Adaptive Numerical CDF for Importance Sampling in BRDF

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Abstract

- Goal:
  - Importance sampling using the measured brdf
- Problem:
  - Storage cost
- Solution:
  - Adaptive numerical CDF
  - Curve Approximation algorithm

Scattering Eq. and Monte Carlo

Scattering Equation:

$$L_o(p,\omega_o) = \int_{S^2} f(p,\omega_o,\omega_i)L_i(p,\omega_i)|cos\theta_i|d\omega_i$$

Monte Carlo Estimator:

$$\int_{x_0}^{x_1}\int_{y_0}^{y_1}\int_{z_0}^{z_1} f(x,y,z)dxdydz \Rightarrow \frac{(x_1-x_0)(y_1-y_0)(z_1-z_0)}{N}\sum_i f(X_i)$$

Measured BRDF

Merl Database (Matusik et al)

Mapping from half-angle and difference vectors to outgoing and half-angle:

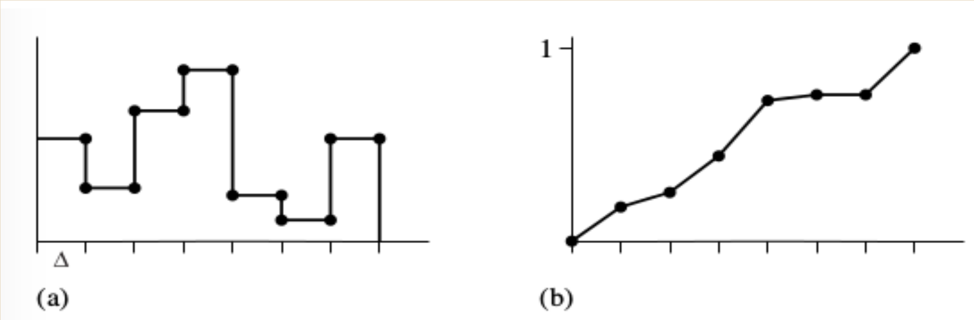
$$(\sqrt{\theta_h}, \theta_d, \phi_d) \rightarrow (\sqrt{\theta_o}, \phi_o, \theta_h, \phi_h)$$

- 90 x 90 x 180 -> 32 x 16 x 256 x 32

Inversion Method in 1D Piecewise-constant functions

Inversion Method:

1. Compute CDF : P(x)



$$f(x)/funcInt$$

2. Compute inverse : $P^{-1}(x)$
3. Compute $X_i = P^{-1}(\square)$

Inversion Method in 2D Piecewise-constant functions

PDF

$$f(u,v)/funcInt$$

Marginal

$$p(v) = \int p(u,v)du = \frac{(1/n_u)\sum_i f[u_i,\tilde{v}]}{funcInt}$$

Conditional

$$p(u|v) = \frac{p(u,v)}{p(v)} = \frac{f[\tilde{u},\tilde{v}]/funcInt}{p[\tilde{v}]}$$

Transforming between distributions

Half-angle to incident vector

$$\frac{d\omega_h}{d\omega_i} = \frac{sin\theta_h d\theta_h d\phi_h}{sin\theta_i d\theta_i d\phi_i}$$

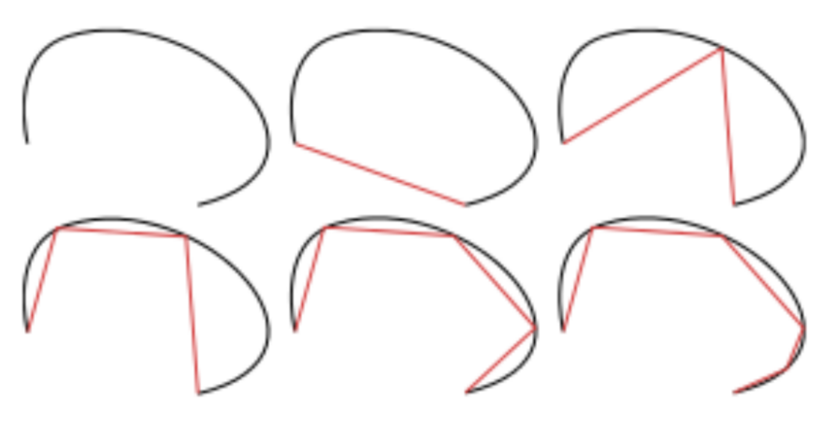
$$\Rightarrow$$

$$\frac{d\omega_h}{d\omega_i} = \frac{1}{4(\omega_o \cdot \omega_h)}$$

$$p(\theta, \phi) = \frac{p_h(\theta, \phi)}{4(\omega_o \cdot \omega_h)}$$

CDF Compression

Douglas-Peucker



CDF

$$p(u|v_i) = \frac{1}{|v_i|} = \int_{v_{i-1}}^{v_i} \frac{p(u, v')}{p(v')}$$

Final Costs

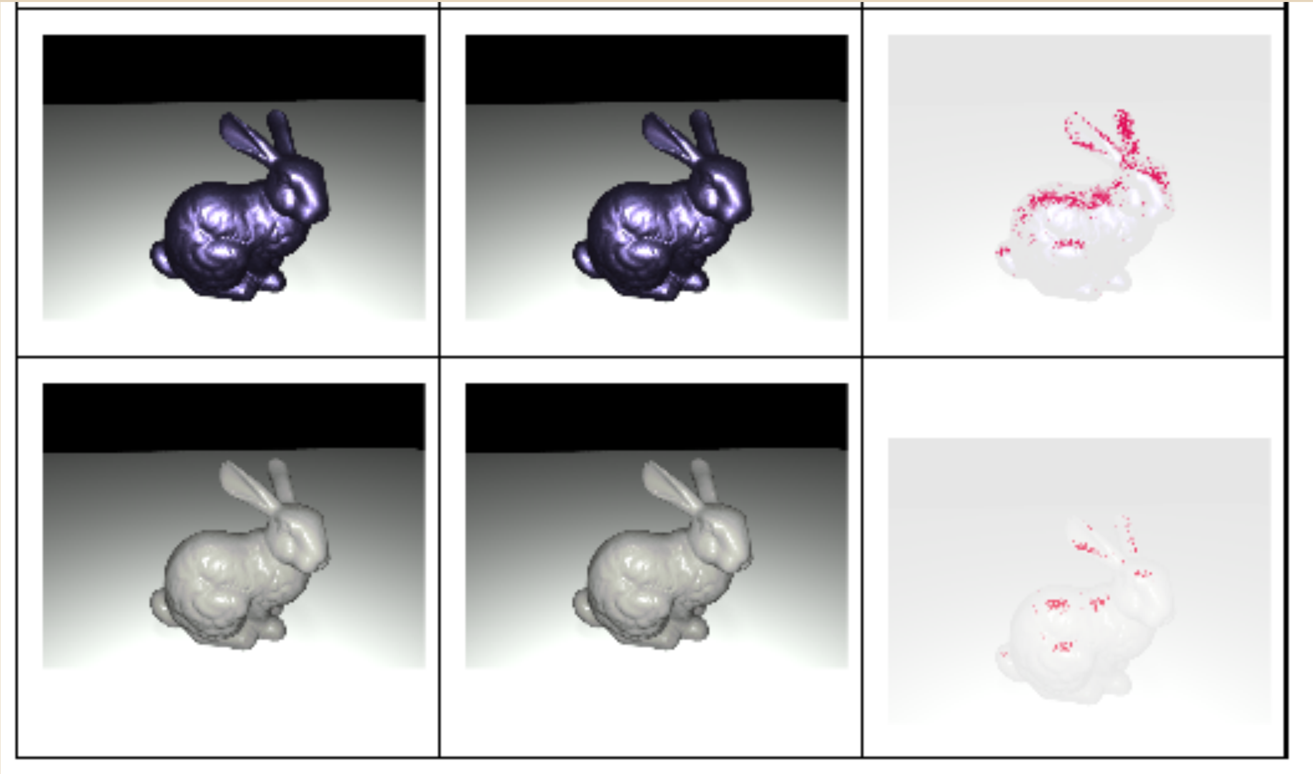
	Uniform	Adaptive
Sample_f	$O(\lg N_U)$	$O(\lg N_A)$
Pdf	$O(1)$	$O(\lg N_A)$

$N_U > N_A$

Results

BRDF	Uniform CDF Resolution	Adaptive CDF Average Resolution	Compression Ratio	Uniform CDF Estimator Efficiency	Adaptive CDF Estimator Efficiency
Measured Nickel	32 x 16 x 256 x 32	32 x 16 x 30 x 10	27:1	8.0415e-06	7.92091e-06
Measured Metallic-Blue	32 x 16 x 256 x 32	32 x 16 x 30 x 12	23:1	5.21638e-06	4.36035e-06
Measured Plastic	32 x 16 x 256 x 32	32 x 16 x 30 x 9	30:1	2.69248e-06	2.7647e-06

Resulting Images



A bit more interesting results...

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Conclusions and future work

- Reduction in storage cost
- Higher quality than Cosine-weighted sampling

- Outgoing angle also adaptive
- Environment maps

Bibliography

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Questions?