Mathematisation – a basic skill for quantitative aptitude

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1) INTRODUCTION:

One of the most important skills you shall require for solving quantitative aptitude problems with ease is the ability to convert given word problem into mathematical equations. Once, that is done, solving such equation is quite easy.

Mathematization refers to the activity of organizing and studying any kind of reality with mathematical means, that is, translating a realistic problem into the symbolic mathematical world.

2) WHAT IS A LINEAR EQUATION?

- **Constant**: Something whose value is fixed – it could be a number or a letter given or assumed as constant.
- **Variable**: Something whose value is not a constant or fixed. It can take various values thus called a variable. We're to assume some unknown quantity as variable and find out which particular value when given to that variable, will satisfy our equations or conditions.
- **Coefficient**: The constant in front of a variable is its coefficient. For example, in 5x, 5 is coefficient of x; in $7xy^3$ 7 is coefficient of xy^3.
- **Term**: Coefficient and variables together joined by multiplication or division. For example: 5x; 2.5xyz etc.
- **Equation**: Equations are mathematical statements containing two algebraic expressions on both sides of an 'equal to (=)' sign. It shows the relationship of equality between the expression written on the left side with the expression written on the right side.
  For example: $5x + 9 = 34; 3y^2 - 4x = 123; 746xy^2 + 64y = 12$ are all equations.
  While, $32x + 12y + z; 12x^2 + 4\ etc.are\ Not\ equations$
- **Degree of equation**: Degree of equation is the highest power of terms in the given equation.
  For example: Degree of $5x + 9 = 34\ is\ 1$;
  Degree of $3y^2 - 4x = 123$ is 2
  Degree of $746xy^2 + 64y = 12$ is 3 (1 for x +2 for y² = 3)
- **Linear equation**: Equations with 1 as the degree (highest exponent/power of variable) are known as linear equations in maths.
  These can be further classified into linear equations in one variable, two-variable linear equations, with three variables, etc.
  The standard form of a linear equation with variables x and y is $ax + by + c = 0$, where a and b are the coefficients of x and y respectively and c is the constant.

Q. Which of the following equations are linear?

1. $5x + 9 = 34$;
2. $3y^2 - 4x = 123$;
3. $746xy^2 + 64y = 12$
4. $5x + 9z = 3y$
5. $2x + 3xy = 8$
6. $3d + 5a = 9bc$
7. $23xyz = yz$
8. $3^2x + 4y = 3z - 2$
9. $3x^2 - 4 = 4x$
10. $y^2 + 2 = y$

3) HOW TO DECIDE UNKNOWN

- **Rule of thumb:** Assume what is being asked as unknown 'x' and just plot the information given in the problem in terms of 'x' – In over 90% questions, this will be the simplest thing to do.

For example:

Q. A sum is divided among 120 men and some women in the ratio 15: 21. If each man gets 5 and each woman gets 4, then find the number of women.

- Let number of women be 'x'
- Each women gets 4, so in total they get 4x
- Each man gets 5, so in total they get $5 \times 120 = 600$
- Ratio is 15: 21 i.e., $\frac{600}{4x} = \frac{15}{21}$

Q. A and B together have Rs. 1210. If 4/15th of A's amount is equal to 2/5th of B's amount, how much amount does B have?

- Let B have amount 'x'
- Let A have amount 'y'
- $\frac{4}{15}y = \frac{2}{5}x$ and $x + y = 1210$

Q. In a mixture 60 litres, the ratio of milk and water 2: 1. If this ratio is to be 1: 2, then the quantity of water to be further added is

- Out of 60 ltr, milk: water is 2:1, so, 40 ltr milk and 20 ltr water
- Now we want to add water and make this ratio 1:2
- If x water is required to do this, then, total mixture is 60+x out of which 40 is milk and 20+x is water
- We also know that, 40*2 = 20+x
- Which gives us, x = 60 litres

Q.

Two candidates X and Y contested an election. 80% of voters cast their vote and there were no invalid votes. There was no NOTA (None of the above) option. X got 56% of the votes cast and won by 1440 votes. What is the total number of voters in the voters list?

- **If problem already has given variables, you don't need to introduce them from your side:**

A rectangle has length and width $(3x - 4)$ cm and $(x + 1)$ cm, respectively. If the perimeter of the rectangle is 34 cm, find the area of the rectangle.

- **Another way –** take that quantity as unknown variable 'x' about which there is maximum information given in the question.

Q. A number is multiplied by 2, the result is then subtracted by 4, and finally is divided by 5. If the final result is 3, what was the final number?
- It is easier to start with assuming the starting number and move forward in question than assuming final number and move back (It is certainly possible though)

4) HOW TO DECIDE NUMBER OF UNKNOWNS

To solve linear equations having 'n' variables, you need 'n' equations. So, you can decide to assume as many variables as number of equations getting formed out of given information. NOTE that, to form an equation, we need an independent information. In other words, if the information is dependent on previous information, it will not yield an equation.

If only 1 equation can be formed, then you can only assume one variable 'x' to form it and solve.

Q. You have a number. The number is subtracted by 4, next the result is divided by 12. If the final result is 13, what was the starting number?
- NOTE that, every subsequent information here depends upon the previous information.
- So, no independent information is there to have more than 1 equation
- So, we can assume only 1 unknown which is to be found as 'x'

If 2 equations can be formed, then you can assume two variables say x and y and solve two equations to get values of x and y. We just have to look for how many different sets of information are given in the question.

For example: Q. A and B together have Rs. 1210. If 4/15th of A's amount is equal to 2/5th of B's amount, how much amount does B have?

- Here we are given two information – 1. Together A and B have 1210 and 2. 4/15th of A's amount is equal to 2/5th of B's
- Now, is the information same or different? Can one be derived from other? – NO
- So, we need to assume two variables.
- Let B have amount 'x'
- Let A have amount 'y'
- $\frac{4}{15}y = \frac{2}{5}x$ and $x + y = 1210$

Suppose the question was: A and B together have Rs. 1210. If double of A's amount and double of B's amount equal 2420, how much amount does B have?

- Here we are given two information are seemingly given – 1. Together A and B have 1210 and 2. double of A's amount and double of B's amount equal 2420
- Now, is the information same or different? Can one be derived from other?
- If yes, it is only one information and not two – we can't solve equation with two unknowns without two equations.
- So, we have two unknowns but only one equation – we can't solve it!

5) PRACTICING THROUGH QUESTIONS:

Convert following word problems into mathematical equations.
1. Amir and Tony together have Rp 30,000. If Amir's amount of money is Rp 4,000 more than Tony's, find each of their amounts.
2. Puja's Plate can hold 8 pieces of mango. Puja took three more pieces to fill it. How many did she already have?
3. Puja has 5 pieces of mango on her plate. She has 2 pieces more than Pushpa. Then how many pieces of mango does Pushpa have?
4. Puja had 5 pieces of mango on her plate. Then she got some more pieces of mango from her mother. Now she has 8 pieces of mango. How many pieces of mango did she get from her mother?

6) COMPREHENSION FOR PRACTICE

The poverty line is quite unsatisfactory when it comes to grasping the extent of poverty in India. It is not only because of its extremely narrow definition of 'who is poor' and the debatable methodology used to count the poor, but also because of a more fundamental assumption underlying it. It exclusively relies on the notion of poverty as insufficient income or insufficient purchasing power. One can better categorize it by calling it income poverty. If

poverty is ultimately about deprivations affecting human well-being, then income poverty is only one aspect of it. Poverty of a life, in our view, lies not merely in the impoverished state in which the person actually lives, but also in the lack of real opportunity given by social constraints as well as personal circumstances–to choose other types of living. Even the relevance of low incomes, meagre possessions, and other aspects of what are standardly seen as economic poverty relate ultimately to their role in curtailing capabilities, i.e., their role in severely restricting the choices people have to lead variable and valued lives.

Q. Why is the methodology adopted in India to count the 'poor' debatable?
(a) There is some confusion regarding what should constitute the 'poverty line'.
(b) There are wide diversities in the condition of the rural and urban poor.
(c) There is no uniform global standard for measuring income poverty.
(d) It is based on the proposition of poverty as meagre income or buying capacity.

Q. Why is income poverty only one measure of counting the 'poor'?
(a) It talks of only one kind of deprivation ignoring all others.
(b) Other deprivations in a human life have nothing to do with lack of purchasing power.
(c) Income poverty is not a permanent condition; it changes from time to time.
(d) Income poverty restricts human choices only at a point of time.

Q. What does the author mean by 'poverty of a life'?
(a) All deprivations in a human life which stem not only from lack of income but lack of real opportunities
(b) Impoverished state of poor people in rural and urban areas
(c) Missed opportunities in diverse personal circumstances
(d) Material as well as non-material deprivations in a human life which
        restrict human choices permanently.

In some places in the world, the productivity of staples such as rice and wheat has reached a plateau. Neither new strains nor fancy agrochemicals are raising the yields. Nor is there much unfarmed land left that is suitable to be brought under the plough. If global temperature continues to rise, some places will become unsuitable for farming. Application of technology can help overcome these problems. Agricultural technology is changing fast. Much of this change is brought about by affluent farmers in the West/Americas. Techniques developed in the West are being adapted in some places to make tropical crops more productive. Technology is of little use if it is not adapted. In the developing world, that applies as much to existing farming techniques as it does to the latest advances in genetic modification. Extending to the smallholders and subsistence farmers of Africa and Asia the best of today's agricultural practices, in such simple matters as how much fertilizers to apply and when, would lead to a greatly increased availability of food for humanity. So would things like better roads and storage facilities, to allow for the carriage of surpluses to markets and reduce wastage.

Q. Based on the above passage, the following assumptions have been made:

1. Development of agricultural technology is confined to developed countries.
2. Agricultural technology is not adapted in developing countries.

Which of the above assumptions is/are valid?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Q. Based on the above passage, the following assumptions have been made:

1. Poor countries need to bring about change in their existing farming techniques.
2. Developed countries have better infrastructure and they waste less food.

Which of the above assumptions is/are valid?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2