

GS FOUNDATION
BATCH FOR CSE 2023
Target Prelims 2023: CSAT
Ace CSAT 2023-Booklet 12
Quantitative Aptitude-8 - Ratio and Proportion

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1) RATIO:

If there are two quantities 'a' and 'b' having same units, then their ratio will be the fraction of the two quantities. So, $a:b = \frac{a}{b}$

NOTE:

- Ratio does not get affected if we multiply both quantities by same quantity

Ex: $a:b = 2a:2b = 5a:5b = na:nb$ i.e., $\frac{a}{b} = \frac{na}{nb}$

Note: division by n is just multiplication by $1/n$ and hence ratio does not change when we divide both quantities by same number

However, $\frac{a}{b} \neq \frac{a+k}{b+k}$

In fact, for a positive k, $\frac{a}{b} < \frac{a+k}{b+k}$ if $\frac{a}{b} < 1$

2) PROPORTION:

When two ratios are equal to each other, this equality is known as proportion.

So, if $a:b = c:d$ (which indicates that two ratios are equal to each other),

We can write the proportionality as, $a:b::c:d$, and then, we can conclude that a, b, c, d are in proportion.

We can write this as $\frac{a}{b} = \frac{c}{d}$ which implies, $a \times d = b \times c$

With this singular fact, we can solve almost all types of problems

3) MERGING TWO RATIOS HAVING ONE COMMON ELEMENT

If both $a:b$ and $b:c$ are known we can merge two ratios into one using common element 'b' as a bridge.

Example: $a:b = 2:3$ & $b:c = 6:11$

Here 'b' is a common element that can be used as a bridge. We try to get same value in both ratios at place of 'b' using multiplication

Here, we can multiply first ratio by $6/3 = 2$ to get

$a:b = 2:3 = 4:6$ and we have $b:c = 6:11$

Thus, $a:b:c = 4:6:11$

4) PROPERTIES OF RATIOS AND PROPORTIONS:

(Proofs follow from cross multiplication)

1. **Invertendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{b}{a} = \frac{d}{c}$

Ex: $3:4 = 9:12$ then $4:3 = 12:9$

2. **Alternendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a}{c} = \frac{b}{d}$

Ex: $3:4 = 9:12$ then, $3:9 = 4:12$

3. **Componendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a+b}{b} = \frac{c+d}{d}$

Ex: $3:4 = 9:12$ then $(3+4):4 = (9+12):12$ i.e., $7:4 = 21:12$

4. **Dividendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a-b}{b} = \frac{c-d}{d}$

Ex: $4:3 = 12:9$ then, $(4-3):3 = (12-9):9$ i.e., $1:3 = 3:9$

Or $3:4 = 9:12$ then $(3-4):4 = (9-12):12$ i.e., $-1:4 = -3:12$

5. **Componendo-Dividendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Ex: $3:4 = 9:12$ then $(3+4):4 = (9+12):12$ i.e.,

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \dots = k$ (say) then, each ratio(k) = $\frac{a+c+e\dots}{b+d+f\dots}$

Ex: $3:4 = 6:8 = 9:12$ then each ratio = $(3+6+9):(4+8+12) = 18:24$

7. Direct proportion:

Direct proportion or direct variation is **the relation between two quantities where the ratio of the two is equal to a constant value.** It is represented by the proportional symbol, \propto .

If one quantity is increased, so does the other

If one quantity is decreased, so does the other

For ex: If $a \propto b$ then, $a = kb$ or $\frac{a}{b} = k$ (constant)

8. Inverse Proportion or inverse variation:

It is the relation between two quantities where, **multiplication of two quantities is constant.**

If one quantity is increased, the other decreases

If one quantity is decreased, the other increases

For ex: If a is inversely proportional to b , then

$ab = k$ (constant)

It is equivalent to saying, a is directly proportional to $1/b$

i.e., $a \propto \frac{1}{b}$ then, $a = \frac{k}{b}$ or $ab = k$ (constant)

5) PRACTICING THROUGH QUESTIONS:

1. If $(2x + 3y):(2x - 3y) = 5:1$, then find $x:y$

We first write this as a division: $\frac{2x+3y}{2x-3y} = \frac{5}{1}$

Method 1: We can observe the similar terms in numerator and denominator and use Componendo-dividendo

Thus, $\frac{2x+3y+(2x-3y)}{2x+3y-(2x-3y)} = \frac{5+1}{5-1}$ or $\frac{4x}{6y} = \frac{6}{4} \Rightarrow \frac{x}{y} = \frac{36}{16} = \frac{9}{4}$

Method 2: We can just cross multiply and simplify without thinking much. $\frac{2x+3y}{2x-3y} = \frac{5}{1}$

$$\Rightarrow 2x + 3y = 5(2x - 3y)$$

$$\Rightarrow 2x + 3y = 10x - 15y$$

$$\Rightarrow 8x = 18y$$

$$\Rightarrow 4x = 9y$$

2. If the ratio of the areas of two squares is 9: 16, then find the ratio of their perimeters. (Note here that, you can solve this question only if you are clear about what area and perimeter of a square is. In actual exam as well, you will always need basic clarity in concepts about arithmetic, geometry, set theory etc. – and this is precisely why we spend some dedicated time in understanding them)

- Let side of squares be 'a' and 'b'
- Thus, areas are a^2 and b^2
- Given: $a^2 : b^2 = 9 : 16$ or $\frac{a^2}{b^2} = \frac{9}{16}$
- Taking square root of both sides: $\frac{a}{b} = \frac{3}{4}$
- Perimeters of squares are $4a$ and $4b$
- Thus, ratio of perimeters is $4a : 4b = a : b$ (multiplication does not change the ratio) = 3: 4

3. Angles of a quadrilateral are in the ratio 6: 8: 10: 12. Find the smallest angle.

Again, we need to know the basic geometry to solve this question. We need to know that, sum of angles of quadrilateral is 360 degrees.

- Let the actual angles be $6x$, $8x$, $10x$ and $12x$
- Thus, $36x = 360$ which implies, $x = 10$
- Thus, smallest angle = $6x = 60$

4. If 20% of $(P + Q) = 50\%$ of $(P - Q)$, then find $P: Q$

Here notice that, two different concepts are mixed i.e. percentages and ratios. Over and above that, you need to know basic arithmetic about how to multiply brackets.

- We can use the fact that $20\% = 20/100 = 1/5$ and $50\% = 1/2$
- $$\frac{P+Q}{5} = \frac{P-Q}{2}$$
- Now we can either cross multiply to arrive at answer
- Or, bring $(P-Q)$ to other side and use Componendo-Dividendo

5. A sum is divided among 120 men and some women in the ratio 15: 21. If each man gets 5 and each woman gets 4, then find the number of women?

- Here let number of women be 'x'
- Since men get 5, they get total 5×120
- Since women get 4, they get total $4x$
- Given:
$$\frac{5 \times 120}{4x} = \frac{15}{21}$$
- Cross multiplying $60x = 120 \times 5 \times 21$
- $$X = \frac{120 \times 105}{60} = 2 \times 105 = 210$$

6. If y varies directly as $(x + 3)$ and $y = 8$ when $x = 1$. Then, find the value of y when $x = 2$?

- Given: $y \propto (x + 3)$
- Which means $y = k(x + 3)$
- Putting given values of y and x , $8 = k(1 + 3)$
- $8 = 4k$ which implies, $k = 2$
- Now, putting $x = 2$, $y = 2 * (2 + 3) = 10$

7. 360 is divided among four friends Sakshi, Rupa, Rohit and Monu in the ratio of 3:4:5:6. What is Sakshi's share of money?

- Very similar to angles of quadrilateral question
8. A and B together have Rs. 1210. If $\frac{4}{15}$ th of A's amount is equal to $\frac{2}{5}$ th of B's amount, how much amount does B have?
 (We just require to convert word problem into mathematical equations)
9. A right circular cone and a right cylinder have the same radius and the same volume.
 Then find the ratio of the height of the cone to that of the cylinder
 (Here you need to know about volume of cone and right cylinder)

6) MILK AND WATER PROBLEMS:

1. In a mixture 60 litres, the ratio of milk and water 2: 1. If this ratio is to be 1: 2, then the quantity of water to be further added is:

(All such problems are very easily solved if we just tract total milk and total water in the mixture)

- Out of 60 ltr, milk: water is 2:1, so, 40 ltr milk and 20 ltr water
 - Now we want to add water and make this ratio 1:2
 - If x water is required to do this, then, total mixture is $60+x$ out of which 40 is milk and $20+x$ is water
 - We also know that, $40*2 = 20+x$
 - Which gives us, $x = 60$ litres
2. The ratio of milk and water in one pot is 3:5 and that in another pot is 6:1. In what ratio should the contents of the two pots be mixed as to make the ratio between milk and water 7:3?
- Here we can take as much quantity of mixtures as we want
 - Let, ' x ' litres of first mixture and ' y ' litres of second mixture be taken and mixed to get the ratio 7: 3
 - Of ' x ' litres from first mixture, $\frac{3x}{8}$ litres is milk and $\frac{5x}{8}$ litre is water
 - Of ' y ' litres from second mixture, $\frac{6y}{7}$ litre is milk and $\frac{y}{7}$ litre is water
 - Total milk: water ratio is $\frac{\frac{3x}{8} + \frac{6y}{7}}{\frac{5x}{8} + \frac{y}{7}} = \frac{7}{3}$ (*given*) – we just cross multiply and simplify
 - $3\left(\frac{3x}{8} + \frac{6y}{7}\right) = 7\left(\frac{5x}{8} + \frac{y}{7}\right)$
 - $x:y = 44:91$
3. Three pots have same volume. Ratios of milk to water are 3: 2, 7: 3, 11: 4 respectively. If all are mixed, find the resulting ratio.

(NOTE: $3+2 = 5$; $7+3 = 10$; $11+4 = 15$; If we assume each mixture to be of 30 litres (lcm of 5, 10, 15), our calculations will become simple)

- In pot 1: 18 litre milk and 12 litres water
- In pot 2: 21 litre milk and 9 litres water
- In pot 3: 22 litre milk and 8 litres water
- In total mixture of 90 litres, 61 litre milk and 29 litres water
- Thus, ratio is 61: 29

7) PARTNERSHIPS

When two or more persons invest their money and run a business jointly, then the persons involved in this transaction are called partners and this transaction is called 'Partnership'. Questions revolve around partners' share in profit or loss when they invest some money for some time in the common venture.

Thus, there are two variables – share in capital & time

TYPES OF QUESTIONS:

1. Only one variable

Here, one of the two – time or share in capital is same for all partners and proportion of profit/loss depends on other variable.

Q. Two partners Amrit and Apurva started a business by investing 6,00,000 and 5,00,000 respectively. After one year, they earned profit of 1,21,000. Find the share which both will get in the profit.

This is the simplest problem based on partnership.

- Here, period of investment is 1 year for both partners
- Investment of Amrit and Apurva is in the ratio 6 lakh: 5 lakh i.e, 6: 5
- Hence, Amrit will get $\frac{6}{11}$ part of profit; $\frac{5}{11}$ part of profit will go to Apurva
- Hence, 66,000 will go to Amrit & 55,000 will go to Apurva

Q. A and B started a business with a total capital of 3,00,000. At the end of the year, they shared the profit in the ratio of their investments. If their capitals were interchanged, then A would have received 130% more than what he actually received. Find the capital of B.

(This is a variant of above problem. Here problem is asked in reverse. Only challenge is to convert the problem into a mathematical equation. This particular skill will come in handy to solve almost all kinds of word problems)

- Let capital of B be 'x' (we often assume the quantity to be determined as unknown 'x')
- Thus, capital of A is $300000 - x$
- Ratio of investments is $(300000 - x): x$, which is same as ratio of profits
- We're given that, if we interchange the investments, A would receive 130% more profit.
- If capitals were x and $(300000 - x)$, A would receive profits in this proportion
- $130\% \text{ of } (300000 - x) + (300000 - x) = x$
- *Solving for x we get, $x = 2,09,090.90$ – which is the capital of B*

2. Two variables:

Here, both initial investment as well as time for which each partner remains invested is variable. All we have to do is calculate total investment made by each partner for the entire period to get the proportion of investment. As earlier, they will get share of profit/loss in proportion of their investment.

Q. Ashwini starts a business with 18,000 and Rajeev joins the business 5 months later with an investment of 36,000. After a year, they earn a profit of 39,000. Find the shares of Ashwini and Rajeev in the profit amount depending on their individual investment.

- Here we note that, Ashwini was invested for 12 months and Rajeev was invested for 7 months
- Investment ratio is $18000 \times 12 : 36000 \times 7 = 6 : 7$

- Share in profit will also be in the ratio 6: 7
- Ashwini will get $\frac{6}{13} \times 39000 = 18000$ and rest 21000 will go to Rajeev

Q. The ratio in which Ajeet and Veena have contributed to the capital of a company is 3: 4. Veena has invested his capital for only 3 months and has received half as much profit as Ajeet, at the end of the year. Find out for how much time has Ajeet invested his capital in the company.

(Here question is reversed and we're to find time Ajeet stayed invested based on the profits they earned. As always, it'll boil down to converting given problem into mathematical equations)

- Let Ajeet invested 3x and Veena invested 4x in the company
- Veena invested for 3 months
- Let, Ajeet invested his money for y months
- Ratio of investments of Ajeet and Veena is $3xy: 12x$
- Now, we're given: $\frac{12x}{3xy+12x} = \frac{1}{2} \times \frac{3xy}{3xy+12x}$
- Thus, $y = 8$

8) PYQS

CSE 2022: Consider the Question and two Statements given below:

Question: What is the age of Manisha?

Statement-1: Manisha is 24 years younger than her mother.

Statement-2: 5 years later, the ages of Manisha and her mother will be in the ratio 3: 5.

Which one of the following is correct in respect of the Question and the Statement?

- Statements-1 alone is sufficient to answer the Question
- Statement-2 alone is sufficient to answer the question
- Both Statement-1 and Statement-2 are sufficient to answer the Question
- Both Statement-1 and Statement-2 are not sufficient to answer the Question

CSE 2022: There are two containers X and Y contains 100 ml of milk and Y contains 100 ml of water. 20 ml of milk from X is transferred to Y. After mixing well, 20 ml of the mixture in Y is transferred back to X. If m denotes the proportion of milk in X and n denotes the proportion of water in Y, then which one of the following is correct?

- $m=n$
- $m > n$
- $m < n$
- Cannot be determined due to insufficient data

CSE 2021: A student appeared in 6 papers. The maximum marks are the same for each paper. His marks in these papers are in the proportion 5 : 6 : 7 : 8 : 9 : 10. Overall he scored 60%. In how many papers did he score less than 60% of the maximum marks?

- 2
- 3
- 4
- 5

CSE 2021: An amount of money was distributed among A, B and c in the ratio p: q: r.

Consider the following statements:

1. A gets the maximum share if p is greater than (q+r).
2. C gets the minimum share if r is less than (p+q).

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

CSE 2020: A bottle contains 20 litres of liquid Q. 4 litres of liquid A is taken out of its and replace by same quantity of liquid B. Again 4 litres of the mixture is taken out and replaced by same quantity of liquid B. What is the ratio of quantity of liquid A to t hat of liquid B in the final mixture?

- (a) 4:1
- (b) 5: 1
- (c) 16: 9
- (d) 17: 8

CSE 2020: A sum of Rs. 2,500 is distributed among X, Y and Z in two ratio $1/2 : 3/4 : 5/6$. What is the difference between the maximum share and the minimum share?

- (a) Rs. 300
- (b) Rs. 350
- (c) Rs. 400
- (d) Rs. 450

CSE 2017: Q. P works thrice as fast as Q, whereas P and Q together can work four times as fast as R. If P, Q and R together work on a job, in what ratio should they share the earnings?

- (a) $3 : 1 : 1$
- (b) $3 : 2 : 4$
- (c) $4 : 3 : 4$
- (d) $3 : 1 : 4$

CSE 2017: The monthly incomes of X and Y are in the ratio of 4 : 3 and their monthly expenses are in the ratio of 3: 2. However, each saves Rs. 6,000 per month. What is their total monthly income?

- (a) Rs. 28,000
- (b) Rs. 42,000
- (c) Rs. 56,000
- (d) Rs. 84,000

CSE 2017: The average rainfall in a city for the first four days was recorded to be 0.40 inch. The rainfall on the last two days was in the ratio of 4: 3. The average of six days was 0.50 inch. What was the rainfall on the fifth day?

- (a) 0.60 inch
- (b) 0.70 inch
- (c) 0.80 inch
- (d) 0.90 inch

CSE 2017: There is a milk sample with 50% water in it. If 1/3rd of this milk is added to equal amount of pure milk, then water in the new mixture will fall down to

- (a) 25%
- (b) 30%
- (c) 35%
- (d) 40%

CSE 2016: The total emoluments of two persons are the same, but one gets allowances to the extent of 65% of his basic pay and the other gets allowances to the extent of 80% of his basic pay. The ratio of the basic pay of the former to the basic pay of the latter is:

- (a) 16: 13
- (b) 5: 4
- (c) 7: 5
- (d) 12: 11

CSE 2016: 30g of sugar was mixed in 180 ml water in a vessel A, 40 g of sugar Was mixed in 280 ml of water in vessel B and 20 g of sugar was mixed in 100 ml of water in vessel C. The solution in vessel B is

- (a) sweeter than that in C
- (b) sweeter than that in A
- (c) as sweet as that in C
- (d) less sweet than that in C

CSE 2015: Two equal glasses of same type are respectively 1/3 and 1/4 full of milk. They are then filled up with water and the contents are mixed in a pot. What is the ratio of milk and water in the pot?

- (a) 7: 17
- (b) 1: 3
- (c) 9: 21
- (d) 11: 23

CSE 2015: The monthly incomes of Peter and Paul are in the ratio of 4: 3. Their expenses are in the ratio of 3 :2. If each save Rs. 6,000 at the end of the month, their monthly incomes respectively are (in Rs.)

- (a) 24,000 and 18,000
- (b) 28,000 and 21,000
- (c) 32,000 and 24,000
- (d) 34,000 and 26,000

CSE 2013: In a rare coin collection, there is one gold coin for every three non-gold coins. 10 more gold coins are added to the collection and the ratio of gold coins to non-gold coins would be 1: 2. Based on the information; the total number of corns in the collection now becomes

- (a) 90
- (b) 80
- (c) 60
- (d) 50

CSE 2013: Out of 120 applications for a post, 70 are male and 80 have a driver's license. What is the ratio between the minimum to maximum number of males having driver's license?

- (a) 1 to 2
- (b) 2 to 3
- (c) 3 to 7
- (d) 5 to 7

CSE 2013: In a garrison, there was food for 1000 soldiers for one month. After 10 days, 1000 more soldiers joined the garrison. How long would the soldiers be able to carry on with the remaining food?

- (a) 25 days
- (b) 20 days
- (c) 15 days
- (d) 10 days

9) COMPREHENSION

Environmental problems cause health problems. Substantial changes in lifestyle can reduce environmental or health problems, but this idea appears almost impossible to adopt. With environmental problems, individual efforts can be perceived as having a negligible effect and therefore lead to inertia. With health, on the other hand, individual choices can make the difference between life and death, literally. And yet, barring a few, there seems to be the same collective lethargy towards making their choices.

Q. Which one of the following statements best implies the most rational assumption that can be made from the passage?

- (a) We are likely to spend more money on cure than prevention.
- (b) It is the job of the government to solve our environmental and public health problems.
- (c) Health can be protected even if environmental problems go on unattended.
- (d) Loss of traditional lifestyle and the influence of western values led to some unhealthy way of living.

Many people are not eating the right food. For some, it is simply a decision to stick with food they enjoy but which is not too healthy. This is leading to an increase in non-communicable diseases. This in turn leads to major burden on our health-care systems that have the potential to derail the economic progress which is essential for the poor to improve their lives. For others, it is about limited access to nutritious food or a lack of affordability, leading to monotonous diets that do not provide the daily nutrients for them to develop fully. Part of the reason nutrition is under threat worldwide is that our food systems are not properly responding to nutritional needs. Somewhere along that long road from farm to fork, there are serious detours taking place.

Q. Which one of the following statements best reflects the crux of the passage?

- (a) The scheme of Universal Basic Income should be implemented worldwide as a way of poverty alleviation.
- (b) We must place food-based nutrition at the centre of our policy debate.
- (c) Nutritional status of food should be improved by creating appropriate genetically modified crops.
- (d) Using modern food processing technologies, we must fortify food items with required nutrient elements

We often hear about conflicts among different States in India over river waters. Of the 20 major river systems, 14 are already water-stressed; 75% of the population lives in water-stressed regions, a third of whom live in water-scarce areas. Climate change, the demands of rising population and the need for agriculture to keep pace, and increased rate of urbanization and industrialization will exacerbate water stress. According to the Constitution of India, water is a State subject and not that of the Union, except for regulation of inter-State rivers. Key to ensuring balance between competing demands of various stakeholders is a basin-based approach to allocate water amongst constituent regions and States. Allocating fair share of water to them requires assessments based on objective criteria, such as specificities of the river basin, size of dependent population, existing water use and demand, efficiency of use, projected future use, etc. while ensuring the environmental needs of the river and aquifers.

Q. Which one of the following statements best reflects the most rational, practical and immediate action required to ensure fair and equitable allocation of water to different stakeholders?

- (a) A national, pragmatic, legal and policy framework for water allocation should be made.
- (b) All river systems of the country should be linked and huge aquifers created.
- (c) Water channels between regions of water surplus and regions of water deficit should be created.
- (d) To mitigate water crisis, water demand of sectors such as agriculture and industry should be reduced

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