GS FOUNDATION 2024

CSAT Booklet – 14 Basics of Arithmetic II

Contents

1) REMAINDERS

The remainder, as its name suggests, is something that "remains" after completing a task.
When we divide one number by another, there's always a remainder. If number is a factor, remainder is 0.
This property is called division algorithm in natural numbers.
Given two numbers a and b, we have two numbers q and r such that,
$\boldsymbol{a = bq + r,}$ here $0 \leq r < b$; q is called quotient and r is called remainder
We've already seen this in division:

$$\boldsymbol{Dividend = Devisor \times Quotient + Remainder}$$

Remainder is always smaller than devisor

- This with some modification solves many of our questions.

Q. A number, when divided by 114, leaves remainder 21. If the same number is divided by 19, then the remainder is:

A. 1
B. 2
C. 7
D. 17

Q. On dividing a number by 13, we get 1 as remainder. If the quotient is divided by 5, we get 3 as remainder. If this number is divided by 65, what will be the remainder?

Reminder when $(\boldsymbol{A_1 + A_2 + \cdots + A_n})$ is divided by B

- Remainder is same as sum of remainders when A_1 *is divided by* B, A_2 *is divided by* B *and so on*
- If sum is more than B, we find remainder when sum is divided by B

Q. Find remainder when $(23123 + 131212 + 1223421)$ *is divided by* 3

Remainder when $(\boldsymbol{A_1 \times A_2 \times \ldots \times A_n})$ **is divided by B**

- Remainder is same as product of remainders when each $\boldsymbol{A_i}$ is divided by B.
- If product is more than B, we find remainder when product is divided by B

Q. Find remainder when $(23 \times 32 \times 5331 \times 125)$ is divided by 3

Q. A number, when divided by 136, leaves remainder 36. If the same number is divided by 17, what will be the remainder?

Q. What will be the remainder when 2^{33} is divided by 10?
- NOTE: remainder of a number when divided by 10 is last digit!
The question is equivalent to finding last digit of the number!

Q. If a perfect square, not divisible by 6, be divided by 6, the remainder will be from which of the following options?

A. 1, 2, 4
B. 1, 2, 5
C. 1, 3, 5
D. 1, 3, 4

Q. What is the remainder when
23456789101112131411161718192021222324252627282930313233343536373839404142
434485 is divided by 45?

A. 0
B. 1
C. 29
D. 44

Q. What is the remainder when 1! + 2! + 3! ... 100! is divided by 18?
When $n! = 1 \times 2 \times 3 \ldots \times (n-1) \times n$

CSE 2023: What is the remainder if 2^{192} is divided by 6?
(a) 0 (b) 1 (c) 2 (d) 4

CSE 2023: A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?
(a) 11 (b) 9 (c) 7 (d) 1

Q. CSE 2022

What is the remainder when

$91 \times 92 \times 93 \times 94 \times 95 \times 96 \times 97 \times 98 \times 99$

is divided by 1261?

(a) 3

(b) 2

(c) 1

(d) 0

Q. CSE 2020: If 3^{2019} is divided by 10, then what is the remainder?
(a) 1
(b) 3
(c) 7
(d) 9

Q. CSE 2020: What is the remainder when 51 x 27 x 35 x 62 x 75 is divided by 100?
(a) 50
(b) 25
(c) 5
(d) 1

CSE 2017:
A 2-digit number is reversed. The larger of the two numbers is divided by the smaller one. What is the largest possible remainder?
(a) 9
(b) 27
(c) 36
(d) 45

Q. If x and y are positive integers such that $(3x + 7y)$ is divisible by 11, then which of the following will be divisible by 11?
A. 4x+6y B. x+y+4 C. 9x+4y D. 4x-9y

Q. What is remainder when $51 \times 27 \times 35 \times 62 \times 75$ is divided by 100?

A. 0

B. 25

C. 50

D. 1

Remainders when large powers are involved:

- If X^k is divided by n,
- We try to represent number X as (an+1) or (an-1)
- Thus, remainder of $X^k = (an \pm 1)^k = (\pm 1)^k$ as $(an)^k$ *is always divisible by n*

Q. Find remainder when 37^{123423} is divided by 9.

- $37 = 9 \times 4 + 1$
- $\frac{37^{123423}}{9} = \frac{(9 \times 4+1)^{123423}}{9}$
- Remainder = 1

Q. Find reminder when 35^{123423} is divided by 9.

- $35 = 9 \times 4 - 1$
- Remainder = $(-1)^{123423} = -1$ equivalent to $-1 + 9 = 8$

Q. Find remainder when $(1! + 2! + \cdots + 1000!)^{40}$ *is divided by* 10.

A. 0

B. 1

C. 2

D. 9

2) SEQUENCE AND SERIES

Sequence:

- The sequence is the group or sequential arrangement of numbers in a particular order or set of rules.

For example: 0, 2, 4, 6, … is sequence of even numbers

- In a sequence, an individual term can be present in many places.

For example: 1, 2, 1, 2, 1 … is a sequence where 1 appears infinitely many times

- Sequences can be of two types, i.e., infinite sequence and finite sequence.

For example: 5,4,3,2,1 is a finite sequence having 5 terms

1,3,5,7, … is an infinite sequence of odd numbers

Series:

Series is formed by adding the terms of a sequence. Sum of infinite terms in a series can be finite as well in some cases.

$2 + 4 + 6 + \cdots$ is a series of even numbers

$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$ is a series that has a finite sum = $\frac{\pi^2}{6}$

There are certain important types of sequences and corresponding series that are relevant for us:

Arithmetic Sequence/Arithmetic Progression

An arithmetic sequence is a sequence where the successive terms are either the addition or subtraction of the common term known as common difference. For example: 1, 4, 7, 10, ... or $50,45,40,35, \ldots$

Sometimes, such sequence is called as Arithmetic progression:

Any sequence of the form: $a, a + d, a + 2d + \cdots a + nd$ is called as an Arithmetic progression

- First term: a
- Common difference: d
- n^th term: $a + (n-1)d$

Sum of first 'n' terms of AP:

Let, AP be: $a, a + d, a + 2d, \ldots, a + (n-1)d$

Then, series would be

$$a + (a + d) + (a + 2d) \ldots + a + (n-1)d = na + (d + 2d + \cdots + (n-1)d)$$
$$= na + \frac{n(n-1)}{2} d$$

CSE 2020: In a race, a competitor has to collect 6 apples which are kept in a straight line on a track and a bucket is placed at the beginning of the track which is a starting point. The condition is that the competitor can pick only one apple at a time, run back with it and drop it in the bucket. If he has to drop all the apples in the bucket, how much total distance he has to run if the bucket is 5 meters from the first apple and all other apples are placed 3 meters apart?

(a) 40 m

(b) 50 m

(c) 75 m

(d) 150 m

CSE 2014: A group of 630 children is seated in rows for a group photo session. Each row contains three less children than the row in front of it. Which one of the following number of rows is not possible?

(a) 3

(b) 4

(c) 5

(d) 6

CSE 2014: A straight line segment is 36 cm long. Points are to be marked on the line from both the end points. From each end, the first point is at a distance of 1 cm from the end, the second point is at a distance of 2 cm from the first point and the third point is at a distance of 3 cm from the second point and so on. If the points on the ends are not counted and the common points are counted as one, what is the number of points?

(a) 10

(b) 12

(c) 14

(d) 16

CSE 2013: A sum of RS. 700 has to be used to give seven cash prizes to the students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, then what is the least value of the prize?

(a) RS. 30

(b) RS. 40

(c) RS. 60

(d) RS. 80

CSE 2011: A contract on construction job specifies a penalty for delay in completion of the work beyond a certain date is as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day etc., the penalty for each succeeding day being 50 more than that of the preceding day. How much penalty should the contractor pay if he delays the work by 10 days?

(a) Rs. 4950

(b) Rs. 4250

(c) Rs. 3600

(d) Rs. 650

Average of finite AP:

Let, AP be: $a, a+d, a+2d, \dots, a+(n-1)d$

- **Case 1) Number of terms is odd: Middle term is the Average**

Thus, Average of above AP is $a + \frac{n-1}{2}d$

Ex: $1,4,7,10,13,16,19$ – Average = 10

- **Case 2) Number of terms is odd: Average is average of two middle terms**

Thus, Average of above AP = $\frac{\left(a+\frac{n-2}{2}d\right)+\left(a+\frac{n}{2}d\right)}{2}$

Ex: $1,4,7,10,13,16,19,22$ – Average = $\frac{10+13}{2} = 11.5$

Geometric Sequence/Geometric Progression

A geometric sequence is a sequence where the successive terms have a common ratio.
For example, 1, 4, 16, 64, ...is a Geometric sequence.
Any Geometric series of the form $a, ar, ar^2, ar^3, \dots ar^{n-1}$ is a Geometric progression

- 1st term: a
- Common ratio: r
- nth term: ar^{n-1}

Such Geometric series can be finite or infinite
$2,4,8,16, \dots ; 1,5,25,125, \dots ; 1,3,9,27, \dots$ are all Geometric progressions

Sum of 1st n terms of GP:

Let**,** $a, ar, ar^2, ar^3, \dots ar^{n-1}$ be a GP.

Then the series would be: $a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = a\frac{r^n-1}{r-1}$

If GP is infinite and |r|<1, then, $+ar + ar^2 + ar^3 + \cdots = a\frac{1}{1-r}$

CSE 2017: If there is a policy that 1/3rd of population of a community has migrated every year from one place, to some other place, what is the leftover population of that community after the sixth year, if there is no further growth in the population during this period?
(a) 16/243rd part of the population
(b) 32/243rd part of the population
(c) 32/729th part of the population
(d) 64/729th part of the population

Harmonic Sequence/Harmonic Progression

A harmonic sequence is a sequence where the sequence is formed by taking the reciprocal of each term of an arithmetic sequence. For example,$1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \dots$ is a harmonic sequence as $1,4,7,10, \dots$ is an arithmetic sequence

Fibonacci Sequence

When every term is an addition of preceding two terms, the Sequence we get is Fibonacci sequence.
$1,1,2,3,5,8,13,21, \dots$

- It is closely related to many objects in nature
- Only important thing to remember for us is: $F_{n+1} = F_n + F_{n-1}$

- If you're asked to find 8^{th},10^{th} or 12^{th} term, just write the series and find it – it'll be quicker

Sum of Fibonacci Series:

- Sum of first n terms = $F_1 + F_{n+1} + \cdots + F_n = F_{n+2} - 1$

Other Important Series:

- **Sum of first n natural numbers:**

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

- **Sum of squares**:

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- **Sum of cubes**:

$$1^3 + 2^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

- **Sum of first 'n' odd numbers:**

$$1 + 3 + \cdots (\text{n numbers}) = \text{n}^2$$

- **Sum of first n even natural numbers**:

$$2 + 4 + \cdots (n\ numbers) = n(n+1)$$

- **Sum of inverses of squares of natural numbers:**

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}$$

- **Sum of inverses of powers of 2**:

$$1 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 2$$

3) IMPORTANT EXPANSIONS

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \cdots + 1)$$
$$For\ n\ odd: x^n + 1 = (x+1)(x^{n-1} - x^{n-2} + x^{n-3} \ldots - x + 1)$$
$$For\ n\ even: x^n + 1\ has\ NO\ real\ root$$

In addition to it, if $n\ is\ even$: $(x^n - 1)$ is divisible by $(x+1)$ as well!

$$As\ (x^{2m} - 1) = (x^m + 1)(x^m - 1);$$
$$If\ m\ is\ odd\ (x+1) is\ factor\ of\ (x^m + 1)\ and\ if\ m\ is\ even\ then\ it\ is\ factor\ of\ (x^m - 1)$$

NOTE: $(x^n - 1)\ is\ always\ divisible\ by\ (x-1)\ and$
$(x^n + 1)\ is\ divisible\ by\ (x+1) when\ n\ is\ odd$
In fact, $(x^n - a^n)\ is\ always\ divisible\ by\ (x-a)\ and$
$(x^n + a^n)\ is\ divisible\ by\ (x+a) when\ n\ is\ odd$

CSE 2021: How many pairs of natural numbers are there such that the difference of whose squares is 63?
(a) 3
(b) 4
(c) 5
(d) 2

1. What is the average of first 50 natural numbers?
A. 25
B. 25.5
C. 27
D. 30

2. What is the value of $11 + 12 + \cdots + 50$?
A. 1140
B. 1160
C. 1200
D. 1220

3. If $x^4 + \frac{1}{x^4} = 47$; what is the value of $x + \frac{1}{x}$
A. 3
B. 4
C. 5
D. 7

4. If $64^2 - 36^2 = 4P$ then what is the value of P?
A. 637

B. 700
C. 600
D. 837

5. What will be the remainder when $67^{67} + 67$ when divided by 68?
A. 0
B. 1
C. 66
D. 67