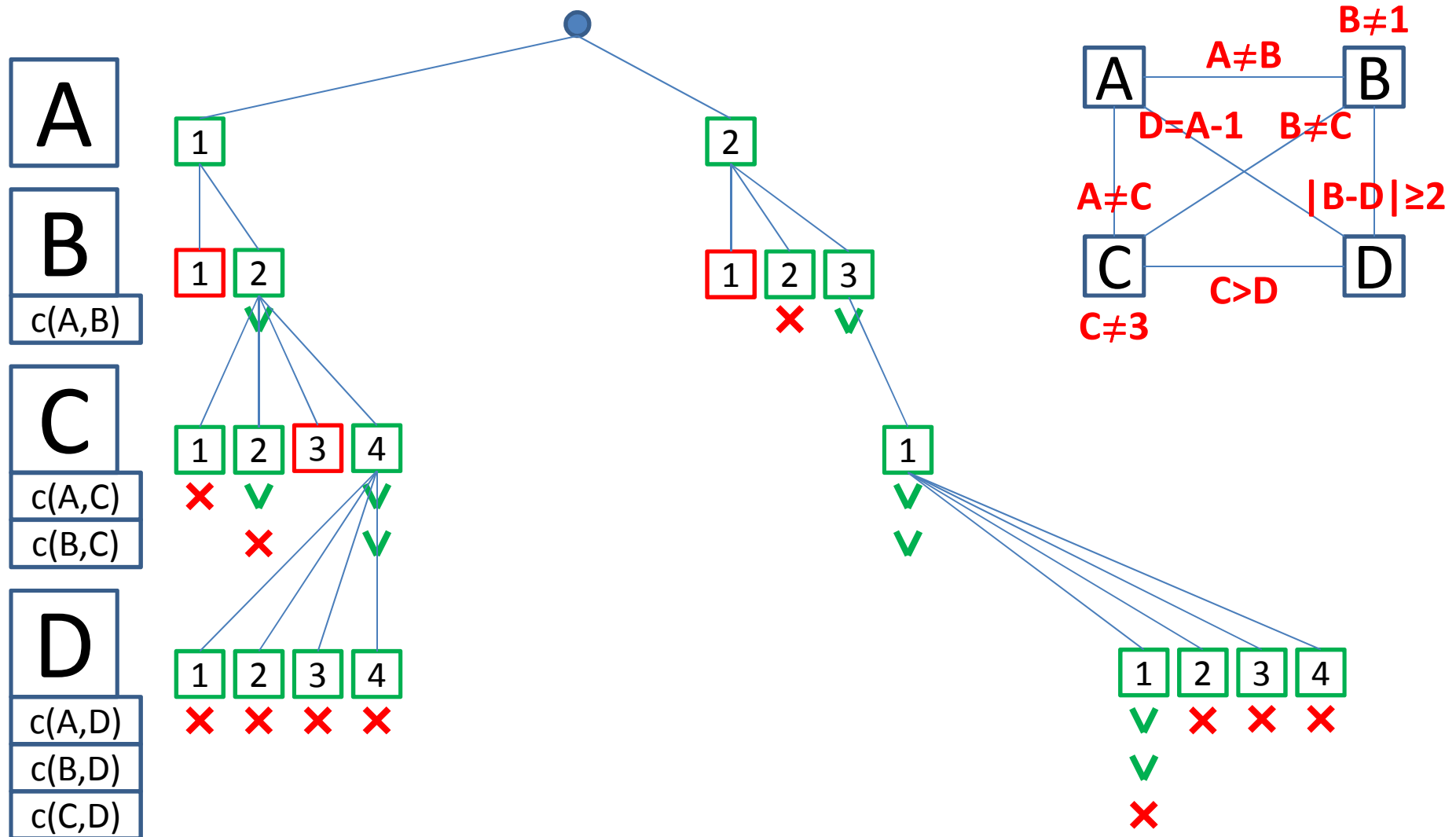
Constraint Processing: Backjumping



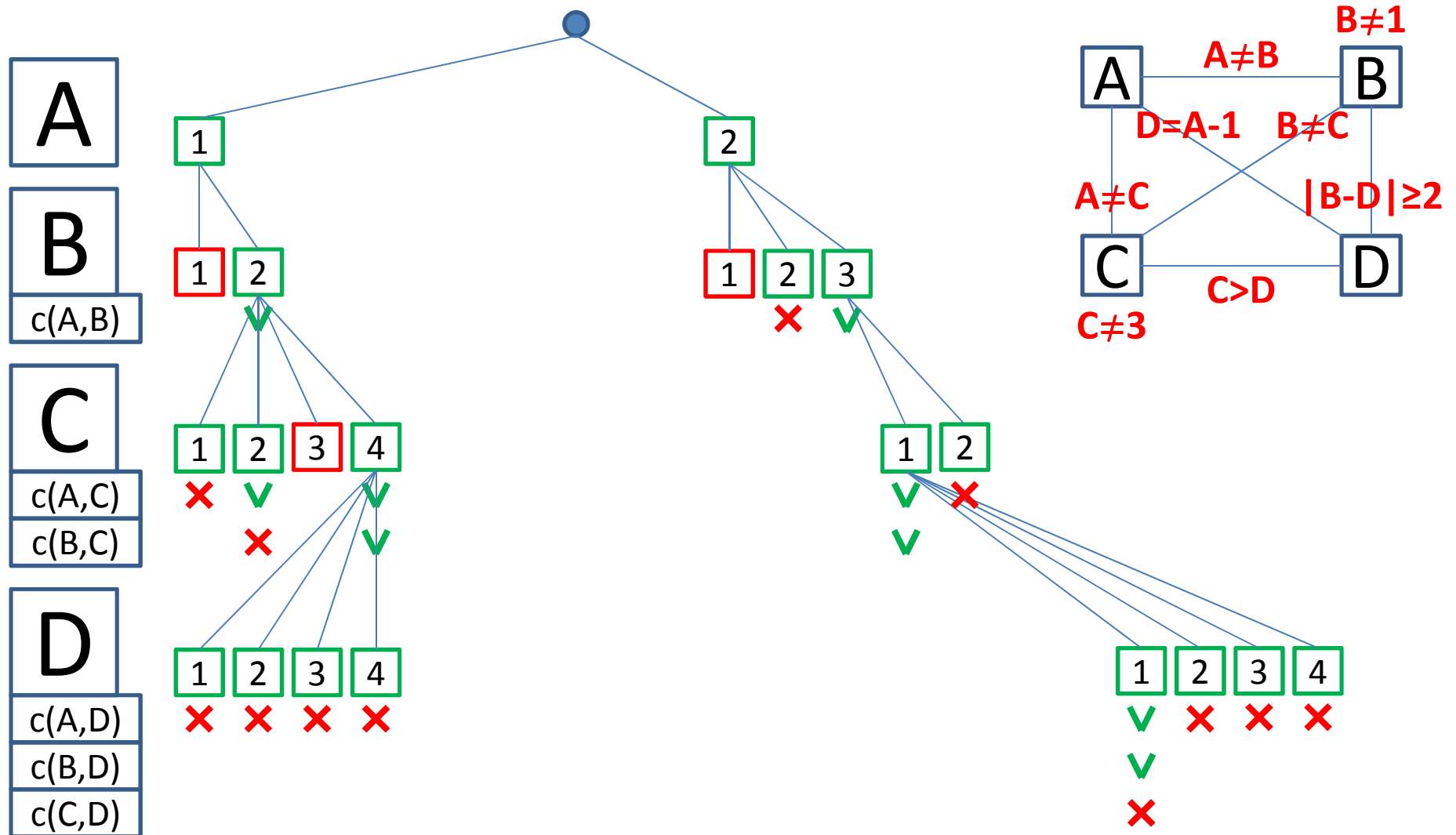
Constraint Processing: Backjumping



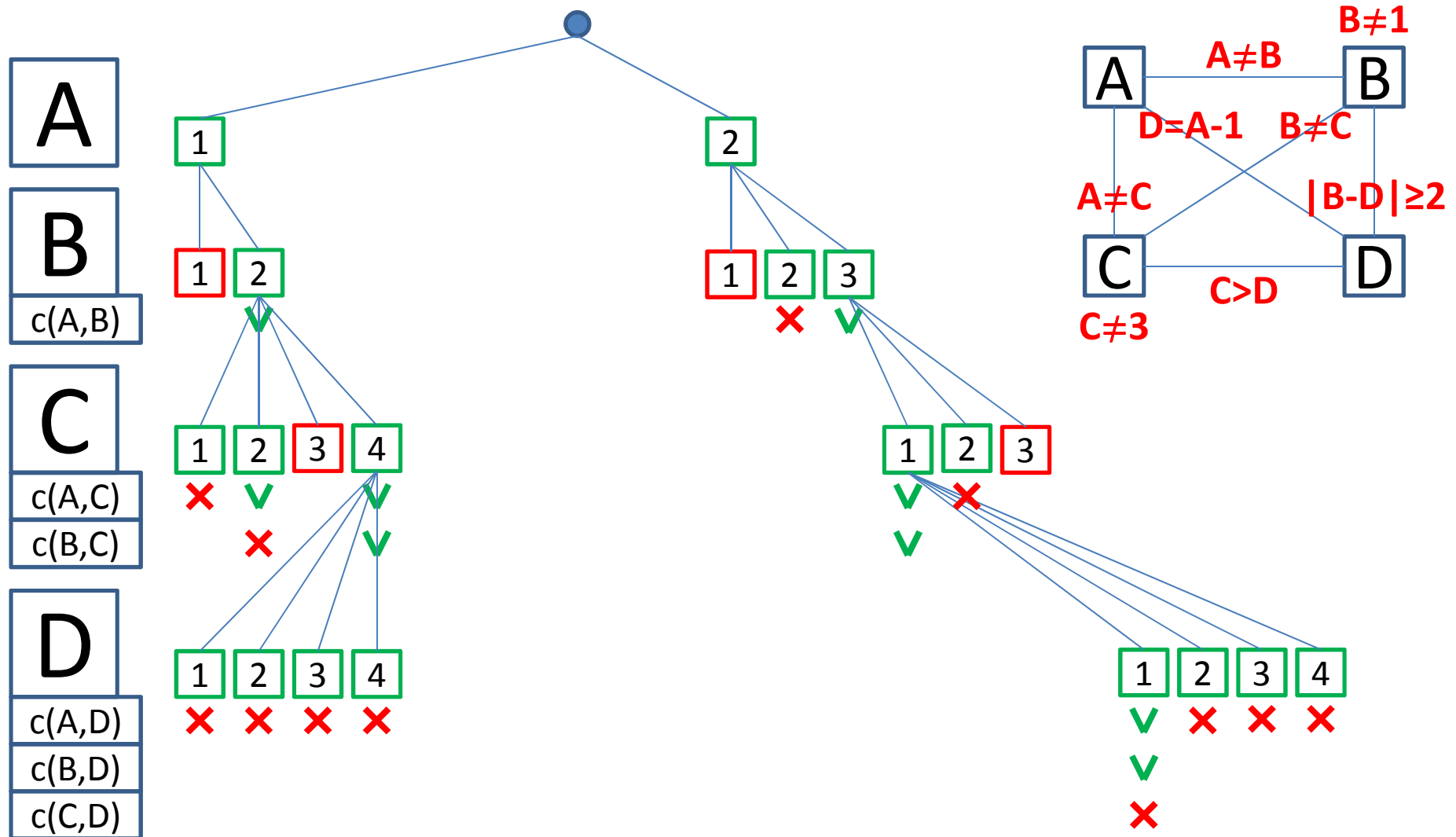
Constraint Processing: Backjumping



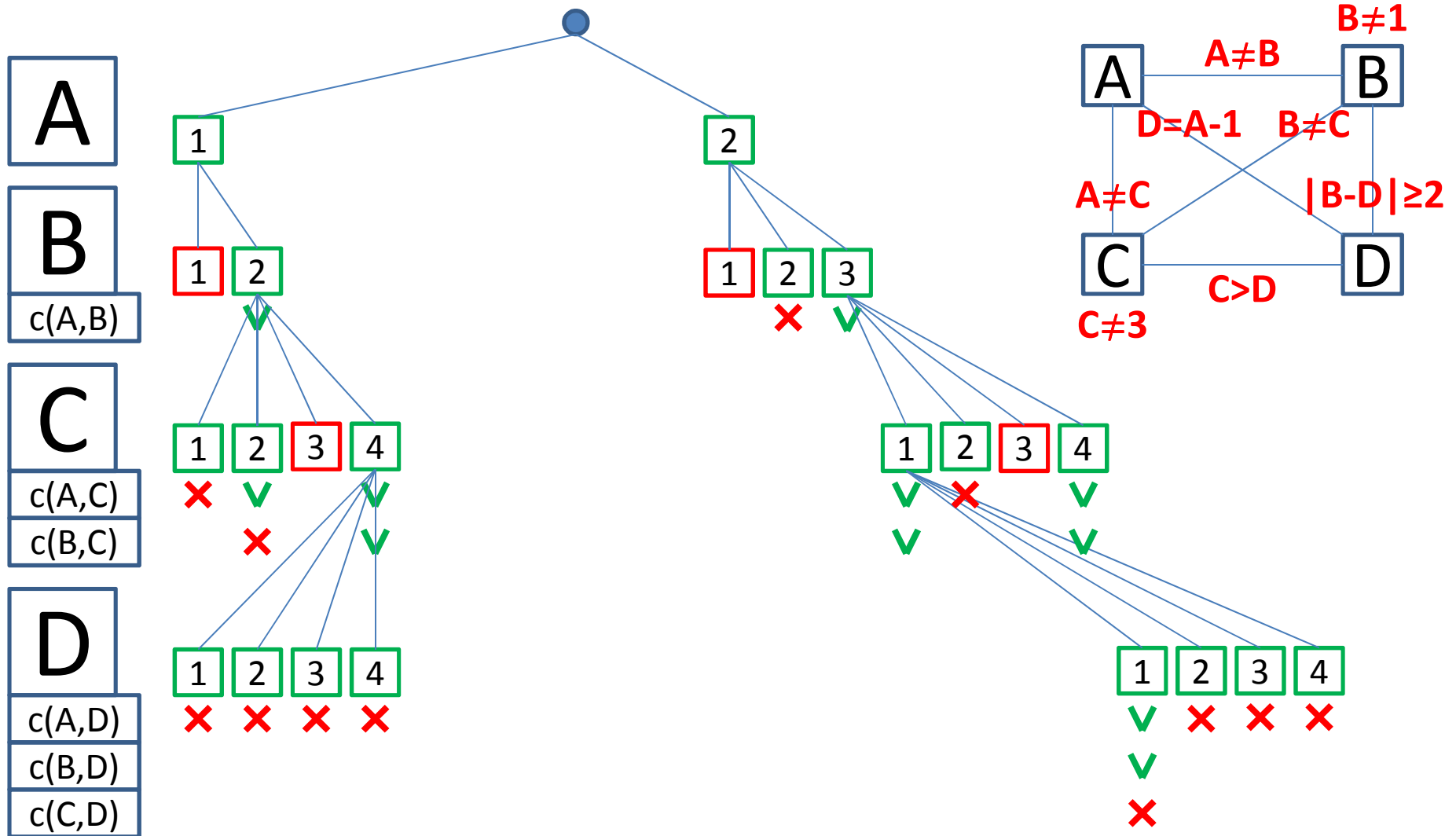
Constraint Processing: Backjumping



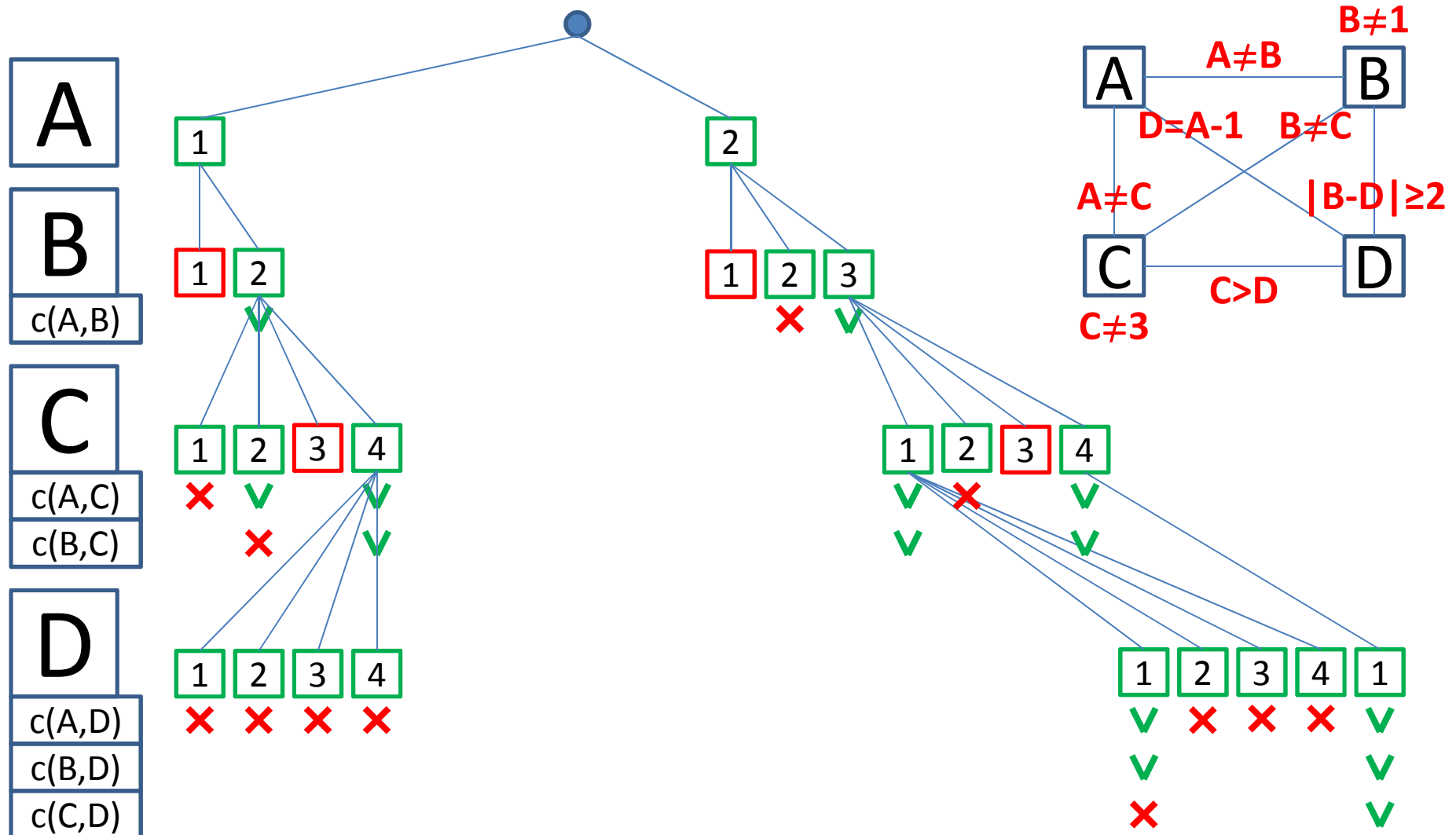
Constraint Processing: Backjumping



Constraint Processing: Backjumping



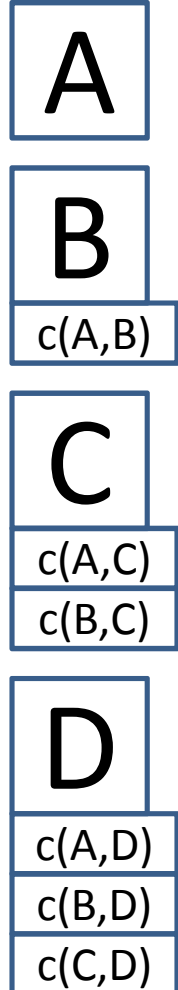
Constraint Processing: Backjumping



MiniMax & Constraint Processing: The 4 Houses problem

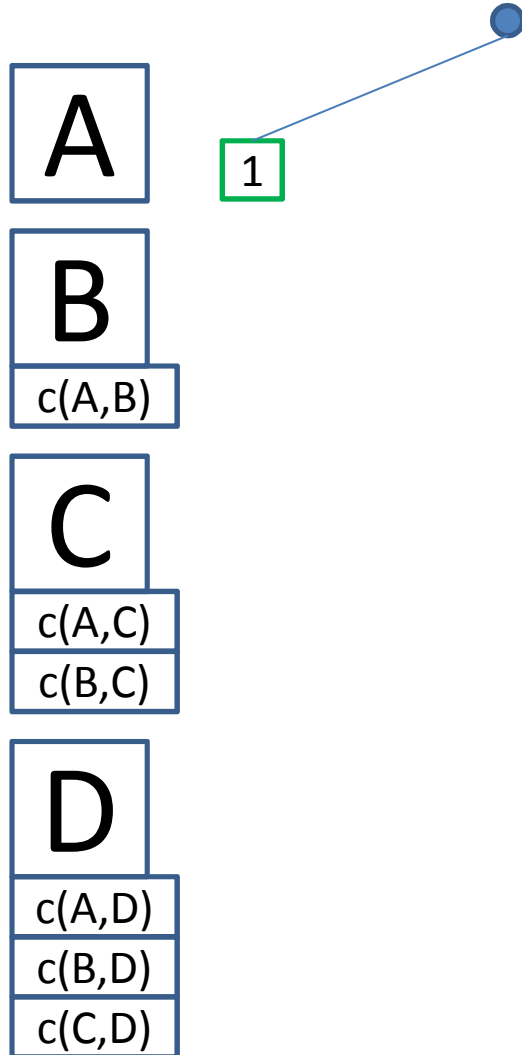
CONSTRAINT PROCESSING: BACKMARKING

Constraint Processing: Backmarking



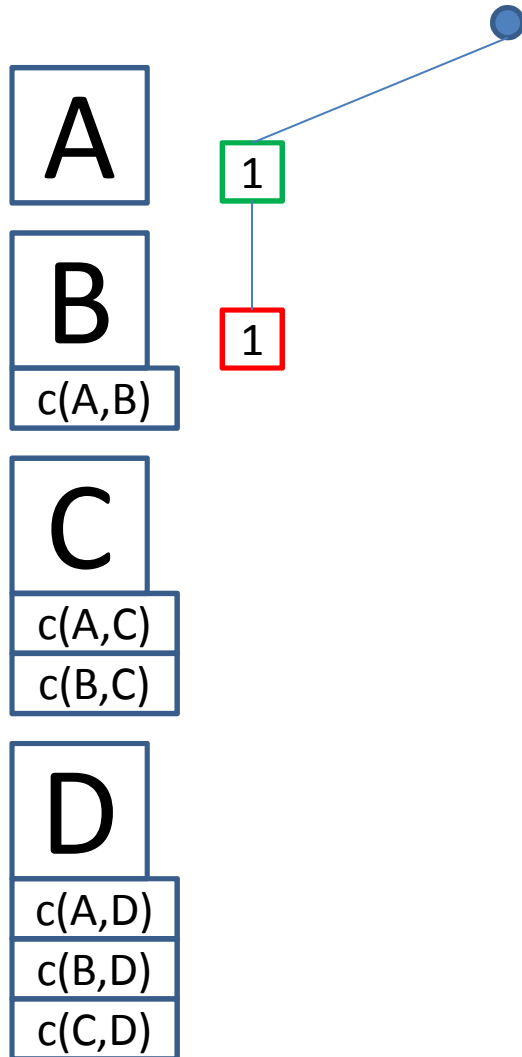
	1	2	3	4	Backup
A	1	1	1	1	1
B	1	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



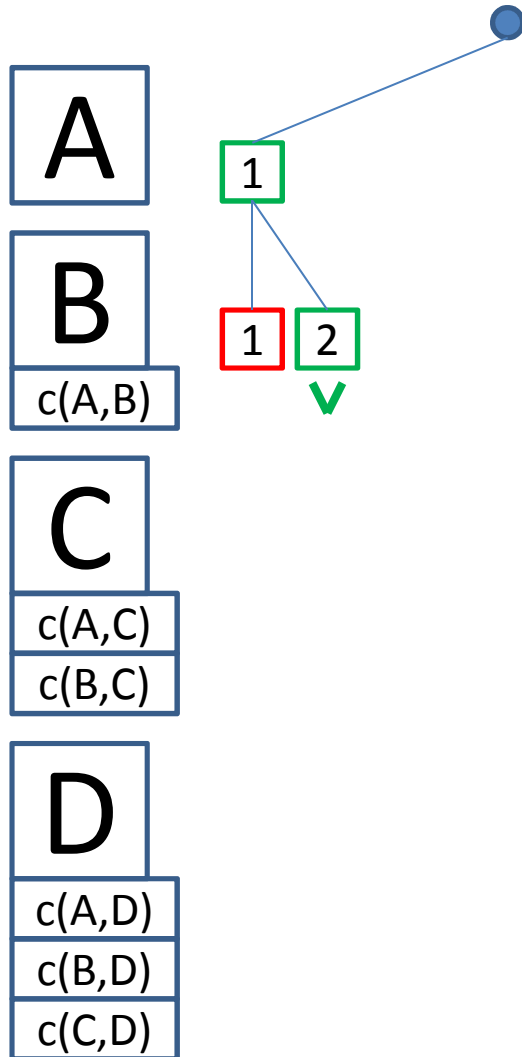
	1	2	3	4	Backup
A	0	1	1	1	1
B	1	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



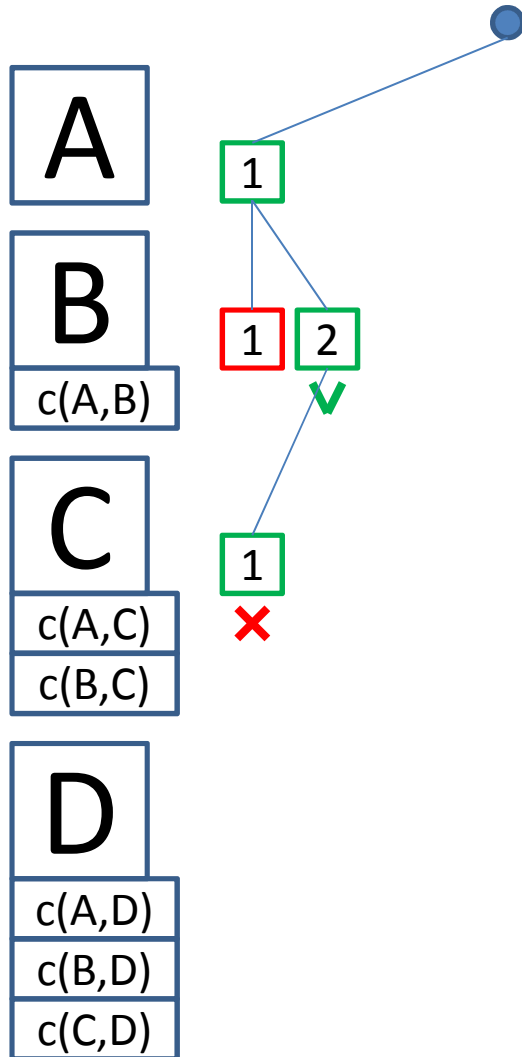
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



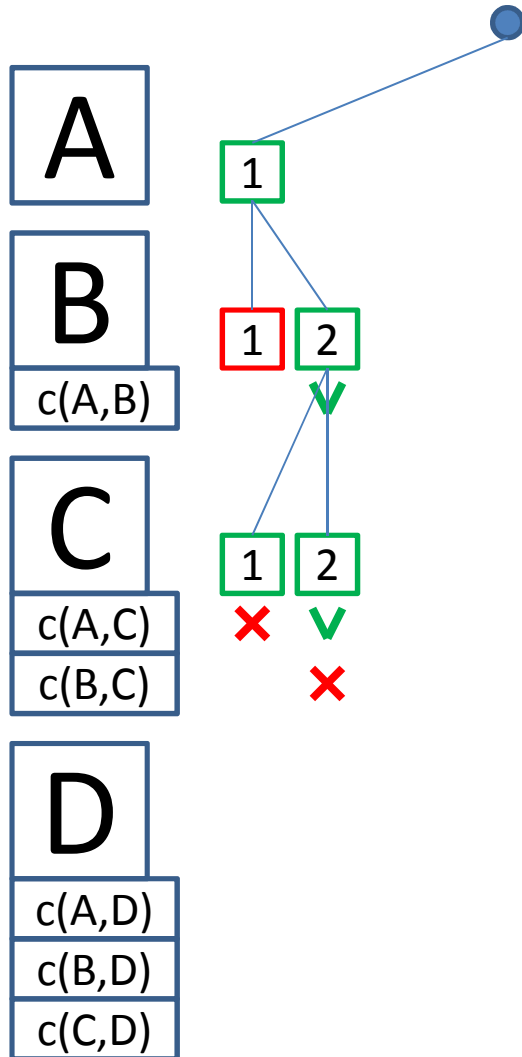
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



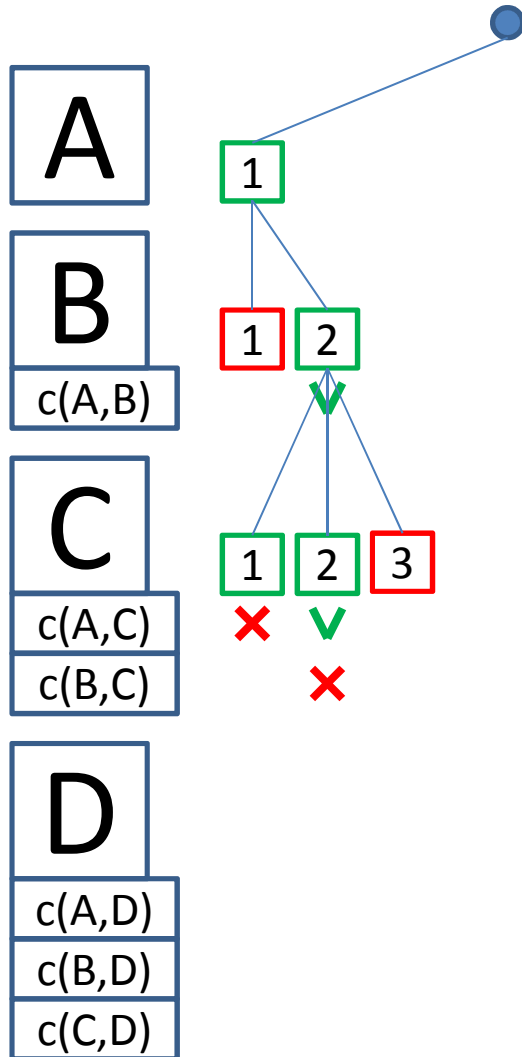
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



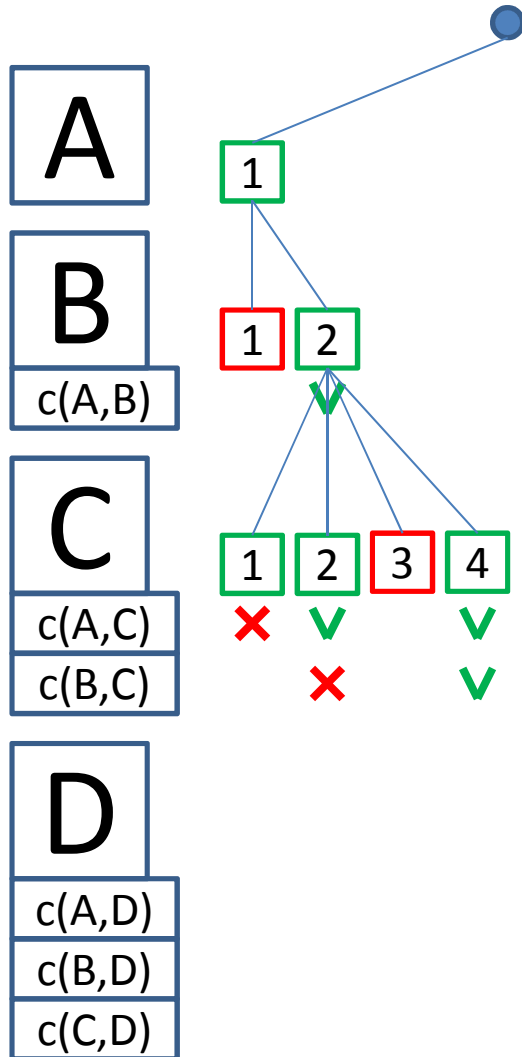
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	1	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



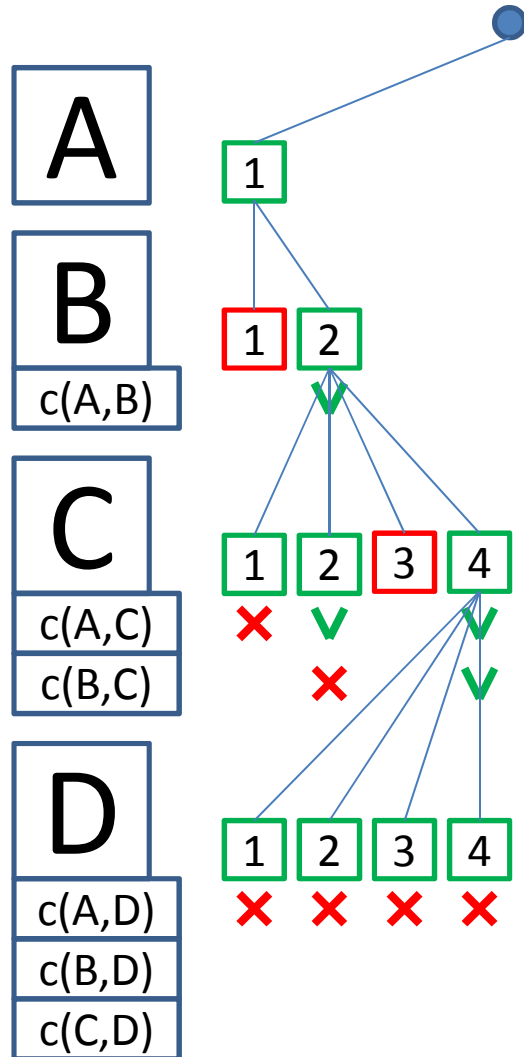
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	1	1
D	1	1	1	1	1

Constraint Processing: Backmarking



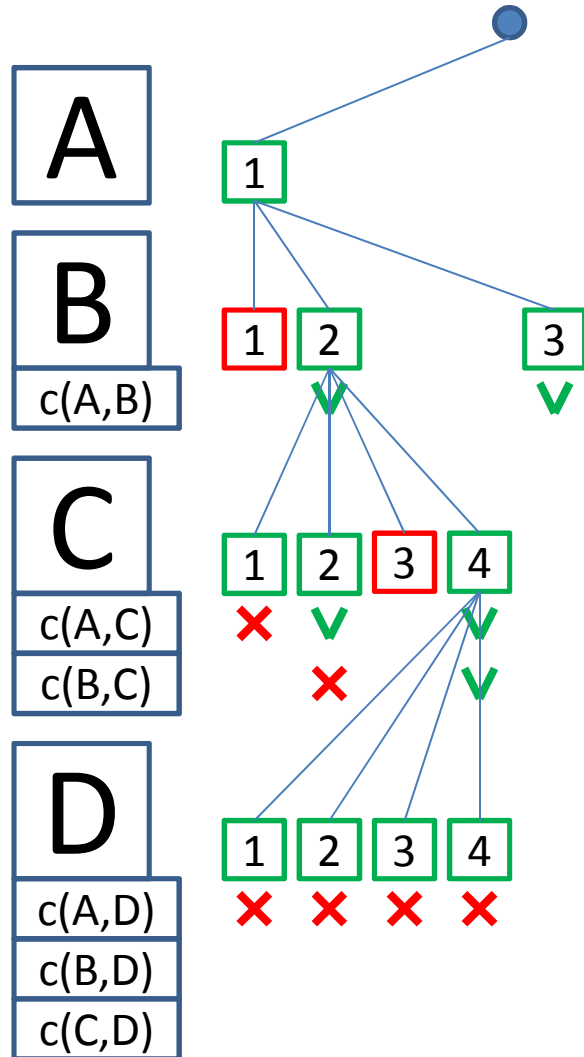
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



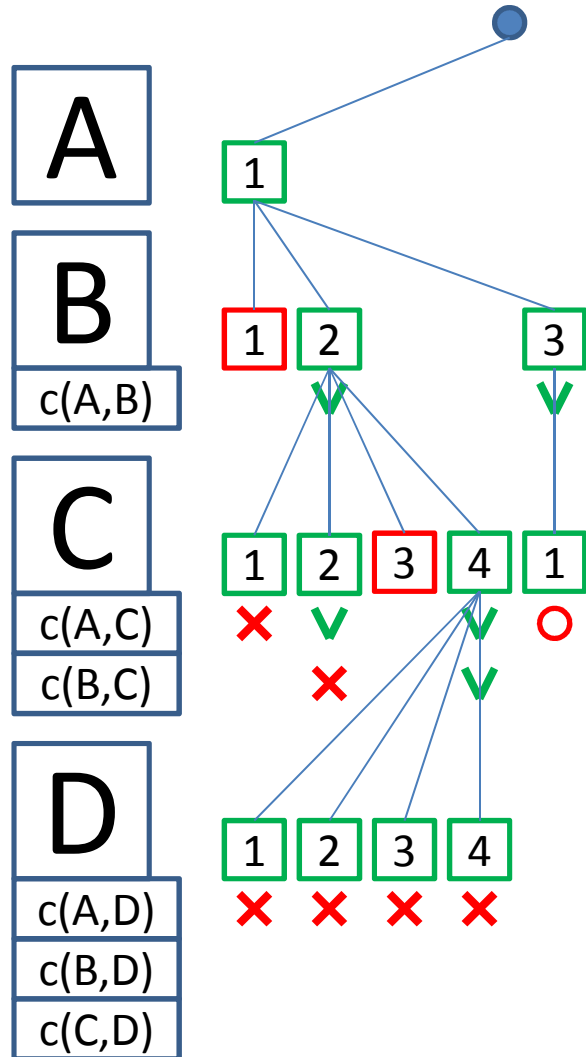
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	1
D	1	1	1	1	1

Constraint Processing: Backmarking



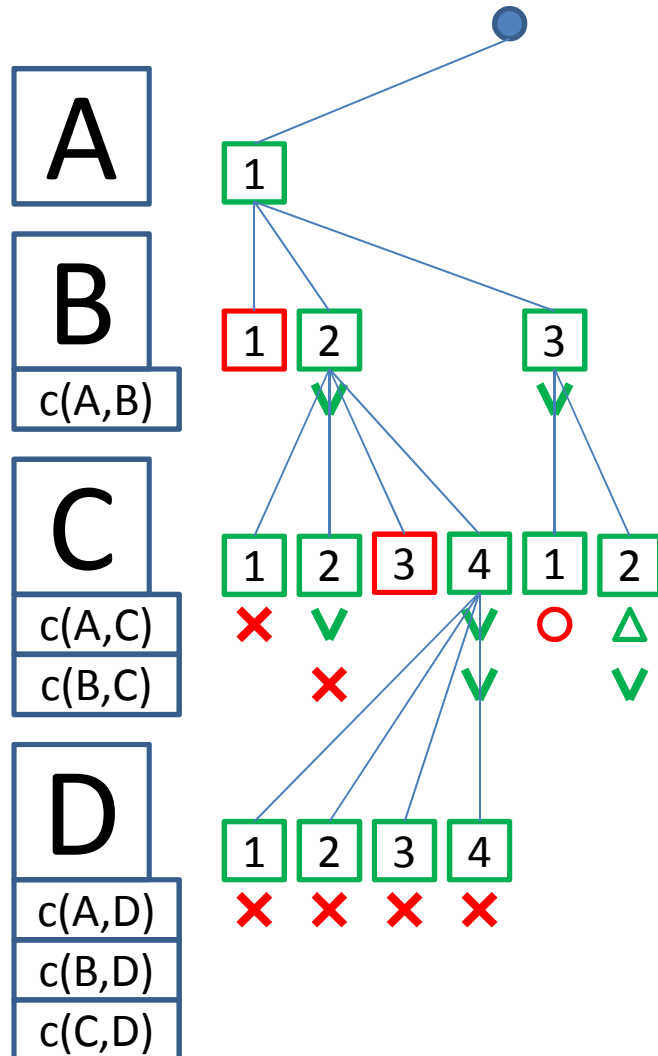
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



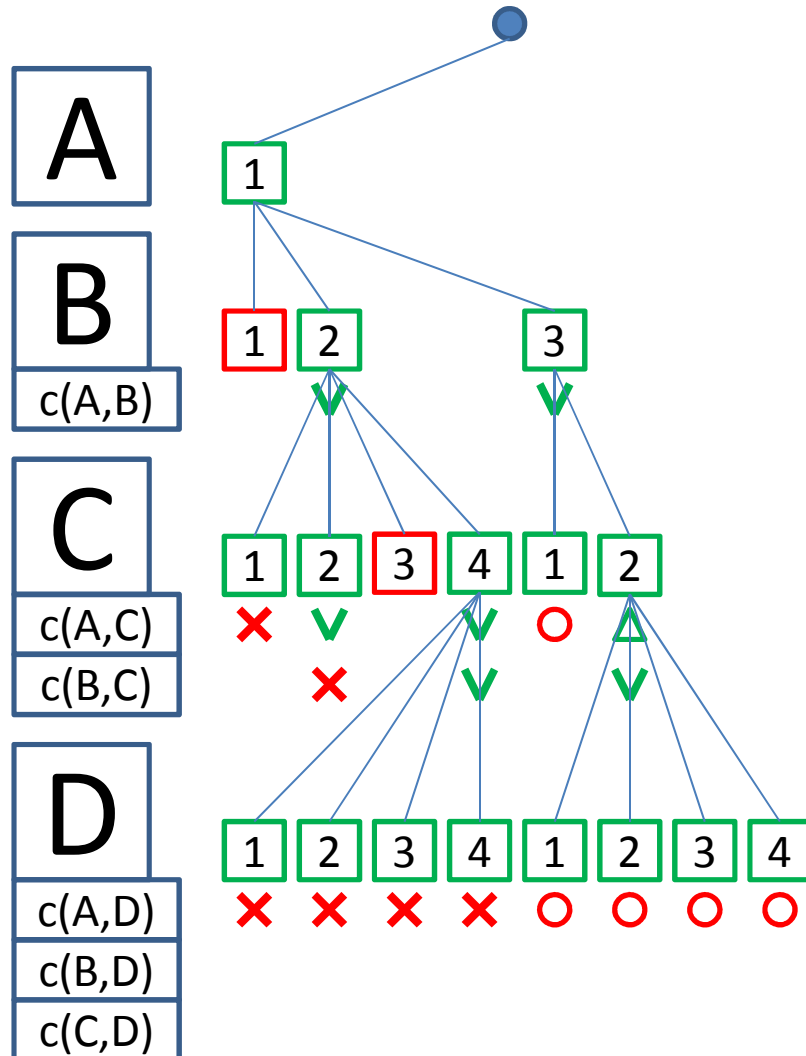
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



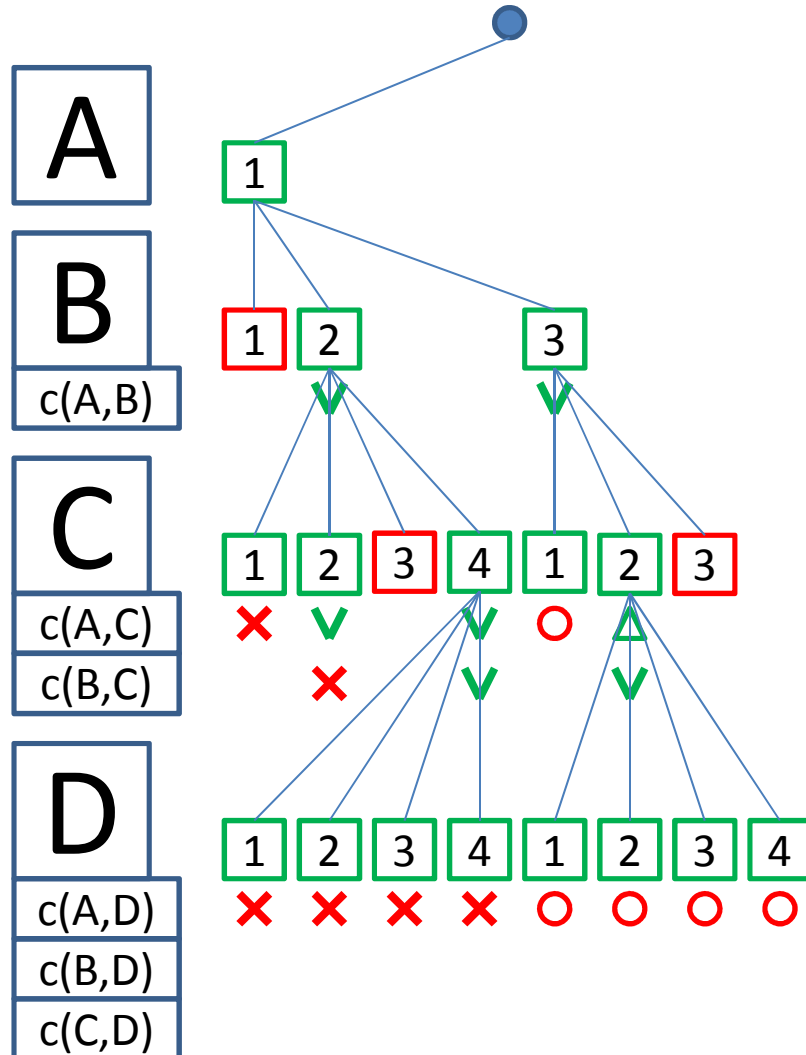
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



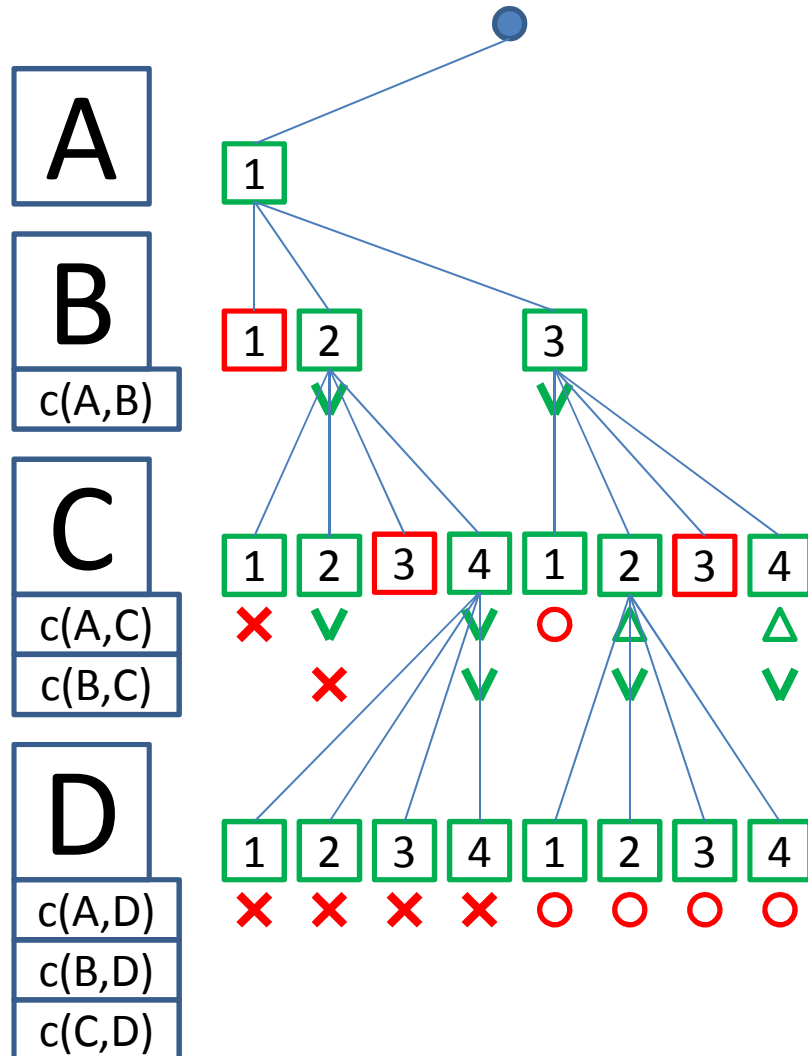
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



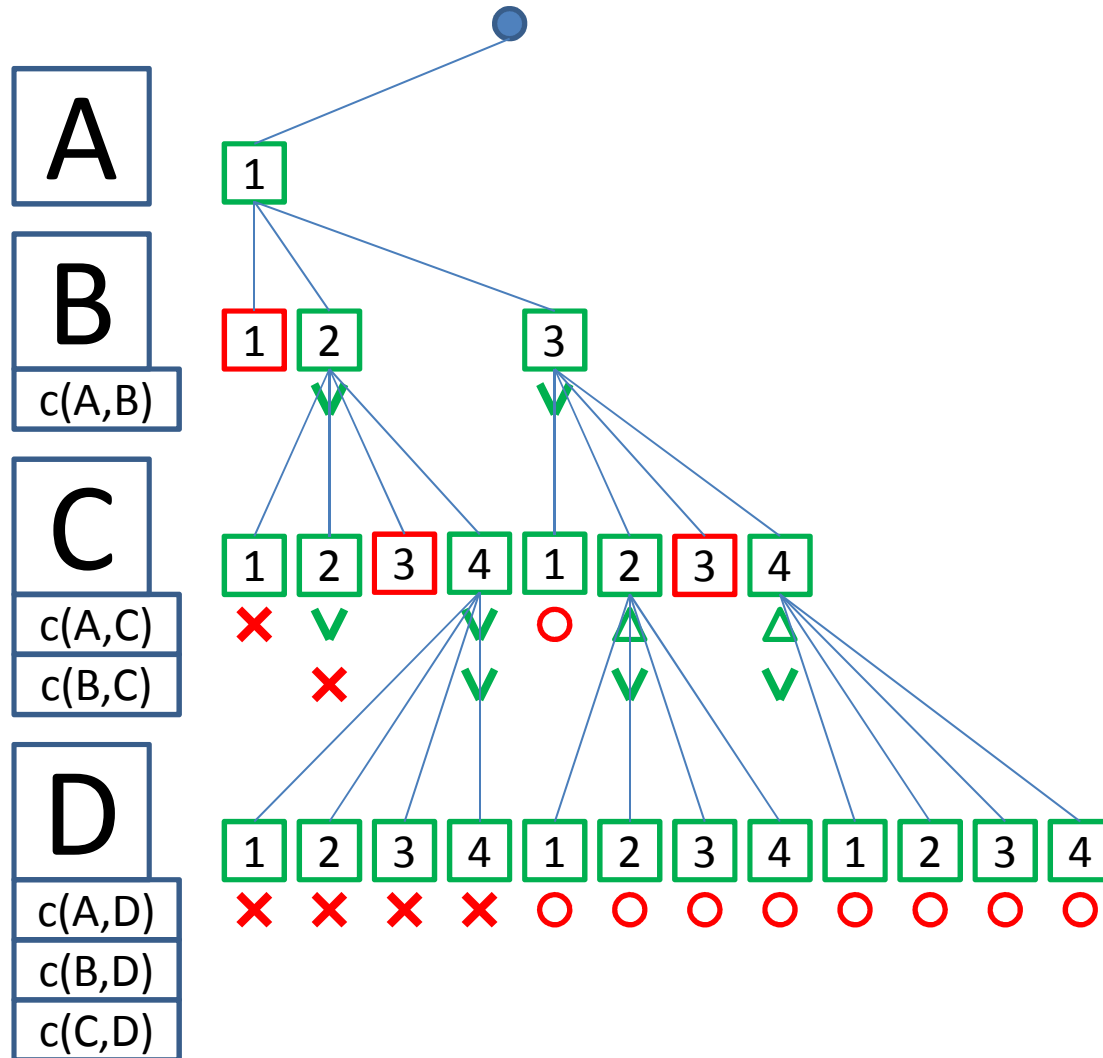
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



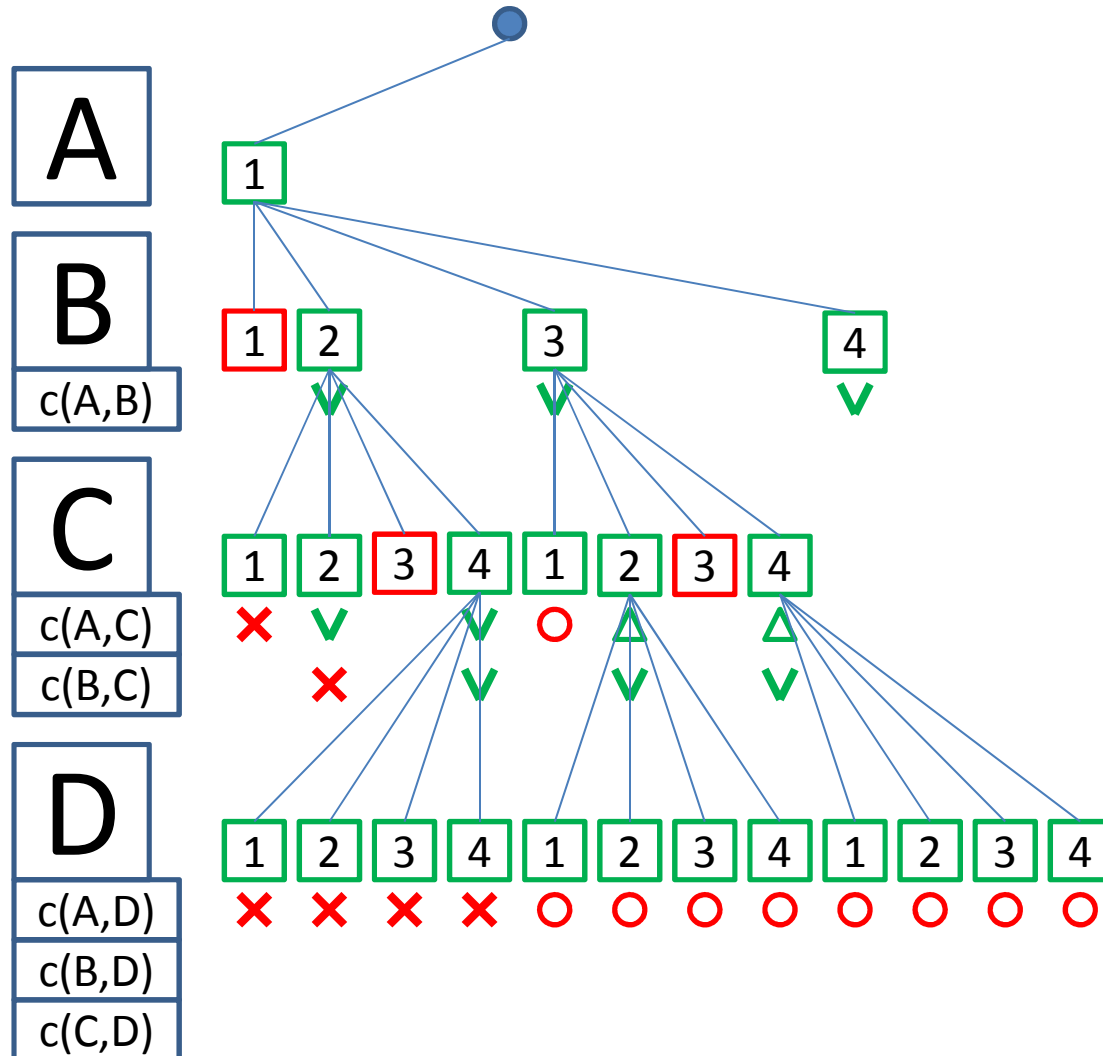
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



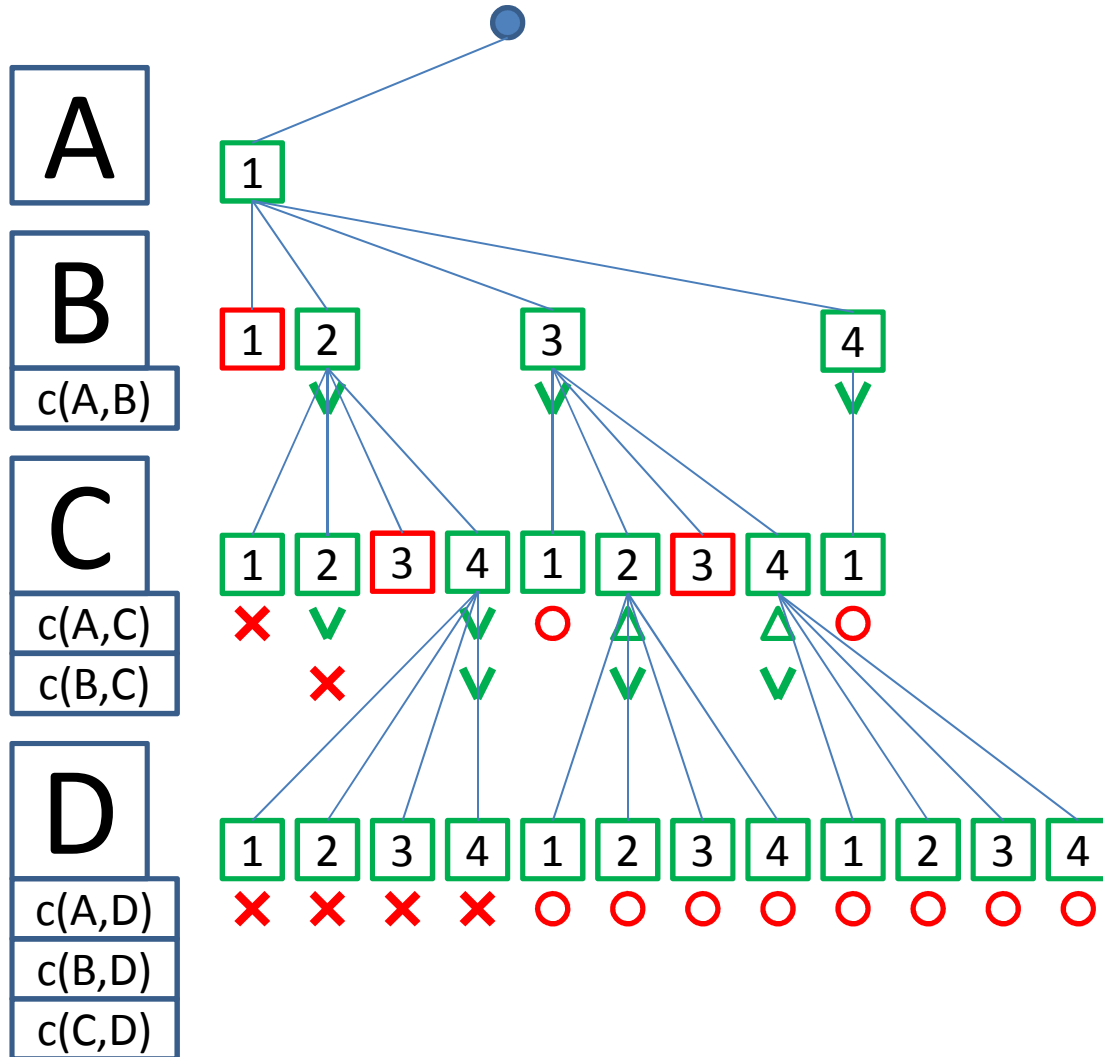
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

Constraint Processing: Backmarking



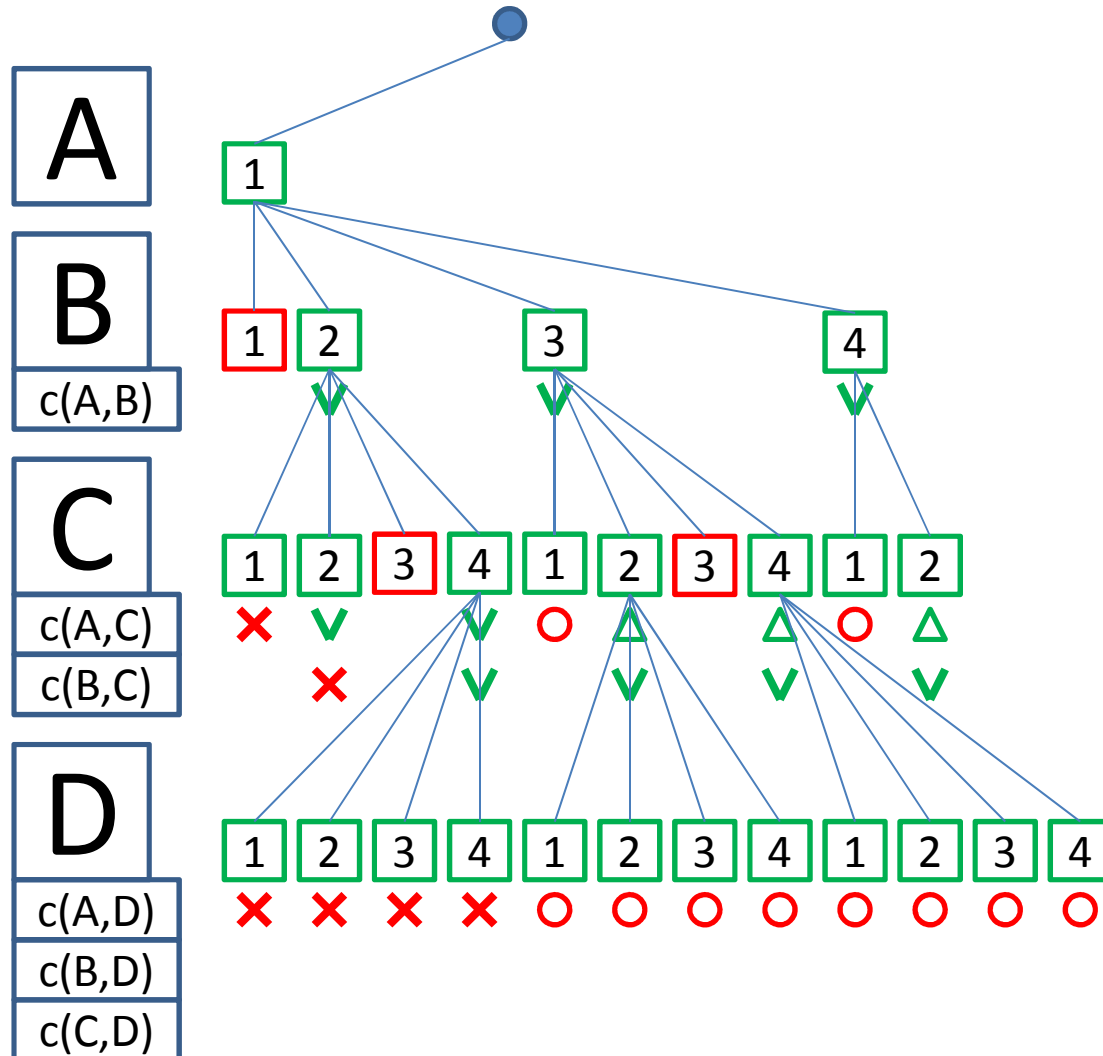
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking



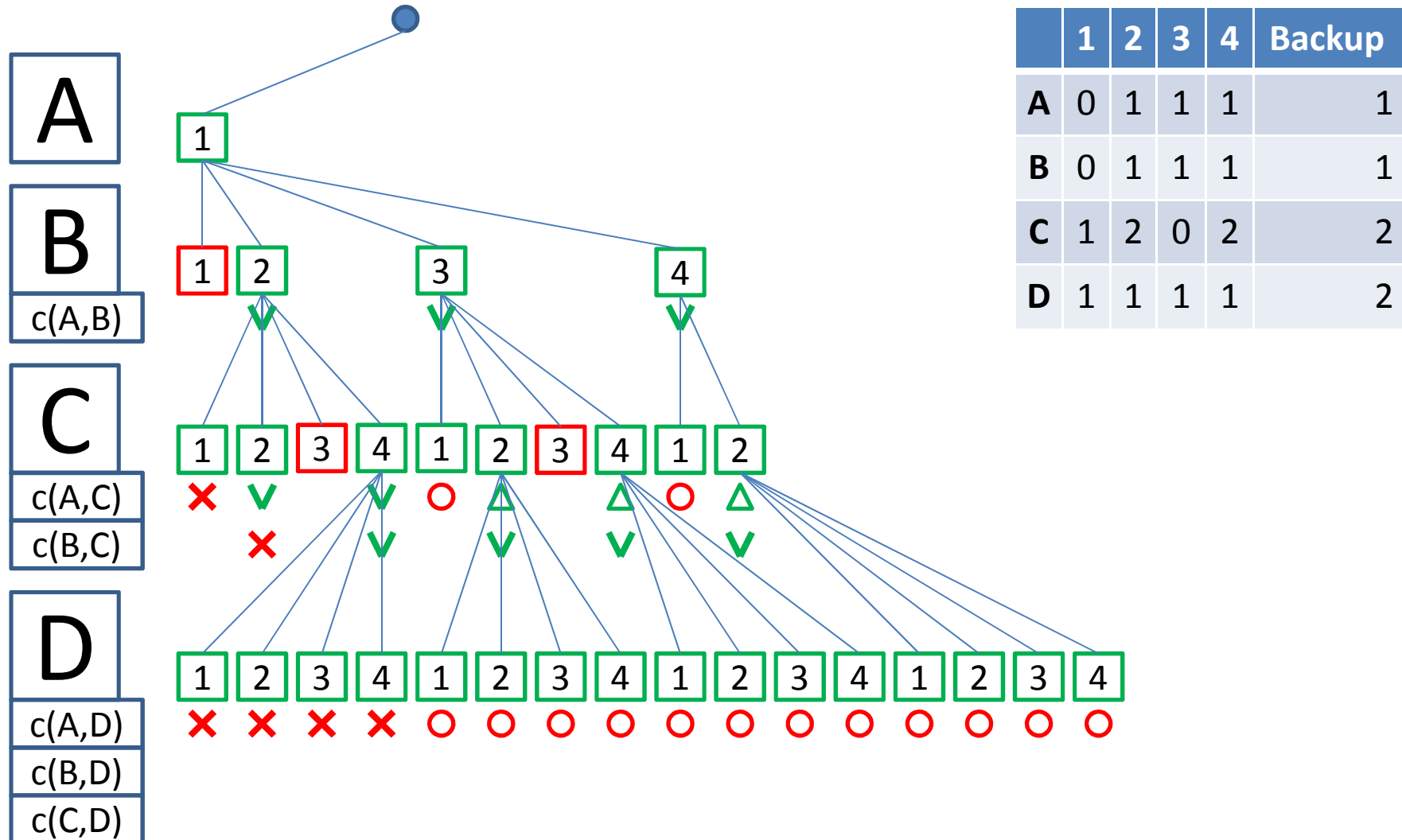
	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking

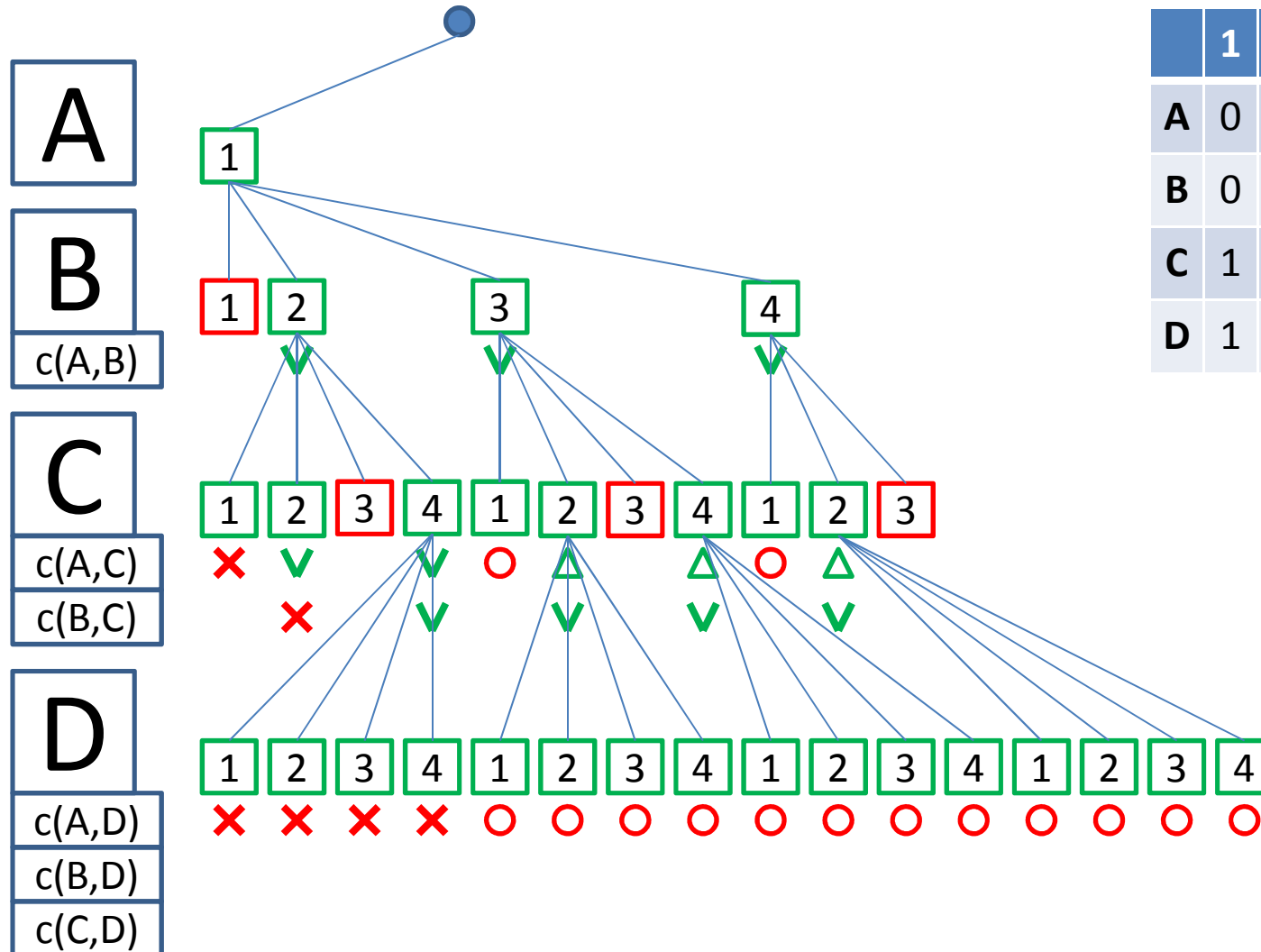


	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	2

Constraint Processing: Backmarking

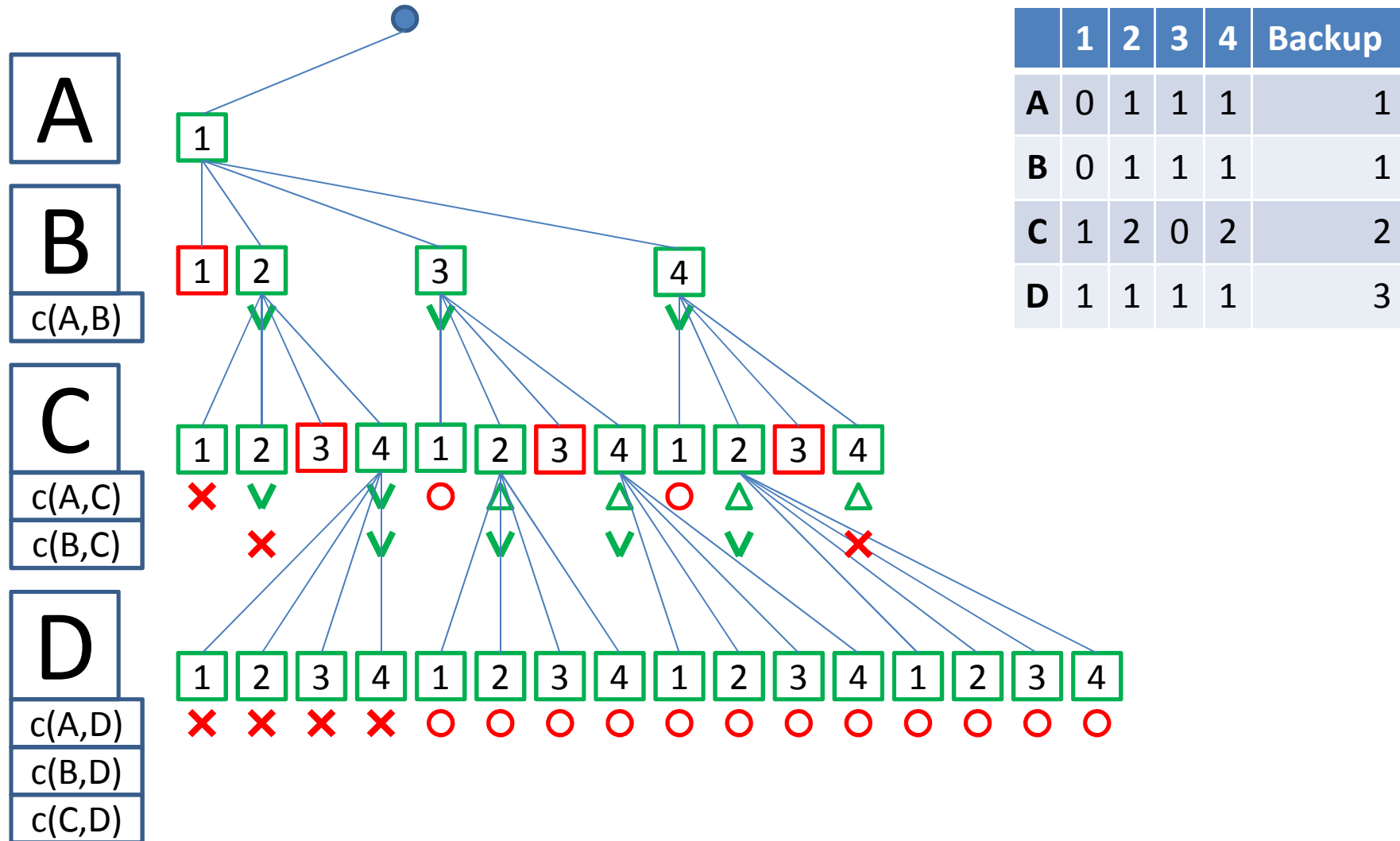


Constraint Processing: Backmarking

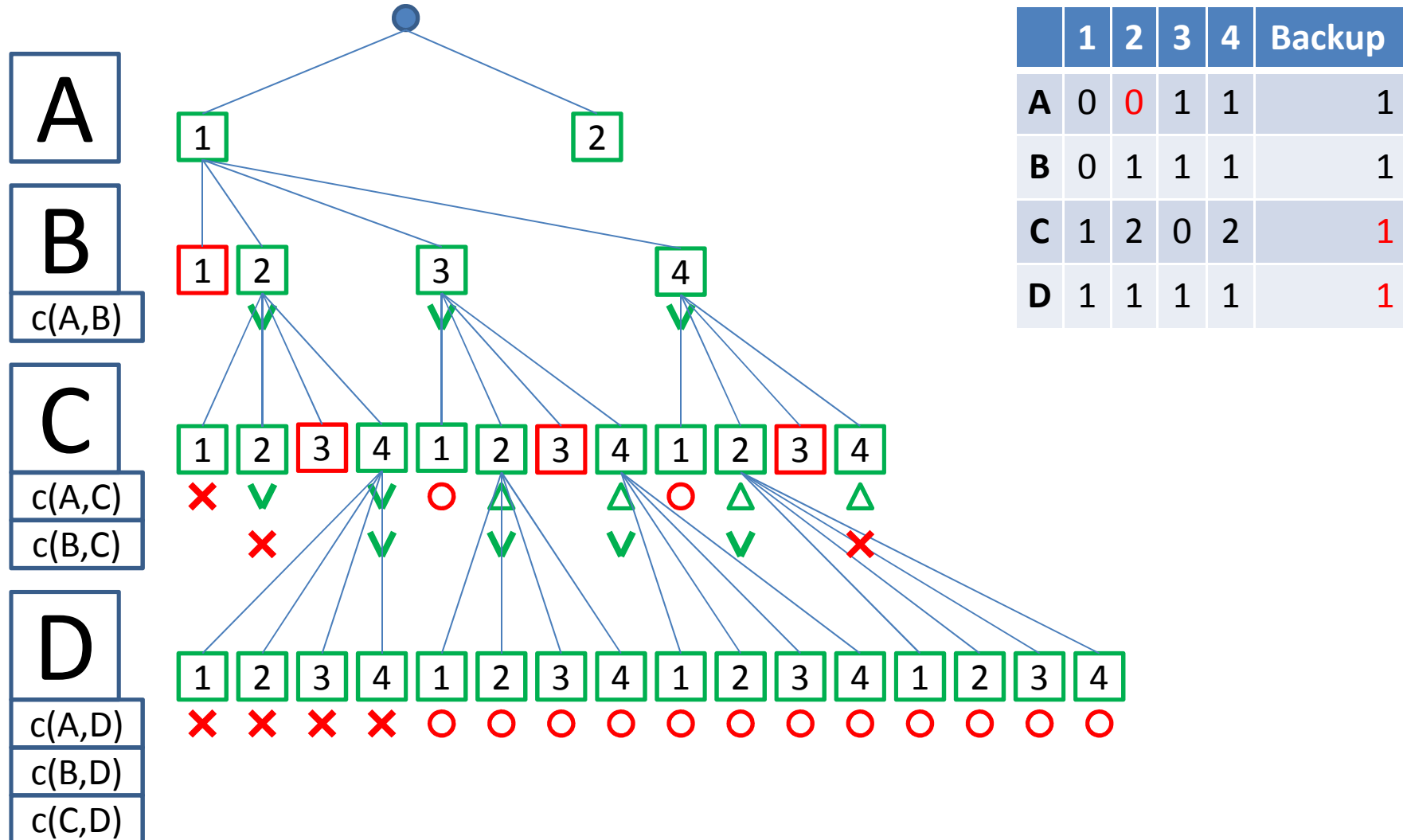


	1	2	3	4	Backup
A	0	1	1	1	1
B	0	1	1	1	1
C	1	2	0	2	2
D	1	1	1	1	3

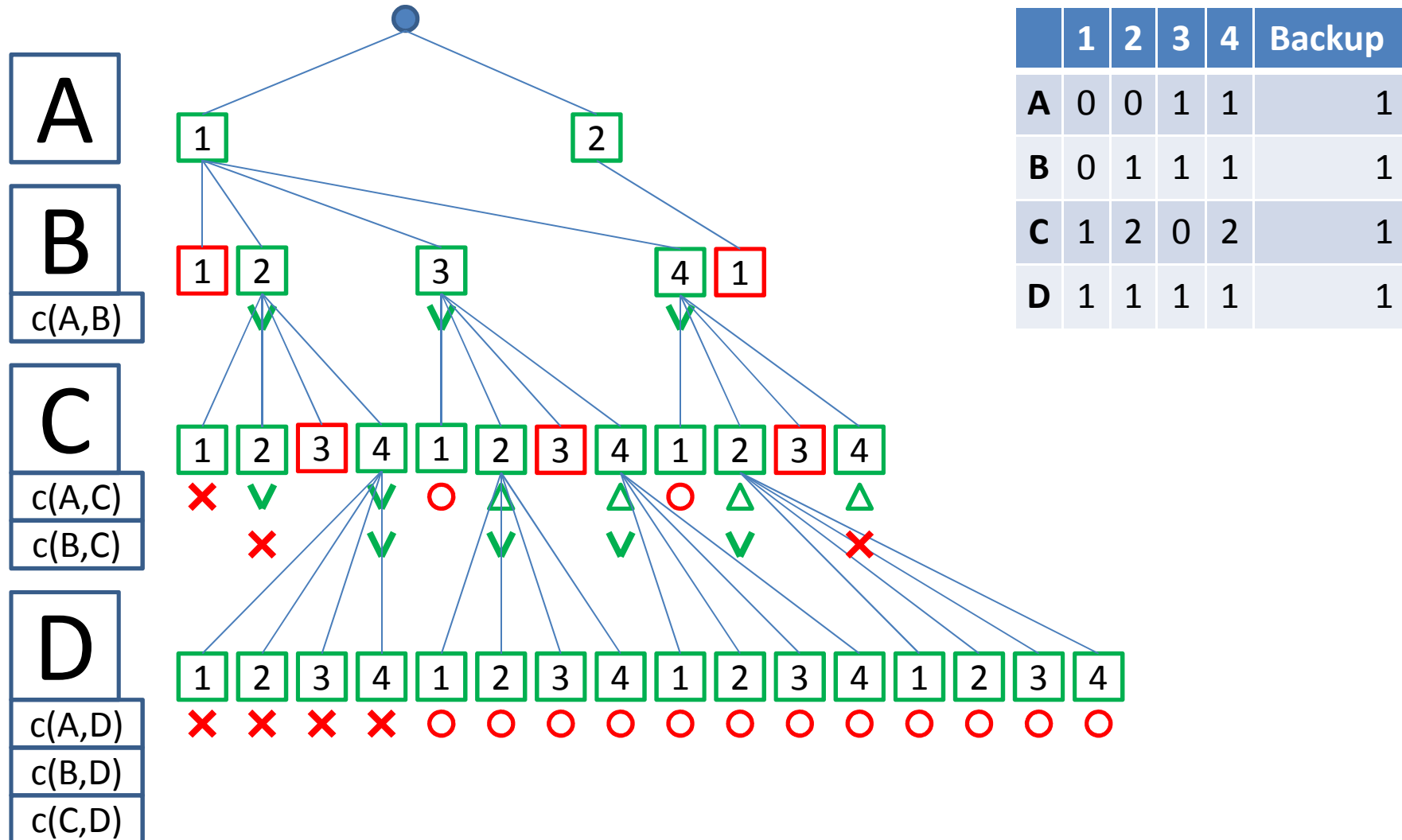
Constraint Processing: Backmarking



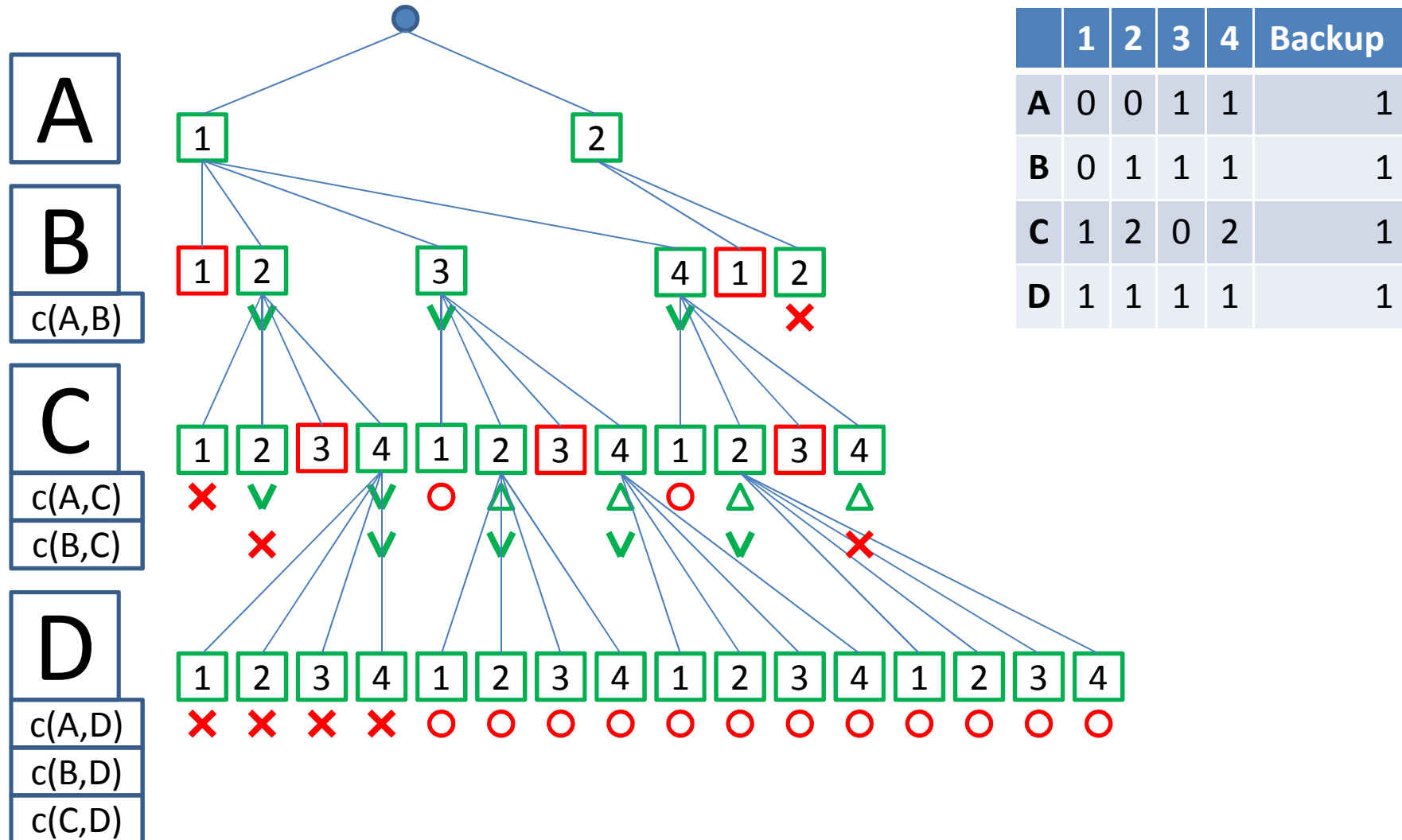
Constraint Processing: Backmarking



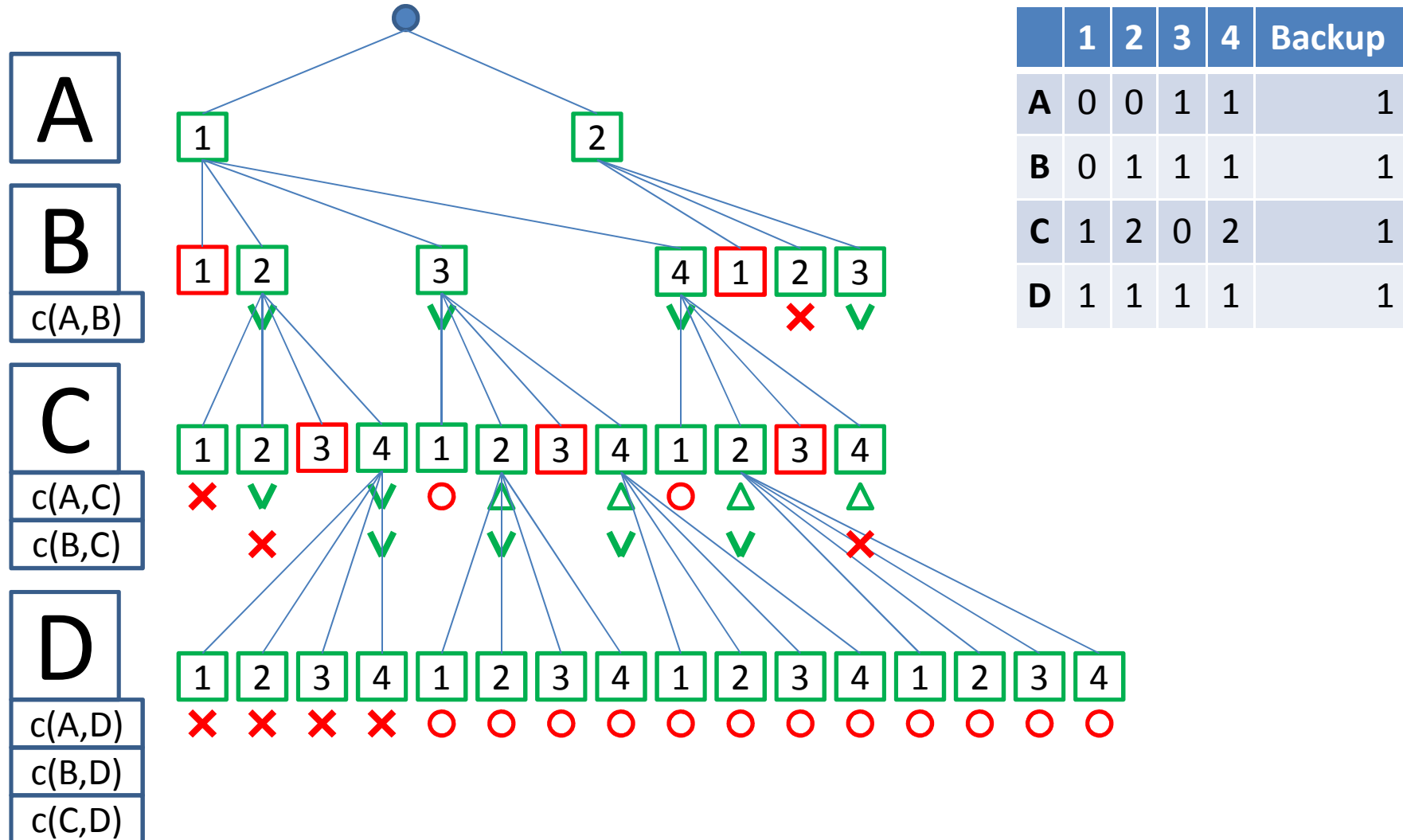
Constraint Processing: Backmarking



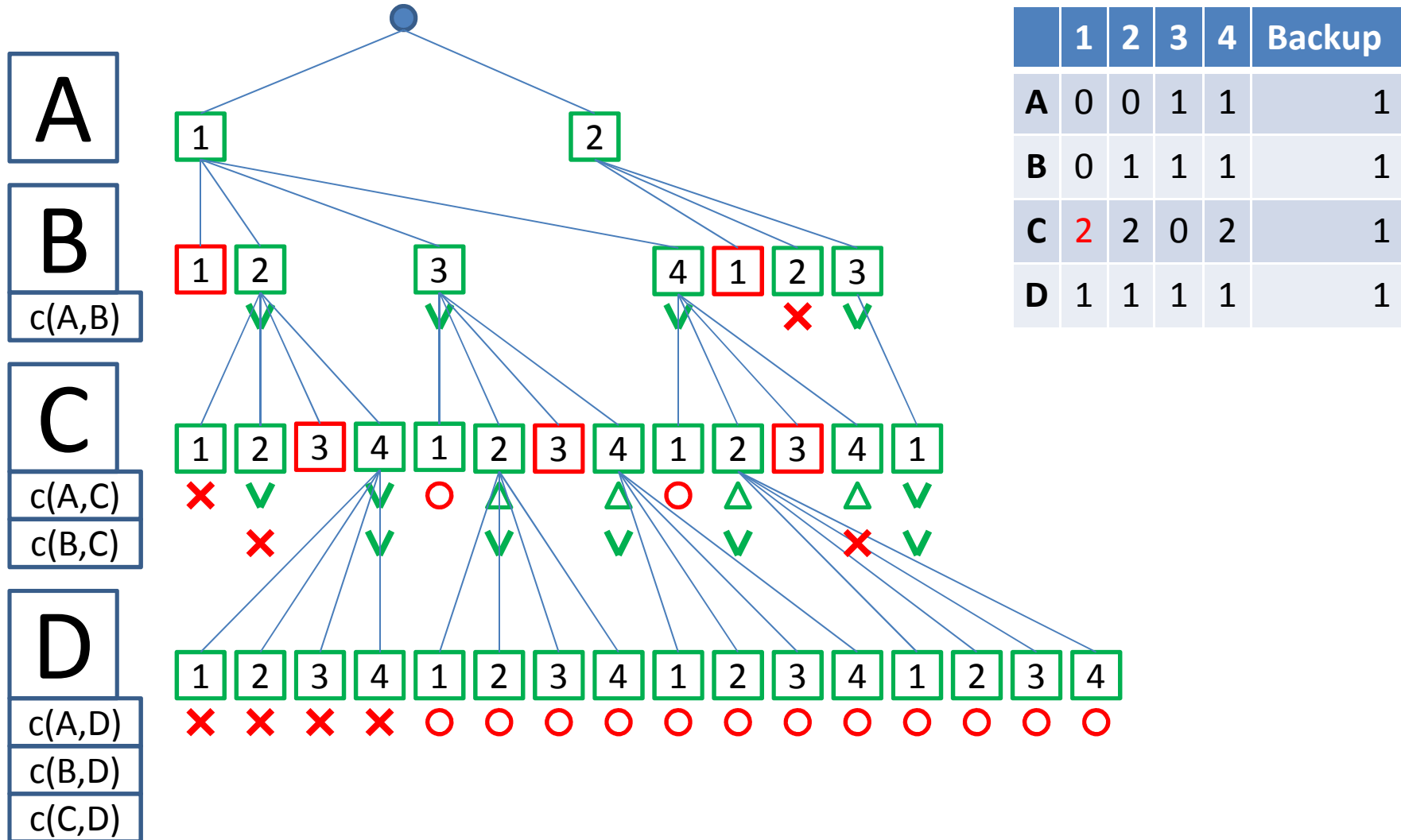
Constraint Processing: Backmarking



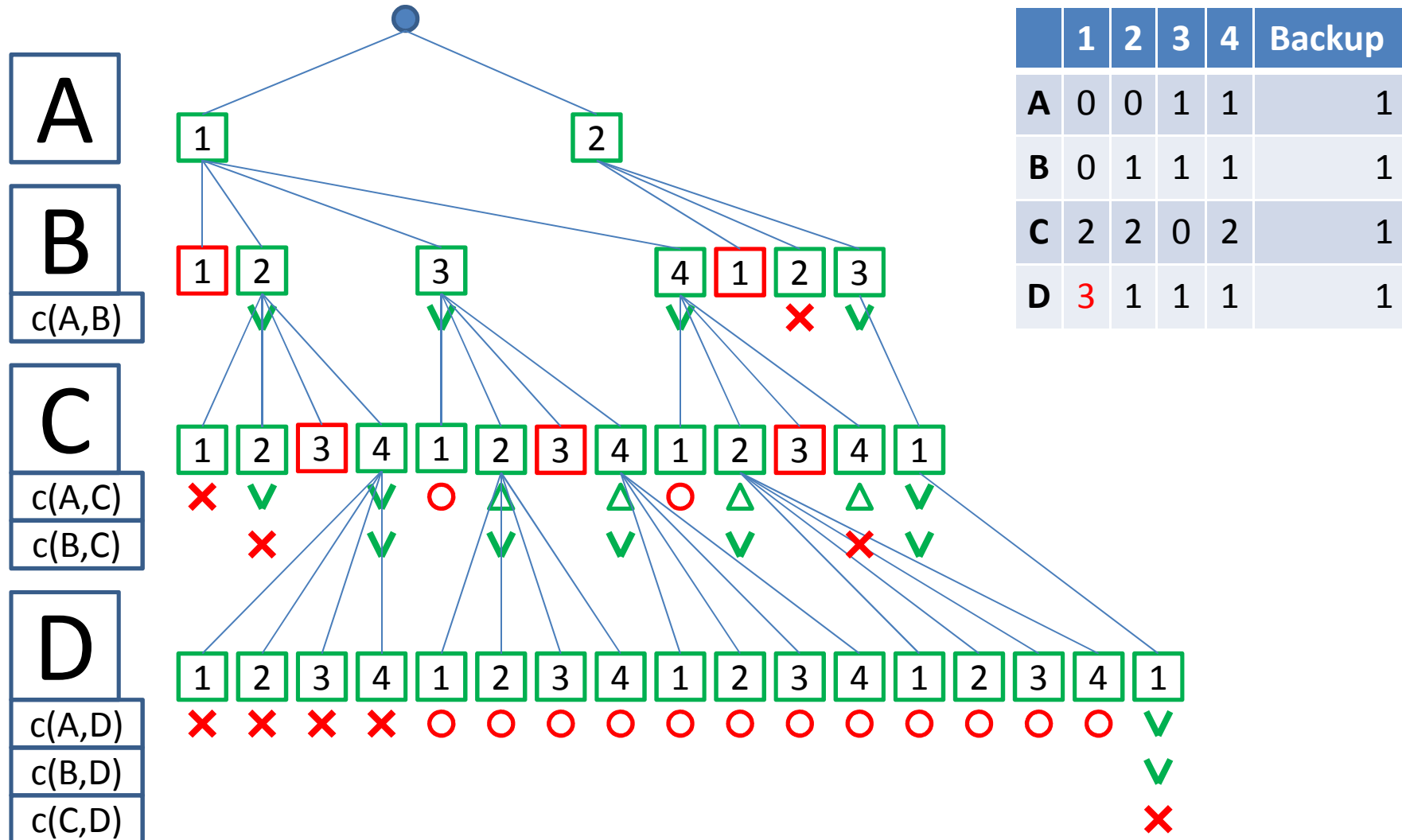
Constraint Processing: Backmarking



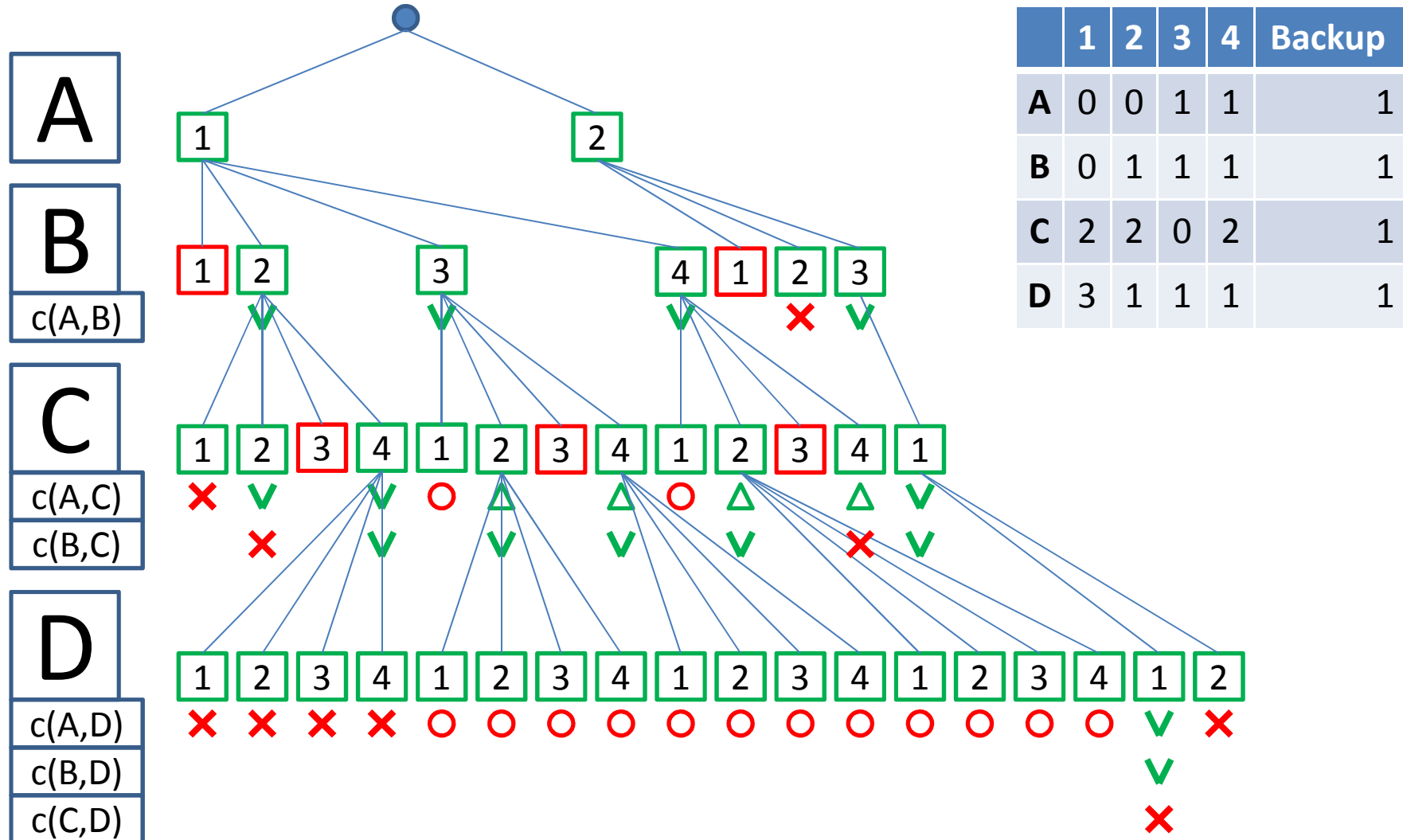
Constraint Processing: Backmarking



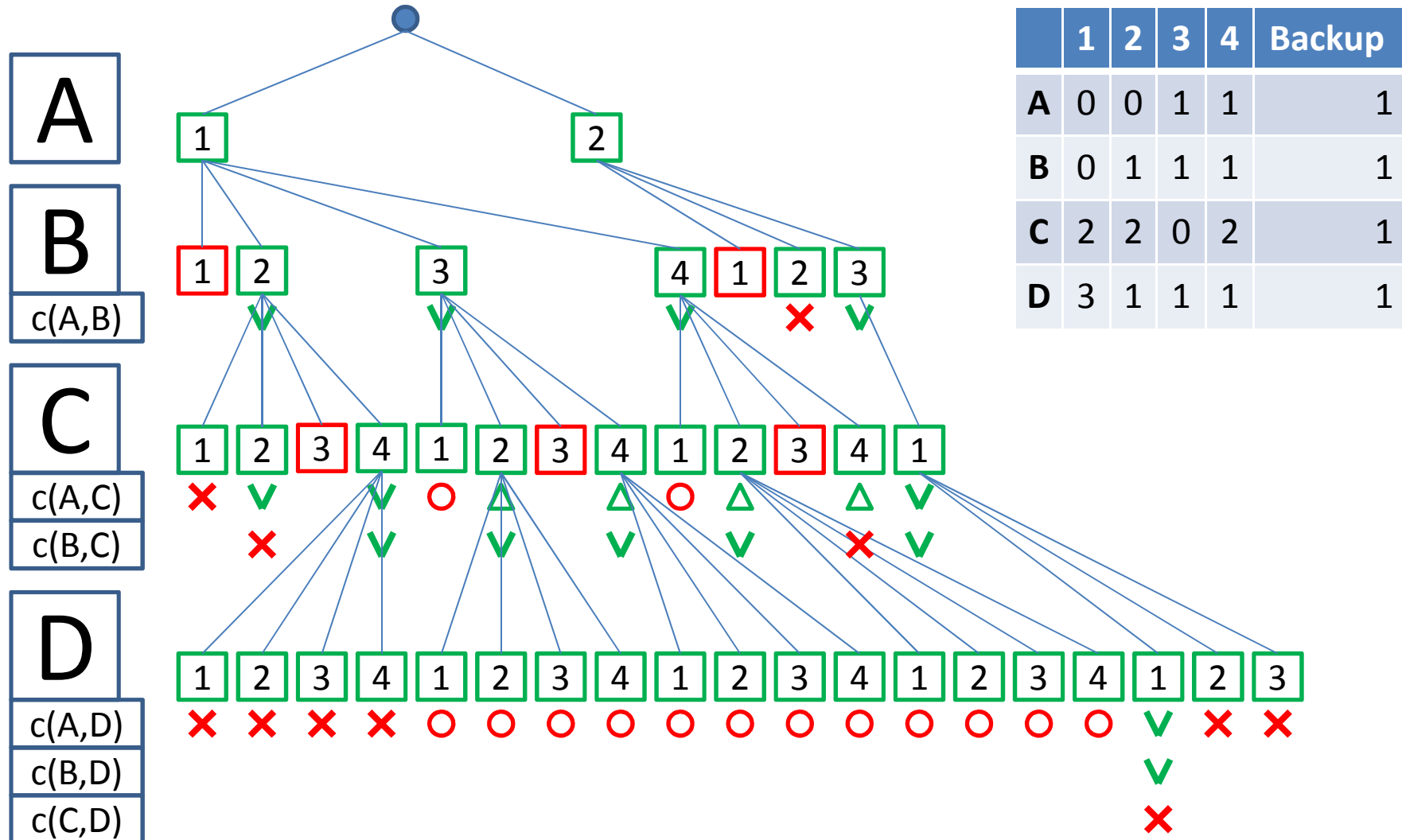
Constraint Processing: Backmarking



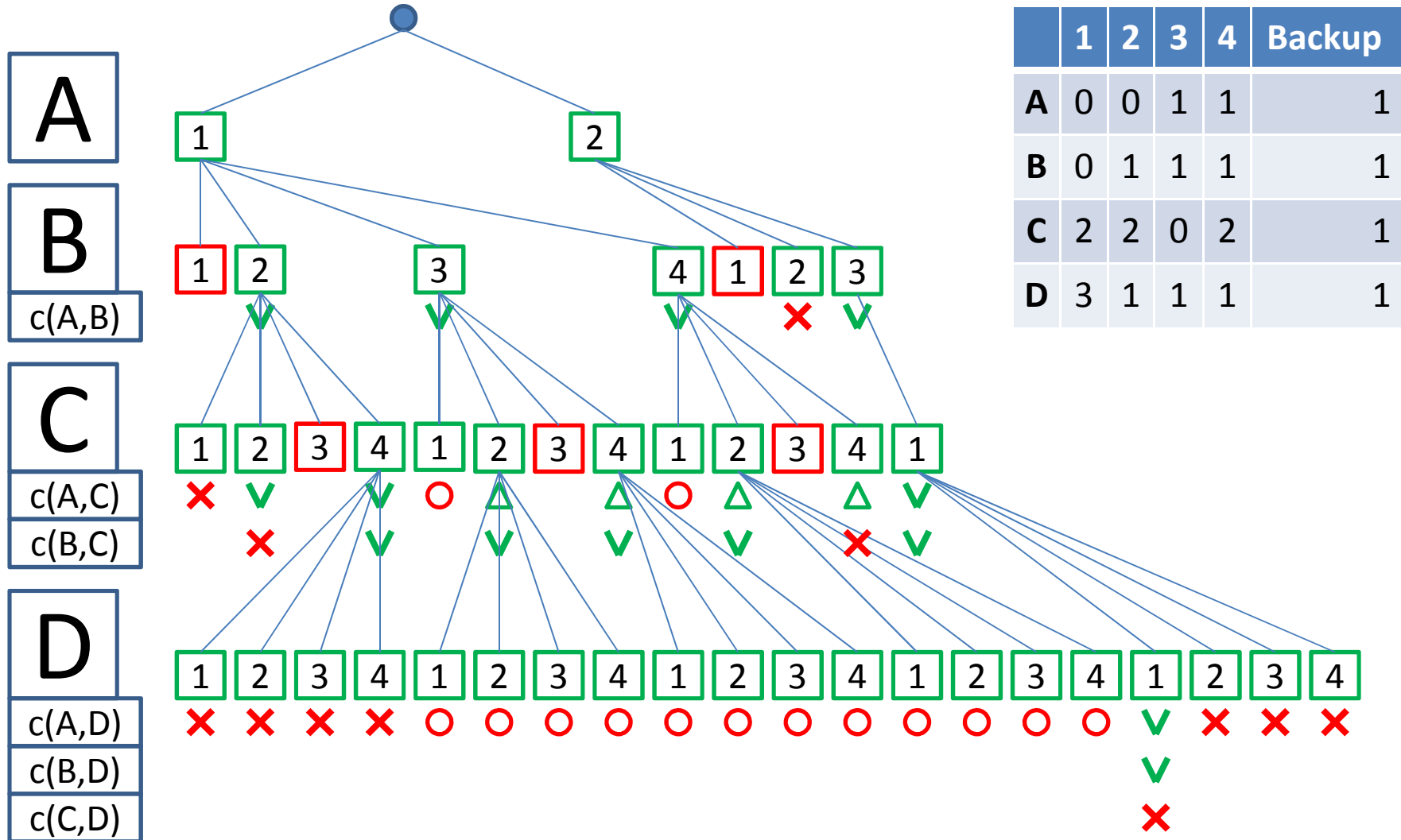
Constraint Processing: Backmarking



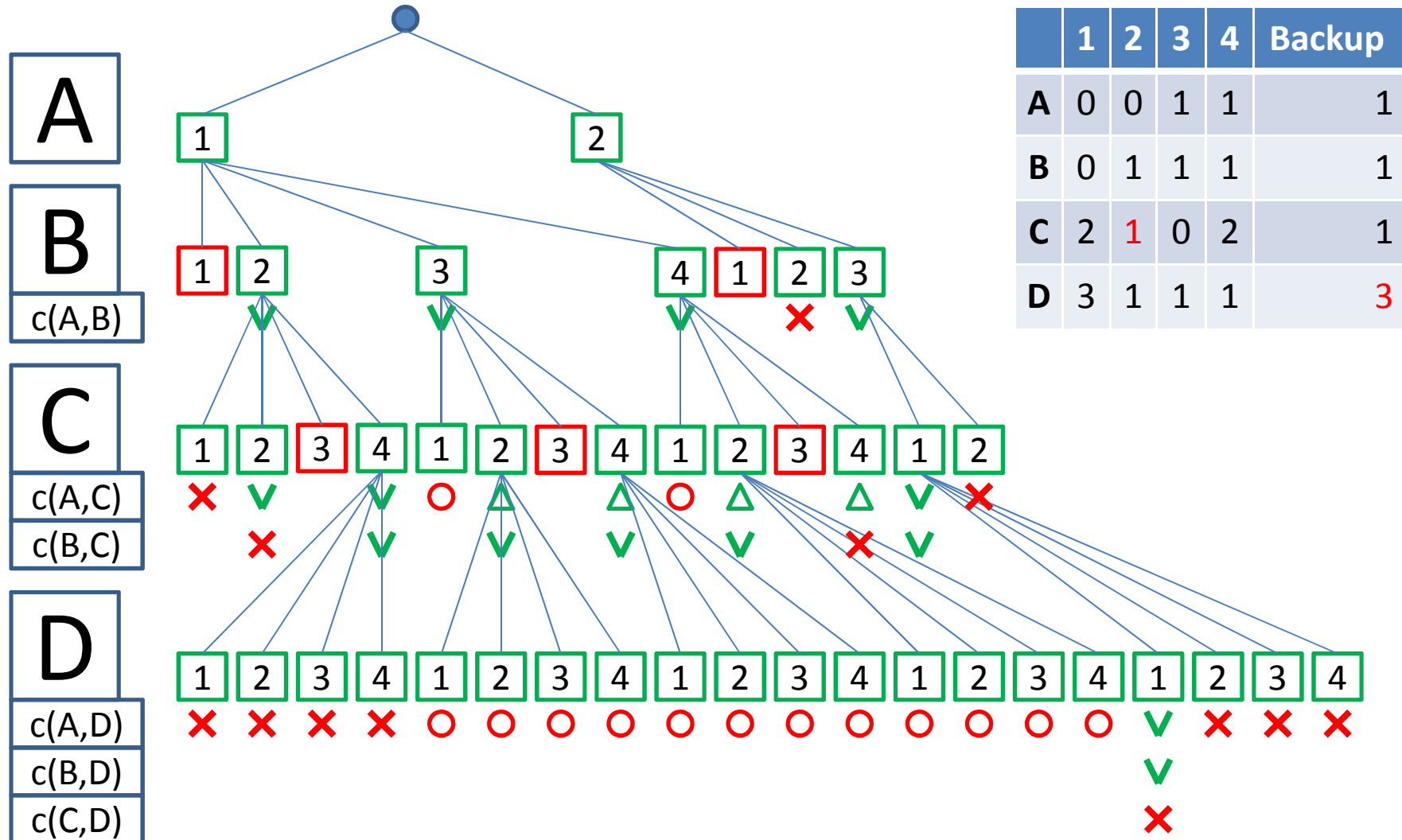
Constraint Processing: Backmarking



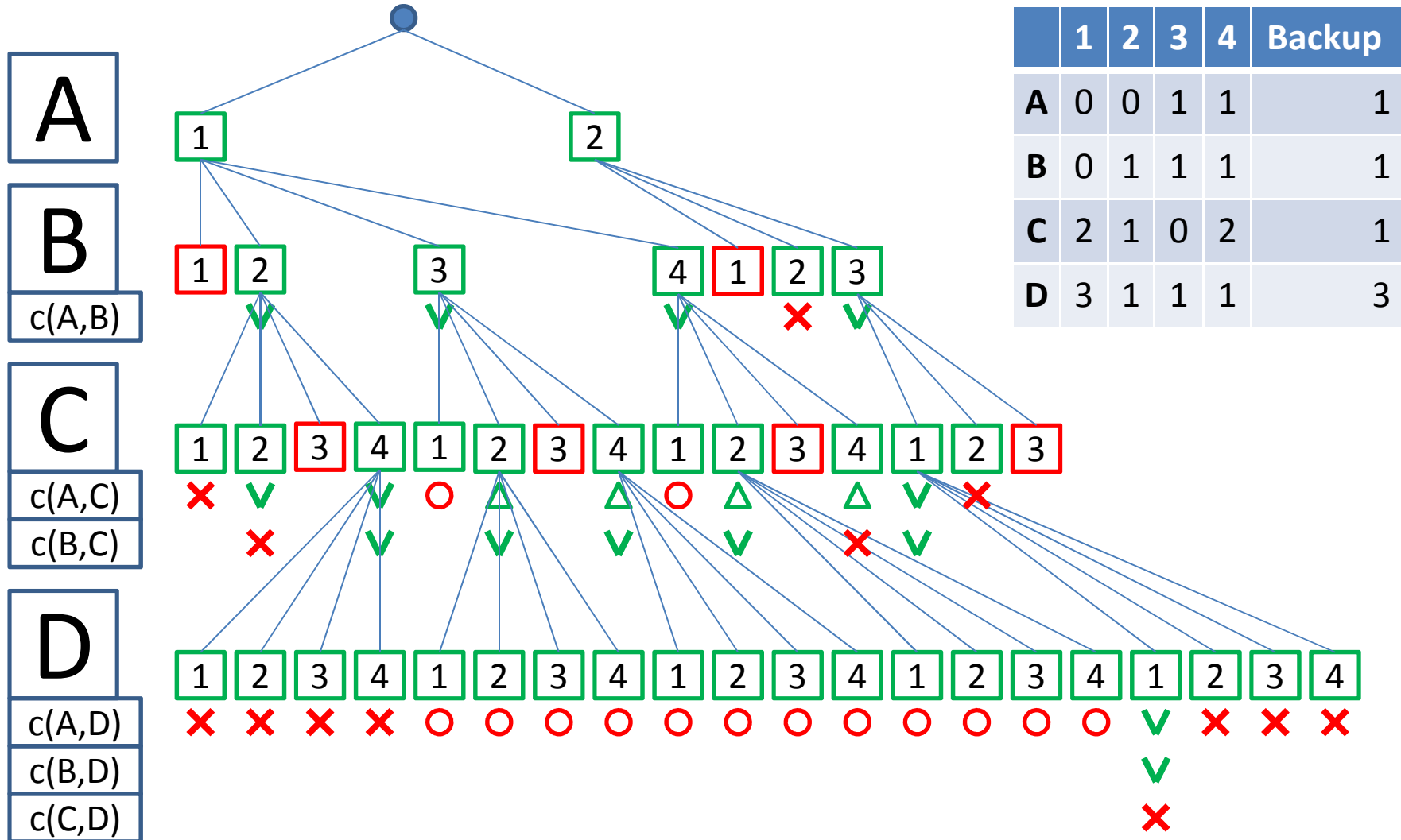
Constraint Processing: Backmarking



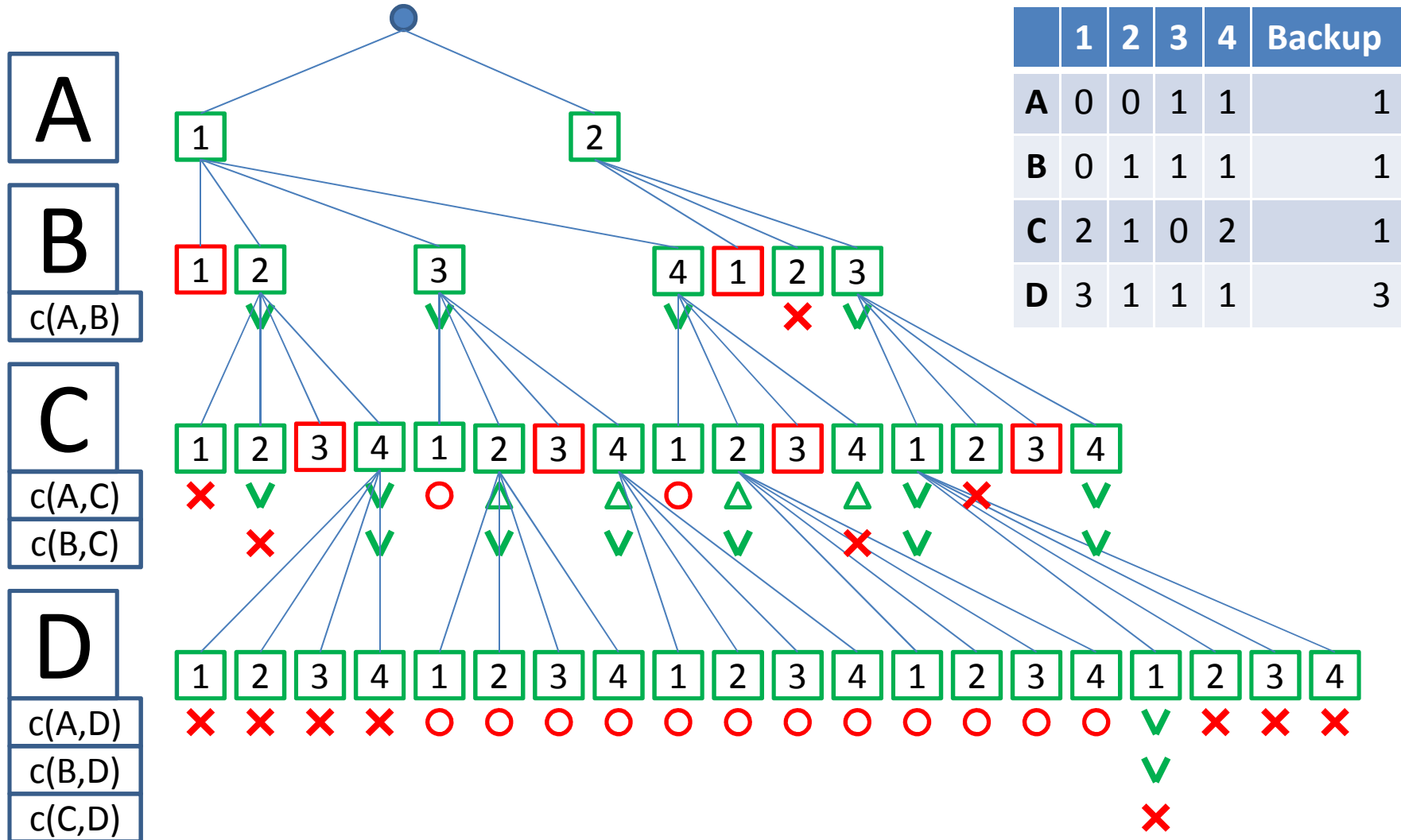
Constraint Processing: Backmarking



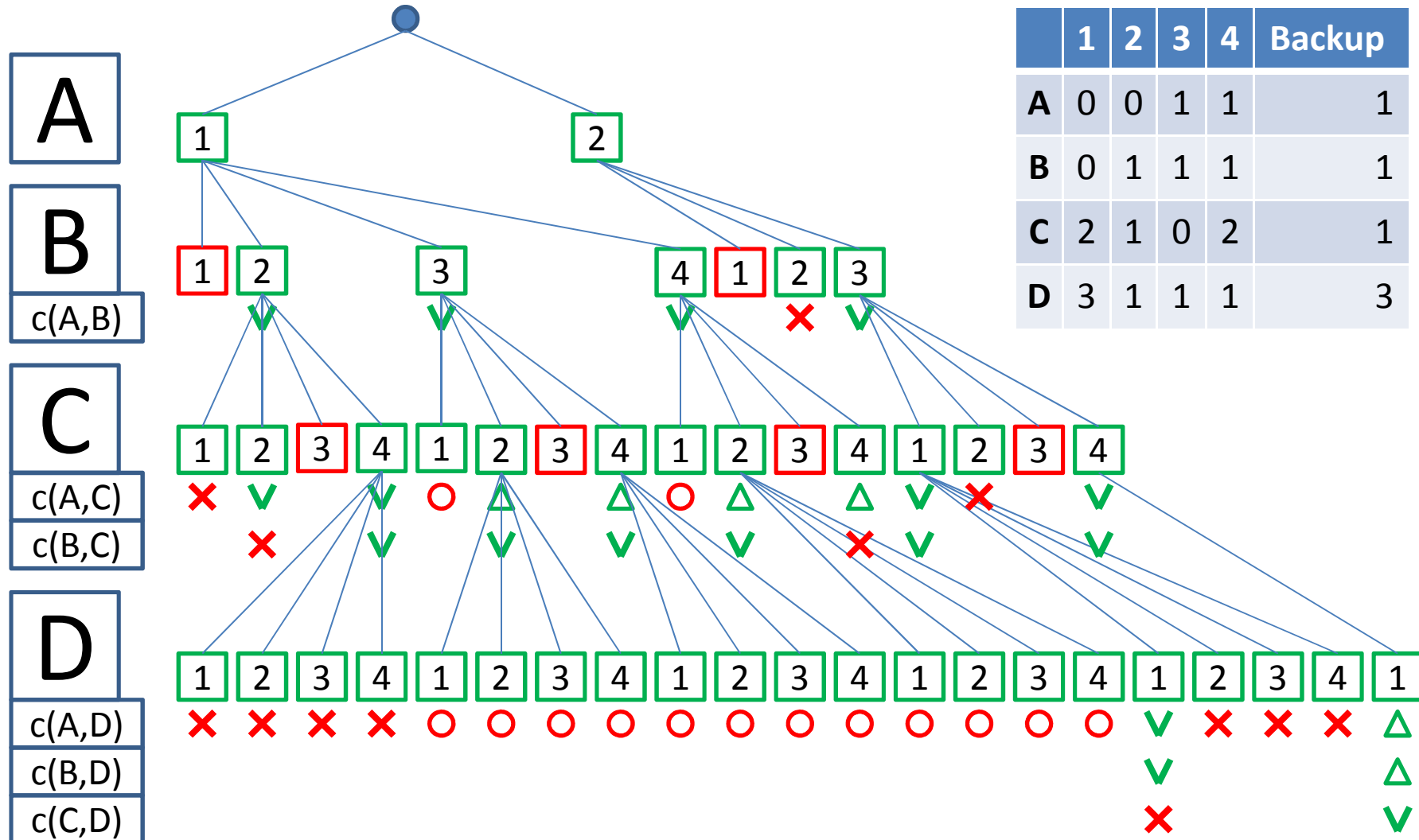
Constraint Processing: Backmarking



Constraint Processing: Backmarking



Constraint Processing: Backmarking



MiniMax & Constraint Processing: The 4 Houses problem

CONSTRAINT PROCESSING: NO- GOODS

Constraint Processing: No-goods

- $\{A=1\}$: No-good
 - No value for D such that $A = D + 1$
- $\{A=2, B=2\}$: No-good
 - A and B should have different houses
- $\{A=2, B=3\}$: Not a no-good: $\{A=2, B=3, C=4, D=1\}$
- $\{A=2, B=3, C=1\}$: No-good
 - $A = D + 1$, thus $D = 1$, but $C = 1$
- $\{A=2, B=4\}$: No-good
 - $A = D + 1$, thus $D = 1$, thus $C = 3$, but C cannot be 3

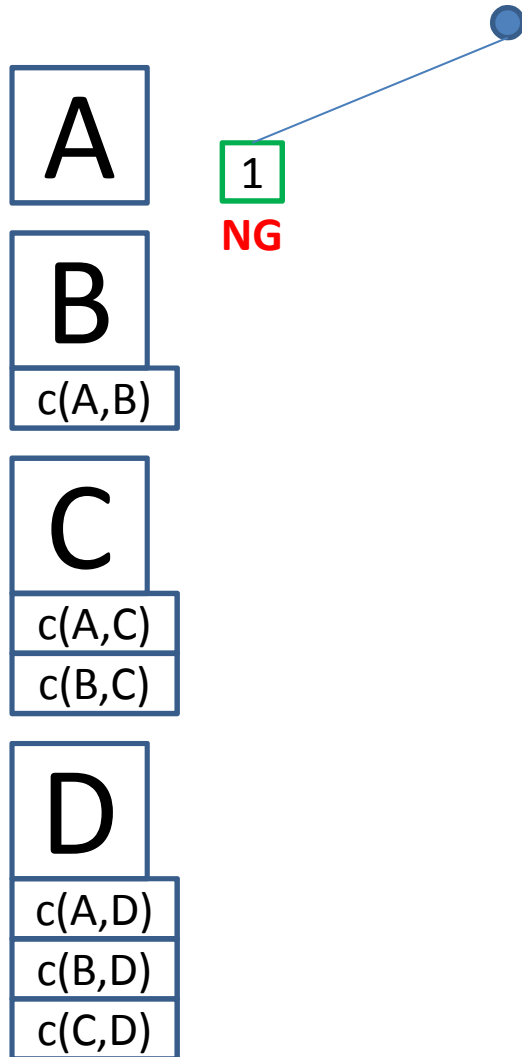
MiniMax & Constraint Processing: The 4 Houses problem

PROBLEM

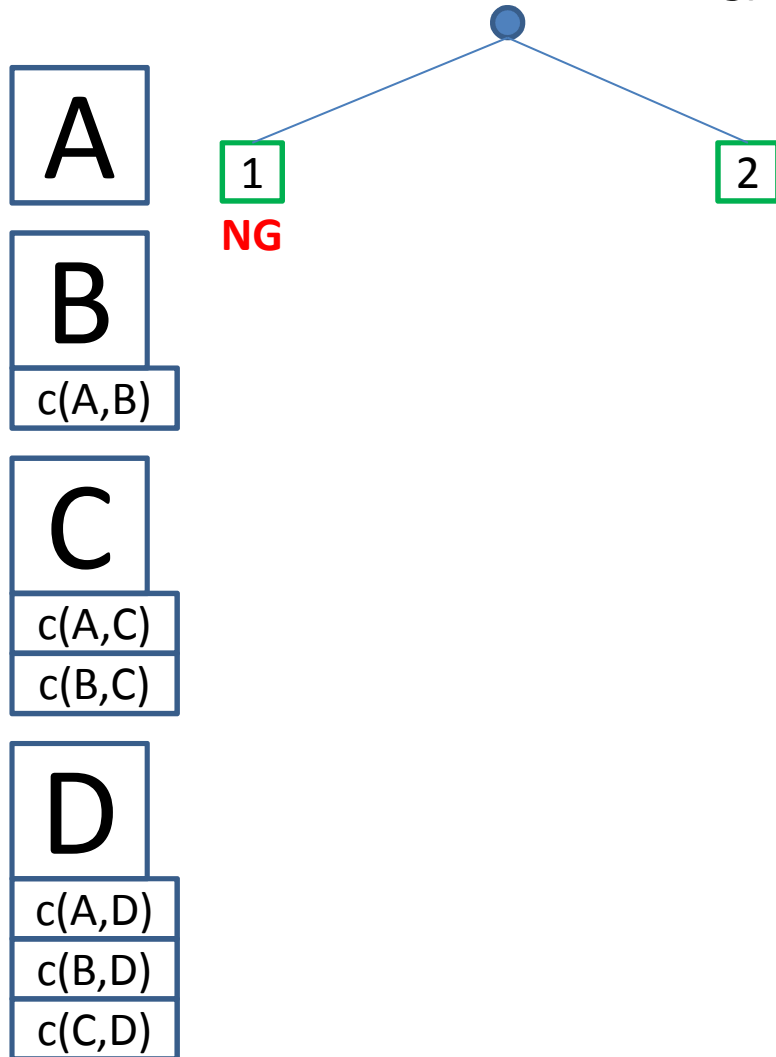
Problem

- Intelligent backtracking:
 - Obtained No-goods:
 - $\{A=1\}, \{A=2, B=2\}, \{A=2, B=3, C=1\}, \{A=2, B=4\}$
 - And the no-goods:
 - $\{A=3, B=2\}, \{A=3, B=4\}, \{A=4, B=2\}, \{A=4, B=3\}, \{A=4, C=2\}$
- Apply standard backtracking
 - Depth-first and left-to-right
 - Don't visit nodes containing no-goods

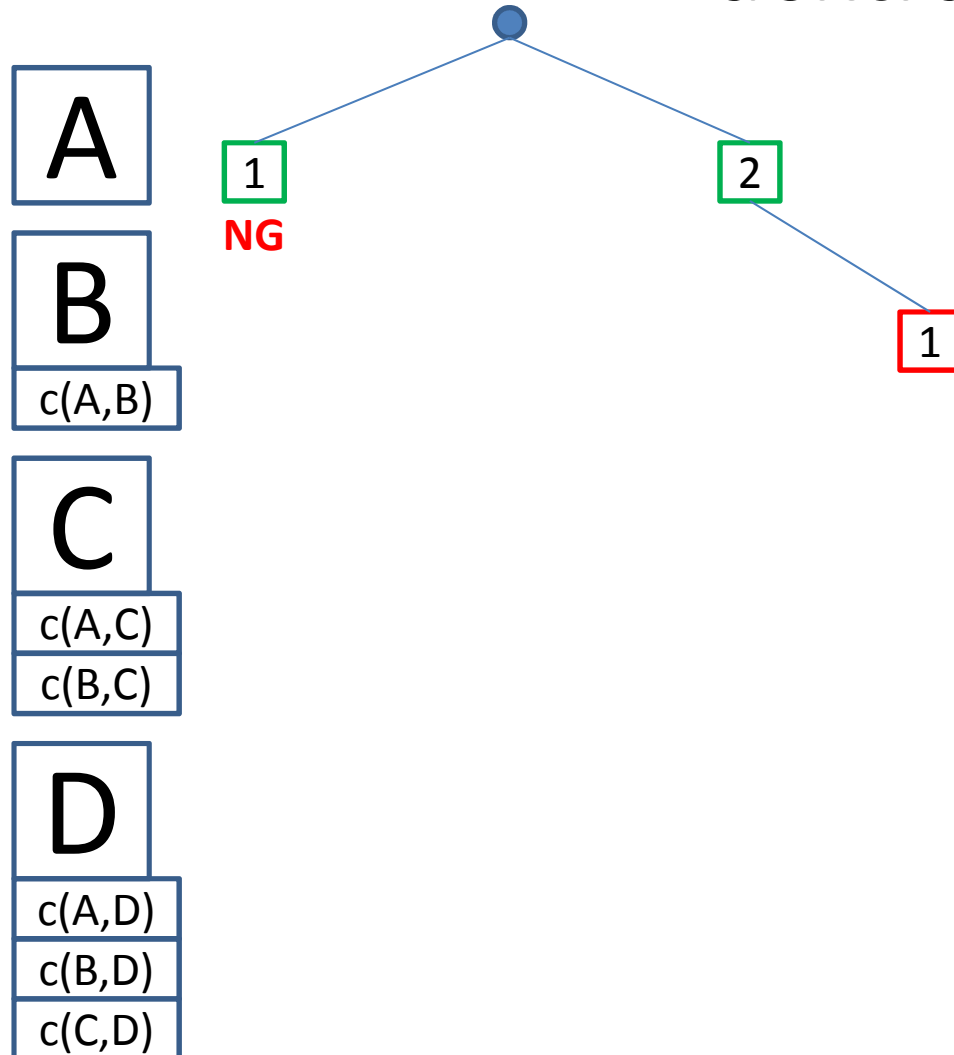
Constraint Processing: Intelligent Backtracking



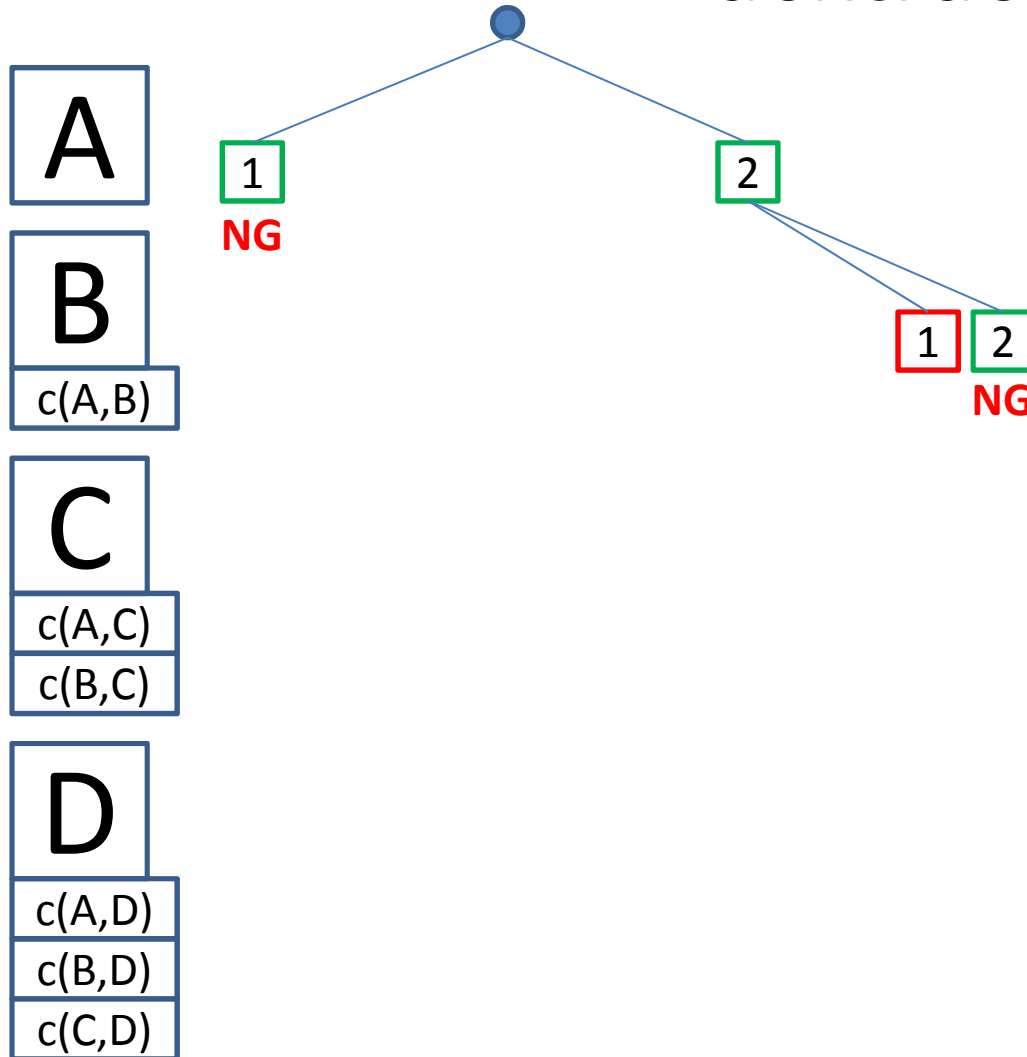
Constraint Processing: Intelligent Backtracking



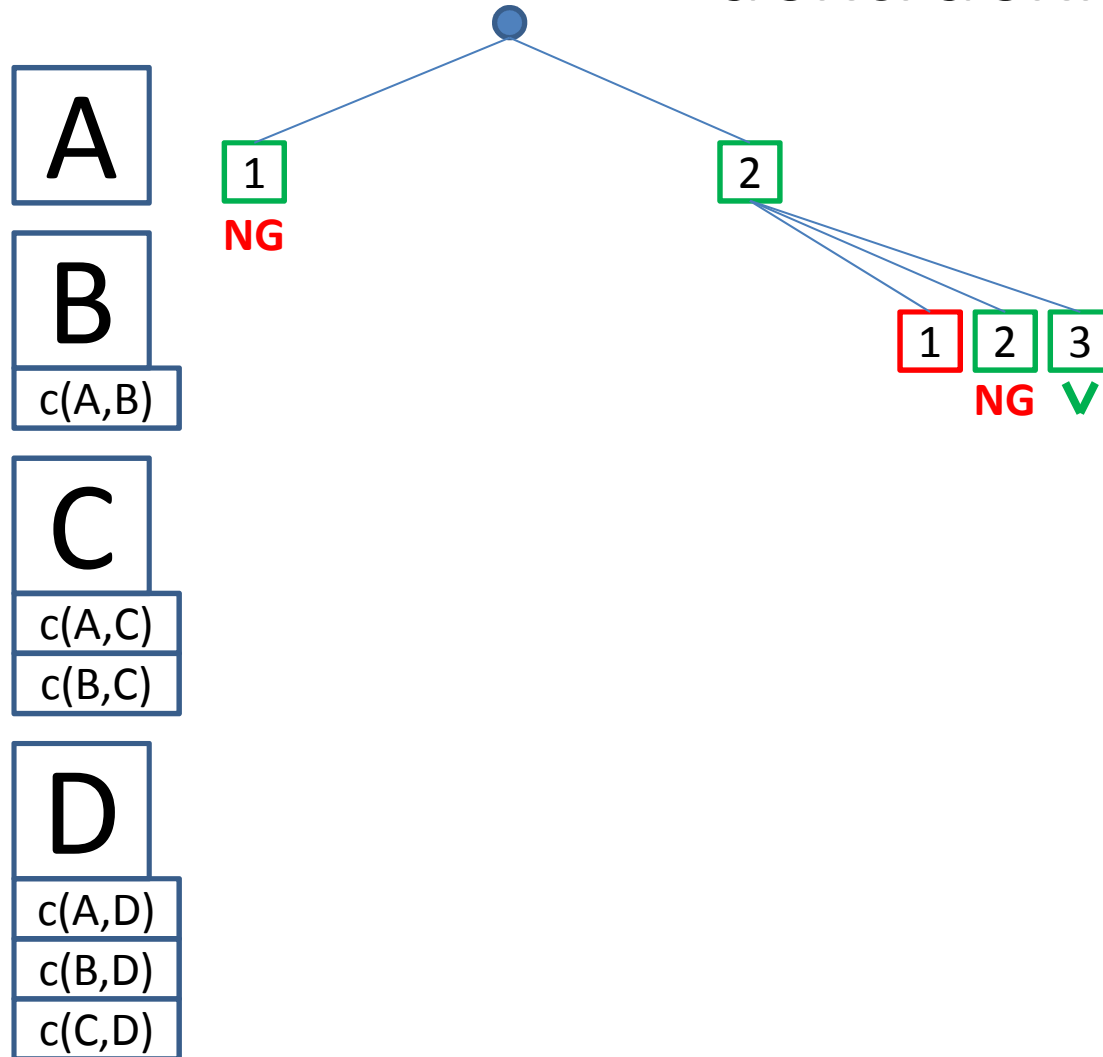
Constraint Processing: Intelligent Backtracking



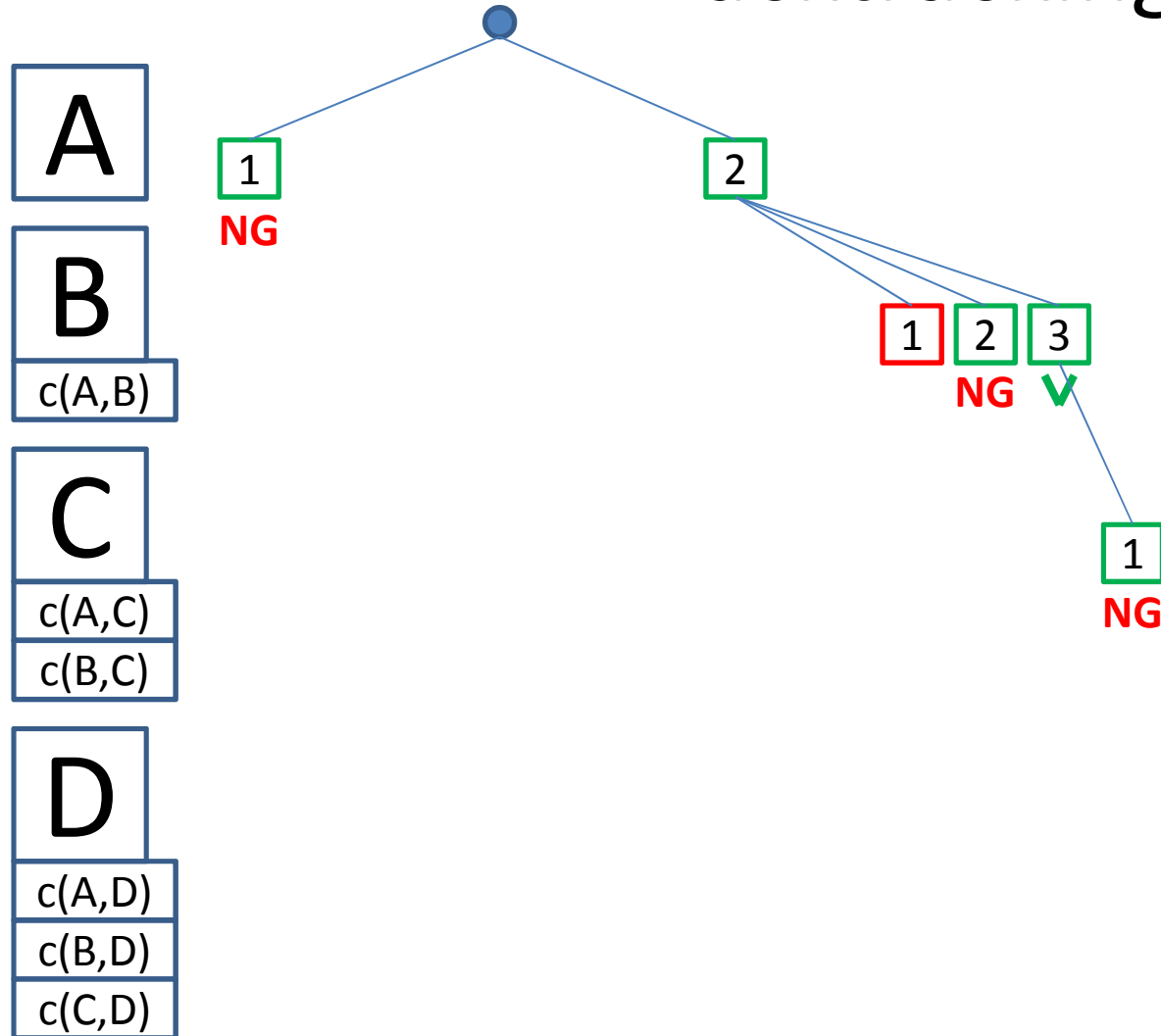
Constraint Processing: Intelligent Backtracking



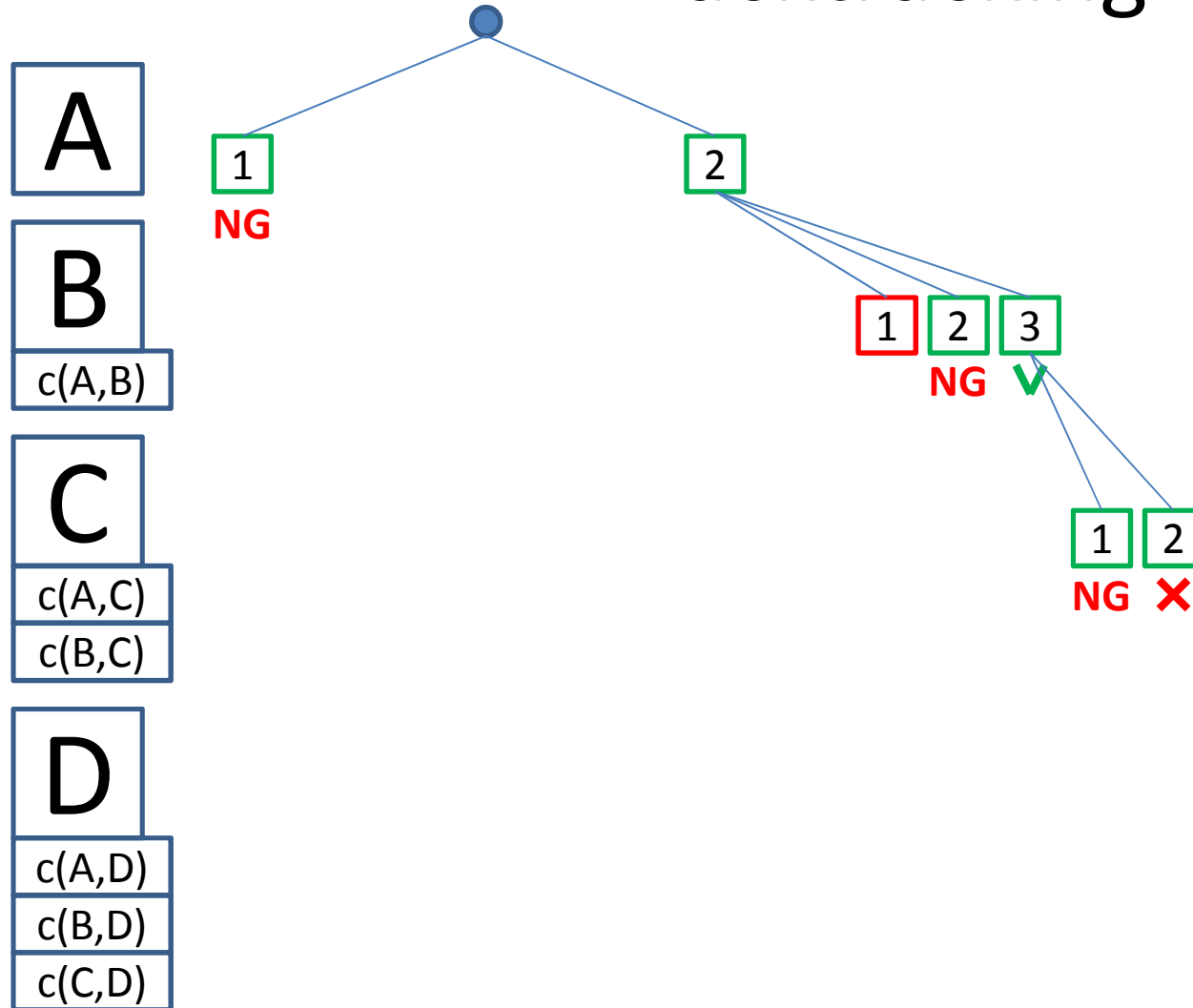
Constraint Processing: Intelligent Backtracking



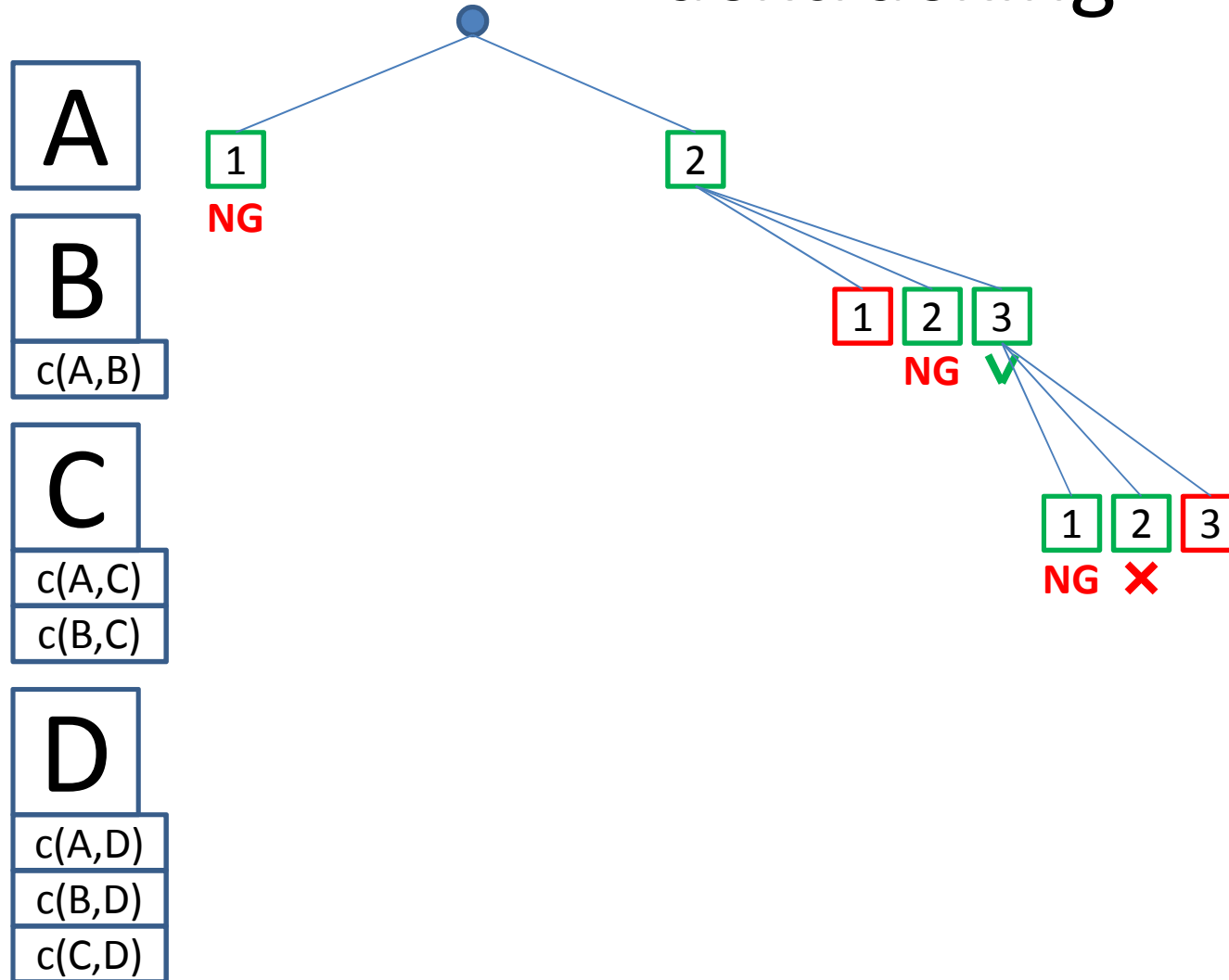
Constraint Processing: Intelligent Backtracking



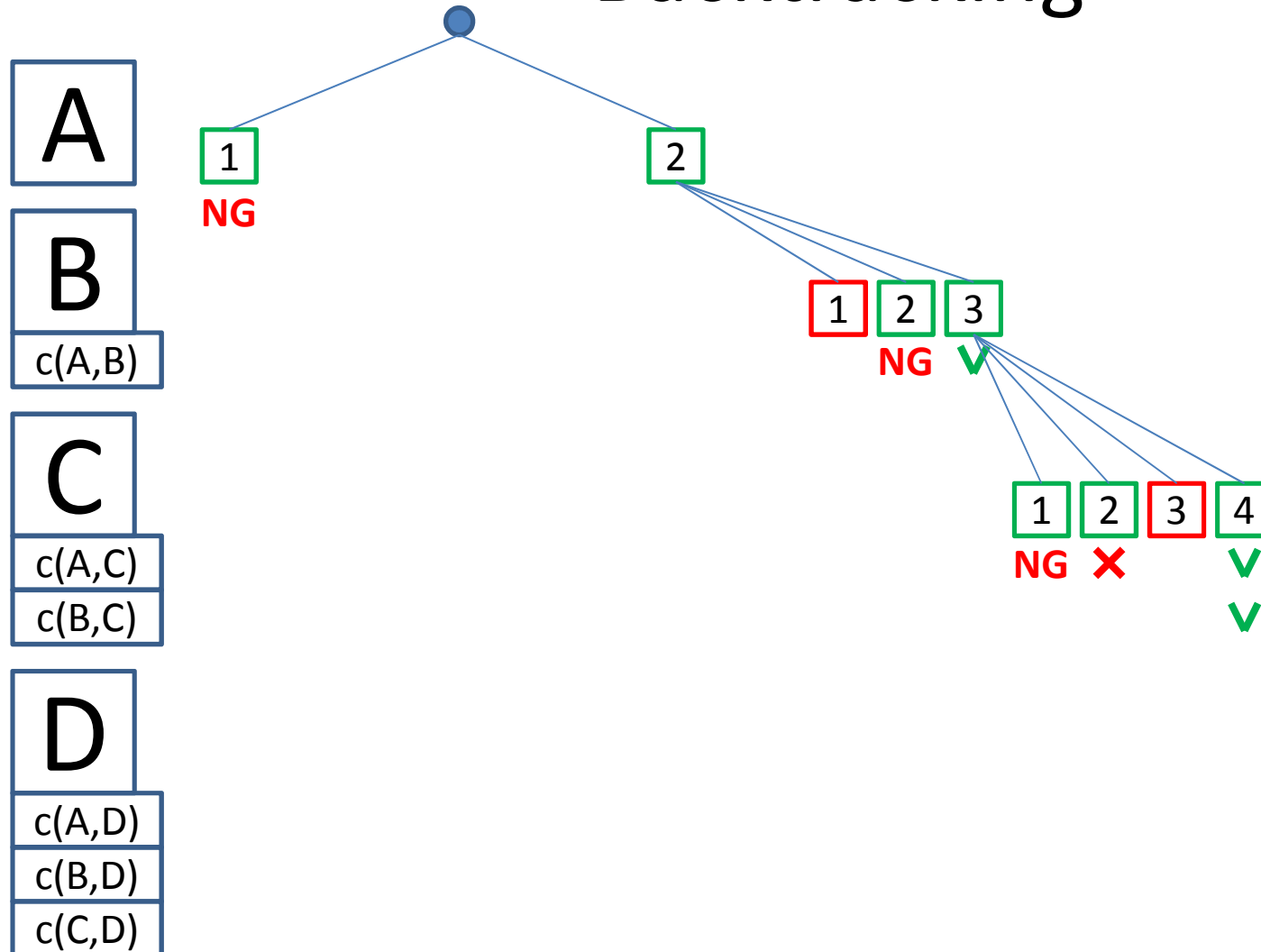
Constraint Processing: Intelligent Backtracking



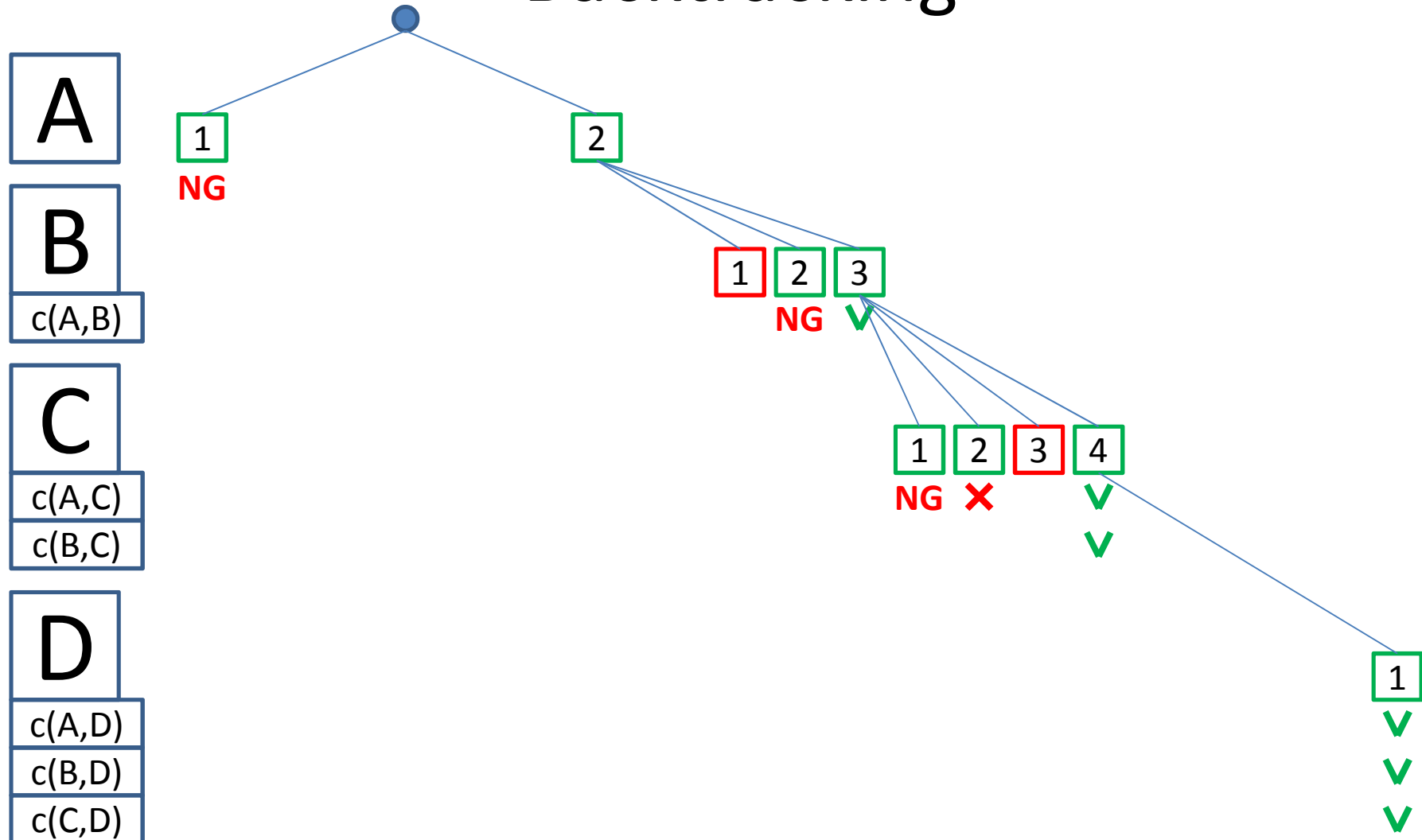
Constraint Processing: Intelligent Backtracking



Constraint Processing: Intelligent Backtracking



Constraint Processing: Intelligent Backtracking



MiniMax & Constraint Processing: The 4 Houses problem

EFFICIENCY

Efficiency

<i>All (One solution)</i>	Opened Nodes	Checks
Standard Backtracking	28 (13)	142 (56)
Backjumping	21 (8)	93 (30)
Backmarking	28 (13)	79 (34)
Intelligent Backtracking	6 (4)	16 (9) + NG