Basics of Geometry

Contents

1) POINT

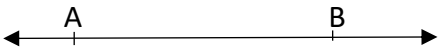
A point is a location in space. It is represented by a dot. Points are usually named with an upper-case letter. For example, we refer to the following as "point A"

. A

2) STRAIGHT LINE, INTERSECTING/CONCURRENT AND PARALLEL LINES

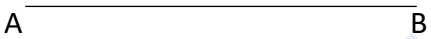
Line:

A line is a collection or set of points that have no width and extend forever. The following is a line. The two arrows are used to show that it extends forever.



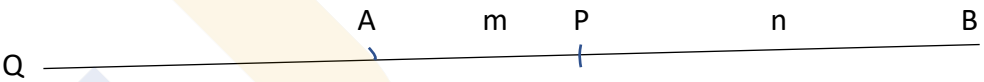
Line Segment:

A line segment is part of a line. A segment has two endpoints. The endpoints in the following segments are A and B. Notice also that the line above has no endpoints.



Since, segment has endpoints, **it has a length** measured in metres, centimetres etc.
Segment also has a midpoint. We **can also divide segment in any given ratio**.

Internal and External division:
Let AB be a line segment. We need to find all points in on the line such that, they divide the segment in the given ratio m: n.
There are two such points.
One of them divides segment internally and other divides it externally.



Here, $\frac{AP}{PB} = \frac{m}{n}$ and $\frac{AQ}{QB} = \frac{m}{n}$ $as\ well$

CSE 2021: There are three points P, Q and R on a straight line such that PQ: QR-3:5. If n is the number of possible values of PQ: PR, then what is n equal to?
(a) 1
(b) 2
(c) 3
(d) 4

Collinear Points:

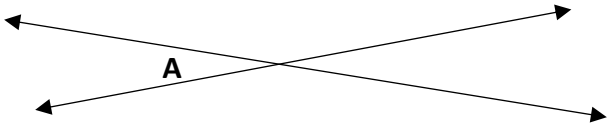
Points which are on the same line are collinear.

NOTE: Two points are always collinear

Intersecting Lines:

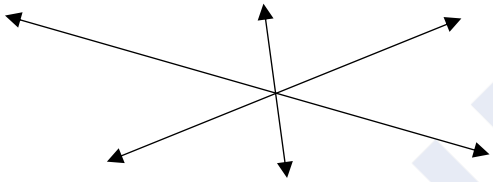
When two or more lines cross each other in a plane, they are called intersecting lines.

The intersecting lines share a common point, which exists on all the intersecting lines, and is called the point of intersection.



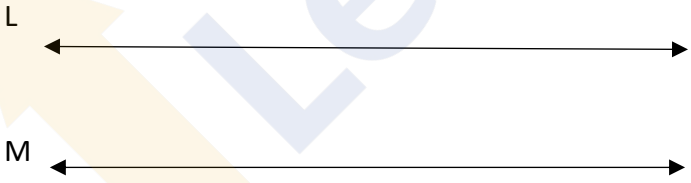
Concurrent Lines:

When two or more lines pass through a single point, in a plane, they are concurrent with each other and are called concurrent lines. A point that is common to all those lines is called the point of concurrency.



Parallel Lines:

Lines that are non-intersecting are parallel. Parallel lines do not intersect each other when extended indefinitely.



CSE 2015: In a plane, line X is perpendicular to line Y and parallel to line Z; line U is perpendicular to both lines V and W; line X is perpendicular to line V.

Which one of the following statements is correct?

(a) Z, U and W are parallel.

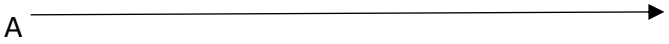
(b) X, V and Y are parallel.

(c) Z, V and U are all perpendicular to W.

(d) Y, V and W are parallel.

Ray:

A ray is a part of the line having one fixed point and the other point does not have an end. It means that a ray has one terminating end and the other end is extending infinitely.



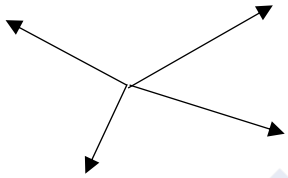
3) ANGLES:

An angle is formed when two straight lines or rays meet at a common endpoint. The common point of contact is called the vertex of an angle.

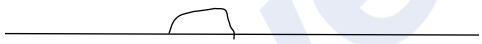
Types of angles:

- **Complete/total angle:**

Total angle around a point is taken as $360°$ (Why 360?)

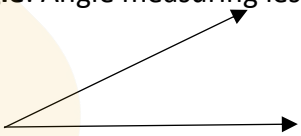


- **Straight angle** – angle of a straight line is half of a total angle which measures as $180°$

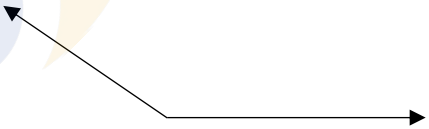


- **Right angle**: Angle measuring $90°$ is called right angle

- **Acute angle**: Angle measuring less than $90°$

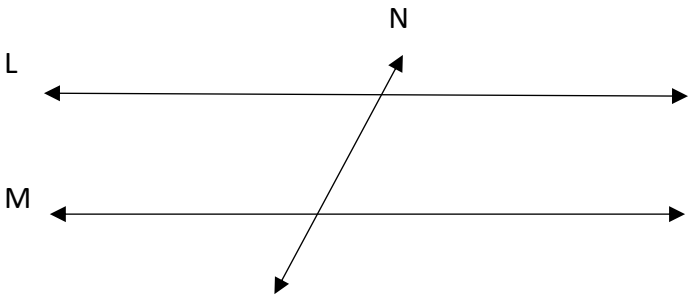


- **Obtuse angle**: Angle measuring more than $90°$ but less than $180°$



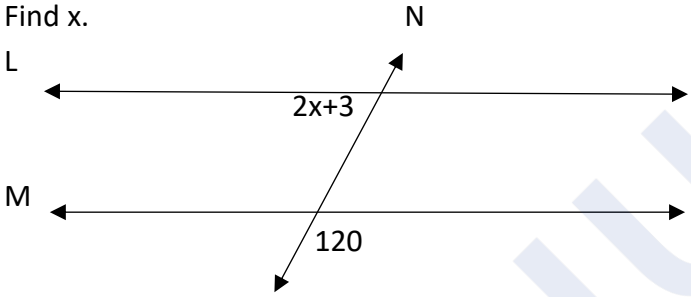
- **Complimentary angle**: 90 – angle
- **Supplementary angle**: 180 – angle

Properties of Angles in Parallel lines:



- Corresponding angles are equal.
- Vertical angles/ Vertically opposite angles are equal.
- Alternate interior angles are equal.
- Alternate exterior angles are equal.
- Pair of interior angles on the same side of the transversal are supplementary

Q. Find x.



4) PLANE:

A plane is a flat, two-dimensional surface that extends indefinitely.

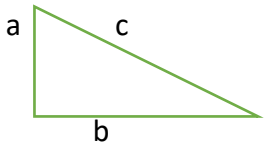
Planar shapes: Shapes on a plane.

Circle:
Set of all points at a constant distance (called radius) from a fixed point (called centre) on a plane.

Triangle:
A closed planar polygon with three edges and three vertices.

Pythagoras Theorem:

In a right-angled triangle,



(Hypotenuse)² = (One side)² + (Second side)² or $a^2 + b^2 = c^2$

Pythagorean triplets:
Set of positive integers, a, b and c that fits the rule: $a^2 + b^2 = c^2$

- (3, 4, 5) is smallest such triplet: $3^2 + 4^2 = 5^2$ i.e., 9 + 16 = 25
- All multiples of this is also a Pythagorean triplet i.e., (3n, 4n, 5n) is also a triplet for all 'n'
- For instance, for n=2, (6, 8, 10) is a triplet
- For n =3, (9, 12, 15) is a triplet etc.

Other triplets:

- (5, 12, 13)
- (6, 8, 10)
- (7, 24, 25)
- (8, 15, 17)
- (9, 40, 41)
- (11, 60, 61)
- 12, 35, 37)

CSE 2019: P, Q and R are three towns. The distance between P and Q is 60 km, whereas the distance between P and R is 80 km. Q is in the West of P and R is in the South of P. What is the distance between Q and R?
a. 140 km
b. 130 km
c. 10 km
d. 100 km

CSE 2016: AB is a vertical trunk of a huge tree with A being the point where the base of the trunk touches the ground. Due to a cyclone, the trunk has been broken at C which is at a height of 12 meters, broken part is partially attached to the vertical portion of the trunk at C. If the end of the broken part B touches the ground at D which is at a distance of 5 meters from A, then the original height of the trunk is:
(a) 20 m
(b) 25 m
(c) 30 m
(d) 35 m

CSE 2016: A person walks 12 km due north, then 15 km due east, after that 19 km due west and then 15 km due south. How far is he from the starting point?
(a) 5 km
(b) 9 km
(c) 37 km
(d) 61 km

Quadrilateral:

A closed planar polygon having 4 edges and 4 vertices

Regular polygons:

A regular polygon is the one where all sides or edges of a polygon are of equal length.
For example – Equilateral triangle, square, regular pentagon etc.

Planar Shape	Properties
Circle	For circle with radius 'r' - Diameter = 2r – diameter passes through centre - Diameter is the largest chord of a circle - Diameter makes right angle with every point on circle - The radius drawn perpendicular to the chord bisects the chord - Tangent: Line that touches circle - Angle subtended by chord at the centre is twice what it subtends at circumference - Circumference = $2\pi r$ - Area = πr^2
Triangle	- Sum of all angles is 180° - Sum of lengths of two sides is always more than the third side - The exterior angle of a triangle is always equal to the sum of the interior opposite angles - Perimeter of triangle = sum of lengths of all three sides - Area of triangle = $\frac{1}{2} \times base \times height$
Equilateral triangle	- All 3 sides of all equal length - All angles are equal and measure $60°$ as sum of 3 is 180° If length of side is 'a', - Perimeter = 3a - Area = $\frac{\sqrt{3}a}{4}$ (Height^2 = a^2 – (a/2)^2)

<table>
<tr><td>Isosceles triangle</td><td>

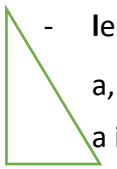
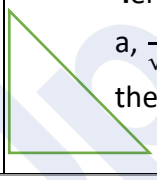
Two sides of a triangle are equal
Bottom two angles are equal


(If two sides are equal, corresponding angles are equal and if two angles are equal, corresponding sides are equal)

Line joining vertex of intersection of equal sides with midpoint of opposite side, bisects it and such line is perpendicular to the side

</td></tr>
<tr><td>Right angled triangle</td><td>

One of the angles is 90°
Lengths of sides of triangle follow Pythagoras theorem i.e., $a^2 + b^2 = c^2$
Most frequently appearing right angled triangles:

<table>
<tr><td><u>30-60-90 triangle</u>
 
- lengths of sides are: a, a/2 and $\frac{\sqrt{3}a}{2}$ where, a is the hypotenuse</td><td><u>45-45-90 triangle</u>
 
- lengths of sides are: a, $\frac{a}{\sqrt{2}}$, $\frac{a}{\sqrt{2}}$ where, a is the hypotenuse</td></tr>
</table>
</td></tr>
<tr><td>Quadrilateral</td><td>

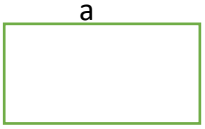
Has 4 sides and 4 angles
Sum of all four angles is 360°
Perimeter = sum of all sides

</td></tr>
<tr><td>Square</td><td>

Quadrilateral whose all 4 sides are equal and all 4 angles are equal
All angles are 90°
Diagonals are also equal and cut each other perpendicularly
If side is of length ‘a’,
Diagonal is of length $\sqrt{2}a$
Perimeter = $4a$
Area = a^2

a
 
</td></tr>
<tr><td>Rectangle</td><td>

All angles are equal and are 90°
Opposite sides are equal (hence, all squares are rectangles but all rectangles are not squares)
Diagonals are equal, bisect each other but do not intersect perpendicularly.
If adjacent sides (called length and breadth) are ‘a’ and ‘b’,
Perimeter = 2a + 2b = 2 (a + b)
Area = ab

a
 
b
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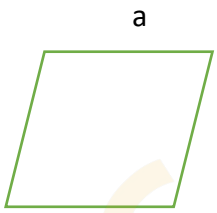
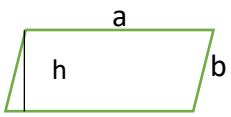
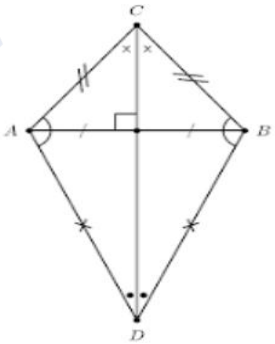
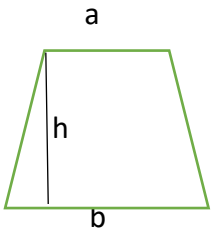
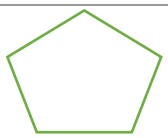
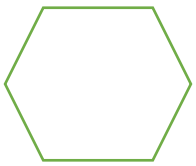
	- Diagonals are of length $\sqrt{a^2 + b^2}$ as per Pythagoras' theorem.
Rhombus	- All sides are equal but all angles are not equal - Opposite angles are equal - Diagonals are not equal but bisect each other perpendicularly - If side of a rhombus is 'a', - Perimeter = 4a - Area – depends on the angle 
Parallelogram	- Quadrilateral with equal opposite sides but all angles are not equal - What rectangle is to square, parallelogram is to rhombus - Opposite sides are parallel to each other hence the name - Diagonals are not equal but bisect each other (not perpendicularly) - Hence, all rectangles, squares and rhombuses are parallelograms - If adjacent sides are 'a' and 'b', - Perimeter = 2a + 2b - Area = ah 
Kite	- Adjacent two sides of Kite are equal - Diagonals bisect each other at right angle 
Trapezoid	- Quadrilateral with two sides parallel - Area = $\frac{1}{2} \times (a + b) \times h$ - Here, a and b are parallel sides and h is a height 
Regular Pentagon	- Has five equal sides and five equal angles - Each angle is 108°

	- 
Regular Hexagon	- Has six equal sides and six equal angles - Each angle is 120° 
Regular Octagon	- Has six equal sides and six equal angles - Each angle is 135° 

CSE 2023: ABCD is a square. One point on each of AB and CD; and two distinct points on each of BC and DA are chosen. How many distinct triangles can be drawn using any three points as vertices out of these six points?
(a) 16 (b) 18 (c) 20 (d) 24

CSE 2022: There are eight equidistant points on a circle. How many right-angled triangles can be drawn using these points as vertices and taking the diameter as one side of the triangle?
(a) 24
(b) 16
(c) 12
(d) 8

CSE 2022: Consider the following statements in respect of a rectangular sheet of length 20 cm and breadth 8 cm:
1. It is possible to cut the sheet exactly into 4 square sheets.
2. It is possible to cut the sheet into 10 triangular sheets of equal area.
Which of the above statements is are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

CSE 2022: A pie chart gives the expenditure on five different items A, B, C, D and E in a household. If B, C, D and E correspond to $90^0, 50^0, 45^0 and\ 75^0$ respectively, then what is the percentage of expenditure on item A?
(a) 112/9
(b) 125/6
(c) 155/9
(d) 250/9

CSE 2021: A pie diagram shows the percentage distribution of proteins, water and on the dry elements in the human body. Given that proteins correspond to 16% and water corresponds to 70%. If both proteins and the other dry elements correspond to p%, then what is the central angle of the sector representing p on the pie diagram?
(a) 54°
(b) 96°
(c) 108°
(d) 120°

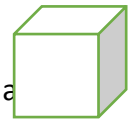
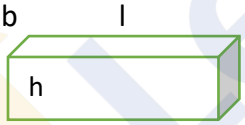
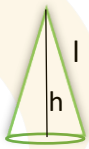
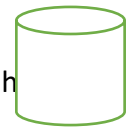
CSE 2020: Consider the following statements:
1. The minimum number of points of intersection of a square and a circle is 2.
2. The maximum number of points of intersection of a square and circle is 8.
Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

CSE 2018: There are 24 equally spaced points lying on the circumference of a circle. What is the maximum number of equilateral triangles that can be drawn by taking sets of three points as the vertices?
(a) 4
(b) 6
(c) 8
(d) 12

CSE 2017: Two walls and a ceiling of a room meet at right angles at a point P. A fly is in the air 1 m from one wall, 8 m from the other wall and 9 m from the point P. How many meters is the fly from the ceiling?
(a) 4
(b) 6
(c) 12
(d) 15

5) AREAS AND VOLUMES

Shape	Area
Circle	πr^2
Triangle	Area of triangle = $\frac{1}{2} \times base \times height$
Equilateral triangle	Area = $\frac{\sqrt{3}a}{4}$; $where\ a\ is\ side$
Square	$a^2; where\ a\ is\ a\ side$
Rectangle	$a * b; where\ a\&b\ are\ length\ and\ breadth$
Parallelogram	$a\ \times height; a\ is\ a\ side\ and\ height\ is\ lenght\ of\ perpendicular\ drawn\ onto\ that\ side$
Regular Hexagon	$\frac{3\sqrt{3}a}{2}; where\ a\ is\ side$

Shape	Diagram	Surface Area	Volume
Sphere (radius = r)		$\boldsymbol{4\pi r^2}$	$\boldsymbol{\frac{4}{3}\pi r^3}$
Cube (side = a)		$\boldsymbol{6a^2}$	$\boldsymbol{a^3}$
Cuboid (Length, breadth, height = l, b, h)		$\boldsymbol{2(lb + bh + lh)}$	$\boldsymbol{lbh}$
Cone (Base circle radius = r; height =h; curved length = l)		**Curved surface area =** $\boldsymbol{\pi rl}$ **Area of base circle =** $\boldsymbol{\pi r^2}$ **Total =** $\boldsymbol{\pi rl + \pi r^2}$	$\boldsymbol{\frac{1}{3}\pi r^2 h}$
Cylinder (Base radius =r; height= h)		**Curved surface area =** $\boldsymbol{2\pi rh}$	$\boldsymbol{\pi r^2 h}$

		Area of 2 circles above and below = $2\pi r^2$ **Total = $2\pi r(h + r)$**	

Questions on this topic often come mixed with other topics like percentages, ratio-proportion etc.

6) QUESTIONS:

CSE 2020: Q. Let x, y be the volumes; m, n be the masses of two metallic cubes P and Q respectively. Each side of Q is two times that of P and mass of Q is two times that of P. Let u=m/x and v=n/y. Which one of the following is correct?
(a) u = 4v
(b) u = 2v
(c) v = u
(d) v = 4u

CSE 2013: A gardener has 1000 plants: He wants to plant them in such a way that the number of rows and the number of columns remains the same. What is the minimum number of plants that he needs more for this purpose?
(a) 14
(b) 24
(c) 32
(d) 34

CSE 2020: If 1 litre of water weighs 1 kg, then how many cubic millimetres of water will weigh 0.1 gm?
(a) 1
(b) 10
(c) 100
(d) 1000

CSE 2018: Twelve equal squares are placed to fit in at rectangle of diagonal 5 cm. There are three rows containing four squares each. No gaps are left between adjacent squares. What is the area of each square?
(a) 5/7 sq cm (b) 7/5 sq cm 5 (c) 1 sq cm (d) 25/12 sq cm

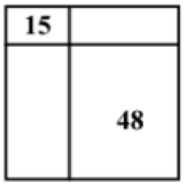
CSE 2016: An agricultural field is in the form of a rectangle having length x_1 meters and breadth x_2 meters (x_1 and x_2 are variable). If $x_1 + x_2 = 40$ meters, then the area of the agricultural field will not exceed which one of the following values?

(a) 400 sq m
(b) 300 sq m
(c) 200 sq m
(d) 80 sq m

CSE 2016: A cylindrical overhead tank of radius 2 m and height 7 m is to be filled from an underground tank of size 5.5m x 4m x 6m. How much portion of the underground tank is still filled with water after filling the overhead tank completely?

(a) 1/3
(b) 1/2
(c) 1/4
(d) 1/6

CSE 2011: Consider the following figure and answer the items that follows:



A square is divided into four rectangles as shown above. The lengths of the sides of rectangles are natural n umbers. The areas of two rectangles are indicated in the figure. What is the length of each side of the square?

(a) 10
(b) 11
(c) 15
(d) Cannot be determined as the given data are Insufficient

CSE 2011: A village having a population of 4000 requires 150 liters of water per head per day. It has a tank measuring 20 m x 15 m x 6 m. The water of this tank will last for

(a) 2 days
(b) 3 days
(c) 4 days
(d) 5 days