

Relative Size of Numbers

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1) TERMS AND BASICS

Sign	Meaning
$X > Y$	X is greater than Y
$X < Y$	X is less than Y
$X = Y$	X is equal to Y
$X \leq Y$	X is less than or equal to Y
$X \geq Y$	X is greater than or equal to Y

• Basic rules of comparison

- *+ve number $>$ -ve number*
- *If $a > b$ and $b > c$ then $a > c$*
- *If $a > b$ then $a + c > b + c$*
- *If $a > b$ then $a - c > b - c$*
- *If $a > b$ and $c > 0$ then $ac > bc$*
- *If $a > b$ and $c < 0$ then $ac < bc$*
- *If $a > b$ and $c > d$ then $a + c > b + d$;*
- *If $a > b$ then $a^n > b^n$ if $n > 0$*
- *If $a > b$ then $\frac{1}{a} < \frac{1}{b}$*
- *If $a > b, \sqrt{a} > \sqrt{b}$, for $a, b \geq 0$*
- *$a^2 > 0$ if $a \neq 0$*
- *If $a, b > 0$ and $a < x < y$ then, $a^2 < x^2 < b^2$*
- *If $a, b < 0$ and $b < x < y$ then, $a^2 < x^2 < b^2$*
- *If $a > 0$ and $-a < x < a$ then, $0 < x^2 < a^2$*
- *If $a, b > 0, b > a$ and $-a < x < b$ then, $0 < x^2 < b^2$*

Tip: Whenever you have any confusion, just take any sample example and verify the inequality

2) COMPARISON OF NATURAL NUMBERS

CSE 2020:

Let p, q, r and s be natural numbers such that

$$P - 2016 = q + 2017 = r - 2018 = s + 2019$$

Which one of the following is the largest natural number?

- (a) P
- (b) q
- (c) r
- (d) s

CSE 2017: The sum of income of A and B is more than that of C and D taken together. The sum of income of A and C is the same as that of B and D taken together. Moreover, A earns half as much as the sum of the income of B and D. Whose income is the highest?

- (a) A
- (b) B
- (c) C
- (d) D

CSE 2017: If A runs less fast than B, and B runs as fast but not faster than C; then, as compared to A, C runs.

- (a) slower than A
- (b) faster than A
- (c) with same speed as A
- (d) Given data is not sufficient to determine

3) COMPARISON OF INTEGERS

Q. If X is between -3 and -1, and Y is between -1 and 1, then $X^2 - Y^2$ is in between which of the following?

- (a) -9 and 1
- (b) -9 and -1
- (c) 0 and 8
- (d) 0 and 9

4) COMPARISON OF TWO FRACTIONS

Case 1) When denominator is same

- When two fractions have same denominator, the one with higher numerator is greater.

Ex: Consider: $\frac{3}{7}$ and $\frac{4}{7}$ – they've same denominator and thus, $\frac{4}{7} > \frac{3}{7}$

Case 2) When numerator is same

- When two fractions have same numerator, the one with lower denominator is greater.

Ex: Consider $\frac{5}{7}$ and $\frac{5}{8}$ – they've same numerator and thus $\frac{5}{7} > \frac{5}{8}$

Case 3) When numerator and denominator is different

- In general, we cross multiply two ratios. The ratio whose numerator yields higher value when multiplied with the denominator of the other ratio, is greater.
- In other words, if $\frac{a}{b}$ and $\frac{c}{d}$ are ratios to be compared,
 - o If $a \times d > c \times b$ then $\frac{a}{b} > \frac{c}{d}$

$$\circ \text{ If } a \times d < c \times b \text{ then } \frac{a}{b} < \frac{c}{d}$$

For example: Consider $\frac{5}{8}$ and $\frac{4}{7}$: $5 \times 7(35) > 8 \times 4(32)$ and thus, $\frac{5}{8} > \frac{4}{7}$

Q. Decide the comparison between following fractions.

1) $\frac{4}{3}; \frac{123}{23}$

2) $\frac{4}{5}; \frac{2}{3}$

3) $\frac{4}{11}; \frac{12}{29}$

4) $\frac{7}{8}; \frac{9}{10}$

5) $\frac{13}{21}; \frac{15}{21}$

6) $\frac{34}{11}; \frac{34}{17}$

7) $-\frac{3}{5}; \frac{4}{77}$

8) $-\frac{8}{13}; -\frac{4}{9}$

NOTE:

- You can use case 3) to solve all problems even if it is of the type of case 1 or case 2.
- In case of $\frac{m}{m+1}$ and $\frac{n}{n+1}$; the fraction with larger numerator is larger
i.e., $\frac{4}{5} > \frac{3}{4}$; $\frac{11}{12} > \frac{4}{5}$; $\frac{56}{57} > \frac{12}{13}$ etc.

CSE 2020: What is the largest number among the following?

(a) $(1/2)^{-6}$

(b) $(1/4)^{-3}$

(c) $(1/3)^{-4}$

(d) $(1/6)^{-2}$

CSE 2019: If the numerator and denominator of a proper fraction are increased by the same positive quantity which is greater than zero, the resulting fraction is

- (a) always less than the original fraction
- (b) always greater than the original fraction
- (c) always equal to the original fraction
- (d) such that nothing can be claimed definitely

CSE 2019: Which one of the following will have minimum change in its value if 5 is added to both numerator and the denominator of the fractions $2/3$, $3/4$, $4/5$ and $5/6$?

(a) $2/3$

(b) $3/4$

(c) $4/5$

(d) $5/6$

CSE 2018: X and Y are natural numbers other than 1, and Y is greater than X. Which of the following represents the largest number?

- (a) XY
- (b) X / Y
- (c) Y / X
- (d) $(X + Y) / XY$

5) COMPARING POWERS

Case 1) When numbers have same base

The one with higher power is larger number.

For example: $2^6 > 2^5 > 2^2 > 2^{1.4894} > 2^{-3}$ etc.

Case 2) When power is same:

If both base numbers are positive: the one with larger base is larger number

For example: $2^3 < 3^3 < 3.127812^3$;

$0.23^{12.12} < 0.74^{12.12} < 4.123^{12.12}$ etc.

Case 3) When neither base nor power is same:

We use above two known cases, known basic results along with basic rules of comparison to arrive at the conclusion.

Known results basically mean cubes and squares known to us from last class.

Q. Which is greater: 2^5 or 3^5

Q. Which is greater: 2^5 or 3^4

Q. Which is smaller: 2^{10} or 3^6

Q. Which is greater: 2^9 or 4^4

Q. Which is smaller: 2^{14} or 8^5

Q. Which is greater: 3^{25} or 27^8

Q. Which is smaller: 3^{27} or 4^{16}

Q. Which is smaller: 2^{34} or 3^{26}

- $2^{34} = 2 \times 8^{11}$
- $3^{26} = 9^{13} = 81 \times 9^{11}$
- $9 > 8$ and thus, $9^{11} > 8^{11}$

Q. Decide the comparison: $\sqrt{2}$ and $\sqrt[3]{3}$

- $2^{\frac{1}{2}}$ and $3^{\frac{1}{3}}$
- Recall that, if $a > b$ then $a^n > b^n$. In other words, taking powers does not change the inequality
- LCM of 2 and 3 = 6. So, we take 6th power of both numbers to get rid of fractional powers without changing inequality
- Numbers become 2^3 and 3^2 i.e. 8 and 9.
- Since $8 < 9$; $\sqrt{2} < \sqrt[3]{3}$

Q. Find the greatest among following:

$$3^{50}, 4^{40}, 5^{30}, 6^{20}$$

- Try to make powers equal using $a^{bc} = (a^b)^c$
- What is HCF of 50, 40, 30, 20?

CSE 2022: Which number amongst $2^{40}, 3^{21}, 4^{18}, 8^{12}$ is smallest?

(a) 2^{40}

(b) 3^{21}

(c) 4^{18}

(d) 8^{12}

Q. Which is larger: 5.1×10^6 or 6.7×10^6

Q. Which is smaller: 2123×10^8 or 0.33×10^{12}

Q. Which is greater: 12×8^6 or 4×9^3