GS FOUNDATION (2023-24) Booklet 16
&
CSAT FOUNDATION 1.0 (2023-24) Booklet 15
Probability

Contents

1) IMPORTANT TERMS

Probability is how likely something is to happen.

- **Random Experiment**

A random experiment is a mechanism that produces a definite outcome that cannot be predicted with certainty.

For example: coin toss; throw of a dice

- **Sample Space:**

The sample space associated with a random experiment is the set of all possible outcomes.

For example: {heads, tails} in case of coin toss; {1, 2, 3, 4, 5, 6} for dice

Q. Construct sample space for two coins being tossed when

1. Coins are indistinguishable
2. Coins are distinguishable

Q. Construct a sample space that describes all three-child families according to the genders of the children with respect to birth order (can draw tree diagram)

- **Event:**

An event is a subset of the sample space.

Example: coin turns out heads; dice shows 1; dice shows even number

2) PROBABILITY:

The probability of an outcome 'e' in a sample space S is a number p between 0 and 1 that measures the likelihood that 'e' will occur on a single trial of the corresponding random experiment.

$$probability\ of\ event\ 'e' = \frac{number\ of\ possibilities\ e\ can\ occur}{Total\ number\ of\ possibilities\ in\ sample\ space}$$

The value p = 0 corresponds to the outcome e being impossible and the value p = 1 corresponds to the outcome e being certain.

The probability of an event A is the sum of the probabilities of the individual outcomes of which it is composed. It is denoted by P(A).

If event E = $\{e_1, e_2, \dots, e_k\}$ where $e_i's$ are events from sample space then

$$P(E) = P(e_1) + P(e_2) + \cdots + P(e_k)$$

Q. Find probability of an even number turning up when a die is rolled.

Q. Find probability of a number greater than 2 turning up when a die is rolled

Q. In a class, 54% students are from science background, 25% are from law, 15% are from commerce and rest are from arts background.

1. What is the probability that student is not from commerce background?
2. What is the probability that student is from either law or science background

Q. In same class from above question, 30% from science, 40% from law, 7/10th from commerce and half from arts background are females.

1. What is the probability that a randomly selected student is a female from law or commerce background?
2. What is the probability that a student is a female but not from science background?

This chapter is very much an extension of Combinatorics chapter. Here also we need to find number of ways of happening particular event and divide by number of all possible outcomes.

3) FAQS

1. A bag contains 5 red and 3 black balls. What is the probability of choosing 3 balls where 2 are red and 1 is black?

(In total there are 8 balls and we're choosing 3 balls out of 8. There're 8C3 ways to do that. Also, there are 5 red balls and we want to choose 2 – 5C2 ways; 3 black balls and we want 1 – 3C1 ways: we've to multiply as there's AND)

Probability = $\frac{5C2 \times 3C1}{8C3}$

2. A bag contains 4 red and 4 black balls. What is the probability of choosing 3 balls where all three are red or all three are black?

(Same story as above – only that – OR is addition. So, number of ways get added. Probability = $\frac{4C3 + 4C3}{8C3}$)

3. A bag contains 4 red and 4 black balls. What is the probability of choosing 3 balls where either 2 balls are red and one is black or all three balls are black?

Probability = $\frac{4C2 \times 4C1 + 4C3}{8C3}$

4. A bag contains 5 red and 4 black balls. What is the probability that in picking 4 balls, only 1 is red?
   - Only 1 is red automatically mean that 1 is red and 3 are black

   Probability = $\frac{5C1 \times 4C3}{9C4}$

5. A bag contains 5 red and 3 black balls. We are choosing 3 balls from the bag. What is the probability that, at least 1 ball is black?

Probability = $\frac{3C1 \times 5C2 + 3C2 \times 5C1 + 3C3}{8C3}$

4) COMPLIMAENT OF AN EVENT

The complement of an event A in a sample space S, denoted A^c, is the collection of all outcomes in S that are not elements of the set A.

$$P(A^c) = 1 - P(A)$$

If there is a 60% chance of rain tomorrow, what is the probability of fair weather? The obvious answer, 40% - that's what compliment is

Q. A die is rolled. E: "The number rolled is even" and T: "the number rolled is greater than two." Find the complement of each.

NOTE: Compliments are especially helpful to find probability of an event when it's difficult to calculate its probability directly.

Q. Find the probability that at least one heads will appear in five tosses of a fair coin.

5) INTERSECTION OF TWO OR MORE EVENTS

The intersection of events A and B, denoted A ∩ B, is the collection of all outcomes that are elements of both of the sets A and B. It corresponds to combining descriptions of the two events using the word "and."

In other words, if event A ∩ B has occurred then both A and B have occurred

Q. In the experiment of rolling a single die, find the intersection $E \cap T$ of the events E: "the number rolled is even" and T: "the number rolled is greater than two." – just write sample space and sets corresponding to E and T and thus $E \cap T$.

Q. Suppose the die has been "loaded" so that P(1)=1⁄12, P(6)=3⁄12, and the remaining four outcomes are equally likely with one another. Now find the probability that the number rolled is both even and greater than two.

Events A and B are mutually exclusive if they have no elements in common or in other words, if A ∩ B has no elements. Thus, P(A ∩ B) = 0

6) UNION OF TWO OR MORE EVENTS

The union of events A and B, denoted A ∪ B, is the collection of all outcomes that are elements of one or the other of the sets A and B, or of both of them.

In other words, element could be in A OR B.

If A and B are two sets then,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

This formula is very useful in finding number of elements in set A ∪ B and thus P (A ∪ B).

Additive rule of probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Q. In the experiment of rolling a single die, E: "the number rolled is even" and T: "the number rolled is greater than two." What is the probability of number rolled is an even number or a number greater than 2?

A two-child family is selected at random. Let B denote the event that at least one child is a boy, let D denote the event that the genders of the two children differ, and let M denote the event that the genders of the two children match. Find P(B ∪ D) and P(B∪M).

Q. Two fair dice are thrown. Find the probabilities of the following events:
1. both dice show a four
2. at least one die shows a four

Q. 63% students need help in mathematics, 34% need help in LR, and 27% need help in both mathematics and LR. What is the probability that a randomly selected student needs help in either mathematics or English?

Q. A jar contains 10 marbles, 7 black and 3 white. Two marbles are drawn without replacement, which means that the first one is not put back before the second one is drawn. What is the probability that both marbles are black?
What is the probability that exactly one marble is black?
What is the probability that at least one marble is black?
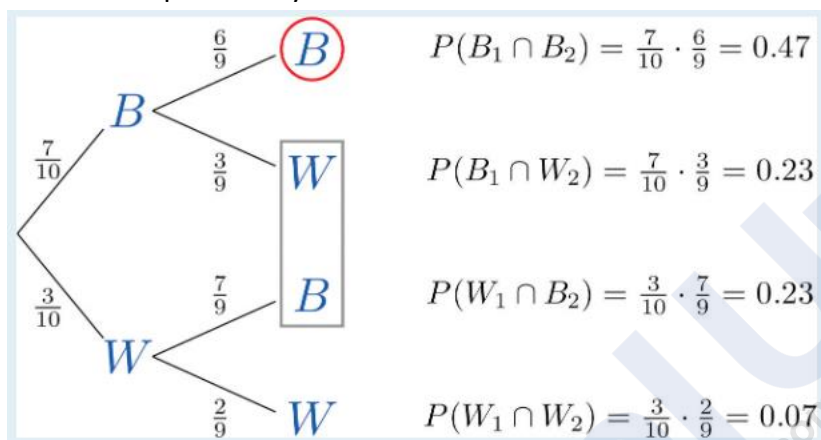
7) PYQS

CSE 2018: A bag contains 15 red balls and 20 black balls. Each ball is numbered either 1 or 2 or 3. 20% of the red balls are numbered 1 and 40% of them are numbered 3. Similarly, among the black balls, 45% are numbered 2 and 30% are numbered 3. A boy picks a ball at random. He wins if the ball is red and numbered 3 or if it is black and numbered 1 or 2. What are the chances of his winning?
(a) 1/2
(b) 4/7
(c) 5/9
(d) 12/13

CSE 2017: A bag contains 20 balls. 8 balls are green, 7 are white and 5 are red. What is the minimum number of balls that must be picked up from the bag blindfolded (without replacing any of it) to be assured of picking at least one ball of each colour?
(a) 17
(b) 16
 (c) 13
(d) 11

CSE 2016: A round archery target of diameter 1 m is marked with four scoring regions from the centre outwards as red, blue, yellow and white. The radius of the red band is 0.20 m. The width of all the remaining bands is equal. If archers throw arrows towards the target, what is the probability, that the arrows fall in the red region of the archery target?

(a) 0.40
(b) 0.20
(c) 0.16
(d) 0.04

8) COMPREHENSION

India has tremendous potential for solar energy. We all realize that we have to stop burning fossil fuels to meet our energy needs. But certain renewable resources are still going through their cost curves and learning curves to get the required amount of output. The Indian Government has strongly committed to its targets of reducing emissions by 33 per cent by 2030, and towards this it has initiated a strong push towards a gas-based economy-and has also invested heavily in renewable energy. However, business houses are wary of investing too heavily in renewable energy at a time when the technology is not yet ready.

Q. Which one of the following is the most logical and rational inference that can be made from the above passage?

   (a) India's commitment to reduce emissions by 33% is unlikely to be achieved
   (b) India should import gas rather than invest in renewable resources
   (c) Getting renewable resources to market too soon may be costly.
   (d) India should put in more efforts in the exploration of natural gas

Economic liberalization in India was shaped largely by the economic problems of the government than by the economic priorities of the people or by the long-term development objectives. Thus, there were limitations in conception and design which have been subsequently validated by experience. Jobless growth, persistent poverty and rising inequality have mounted as problems since economic liberalization began. And all these years later, four quite crises confront the economy; agriculture, infrastructure, industrialization and education as constraints on the country's future prospects. These problems must be resolved in economic growth has to be sustained and transformed into meaningful development.

Q. Which of the following is/are the most rational and logical inference/inferences that can be made from the passage?

1. It is essential to rethink and redefine the economic role of the State in the quest for development.

2. India has not made effective implementation of its policies in social sectors nor made sufficient investments in them.

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2.
(d) Neither 1 nor 2

Q. The 103rd Amendment inserting Articles 15 (6) and 16(6) to the Constitution, permits 10% reservation in educational institutions and public employment for those from the EWS. This reservation explicitly excludes persons from the Scheduled Castes (SC), Scheduled Tribes (ST) and the Other Backward Classes (OBC) categories. The majority judgment of Justices Dinesh Maheshwari, Bela M. Trivedi and J.B. Pardiwala upheld the constitutionality of the amendment and held that such exclusion was justified because the SC, ST and OBC categories had reservations under Articles 15(4), 15(5) and 16(4). They held that a 'mere violation of the rule of equality does not violate the basic structure of the Constitution unless the violation is shocking, an unconscionable or unscrupulous travesty of the quintessence of equal justice' and that 'if any constitutional amendment moderately abridges or alters the equality principles, it cannot be said to be a violation of the basic structure'. This brings us to the principle of equality and its place in the Constitution. Can the guarantee of equality be violated and if so, would the identity of the Constitution survive? The Supreme Court of India has held that in understanding what forms part of the 'basic structure' of the Constitution, that inviolable part which can never be tampered with or altered, equality is an integral part of it. It is one of those core features without which the Constitution will not be recognisable and can never be taken away, however 'minor' such violation may be.

Q. Which of the following is the most logical rational inference of the passage?
A. With the Supreme Court judgment, all debates around EWS should now rest and we should move towards era of equality
B. Violation of constitution in EWS reservation is not a violation of basic structure of constitution
C. Violation of equality even if minor, would render constitution unrecognisable
D. EWS judgement goes to the very heart of equality debate in India