



GS FOUNDATION BATCH FOR CSE 2024

CSAT

**Booklet-1 : Introduction
to CSAT**

Booklet-1: Introduction to CSAT

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1) CSAT IN CSE

- In 2011, UPSC changed the pattern of examination wherein optional papers in prelims were replaced by CSAT paper
- Objective was:
 - o Test candidate based on innate abilities (aptitude) and intelligence rather than knowledge – which is tested in mains
 - o To have common test for all – rather than different optional
- It was introduced considering expert opinions which held that, aptitude is more important for a potentially good civil servant rather than domain specific knowledge
- Aptitude tests are aimed at evaluating innate ability and intelligence of students to assess their suitability towards skills like analytical ability, comprehension skills, abstract thinking, sense of patterns and rhythms, linguistic skills and thinking on the feet – all are important in career of a civil servant
- In 2015 – CSAT was made qualifying requiring 1/3rd marks to pass
- Since 2019, UPSC has significantly increased difficulty level of CSAT paper. Comprehension passages are complex, have only 1-2 questions per passage and options are confusing. Moreover, quant questions involve application of multiple concepts which require conceptual clarity.
- Thus, despite being a qualifying paper, CSAT requires specific attention to gain the confidence and surety of passing prelims.

2) WHAT IT ENTAILS: SYLLABUS

- **Comprehension**
- **Interpersonal skills including communication skills**
- **Logical reasoning and analytical ability:** Include – arrangements, rankings, sequence-series, blood relations, clocks, calendars, directions, puzzles, cubes and dice, non-verbal reasoning
- **Decision making and problem solving** – No question asked after 2014 (since CSAT became qualifying). Questions like which option is best course of action in given situation are asked.
- **General mental ability:** Include – syllogisms-Venn diagrams, statement – assumption – conclusion, assertion – reason, routes and networks, probability, combinatorics
- **Basic numeracy** (numbers and their relations, orders of magnitude, etc.) (Class X level),
- **Data interpretation** (charts, graphs, tables, data sufficiency etc. — Class X level)

3) HOW MUCH TO PREPARE: WEIGHTAGE OF TOPICS

	2015	2016	2017	2018	2019	2020	2021	2022	2023
Comprehension	32	27	31	26	30	25	27	27	27
LR + AR	14	18	13	10	9	6	14	13	13
General Mental Ability	9	8	8	14	9	8	5	11	10
Basic Numeracy	20	27	25	16	29	38	26	22	24
DI	5	0	3	14	3	3	8	7	6

- Comprehension questions will be around 25-30
- Basic Maths will have around 30 questions
- Reasoning will have around 15 questions
- If you get 40 questions right out of 45 of Maths + Reasoning – you'll pass prelims with 100% surely
- Or if you get 20 right from comprehension and 20 from LR+DI+ some basic numeracy – you'll pass

4) CHANGING PATTERN

1. 2011-2015
2. 2015-2019
3. 2019-2022
4. 2023-??

What exactly is changing?

- Focus on basic concepts especially: Arithmetic, Geometry, Mathematization, Counting etc.
- Mixing of concepts like calendar with division – need clarity
- Statement based questions like paper-1
- Comprehension – tougher language and questions

2023: Some change but mostly continuity

- Change was with regard to lengthiness of questions rather than difficulty
- Comprehension was on similar lines
- Focus seemed much more on Arithmetic, PnC, Linear equations
- Even traditional questions were mixed with Arithmetic

5) SKILLS NEEDED: DECIPHERING FAQS AMONG PYQS

Most important skills:

1. Basic understanding & clarity in arithmetic – from number system, place values to operating with fractions
2. Basic understanding of Geometry – basic shapes, area-volume etc. – get used in quant as well as reasoning questions.
3. Mathematisation – ability to convert given word problem into equations
4. Understanding different mathematical concepts like percentage, ratio, average, HCF-LCM etc. and ability to use them with clarity
5. Basics of reasoning – standard questions get asked
6. Put values in question/equation to get answer quickly
7. Ability to use options given to get quick answer without calculation
8. Comprehension skills – Understand the context and assumptions the author has made; Understand the vocabulary used; Understand the core message and inference of the author; Choosing the correct option from similar looking alternatives; Reading speed
9. Choice of questions becoming crucial
10. Balance between comprehension and other questions
11. Reading speed
12. Thinking on the feet and analytical ability – not enough to mug up formulae
13. Lots of practice to gain confidence

6) PLAN OF ACTION:

What to prepare

- We've to prepare everything mentioned in the syllabus but some areas we can focus more
- From our above analysis of areas where questions are getting asked, we know that we cannot avoid understanding basic mathematics.
- Moreover, it is linked with reasoning, data sufficiency, data interpretation as well as general mental ability
- Hence, that will be our priority number one.

- Apart from that, comprehension has the largest chunk of questions – while it is tougher to master in short time, we'll have to practice

Course plan

- We shall begin with basic arithmetic, basic geometry and mathematisation: as these are the basic skills required for all topics to be discussed in the course
- It will be followed by a lesson on basics of comprehension – where we shall understand what it is all about and what kind of questions get asked
- Quantitative aptitude and Reasoning will go together so that you don't get bored with one section
- In every lecture we shall discuss 1/2 comprehensions at the end – to ensure continual practice. We shall preferably discuss some passage from editorial of the day or some previously asked passage by the UPSC
- Every class sheet will have some assignment problems

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CSAT - 02

**(Quantitative Aptitude_1_Basics of
Arithmetic-)**

Ace CSAT 2023

Booklet-2: Quantitative aptitude-1

Basics of Arithmetic

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Quantitative Aptitude Introduction

Quantitative aptitude is a surest pathway towards ensuring qualification in CSAT. And fortunately, it is also the easiest.

UPSC expects: 10th level understanding of basic numeracy. To be specific:

- Basic numeracy (numbers and their relations, orders of magnitude, etc. (Class X level),
- Data interpretation (charts, graphs, tables, data sufficiency etc. — Class X level)

But remember that, this syllabus is indicative and not complete. For instance, percentages, averages, ratio-proportion, probability, combinatorics etc. are topics getting asked in the exam but not explicitly mentioned in the syllabus.

1) NUMBER SYSTEM

1.1 What is Number System and Number Line

Mathematics is about finding patterns, structures, regularity, rules that govern what we see and representing these patterns in a language and if there's no language, inventing a new one. A number system is such a language.

A number system is a writing system for expressing numbers; that is, a mathematical notation for representing numbers of a given set, using digits or other symbols in a consistent manner.

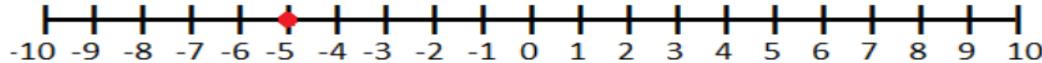
Number System is a method of representing Numbers on the Number Line with the help of a set of Symbols and rules.

Number Line:

A Number line is a representation of Numbers with a fixed interval in between on a straight line. A Number line contains all the types of numbers like natural numbers, Integers, etc. Numbers on the number line increase while moving Left to Right and decrease while moving from right to left.

Ends of a number line are not defined i.e., numbers on a number line range from infinity on the left side of the zero to infinity on the right side of the zero.

1.2 Natural Numbers



1, 2, 3, 4... are called natural numbers on number line. Natural numbers are called natural because they are used for counting naturally.

The set of natural numbers is the most basic system of numbers because it is intuitive, or natural, and hence the name. We use natural numbers in our everyday life, in counting discrete objects, that is, objects which can be counted like number of benches in class or number of sheep on a farm.

Every Natural number has a successor and every natural number except 1 has a predecessor.

Even Numbers: 2, 4, 6, ...

Odd numbers: 1, 3, 5, ...

1.3 Whole Numbers

All natural numbers except 1 have predecessors. So, to the collection of natural numbers we add zero as the predecessor for 1.

The resulting set is that of Whole numbers

i.e. 0, 1, 2, 3, ...

Even Numbers: 0, 2, 4, 6, ...

Odd numbers: 1, 3, 5, ...

$$0 + 0 = 0$$

$$0 + 0 = 0$$

$$0 + 0 = 0$$

CSE 2023: Three of the five positive integers p, q, r, s, t are even and two of them are odd (not necessarily in order).

Consider the following:

1. $p + q + r - s - t$ is definitely even. ✓

2. $2p + q + 2r - 2s + t$ is definitely odd. ✗

Which of the above statements is/are correct?

- (a) 1 Only (b) 2 Only (c) Both 1 and 2 (d) Neither 1 nor 2

$$p + q + r - s - t$$

e	e	e	o	o
e	o	e	o	e

CSE 2020: Q. Consider the following sequence of numbers:

5 1 4 7 3 9 8 5 7 2 6 3 1 5 8 6 3 8 5 2 2 4 3 4 9 6

How many odd numbers are followed by the odd number in the above sequence?

- (a) 5
- (b) 6
- (c) 7
- (d) 8

1.4 Integers:

Integers are the collection of Whole Numbers plus the negative values of the Natural Numbers.

i.e. ..., -3, -2, -1, 0, 1, 2, 3, ...

Apart from this, there are rational numbers and irrational numbers on the real number line.

The sum of three consecutive integers is equal to their product. How many such possibilities are there?

- (a) Only one
- (b) Only two
- (c) Only three
- (d) No such possibility is there

$$\begin{aligned}
 & 3u = u(u+1)(u-1) \\
 & 3u = u^3 - u \\
 & u^3 - u = 0 \\
 & u(u^2 - 1) = 0 \\
 & u(u-1)(u+1) = 0
 \end{aligned}$$

$\boxed{u=0}$

$\boxed{u=\pm 1}$

2) PLACE VALUES IN DECIMAL SYSTEM

A decimal number is made up of an integer and a fractional part that is separated by a dot called the decimal point.

For example, 3.19 is a decimal number in which 3 is the integer part and .37 is the fractional part; in -57.3948, -57 is an integer part and .3948 is the fractional part

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	-	tenths	hundredths	thousandths	ten thousandths	hundred thousandths	
HTH	TTTh	Th	H	T	O	-	t	h	th	tth	hth	
100,000	10,000	1,000	100	10	1	-	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$	$\frac{1}{10,000}$	$\frac{1}{100,000}$	
Whole Number Part						Decimal Point	Fractional Part					

$$\begin{aligned}
 & u_1y_2u_1y_2 \times 1000 \\
 & + u_1y_2
 \end{aligned}$$

$u_1y_2 \times 100$

CSE 2023: For any choices of values of X, Y and Z, the 6-digit number of the form XYZXYZ is divisible by:

- (a) 7 and 11 only (b) 11 and 13 only (c) 7 and 13 only (d) 7, 11 and 13

4 9
1 9
0 9

CSE 2020: Q. The difference between a 2-digit number and the number obtained by interchanging the positions of the digits is 54.

$$xy - yx = 54$$

1. The sum of the two digits of a number can be determined only if the product of the two digits is known.

2. The difference between the two digits of the number can be determined.

Which of the above statements is/are correct?

- (a) 1 only
(b) 2 only
~~(c) Both 1 and 2~~
(d) Neither 1 nor 2

$$\begin{array}{r} 10x+y \\ - 10y-x \\ \hline 9x-y \end{array} = 54$$

$$90 - 9$$

$$91 - 19$$

$$82$$

CSE 2019: Q. The ratio of a two-digit natural number to a number formed by reversing its digits is 4/7. The number of such pairs is

- a. 5
b. 4
c. 3
d. 2

$$xy - yx = 54$$

$$\frac{xy}{yx} = \frac{4}{7}$$

$$\cancel{x-y} = 6$$

$$\frac{10x+y}{10y+x} = \frac{4}{7}$$

3) MATHEMATICAL OPERATIONS

2.1 What is Mathematical operation?

Mathematical operation is a function which takes input values to a well-defined output value.

There are five fundamental operations that are of special use to us. They're addition, subtraction, multiplication, division and power

$$70x+7y = \frac{40y}{+4x}$$

$$66x = 33y$$

$$2x = y$$

0

1

2

3

4

2.2 Addition on number line:

The addition of two numbers results in the total amount or sum of those values combined.

Addition operation represented as '+' takes given number to the right side on number line by unit value of number being added to it.

Properties:

- Addition of two even numbers is even number
- Addition of two odd numbers is an even number
- Addition of odd number and even number is odd number
- Addition of two integers is always an integer
- Addition of two natural numbers is always a natural number
- Addition is associative and commutative.

- Addition with decimals

For example: $0.35 + 0.5 = 0.85$

2.3 Subtraction

Subtraction is the process of taking away a number from another. It is a primary arithmetic operation that is denoted by a subtraction symbol (-) and is the method of calculating the difference between two numbers.

Subtraction is a reverse of addition operation as here we go towards the left on number line.

Properties:

- Subtraction of two even numbers is even number
- Subtraction of two odd numbers is an even number
- Subtraction of odd number and even number is odd number
- Subtraction of two integers is always an integer
- Subtraction of two natural numbers is NOT always a natural number
- Subtraction is associative but NOT commutative
- **Subtraction with decimals: similar to addition**

$$\begin{array}{rcl} e - e & = & e \\ o - o & = & e \\ e - o & = & o \end{array}$$

2.4 Multiplication

Multiplication is an operation that represents the basic idea of repeated addition of the same number.

The numbers that are multiplied are called the factors and the result that is obtained after the multiplication of two or more numbers is known as the product of those numbers.

Multiplication is used to simplify the task of repeated addition of the same number.

Ex: $9 \times 6 = 9 + 9 + 9 + 9 + 9 + 9 = 54$

- I assume you're familiar with process to multiply two numbers say 569×63
- I also hope you recall your multiplication tables at least till 20!
- What is number multiplied by 1?
- What is number multiplied by 0?
- What is number multiplied by 10, 100, 1000?
- Rules for sign of **multiplication with negative numbers**:

$$\begin{aligned} \text{negative number} \times \text{positive number} &= \text{negative number} \\ \text{negative number} \times \text{negative number} &= \text{positive number} \end{aligned}$$

So, $(-2) \times 2 = -4$ and $(-2) \times (-2) = 4$

Other Properties:

- Multiplication of two numbers if one or both are even is even number
- Multiplication of two odd numbers is an odd number

- Multiplication of two integers is always an integer
- Multiplication of two natural numbers is always a natural number
- Multiplication is associative and commutative

Multiplication with decimals:

$0.2 \times 3.1 = 0.62$; $0.2 \times 31 = 6.2$; $2 \times 0.31 = 0.62$; $2 \times 3.1 = 6.2$ etc.

Rule:

- Write all numbers removing zeros at the end. For example: 0.30 is to be written as 0.3; 1001.00900 is to be written as 1001.009 etc.
- Multiply two or more numbers disregarding the decimal points
- Count the total digits after decimal point in all numbers. If it is 'n', put the decimal point in product after 'n' digits from right.

2.5 Division

The division is one of the basic arithmetic operations in math in which a larger number is broken down into smaller groups having the same number of items.

It is the inverse of the multiplication operation. While dividing numbers, we break down a larger number into smaller numbers such that the multiplication of those smaller numbers will be equal to the larger number taken.

- I assume you're familiar with long division process
- I assume you know the meaning of terms – dividend, divisor, quotient, remainder, factor

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

- Dividend is divisible by divisor if remainder is zero. In other words, it is a divisible if divisor is a factor of dividend.
- What is number divided by 1?
(Remainder is always 0 – Hence every number is divisible by 1!)
- What is number divided by itself?
(Remainder is always 0 – Hence every number is divisible by itself)
- What is number divided by 10, 100, 1000 etc.
- What is 0 divided by any number?
- What is number divided by 0?
Q. Why can't we divide by 0?
- Rules for negative numbers are same as that of multiplication as division is just inverse operation of multiplication. To be specific, division by 'n' is multiplication by $1/n$.

A B C D E f or, digit

→ A B C
 D E F

$C+f = 11$ digit
 $B+E = 10$ } only logic
 $A+D = 10$ forming .

Division with decimal point:

Very similar to multiplication only in reverse. In multiplication, we add total digits after decimal point, here we subtract. But better way is to just remove decimal point by multiplying both numerator and denominator with appropriate power of 10 like 10, 100, 1000 etc. as it just shifts decimal point to right.

For example:

$$\frac{0.62}{0.2} = 3.1; \frac{0.62}{2} = 0.31; \frac{6.2}{2} = 3.1; \frac{6.2}{0.2} = 31; \frac{62}{0.2} = 310$$

CSE 2023: What is the sum of all 4-digit numbers less than 2000 formed by the digits 1, 2, 3 and 4, where none of the digits is repeated?

- (a) 7998 (b) 8028 (c) 8878 (d) 9238

1 - 2 - 4 - 3 *Punjabi*

1 2 3 9
8 2 4

3
2
1

CSE 2023: If ABC and DEF are both 3-digit numbers such that A, B, C, D, E, and F are distinct non-zero digits such that ABC + DEF = 1111, then what is the value of A+B+C+D+E+F?

- (a) 28 (b) 29 (c) 30 (d) 31

CSE 2023: A 3-digit number ABC, on multiplication with D gives 37DD where A, B, C and D are different non-zero digits. What is the value of A+B+C?

- (a) 18 (b) 16 (c) 15 (d) Cannot be determined due to insufficient data

Google min

1 1 7 9 9 8
7 9 9 8

CSE 2023: Let pp, qq and rr be 2 digit numbers where p < q < r. If pp + qq + rr = tt0, where tt0 is a 3-digit number ending with zero,

consider the following statements:

1. The number of possible values of p is 5.
2. The number of possible values of q is 6

Which of the above statements is/are correct?

- (a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

CSE 2023: Each digit of a 9-digit number is 1. It is multiplied by itself. What is the sum of the digits of the resulting number?

- (a) 64 (b) 80 (c) 81 (d) 100

CSE 2023: AB and CD are 2-digit numbers. Multiplying AB with CD results in a 3-digit number DEF. Adding DEF to another 3-digit number GHI results in 975. Further A, B, C, D, E, F, G, H, I are distinct digits. If E= 0, F=8, then what is A+B+C equal to?

- (a) 6 (b) 7 (c) 8 (d) 9

CSE 2023: If p, q, r and s are distinct single digit positive numbers, then what is the greatest value of (p + q)(r + s)?

- (a) 230 (b) 225 (c) 224 (d) 221

CSE 2023: What is the sum of all digits which appear in all the integers from 10 to 100?

- (a) 855 (b) 856 (c) 910 (d) 911

CSE 2023: D is a 3-digit number such that the ratio of the number to the sum of its digits is least. What is the difference between the digit at the hundred's place and the digit at the unit's place of D?

- (a) 0 (b) 7 (c) 8 (d) 9

CSE 2020: Consider the following addition problem: $3P+4P+PP+PP=RQ2$; where P, Q and R are different digits.

What is the arithmetic mean of all such possible sums?

- (a) 102
(b) 120
(c) 202
(d) 220

CSE 2020: Consider the following multiplication problem:

$(PQ) \times 3 = RQQ$, where P, Q and R are different digits and $R \neq 0$.

What is the value of $(P+R) \div Q$?

- (a) 1
(b) 2
(c) 5
(d) Cannot be determined due to insufficient data

CSE 2020: How many zeroes are there at the end of the following product?

$$1 \times 5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50 \times 55 \times 60$$

- (a) 10
(b) 12
(c) 14
(d) 15

CSE 2017: Certain 3-digit numbers following characteristics: 1. All the three digits are different. 2. The number is divisible by 7. 3. The number on reversing the digits is also divisible by 7. How many such 3-digit numbers are there?

- (a) 2
(b) 4
(c) 6
(d) 8

CSE 2015: If ABC x DEED = ABCABC; where A, B, C, D and E are different digits, what are the values of D and E?

- (a) D = 2, E = 0
- (b) D = 0, E = 1
- (c) D = 1, E = 0
- (d) D = 1, E = 2

Is multiple of an integer an integer?

Is fraction of an integer an integer?

Is addition/ multiplication/ subtraction/ division of two integers an integer?

4) EXPONENTS

The exponent or power of a number shows how many times the number is multiplied by itself.

For example, $2 \times 2 \times 2 \times 2$ can be written as 2^4 , as 2 is multiplied by itself 4 times.

Here, 2 is called the "base" and 4 is called the "exponent" or "power."

We call it '2 raised to 4' or '2 to the power 4'

Similarly, $9 \times 9 \times 9 = 9^3$

In general, x^n means that x is multiplied by itself for n times.

CSE 2020: For what value of n, the sum of digits in the number $(10^n + 1)$ is 2?

- (a) For n = 0 only
- (b) For any whole number n
- (c) For any positive integer n only
- (d) For any real number n

SQAURE-SQUARE ROOT/ CUBE-CUBE ROOT

- A number's 2nd power is called its square and its third power is called its cube

For example:

square of 2 = $2^2 = 2 \times 2 = 4$ and square root of 4 is 2 or $\sqrt{4} = \sqrt[2]{4} = 2$

Cube of 2 = $2^3 = 2 \times 2 \times 2 = 8$ and cube root of 8 is 2 or $\sqrt[3]{8} = 2$

IMPORTANT NOTE:

Square root of a number has 2 possible values. Square root of 4 for instance can be 2 as well as -2.

$9^{\frac{1}{2}} = \pm 3$ and so on.

Important Squares and Cubes

Number	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	91
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625
26	676
27	729
28	784
29	841

Number	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

NOTE: Square of an integer is always positive. **Unit's place for any square will always be one of 0, 1, 4, 5, 6 or 9.**

In other words, numbers ending with 2, 3, 7 or 8 can NEVER be squares.

CSE 2017: The age of Mr. X last year was the square of a number and it would be the cube of a number next year. What the least number is of years he must wait for his age to become the cube of a number again? (CSE CSAT-2017)

- (a) 42
- (b) 38
- (c) 25
- (d) 16

PROPERTIES OF EXPONENTS

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $a^0 = 1$ for all 'a'
- $a^1 = a$
- $a^{-1} = \frac{1}{a}$
- $a^{-n} = \frac{1}{a^n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $a^{1/n} = \sqrt[n]{a}$

EXPONENTS WITH FRACTIONS:

- $a^{m/n} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$

Unit's digit in exponent

Regardless of powers or bases involved, all we have to look for are powers of unit's place digit of the base. It follows directly from the definition of powers.

- Q. What is the unit's digit in 125^4
- Q. What is unit's digit in 21321325^{12321}
- Q. What is unit's digit in 12^{33}
- Q. What is unit's digit in 1223^{45}
- Q. What is unit's digit in $1223^{45} \times 125^{123} \times 46572^{22}$

Exponents with decimals:

We just have to combine multiplication with decimals with definition of exponents to deal with powers of decimals

For example: $0.2^3 = 0.008$; $1.2^2 = 1.44$; $(1.69)^{\frac{1}{2}} = \pm 1.3$;

Find $(0.04)^{-\left(\frac{1}{2}\right)}$. Take only positive value of power.

$$(0.04)^{-\left(\frac{1}{2}\right)} = \frac{1}{0.04^{\frac{1}{2}}} = \frac{1}{\pm 0.2} = \pm 5$$

CSE 2023: What is the unit digit in the expansion of $(57242)^{9 \times 7 \times 5 \times 3 \times 1}$?

- (a) 2 (b) 4 (c) 6 (d) 8

5) PRIME NUMBERS AND PRIME FACTORIZATION

- **Prime number** is a number that is divisible only by itself and 1.
 - Numbers that have other factors than 1 and itself are called **Composite numbers**.
- For example: 2, 3, 5, 7
- Is 6 a prime number? Is 15 a prime number?
- Is 1 a prime number? Is 1 a composite number? Is 1 unique?
- A number greater than 1 with exactly two factors, i.e., 1 and the number itself is a prime number
 - Enlist prime numbers from 1 to 100:
 - Are there even prime numbers or do prime numbers always have to be odd?

Q. CSE 2023: Choose the group which is different from the others:

- (a) 17, 37, 47, 97
 (b) 31, 41, 53, 67
 (c) 71, 73, 79, 83
 (d) 83, 89, 91, 97

CSE 2023: Consider the following in respect of prime number p and composite number c.

1. $p+c / p-c$ can be even.
2. $2p+c$ can be odd.
3. pc can be odd.

Which of the statements given above are correct?

- (a) 1 and 2 only (b) 2 and 3 only (c) 1 and 3 only (d) 1, 2 and 3

Q. CSE 2022: Consider the following statements in respect of two natural numbers p and q such that p is a prime number and q is a composite number:

1. $p \times q$ can be an odd number.
2. q / p can be a prime number.
3. $p + q$ can be a prime number.

Which of the above statements are correct?

- (a) 1 and 2 only
 (b) 2 and 3 only
 (c) 1 and 3 only

(d) 1, 2 and 3

CSE 2020:

Q. Two Statements S1 and S2 are given below followed by a Question:

S1: n is a prime number.

S2: n leaves a remainder of 1 when divided by 4.

Question: If n is a unique natural number between 10 and 20, then what is n? Which one of the following is correct in respect of above Statements and the Question?

- (a) S1 alone is sufficient to answer the Question.
- (b) S2 alone is sufficient to answer the Question.
- (c) S1 and S2 together are sufficient to answer the Question, but neither S1 alone nor S2 alone is sufficient to answer the Question.
- (d) S1 and S2 together are not sufficient to answer the Question.

CSE 2019: Consider two statements S1 and S2 followed by a question:

S1: p and q both are prime numbers.

S2: p+q is an odd integer.

Question: Is pq an odd integer?

Which one of the following is correct?

- a. S1 alone is sufficient to answer the question
- b. S2 alone is sufficient to answer the question
- c. Both S1 and S2 taken together are not sufficient to answer the question
- d. Both S1 and S2 are necessary to answer the question

PRIME FACTORIZATION:

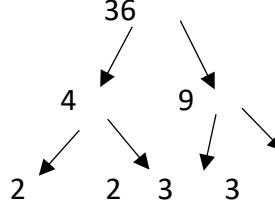
Note that a number greater than 1 will always have a prime factor that is a number will be divisible by some prime number.

- If number is itself a prime number, it itself is a prime factor
- If number is a composite number, then it has factors other than 1 and itself, those factors can be prime or composite and thus they also will have some other factors – this way we will get some prime number as a factor of a given number

The process of writing a number as the product of powers of prime numbers is prime factorization.

For example,

- Prime factorization of 3 (prime number) is number itself
- Prime factorization of a square of prime number like $9 = 3 \times 3 = 3^2$
- the prime factorization of composite number like 36 can be done in the following way:

Division Method (starting with the smallest prime factor)	Factor Tree method
$ \begin{array}{r rr} 2 & 36 \\ \hline 2 & 18 \\ & 9 \\ \hline 3 & 3 \\ & 1 \end{array} $	

$$36 = 2 \times 2 \times 3 \times 3$$

Divisibility can now be better understood with this understanding.

A number N is divisible by a prime number 'p' if and only if 'p' appears in the prime factorisation of N.

This is because, when we're writing prime factorisation, we're basically enlisting all the prime factors of a number and writing all of them in expanded form. If prime number does not appear here, it is certainly not a factor of given number.

Further, a number N is divisible by another number M if and only if all the prime factors of M are also prime factors of N with respective powers.

Number of Factors:

Any number of the form $p^a q^b r^c$ will have $(a+1)(b+1)(c+1)$ factors, where p, q, r are prime numbers.

For example: $6 = 2 \times 3$ will have $(1+1)(1+1) = 4$ factors namely: 1,2,3,6

$24 = 2^3 \times 3$ will have $(3+1)(1+1) = 8$ factors

- A prime number will always have only 2 factors
- If a number has exactly 3 factors, it is always of the form p^2 where 'p' is a prime number

Q. CSE 2022

What is the remainder when $91 \times 92 \times 93 \times 94 \times 95 \times 96 \times 97 \times 98 \times 99$ is divided by 1261?

- (a) 3
- (b) 2
- (c) 1
- (d) 0

CSE 2023: What is the remainder when $85 \times 87 \times 89 \times 91 \times 95 \times 96$ is divided by 100?

- (a) 0 (b) 1 (c) 2 (d) 4

CSE 2022: Q. $15 \times 14 \times 13 \times \dots \times 3 \times 2 \times 1 = 3^m \times n$

Where m and n are positive integers, then what is the maximum value of m?

- (a) 7

- (b) 6
 - (c) 5
 - (d) 4

CSE 2022: Let p be a two-digit number and q be the number consisting of same digits written in reverse order. If $p \times q = 2430$, then what is the difference between p and q ?

- (a) 45
 - (b) 27
 - (c) 18
 - (d) 9

6) DIVISIBILITY TESTS

1. Divisibility Test of 2

A number having, 0, 2, 4, 6, 8 at unit's place is divisible by 2. All such numbers are called even numbers.

Can any even number greater than 2 be prime?

2. Divisibility Test of 3

If sum of all the digits of a number is divisible by 3, number is divisible by 3.

For example: 434322 – sum of digits is 18 which is divisible by 3 hence the number is divisible by 3.

Q. Which of the following are divisible by 3?

Q. How many prime numbers can you form from digits 1,2,3,4,5?

3. Divisibility Test of 4

If a two-digit number created by the last two digits of a number is divisible by 4, the number is divisible by 4.

In short, we just have to look at last two digits and divide those by 4. If they're divisible by 4, so is the number.

For example: 234504; 234508; 23358912; 12345600; 13424340; 120 are all divisible by 4
While, 12213210; 123123; 95498330; 123142 are not divisible by 4.

Q. Which of the following numbers are divisible by 4?

1. 2389739816
 2. 1781418
 3. 1222222
 4. 444444
 5. 8888
 6. 100000
 7. 189898988
 8. 1982187
 9. 1231289123
 10. 213123176
- Will any odd number be divisible by 4?
 - Will every number divisible by 4, divisible by 2?
 - If you know all the 2-digit numbers divisible by 4, will it suffice to test if the given number is divisible by 4 or not?

4. Divisibility Test of 5

If a number has 0 or 5 at unit's place, it is divisible by 5. The test for 5 is thus very simple and straightforward. You just have to examine unit's place.

For example: 12345; 231230; 1231235; 8485895; 43243200 etc. are all divisible by 5. While, 10000001; 24384234; 3432842; 3289219 etc. are not.

5. Divisibility Test of 6

We recall the prime factorisation of 6. We've $6 = 2 \times 3$. Hence, if a number is divisible by both 2 and 3, it is also divisible by 6. There's no separate test for 6 as such.

NOTE: we can check divisibility by any number using this technique. We simply write number's prime factorization or any factorization and check if all the factors with corresponding powers are factors of any other given number.

For instance, if you want to check if 864292324 is divisible by 24, we just write $24 = 2^3 \times 3$. We just check if number is divisible by $2^3 = 8$ and 3

(You can use divisibility tests or just directly divide. Since, the number is large, direct division would take more time, it is better to use tests.)

6. Divisibility Test of 7

We isolate last digit (units place) of given number. We double it and subtract from remaining number. If this difference is divisible by 7, the number is divisible by 7. If the difference is too large, we repeat the above process as many times as we want.

Examples:

1234: units place is 4; its double is 8. We subtract 8 from 123. We get 115 which is not divisible by 7. So, 1234 is not divisible by 7.

2345679:

- Units place is 9.
- $9 \times 2 = 18$
- $234567 - 18 = 234549$
- Now for 234549, units place is 9 and its double is 18
- $23454 - 18 = 23436$
- Now for 23436, units place is 6
- $6 \times 2 = 12$
- $2343 - 12 = 2331$
- For 2331, units place is 1
- $1 \times 2 = 2$
- $233 - 2 = 231$
- For units place is 1, its double is 2
- $23 - 2 = 21$
- Since, 21 is divisible by 7, 2345679 is divisible by 7

7. Divisibility Test of 8

If the number formed by last 3 digits of a given number (hundred's, tens and unit's place) is divisible by 8, the number is divisible by 8.

Note that, for 2 we looked at last digit; for 4 we looked at last 2 digits and now for 8 we look at last 3 digits. Can you see the pattern and think what would we be looking at for checking divisibility of 16, 32, 64?

Q. which of the following are divisible by 8?

213892189333212; 21234; 83737888; 223981222; 2389000; 238914344; 29831008;
12831900; 12391777; 37817064

8. Divisibility Test of 9

If sum of all the digits of a number is divisible by 9, number is divisible by 9.

Thus, the test for 9 is very similar to test for 3.

Which of the following are divisible by 9?

1. 456738

9. Divisibility Test of 10

If the number has 0 at unit's place, the number is divisible by 10. This is perhaps the simplest test; you just have to see if the last digit is 0 or not.

10. Divisibility Test of 11

For a given number, if the difference between sums of alternate digits is divisible by 11, the number is divisible by 11.

This is a very peculiar test.

Let, the number be some abcdefghijklm

We look at sums of alternate digits. So, we have $(a + c + e + g + i + k + m)$ and $(b + d + f + h + j + l)$ as sums. If the difference between them, i.e.,

$(a + c + e + g + i + k + m) - (b + d + f + h + j + l)$ is divisible by 11 then the number abcdefghijklm is divisible by 11.

For instance,

let's take 387249254394938100 – the difference between sums is $46 - 35 = 11$, which is divisible by 11. Hence the number is divisible by 11.

Let's take 333333; difference between sums is 0 – which is divisible by 11 (0 is divisible by all numbers – as remainder is always 0)

Q. Which of the following are divisible by 11?

1. 121
 2. 1289319
 3. 1331
 4. 31298
 5. 1111
 6. 8573930
 7. 29832139801001203

8. 3298432993
9. 2893123
10. 999999999999
11. 2222222222
12. 18181818181818
13. 1221122112211221

11. Divisibility Test of 25

If the last two digits of a number are 00, 25, 50 or 75, then the number

Q. UPSC 2022:

An Identity Card has the number ABCDEFG, not necessarily in that order, where each letter represents a distinct digit (1, 2, 4, 5, 7, 8, 9 only). The number is divisible by 9. After deleting the first digit from the right, the resulting number is divisible by 6. After deleting two digits from the right of original number, the resulting number is divisible by 5. After deleting three digits from the right of original number, the resulting number is divisible by 4. After deleting four digits from the right of original number, the resulting number is divisible by 3. After deleting five digits from the right of original number, the resulting number is divisible by 2.

Which of the following is a possible value for the sum of the middle three digits of the number?

- (a) 8
- (b) 9
- (c) 11
- (d) 12

CSE 2021: Q. Consider all 3-digit numbers (without repetition of digits) obtained using three non-zero digits which are multiples of 3. Let S be their sum.

Which of the following is/are correct?

- 1. S is always divisible by 74.
- 2. S is always divisible by 9.

Select the correct answer using the code given below:

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

CSE 2021: Q. The number 3798125P369 is divisible by 7. What is the value of the digit P?

- (a) 1
- (b) 6
- (c) 7
- (d) 9

CSE 2021: Q. When a certain number is multiplied by 7, the product entirely comprises ones only (1111...). What is the smallest such number?

- (a) 15713
- (b) 15723
- (c) 15783
- (d) 15873

CSE 2020: Let XYZ be a three-digit number, where $(X + Y + Z)$ is not a multiple of 3. Then $(XYZ + YZX + ZXY)$ is not divisible by

- (a) 3
- (b) 9
- (c) 37
- (d) $(X + Y + Z)$

CSE 2020: Two statements are given followed by two Conclusions:

Statements:

All numbers are divisible by 2. All numbers are divisible by 3.

Conclusion-I: All numbers are divisible by 6.

Conclusion-II: All numbers are divisible by 4.

Which of the above Conclusions logically follows/follow from the two given Statements?

- (a) Only Conclusion-I
- (b) Only Conclusion-II
- (c) Neither Conclusion-I nor Conclusion-II
- (d) Both Conclusion-I and Conclusion-II

CSE 2020: Q. How many integers are there between 1 and 100 which have 4 as a digit but are not divisible by 4?

- (a) 5
- (b) 11
- (c) 12
- (d) 13

CSE 2020: Q. How many five-digit prime numbers can be obtained by using all the digits 1, 2, 3, 4 and 5 without repetition of digits?

- (a) Zero
- (b) One
- (c) Nine
- (d) Ten

Q. CSE 2020: Q. A digit $n > 3$ is divisible by 3 but not divisible by 6. Which one of the following is divisible by 4?

- (a) $2n$
- (b) $3n$
- (c) $2n + 4$
- (d) $3n + 1$

CSE 2020: Q. An 8-digit number 4252746B leaves remainder 0 when divided by 3. How many values of B are possible?

- a. 2
- b. 3
- c. 4
- d. 6

CSE 2019: Q. Number 136 is added to 5B7 and the sum obtained is 7A3, where A and B are integers. It is given that 7A3 is exactly divisible by 3. The only possible value of B is

- a. 2
- b. 5
- c. 7
- d. 8

CSE 2016: If R and S are different integers both divisible by 5, then which of the following is not necessarily true?

- (a) $R - S$ is divisible by 5
- (b) $R + S$ is divisible by 10
- (c) $R \times S$ is divisible by 25
- (d) $R^2 + S^2$ is divisible by 5

7) BODMAS RULE

3.1 Order of applying mathematical operations: BODMAS rule

BODMAS stands for **Bracket, Order, Division, Multiplication, Addition, and Subtraction**.

This is the fundamental rule we study in arithmetic to apply when there is more than one operation involved.

- So, first we've to resolve bracket; then we solve exponents or roots
- Thereafter, we solve division and multiplication from left to right
- And then we solve addition or subtraction from left to right

For instance, if the value of $3 + 5 \times 2 \div 2 - 6 + 2^3 \times (4 - 2)$ is asked, we use above BODMAS rule to find the value.

Other useful properties:

- $a - (b + c) = a - b - c$
- $a \times (b + c) = a \times b + a \times c$
- $a \times (b - c) = a \times b - a \times c$
- $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$

Q. CSE 2020: If \$ means divided by: @ means multiplied by: # means 'minus', then the value of 10#5@1\$5 is

- a. 0
- b. 1
- c. 2
- d. 9

8) HCF/GCD AND LCM

4.1 Factor or divisor of a number and multiple of a number

- We've already seen what is the meaning of factor or divisor of a number: M is factor of N if N is divisible by M.
- Multiple of a number N is all such number mN where m is any natural number. So, multiples of 3 would be 6, 9, 12, 15 and so on.

4.2 HCF (Highest Common Factor) or GCD (Greatest Common Divisor) of two numbers

- HCF or GCD of two numbers is the highest possible number that divides both the numbers or that is factor of the both numbers.
- HCF can be defined for more than two numbers as well. In that case, HCF would be highest number among all the common factors of the given numbers.

Example: Consider 18 and 27.

- All the factors of 18 are 1, 2, 3, 6, 9 and 18
- All the factors of 27 are 1, 3, 9 and 27
- Highest common factor is 9 and thus HCF/GCD of 18 and 27 is 9.

Finding HCF/GCD:

There are various methods of finding HCF of two numbers. But most efficient and simple is the method of prime factorisation.

1. Method of Prime-factorization:

In this method, we write prime factorization of given numbers. We then write all the common factors with least power amongst the given numbers. Number we obtain after multiplying them is the required HCF or GCD.

Q. Find GCD of 54 and 117.

- $54 = 2 \times 3^3$
- $117 = 3^2 \times 13$
- Common factor is 3 and its least power is 2.
- GCD is $3^2 = 9$

Q. Find HCF of 512 and 288.

- $512 = 8^3 = (2^3)^3 = 2^9$
- $288 = 9 \times 32 = 2^5 \times 3^2$
- Common factor is 2 and least power is 5.
- HCF is $2^5 = 32$

2. Method of listing Factors

In this method, we simply list all the possible factors of both or all numbers in increasing order, the highest common factor listed is the HCF or GCD.

Q. Find GCD of 54 and 117

- Factors of 54 are 1, 2, 3, 6, 9, 18, 27, 54
- Factors of 117 are 1, 3, 9, 13, 39, 117
- HCF is 9.

***NOTE:** listing all the factors can be time consuming for larger numbers. Generally, we have to check all prime numbers till half of the number to check if they are the factor.

For smaller numbers, this method is quite simple and clear.

3. Method of division:

It involves following steps:

- Step 1: We divide larger number by smaller number and check the remainder. If the remainder is 0, smaller number is a factor of larger number and thus smaller number itself is HCF. If not, we go to step 2.
- Step 2: This remainder becomes new divisor and the earlier divisor becomes new dividend
- Step 3: We continue step 2 till we get remainder as 0. Once we get that, last divisor will be HCF of two numbers.

Example:

Q. Find GCD of 54 and 117

$$\begin{array}{r}
 2 \\
 54) \overline{117} \\
 -\underline{108} \quad 6 \\
 9 \overline{)54} \\
 -\underline{54} \\
 0
 \end{array}$$

Hence, we have the 0 remainder. Last divisor i.e., 9 is the gcd or hcf

Q. Find HCF of 512 and 288.

$$\begin{array}{r}
 1 \\
 288) \overline{512} \\
 -\underline{288} \quad 1 \\
 224 \overline{)288} \\
 -\underline{224} \quad 3 \\
 64 \overline{)224} \\
 -\underline{192} \quad 2 \\
 32 \overline{)64} \\
 -\underline{64} \\
 0
 \end{array}$$

Hence, we've 0 reminder. Last divisor 32 is the GCD or HCF.

Properties of HCF:

- HCF of two or more numbers is always a factor of all the numbers
- If any number divides all the given numbers, it also divides the HCF of the given numbers.
- HCF is always less than or equal to all the given numbers
- HCF of two prime numbers is always 1
- HCF of a prime number and the number which is not a multiple of it, is always 1. HCF of a prime number and some multiple of a prime number is that prime number.

(Example: HCF of 2, 9 = 1; HCF of 17, 51 = 17)

Coprime Numbers: A pair of numbers are coprime if their HCF is 1. The numbers do not have to be primes to be coprime with each other.

For Example: (2, 5); (4, 9); (245, 3) are all pairs of coprime numbers

- NOTE that: If one of the two numbers is prime, then numbers are coprime if the other number is not divisible by prime. (As only factors of prime are 1 and itself, if prime is not a factor of a given number, then HCF would be automatically 1)
- Two even numbers can never be coprime
- Two consecutive numbers are always coprime

4.3 LCM (Least Common Multiple) of two numbers

- The LCM of two numbers or the least common multiple of two numbers is the smallest number which is a multiple of those two numbers.
- If the numbers are, n and m, then we have n, 2n, 3n, 4n, ... as multiples of n and m, 2m, 3m, ... as multiples of m.
- Smallest common multiple in these two lists would be our LCM.
- NOTE that, multiplication of two numbers will always yield a common multiple. In other words, mn will always be a multiple of both m and n (common multiple) – in LCM we want a smallest such common multiple. At the most LCM can be multiplication of given numbers.

Example: LCM of 6 and 9.

- Multiples of 6 are – 6, 12, 18, 24, 30, 36 ...
- Multiples of 9 are – 9, 18, 27, 36, ...
- Common multiples are – 18, 36, ...
- Least common multiple thus is 18.

Finding LCM

Like for HCF, there are various methods to find LCM of two or more numbers. Most efficient method is using prime factorization here as well. But we shall see all the methods and you can decide which you find the easiest.

1. Method of listing multiples:

Here, we simply enlist multiples of both (or all) the given numbers. We then identify the smallest of the common multiples and that is our LCM.

2. Method of prime factorization:

We write the prime factorization of both the numbers. We then multiply each factor with the highest powers to calculate the LCM of two numbers.

Example: Find LCM of 40 and 54.

- $40 = 2^3 \times 5$
- $54 = 2 \times 3^3$
- Factors are – 2, 3 and 5. And their highest powers are 3, 3 and 1
- $LCM = 2^3 \times 3^3 \times 5 = 8 \times 27 \times 5 = 1080$

3. Finding LCM using HCF (and vice versa)

We use the relation: **$HCF \times LCM = Product\ of\ two\ numbers$**

$$\text{Hence, } LCM = \frac{\text{Product of two numbers}}{HCF} \quad \text{Similarly, } HCF = \frac{\text{Product of two numbers}}{LCM}$$

Example: Find LCM of 40 and 54

- HCF of 40 and 54 is 2
- $LCM = \frac{40 \times 54}{2} = 1080$

Properties of LCM:

- All the numbers are factor of their LCM
- LCM is always greater than or equal to all the given numbers
- LCM of two numbers is one of the numbers if and only if, the other number is its factor (For example: lcm of 4 and 16 is 16 as 4 is factor of 16; lcm of 8 and 56 is 56 as 8 is factor of 56 and so on)
- LCM of two coprime numbers is their product

CSE 2023: There are three traffic signals. Each signal changes colour from green to red and then from red to green. The first signal takes 25 seconds, the second signal takes 39 seconds and the third signal takes 60 seconds to change the colour from green to red. The durations for green and red colours are same. At 2:00 p.m, they together turn green. At what time will they change to green next, simultaneously?

- (a) 4:00 p.m. (b) 4:10 p.m. (c) 4:20 p.m. (d) 4:30 p.m

CSE 2022: Q. What is the smallest number greater than 1000 that when divided by any one of the numbers 6, 9, 12, 15, 18 leaves a remainder of 3?

- (a) 1063
 (b) 1073
 (c) 1083
 (d) 1183

CSE 2020: What is the least four-digit number when divided by 3, 4, 5 and 6 leaves a remainder 2 in each case?

- (a) 1012
 (b) 1022
 (c) 1122
 (d) 1222

CSE 2020: Q. Joseph visits the club on every 5th day, Harsh visits on every 24th day, while Sumit visits on every 9th day. If all three of them met at the club on a Sunday, then on which day will all three of them meet again?

- (a) Monday
- (b) Wednesday
- (c) Thursday
- (d) Sunday

CSE 2019: Q. In a school every student is assigned a unique identification number. A student is a football player if and only if the identification number is divisible by 4, whereas a student is a cricketer if and only if the identification number is divisible by 6. If every number from 1 to 100 is assigned to a student, then how many of them play cricket as well as football?

- (a) 4
- (b) 8
- (c) 10
- (d) 12

CSE 2019: Seeta and Geeta go for a swim after a gap of every 2 days and every 3 days respectively. If on 1st January both of them went for a swim together, when will they go together next?

- (a) 7th January
- (b) 8th January
- (c) 12th January
- (d) 13th January

CSE 2016: There are five hobby clubs in a college —photography, yachting, chess, electronics and gardening. The gardening group meets every second day, the electronics group meets every third day, the chess group meets every fourth day, the yachting group meets every fifth day and the photography group meets every sixth day. How many times do all the five groups meet on the same day within 180 days?

- (a) 5
- (b) 18
- (c) 10
- (d) 3

CSE 2015: There are five hobby clubs in a college viz, photography, yachting, chess, electronics and gardening. The gardening group meets every second day, the electronics group meets every third day, the chess group meets every fourth day, the yachting group meets every fifth day and the photography group meets every sixth day. How many times do all the five groups meet on the same day within 180 days?

- (a) 3
- (b) 5
- (c) 10
- (d) 18

CSE 2014: A bell rings every 18 minutes. A second bell rings every 24 minutes. A third bell rings every 32 minutes. If all the three bells ring at the same time at 8 o'clock in the morning, at what other time will they all ring together?

- (a) 12: 40 hrs
- (b) 12: 48 hrs
- (c) 12: 56 hrs
- (d) 13: 04 hrs

CSE 2014: Five persons fire bullets at a target at an interval of 6, 7, 8, 9 and 12 seconds respectively. The number of times they would fire the bullets together at the target in an hour is

- (a) 6
- (b) 7
- (c) 8
- (d) 9

CSE 2011: Q. Three persons start walking together and their steps measure 40 cm, 42 cm and 45 cm

respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

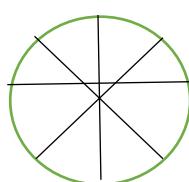
- (a) 25 m 20 cm
- (b) 50 m 40 cm
- (c) 75 m 60 cm
- (d) 100 m 80 cm

9) FRACTIONS

1.1 What is fraction

A fraction shows part of a whole. A fraction can be a portion or section of any quantity out of a whole, where the whole can be any number, a specific value, or a thing.

For example: If we have a cake. We cut it into say 8 equal pieces. Each piece is equivalent to $1/8$ part of a whole cake.



Fraction : $\frac{\text{Numerator}}{\text{Denominator}}$

Here both numerator and denominator are integers.

Denominator is never 0 as fraction is essentially a division and we never divide by 0.

- **Proper Fraction:** Numerator < Denominator Ex: $\frac{2}{7}, \frac{456}{666}, \frac{123}{1222}$ etc.
- **Improper fraction:** Denominator < Numerator Ex: $\frac{3}{2}, \frac{45}{23}, \frac{31233143}{12312}$ etc.
- **Mixed fractions:** Whole number and a proper fraction $4\frac{1}{4}, 32\frac{2}{3}, 3\frac{3}{5}$ etc.

Properties of fractions:

- If denominator = 1, fraction reduces to an integer
- If numerator = 0, fraction is 0 (as 0 divided by anything is 0).
- Fraction doesn't change if both numerator and denominator are multiplied or divided by same number

$$\frac{a}{b} = \frac{n \times a}{n \times b} = \frac{a \div n}{b \div n}$$

Thus, $\frac{2}{3} = \frac{4}{6} = \frac{20}{30} = \frac{30}{45}$ and $\frac{30}{40} = \frac{6}{8} = \frac{3}{4} = \frac{3/5}{4/5}$ etc.

1.2 Equivalence of fractions:

- When are two fractions equivalent?

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } a \times d = b \times c$$

Example: $\frac{1}{4}$ & $\frac{48}{192}$: Since $1 \times 192 = 48 \times 4 = 192$ Thus, fractions are equal

Q. Which of the following pair of fractions are equivalent?

- 1) $\frac{3}{2}; \frac{45}{30}$
- 2) $\frac{5}{17}; \frac{4}{15}$
- 3) $\frac{8}{23}; \frac{16}{46}$
- 4) $\frac{4/7}{3/28}; \frac{16}{3}$
- 5) $\frac{-4}{13}; \frac{8}{-26}$

1.3 Terminating and Non-terminating fractions:

- Try to convert fractions $\frac{1}{2}$ and $\frac{1}{3}$ into decimals by direct division.
- What is the difference?

- Now, try $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ – Which of these terminate? Which do not terminate?

Non-terminating but Recurring/repeating and non-repeating fractions:

- Recurring fractions are those where a group of numbers tend to repeat indefinitely after certain digits after decimal points.
- For ex: 123.121212121212...
- 0.22345634563456...

We show recurring fractions with small bar overhead.

In above examples, numbers would be 123. $\overline{12}$, 0.2 $\overline{23456}$

1.4 Converting fractions into decimals

- Terminating fractions:

- Q. Convert 0.5 into fractions.
Q. Convert 12.123 into fraction.

- Non-terminating but recurring fractions:

Type:1 – recurring starts immediately after decimal point

- Q. Convert 0.3333333... into fraction
 - We write: $x = 0.\overline{3}$ as only one digit is repeating, we multiply both sides by 10
 - $10x = 3.\overline{3}$ We now subtract x from $10x$ to get $9x = 3$.

- Q. Convert 0.12121212... into fraction
 Q. Convert 0.432432432... into fraction
 Q. Convert 1.232323... into fraction
 Q. Convert 0. $\overline{12333}$ into fraction

(We divide by 9 or 99 or 999 etc. depending on how many digits are recurring)

Type:2 – recurring part starts after few digits after decimal

- Q. Convert 0.1333... into fraction.
- We write $x = 0.1\overline{3} = 0.1\bar{3}$
 - Like earlier, we want to eliminate recurring part
 - We multiply by 10 to get only recurring part after decimal:
 - $10x = 1.333 \dots$ Now, can we eliminate recurring part by subtracting x from $10x$ – NO! as there's 1 after decimal. But, like earlier, we've recurring part just after decimal point.
 - Now, we can multiply by 10 to this equation as earlier
 - $100x = 13.333 \dots$ Now, subtracting:

- $90x = 12 \text{ or } x = \frac{12}{90} = \frac{2}{15}$

RULE:

- Write as many numbers of 9's in the denominator as the recurring digits followed by certain number of 0's
- Number of 0's is as many digits there are between recurring part and decimal
In above case, 9 followed by 1 zero i.e. 90 in denominator
- Numerator is difference between number after decimal taking recurring part only once and non-recurring part
In above case: $13 - 1 = 12$ in numerator
- Example: $0.\overline{345} = \frac{345 - 33}{990} = \frac{312}{990}$

- Q. Convert 0.1205050505... into fraction
 Q. Convert 0.22434343... into fraction
 Q. Convert 0.5123412341234... into fraction
 Q. 0.35555...
 Q. 0.2484848...
 Q. 0.12898989...

Type:3 – Number has non-zero number before decimal point

- This is simple extension of earlier rule.
- We have to include number before decimal point also as a part of non-recurring part of number while subtracting

For ex: $2.\overline{3} = \frac{23 - 2}{9} = \frac{21}{9} = \frac{7}{3}$

$3.2\overline{34} = \frac{32343 - 323}{9900} = \frac{32020}{9900}$

$$\text{Fraction} = \frac{\text{Total Number taking recurring part only once} - \text{Non recurring part}}{\text{'9's followed by '0's}}$$

- Q. Convert following decimals into fractions:
1. 1.234444...
 2. 2.1232323...
 3. 44.44444...
 4. 1.111222222....
 5. 6.00121212...

Q. CSE 2020: The recurring decimal representation 1.272727 ... is equivalent to

- (a) 13/11
 (b) 14/11
 (c) 127/99

(d) 137/99

1.5 Operating on fractions: addition, subtraction, multiplication, division, powers

- Addition of fractions

Case 1) Denominator of both/all fractions is same

- In this case, we simply add the numerators and keep denominator same to get the answer

For example:

$$\frac{5}{7} + \frac{1}{7} = \frac{5+1}{7} = \frac{6}{7}$$

$$\frac{12}{4} + \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$$

$$\frac{2}{7} + \frac{3}{7} + \frac{5}{7} = \frac{10}{7}$$

Case 2) Denominator is NOT same

- In this case, we've to make the denominator same and use case 1.
- We recall that, fraction does not change when we multiply or divide it by the same number.
- We use this property to multiply denominators by such numbers that they become equal.

For example:

$$\text{Consider } \frac{2}{3} + \frac{3}{4};$$

Here denominators are 3 and 4.

If we multiply 3 by 4 and 4 by 3, we shall get same number 12.

- We multiply numerator and denominator of first ratio by 4 and
- We multiply numerator and denominator of second ratio by 3

$$\begin{aligned}\frac{2}{3} + \frac{3}{4} &= \frac{2 \times 4}{3 \times 4} + \frac{3 \times 3}{4 \times 3} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} \\ \frac{2}{3} + \frac{3}{4} &= \frac{2 \times 4 + 3 \times 3}{3 \times 4} = \frac{17}{12}\end{aligned}$$

NOTE: In making denominators equal, we're essentially finding a common multiple of denominators. Product of two numbers will always be their common multiple as we've seen. But, have studied anything else involving common multiples?

So, we've two options,

- We just blindly use product of denominators as common multiple and change the ratios accordingly (Works well for most small ratios. But becomes very tough in case denominators are large)
- Or we can find LCM of two numbers and change the fraction accordingly

For example: $\frac{3}{80} + \frac{5}{16}$

- $\frac{3 \times 16 + 5 \times 80}{80 \times 16} = \frac{48 + 400}{80 \times 16} = \frac{448}{80 \times 16} = \frac{7}{20}$
- Or note that, LCM = 80 as 16 is a factor of 80
- $\frac{3}{80} + \frac{5 \times 5}{80} = \frac{28}{80} = \frac{7}{20}$
- Suggestion:
 - o If LCM is clearly visible like above or is easy to find (when one denominator is factor of other) OR if the numbers in denominator are too large to multiply quickly – go for LCM method
 - o Otherwise just cross multiply quickly and reduce the fraction you get at the end

Q. Find addition of following fractions:

- 1) $\frac{4}{3}; \frac{123}{23}$
- 2) $\frac{4}{5}; \frac{2}{3}$
- 3) $\frac{4}{11}; \frac{12}{29}$
- 4) $\frac{7}{8}; \frac{9}{10}$
- 5) $\frac{13}{21}; \frac{15}{21}$
- 6) $\frac{34}{102}, \frac{3}{17}$
- 7) $-\frac{3}{5}; \frac{4}{77}$
- 8) $-\frac{8}{13}; -\frac{4}{9}$

- Subtraction of fractions:

It is exactly same as addition of fractions.

- If denominators are same, we simply subtract second numerator from first

For example: $\frac{5}{12} - \frac{3}{12} = \frac{2}{12} = \frac{1}{6}$; $\frac{4}{7} - \frac{6}{7} = -\frac{2}{7}$

- If denominators are not same, we make them same like in addition by cross multiplication or with help of LCM and do the subtraction

For example:

$$\frac{6}{7} - \frac{2}{3} = \frac{6 \times 3 - 7 \times 2}{7 \times 3} = \frac{4}{21}$$

$$\frac{5}{16} - \frac{3}{80} = \frac{5 \times 5}{16 \times 5} - \frac{3}{80} = \frac{22}{80} = \frac{11}{40}$$

$$\frac{2}{5} - \frac{6}{7} = \frac{2 \times 7 - 6 \times 5}{5 \times 7} = -\frac{16}{35}$$

Multiplication of fractions:

Multiplication of fractions is simply multiplication of corresponding numerators and denominators.

In other words, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

For example: $\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$; $-\frac{3}{4} \times \frac{2}{5} = -\frac{6}{20} = -\frac{3}{10}$

Rules for multiplication with negative numbers hold for fractions as well
i.e., $-ve \times -ve = +ve$; $-ve \times +ve = +ve \times -ve = -ve$

- Division of fractions:

Division is simply reverse multiplication.

Thus, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ OR $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

For example: $\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{5}{4}$; $\frac{3/4}{2/3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$

- Powers of fractions:

Power of a fraction is fraction of powers of numerator and denominator. In other words,

$$\left(\frac{\text{Numerator}}{\text{Denominator}}\right)^a = \frac{\text{Numerator}^a}{\text{Denominator}^a}$$

- Rest all the properties of powers follow as it is for fractions as well

PROPERTIES OF EXPONENTS

- $\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n} = \frac{a^{m+n}}{b^{m+n}}$
- $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$
- $\left(\frac{a}{b}\right)^0 = 1$ for all ' $\left(\frac{a}{b}\right)$ '
- $\left(\frac{a}{b}\right)^1 = \left(\frac{a}{b}\right)$
- $\left(\frac{a}{b}\right)^{-1} = \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a}$
- $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \left(\frac{b}{a}\right)^n$
- $\left(\left(\frac{a}{b}\right)^m\right)^n = \left(\frac{a}{b}\right)^{mn}$
- $\left(\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)\right)^n = \left(\frac{a}{b}\right)^n \left(\frac{c}{d}\right)^n$

- $\left(\frac{a}{b}\right)^{1/n} = \sqrt[n]{\left(\frac{a}{b}\right)}$

EXONENTS WITH FRACTIONS:

- $\left(\frac{a}{b}\right)^{m/n} = \left(\left(\frac{a}{b}\right)^m\right)^{\frac{1}{n}} = \sqrt[n]{\left(\frac{a}{b}\right)^m}$

Q. What is $\left(\frac{a}{b}\right)^{5p/q} \times \left(\frac{a}{b}\right)^{9p/q}$?

Q. Find $(\frac{1}{3})^{15} \div (\frac{1}{6})^{15}$

Q. What is $\left(\frac{1}{4}\right)^{-5} \times 2^{-3}$

Q. What is $(37^5 \times (\frac{1}{3})^{3.453} \div 519^{34/3})^0$

Q. Find $((\frac{1}{3})^4)^2 \div 9$

Q. Find $(0.5)^4 \times 2^5$

Q. Simplify $\left(\frac{1}{b}\right)^{5p/q} \times \left(\frac{a}{b}\right)^{9/q}$

1.6 BODMAS rule with fractions

It is exactly same as BODMAS rule for integers. No change what so ever. In the worksheet you'll find problems on this part which only require you to use operating with fractions.

1.7 Finding fraction of a Number

Fraction of a number is a part of a number. So, we fundamentally divide the number into smaller portion.

Fraction of a number is obtained by simply multiplying number by the fraction.

For example: 2/5 of 25 = $\frac{2}{5} \times 25 = 10$; 3/8 of 40 = $\frac{3}{8} \times 40 = 15$ and so on.

Another terminology is used sometimes. Sometimes the question may say that, 5th portion of a number is 30. It only means that, a number when divided into 5 gives 30 or $\frac{1}{5} \times \text{number} = 30$. Thus number is $5 \times 30 = 150$.

Questions:

1. 4/7 of a number is 84. Find the number
2. Rachel took 1/2 hour to paint a table and 1/3 hour to paint a chair. How much time did she take in all?
3. If 3 $\frac{1}{2}$ m of wire is cut from a piece of 10 m long wire, how much of wire is left?
4. One half of the students in a school are girls, 3/5 of these girls are studying in lower classes. What fraction of girls are studying in lower classes?

5. A herd of cows gives 4 litres of milk each day. But each cow gives one-third of total milk each day. They give 24 litres milk in six days. How many cows are there in the herd?
6. Shelly walked $\frac{1}{3}$ km. Kelly walked $\frac{4}{15}$ km. Who walked farther? How much farther did one walk than the other?

Note on Irrational numbers:

- Irrational numbers are those numbers that cannot be written as a fraction or a ratio of two integers.
- Most popular examples: $\sqrt{2}, \sqrt{3}, \pi, e$
- Square roots of all non-square rational numbers, cube-roots of all non-cube rational numbers, multiples of irrational numbers like π, e – are all rational numbers
- Addition/subtraction of two different irrational numbers is always an irrational number
- Multiplication of two irrational numbers can be rational as well as irrational
Ex. $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$
- Division of two irrational numbers also can be rational as well as irrational
Ex: $\frac{3\pi}{\pi} = 3; \frac{\sqrt{54}}{\sqrt{6}} = \sqrt{9} = 3$
- Power of irrational number can be rational or irrational
Ex: $(\sqrt{5})^2 = 5; \left(\sqrt[4]{\frac{5}{7}}\right)^{12} = \left(\frac{5}{7}\right)^3 = \frac{125}{343}$
- Irrational and rational numbers together make real number (all numbers on the number line)



GS FOUNDATION BATCH FOR CSE 2024

Worksheet CSAT - 01 (Basics of Arithmetic)

Worksheet 1: Basics of Arithmetic – 1

1. Consider the following statements:

- I. If 'n' is an integer then '5n' is always an integer
- II. If '5n' is an integer then 'n' is always an integer

Which of the statements above is/are correct?

- A. 1 only
- B. 2 only
- C. Both 1 and 2
- D. Neither 1 nor 2

2. Let 'p' be a two-digit number divisible by 5 and q obtained by reversing the digits of p is also a two-digit number and divisible by 7. What is the sum of digits of 'p'?

- A. 7
- B. 9
- C. 11
- D. 12

3. What is the place (location) value of 5 in 3254710?

- A. 10000
- B. 5
- C. 54710
- D. 50000

4. The smallest number that should be subtracted from 2085, so that the new number is completely divisible by 23 is

- A. 9
- B. 15
- C. 20
- D. 19

5. What is the difference between place values of 3 in 123454321

- A. 2999700
- B. 299700
- C. 999900
- D. 99900

6. Consider the following statements:

- I. Smallest natural number is same as smallest positive integer
 - II. Subtraction of two natural numbers is always a whole number
- Which of the statements above is/are correct?

- A. 1 only

- B. 2 only
- C. Both 1 and 2
- D. Neither 1 nor 2

7. Two consecutive even positive integers, sum of the squares of which is 1060 are:

- A. 12, 14
- B. 16, 18
- C. 20, 22
- D. 22, 24

8. What is the value of $(0.027)^{-\frac{2}{3}}$

- A. $\frac{100}{9}$
- B. $-\frac{100}{9}$
- C. 0.3
- D. -0.3

9. What is the unit's digit in $6374^{193} \times 12345^{127} \times 111^{111}$

- A. 0
- B. 2
- C. 3
- D. 5

10. If $n = 1 + x$, where x is the product of four consecutive positive integers, consider the following statements:

- I. n is an even number
- II. n is prime number
- III. n is a perfect square
- IV. n is odd number

Which of the statements above are correct?

- A. I, III
- B. I, II
- C. I, IV
- D. III, IV

11. Consider following statements:

- I. Sum of first 4 prime numbers is less than 15
- II. A prime number is never divisible by 2
- III. Prime numbers are evenly distributed from 1 to 100

Which of the statements above are incorrect?

- A. 1 and 2 only
- B. 2 only

- C. 1, 2 and 3
- D. 1 and 3 only

12. Let x be the greatest 4-digit number, which when divided by 15, 20 and 28 leaves in each case the remainder 2. The sum of digits of x is

- A. 19
- B. 21
- C. 23
- D. 25

13. Consider $P = 4^{61} + 4^{62} + 4^{63} + 4^{64}$. Which of the following is a factor of P ?

- A. 9
- B. 10
- C. 11
- D. 12

14. How many of the following numbers are divisible by 132?

264, 396, 462, 792, 968, 2178, 5184, 6336

- A. 4
- B. 5
- C. 6
- D. 7

15. Number of students who have opted for the subjects A, B and C are 60, 84 and 108 respectively. The examination is to be conducted for these students such that only the students of the same subject are allowed in one room. Also, the number of students in each room must be same. What is the minimum number of rooms that should be arranged to meet all these conditions?

- A. 28
- B. 60
- C. 12
- D. 21

16. A student was asked to simplify the expression:

$$\left(\frac{0.1216 \times 0.105 \times 0.0002}{0.625 \times 0.08512 \times 0.039 \times 0.16} \right)^0$$

His answer was $\frac{1}{65}$. What is the difference between his answer and the correct answer?

- A. $\frac{1}{65}$
- B. $\frac{64}{65}$
- C. $\frac{1}{26}$

D. $\frac{7}{26}$

17. A student was asked to simplify the expression:

$$\left(\frac{0.1216 \times 0.105 \times 0.0002 \times 0}{0.625 \times 0.08512 \times 0.039 \times 0.16} \right)^1$$

His answer was $\frac{1}{65}$. What is the difference between his answer and the correct answer?

- A. $\frac{1}{65}$
- B. $\frac{64}{65}$
- C. $\frac{1}{130}$
- D. $\frac{3}{130}$

18. What is the value of $\left(\frac{1331}{729}\right)^{-\frac{1}{3}}$?

- A. $-\frac{11}{9}$
- B. 0.81
- C. 0.81
- D. $\frac{11}{9}$

19. If $\left(\frac{9}{7}\right)^3 \times \left(\frac{49}{81}\right)^{\frac{x}{2}} = \left(\frac{7}{9}\right)^8$. What is x?

- A. 5
- B. 8
- C. 11
- D. 12

20. If $2^x = \left(\frac{1}{8}\right)^{-3} \times 16^{\frac{3}{4}} \div 4^{-\frac{3}{2}}$. What is x?

- A. -6
- B. 11
- C. -12
- D. 15

Worksheet Solutions

Worksheet 1: Basics of Arithmetic

1. A
2. C
3. D
4. B
5. A
6. A
7. D
8. A
9. A
10. D
11. C
12. C
13. B
14. A
15. D
16. B
17. A
18. C
19. C
20. D

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GS FOUNDATION BATCH FOR CSE 2024

CSAT Booklet – 03

**(Quantitative Aptitude 2 Relative
Size of Numbers)**

Relative Size of Numbers

Contents

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1) TERMS AND BASICS

Sign	Meaning
$X > Y$	X is greater than Y
$X < Y$	X is less than Y
$X = Y$	X is equal to Y
$X \leq Y$	X is less than or equal to Y
$X \geq Y$	X is greater than or equal to Y

- Basic rules of comparison

- +ve number $>$ -ve number
- If $a > b$ and $b > c$ then $a > c$
- If $a > b$ then $a + c > b + c$
- If $a > b$ then $a - c > b - c$
- If $a > b$ and $c > 0$ then $ac > bc$
- If $a > b$ and $c < 0$ then $ac < bc$
- If $a > b$ and $c > d$ then $a + c > b + d$;
- If $a > b$ then $a^n > b^n$ if $n > 0$
- If $a > b$ then $\frac{1}{a} < \frac{1}{b}$
- If $a > b$, $\sqrt{a} > \sqrt{b}$, for $a, b \geq 0$
- $a^2 > 0$ if $a \neq 0$
- If $a, b > 0$ and $a < x < y$ then, $a^2 < x^2 < b^2$
- If $a, b < 0$ and $b < x < y$ then, $a^2 < x^2 < b^2$
- If $a > 0$ and $-a < x < a$ then, $0 < x^2 < a^2$
- If $a, b > 0$, $b > a$ and $-a < x < b$ then, $0 < x^2 < b^2$

Tip: Whenever you have any confusion, just take any sample example and verify the inequality

2) COMPARISON OF NATURAL NUMBERS

CSE 2020:

Let p, q, r and s be natural numbers such that

$$P - 2016 = q + 2017 = r - 2018 = s + 2019$$

Which one of the following is the largest natural number?

- (a) P
- (b) q
- (c) r
- (d) s

2017
2018
NOT
Consider

$$\begin{aligned} P &\geq q + 13 \\ R &= q + 15 \end{aligned}$$

CSE 2017: The sum of income of A and B is more than that of C and D taken together. The sum of income of A and C is the same as that of B and D taken together. Moreover, A earns half as much as the sum of the income of B and D. Whose income is the highest?

- (a) A
- (b) B
- (c) C
- (d) D

CSE 2017: If A runs less fast than B, and B runs as fast but not faster than C; then, as compared to A, C runs.

- (a) slower than A
- (b) faster than A
- (c) with same speed as A
- (d) Given data is not sufficient to determine

3) COMPARISON OF INTEGERS

Q. If X is between -3 and -1, and Y is between -1 and 1, then $X^2 - Y^2$ is in between which of the following?

- (a) -9 and 1
- (b) -9 and -1
- (c) 0 and 8
- (d) 0 and 9

4) COMPARISON OF TWO FRACTIONS

Case 1) When denominator is same

- When two fractions have same denominator, the one with higher numerator is greater.

Ex: Consider: $\frac{3}{7}$ and $\frac{4}{7}$ – they've same numerator and thus, $\frac{4}{7} > \frac{3}{7}$

Case 2) When numerator is same

- When two fractions have same numerator, the one with lower denominator is greater.

Ex: Consider $\frac{5}{7}$ and $\frac{5}{8}$ – they've same numerator and thus $\frac{5}{7} > \frac{5}{8}$

Case 3) When numerator and denominator is different

- In general, we cross multiply two ratios. The ratio whose numerator yields higher value when multiplied with the denominator of the other ratio, is greater.
- In other words, if $\frac{a}{b}$ and $\frac{c}{d}$ are ratios to be compared,
 - o If $a \times d > c \times b$ then $\frac{a}{b} > \frac{c}{d}$

- If $a \times d < c \times b$ then $\frac{a}{b} < \frac{c}{d}$

For example: Consider $\frac{5}{8}$ and $\frac{4}{7}$: $5 \times 7(35) > 8 \times 4(32)$ and thus, $\frac{5}{8} > \frac{4}{7}$

Q. Decide the comparison between following fractions.

- 1) $\frac{4}{3}; \frac{123}{23}$
- 2) $\frac{4}{5}; \frac{2}{3}$
- 3) $\frac{4}{11}; \frac{12}{29}$
- 4) $\frac{7}{8}; \frac{9}{10}$
- 5) $\frac{13}{21}; \frac{15}{21}$
- 6) $\frac{34}{11}; \frac{34}{17}$
- 7) $-\frac{3}{5}; \frac{4}{77}$
- 8) $-\frac{8}{13}; -\frac{4}{9}$

NOTE:

- You can use case 3) to solve all problems even if it is of the type of case 1 or case 2.
- In case of $\frac{m}{m+1}$ and $\frac{n}{n+1}$; the fraction with larger numerator is larger
i.e., $\frac{4}{5} > \frac{3}{4}$; $\frac{11}{12} > \frac{4}{5}$; $\frac{56}{57} > \frac{12}{13}$ etc.

CSE 2020: What is the largest number among the following?

- (a) $(1/2)^{-6}$ $\approx 2^6$
 (b) $(1/4)^{-3}$ $\approx 4^3$
 (c) $(1/3)^{-4}$ $\approx 3^4$
 (d) $(1/6)^{-2}$ $\approx 6^2$

CSE 2019: If the numerator and denominator of a proper fraction are increased by the same positive quantity which is greater than zero, the resulting fraction is

- (a) always less than the original fraction
- (b) always greater than the original fraction
- (c) always equal to the original fraction
- (d) such that nothing can be claimed definitely

CSE 2019: Which one of the following will have minimum change in its value if 5 is added to both numerator and the denominator of the fractions $2/3$, $3/4$, $4/5$ and $5/6$?

- (a) $2/3$
- (b) $3/4$
- (c) $4/5$
- (d) $5/6$

CSE 2018: X and Y are natural numbers other than 1, and Y is greater than X. Which of the following represents the largest number?

- (a) XY
- (b) X / Y
- (c) Y / X
- (d) (X + Y) / XY

5) COMPARING POWERS

Case 1) When numbers have same base

The one with higher power is larger number.

For example: $2^6 > 2^5 > 2^2 > 2^{1.4894} > 2^{-3}$ etc.

Case 2) When power is same:

If both base numbers are positive: the one with larger base is larger number

For example: $2^3 < 3^3 < 3.127812^3$;

$0.23^{12.12} < 0.74^{12.12} < 4.123^{12.12}$ etc.

Case 3) When neither base nor power is same:

We use above two known cases, known basic results along with basic rules of comparison to arrive at the conclusion.

Known results basically mean cubes and squares known to us from last class.

Q. Which is greater: 2^5 or 3^5

Q. Which is greater: 2^5 or 3^4

Q. Which is smaller: 2^{10} or 3^6

Q. Which is greater: 2^9 or 4^4

Q. Which is smaller: 2^{14} or 8^5

Q. Which is greater: 3^{25} or 27^8

Q. Which is smaller: 3^{27} or 4^{16}

Q. Which is smaller: 2^{34} or 3^{26}

- $2^{34} = 2 \times 8^{11}$
- $3^{26} = 9^{13} = 81 \times 9^{11}$
- $9 > 8$ and thus, $9^{11} > 8^{11}$

Q. Decide the comparison: $\sqrt{2}$ and $\sqrt[3]{3}$

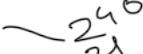
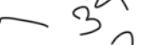
- $2^{\frac{1}{2}}$ and $3^{\frac{1}{3}}$
- Recall that, if $a > b$ then $a^n > b^n$. In other words, taking powers does not change the inequality
- LCM of 2 and 3 = 6. So, we take 6th power of both numbers to get rid of fractional powers without changing inequality
- Numbers become 2^3 and 3^2 i.e. 8 and 9.
- Since $8 < 9$; $\sqrt{2} < \sqrt[3]{3}$

Q. Find the greatest among following:

$$3^{50}, 4^{40}, 5^{30}, 6^{20}$$

- Try to make powers equal using $a^{bc} = (a^b)^c$
- What is HCF of 50, 40, 30, 20?

CSE 2022: Which number amongst $2^{40}, 3^{21}, 4^{18}, 8^{12}$ is smallest?

- (a) 2^{40} 
- (b) 3^{21} 
- (c) 4^{18} 
- (d) 8^{12} 

Q. Which is larger: 5.1×10^6 or 6.7×10^6

Q. Which is smaller: 2123×10^8 or 0.33×10^{12}

Q. Which is greater: 12×8^6 or 4×9^3

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GS FOUNDATION BATCH FOR CSE 2024

Worksheet - 2

(Relative size of numbers)

Worksheet 2: Relative Size of Numbers

1. Consider the following statements:

- 1) $a + b > a$ for all natural numbers a and b
- 2) $a + b > a$ for all whole numbers a and b
- 3) $a + b > a$ for all non-zero integers a and b

Which of the statements above is/are correct?

- A. 1 only
- B. 1 and 2 only
- C. 2 and 3 only
- D. 3 only

2. Given $x + \frac{1}{x} = 4$ and $y + \frac{1}{y} = 7$ which of the following is true?

- A. $\frac{1}{x+\frac{1}{x}} \geq y + \frac{1}{y}$
- B. $\frac{1}{x+\frac{1}{x}} > \frac{1}{y+\frac{1}{y}}$
- C. $\frac{1}{x+\frac{1}{x}} < \frac{1}{y+\frac{1}{y}}$
- D. $x + \frac{1}{x} \geq \frac{1}{y+\frac{1}{y}}$

3. Consider the following statements:

- 1) As $5 > 3, 5^n > 3^n$ for all $n > 0$
- 2) As $5 > 3, \sqrt{5} > \sqrt{3}$
- 3) $(123.123)^2, (-123.123)^2, (-0.123)^2$ are all positive

Which of the statements above is/are correct?

- A. 1 and 2 only
- B. 1 and 3 only
- C. 2 and 3 only
- D. 1, 2 and 3

4. Consider the following statements:

- 1) $\frac{m}{m+1} < \frac{n}{n+1}$ if $n > m$ and m, n are natural numbers
- 2) $\frac{56}{57} > \frac{12}{13}$

Which of the statements above is/are correct?

- A. 1 only
- B. 2 only
- C. Both 1 and 2
- D. Neither 1 nor 2

5. Amongst $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$ which number is the highest?
- $\sqrt{2}$,
 - $\sqrt[3]{3}$
 - $\sqrt[4]{4}$
 - Two numbers of the three are equal and are combined highest*
6. Find the greatest among following:
- $$4^{50}, 5^{40}, 6^{30}, 7^{20}$$
- 4^{50} ,
 - 5^{40} ,
 - 6^{30} ,
 - 7^{20}
7. Which number amongst 3^{14} , 2^{18} , 4^{13} , 128472^{-3} is the smallest?
- 3^{14} ,
 - 2^{18} ,
 - 4^{13} ,
 - 128472^{-3}
8. What is the correct arrangement in ascending order of fractions:
- $$\frac{42}{491}, \frac{30}{313}, \frac{35}{367}$$
- $\frac{42}{491}, \frac{30}{313}, \frac{35}{367}$
 - $\frac{42}{491}, \frac{35}{367}, \frac{30}{313}$
 - $\frac{35}{367}, \frac{30}{313}, \frac{42}{491}$
 - $\frac{35}{367}, \frac{42}{491}, \frac{30}{313}$
9. Which of the following options has numbers in relational order?
- $12 \times 8^6 < 4 \times 9^3$
 - $7658 \times 10^8 > 0.7658 \times 10^{13}$
 - $\frac{2345}{2346} < \frac{8976}{8977}$
 - $\sqrt{78} < -\sqrt{98}$
10. Which amongst the 2^{50} , 3^{40} , 5^{25} , 7^{21} is smallest?
- 2^{50} ,
 - 3^{40} ,
 - 5^{25} ,
 - 7^{21}

11. If A is between -4 and 1, and B is between 1 and 3, then $X^2 - Y^2$ is in between which of the following?
- A. 0 to 16
 - B. -16 to 9
 - C. 8 to 15
 - D. -7 to 15

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GS FOUNDATION BATCH FOR CSE 2024

CSAT Booklet – 04

**(Quantitative Aptitude_3_Basics of
Geometry)**

Basics of Geometry

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1) POINT

A point is a location in space. It is represented by a dot. Points are usually named with an upper-case letter. For example, we refer to the following as "point A"

. A

2) STRAIGHT LINE, INTERSECTING/CONCURRENT AND PARALLEL LINES

Line:

A line is a collection or set of points that have no width and extend forever. The following is a line. The two arrows are used to show that it extends forever.



Line Segment:

A line segment is part of a line. A segment has two endpoints. The endpoints in the following segments are A and B. Notice also that the line above has no endpoints.



Since, segment has endpoints, **it has a length** measured in metres, centimetres etc.

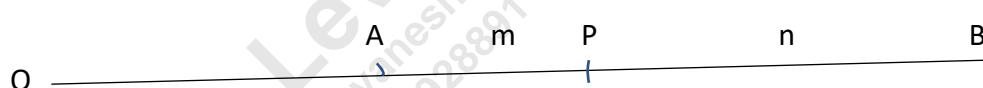
Segment also has a midpoint. We can also divide segment in any given ratio.

Internal and External division:

Let AB be a line segment. We need to find all points in on the line such that, they divide the segment in the given ratio m: n.

There are two such points.

One of them divides segment internally and other divides it externally.



Here, $\frac{AP}{PB} = \frac{m}{n}$ and $\frac{AQ}{QB} = \frac{m}{n}$ as well

CSE 2021: There are three points P, Q and R on a straight line such that PQ: QR=3:5. If n is the number of possible values of PQ: PR, then what is n equal to?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Collinear Points:

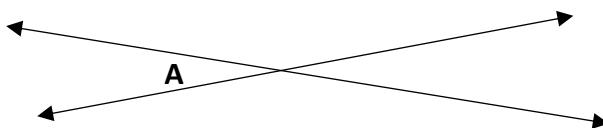
Points which are on the same line are collinear.

NOTE: Two points are always collinear

Intersecting Lines:

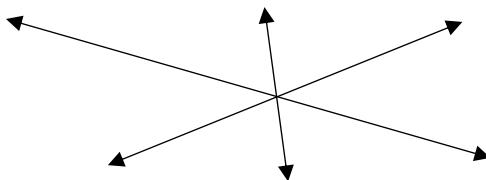
When two or more lines cross each other in a plane, they are called intersecting lines.

The intersecting lines share a common point, which exists on all the intersecting lines, and is called the point of intersection.



Concurrent Lines:

When two or more lines pass through a single point, in a plane, they are concurrent with each other and are called concurrent lines. A point that is common to all those lines is called the point of concurrency.



Parallel Lines:

Lines that are non-intersecting are parallel. Parallel lines do not intersect each other when extended indefinitely.



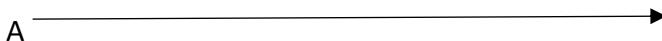
CSE 2015: In a plane, line X is perpendicular to line Y and parallel to line Z; line U is perpendicular to both lines V and W; line X is perpendicular to line V.

Which one of the following statements is correct?

- (a) Z, U and W are parallel.
- (b) X, V and Y are parallel.
- (c) Z, V and U are all perpendicular to W.
- (d) Y, V and W are parallel.

Ray:

A ray is a part of the line having one fixed point and the other point does not have an end. It means that a ray has one terminating end and the other end is extending infinitely.



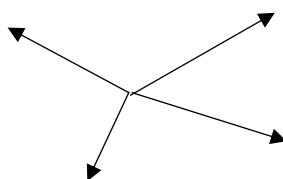
3) ANGLES:

An angle is formed when two straight lines or rays meet at a common endpoint. The common point of contact is called the vertex of an angle.

Types of angles:

- **Complete/total angle:**

Total angle around a point is taken as 360° (Why 360° ?)



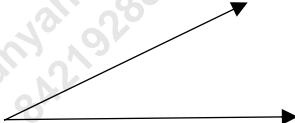
- **Straight angle** – angle of a straight line is half of a total angle which measures as 180°



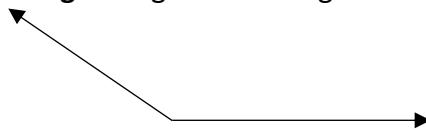
- **Right angle:** Angle measuring 90° is called right angle



- **Acute angle:** Angle measuring less than 90°



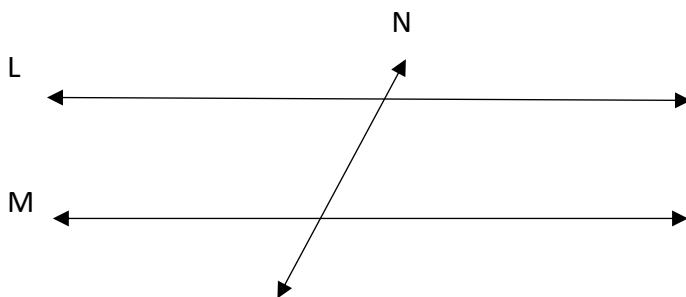
- **Obtuse angle:** Angle measuring more than 90° but less than 180°



- **Complimentary angle:** $90 - \text{angle}$

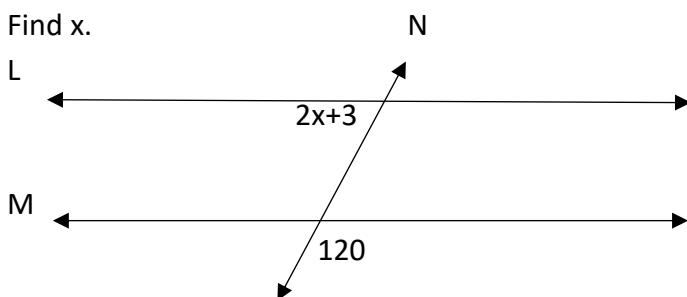
- **Supplementary angle:** $180 - \text{angle}$

Properties of Angles in Parallel lines:



- Corresponding angles are equal.
- Vertical angles/ Vertically opposite angles are equal.
- Alternate interior angles are equal.
- Alternate exterior angles are equal.
- Pair of interior angles on the same side of the transversal are supplementary

Q. Find x.



4) PLANE:

A plane is a flat, two-dimensional surface that extends indefinitely.

Planar shapes: Shapes on a plane.

Circle:

Set of all points at a constant distance (called radius) from a fixed point (called centre) on a plane.



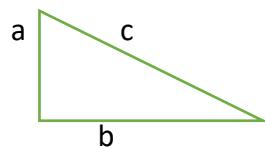
Triangle:

A closed planar polygon with three edges and three vertices.



Pythagoras Theorem:

In a right-angled triangle,



$$(\text{Hypotenuse})^2 = (\text{One side})^2 + (\text{Second side})^2 \text{ or } a^2 + b^2 = c^2$$

Pythagorean triplets:

Set of positive integers, a, b and c that fits the rule: $a^2 + b^2 = c^2$

- (3, 4, 5) is smallest such triplet: $3^2 + 4^2 = 5^2$ i.e., $9 + 16 = 25$
- All multiples of this is also a Pythagorean triplet i.e., (3n, 4n, 5n) is also a triplet for all 'n'
- For instance, for n=2, (6, 8, 10) is a triplet
- For n =3, (9, 12, 15) is a triplet etc.

Other triplets:

- (5, 12, 13)
- (6, 8, 10)
- (7, 24, 25)
- (8, 15, 17)
- (9, 40, 41)
- (11, 60, 61)
- (12, 35, 37)

CSE 2019: P, Q and R are three towns. The distance between P and Q is 60 km, whereas the distance between P and R is 80 km. Q is in the West of P and R is in the South of P. What is the distance between Q and R?

- a. 140 km
- b. 130 km
- c. 10 km
- d. 100 km

CSE 2016: AB is a vertical trunk of a huge tree with A being the point where the base of the trunk touches the ground. Due to a cyclone, the trunk has been broken at C which is at a height of 12 meters, broken part is partially attached to the vertical portion of the trunk at C. If the end of the broken part B touches the ground at D which is at a distance of 5 meters from A, then the original height of the trunk is:

- (a) 20 m
- (b) 25 m
- (c) 30 m
- (d) 35 m

CSE 2016: A person walks 12 km due north, then 15 km due east, after that 19 km due west and then 15 km due south. How far is he from the starting point?

- (a) 5 km
- (b) 9 km
- (c) 37 km
- (d) 61 km

Quadrilateral:

A closed planar polygon having 4 edges and 4 vertices

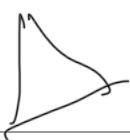


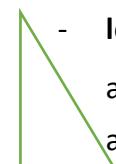
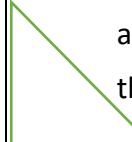
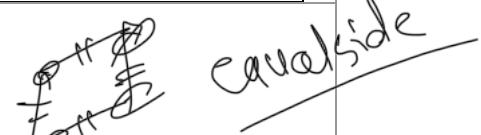
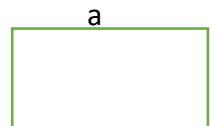
Regular polygons:

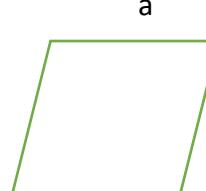
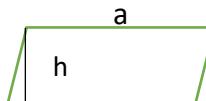
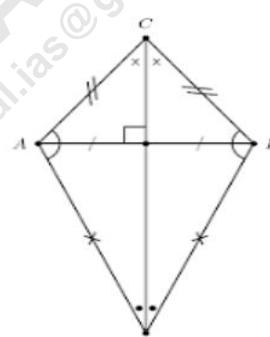
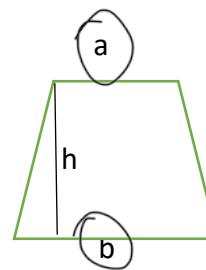
A regular polygon is the one where all sides or edges of a polygon are of equal length.

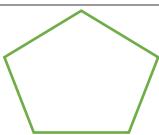
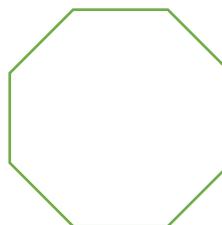
For example – Equilateral triangle, square, regular pentagon etc.

Planar Shape	Properties
Circle	<p>For circle with radius 'r'</p> <ul style="list-style-type: none"> - Diameter = $2r$ – diameter passes through centre - Diameter is the largest chord of a circle - Diameter makes right angle with every point on circle - The radius drawn perpendicular to the chord bisects the chord - Tangent: Line that touches circle - Angle subtended by chord at the centre is twice what it subtends at circumference - Circumference = $2\pi r$ - Area = πr^2
Triangle	<ul style="list-style-type: none"> - Sum of all angles is 180° - Sum of lengths of two sides is always more than the third side - The exterior angle of a triangle is always equal to the sum of the interior opposite angles - Perimeter of triangle = sum of lengths of all three sides - Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
Equilateral triangle	<ul style="list-style-type: none"> - All 3 sides of all equal length - All angles are equal and measure 60° as sum of 3 is 180° <p>If length of side is 'a',</p> <ul style="list-style-type: none"> - Perimeter = $3a$ - Area = $\frac{\sqrt{3}a}{4}$ ($\text{Height}^2 = a^2 - (a/2)^2$)



Isosceles triangle	<ul style="list-style-type: none"> - Two sides of a triangle are equal - Bottom two angles are equal <p>(If two sides are equal, corresponding angles are equal and if two angles are equal, corresponding sides are equal)</p> <ul style="list-style-type: none"> - Line joining vertex of intersection of equal sides with midpoint of opposite side, bisects it and such line is perpendicular to the side 	
Right angled triangle	<ul style="list-style-type: none"> - One of the angles is 90° - Lengths of sides of triangle follow Pythagoras theorem i.e., $a^2 + b^2 = c^2$ - Most frequently appearing right angled triangles: 	
30-60-90 triangle 	45-45-90 triangle 	
Quadrilateral	<ul style="list-style-type: none"> - Has 4 sides and 4 angles - Sum of all four angles is 360° - Perimeter = sum of all sides 	
Square	<ul style="list-style-type: none"> - Quadrilateral whose all 4 sides are equal and all 4 angles are equal - All angles are 90° - Diagonals are also equal and cut each other perpendicularly - If side is of length 'a', - Diagonal is of length $\sqrt{2}a$ - Perimeter = $4a$ - Area = a^2 	
Rectangle	<ul style="list-style-type: none"> - All angles are equal and are 90° - Opposite sides are equal (hence, all squares are rectangles but all rectangles are not squares) - Diagonals are equal, bisect each other but do not intersect perpendicularly. - If adjacent sides (called length and breadth) are 'a' and 'b', - Perimeter = $2a + 2b = 2(a + b)$ - Area = ab 	

	<ul style="list-style-type: none"> - Diagonals are of length $\sqrt{a^2 + b^2}$ as per Pythagoras' theorem.
Rhombus	<ul style="list-style-type: none"> - All sides are equal but all angles are not equal - Opposite angles are equal - Diagonals are not equal but bisect each other perpendicularly - If side of a rhombus is 'a', - Perimeter = $4a$ - Area – depends on the angle
	
Parallelogram	<ul style="list-style-type: none"> - Quadrilateral with equal opposite sides but all angles are not equal - What rectangle is to square, parallelogram is to rhombus - Opposite sides are parallel to each other hence the name - Diagonals are not equal but bisect each other (not perpendicularly) - Hence, all rectangles, squares and rhombuses are parallelograms - If adjacent sides are 'a' and 'b', - Perimeter = $2a + 2b$ - Area = ah
	
Kite	<ul style="list-style-type: none"> - Adjacent two sides of Kite are equal - Diagonals bisect each other at right angle
	
Trapezoid	<ul style="list-style-type: none"> - Quadrilateral with two sides parallel - Area = $\frac{1}{2} \times (a + b) \times h$ - Here, a and b are parallel sides and h is a height
	
Regular Pentagon	<ul style="list-style-type: none"> - Has five equal sides and five equal angles - Each angle is 108°

	-	
Regular Hexagon	<ul style="list-style-type: none"> - Has six equal sides and six equal angles - Each angle is 120° 	
Regular Octagon	<ul style="list-style-type: none"> - Has six equal sides and six equal angles - Each angle is 135° 	

CSE 2023: ABCD is a square. One point on each of AB and CD; and two distinct points on each of BC and DA are chosen. How many distinct triangles can be drawn using any three points as vertices out of these six points?

- (a) 16 (b) 18 (c) 20 (d) 24

CSE 2022: There are eight equidistant points on a circle. How many right-angled triangles can be drawn using these points as vertices and taking the diameter as one side of the triangle?

- (a) 24
 (b) 16
 (c) 12
 (d) 8

CSE 2022: Consider the following statements in respect of a rectangular sheet of length 20 cm and breadth 8 cm:

1. It is possible to cut the sheet exactly into 4 square sheets.
2. It is possible to cut the sheet into 10 triangular sheets of equal area.

Which of the above statements are correct?

- (a) 1 only
 (b) 2 only
 (c) Both 1 and 2
 (d) Neither 1 nor 2

CSE 2022: A pie chart gives the expenditure on five different items A, B, C, D and E in a household. If B, C, D and E correspond to 90° , 50° , 45° and 75° respectively, then what is the percentage of expenditure on item A?

- (a) $112/9$
- (b) $125/6$
- (c) $155/9$
- (d) $250/9$

CSE 2021: A pie diagram shows the percentage distribution of proteins, water and on the dry elements in the human body. Given that proteins correspond to 16% and water corresponds to 70%. If both proteins and the other dry elements correspond to p%, then what is the central angle of the sector representing p on the pie diagram?

- (a) 54°
- (b) 96°
- (c) 108°
- (d) 120°

CSE 2020: Consider the following statements:

- 1. The minimum number of points of intersection of a square and a circle is 2.
- 2. The maximum number of points of intersection of a square and circle is 8.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

CSE 2018: There are 24 equally spaced points lying on the circumference of a circle. What is the maximum number of equilateral triangles that can be drawn by taking sets of three points as the vertices?

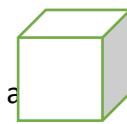
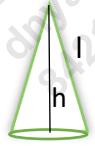
- (a) 4
- (b) 6
- (c) 8
- (d) 12

CSE 2017: Two walls and a ceiling of a room meet at right angles at a point P. A fly is in the air 1 m from one wall, 8 m from the other wall and 9 m from the point P. How many meters is the fly from the ceiling?

- (a) 4
- (b) 6
- (c) 12
- (d) 15

5) AREAS AND VOLUMES

Shape	Area
Circle	πr^2
Triangle	Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
Equilateral triangle	Area = $\frac{\sqrt{3}a}{4}$; where a is side
Square	a^2 ; where a is a side
Rectangle	$a * b$; where a & b are length and breadth
Parallelogram	$a \times \text{height}$; a is a side and height is length of perpendicular drawn onto that side
Regular Hexagon	$\frac{3\sqrt{3}a}{2}$; where a is side

Shape	Diagram	Surface Area	Volume
Sphere (radius = r)		$4\pi r^2$	$\frac{4}{3}\pi r^3$
Cube (side = a)		$6a^2$	a^3
Cuboid (Length, breadth, height = l , b , h)		$2(lb + bh + lh)$	lbh
Cone (Base circle radius = r ; height = h ; curved length = l)		<p>Curved surface area = $\pi r l$</p> <p>Area of base circle = πr^2</p> <p>Total = $\pi r l + \pi r^2$</p>	$\frac{1}{3}\pi r^2 h$
Cylinder (Base radius = r ; height = h)		Curved surface area = $2\pi r h$	$\pi r^2 h$

		Area of 2 circles above and below $= 2\pi r^2$ Total = $2\pi r(h + r)$	
--	--	---	--

Questions on this topic often come mixed with other topics like percentages, ratio-proportion etc.

6) QUESTIONS:

CSE 2020: Q. Let x, y be the volumes; m, n be the masses of two metallic cubes P and Q respectively. Each side of Q is two times that of P and mass of Q is two times that of P. Let $u=m/x$ and $v=n/y$. Which one of the following is correct?

- (a) $u = 4v$
- (b) $u = 2v$
- (c) $v = u$
- (d) $v = 4u$

CSE 2013: A gardener has 1000 plants: He wants to plant them in such a way that the number of rows and the number of columns remains the same. What is the minimum number of plants that he needs more for this purpose?

- (a) 14
- (b) 24
- (c) 32
- (d) 34

CSE 2020: If 1 litre of water weighs 1 kg, then how many cubic millimetres of water will weigh 0.1 gm?

- (a) 1
- (b) 10
- (c) 100
- (d) 1000

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$$0.1 \text{ gm} \quad | \quad 1 \text{ kg} = 10^3 \text{ gm}$$

(Handwritten note: 10³ gm = 1 kg)

CSE 2018: Twelve equal squares are placed to fit in at rectangle of diagonal 5 cm. There are three rows containing four squares each. No gaps are left between adjacent squares. What is the area of each square?

- (a) $5/7$ sq cm
- (b) $7/5$ sq cm
- (c) 1 sq cm
- (d) $25/12$ sq cm

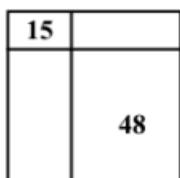
CSE 2016: An agricultural field is in the form of a rectangle having length x_1 meters and breadth x_2 meters (x_1 and x_2 are variable). If $x_1 + x_2 = 40$ meters, then the area of the agricultural field will not exceed which one of the following values?

- (a) 400 sq m
- (b) 300 sq m
- (c) 200 sq m
- (d) 80 sq m

CSE 2016: A cylindrical overhead tank of radius 2 m and height 7 m is to be filled from an underground tank of size 5.5m x 4m x 6m. How much portion of the underground tank is still filled with water after filling the overhead tank completely?

- (a) 1/3
- (b) 1/2
- (c) 1/4
- (d) 1/6

CSE 2011: Consider the following figure and answer the items that follows:



A square is divided into four rectangles as shown above. The lengths of the sides of rectangles are natural numbers. The areas of two rectangles are indicated in the figure. What is the length of each side of the square?

- (a) 10
- (b) 11
- (c) 15
- (d) Cannot be determined as the given data are Insufficient

CSE 2011: A village having a population of 4000 requires 150 liters of water per head per day. It has a tank measuring 20 m x 15 m x 6 m. The water of this tank will last for

- (a) 2 days
- (b) 3 days
- (c) 4 days
- (d) 5 days

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Worksheet CSAT - 03 (Basic Geometry)

Worksheet 3 Basic Geometry

1. Consider following statements:

- I. Two points are always collinear
- II. If three points in a plane are not collinear then they always form a triangle
- III. Two parallel lines are always concurrent

Which of the statements above is/are correct?

- A. 1 and 2 only
- B. 1 and 3 only
- C. 2 and 3 only
- D. 1, 2 and 3

2. What would be the measure of supplementary angle of complimentary angle of 60 degree in degrees?

- A. 30
- B. 60
- C. 120
- D. 150

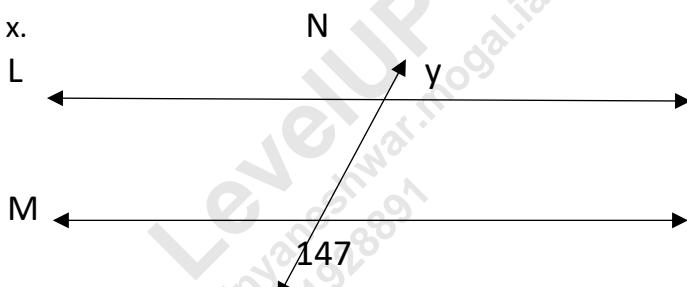
3. Consider following statements:

- I. 3 right angles make a complete angle while 2 right angles make a straight angle
- II. Sum of largest acute integer angle and smallest obtuse natural number angle can never be more than straight angle

Which of the statements above is/are correct?

- A. 1 only
- B. 2 only
- C. Both 1 and 2
- D. Neither 1 nor 2

4. Find x.



What is the measure of angle 'y'

- A. 23.33
- B. 33
- C. 43
- D. 53

5. A pole of length 15 yards is fixed by the municipality of Delhi. Kritika is walking past the pole and after walking 8 yards from the pole she realises that, there's a Siberian crane (is it endangered or critically endangered she wondered!) on top of the pole. What is the direct distance of the bird from her?

- A. 25 yards
- B. 20 yards
- C. 17 yards
- D. 16.66 yards

6. A regular hexagon ABCDEF is inscribed inside a circle with centre 'O'. A chord AB will subtend some angle X at centre O. At how many vertices of hexagon will the same chord subtend angle of X/3?

- A. 0
- B. 2
- C. 4
- D. 6

7. Quadrilateral ABCD is such that, diagonal AC passes through a centre of a circle. Which of the following will always be true about ABCD?

- A. ABCD is a kite
- B. Adjacent angles of ABCD are equal
- C. Opposite angles of ABCD are supplementary
- D. At most one angle of ABCD can be a right angle

8. If length, breadth and height of a cuboid are increased by 10%, 15% and 20% respectively, what will be the ratio of new volume to the old volume

- A. 759: 500
- B. 3: 1
- C. 533: 300
- D. 145: 100

9. If Volume of the cube of side 'a' is to be trebled, what should be the new side length?

- A. $\sqrt{3}a$
- B. $\sqrt[3]{3}a$
- C. $3a$
- D. $\frac{a}{3}$

10. If a mosquito is to travel from the centre of a cuboidal room to a corner, what is the least distance he needs to travel given that, length, breadth and height of room are 8 metres, 6 metres and 24 metres

- A. 19 metres
- B. 16.66 metres
- C. 15 metres
- D. 13 metres

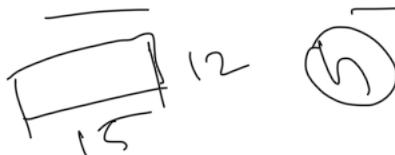
11. The slant height of a right circular cone is 10 m and its height is 8 m. Find the area of its curved surface.
- A. 30 m²
 - B. 40 m²
 - C. 60 m²
 - D. 80 m²
12. What is the total surface area of a right circular cone of height 14 cm and base radius 7 cm?
- A. 344.35 cm²
 - B. 462 cm²
 - C. 498.35 cm²
 - D. None of these
13. How many bricks, each measuring 25 cm x 11.25 cm x 6 cm, will be needed to build a wall of 8 m x 6 m x 22.5 cm?
- A. 5600
 - B. 6000
 - C. 6400
 - D. 7200
14. If length, breadth and height of a cuboid is increased by 10% each, by what percentage the volume would increase?
- A. 10%
 - B. 30%
 - C. 33.1%
 - D. 33.33%
15. If volume of a cylinder becomes 64 times when its height is kept the same, by what amount radius would have been increased?
- A. Radius doubled
 - B. Radius became 4 times
 - C. Radius became 8 times
 - D. Radius became 16 times
16. Height and radius of a cylinder are in the ratio 3:2. If height is increased by 20% and radius is decreased by 10%, what would be the ratio of new volume to old volume of the cylinder.
- A. 123: 125
 - B. 443: 500
 - C. 987: 1000
 - D. 243: 250
17. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated the side of 3 cm to form a cone. The volume of the cone so formed is
- A. $12\pi\text{cm}^3$
 - B. $18\pi\text{cm}^3$
 - C. $24\pi\text{cm}^3$
 - D. $30\pi\text{cm}^3$

18. In a shower, 5 cm of rain falls. The volume of water that falls on 1.5 hectares of ground is
 (1 hectare = 10000 m^2)

- A. 75 m^3
- B. 750 m^3
- C. 7500 m^3
- D. 7.5 m^3

19. A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of the areas of four walls, the volume of the hall is:

- A. 720
- B. 900
- C. 1200
- D. 1800



20. 66 cubic centimetres of silver is drawn into a wire 1 mm in diameter. The length of the wire in metres will be:

- A. 84
- B. 90
- C. 168
- D. 336

21. A hollow iron pipe is 21 cm long and its external diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weighs 8 g/cm^3 , then the weight of the pipe is:

- A. 3.6 kg
- B. 3.696 kg
- C. 36 kg
- D. 36.9 kg

22. A boat having a length 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm when a man gets on it. The mass of the man is: (Take density of water = 1000 kg/m^3)

- A. 12 kg
- B. 60 kg
- C. 72 kg
- D. 96 kg

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Solutions:

1. A
2. D
3. B
4. B
5. C
6. A
7. C
8. A
9. B
10. D
11. C
12. C
13. C
14. C
15. C
16. D
17. A
18. B
19. C
20. A
21. B
22. B

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GS FOUNDATION
BATCH FOR CSE 2023
Ace CSAT 2023-Worksheet 4
Mathematisation and Linear Equations

1. You have a number. The number is subtracted by 7, next the result is divided by 5. If the final result is 11, what was the starting number?
 - A. 52
 - B. 62
 - C. 57
 - D. 77

2. The difference of the distances from Yash's and Wani's homes to their school divided by 2 is equal to twice the distance from Baba's home to the school. If Baba's and Wani's homes are at distances are 1 km and 2 km from school, respectively, find the distance between Yash's home and the school.
 - A. 2 km
 - B. 4 km
 - C. 6 km
 - D. Cannot be uniquely determined

3. Wani's and Yani's ages together are 27. If Yani is 9 years younger than Wani, how old is Wani?
 - A. 14
 - B. 17.5
 - C. 18
 - D. 19.5

4. The sum of distances of Tom's and Jerry's homes to the city center divided by 9 is equal to the distance of three times Udin's home to the city center. If Udin's and Jerry's home distances from city centre are 1 km and 7 km, respectively, find the distance between Tom's home and the city center.
 - A. 8 km
 - B. 12 km
 - C. 15 km
 - D. 20 km

5. Two friends Ram and Shyam go out to buy food. Ram buys pizza and a burger for Rs. 150. Shyam being more health conscious, buys Salad and Protein shake for Rs. 500. Had Ram bought Salad instead of burger, he would have to pay 150 more. And if he were to buy salad instead of pizza, he'd have to pay 100 more. How much does the protein shake costs in rupees?
- A. 150
 - B. 200
 - C. 250
 - D. 300
6. A and B together have Rs. 470. If $\frac{4}{3}$ rd of A's amount is equal to $\frac{5}{8}$ th of B's amount, how much amount does B have?
- A. 256
 - B. 235
 - C. 300
 - D. 320
7. Ram's age is twice that of his daughter's age. In 5 years, sum of their ages will be 85. What will be Ram's age after 20 years?
- A. 57
 - B. 60
 - C. 65
 - D. 70
8. Sita travelled for 1500km to Srilanka with Ravana. Ram and Hanuman want to reach there along the same path. If Ram wants to reach there in 12 hours, what should be his speed?
- A. 100 kmph
 - B. 80 kmph
 - C. 150 kmph
 - D. 100 kmph
9. 'A' wants to distribute 40 pens and some notebooks to his 4 children X, Y, Z and W. In total every child should receive half the number of notebooks than pens. X gets twice the number of pens as that of Y and $\frac{1}{3}$ rd that of Z. Total pens X and W receive is 12. How many notebooks does Z get?
- A. 4
 - B. 6
 - C. 8
 - D. 12

10. Let p and q be three-digit numbers such that, q is obtained by reversing digits of p .

Consider following statements:

1. $(p - q)$ is always divisible by 11
2. $(P + q)$ is always divisible by 37

Which of the statements above is/are correct?

- A. 1 only
- B. 2 only
- C. Both 1 and 2
- D. Neither 1 nor 2

11. In an objective type test of 80 questions, 2.5 marks are allotted for every correct answer and $1/3^{\text{rd}}$ marks are deducted for every wrong answer. After attempting all the 80 questions, a student got a total of 120 marks. What is the number of incorrect responses?

- A. 12
- B. 24
- C. 16
- D. 20

12. Two Statements S1 and S2 are given below with regard to two numbers followed by a Question:

S1: Their product is 21.

S2: 7 times their product is 147.

Question: What are the two numbers?

Which one of the following is correct in respect of the above Statements and the Question?

- A. S1 alone is sufficient to answer the Question.
- B. S2 alone is sufficient to answer the Question.
- C. S1 and S2 together are sufficient to answer the Question, but neither S1 alone nor S2 alone is sufficient to answer the Question.
- D. S1 and S2 together are not sufficient to answer the Question.

13. $1yy + 2yy + y0y + y4 + yy0 = 10yy$ for which digit does the y stand?

- A. 2
- B. 3
- C. 4
- D. 5

14. N ran for $(3x + 1)$ hours at constant speed $(2w + 7)$ and M ran at speed $(5w - 3)$ for $(9 - x)$ hours. What is the total distance they together cover?
- A. $51wx + 24x + 25w - 20$
 - B. $xw + 16x + 47w - 10$
 - C. $51xw + 16x - 25w + 10$
 - D. $xw + 24x - 20 + 47w$
15. For a charity show, the total tickets sold were 630. Half of these tickets were sold at the rate of Rs. 7.5 each, one-third at the rate of Rs. 4.5 each and the rest for Rs. 3 each. What was the total amount received?
- A. Rs. 1350
 - B. Rs. 2310
 - C. Rs. 2415
 - D. Rs. 3000

Solutions:

- 1. B
- 2. C
- 3. C
- 4. D
- 5. D
- 6. D
- 7. D
- 8. D
- 9. D
- 10. A
- 11. B
- 12. D
- 13. B
- 14. D
- 15. C

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GS FOUNDATION
BATCH FOR CSE 2024
Ace CSAT 2023 – Booklet 5
Comprehension Basics

Comprehension

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1) INTRODUCTION

There's no fixed pattern to teach comprehension. It's a matter of practice and efforts to go through what is being said, understand the context, understand the core message and draw the core inferences. And that is where UPSC is coming from.

They don't want you to only answer questions based on text of passage. That you can do anyway. They want you to comprehend the context, content and inferences of the passage. And do it all in speed.

2) ASPECTS OF COMPREHENSION

There are five aspects to get to right answer of given comprehension:

1. Understand the context and assumptions the author has made
2. Understand the vocabulary used
3. Understand the core message and inference of the author – it typically comes in later part of the passage
4. Choosing the correct option from similar looking alternatives
5. Reading speed

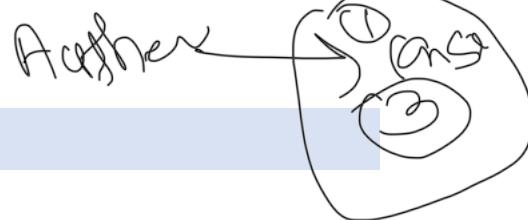
Important thing to remember here is that, you've to answer based on passage given and NOT based on what you otherwise know about the topic.

Another important thing to remember is to read the statements very carefully. Special attention has to be given to words like 'only', 'all', 'exclusively', 'uniquely', 'most', 'almost', 'not', 'definitely', 'larger', 'largest' etc. – for these words change the meaning of passage or underlying tone.

3) TYPES OF QUESTIONS ASKED

Now let see what kind of questions UPSC keeps asking us:

1. Which option best reflects the message of the author/crux of the passage?
2. What does author mean by XYZ?
3. Based on the passage following assumptions have been made: (context)
4. According to passage, which of the following is true? (Understanding and reading carefully)
5. Passage is based on which idea? (context)
6. Which of the following is definite consequence of the passage?
7. Which of the following best describes underlying tone of the passage?
8. The author is against which of the following?
9. Which of the following is implied by the passage? (inference)
10. What is the main idea of the passage? ✓
11. Which of the following reflects practical, lasting, rational solution as per passage?



4) MEANING OF THE QUESTIONS

We firstly need to understand what do these questions mean?

1. **Assumption:** Something that is implicit or inherently true while making a statement. An assumption is something which is assumed, supposed and taken for granted. When someone says something, he may not be able to convey his entire idea into words. The particular part of his idea, not expressed in words or to be taken for granted is an assumption. The author has some prior beliefs based on which he/she writes the passage. In doing so, some assumptions are inherently made by the author – student has to use their analytical mind to unearth those based on the passage only.
2. **Crux:** It is the core idea or the message of the passage. Author has tried to make few arguments in the passage to come to some ultimate conclusion. He/she wants to convince reader of some of his idea or opinion. That is what the crux is.
3. **Conclusion:** Something which directly follows from the statements or arguments made by the author. Here you cannot assume anything and cannot use your prior knowledge or experience.
4. **Inference:** These are the set of conclusions that can be drawn from arguments made by the author in the passage. There can be more than one inference. You may not get answer directly in the passage but have to use your analytical mind to draw out inference. You may use your prior knowledge, experience to draw out inference from the passage if you can't derive inference from the passage.
5. **Underlying tone:** In the passage the author tried to defend his ideas. In doing so he/she ends up lending a specific direction or tone to the passage. The passage could criticise something or someone; it could praise something or someone etc. Underlying tone is about reader's understanding of author's mind or opinions.
6. **Implication:** Something which can be understood or inferred from the given information. The word implication follows from implicit which means implied indirectly or being an integral part of something stated. It is the unexpressed part of the statement. Sometimes implications are also taken as assumptions.
7. **Corollary:** If some statement, act or situation is given and we can derive some other statement or act or situation as a side-effect from it, it is called a corollary. Also, what options one might have based on given situation lead us to corollary.
For instance, if your Quant and Reasoning is poor, corollary is you will focus more on comprehension.
Or, if you are very confident in your comprehension abilities, corollary is that, you will attempt comprehension first.

5) PRACTICE QUESTIONS

The main threat to maintaining progress in human development comes from the increasingly evident unsustainability of production and consumption patterns. Current production models rely heavily on fossil fuels. We now know that this is unsustainable because the resources are finite. The close link between economic growth and greenhouse gas emissions needs to be served for human development to become truly sustainable. Some developed countries have

begun to alleviate the worst effects by expanding recycling and investing in public transport and infrastructure. But most developing countries are hampered by the high costs and low availability of clean energy sources. Developed countries need to support developing countries' transition to sustainable human development.

Q. Unsustainability in production pattern is due to which of the following?

1. Heavy dependence on fossil fuels
2. Limited availability of resources
3. Expansion of recycling

Select the correct answer using the code given below.

- (a) 1 and 2 only
(b) 2 only
(c) 1 and 3 only
(d) 1, 2 and 3

Consider the following statements: Developed countries can support developing countries' transition to sustainable human development by

1. making clean energy sources available at low cost
2. providing loans for improving their public transport at nominal interest rates
3. encouraging them to change their production and consumption patterns

Which of the statements given above is/are correct?

1. 1 only
2. 1 and 2 only
(c) 2 and 3 only
(d) 1, 2 and 3

Unless the forces and tendencies which are responsible for destroying the country's environment are checked in the near future and afforestation of denuded areas is taken up on a massive scale, the harshness of the climatic conditions and soil erosion by wind and water will increase to such an extent that agriculture, which is the mainstay of our people, will gradually become impossible. The desert countries of the world and our own desert areas in Rajasthan are a grim reminder of the consequences of large-scale deforestation. Pockets of desert-like landscape are now appearing in other parts of the country including the Sutlej-Ganga Plains and Deccan Plateau. Where only a few decades back there used to be lush green forests with perennial streams and springs, there is only brown earth, bare of vegetation, without any water in the streams and springs except in the rainy season.

Q. According to the passage given above, deforestation and denudation will ultimately lead to which of the following?

1. Depletion of soil resource
2. Shortage of land for the common man
3. Lack of water for cultivation

Select the correct answer using the code given below.

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

Economic science has dominated public policy since the 20th century. Debates have raged between “Keynesian” economists and “Friedman” economists: between “welfarists” who see the need for a government hand in the economy and “monetarists” who want governments out of the way to let private entrepreneurs loose and let an “invisible hand” produce good outcomes for all. Both sides agree that growth in GDP — the size of the economy measured in money terms — is essential. Far-sighted systems thinkers in the Club of Rome gave a wake-up call in 1972. They showed that pursuit of GDP growth was destroying the earth’s capacity to renew itself and provide resources for unbridled economic growth. They introduced the health of the planet into calculations of profit and growth. Meanwhile, economists continue to treat the natural environment as external to the economy. Pleas by communities to protect it are dismissed as impediments to “ease of doing business” and GDP growth.

Q. Which of the following is the crux of the message that author intends to give?

- (a) Every country needs to pursue higher GDP as Keynesian and Friedman economists both agree on it
- (b) Pursuit of GDP should be given up as it is destroying the Earth’s natural systems
- (c) Natural environment is part of our economic systems and impact on it must be part of our calculations of growth
- (d) Voice of the community must always be heard and not be discarded as against growth or ease of doing business

"In simple matters like shoe-making, we think only a specially trained person will serve our purpose, but in politics, we presume that everyone who knows how to get votes knows how to administer a State. When we are ill, is a guarantee of specific preparation and technical competence—we do not ask for the handsomest physician, or the most eloquent one: well then, when the whole State is ill should we not look for the service and guidance of the wisest and the best?"

Q. Which one of the following statements best reflects the message of the author of the passage?

- (a) We assume that in a democracy, any politician is qualified to administer a State.
- (b) Politicians should be selected from those trained in administration.
- (c) We need to devise a method of barring incompetence from public office.
- (d) As voters select their administrators, the eligibility of politicians to administer a State cannot be questioned.

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BATCH FOR CSE 2023
Ace CSAT 2023 – Booklet 6
Quantitative Aptitude 4 Mathematisation

Mathematisation – a basic skill for quantitative aptitude

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1) INTRODUCTION:

One of the most important skills you shall require for solving quantitative aptitude problems with ease is the ability to convert given word problem into mathematical equations. Once, that is done, solving such equation is quite easy.

Mathematization refers to the activity of organizing and studying any kind of reality with mathematical means, that is, translating a realistic problem into the symbolic mathematical world.

2) WHAT IS A LINEAR EQUATION?

- **Constant:** Something whose value is fixed – it could be a number or a letter given or assumed as constant.
- **Variable:** Something whose value is not a constant or fixed. It can take various values thus called a variable. We're to assume some unknown quantity as variable and find out which particular value when given to that variable, will satisfy our equations or conditions.
- **Coefficient:** The constant in front of a variable is its coefficient. For example, in $5x$, 5 is coefficient of x ; in $7xy^3$ 7 is coefficient of xy^3 .
- **Term:** Coefficient and variables together joined by multiplication or division. For example: $5x$; $2.5xyz$ etc.
- **Equation:** Equations are mathematical statements containing two algebraic expressions on both sides of an 'equal to (=)' sign. It shows the relationship of equality between the expression written on the left side with the expression written on the right side.

For example: $5x + 9 = 34$; $3y^2 - 4x = 123$; $746xy^2 + 64y = 12$ are all equations.

While, $32x + 12y + z$; $12x^2 + 4$ etc. are Not equations

- **Degree of equation:** Degree of equation is the highest power of terms in the given equation.

For example: Degree of $5x + 9 = 34$ is 1;

Degree of $3y^2 - 4x = 123$ is 2

Degree of $746xy^2 + 64y = 12$ is 3 (1 for x + 2 for y^2 = 3)

- **Linear equation:** Equations with 1 as the degree (highest exponent/power of variable) are known as linear equations in maths.

These can be further classified into linear equations in one variable, two-variable linear equations, with three variables, etc.

The standard form of a linear equation with variables x and y is $ax + by + c = 0$, where a and b are the coefficients of x and y respectively and c is the constant.

Q. Which of the following equations are linear?

1. $5x + 9 = 34$;
2. $3y^2 - 4x = 123$;
3. $746xy^2 + 64y = 12$
4. $5x + 9z = 3y$
5. $2x + 3xy = 8$
6. $3d + 5a = 9bc$
7. $23xyz = yz$
8. $3^2x + 4y = 3z - 2$
9. $3x^2 - 4 = 4x$
10. $y^2 + 2 = y$

3) HOW TO DECIDE UNKNOWN

- **Rule of thumb:** Assume what is being asked as unknown 'x' and just plot the information given in the problem in terms of 'x' – In over 90% questions, this will be the simplest thing to do.

For example:

Q. A sum is divided among 120 men and some women in the ratio 15: 21. If each man gets 5 and each woman gets 4, then find the number of women.

- Let number of women be 'x'
- Each women gets 4, so in total they get $4x$
- Each man gets 5, so in total they get $5 \times 120 = 600$
- Ratio is 15: 21 i.e., $\frac{600}{4x} = \frac{15}{21}$

Q. A and B together have Rs. 1210. If $4/15$ th of A's amount is equal to $2/5$ th of B's amount, how much amount does B have?

- Let B have amount 'x'
- Let A have amount 'y'
- $\frac{4}{15}y = \frac{2}{5}x$ and $x + y = 1210$

Q. In a mixture 60 litres, the ratio of milk and water 2: 1. If this ratio is to be 1: 2, then the quantity of water to be further added is

- Out of 60 ltr, milk: water is 2:1, so, 40 ltr milk and 20 ltr water
- Now we want to add water and make this ratio 1:2
- If x water is required to do this, then, total mixture is $60+x$ out of which 40 is milk and $20+x$ is water
- We also know that, $40*2 = 20+x$
- Which gives us, $x = 60$ litres

Q.

Two candidates X and Y contested an election. 80% of voters cast their vote and there were no invalid votes. There was no NOTA (None of the above) option. X got 56% of the votes cast and won by 1440 votes. What is the total number of voters in the voters list?

- If problem already has given variables, you don't need to introduce them from your side:

A rectangle has length and width $(3x - 4)$ cm and $(x + 1)$ cm, respectively. If the perimeter of the rectangle is 34 cm, find the area of the rectangle.

- Another way – take that quantity as unknown variable 'x' about which there is maximum information given in the question.

Q. A number is multiplied by 2, the result is then subtracted by 4, and finally is divided by 5. If the final result is 3, what was the final number?

- It is easier to start with assuming the starting number and move forward in question than assuming final number and move back (It is certainly possible though)

4) HOW TO DECIDE NUMBER OF UNKNOWNS

To solve linear equations having 'n' variables, you need 'n' equations. So, you can decide to assume as many variables as number of equations getting formed out of given information. NOTE that, to form an equation, we need an independent information. In other words, if the information is dependent on previous information, it will not yield an equation.

If only 1 equation can be formed, then you can only assume one variable 'x' to form it and solve.

Q. You have a number. The number is subtracted by 4, next the result is divided by 12. If the final result is 13, what was the starting number?

- NOTE that, every subsequent information here depends upon the previous information.
- So, no independent information is there to have more than 1 equation
- So, we can assume only 1 unknown which is to be found as 'x'

If 2 equations can be formed, then you can assume two variables say x and y and solve two equations to get values of x and y. We just have to look for how many different sets of information are given in the question.

For example: Q. A and B together have Rs. 1210. If $\frac{4}{15}$ th of A's amount is equal to $\frac{2}{5}$ th of B's amount, how much amount does B have?

- Here we are given two information – 1. Together A and B have 1210 and 2. $\frac{4}{15}$ th of A's amount is equal to $\frac{2}{5}$ th of B's
- Now, is the information same or different? Can one be derived from other? – NO
- So, we need to assume two variables.
- Let B have amount 'x'
- Let A have amount 'y'
- $\frac{4}{15}y = \frac{2}{5}x$ and $x + y = 1210$

Suppose the question was: A and B together have Rs. 1210. If double of A's amount and double of B's amount equal 2420, how much amount does B have?

- Here we are given two information are seemingly given – 1. Together A and B have 1210 and 2. double of A's amount and double of B's amount equal 2420
- Now, is the information same or different? Can one be derived from other?
- If yes, it is only one information and not two – we can't solve equation with two unknowns without two equations.
- So, we have two unknowns but only one equation – we can't solve it!

5) PRACTICING THROUGH QUESTIONS:

Convert following word problems into mathematical equations.

1. Amir and Tony together have Rp 30,000. If Amir's amount of money is Rp 4,000 more than Tony's, find each of their amounts.
2. Puja's Plate can hold 8 pieces of mango. Puja took three more pieces to fill it. How many did she already have?
3. Puja has 5 pieces of mango on her plate. She has 2 pieces more than Pushpa. Then how many pieces of mango does Pushpa have?
4. Puja had 5 pieces of mango on her plate. Then she got some more pieces of mango from her mother. Now she has 8 pieces of mango. How many pieces of mango did she get from her mother?

6) COMPREHENSION FOR PRACTICE

The poverty line is quite unsatisfactory when it comes to grasping the extent of poverty in India. It is not only because of its extremely narrow definition of 'who is poor' and the debatable methodology used to count the poor, but also because of a more fundamental assumption underlying it. It exclusively relies on the notion of poverty as insufficient income or insufficient purchasing power. One can better categorize it by calling it income poverty. If

poverty is ultimately about deprivations affecting human well-being, then income poverty is only one aspect of it. Poverty of a life, in our view, lies not merely in the impoverished state in which the person actually lives, but also in the lack of real opportunity given by social constraints as well as personal circumstances—to choose other types of living. Even the relevance of low incomes, meagre possessions, and other aspects of what are standardly seen as economic poverty relate ultimately to their role in curtailing capabilities, i.e., their role in severely restricting the choices people have to lead variable and valued lives.

Q. Why is the methodology adopted in India to count the 'poor' debatable?

- (a) There is some confusion regarding what should constitute the 'poverty line'.
- (b) There are wide diversities in the condition of the rural and urban poor.
- (c) There is no uniform global standard for measuring income poverty.
- (d) It is based on the proposition of poverty as meagre income or buying capacity.

Q. Why is income poverty only one measure of counting the 'poor'?

- (a) It talks of only one kind of deprivation ignoring all others.
- (b) Other deprivations in a human life have nothing to do with lack of purchasing power.
- (c) Income poverty is not a permanent condition; it changes from time to time.
- (d) Income poverty restricts human choices only at a point of time.

Q. What does the author mean by 'poverty of a life'?

- (a) All deprivations in a human life which stem not only from lack of income but lack of real opportunities
- (b) Impoverished state of poor people in rural and urban areas
- (c) Missed opportunities in diverse personal circumstances
- (d) Material as well as non-material deprivations in a human life which restrict human choices permanently.

In some places in the world, the productivity of staples such as rice and wheat has reached a plateau. Neither new strains nor fancy agrochemicals are raising the yields. Nor is there much unfarmed land left that is suitable to be brought under the plough. If global temperature continues to rise, some places will become unsuitable for farming. Application of technology can help overcome these problems. Agricultural technology is changing fast. Much of this change is brought about by affluent farmers in the West/Americas. Techniques developed in the West are being adapted in some places to make tropical crops more productive. Technology is of little use if it is not adapted. In the developing world, that applies as much to existing farming techniques as it does to the latest advances in genetic modification. Extending to the smallholders and subsistence farmers of Africa and Asia the best of today's agricultural practices, in such simple matters as how much fertilizers to apply and when, would lead to a greatly increased availability of food for humanity. So would things like better roads and storage facilities, to allow for the carriage of surpluses to markets and reduce wastage.

Q. Based on the above passage, the following assumptions have been made:

1. Development of agricultural technology is confined to developed countries.
2. Agricultural technology is not adapted in developing countries.

Which of the above assumptions is/are valid?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

Q. Based on the above passage, the following assumptions have been made:

1. Poor countries need to bring about change in their existing farming techniques.
2. Developed countries have better infrastructure and they waste less food.

Which of the above assumptions is/are valid?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

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GS FOUNDATION
BATCH FOR CSE 2023
Ace CSTA 2023 - Worksheet - 6
Percentages

1. Express the following as a fraction: (i) 55% (ii) 0.6%
 - A. $\frac{11}{20}; \frac{3}{5}$
 - B. $\frac{11}{20}; \frac{2}{5}$
 - C. $\frac{9}{20}; \frac{3}{5}$
 - D. $\frac{9}{20}; \frac{2}{5}$

2. If Aakash earns 10% less than Aayush, then how many per cent more does Aayush earn than Aakash?
 - A. 11.11%
 - B. 22.22%
 - C. 13.27%
 - D. 16.28%

3. What percentage of 70 is 14?
 - A. 10%
 - B. 20%
 - C. 16.66%
 - D. 25%

4. In a Film Festival 16 2 3 % of the movies shown were Indian. If the total no of films shown in the festival were 360, find the number of movies shown which were other than Indian.
 - A. 240
 - B. 300
 - C. 360
 - D. 400

5. In a Film Festival 16 2 3 % of the movies shown were Indian. If the total no of films shown in the festival were 360, find the number of movies shown which were other than Indian.
 - A. 11.27%
 - B. 13.64%
 - C. 16.28%
 - D. 15%

6. What percentage of 200 will be equal to 25% of 400?
 - A. 25%
 - B. 40%
 - C. 50%
 - D. 75%

7. A town has a population of 20,000. It's increasing at the rate of 5% per annum. What will be the population of town after 2 years?
- 22050
 - 23450
 - 21300
 - 27600
8. The value of a mobile phone depreciates at the rate of 40% per annum and its present price is `6,000. Calculate its price 2 years ago.
- 18000
 - 15666.67
 - 23333.67
 - 16666.67
9. Out of the total income, X spends 20% on house rent and 70% of the remaining amount on household expenditure. If X saves 1800, the total income is ___
- 8000
 - 9500
 - 7500
 - 8500
10. Out of the total income, X spends 20% on house rent and 70% of the remaining amount on household expenditure. If X saves 1800, the total income is ___
- 2.8
 - 3.0
 - 3.2
 - 3.6
11. The population of a city is 250 thousand. It is increasing at the rate of 2% every year. The growth in the population after 2 years is:
- 50500
 - 10100
 - 20200
 - 30300
12. If $80\% \text{ of } A = 20\% \text{ of } B$ and $B = 5x\% \text{ of } A$, then the value of x is:
- 75
 - 80
 - 85
 - 90
13. Which one of the following is same as 30% of 40% of 560?
- 60% of 40% of 280
 - 15% of 80% of 280
 - 30% of 40% of 280
 - 15% of 80% of 140

14. In a certain class, 72% of the students prefer cold coffee and 44% prefer fruit juice. If each of them prefers cold coffee or fruit juice and 48 likes both, the total number of students in the class is –
- 240
 - 200
 - 300
 - 250
15. Population of a city is 2,96,000 out of which 1,66,000 are males. 50% population is literate. If 70% males are literate, then the number of women who are literate is –
- 48,000
 - 66,400
 - 32,200
 - 31,800

Solutions:

- A
- A
- B
- B
- B
- C
- A
- D
- C
- C
- B
- B
- A
- C
- D

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GS FOUNDATION
BATCH FOR CSE 2023
Ace CSAT 2023 – Booklet 7

Quantitative Aptitude 5 Solving Linear and Quadratic Equations

Solving Linear and Quadratic Equations

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1) SOLUTION OF AN EQUATION

Solving equation means finding value of unknown variable which satisfies the equation.

For example: $x + 2 = 5$ is a linear equation satisfied by $x = 3$. We say that, $x = 3$ solves the equation.

2) SOLVING LINEAR EQUATIONS

Linear Equations in one variable

This equation is of the type $ax + b = 0$.

Example: $3x - 6 = 0$

$3x = 6$ (*adding 6 on both sides*)

$$x = \frac{6}{3} = 2$$

- In case, you have variable on both sides, bring variables on one side and constants on other

For example: $2x + 8 = 5x - 1$

$$8 + 1 = 5x - 2x \Rightarrow 3x = 9 \Rightarrow x = 3$$

Linear Equation in one variable with fractions

We simply find LCM of denominators and multiply by that throughout to clear the denominator. Alternatively, we can multiply by the product of all the denominators if they aren't large to complicate calculations.

Example:

$\frac{2}{7}x - \frac{3}{2} = 0$: Here, 7 and 2 are not large – so we simply multiply throughout by their product

(14)

$$14 \times \frac{2}{7}x - 14 \times \frac{3}{2} = 0 \Rightarrow 4x - 21 = 0 \Rightarrow x = \frac{21}{4}$$

Q. Solve following linear equations:

1. $5x + 10 = 15$
2. $2y - 9 = \frac{5}{2}$
3. $x - \frac{3}{8}x = 5x + \frac{2}{3}x - 11$
4. $2.3x - 2.3 = 4.6$
5. $0.25y - 10 = -5$
6. $10 - (5x + 2) = 2 - x$

Linear Equation with two variables:

A general linear equation with two variables will look like: $ax + by + c = 0$

For example: $x + 2y - 3 = 0$ or $2p - 2q + 5 = 0$

Can you find values of two variables from these equations?

We need at least 'n' equations to solve equations with 'n' variables

- So, we need two equations to solve above equation i.e., $x + 2y - 3 = 0$
- Consider another equation: $4x - 3y - 1 = 0$.
- Can we solve the equations now?

$$4x - 3y - 1 = 0$$

$$x + 2y - 3 = 0$$

- Multiply equation 2 by 4 and subtract it from equation 1
- Now we have linear equation with one variable.

Q. Solve following linear equations in two variables:

1. $3x + 5y - 11 = 0; 2x - y = 3$
2. $p + q - 7 = 0; q - p + 1 = 0$
3. $3x + y = 13; 2x + 3y = 18$
4. $\frac{2}{3}x - \frac{5}{8}y = \frac{1}{24}; x - y = 0$
5. $\frac{1}{3}x + \frac{2}{5}y - 3 = 0; \frac{23}{3}x - \frac{3}{5}y = 20$

3) QUADRATIC EQUATIONS

Equation with degree 2 is quadratic equation.

Example: $xy = 2; x^2 + 3 = 9$ etc.

Quadratic Equation in one variable: $ax^2 + bx + c = 0$

Solutions are:
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For example: For equation $x^2 - 2x - 8 = 0$ – calculate solutions

4) PYQS

CSE 2023: For five children with ages $a < b < c < d < e$; any two successive ages differ by 2 years.

Question: What is the age of the youngest child?

Statement-1: The age of the eldest is 3 times the youngest.

Statement-2: The average age of the children is 8 years

Which one of the following is correct in respect of the above Question and the Statements?

- (a) The Question can be answered by using one of the Statement alone.
- (b) The Question can be answered by using either Statement alone.
- (c) The Question can be answered by using both the Statement together, but cannot be answered using either Statement alone.
- (d) The Question cannot be answered even by using both the Statements together.

CSE 2023: In a party, 75 persons took tea, 60 persons took coffee and 15 persons took both tea and coffee. No one taking milk takes tea. Each person takes at least one drink.

Question: how many persons attended the party?

Statement-1: 50 persons took milk.

Statement-2: Number of persons who attended the party is five times the number of persons who took milk only.

Which one of the following is correct in respect of the above Question and the Statements?

- (a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone.
- (b) The Question can be answered by using either Statement alone.
- (c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone.
- (d) The Question cannot be answered even by using both the Statements together.

CSE 2022: The sum of three consecutive integers is equal to their product. How many such possibilities are there?

- (a) Only one
- (b) Only two
- (c) Only three
- (d) No such possibility is there

CSE 2022:

Consider the Question Statements given below:

Question: What is the age of Manisha?

Statement-1: Manisha is 24 years younger than her mother.

Statement-2: 5 years later, the ages of Manisha and her mother will be in the ratio 3:5.

Which one of the following is correct in respect of the Question and the Statements?

- (a) Statement-1 alone is sufficient to answer the Question
- (b) Statement-2 alone is sufficient to answer the Question
- (c) Both Statement and Statement-2 are sufficient to answer the Question
- (d) Both Statement-1 and Statement-2 are not sufficient to answer the Question

CSE 2022:

A bill for 1,840 is paid in the denominations of 50, 20 and 10 notes. 50 notes in all are used.

Consider the following statements:

1. 25 notes of 50 are used and the remaining are in the denominations of 20 and 10.
2. 35 notes of 20 are used and the remaining are in the denominations of 50 and 10.
3. 20 notes of 10 are used and the remaining are in the denominations of 50 and 20.

Which of the above statements are **not** correct

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

CSE 2022:

A person X wants to distribute some pens among six children A B C D E and F. Suppose A gets twice the number of pens received by three times that of four times that of D, five times that of E and six times that of F. What is the minimum number of pens X should buy so that the number of pens each one gets is an even number?

- (a) 147
- (b) 150
- (c) 294
- (d) 300

CSE 2022:

Five friends P, O, X, Y and Z purchased some notebooks. The relevant information is given below:

- 1. Z purchased 8 notebooks more than X did.
- 2. P and Q together purchased 21 notebooks.
- 3. O purchased 5 notebooks less than P did.
- 4. X and Y together purchased 28 notebooks.
- 5. P purchased 5 notebooks more than X did.

If each notebook is priced 40, then what is the total cost of all the notebooks?

- (a) 2,600
- (b) 2,400
- (c) 2,360
- (d) 2,320

CSE 2022: A has some coins. He gives half of the coins and 2 more to B. B gives half of the coins and 2 more to C. C gives half of the coins and 2 more to D. The number of coins D has now, is the smallest two-digit number. How many coins does A have in the beginning?

- (a) 76
- (b) 68
- (c) 60
- (d) 52

CSE 2021: The difference between a 2-digit number and the number obtained by interchanging the positions of the digits is 54.

1. The sum of the two digits of a number can be determined only if the product of the two digits is known.

2. The difference between the two digits of the number can be determined.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

CSE 2021: X said to Y, "At the time of your birth I was twice as old as you are at present." If the present age of X is 42 years, then consider the following statements:

1. 8 years ago, the age of X was five times the age of Y.
2. After 14 years, the age of X would be two times the age of Y.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

CSE 2021: A person P asks one of his three friends X as to how much money he had. X replied, "If Y given me Rs. 40, then Y will have half of as much as Z, but if Z gives me Rs. 40, then three of us will have equal amount." What is the total amount of money that X, Y and Z have?

- (a) Rs. 420
- (b) Rs. 360
- (c) Rs. 300
- (d) Rs. 270

CSE 2021: In an objective type test of 90 questions, 5 marks are allotted for every correct answer and 2 marks are deducted for every wrong answer. After attempting all the 90 questions, a student got a total of 387 marks. What is the number of incorrect responses?

- (a) 9
- (b) 13
- (c) 27
- (d) 43

CSE 2021:

Q. Jay and Vijay spent an equal amount of money to buy some pens and special pencils of the same quality from the same store. If Jay bought 3 pens and 5 pencils, and Vijay bought 2 pens and 7 pencils, then which one of the following is correct?

- (a) A pencil costs more than a pen.
- (b) The price of a pencil is equal to that of a pen
- (c) The price of a pen is two times the price of a pencil
- (d) The price of a pen is three times the price of a pencil

CSE 2021:

A person P asks one of his three friends X as to how much money he had. X replied, "If Y given me Rs. 40, then Y will have half of as much as Z, but if Z gives me Rs. 40, then three of us will have equal amount." What is the total amount of money that X, Y and Z have?

- (a) Rs. 420
- (b) Rs. 360
- (c) Rs. 300
- (d) Rs. 270

CSE 2020: Two Statements S1 and S2 are given below with regard to two numbers followed by a Question:

S1: Their product is 21.

S2: Their sum is 10.

Question: What are the two numbers? Which one of the following is correct in respect of the above Statements and the Question?

(a) S1 alone is sufficient to answer the Question.

(b) S2 alone is sufficient to answer the Question.

(c) S1 and S2 together are sufficient to answer the Question, but neither S1 alone nor S2 alone is sufficient to answer the Question.

(d) S1 and S2 together are not sufficient to answer the Question.

CSE 2020: There are some nectar-filled flowers on a tree and some bees are hovering on it. If one bee lands on each flower, one bee will be left out. If two bees land on each flower, one flower will be left out. The number of flowers and bees respectively are:

(a) 2 and 4

(b) 3 and 2

(c) 3 and 4

(d) 4 and 3

CSE 2020: In aid of charity, every student in a class contributes as many rupees as the number of students in that class. With the additional contribution of Rs. 2 by one student only, the total collection is Rs. 443. Then how many students are there in the class?

(a) 12

(b) 21

(c) 43

(d) 45

CSE 2020: A vessel full of water weighs 40 kg. If its is one third filled, its weight becomes 20 kg. What is the weight of the empty vessel?

(a) 10 kg

(b) 15 kg

(c) 20 kg

(d) 25 kg

CSE 2020: In the sum

$$X + 1X + 5X + XX + X = 1XX$$

for which digit does the X stand?

(a) 2

(b) 3

(c) 4

(d) 5

$$(x + 10 + x + 50 + x + 10x + x + x = 100 + 10x + x)$$

CSE 2019: Rakesh and Rajesh together bought 10 balls and 10 rackets. Rakesh spent 1300 and Rajesh spent 1500 If each racket costs three times a ball does, then what is the price of a racket?

- (a) Rs. 70
- (b) Rs. 50
- (c) Rs. 210
- (d) Rs. 240

CSE 2019: Ena was born 4 years after her parents' marriage. Her mother is three years younger than her father and 24 years older than Ena, who is 13 years old. At what age did Ena's father get married?

- (a) 22 years
- (b) 23 years
- (c) 24 years
- (d) 25 years

CSE 2018: Consider the following sum:

$$x + 1x + 2x + x3 + x1 = 21x$$

In the above sum, x stands for

- (a) 4
- (b) 5
- (c) 6
- (d) 8

CSE 2018: A number consists of three digits of which the middle one is zero and their sum is 4. If the number formed by interchanging the first and last digits is greater than the number itself by 198, the difference between the first and last digits is

- (a) 1
- (b) 2
- (c) 3
- (d) 4

CSE 2018: A bookseller sold 'a' number of Geography textbooks at the rate of `x per book, 'a + 2' number of History textbooks at the rate of `(x + 2) per book and 'a - 2' number of Mathematics textbooks at the rate of `(x - 2) per book. What is his total sale in Rs?

- (a) $3x + 3a$
- (b) $3ax + 8$
- (c) $9ax$
- (d) $3a3x$

CSE 2017: There are certain 2-digit numbers. The difference between the number and the one obtained on reversing it is always 27. How many such maximum 2-digit numbers are there?

- (a) 3
- (b) 4
- (c) 5
- (d) None of the above

CSE 2017: 10. Each of A., B, C and D has Rs 100. A pays Rs 20 to B, who pays Rs 10 to C, who gets Rs 30 from D. In this context, which one of the following statements is not correct?

- (a) C is the richest
- (b) D is the poorest.
- (c) C has more than what A and D have together.
- (d) B is richer than D

CSE 2016: In a class, there are 18 very tall boys. If these constitute three-fourths of the boys and the total number of boys is two-thirds of the total number of students in the class, what is the number of girls in the class?

- (a) 6
- (b) 12
- (c) 18
- (d) 21

CSE 2015: In a parking area, the total number of wheels of all the cars (four-wheelers) and scooters/motorbikes (two-wheelers) is 100 more than twice the number of parked vehicles. The number of cars parked is

- (a) 35
- (b) 45
- (c) 50
- (d) 55

CSE 2015: A father is nine times as old as his son and the mother is eight times as old as the son. The sum of the father's and the mother's age is 51 years. What is the age of the son?

- (a) 7 year
- (b) 5 year
- (d) 4 year
- (d) 3 year

CSE 2015: In a box of marbles, there are three less white marbles than the red ones and five more white marbles than the green ones. If there are a total of 10 white marbles, how many marbles are there in the box?

- (a) 26
- (b) 28
- (c) 32
- (d) 36

CSE 2015: A person ordered 5 pairs of black socks and some pairs of brown socks. The price of a black pair was thrice that of a brown pair. While preparing the bill, the bill clerk interchanged the number of black and brown pairs by mistake which increased the bill by 100%. What was the number of pairs of brown socks in the original order?

- (a) 10
- (b) 15
- (c) 20
- (d) 25

CSE 2014: As per agreement with a bank, a businessman had to refund a loan in some equal instalments without interest. After paying 18 instalments he found that 60 percent of his loan was refunded. How many instalments were there in the agreement?

- (a) 22
- (b) 24
- (c) 30
- (d) 33

CSE 2014: For a charity show, the total tickets sold were 420. Half of these tickets were sold at the rate of Rs. 5 each, one-third at the rate of Rs. 3 each and the rest for Rs. 2 each. What was the total amount received?

- (a) Rs. 900
- (b) Rs. 1,540
- (c) Rs. 1,610
- (d) Rs. 2,000

CSE 2013: There are some balls of red, green and yellow colour lying on a table. There are as many red balls as there are yellow balls. There are twice as many yellow balls as there are green ones. The number of red balls

- (a) is equal to the sum of yellow and green balls.
- (b) is double the number of green balls.
- (c) is equal to yellow balls minus green balls.
- (d) cannot be ascertained

CSE 2011: A person has only Rs. 1 and Rs. 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs. 75, then the number of Rs. 1 and Rs. 2 coins are

- respectively
- (a) 15 and 35
 - (b) 35 and 15
 - (e) 30 and 20
 - (d) 25 and 25

GS FOUNDATION
BATCH FOR CSE 2023
Ace CSAT 2023 – Booklet 8
Logical_Reasoning_1_Calendars

Calendars

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1) INTRODUCTION:

A calendar is a system of organizing days. This is done by giving names to periods of time, typically days, weeks, months and years. A date is the designation of a single, specific day within such a system.

Gregorian calendar is the most widely used calendar across the world and we in India as well use it as a national calendar.

In LR section, questions on calendar are very frequently asked. This is one of the easiest topics once we get hang of how the Gregorian calendar is constructed, how the leap year is incorporated and how it affects days of various years.

2) GREGORIAN CALENDAR:

In this type of calendar, a day is basic unit consisting of 24 hours (time Earth takes to complete one rotation around its own axis)

7 days make a week and

365 days (usually) make a year.

A year is the time Earth takes to complete one revolution around the Sun. To be precise, actual time for one revolution is **365 days, 5 hours, 59 minutes and 16 seconds.**

So, every four years, to incorporate this extra 6 hours occurring every year, we add an extra day in the calendar every 4 years. We call such a year a LEAP YEAR.

So, leap year has 366 days instead of 365. It occurs in a year which is divisible by 4. (What is test for divisibility by 4?)

List down other 2-digit numbers divisible by 4.

So, 1948, 2016, 2224 – are all examples of leap year.

However, there's a catch. We're rounding off 5 hours, 59 minutes and 16 seconds to 6 hours. We're essentially adding little extra every 4 years.

To compensate for that, we don't add an extra day if year is a multiple of 100 unless it is multiple of 400.

So, 1900 is NOT a leap year even though it is divisible by 4 as it is NOT divisible by 400. But, 1600 is a leap year and so is 2000.

Examples: Which of the following are leap year?

1. 2022
2. 2020
3. 2024
4. 2030
5. 1234
6. 2345
7. 8573
8. 8888
9. 2424
10. 2009

11. 1000
12. 12000
13. 3400
14. 2348
15. 9998
16. 10000
17. 1212
18. 1256
19. 200
20. 3000

Number of days in a month:

January	31	July	31
February	28 or 29	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

3) DIVIDING BY 7 – CONCEPT OF ‘EXTRA’ DAYS

A week has 7 days and a day repeats every 8th day. So, if today 1st August is Sunday, there will be Sunday on 8th August, 15th August and so on.

So, in a month having 31 days, there shall be 4 full weeks and 3 extra days

In a month having 30 days, there shall be 4 full weeks and 2 extra days

In a month having 28 days there shall be 4 full weeks

In a month having 29 days there shall be 4 full weeks and 1 extra day

In a year having 365 days, there shall be 52 full weeks and 1 extra day

In a leap year with 366 days, there shall be 52 weeks and 2 extra days.

In 100 years (ending on non-leap year) there are 24 leap years and 76 normal years. So, there are $76 + 48 = 124$ extra day which reduces to 5 extra days

In 200 years, there will 10 extra days which is reduced to 3 extra days

In 300 years, there will be 15 extra days which is reduced to 1 extra day

In 400 years, there will be $20 + 1 = 21$ extra days (as 400 itself is leap year) which is reduced to 0 extra days

4) TYPES OF QUESTIONS:

1. Which day will be there after X days:

In this type, today's day is given and we're asked to find out which day will be there on Xth day or after X days.

Q. If today is Thursday, after 68 days, it will be __

Here, we only have to use the fact that there are 7 days in a week. We simply divide 68 by 7 and look at what is the remainder.

Note that, 63 is divisible by 7 and thus remainder is 5. So, it will be Tuesday.

Alternatively, 70 is divisible by 7. So, after 70 days it will be Thursday. Thus, going back two days, it will be Tuesday after 68 days.

Q. If today is Monday, what will be day after 1000 days?

Q. If today is Sunday, what will be day after 12345 days?

2. Which day will be there after X years?

In this type of questions, today's day will be given and we're to find out which day will be there after X years on the same date.

Here we again note that, there is 1 extra day in a normal year and 2 extra days in a leap year.

Q. If today on 1st August 2022 it is Monday, what day will it be on 1st August 2023?

Only question we've to ask is there any involvement of leap year? Here clearly 2023 is NOT a leap year. So, between the given period of 1 year, only 1 extra day shall occur.

So, on 1st August 2023, it will be TUESDAY.

Q. If today on 1st January 2020 it is Wednesday, what day will it be on 1st January 2021?

Now, is there leap year involved? – YES, 2020 is a leap year and extra day of 29th February is between the given period. So, there are 2 extra days.

It will be Friday on 1st January 2021.

Q. If today on 1st August 2022 it is Monday, what day will it be on 1st August 2048?

We're asked day after exactly 26 years. We just need how many extra days occur in that period.

For that, we just need to find out how many leap years occur in this period.

It is clearly 7. So extra days are $19 + 7*2 = 19 + 14 = 33$ which is same as 5.

So, it will be Saturday on 1st August 2048.

Q. If today on 1st January 2022 it is Saturday, what day will it be on 1st January 2048?

We again want to find out extra days in the period. Similar to above, how many leap years occur inside the period.

Does extra day of 29th February 2048 occur inside the period? – NO

So, actually only 6 leap years occur inside the given period.

Extra days = $20 + 6*2 = 32$, which is same as 4.

So, it will be Wednesday.

Q. If today on 1st August 2022 it is Monday, what day will it be on 1st August 2099?

Q. If today on 1st August 2022 it is Monday, what day will it be on 1st August 3022?

NOTE that, this is a period of 100 years. We've to just look for how many leap years occur in this period. Is it 24 or 25?

When is number 24 and when is it 25?

Here it is 24. We already know that in such a case, there are 5 extra days. So, it will be Saturday.

Q. If today on 1st August 2022 it is Monday, what day will it be on 1st February 2030?

3. Which day was there on particular date in the past?

Here you're either given today's day or you may have to assume 1st January 0000 as a Monday if nothing is given. This is where extra days per 100, 200, 300, 400 years calculation we did above comes in handy.

Q. What was the day on 25th January, 1975?

NOTE: No other information is given. So, we use known information that is 1st January 0000 was Monday.

We also know that in 400 years there are 0 extra days. So, after 1600 years, there are no extra days.

$$1974 = 1600 + 300 + 74$$

In 300 years, there is 1 extra day.

In next 74 years, there are 18 leap years. So, extra days = $56 + 18 \times 2 = 92$. Which is same as 1 extra day.

So, after 1974 years there are $0 + 1 + 1 = 2$ extra days. So, on 1st January 1975 it was Wednesday. Thus, on 22 January it was Wednesday.

Thus, on 25 January 1975 was Saturday.

Q. What day of the week was 20th June 1837?

1836 complete years + first 5 months of the year 1837 + 20 days of June

$$1836 = 1600 + 200 + 36$$

In 1600 years – no extra days; In 200 years 3 extra days;

In 36 years, there are 9 leap years – $27+18=45$ extra days – same as 3 extra days

First 5 months of 1837: $3+0+3+2+3=11$ extra days – same as 4 extra days

In total: $0 + 3 + 3 + 4 = 10$ extra days – same as 3 extra days

So, it was Thursday on 1st June 1837 and thus on 22nd June 1837.

So, it was Tuesday in 20th June 1837.

Q. If today 10th August is Wednesday, what is the day on 2nd February 1990?

Assume there was some day X on given day and move similarly with calculating extra days till 10th August 2022.

Ans. Friday

Q. The last day of a century cannot be __

- (a) Monday
- (b) Wednesday
- (c) Tuesday
- (d) Friday

4. Repetition of calendar of particular year

In this type of questions, we are given a reference year and we're asked after what minimum time will this calendar will repeat in the future.

There are ONLY 14 different calendars possible in Gregorian system. So, calendar indeed repeats after certain number of years.

Solving Technique:

We look whether the given year is leap or not.

If not leap, then we look at the remainder year has when divided by 4.

YEAR	Repetition after years
Leap year	28
Leap year + 1 ($4n + 1$)	6
Leap year + 2	11
Leap year + 3	11

Q. After how many years will the calendar of 2001 repeat?

NOTE: 2001 is leap year+1 – So the calendar will repeat after 6 years i.e. 2007

Q. After how many years will the calendar of 2011 repeat?

2011 is leap year + 3: repeat after 11 years i.e. 2022

Q. After how many years will the calendar of 2013 repeat?

Q. After how many years will the calendar of 2022 repeat?

Q. After how many years will the calendar of 2035 repeat?

Q. After how many years will the calendar of 2040 repeat?

Q. After how many years will the calendar of 2072 repeat?

SPECIAL CASE: If leap year + 28 is NOT a leap year – In this case answer is +40

Thus, for 2072 – repetition will happen in 2012 after 40 years

Q. In case of following years find the year when the calendar repeats?

1. 1845
2. 1345
3. 9867

- 4. 3000
- 5. 1234
- 6. 2020
- 7. 3434
- 8. 1972
- 9. 2172
- 10. 2472
- 11. 7777
- 12. 1111
- 13. 2323
- 14. 7444
- 15. 860
- 16. 0000
- 17. 0001
- 18. 0040
- 19. 1212
- 20. 9091

5) PYQS:

CSE 2023: If today is Sunday, then which day is it exactly on 10^{10} th day?

- (a) Wednesday
- (b) Thursday
- (c) Friday
- (d) Saturday

CSE 2022: Which date of June 2099 among the following is Sunday?

- (a) 4
- (b) 5
- (c) 6
- (d) 7

CSE 2021: Consider two Statements and a Question:

Statement 1: The last day of the month is a Wednesday.

Statement 2: The third Saturday of the month was the seventeenth day.

Question: What day is the fourteenth of the given month?

Which one of the following is correct in respect of the Statements and the Question?

- (a) Statement 1 alone is sufficient to the answer of the Question
- (b) Statement 2 alone is sufficient to answer the Question
- (c) Both Statement 1 and Statement 2 are required to answer the Question
- (d) Neither Statement 1 alone nor Statement 2 alone is sufficient to answer the Question

CSE 2021: Which day is 10th October, 2027?

- (a) Sunday
- (b) Monday
- (c) Tuesday
- (d) Saturday

CSE 2021: From January 1, 2021, the price of petrol (in Rupees per litre) on mth day of the year is $80 + 0.1m$, where $m = 1, 2, 3, \dots, 100$ and thereafter remains constant. On the other hand, the price of diesel (in Rupees per litre) on nth day of 2021 is $69 + 0.15n$ for any n. On which date in the year 2021 are the prices of these two fuels equal?

- (a) 21st May
- (b) 20th May
- (c) 19th May
- (d) 18th May

CSE 2020: Q. In the particular year 12th January is a Sunday, then which one of the following is correct?

- (a) 15th July is a Sunday if the year is a leap year.
- (b) 15th July is a Sunday if the year is not a leap year.
- (c) 12th July is Sunday if the year is a leap year.
- (d) 12th July is not Sunday if the year is a leap year.

CSE 2019: Which year has the same calendar as that of 2009?

- (a) a. 2018
- (b) b. 2017
- (c) c. 2016
- (d) d. 2015

CSE 2019: Mr 'X' has three children. The birthday of the first child falls on the 5th Monday of April, that of the second one falls on the 5th Thursday of November. On which day is the birthday of his third child, which falls on 20th December?

- (a) Monday
- (b) Thursday
- (c) Saturday
- (d) Sunday

CSE 2019: In 2002, Meenu's age was one-third of the age of Meera, whereas in 2010, Meenu's age was half the age of Meera, what is Meenu's year of birth?

- (a) 1992
- (b) 1994
- (c) 1996
- (d) 1998

CSE 2017: If second and fourth Saturdays and ,all the Sundays are taken as only holidays for an office, what would be the minimum number of possible working days of any month of any year?

- (a) 23
- (b) 22
- (c) 21
- (d) 20

CSE 2014: If the 3rd day of a month is Monday, which one of the following will be the fifth day from 21st of this month?

- (a) Monday
- (b) Tuesday
- (c) Wednesday
- (d) Friday

6) COMPREHENSION

Private investment in general is volatile. Foreign private investment is more volatile because the available investment avenues are significantly greater (i.e., the entire world). Therefore, the responsibility of providing employment cannot be left to Foreign District Investment (FDI). The current FDI inflows are volatile over time and across sectors and regions, which is a necessary consequence of their search for the highest returns. The adverse consequences are unstable employment and an accentuation of income and regional inequalities. A probable positive consequence of foreign investment is the inflow of new technology and its subsequent diffusion. However, the technology diffusion is not at all certain because the existing state of physical and human capital in India may prove inadequate for the diffusion.

Q. With reference to the above passage, the following assumptions have been made:

1. Relying on foreign investment in the long run is not an economically sound policy.
2. Policies must be undertaken to reduce volatility in foreign private investment.
3. Policies must be undertaken to strengthen domestic private investment.
4. Public investment should be given priority over private investment.
5. Substantial public investment in education and health should be undertaken.

Which of the above assumptions is/are valid?

- (a) 1, 2 and 4
- (b) 1, 3 and 5
- (c) 2, 4 and 5
- (d) 3 only

Many opportunities to harness the highly skewed, seasonal and spatial distribution of monsoon floes, which occur in a four-month period from June to September annually, have been lost. Since these few months account for most of the rainfall and consequent freshwater availability, the need for holding rainwater in reservoirs, for subsequently releasing it for use over the year, is a necessity nobody can afford to overlook. Climate change will continue to

affect weather conditions and create water shortages and excesses. While millions suffer from droughts and floods, waters in the country's many rivers flow unutilized, and are discharged into the sea every year

Q. With reference to the above passage, which of the following could be the most rational and practical implications for India?

1. Inter-linking of rivers should be undertaken.
2. A network of dams and canals should be built across the country for proper distribution of water.
3. Farmers should be provided easy loans for digging borewells.
4. Usage of water for agriculture should be regulated by law.
5. Distribution of river water among regions should be regulated by the Union Government.

Select the correct answer using the code given below.

- (a) 1 and 2
(b) 2, 4 and 5
(c) 1, 3 and 4
(d) 2, 3 and 5

In India, the segregation of municipal waste at source is rare. Recycling is mostly with the informal sector. More than three-fourths of the municipal budget goes into collection and transportation, which leaves very little for processing/resource recovery and disposal. Where does waste-to-energy fit into all this? Ideally it fits in the chain after segregation (between wet waste and rest), collection, recycling, and before getting to the landfill. Which technology is most appropriate in converting waste to energy depends on what is in the waste (that is biodegradable versus non-biodegradable component) and its calorific value. The biodegradable component of India's municipal solid waste is a little over 50 per cent, and bio-methanation offers a major solution for processing this.

Q. Based on the above passage, the following assumptions have been made:

1. Collection, processing and segregation of municipal waste should be with government agencies.
2. Resource recovery and recycling require technological inputs that can be best handled by private sector enterprises.

Which of the assumptions given above is/are correct?

- (a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

Q. Which one of the following statements best reflects the crux of the passage?

- (a) Generation of energy from municipal solid waste is inexpensive.
(b) Bio-methanation is the most ideal way of generating energy from municipal solid waste.
(c) Segregation of municipal solid waste is the first step in ensuring the success of waste-to-energy plants.
(d) The biodegradable component of India's municipal solid waste is not adequate to provide energy from waste efficiently/effectively.

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BATCH FOR CSE 2023
Ace CSAT 2023 -Worksheet 5
Calendars

1. Which will be the first day of year 2345?
 - A. Monday
 - B. Wednesday
 - C. Thursday
 - D. Saturday

2. Gandhi Jayanti of year 2178 will fall on which day?
 - A. Friday
 - B. Thursday
 - C. Monday
 - D. None of the above

3. In what year will the calendar of 872 repeat?
 - A. 900
 - B. 893
 - C. 1022
 - D. 1040

4. It was Sunday on Jan 1, 2006. What was the day of the week Jan 1, 2010?
 - A. Tuesday
 - B. Friday
 - C. Sunday
 - D. Monday

5. Which day of the week will it be on the middle day of the year 2065?
 - A. Tuesday
 - B. Wednesday
 - C. Friday
 - D. Sunday

6. Which day of the week was on 11th November, 1662?
 - A. Tuesday
 - B. Friday
 - C. Saturday
 - D. Monday

7. If 27th March, 2011 was Sunday, what was the day on 27th June, 2011?
 - A. Sunday
 - B. Tuesday
 - C. Monday
 - D. Saturday

8. The birthday of Ms. Y was celebrated six days before Ms. X, who was born on 4th October 1999. The Independence Day of that year fell on Sunday. On which day did Ms. Y celebrate her birthday, if it was not a leap year?
 - A. Sunday
 - B. Monday
 - C. Wednesday
 - D. Tuesday

9. Today is Monday. After 61 days, it will be __
 - A. Friday
 - B. Saturday
 - C. Sunday
 - D. Monday

10. How many days are there in x weeks x days?
 - A. $7x^2$
 - B. 7
 - C. $8x$
 - D. $14x$

Solutions:

1. A
2. A
3. D
4. B
5. B
6. C
7. C
8. D
9. B
10. C

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GS FOUNDATION
BATCH FOR CSE (2023-24)
ACE CSAT - WORKSHEET 08
ARITHMETIC II

1. What will be the remainder when 3^{436} is divided by 10?
A. 1
B. 3
C. 9
D. 7

2. The sum of the digits of a two-digit number is $1/7$ of the number. The units digit is 4 less than the tens digit. If number obtained by reversing the digits is divided by 7, what will be the remainder?
A. 1
B. 4
C. 5
D. 6

3. A positive integer when divided by 425 gives a remainder 45. When the same number is divided by 17, what will be the remainder?
A. 7
B. 11
C. 13
D. 16

4. What is the remainder when 37^{123423} is divided by 4?
A. 0
B. 1
C. 2
D. 3

5. What is the remainder when $(23123 + 131212 + 1223421)$ is divided by 7?
A. 0
B. 1
C. 2
D. 5

6. What is the remainder when $(23 \times 32 \times 5331 \times 125)$ is divided by 13?
A. 1
B. 5
C. 8
D. 12

7. If n is a natural number, consider following statements

1. $6n^2 + 6n$ will always be divisible by 6
2. $6n^2 + 6n$ will always be divisible by 12
3. $6n^2 + 6n$ will always be divisible by 18

Which of the statements above are correct?

- A. 1 and 2 only
- B. 1 and 3 only
- C. 2 and 3 only
- D. 1, 2 and 3

8. What is the value of $107 \times 107 + 93 \times 93$?

- A. 19573
- B. 19876
- C. 20098
- D. 21284

9. A fly starts to travel from point P to Q. It then comes back to P. In next round, fly travels to midpoint of PQ, which is R and comes back to P. In the next step, fly travels to midpoint of PR and returns back. In each subsequent round, fly reduces its target point to go and come back by half. Fly makes trips indefinitely. What is the total distance travelled by the fly if distance between P and Q is 1?

- A. 2
- B. 4
- C. 8
- D. Fly will travel infinite distance as it keeps on travelling indefinitely

10. How many 3-digit numbers are completely divisible by 6?

- A. 149
- B. 150
- C. 151
- D. 166

11. What is the value of $\frac{(376+843)^2+(376-843)^2}{376 \times 376 + 843 \times 843}$?

- A. 0
- B. 1
- C. 2
- D. 756

12. What is the 10th term of an Arithmetic progression with initial term 3 and square of a smallest prime number as common difference?

- A. 12
- B. 21
- C. 39
- D. 84

13. Which of the following is common factor of $(43^{43} + 47^{43})$ and $(43^{47} + 47^{47})$?

- A. 85
- B. 90
- C. $(43^{43} + 47^{43})$
- D. None of the above

14. What least number must be subtracted from 13601, so that the remainder is divisible by 87?

- A. 23
- B. 31
- C. 29
- D. 37

15. 476XY0 is divisible by both 3 and 11. Which of the following digits could be there in place of X and Y?

- A. 8 and 5
- B. 2 and 7
- C. 1 and 3
- D. 0 and 1

16. On dividing 2272 as well as 875 by 3-digit number N, we get the same remainder. The sum of the digits of N is

- A. 10
- B. 11
- C. 12
- D. 13

17. If $N = 71 + 72 + 73 + \dots + 99$; What is N?

- A. 1736
- B. 2465
- C. 3345
- D. 2284

18. Which of the following will completely divide $(3^{31} + 3^{32} + 3^{33} + 3^{34})$

- A. 11
- B. 16
- C. 30
- D. 50

19. The sum of how many terms of the series $6 + 12 + 18 + 24 + \dots$ is 1800?

- A. 18
- B. 20
- C. 22
- D. 24

20. What is the value of $3^{13} - 2 \times (3 + 3^2 + 3^3 + \dots + 3^{12})$

- A. -2
- B. 3
- C. -3
- D. 2

21. Which of the following is not a square of a natural number?

- A. 276676
- B. 113569
- C. 396412
- D. 622521

Solutions:

- | | |
|-------|-------|
| 1. A | 15. A |
| 2. C | 16. A |
| 3. B | 17. B |
| 4. B | 18. C |
| 5. C | 19. B |
| 6. D | 20. B |
| 7. A | 21. C |
| 8. C | |
| 9. B | |
| 10. B | |
| 11. C | |
| 12. C | |
| 13. B | |
| 14. C | |

GS FOUNDATION
BATCH FOR CSE 2023
Ace CSAT 2023 – Booklet 9
Quantitative aptitude - 3 Percentages

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1) INTRODUCTION:

Percent means per hundred. So, it is one-hundredth part of something (cent is hundred).

So, 1% will mean $1/100$

5% will mean $5/100$

20% will mean $20/100$

Percentage is a fraction whose denominator is 100 and numerator is called the rate

2) CONVERTING PERCENTAGE INTO FRACTION:

Here we simply divide given percent by 100 and delete % sign

For example:

Fraction form of 2% is $2/100$ which is same as $1/50$

Fraction form of 10% is $10/100$ which is same as $1/10$

Fraction form of 120% is $120/100$ which is same as $6/5$

3) CONVERTING FRACTION TO PERCENTAGE

Here we just multiply given fraction by 100 and add a percentage sign next to it

So, a/b is same as $(a/b * 100) \%$

Q. Express $\frac{3}{4}$ into percentage

Q. Express $\frac{3}{8}$ into percentage

Q. Express $\frac{1}{2}$ into percentage

4) PERCENTAGE OF A PARTICULAR NUMBER

P% of a number = result

$P/100 * \text{number} = \text{result}$

Number = $\text{result}/P * 100$

Q. 9% of what number is 36?

Q. 15% of what number is 150?

Q. 20% of what number is 50?

Q. If 30% of a number is 48, then what is 70% of that number?

CSE 2022: When 70% of a number x is added to another number y , the sum becomes 165% of the value of y . When 60% of the number x is added to another number z , then the sum becomes 165% of the value of z . Which one of the following is correct?

(a) $z < x < y$

(b) $x < y < z$

(c) $y < x < z$

(d) $z < y < x$

5) TWO DIFFERENT PERCENTAGES OF A NUMBER

Two different percentages of same number will lead to two different results.

Let, $p\%$ of $N = x$ and $q\%$ of $N = y$

Then, $(p + q)\%$ of $N = x + y$ and $(p - q)\%$ of $N = x - y$

Q. If 40% of a number exceeds the 25% of the same number by 54, find the number.

Q. If 50% of the number is 12 more than 40% of the number, find the number.

6) GIVEN QUANTITY AS A PERCENTAGE OF ANOTHER QUANTITY

Here, we're given two quantities x and y and have to find what percentage of x is y .

That is, we've to express y as a percentage of another quantity x .

So, $y = p\%$ of x

i.e. $y = p/100 * x$

i.e. $p = y/x * 100$

Q. Find 30 is what percentage of 150.

Q. What percentage of 200 is 25?

Q. What percentage of 100 is 37.5?

7) CONVERTING PERCENTAGE INTO DECIMALS:

Since, $p\% = p/100$, we just shift decimal point by two places to the left

Q. What is 67% into decimal?

Q. Represent 4.7% into decimal.

Q. What is 345% into decimal?

Q. Represent 1234% into decimal.

Q. Convert 3% into decimal.

8) CONVERTING DECIMAL INTO PERCENTAGE

In this case, above method is just reversed. We just shift decimal point to right by two places and % sign is put next to the number.

Q. Express 0.3 as a percentage

Q. What is 4.5 as a percentage

Q. Express 1234 as a percentage

Q. What is 0.046 as a percentage?

9) SOME IMPORTANT PERCENTAGE TO FRACTION CONVERSIONS TO DECIMAL CONVERSIONS:

Fraction	Decimal	Percentage
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%
$\frac{1}{5}$	0.2	20%
$\frac{1}{7}$	0.142857	14.2857%
$\frac{1}{15}$	0.0667	6.67%
$\frac{1}{11}$	0.0909	9.09%
$\frac{1}{20}$	0.05	5%

10) EFFECT OF PERCENTAGE CHANGE ON ANY NUMBER

If number N is increased by x% then we want to find the resulting new number

$$\text{New number} = N + x\% \text{ of } N = N * (1 + x\%) = N * \left(\frac{100+x}{100}\right)$$

Q. Salary of Balbir is 1000. It is increased by 10%. What is his new salary?

Q. What is the resulting number if 10 is increased by 150%.

If any number N is decreased by x%, find the resulting number:

$$\text{New number} = N - x\% \text{ of } N = N * (1 - x\%) = N * \left(\frac{100-x}{100}\right)$$

Q. What is resulting number if 500 is decreased by 2.5%

Q. What is resulting number if 200 is decreased by 100%

11) NET CHANGE OF PERCENTAGE

In this type of questions, percentage is changed in subsequent stages.

In first step, number is changed (increased or decreased) by $x\%$ and in second step it is changed (increased or decreased) by $y\%$. we're asked to find the net change in percentage.

If N is a given number,

New number after step 1 is: $N * \left(\frac{100+x}{100}\right)$; Let's call it M

Thus, the new number after step 2 is: $M * \left(\frac{100+y}{100}\right)$ which is same as $N * \left(\frac{100+x}{100}\right) \left(\frac{100+y}{100}\right)$

$$\begin{aligned} \text{Thus, change is } & N * \left(\frac{100+x}{100}\right) \left(\frac{100+y}{100}\right) - N = N ((1+x\%) (1+y\%) - 1) \\ & = N (x\% + y\% + x\% y\%) \end{aligned}$$

$$\text{Thus, percentage change is } = \frac{N (x\% + y\% + x\% y\%)}{N} \times 100 = x + y + \frac{xy}{100}$$

Q. If a number is increased by 12% and then decreased by 18%, find the net change in percentage.

Q. If Salary of Virat is increased by 10% and then again by 20%, find the net increase in the salary.

Q. If length of a rectangle is increased by 30% and breadth is decreased by 20%, find the net change in the area of rectangle.

CSE 2022:

The increase in the price of a certain item was 25%. Then the price was decreased by 20% then again increased by 10%. What is the resultant increase in the price?

- (a) 5%
- (b) 10%
- (c) 12.5%
- (d) 15%

CSE 2021: If the price of an article is decreased by 20% and then the new price is increased by 25%, then what is the net change in the price?

- (a) 0%
- (b) 5% increase
- (c) 5% decrease
- (d) Cannot be determined due to insufficient data

12) KEEPING EXPENDITURE FIXED

Expenditure = consumption * rate of item

So, to keep expenditure fixed, in case rate increases, we've to reduce consumption and if the rate falls, we've to increase consumption

If rate is changed by $r\%$, we've to change consumption by say $p\%$

According to formula above, new rate and new consumption are

Old rate* $(\frac{100+r}{100})$ and old consumption* $(\frac{100+p}{100})$.

And since new and old expenditure are same

Old rate*old consumption = new rate* new consumption

Old rate*old consumption = Old rate* $(\frac{100+r}{100})$ *old consumption* $(\frac{100+p}{100})$.

Here, we want to find ' p ' as ' r ' is known

$$p = \left(\frac{r}{100+r} \right) \times 100$$

$$\% \text{ Change in consumption (p)} = \left(\frac{\text{percentage change in rate}(r)}{100 + \% \text{ change in rate}(r)} \right) \times 100$$

Observe that expenditure = consumption * rate and consumption is being kept constant

Similarly, for area of rectangle = length * breadth

Distance = speed * time; same formula can be used if we keep area or distance as constant

Q. If price of watch is increased by 10%, by how much percentage must a shopkeeper reduce his consumption to have no extra expenditure

Q. If price is reduced by 5%, and then increased by 20%, by how much percentage must consumption change to have same expenditure?

13) % EXCESS OR % SHORTNESS

Here we're given 2 numbers A and B. we've to tell by how much percentage, A is more than B and by how much B is less than A.

If A exceeds B by $x\%$, then, by our formula

$$A = B * \left(\frac{100+x}{100} \right)$$

And if B is short of A by $y\%$,

$$B = A * \left(\frac{100+y}{100} \right); \text{ putting the value of B from above,}$$

$$A * \left(\frac{100}{100+x} \right) = A * \left(\frac{100+y}{100} \right), \text{ solving for } y,$$

$$\text{Percentage shortness of B (y)} = \left(\frac{x}{100+x} \right) \times 100$$

Similarly, if y or shortness of B is given, then,

A is more than B by $(\frac{y}{100-y}) \times 100$

Q. If income of Amar is 20% more than Samar, by what percentage is Samar's income less than that of Amar?

Q. If A is 20% less than B, then by how much percentage, B is more than A?

14) INCREASING POPULATION OF A CITY: INCREASE AT A CONSTANT RATE

If there's a city whose population is increasing at a constant rate R every year, we are asked to find population after certain number of years.

This calculation is much similar to compound interest calculation where capital or base amount on which interest is given keeps on increasing every year/period.

If a population of city is X and it is increasing at the rate of R% per year.

$$\text{Population after } N \text{ years} = X \times \left(1 + \frac{R}{100}\right)^N$$

If a population of city is X and we are to find out population before N years or N years ago.

If P is the population N years ago.

$$\text{Population after } N \text{ years} = P \times \left(1 + \frac{R}{100}\right)^N = X \text{ (given)}$$

$$\text{So, } P = \frac{X}{\left(1 + \frac{R}{100}\right)^N}$$

15) PYQS

CSE 2022: Two candidates X and Y contested an election. 80% of voters cast their vote and there were no invalid votes. There was no NOTA (None of the above) option. X got 56% of the votes cast and won by 1440 votes. What is the total number of voters in the voters list?

- (a) 15000
- (b) 12000
- (c) 9600
- (d) 5000

CSE 2022: A pie chart gives the expenditure on five different items A, B, C, D and E in a household. If B, C, D and E correspond to 90° , 50° , 45° and 75° respectively, then what is the percentage of expenditure on item A?

- (a) $112/9$
- (b) $125/6$
- (c) $155/9$
- (d) $250/9$

CSE 2021: In a class, 60% of students are from India and 50% of the students are girls. If 30% of the Indian students are girls, then what percentage of foreign students are boys?

- (a) 45%
- (b) 40%
- (c) 30%
- (d) 20%

CSE 2021: P scored 40 marks more than Q in an examination. If Q scored 10% less marks than P, then how much did Q score.

- (a) 360
- (b) 380
- (c) 400
- (d) 420

CSE 2019: A and B are two heavy steel blocks. If B is placed on the top of A. the weight increases by 60% How much weight will reduce with respect to the total weight of A and B, if B is removed from the top of A?

- (a) 60 %
- (b) 45.5%
- (c) 40%
- (d) 37.5%

CSE 2019: In an examination. A has scored 20 marks more than 11. If has scored 3% less marks than A. how much has B scored

- (a) 360
- (b) 380
- (c) 400
- (d) 420

CSE 2018: A student has to get 40% marks to pass in an examination. Suppose he gets 30 marks and fails by 30 marks, then what are the maximum marks in the examination??

- (a) 100
- (b) 120
- (c) 150
- (d) 300

CSE 2017: $P = (40\% \text{ of } A) + (65\% \text{ of } B)$ and $Q = (50\% \text{ of } A) + (50\% \text{ of } B)$, where A is greater than B. In this context,

which of the following statements is correct?

- (a) P is greater than Q.
- (b) Q is greater than P.
- (c) P is equal to Q.
- (d) None of the above can be concluded with certainty

CSE 2017: In a city, 12% of households earn less than Rs. 30,000 per year, 6% households earn more than Rs. 2,00,000 per year, 22% households earn more than Rs. 1,00,000 per year and 990 households earn between Rs. 30,000 and Rs. 1,00,000 per year. How many households earn between Rs. 1,00,000 and Rs. 2,00,000 per year?

- (a) 250
- (b) 240
- (c) 230
- (d) 225

CSE 2016: Anita's mathematics test had 70 problems carrying equal marks i.e., 10 arithmetic, 30 algebra and 30 Geometry. Although she answered 70% of the arithmetic, 40% of the algebra and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% marks. The number of more questions she would have to answer correctly to earn a 60% passing marks is:

- (a) 1
- (b) 5
- (c) 7
- (d) 9

CSE 2015: In a test, a candidate attempted only 8 questions and secured 50% marks in each of the questions. If he obtained a total of 40% in the test and all questions in the test carried equal marks, how many questions were there in the test?

- (a) 8
- (b) 10
- (c) 15
- (d) 16

CSE 2015: Candidates in a competitive examination consisted of 60% men and 40% women. 70% men and 75% women cleared the qualifying test and entered the final test where 80% men and 70% women were successful.

Which of the following statements is correct?

- (a) Success rate is higher for women.
- (b) Overall success rate is below 50%.
- (c) More men cleared the examination than women.
- (d) Both (a) and (b) above are correct

CSE 2015: A and B decide to travel from place X to place Y by bus. A has Rs. 10 with him and he finds that it is 80% of the bus fare for two persons. B finds that he has Rs. 3 with him and hands it over to A. In this context, which one of the following statements is correct?

- (a) Now the money A has just enough to buy two tickets.
- (b) A still needs Rs. 2 for buying the tickets
- (c) After buying the two tickets A will be left with 50 paise.
- (d) The money A now has is still not sufficient to buy two tickets.

CSE 2014: As per agreement with a bank, a businessman had to refund a loan in some equal instalments without interest. After paying 18 instalments he found that 60 percent of his loan was refunded. How many instalments were there in the agreement?

- (a) 22
- (b) 24
- (c) 30
- (d) 33

CSE 2014: A gardener increased the area of his rectangular garden by increasing its length by 40% and decreasing its width by 20%. The area of the new garden

- (a) has increased by 20%.
- (b) has increased by 12%.
- (c) has increased by 8%.
- (d) is exactly the same as the old area.

CSE 2011: In a group of persons, 70% of the persons are male and 30% of the persons are married. If two sevenths of males are married, what fraction of the females is single?

- (a) $2/7$
- (b) $1/3$
- (c) $3/7$
- (d) $2/3$

16) COMPREHENSION

There is a claim that organic farming is inherently safer and healthier. The reality is that because the organic farming industry is still young and not well-regulated in India, farmers and consumers, alike, are not only confused about what products are best for them, but sometimes use products in ways that could harm them as well. For example, since organic fertilizers are difficult to obtain on a large scale in India, farmers often use farmyard manure, which may contain toxic chemicals and heavy metals. Certain plant sprays, such as Datura flower and leaf spray, have an element called atropine. If it is not applied in the right dose, it can act on the nervous system of the consumer. Unfortunately, how much and when to use it are not well-researched or regulated issues.

Q. Based on the above passage, the following assumptions have been made:

- 1. Organic farming is inherently unsafe for both farmers and consumers.
- 2. Farmers and consumers need to be educated about eco-friendly food.

Which of the assumptions given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

Q. Which one of the following statements best reflects the most logical, rational and practical message conveyed by the author of the passage?

- (a) In India, organic farming should not be promoted as a substitute for conventional farming.
- (b) There are no safe organic alternatives to chemical fertilizers.
- (c) In India, farmers need to be guided and helped to make their organic farming sustainable.
- (d) The aim of organic farming should not be to generate huge profits as there is still no global market for its products.

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GS FOUNDATION
BATCH FOR CSE 2023
Ace CSAT 2023-Booklet - 10
Logical Reasoning Blood Relations

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1) INTRODUCTION

Blood relation simply mean any relation by birth or by marriage. So, father, mother, sister, wife, husband etc. are all types of blood relations.

The most important thing to remember in blood relation problems is that, **YOU MUST NOT ASSUME GENDER UNLESS SPECIFIED IN QUESTION.** If question says that A is son of B then B can be mother or father of A who is a male.

Secondly, you've to choose an option which is correct in all cases and not just in one particular case. We shall see examples where all the options are correct in some specific scenario but only one option is always correct.

2) TYPE OF BLOOD RELATIONS

Blood relations can be from paternal side, maternal side, spouse's side or inter-generational. For sake of completion, let us enlist them.

1. From paternal side:

Father's father	Grandfather
Father's mother	Grandmother
Father's sister/uncle's wife	Aunt
Father's brother/aunt's husband	Uncle
Children of uncle/aunt	Cousin

2. From maternal side:

Mother's father	Grandfather
Mother's mother	Grandmother
Mother's sister	Aunt
Mother's brother/aunt's husband	Uncle (maternal uncle)
Maternal uncle's wife	maternal aunt
Children of uncle/aunt	Cousin

3. From spouse's side:

Spouse's father	Father-in-law
Spouse's mother	Mother-in-law
Spouse's brother	Brother-in-law
Spouse's sister	Sister-in-law

4. Inter-generational:

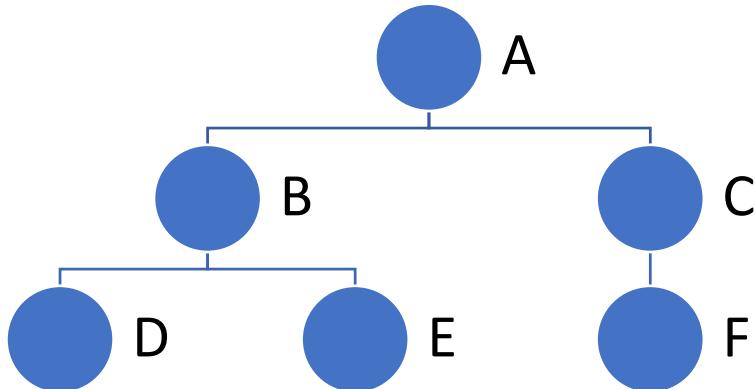
Own children	Son/Daughter
Brother's/sister's children	Niece/Nephew
Children of children	Grandson/granddaughter

3) DRAWING FAMILY TREE

There are various ways you can approach the blood relation questions. You can try to superimpose the question in your family to arrive at answer, you can enlist all relations as per questions, you can look at options given and look for relevant information in passage. But most quick and effective method is to draw family tree.

Family tree is basically a chart showing intergenerational, intra-generational and spousal information in concise manner.

Typically, such tree will look like this:



There are few rules we shall follow for our benefit:

1. Same generation is represented at same level
2. Members of higher generation are represented above in the chart than lower generation
3. Male members are given small '+' sign and female members are given small '-' sign to distinguish them clearly
4. Spousal relationship is shown by bi-directional arrow \leftrightarrow and brother-sister, brother-brother, sister-sister relationships is shown by non-directional line connecting them

Practice problems to draw family tree:

1. In a family, there are six members A, B, C, D, E and F. A and B are a married couple, A being the male member. D is the only son of C, who is the brother of A. E is the sister of D. B is the daughter-in-law of F, draw a family tree.
2. A has 3 children. B is the brother of C and C is the sister of D, E who is the wife of A is the mother of D. There is only one daughter of the husband of E.
3. A is one of two sons of B. D is only sister of E. C, who is mother of E is also a grandmother of A and mother-in-law of B. A and E are children of C. Draw a family tree.
4. There are six persons A, B, C, D, E and F. C is the sister of F. B is the brother of E's husband. D is the father of A and grandfather of F. There are two fathers, three brothers and a mother in the group.

4) TYPES OF PROBLEMS

1. Decoding description

In this type, description is given about relations amongst various persons. Sometimes, multiple families might be involved. Here, we start at the top and plot family tree line by line.

Q. There are six persons A, B, C, D, E and F. C is the sister of F. B is the brother of E's husband. D is the father of A and grandfather of F. There are two fathers, three brothers and a mother in the group. Who is the mother?

- A. A
- B. C
- C. E
- D. B

2. Dialogue based

Here, one person describes some information about some other person or persons and finally the relation between two of them is asked.

We start with the person speaking and work our way through family tree as the person speaks.

Q. Pointing to a person, a man said to a woman, "His mother is the only daughter of your father." How was the woman related to the person?

- A. Daughter
- B. Mother
- C. Wife
- D. Aunt

Q. Pointing to a photograph of a boy Mr. Ram said, "He is the son of the only son of my mother." How is Mr Ram related to that boy?

- A. Brother
- B. Uncle
- C. Cousin
- D. Father

3. Coding – decoding blood relations

Here various relations are represented in codified form. We have to decode the code and solve the question.

- Q. If A + B means A is the mother of B;
- A x B means A is the father of B;
- A \$ B means A is the brother of B and
- A @ B means A is the sister of B

Then which of the following means P is the son of Q?

- (A) Q + R @ P @ N
- (B) Q + R * P @ N
- (C) Q x R \$ P @ N
- (D) Q x R \$ P \$ N

In all the above types, method to solve remains the same. We just quickly draw family tree and answer whatever the question demands.

5) SAMPLE QUESTIONS

1. P, Q, R, S, T, U are six members of a family. R is a spouse of Q. U is the mother of T and S is the daughter of U. P's daughter is T and R's son is P. There are two couples in the family. Which of the following is correct?
 - A. Q is grandfather of T
 - B. Q is grandmother of T
 - C. R is mother of P
 - D. T is granddaughter of Q

2. A has 3 children. B is the brother of C and C is the sister of D, E who is the wife of A is the mother of D. There is only one daughter of the husband of E. what is the relation between D and B?
 - A. Brothers
 - B. Father-son
 - C. Brother-sister
 - D. Mother-son

3. A girl introduced a boy as the son of' the daughter of the father of her uncle. The boy is girl's
 - A. Uncle
 - B. Son
 - C. Son-in-law
 - D. Brother

4. Looking at a portrait of a man, Harsh said, "His mother is the wife of my father's son. Brothers and sisters, I have none." At whose portrait was Harsh looking?
 - A. His son
 - B. His father
 - C. His grandfather
 - D. His nephew

5. Pointing out to a lady, a girl said, "She is the daughter-in-law of the grandmother of my father's only son". How is the lady related to the girl?
 - A. Sister-in-law
 - B. Mother
 - C. Aunt
 - D. Can't be determined
6. A's mother is a sister of B and daughter of C. D is daughter of B and sister of E. How is C related to E?
 - A. Father
 - B. Grandfather
 - C. Grandmother
 - D. Either grandfather or grandmother
7. Directions: Study the following information carefully and answer the questions given beside:
The Mishra family went to the photographer for their family photograph. The photographer arranged them in a straight row facing the North. The family members were Anshu, Rama, Krishna, Deva and Sonu.
 - Anshu was sitting third to the right of his daughter
 - Rama was sitting on the immediate right of Krishna, whose husband was the brother of Rama.
 - Anshu and Sonu were immediate neighbours.
 - Deva was not an immediate neighbour of Anshu's son.
 - Deva was sitting at the left end and the mother of Deva is Krishna, who was the wife of Anshu.
 - Rama and Anshu were immediate neighbours.

Q. How is Deva related to the person who sits in the middle of the row?

- A. Daughter
- B. Son
- C. Aunt
- D. None of these

8. If A+B means A is mother of B;

A-B means A is brother of B

A%B means A is the father of B

A*B means A is sister of B

Which of the following shows that P is maternal uncle of Q?

- A. Q - N + M * P
- B. P + S * N - Q

- C. $P - M + N * Q$
- D. $Q - S \% P$

6) PYQS

CSE 2022: Consider the Question and two Statements given below:

Question: Is Z brother of X.

Statement 1: X is a brother of Y and Y is a brother of Z.

Statement 2: X, Y and Z are siblings.

Which one of the following is correct in respect of the Question and the Statements?

- (a) Statement 1 alone is sufficient to answer the Question
- (b) Statement 2 alone is sufficient to answer the Question
- (c) Both Statement 1 and Statement 2 are sufficient to answer the Question
- (d) Both Statement 1 and Statement 2 are not sufficient to answer the Question

CSE 2022: P, Q, R, S, T and U are six members of a family. R is the spouse of Q; U is the mother of T and S is the daughter of U. P's daughter is T and R's son is P. There are two couples in the family.

Which one of the following is correct?

- (a) Q is the grandfather of T
- (b) Q is the grandmother of T
- (c) R is the mother of P.
- (d) T is the granddaughter of Q

CSE 2020: A family of two generations consisting of six members P, Q, R, S, T and U has three males and three females. There are two married couples and two unmarried siblings. U is P's daughter and Q is R's mother-in-law. T is an unmarried male and S is a male. Which one of the following is correct?

- (a) R is U's husband.
- (b) R is S's wife.
- (c) S is unmarried
- (d) None of the above

CSE 2019: A joint family consists of seven members A. B. C. D, E, F and G with three females. G is a widow and sister-in-law at the father F. B and D are siblings and A is daughter of B. C is cousin of 1. Who is E?

1. Wife of t
2. Grandmother of A
3. Aunt of C

Select the correct answer using the code given below:

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only

(d) 1.2 and3

CSE 2017: Consider the following relationships among members of a family of six persons A, B, C, D, E and F:

1. The number of males equals that of females.
2. A and E are sons of F.
3. D is the mother of two, one boy and one girl.
4. B is the son of A.
5. There is only one married couple in the family at present.

Which one of the following inferences can be drawn from the above?

- (a) A, B and C are all females.
- (b) A is the husband of D.
- (c) E and F are children of D.
- (d) D is the daughter of F.

CSE 2017: Consider the following: A+ B means A is the son of B. A - B means A is the wife of B.

What does the expression P + R – Q mean?

- (a) Q is the son of P.
- (b) Q is the wife of P.
- (c) Q is the father of P.
- (d) None of the above

CSE 2015: Four persons A, B, C and D consisting of two married couples are in a group. Both the women are shorter than their respective husbands. A is the tallest among the four. C is taller than B. D is B's brother. In this context, which one of the following statements is not correct?

- (a) All four have family ties.
- (b) B is the shortest among the four.
- (c) C is taller than D.
- (d) A is B's husband

CSE 2014: Read the passage given below and answer the items that follow.

A, B, C, D, E, F are members of a family. They are engineer, stenographer, doctor, draughtsman, lawyer and judge (not in order). A, the engineer is married to the lady stenographer. The judge is married to the lawyer. F, the draughtsman is the son of B and brother of E. C, the lawyer is the daughter-in-law of D. E is the unmarried doctor. D is the grandmother of F. There are two married couples in the family.

Q. What is the profession of B?

- (a) Judge
- (b) Lawyer
- (c) Draughtsman
- (d) Cannot be determined

Q. Which of the following is/are couple/couples?

- (a) AD only
- (b) BC only
- (c) Both AD and BC
- (d) Both AC and BD

Q. What is the profession of D?

- (a) Judge
- (b) Stenographer
- (c) Doctor
- (d) Cannot be determined

CSE 2014: Given that, 1. A is the brother of B 2. C is the father of A. 3. D is brother of E. 4. E is the daughter of B Then, the uncle of D is?

- (a) A
- (b) B
- (c) C
- (d) E

CSE 2011: Read the following passage and answer (three) items that follow:

A, B, C, D and E are members of the same family. There are two fathers, two sons, two wives, three males and two females. The teacher was the, wife of a lawyer who was the son of a doctor. E is not male, neither also a wife of a professional. C is the youngest person in the family and D is the eldest. B is a male.

Q. How is D related to E?

- (a) Husband
- (b) Son
- (c) Father
- (d) Wife

Q. Who are the females in the group?

- (a) C and E
- (b) C and D
- (c) E and A
- (d) D and E

Q. Whose wife is the teacher?

- (a) C
- (b) D
- (c) A
- (d) B

7) COMPREHENSION

As the 21st century advances, a new danger — the cyber threat — is becoming a hydra-headed monster. It is hardly confined to any one domain though the military is the one most often touted. Rather, it is the civilian sphere where the cyber threat is becoming more all-pervading today and, in turn, a serious menace. It is beginning to have a cascading effect with questions being raised on how this would fit in with our belief in, and need for, a well-regulated world order. What is most unfortunate is that not enough attention is being bestowed on the ‘all-encompassing nature’ of the cyber threat. In the wake of the Russia-Ukraine conflict, the world seems awash with papers on artificial intelligence (AI)-driven military innovations and ‘potential crisis hot zones’, along with stray references to new forms of hybrid warfare. But there is very little about the threat posed by cyber-attacks. Ignored also is the new reality of the ‘weaponization of everything’ which has entered the vocabulary of threats. The latter clearly demands a ‘proto-revolutionary’ outlook on the part of policymakers, which is evidently lacking. Lost in translation is also the nature of today’s weapon of choice, viz., cyber. This lack of awareness is unfortunate at a time when states clearly lack the necessary resilience to face a variety of multi-vector threats.

Q. Which of the following opinions would author will disagree with most?

- A. Threat of cyber-security has not remained confined to threat to critical infrastructure
- B. Discussion about future of wars is incomplete without discussion cyber warfare and weaponization of everything
- C. States and policymakers are taking substantial steps however lot more needs to be done
- D. Aftermath of Russia-Ukraine crisis has failed to brought into focus cyber security as much as it should have

Natural selection cannot anticipate future environments on the earth. Therefore, the set of existing organisms can never be fully prepared for environmental catastrophes that await life. An outcome of this is the extinction of those species which cannot overcome environmental adversity. This failure to survive, in modern terms, can be attributed to the genomes which are unable to withstand geological vagaries or biological mishaps infections, diseases and so on). In biological evolution on the earth, extinction of species has been a major feature. The earth may presently have up to ten million species, yet more than 90% of species that have ever lived on the earth are now extinct. Once again, the creationist doctrines fail to satisfactorily address why a divine creator will firstly bother to create millions of species and then allow them to perish. The Darwinian explanation for extinct life is once again simple, elegant and at once convincing organisms go extinct as a function of environmental or biological assaults for which their inheritance deems them ill-equipped. Therefore, the so-called Darwinian theory of evolution is not a theory at all. Evolution happens-this is a fact. The mechanism of evolution (Darwin proposed natural selection) is amply supported by scientific data. Indeed, to date no single zoological, botanical, geological, paleontological, genetic or physical evidence has refuted either of the central two main Darwinian ideas. If religion is not taken into consideration, Darwinian laws are acceptable just like the laws proposed by

Copernicus, Galileo, Newton and Einstein-sets of natural laws that explain natural phenomena in the universe.

Q. According to the passage, natural selection cannot anticipate future environments on the earth as

1. species not fully prepared to face the environmental changes that await them will face extinction
2. all the existing species would get extinct as their genomes will not withstand biological mishaps
3. inability of the genome to withstand environmental changes would result in extinction
4. extinction of species is a common feature

Select the correct answer using the code given below.

- (a) 1, 2 and 3
- (b) 2, 3 and 4
- (c) 1, 3 and 4
- (d) 1, 2 and 4

Q. The passage suggests that Darwinian theory of evolution is not a theory at all because

- (a) it does not satisfy the creationist doctrine
- (b) extinction is a function of environment and biological assaults
- (c) there are no evidences to refute it
- (d) existence of organisms is attributed to a creator

Q. With reference to the passage, the following assumptions have been made:

1. Only species that have the ability to overcome environmental catastrophes will survive and perpetuate.
2. More than 90% of the species on the earth are in the danger of getting extinct due to drastic changes in the environment.
3. Darwin's theory explains all the natural phenomena,

Which of the above assumptions is/are valid?

- (a) 1 only
- (b) 1 and 2 only
- (c) 3 only
- (d) 1, 2 and 3

Q. People will invest in education whenever they are granted the economic freedom to fully enjoy its benefits. Again, this is for the obvious reason that the return on education increases as the level of economic freedom rises. When people, thanks to lower tax rates, are allowed to retain most of the higher income that they gain from each incremental level of education, it makes eminent sense to invest in education. On the other hand, when the government decides to tax the higher income of educated individuals at even higher rates, it makes very little sense to invest in educating oneself further. The same incentives apply to parents who decide on whether to invest in their children's education.

Which references to the above passage, the following assumptions have been made:

1. Lower tax rates in a country invariably translate into greater investments in higher education.
2. Investment in the education of children ensures their economic freedom.
3. Economic freedom has a positive impact on building up human capital.

Which of the above assumptions is/are valid?

- (a) 1 only
- (b) 2 only
- (c) 3 only.
- (d) 1, 2 and

To tackle the problem of pollution in cities, policy makers think that drastic actions like temporary use of odd-even number scheme for vehicles, closing schools, factories, construction activities, and banning the use of certain type of vehicles are a way forward. Even then the air is not clean. Vehicles more than 15 years old comprise one percent of the total; and taking them off the road will not make any difference. Banning certain fuels and car types arbitrarily is not proper. Diesel engines produce more PM 2.5 and less CO₂ than petrol or CNG engines. On the other hand, both diesel and CNG engines produce more NO_x than petrol engines. No one has measured the amount of NO_x that CNG engines are emitting. Arbitrary bans on vehicles that have passed mandated fitness tests and periodic pollution tests are unfair. What is needed is the scientific and reliable information about the source of pollutants on a continuing basis and the technologies that will work to reduce pollution from them.

Q. Which one of the following statements best reflects the most logical and rational implication conveyed by the passage?

- (a) Arbitrary curbs on vehicles to reduce pollution are difficult to implement.
- (b) Knee-jerk reactions cannot but an evidence-based approach will be more effective.
- (c) A heavy penalty should be enforced on those driving without periodic pollution tests.
- (d) In the absence of laws to deal with the problem of pollution, the administration tends to make arbitrary decisions.

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BATCH FOR CSE 2023
Ace CSAT 2023-Worksheet 17
Blood relations: Class Test – 1

1. 1 A3P means A is the mother of P

A4P means A is the brother of P

A9P means A is the husband of P

A5P means A is the daughter of P

Which of the following means that K is the mother-in-law of M?

A. M9N3K4J

B. M9N5K3J

C. K5J9M3N

D. K3J9N4M

2. A is the brother of B. B is the brother of C. D is the father of A. Based on these three statements, which of the following statements cannot be definitely true?

A. B is son of D

B. A is brother of C

C. C is brother of A

D. B is brother of A

3. A family has six members – P, Q, R, S, T and U. Q is brother-in-law of R's mother. R is father of U. S is father of P and U is grandson of S. There are two females in family.

Who is mother of R?

A. Q

B. T

C. P

D. Cannot be determined

4. Which of the following options represent females of the family?

A. Q and R

B. P and T

C. R and S

D. Cannot be determined

5. Members of the family belong to how many generations?

A. 2

B. 3

C. 4

D. 2 or 3

6. P, Q, R, S, T, U and V are seven members of a family belonging to three generations. There are two married couples – one each of first and second generation. R is a granddaughter. T is father of Q. S is granddaughter of U. P has a daughter.

How many females are there in the family?

- A. 3
- B. 4
- C. 5
- D. None of the above

7. Which of the following is a married couple?

- A. QR
- B. TU
- C. QS
- D. PQ

8. A + B means A is daughter of B

A – B means A is husband of B

A*B means A is brother of B

P*Q – R means?

- A. P is son-in-law of R
- B. P is brother of R
- C. R is wife of P
- D. P is husband of Q

9. H + I * J + K * L + M * N then what is the generation of H assuming oldest generation among them as first?

- A. 1st
- B. 2nd
- C. 3rd
- D. 4th

10. Which of the options below do not make sense?

- A. A + B + C
- B. A – B + C
- C. A + B – C
- D. A*B – C

11. A*B*C*D – which option is NOT necessarily true?

- A. B is brother of A
- B. C is brother of A
- C. D is brother of C
- D. A is brother of B

12. Pointing to a girl in the photograph, Amar said, "Her mother's brother is the only son of my mother's father." How is the girl's mother related to Amar?

- A. Mother
- B. Sister
- C. Aunt
- D. Grandmother

13. A, B, C, D, E and F are members of a family. A is son of B and E. C and D are married couple and have two children. F is a female and not a sister of B. C is grandmother of A.

What is relationship between D and B?

- A. Mother-son
- B. Father-son
- C. Father-in-law
- D. Mother-in-law

14. How many persons of the above family are females?

- A. 1
- B. 2
- C. 3
- D. Cannot be determined

15. Who amongst the following is youngest family member?

- A. A
- B. D
- C. F
- D. Cannot be determined

16. If D is the brother of B, how B is related to C? To answer this question which of the statements is/are necessary?

1. The son of D is the grandson of C.

2. B is the sister of D.

A. 1 only

B. 2 only

C. 1 and 2 both

D. Neither 1 nor 2

17. A and B are children of D. Who is the father of A?

To answer this question which of the statements (1) and (2) is necessary?

1. C is the brother of A and the son of E.

2. F is the mother B.

A. Only (1)

B. Only (2)

- C. Either (1) or (2)
- D. (1) and (2) both

18. Q is the brother of R; P is the sister of Q; T is the brother of S; S is the daughter of R. Who are the cousins of Q?

- A. R, P
- B. S, T
- C. Q, T
- D. P, T

19. A, B, C, D, E, F are family of six. Three are males and three are females. There are two married couples and two persons are unmarried. E is mother-in-law of A, who is wife of C. D is father of F. B is sister of F.

How is F related to E?

- A. Brother
- B. Son
- C. Father
- D. Daughter

20. Which is one of the married couples?

- A. B and D
- B. B and F
- C. E and F
- D. D and E

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GS FOUNDATION
BATCH FOR CSE 2023
Ace CSAT 2023-Booklet 11
Averages

Contents

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1) MEANING OF AVERAGE OR MEAN

Concept of average is not new to us. (Sachin's average, Kohli's average etc.)

Average is simply sum of all items/units divided by number of units.

For example: Average marks of a class = $\frac{\text{sum of marks of all students}}{\text{number of students}}$

In general,

$$\text{Average} = \frac{\text{Sum of items}}{\text{Number of items}}$$

Also,

$$\text{Number of items} = \frac{\text{Sum of items}}{\text{Average}}$$

And,

$$\text{Sum of items} = (\text{Number of items}) \times \text{Average}$$

Q. Find average/mean of following:

1. 23,34,45,65,13
2. 453,33,567
3. 12,12,12,12,12,12

2) AVERAGE OF COMBINED GROUP

If average of two groups separately is known and the average of combined group is to be determined:

- If there are m items in group 1 and n in group 2. And averages of two groups is a and b respectively.
- Then, sum of items in group 1 is ma and those in group 2 in nb.
- For the combined group, sum of items = $ma + nb$ and total number of items is $m + n$.
- Average = $\frac{ma+nb}{m+n}$

Q. Average weight of 30 girls in class is 1500kg and 40 boys in class is 2400kg. Find the average weight of the class.

Q. Average salary of 30 workers is 6000 and average salary of 12 managers is 15000. What's their combined average salary.

3) ITEM/PERSON IS REPLACED IN A GROUP

If a group has 'n' items. And we replace one item with another of different value, average will change.

$$\text{New average} = \frac{\text{Old total sum} - \text{value of item left} + \text{value of item which replaced old}}{n}$$

NOTE: Old total sum can be found using old average.

If new average is given and we're to find out value of new item:

- If the average increases,

$$\text{Value of new item} = \text{Value of old item} + (\text{Increase in average} \times \text{Total number of items})$$

- If the average decreases,

$$\text{Value of new item} = \text{Value of old item} - (\text{Decrease in average} \times \text{Total number of items})$$

Q. If average age of a class is 17. Raju a boy aged 18 leaves class due to transfer of his father. After 3 weeks, Meera, joins the class making the average age 17.5. What is the age of Meera if there were 19 students in class after Raju left.

4) AN ITEM/PERSON JOINS THE GROUP

If a group has 'n' items and another one is added to the group. It'll increase both the total sum and total number of items. Average may get affected.

$$\text{New average} = \frac{\text{Old total sum} + \text{value of new item}}{n + 1}$$

NOTE: Old total sum can be found using old average.

If new average is given and we're to find out value of added item:

- If average increases:

$$\text{Value of added item} = \text{Previous average} + \text{Increase in average} \times (n + 1)$$

$$\text{Value} = A + (n + 1)x$$

Where, A is old average and 'x' is the increase

- If average decreases:

$$\text{Value of added item} = \text{Previous average} - \text{Decrease in average} \times (n + 1)$$

$$\text{Value} = A - (n + 1)x$$

5) AN ITEM/PERSON LEAVES THE GROUP

If a group has 'n' items and one of them leaves the group reducing both the total sum and the number of items. It may change the average as well.

$$\text{New Average} = \frac{\text{Old total sum} - \text{value of item left}}{n - 1}$$

NOTE: Old total sum can be found using old average

If new average is given and we're to find out value of item left:

- **If average increases:**

Value of item removed = Previous average + Increase in average × (1 - n)

$$\text{Value} = A + (1 - n)x$$

Where, A is old average and 'x' is the increase

- **If average decreases:**

Value of item removed = Previous average - Decrease in average × (1 - n)

$$\text{Value} = A - (1 - n)x$$

6) QUESTIONS:

1. If the sum of few numbers is 630 and their mean is 90. If another number 120 is included, what is the new mean?
 - A. 75
 - B. 86.25
 - C. 92.75
 - D. 98.125
2. The average of 9 quantities is 8. The average of 4 of them is 10. What is the average of remaining 5?
 - A. 8
 - B. 6.4
 - C. 6.5
 - D. 6.66
3. The average temperature on 1st, 2nd and 3rd September was 37 degrees and the average temperature on 2nd, 3rd and 4th September was 38 degrees. If the temperature on 4th was 39, what was the temperature on 1st?
 - A. 36
 - B. 37
 - C. 38
 - D. 39
4. The average weight of the class was 48 kg for a class of 29 students. If the teacher's weight is included, the average increases by 0.5kg. What is the weight of the teacher?
 - A. 54 kg
 - B. 60 kg
 - C. 63 kg
 - D. 72 kg
5. What is the average of all prime numbers from 10 to 30?

- A. 18.67
- B. 20.33
- C. 21
- D. 24.67

7) PYQS

CSE 2022: The average weight of A, B, C is 40 kg, the average weight of B, D, E is 42 kg and the weight of F is equal to that of B. What is the average weight of A, B, C, D, E and F?

- (a) 40.5 kg
- (b) 40.8 kg
- (c) 41 kg
- (d) Cannot be determined as data is inadequate

CSE 2021: There are two Classes A and B having 25 and 30 students respectively. In Class-A the highest score is 21 and lowest score is 17. In Class-B the highest score is 30 and lowest score is 22. Four students are shifted from Class-A to Class-B.

Consider the following statements:

- 1. The average score of Class-B will definitely decrease.
- 2. The average score of Class-A will definitely increase.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

CSE 2021: The average age of a teacher and three students is 20 years. If all the three students are of same age and the difference between the age of the teachers and each student is 20 years, then what is the age of the teacher?

- (a) 25 years
- (b) 30 years
- (c) 35 years
- (d) 45 years

CSE 2021: The average score of a batsman after his 50th innings was 46.4. After 60th innings, his average score increases by 2.6. What was his average score in the last ten innings?

- (a) 122
- (b) 91
- (c) 62
- (d) 49

CSE 2021: In a class, there are three groups A, B and C. If one student from group A and two students from group B are shifted to group C, then what happens to the average weight of the students of the class?

- (a) It increases.
- (b) It decreases.
- (c) It remains the same.
- (d) No conclusion can be drawn

CSE 2019: A family has two children along with their parents. The average of the weights of the children and their mother is 50 kg. The average of the weights of the children and their father is 52 kg. If the weight of the father is 60 kg, then what is the weight of the mother?

- (a) 48 kg
- (b) 50 kg
- (c) 52 kg
- (d) 34 kg

CSE 2019: The average marks of 100 students are given to be 40. It was found later that marks of one student were 53 which were misread as 83. The corrected mean marks are

- (a) 39
- (b) 39.7
- (c) 40
- (d) 40

CSE 2017: Suppose the average weight of 9 persons is 50 kg. The average weight of the first 5 persons is 45 kg, whereas the average weight of the last 5 persons is 55 kg. Then the weight of the 5th person will be

- (a) 45 kg
- (b) 47.5 kg
- (c) 50 kg
- (d) 52.5 Kg

CSE 2017: There are thirteen 2-digit consecutive odd numbers. If 39 is the mean of the first Five such numbers, then what is the mean of all the thirteen numbers?

- (a) 47
- (b) 49
- (c) 51
- (d) 45

CSE 2016: The sum of the ages of 5 members comprising a family, 3 years ago was 80 years. The average age of the family today is the same as it was 3 years ago, because of an addition of a baby during the intervening period. How old is the baby?

- (a) 6 months
- (b) 1 year
- (c) 2 years
- (d) 2 years and 6 months

CSE 2016: The average monthly income of a person in a certain family of 5 is Rs. 10,000. What will be the average monthly income of a person in the same family if the income of one person increased by Rs. 1,20,000 per year?

- (a) Rs. 12,000
- (b) Rs. 16,000
- (c) Rs. 20,000
- (d) Rs. 34,000

CSE 2016: The monthly average salary paid to all the employees of a company was Rs. 5000. The monthly average salary paid to male and female employees was Rs. 5200 and Rs. 4200 respectively. Then the percentage of males employed in the company is

- (a) 75%
- (b) 80%
- (c) 85%
- (d) 90%

CSE 2014: The following table shows the marks obtained by two students in different subjects:

Subjects	Student A	Maximum marks	Student B	Maximum marks
English	60	100	80	150
Psychology	70	100	70	100
History	30	100	60	100
Sanskrit	50	50	15	25

The difference in the mean aggregate percentage marks of the students is

- (a) 2.5%
- (b) 13.75%
- (c) 1.25%
- (d) Zero

CSE 2011: A student on her first 3 tests receives on an average score of N points. If she exceeds her previous average score by 20 points on her fourth test, then what is the average score for the first 4 tests?

- (a) $N + 20$
- (b) $N + 10$
- (c) $N + 4$
- (d) $N + 5$

8) COMPREHENSION

Good corporate governance structures encourage companies to provide accountability and control. A fundamental reason why corporate governance has moved onto the economic and political agenda worldwide has been the rapid growth in international capital markets. Effective corporate governance enhance access to external financing by firms, leading to greater investment, higher growth and employment. Investors look to place their funds where the standards of disclosure, of timely and accurate financial reporting, and of equal treatment to all stakeholders are met.

Q. Which of the following statements best reflects the logical inference from the passage given above?

- (a) It is an important agenda of the countries around the world to ensure access to good external financing.
- (b) Good corporate governance improves the credibility of the firms.
- (c) International capital markets ensure that the firms maintain good corporate governance.
- (d) Good corporate governance paves the way for robust supply chains

Elephants are landscape architects, creating clearings in the forest, preventing overgrowth of certain plant species and allowing space for the regeneration of others, which in turn provide sustenance to others herbivorous animals. Elephants eat plants, fruits and seeds, propagating the seeds when they defecate in other places as they travel. Elephant dung provides nourishment to plants and animals and acts as a breeding ground for insects. In times of drought, they access water by digging holes which benefits other wildlife.

Q. Which one of the following statements best reflects the most logical and rational inference that can be drawn from the passage?

- (a) The home range of elephants needs to be a vast area of rich biodiversity.
- (b) Elephants are the keystone species and they benefit the biodiversity.
- (c) Rich biodiversity cannot be maintained in the forests without the presence of elephants.
- (d) Elephants are capable of regenerating forests with species as per their requirement.

The emissions humans put into the atmosphere now will affect the climate in the middle of the century and onwards. Technological change, meanwhile, could make a future transition away from fossil fuels cheap or it might not, leaving the world with a terrible choice between sharply reducing emissions at huge cost or suffering through the effects of unabated warming. Businesses that do not hedge against the threat of uncertain outcomes fail. The world cannot afford such recklessness on climate change.

Q. Which one of the following statements best reflects the crucial message conveyed by the author of the passage?

- (a) Businesses that cause emissions may need to close down or pay for pollution in future.
- (b) The only solution is technological development related to the issues of climate change.
- (c) Waiting to deal with carbon emissions until technology improves is not a wise strategy.
- (d) Since future technological change is uncertain, new industries should be based on renewable energy sources.

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Target Prelims 2023: CSAT
Ace CSAT 2023-Booklet 12
Quantitative Aptitude-8 - Ratio and Proportion

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1) RATIO:

If there are two quantities 'a' and 'b' having same units, then their ratio will be the fraction of the two quantities. So, $a:b = \frac{a}{b}$

NOTE:

- Ratio does not get affected if we multiply both quantities by same quantity
Ex: $a:b = 2a:2b = 5a:5b = na:nb$ i.e., $\frac{a}{b} = \frac{na}{nb}$

Note: division by n is just multiplication by $1/n$ and hence ratio does not change when we divide both quantities by same number

However, $\frac{a}{b} \neq \frac{a+k}{b+k}$

In fact, for a positive k, $\frac{a}{b} < \frac{a+k}{b+k}$ if $\frac{a}{b} < 1$

2) PROPORTION:

When two ratios are equal to each other, this equality is known as proportion.

So, if $a:b = c:d$ (which indicates that two ratios are equal to each other),

We can write the proportionality as, $a:b :: c:d$, and then, we can conclude that a, b, c, d are in proportion.

We can write this as $\frac{a}{b} = \frac{c}{d}$ which implies, $a \times d = b \times c$

With this singular fact, we can solve almost all types of problems

3) MERGING TWO RATIOS HAVING ONE COMMON ELEMENT

If both $a:b$ and $b:c$ are known we can merge two ratios into one using common element 'b' as a bridge.

Example: $a:b = 2:3$ & $b:c = 6:11$

Here 'b' is a common element that can be used as a bridge. We try to get same value in both ratios at place of 'b' using multiplication

Here, we can multiply first ratio by $6/3 = 2$ to get

$a:b = 2:3 = 4:6$ and we have $b:c = 6:11$

Thus, $a:b:c = 4:6:11$

4) PROPERTIES OF RATIOS AND PROPORTIONS:

(Proofs follow from cross multiplication)

1. **Invertendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{b}{a} = \frac{d}{c}$

Ex: $3:4 = 9:12$ then $4:3 = 12:9$

2. **Alternendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a}{c} = \frac{b}{d}$

Ex: $3:4 = 9:12$ then, $3:9 = 4:12$

3. **Componendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a+b}{b} = \frac{c+d}{d}$

Ex: $3:4 = 9:12$ then $(3+4):4 = (9+12):12$ i.e., $7:4 = 21:12$

4. **Dividendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a-b}{b} = \frac{c-d}{d}$

Ex: $4:3 = 12:9$ then, $(4-3):3 = (12-9):9$ i.e., $1:3 = 3:9$

Or $3:4 = 9:12$ then $(3-4):4 = (9-12):12$ i.e., $-1:4 = -3:12$

5. **Componendo-Dividendo:** If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Ex: $3:4 = 9:12$ then $(3+4):4 = (9+12):12$ i.e.,

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \dots = k$ (say) then, each ratio(k) = $\frac{a+c+e\dots}{b+d+f\dots}$

Ex: $3:4 = 6:8 = 9:12$ then each ratio = $(3+6+9):(4+8+12) = 18:24$

7. Direct proportion:

Direct proportion or direct variation is **the relation between two quantities where the ratio of the two is equal to a constant value**. It is represented by the proportional symbol, \propto .

If one quantity is increased, so does the other

If one quantity is decreased, so does the other

For ex: If $a \propto b$ then, $a = kb$ or $\frac{a}{b} = k$ (constant)

8. Inverse Proportion or inverse variation:

It is the relation between two quantities where, **multiplication of two quantities is constant**.

If one quantity is increased, the other decreases

If one quantity is decreased, the other increases

For ex: If a is inversely proportional to b , then

$ab = k$ (constant)

It is equivalent to saying, a is directly proportional to $1/b$

i.e., $a \propto \frac{1}{b}$ then, $a = \frac{k}{b}$ or $ab = k$ (constant)

5) PRACTICING THROUGH QUESTIONS:

1. If $(2x + 3y):(2x - 3y) = 5:1$, then find $x:y$

We first write this as a division: $\frac{2x+3y}{2x-3y} = \frac{5}{1}$

Method 1: We can observe the similar terms in numerator and denominator and use Componendo-dividendo

Thus, $\frac{2x+3y+(2x-3y)}{2x+3y-(2x-3y)} = \frac{5+1}{5-1}$ or $\frac{4x}{6y} = \frac{6}{4} \Rightarrow \frac{x}{y} = \frac{36}{16} = \frac{9}{4}$

Method 2: We can just cross multiply and simplify without thinking much. $\frac{2x+3y}{2x-3y} = \frac{5}{1}$

$$\Rightarrow 2x + 3y = 5(2x - 3y)$$

$$\Rightarrow 2x + 3y = 10x - 15y$$

$$\Rightarrow 8x = 18y$$

$$\Rightarrow 4x = 9y$$

2. If the ratio of the areas of two squares is 9: 16, then find the ratio of their perimeters. (Note here that, you can solve this question only if you are clear about what area and perimeter of a square is. In actual exam as well, you will always need basic clarity in concepts about arithmetic, geometry, set theory etc. – and this is precisely why we spend some dedicated time in understanding them)

- Let side of squares be 'a' and 'b'
- Thus, areas are a^2 and b^2
- Given: $a^2 : b^2 = 9 : 16$ or $\frac{a^2}{b^2} = \frac{9}{16}$
- Taking square root of both sides: $\frac{a}{b} = \frac{3}{4}$
- Perimeters of squares are $4a$ and $4b$
- Thus, ratio of perimeters is $4a : 4b = a : b$ (multiplication does not change the ratio) = 3: 4

3. Angles of a quadrilateral are in the ratio 6: 8: 10: 12. Find the smallest angle.

Again, we need to know the basic geometry to solve this question. We need to know that, sum of angles of quadrilateral is 360 degrees.

- Let the actual angles be $6x, 8x, 10x$ and $12x$
- Thus, $36x = 360$ which implies, $x = 10$
- Thus, smallest angle = $6x = 60$

4. If 20% of $(P + Q) = 50\%$ of $(P - Q)$, then find $P: Q$

Here notice that, two different concepts are mixed i.e. percentages and ratios. Over and above that, you need to know basic arithmetic about how to multiply brackets.

- We can use the fact that $20\% = 20/100 = 1/5$ and $50\% = 1/2$
- $\frac{P+Q}{5} = \frac{P-Q}{2}$
- Now we can either cross multiply to arrive at answer
- Or, bring $(P-Q)$ to other side and use Componendo-Dividendo

5. A sum is divided among 120 men and some women in the ratio 15: 21. If each man gets 5 and each woman gets 4, then find the number of women?

- Here let number of women be 'x'
- Since men get 5, they get total 5×120
- Since women get 4, they get total $4x$
- Given: $\frac{5 \times 120}{4x} = \frac{15}{21}$
- Cross multiplying $60x = 120 \times 5 \times 21$
- $x = \frac{120 \times 105}{60} = 2 \times 105 = 210$

6. If y varies directly as $(x + 3)$ and $y = 8$ when $x = 1$. Then, find the value of y when $x = 2$?

- Given: $y \propto (x + 3)$
- Which means $y = k(x + 3)$
- Putting given values of y and x , $8 = k(1 + 3)$
- $8 = 4k$ which implies, $k = 2$
- Now, putting $x = 2$, $y = 2 * (2 + 3) = 10$

7. 360 is divided among four friends Sakshi, Rupa, Rohit and Monu in the ratio of 3:4:5:6. What is Sakshi's share of money?

- Very similar to angles of quadrilateral question
8. A and B together have Rs. 1210. If $\frac{4}{15}$ th of A's amount is equal to $\frac{2}{5}$ th of B's amount, how much amount does B have?

(We just require to convert word problem into mathematical equations)

9. A right circular cone and a right cylinder have the same radius and the same volume. Then find the ratio of the height of the cone to that of the cylinder

(Here you need to know about volume of cone and right cylinder)

6) MILK AND WATER PROBLEMS:

1. In a mixture 60 litres, the ratio of milk and water 2: 1. If this ratio is to be 1: 2, then the quantity of water to be further added is:

(All such problems are very easily solved if we just tract total milk and total water in the mixture)

- Out of 60 ltr, milk: water is 2:1, so, 40 ltr milk and 20 ltr water
 - Now we want to add water and make this ratio 1:2
 - If x water is required to do this, then, total mixture is $60+x$ out of which 40 is milk and $20+x$ is water
 - We also know that, $40*2 = 20+x$
 - Which gives us, $x = 60$ litres
2. The ratio of milk and water in one pot is 3:5 and that in another pot is 6:1. In what ratio should the contents of the two pots be mixed as to make the ratio between milk and water 7:3?
- Here we can take as much quantity of mixtures as we want
 - Let, ' x ' litres of first mixture and ' y ' litres of second mixture be taken and mixed to get the ratio 7: 3
 - Of ' x ' litres from first mixture, $\frac{3x}{8}$ litres is milk and $\frac{5x}{8}$ litre is water
 - Of ' y ' litres from second mixture, $\frac{6y}{7}$ litre is milk and $\frac{y}{7}$ litre is water
 - Total milk: water ratio is $\frac{\frac{3x}{8} + \frac{6y}{7}}{\frac{5x}{8} + \frac{y}{7}} = \frac{7}{3}$ (given) – we just cross multiply and simplify
 - $3\left(\frac{3x}{8} + \frac{6y}{7}\right) = 7\left(\frac{5x}{8} + \frac{y}{7}\right)$
 - $x:y = 44:91$
3. Three pots have same volume. Ratios of milk to water are 3: 2, 7: 3, 11: 4 respectively. If all are mixed, find the resulting ratio.

(NOTE: $3+2 = 5$; $7+3 = 10$; $11+4 = 15$; If we assume each mixture to be of 30 litres (lcm of 5, 10, 15), our calculations will become simple)

- In pot 1: 18 litre milk and 12 litres water
- In pot 2: 21 litre milk and 9 litres water
- In pot 3: 22 litre milk and 8 litres water
- In total mixture of 90 litres, 61 litre milk and 29 litres water
- Thus, ratio is 61: 29

7) PARTNERSHIPS

When two or more persons invest their money and run a business jointly, then the persons involved in this transaction are called partners and this transaction is called 'Partnership'. Questions revolve around partners' share in profit or loss when they invest some money for some time in the common venture.

Thus, there are two variables – share in capital & time

TYPES OF QUESTIONS:

1. Only one variable

Here, one of the two – time or share in capital is same for all partners and proportion of profit/loss depends on other variable.

Q. Two partners Amrit and Apurva started a business by investing 6,00,000 and 5,00,000 respectively. After one year, they earned profit of 1,21,000. Find the share which both will get in the profit.

This is the simplest problem based on partnership.

- Here, period of investment is 1 year for both partners
- Investment of Amrit and Apurva is in the ratio 6 lakh: 5 lakh i.e, 6: 5
- Hence, Amrit will get $\frac{6}{11}$ part of profit; $\frac{5}{11}$ part of profit will go to Apurva
- Hence, 66,000 will go to Amrit & 55,000 will go to Apurva

Q. A and B started a business with a total capital of 3,00,000. At the end of the year, they shared the profit in the ratio of their investments. If their capitals were interchanged, then A would have received 130% more than what he actually received. Find the capital of B.

(This is a variant of above problem. Here problem is asked in reverse. Only challenge is to convert the problem into a mathematical equation. This particular skill will come in handy to solve almost all kinds of word problems)

- Let capital of B be 'x' (we often assume the quantity to be determined as unknown 'x')
- Thus, capital of A is $300000 - x$
- Ratio of investments is $(300000 - x): x$, which is same as ratio of profits
- We're given that, if we interchange the investments, A would receive 130% more profit.
- If capitals were x and $(300000 - x)$, A would receive profits in this proportion
- $130\% \text{ of } (300000 - x) + (300000 - x) = x$
- *Solving for x we get, $x = 2,09,090.90$ – which is the capital of B*

2. Two variables:

Here, both initial investment as well as time for which each partner remains invested is variable. All we have to do is calculate total investment made by each partner for the entire period to get the proportion of investment. As earlier, they will get share of profit/loss in proportion of their investment.

Q. Ashwini starts a business with 18,000 and Rajeev joins the business 5 months later with an investment of 36,000. After a year, they earn a profit of 39,000. Find the shares of Ashwini and Rajeev in the profit amount depending on their individual investment.

- Here we note that, Ashwini was invested for 12 months and Rajeev was invested for 7 months
- Investment ratio is $- 18000 \times 12 : 36000 \times 7 = 6: 7$

- Share in profit will also be in the ratio 6: 7
- Ashwini will get $\frac{6}{13} \times 39000 = 18000$ and rest 21000 will go to Rajeev

Q. The ratio in which Ajeet and Veena have contributed to the capital of a company is 3: 4. Veena has invested his capital for only 3 months and has received half as much profit as Ajeet, at the end of the year. Find out for how much time has Ajeet invested his capital in the company.

(Here question is reversed and we're to find time Ajeet stayed invested based on the profits they earned. As always, it'll boil down to converting given problem into mathematical equations)

- Let Ajeet invested 3x and Veena invested 4x in the company
- Veena invested for 3 months
- Let, Ajeet invested his money for y months
- Ratio of investments of Ajeet and Veena is $3xy : 12x$
- Now, we're given: $\frac{12x}{3xy+12x} = \frac{1}{2} \times \frac{3xy}{3xy+12x}$
- Thus, $y = 8$

8) PYQS

CSE 2022: Consider the Question and two Statements given below:

Question: What is the age of Manisha?

Statement-1: Manisha is 24 years younger than her mother.

Statement-2: 5 years later, the ages of Manisha and her mother will be in the ratio 3: 5.

Which one of the following is correct in respect of the Question and the Statement?

- (a) Statements-1 alone is sufficient to answer the Question
- (b) Statement-2 alone is sufficient to answer the question
- (c) Both Statement-1 and Statement-2 are sufficient to answer the Question
- (d) Both Statement-1 and Statement-2 are not sufficient to answer the Question

CSE 2022: There are two containers X and Y contains 100 ml of milk and Y contains 100 ml of water. 20 ml of milk from X is transferred to Y. After mixing well, 20 ml of the mixture in Y is transferred back to X. If m denotes the proportion of milk in X and n denotes the proportion of water in Y, then which one of the following is correct?

- (a) $m=n$
- (b) $m > n$
- (c) $m < n$
- (d) Cannot be determined due to insufficient data

CSE 2021: A student appeared in 6 papers. The maximum marks are the same for each paper. His marks in these papers are in the proportion 5 : 6 : 7 : 8 : 9 : 10. Overall he scored 60%. In how many papers did he score less than 60% of the maximum marks?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

CSE 2021: An amount of money was distributed among A, B and c in the ratio p: q: r.

Consider the following statements:

1. A gets the maximum share if p is greater than (q+r).
2. C gets the minimum share if r is less than (p+q).

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

CSE 2020: A bottle contains 20 litres of liquid Q. 4 litres of liquid A is taken out of its and replace by same quantity of liquid B. Again 4 litres of the mixture is taken out and replaced by same quantity of liquid B. What is the ratio of quantity of liquid A to t hat of liquid B in the final mixture?

- (a) 4:1
- (b) 5: 1
- (c) 16: 9
- (d) 17: 8

CSE 2020: A sum of Rs. 2,500 is distributed among X, Y and Z in two ratio 1/ 2 : 3/4 : 5/6. What is the difference between the maximum share and the minimum share?

- (a) Rs. 300
- (b) Rs. 350
- (c) Rs. 400
- (d) Rs. 450

CSE 2017: Q. P works thrice as fast as Q, whereas P and Q together can work four times as fast as R. If P, Q and R together work on a job, in what ratio should they share the earnings?

- (a) 3 : 1 : 1
- (b) 3 : 2 : 4
- (c) 4 : 3 : 4
- (d) 3 : 1 : 4

CSE 2017: The monthly incomes of X and Y are in the ratio of 4 : 3 and their monthly expenses are in the ratio of 3: 2. However, each saves Rs. 6,000 per month. What is their total monthly income?

- (a) Rs. 28,000
- (b) Rs. 42,000
- (c) Rs. 56,000
- (d) Rs. 84,000

CSE 2017: The average rainfall in a city for the first four days was recorded to be 0.40 inch. The rainfall on the last two days was in the ratio of 4: 3. The average of six days was 0.50 inch. What was the rainfall on the fifth day?

- (a) 0.60 inch
- (b) 0.70 inch
- (c) 0.80 inch
- (d) 0.90 inch

CSE 2017: There is a milk sample with 50% water in it. If 1/3rd of this milk is added to equal amount of pure milk, then water in the new mixture will fall down to

- (a) 25%
- (b) 30%
- (c) 35%
- (d) 40%

CSE 2016: The total emoluments of two persons are the same, but one gets allowances to the extent of 65% of his basic pay and the other gets allowances to the extent of 80% of his basic pay. The ratio of the basic pay of the former to the basic pay of the latter is:

- (a) 16: 13
- (b) 5: 4
- (c) 7: 5
- (d) 12: 11

CSE 2016: 30g of sugar was mixed in 180 ml water in a vessel A, 40 g of sugar Was mixed in 280 ml of water in vessel B and 20 g of sugar was mixed in 100 ml of water in vessel C. The solution in vessel B is

- (a) sweeter than that in C
- (b) sweeter than that in A
- (c) as sweet as that in C
- (d) less sweet than that in C

CSE 2015: Two equal glasses of same type are respectively 1/3 and 1/4 full of milk. They are then filled up with water and the contents are mixed in a pot. What is the ratio of milk and water in the pot?

- (a) 7: 17
- (b) 1: 3
- (c) 9: 21
- (d) 11: 23

CSE 2015: The monthly incomes of Peter and Paul are in the ratio of 4: 3. Their expenses are in the ratio of 3 :2. If each save Rs. 6,000 at the end of the month, their monthly incomes respectively are (in Rs.)

- (a) 24,000 and 18,000
- (b) 28,000 and 21,000
- (c) 32,000 and 24,000
- (d) 34,000 and 26,000

CSE 2013: In a rare coin collection, there is one gold coin for every three non-gold coins. 10 more gold coins are added to the collection and the ratio of gold coins to non-gold coins would be 1: 2. Based on the information; the total number of coins in the collection now becomes

- (a) 90
- (b) 80
- (c) 60
- (d) 50

CSE 2013: Out of 120 applications for a post, 70 are male and 80 have a driver's license. What is the ratio between the minimum to maximum number of males having driver's license?

- (a) 1 to 2
- (b) 2 to 3
- (c) 3 to 7
- (d) 5 to 7

CSE 2013: In a garrison, there was food for 1000 soldiers for one month. After 10 days, 1000 more soldiers joined the garrison. How long would the soldiers be able to carry on with the remaining food?

- (a) 25 days
- (b) 20 days
- (c) 15 days
- (d) 10 days

9) COMPREHENSION

Environmental problems cause health problems. Substantial changes in lifestyle can reduce environmental or health problems, but this idea appears almost impossible to adopt. With environmental problems, individual efforts can be perceived as having a negligible effect and therefore lead to inertia. With health, on the other hand, individual choices can make the difference between life and death, literally. And yet, barring a few, there seems to be the same collective lethargy towards making their choices.

Q. Which one of the following statements best implies the most rational assumption that can be made from the passage?

- (a) We are likely to spend more money on cure than prevention.
- (b) It is the job of the government to solve our environmental and public health problems.
- (c) Health can be protected even if environmental problems go on unattended.
- (d) Loss of traditional lifestyle and the influence of western values led to some unhealthy way of living.

Many people are not eating the right food. For some, it is simply a decision to stick with food they enjoy but which is not too healthy. This is leading to an increase in non-communicable diseases. This in turn leads to major burden on our health-care systems that have the potential to derail the economic progress which is essential for the poor to improve their lives. For others, it is about limited access to nutritious food or a lack of affordability, leading to monotonous diets that do not provide the daily nutrients for them to develop fully. Part of the reason nutrition is under threat worldwide is that our food systems are not properly responding to nutritional needs. Somewhere along that long road from farm to fork, there are serious detours taking place.

Q. Which one of the following statements best reflects the crux of the passage?

- (a) The scheme of Universal Basic Income should be implemented worldwide as a way of poverty alleviation.
- (b) We must place food-based nutrition at the centre of our policy debate.
- (c) Nutritional status of food should be improved by creating appropriate genetically modified crops.
- (d) Using modern food processing technologies, we must fortify food items with required nutrient elements

We often hear about conflicts among different States in India over river waters. Of the 20 major river systems, 14 are already water-stressed; 75% of the population lives in water-stressed regions, a third of whom live in water-scarce areas. Climate change, the demands of rising population and the need for agriculture to keep pace, and increased rate of urbanization and industrialization will exacerbate water stress. According to the Constitution of India, water is a State subject and not that of the Union, except for regulation of inter-State rivers. Key to ensuring balance between competing demands of various stakeholders is a basin-based approach to allocate water amongst constituent regions and States. Allocating fair share of water to them requires assessments based on objective criteria, such as specificities of the river basin, size of dependent population, existing water use and demand, efficiency of use, projected future use, etc. while ensuring the environmental needs of the river and aquifers.

Q. Which one of the following statements best reflects the most rational, practical and immediate action required to ensure fair and equitable allocation of water to different stakeholders?

- (a) A national, pragmatic, legal and policy framework for water allocation should be made.
- (b) All river systems of the country should be linked and huge aquifers created.
- (c) Water channels between regions of water surplus and regions of water deficit should be created.
- (d) To mitigate water crisis, water demand of sectors such as agriculture and industry should be reduced

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Basics of Arithmetic II

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1) REMAINDERS

The remainder, as its name suggests, is something that "remains" after completing a task. When we divide one number by another, there's always a remainder. If number is a factor, remainder is 0.

This property is called division algorithm in natural numbers.

Given two numbers a and b , we have two numbers q and r such that,

$$a = bq + r, \text{ here } 0 \leq r < b; q \text{ is called quotient and } r \text{ is called remainder}$$

We've already seen this in division:

$$\text{Dividend} = \text{Devisor} \times \text{Quotient} + \text{Remainder}$$

Remainder is always smaller than divisor

- This with some modification solves many of our questions.

Q. A number, when divided by 114, leaves remainder 21. If the same number is divided by 19, then the remainder is:

- A. 1
- B. 2
- C. 7
- D. 17

Q. On dividing a number by 13, we get 1 as remainder. If the quotient is divided by 5, we get 3 as remainder. If this number is divided by 65, what will be the remainder?

Reminder when $(A_1 + A_2 + \dots + A_n)$ is divided by B

- Remainder is same as sum of remainders when A_1 is divided by B, A_2 is divided by B and so on
- If sum is more than B, we find remainder when sum is divided by B

Q. Find remainder when $(23123 + 131212 + 1223421)$ is divided by 3

Reminder when $(A_1 \times A_2 \times \dots \times A_n)$ is divided by B

- Remainder is same as product of remainders when each A_i is divided by B.
- If product is more than B, we find remainder when product is divided by B

Q. Find remainder when $(23 \times 32 \times 5331 \times 125)$ is divided by 3

Q. A number, when divided by 136, leaves remainder 36. If the same number is divided by 17, what will be the remainder?

Q. What will be the remainder when 2^{33} is divided by 10?

- NOTE: remainder of a number when divided by 10 is last digit!

The question is equivalent to finding last digit of the number!

Q. If a perfect square, not divisible by 6, be divided by 6, the remainder will be from which of the following options?

- A. 1, 2, 4
- B. 1, 2, 5
- C. 1, 3, 5
- D. 1, 3, 4

Q. What is the remainder when

23456789101112131411161718192021222324252627282930313233343536373839404142
434485 is divided by 45?

- A. 0
- B. 1
- C. 29
- D. 44

Q. What is the remainder when $1! + 2! + 3! \dots 100!$ is divided by 18?

When $n! = 1 \times 2 \times 3 \dots \times (n - 1) \times n$

CSE 2023: What is the remainder if 2^{192} is divided by 6?

- (a) 0 (b) 1 (c) 2 (d) 4

CSE 2023: A number N is formed by writing 9 for 99 times. What is the remainder if N is divided by 13?

- (a) 11 (b) 9 (c) 7 (d) 1

Q. CSE 2022

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What is the remainder when

$$91 \times 92 \times 93 \times 94 \times 95 \times 96 \times 97 \times 98 \times 99$$

is divided by 1261?

(a) 3

(b) 2

(c) 1

(d) 0

Q. CSE 2020: If 3^{2019} is divided by 10, then what is the remainder?

(a) 1

(b) 3

(c) 7

(d) 9

Q. CSE 2020: What is the remainder when $51 \times 27 \times 35 \times 62 \times 75$ is divided by 100?

(a) 50

(b) 25

(c) 5

(d) 1

CSE 2017:

A 2-digit number is reversed. The larger of the two numbers is divided by the smaller one.

What is the largest possible remainder?

(a) 9

(b) 27

(c) 36

(d) 45

Q. If x and y are positive integers such that $(3x + 7y)$ is divisible by 11, then which of the following will be divisible by 11?

- A. $4x+6y$ B. $x+y+4$ C. $9x+4y$ D. $4x-9y$

Q. What is remainder when $51 \times 27 \times 35 \times 62 \times 75$ is divided by 100?

- A. 0
- B. 25
- C. 50
- D. 1

Remainders when large powers are involved:

- If X^k is divided by n,
- We try to represent number X as $(an+1)$ or $(an-1)$
- Thus, remainder of $X^k = (an \pm 1)^k = (\pm 1)^k$ as $(an)^k$ is always divisible by n

Q. Find remainder when 37^{123423} is divided by 9.

- $37 = 9 \times 4 + 1$
- $\frac{37^{123423}}{9} = \frac{(9 \times 4 + 1)^{123423}}{9}$
- Remainder = 1

Q. Find remainder when 35^{123423} is divided by 9.

- $35 = 9 \times 4 - 1$
- Remainder = $(-1)^{123423} = -1$ equivalent to $-1 + 9 = 8$

Q. Find remainder when $(1! + 2! + \dots + 1000!)^{40}$ is divided by 10.

- A. 0
- B. 1
- C. 2
- D. 9

2) SEQUENCE AND SERIES

Sequence:

- The sequence is the group or sequential arrangement of numbers in a particular order or set of rules.

For example: 0, 2, 4, 6, ... is sequence of even numbers

- In a sequence, an individual term can be present in many places.

For example: 1, 2, 1, 2, 1 ... is a sequence where 1 appears infinitely many times

- Sequences can be of two types, i.e., infinite sequence and finite sequence.

For example: 5,4,3,2,1 is a finite sequence having 5 terms

1,3,5,7, ... is an infinite sequence of odd numbers

Series:

Series is formed by adding the terms of a sequence. Sum of infinite terms in a series can be finite as well in some cases.

$2 + 4 + 6 + \dots$ is a series of even numbers

$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$ is a series that has a finite sum = $\frac{\pi^2}{6}$

There are certain important types of sequences and corresponding series that are relevant for us:

Arithmetic Sequence/Arithmetic Progression

An arithmetic sequence is a sequence where the successive terms are either the addition or subtraction of the common term known as common difference. For example: 1, 4, 7, 10, ... or 50, 45, 40, 35, ...

Sometimes, such sequence is called as Arithmetic progression:

Any sequence of the form: $a, a + d, a + 2d + \dots, a + nd$ is called as an Arithmetic progression

- First term: a
- Common difference: d
- n^{th} term: $a + (n - 1)d$

Sum of first 'n' terms of AP:

Let, AP be: $a, a + d, a + 2d, \dots, a + (n - 1)d$

Then, series would be

$$\begin{aligned} a + (a + d) + (a + 2d) \dots + a + (n - 1)d &= na + (d + 2d + \dots + (n - 1)d) \\ &= na + \frac{n(n - 1)}{2}d \end{aligned}$$

CSE 2020: In a race, a competitor has to collect 6 apples which are kept in a straight line on a track and a bucket is placed at the beginning of the track which is a starting point. The condition is that the competitor can pick only one apple at a time, run back with it and drop it in the bucket. If he has to drop all the apples in the bucket, how much total distance he has to run if the bucket is 5 meters from the first apple and all other apples are placed 3 meters apart?

- (a) 40 m
- (b) 50 m
- (c) 75 m
- (d) 150 m

CSE 2014: A group of 630 children is seated in rows for a group photo session. Each row contains three less children than the row in front of it. Which one of the following number of rows is not possible?

- (a) 3
- (b) 4
- (c) 5
- (d) 6

CSE 2014: A straight line segment is 36 cm long. Points are to be marked on the line from both the end points. From each end, the first point is at a distance of 1 cm from the end, the second point is at a distance of 2 cm from the first point and the third point is at a distance of 3 cm from the second point and so on. If the points on the ends are not counted and the common points are counted as one, what is the number of points?

- (a) 10
- (b) 12
- (c) 14
- (d) 16

CSE 2013: A sum of RS. 700 has to be used to give seven cash prizes to the students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, then what is the least value of the prize?

- (a) RS. 30
- (b) RS. 40
- (c) RS. 60
- (d) RS. 80

CSE 2011: A contract on construction job specifies a penalty for delay in completion of the work beyond a certain date is as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day etc., the penalty for each succeeding day being 50 more than that of the preceding day. How much penalty should the contractor pay if he delays the work by 10 days?

- (a) Rs. 4950
- (b) Rs. 4250
- (c) Rs. 3600
- (d) Rs. 650

Average of finite AP:

Let, AP be: $a, a + d, a + 2d, \dots, a + (n - 1)d$

- Case 1) Number of terms is odd: Middle term is the Average

Thus, Average of above AP is $a + \frac{n-1}{2}d$

Ex: 1,4,7,10,13,16,19 – Average = 10

- **Case 2) Number of terms is odd: Average is average of two middle terms**

Thus, Average of above AP = $\frac{(a+\frac{n-2}{2}d)+(a+\frac{n}{2}d)}{2}$

Ex: 1,4,7,10,13,16,19,22 – Average = $\frac{10+13}{2} = 11.5$

Geometric Sequence/Geometric Progression

A geometric sequence is a sequence where the successive terms have a common ratio.

For example, 1, 4, 16, 64, ... is a Geometric sequence.

Any Geometric series of the form $a, ar, ar^2, ar^3, \dots ar^{n-1}$ is a Geometric progression

- 1st term: a
- Common ratio: r
- nth term: ar^{n-1}

Such Geometric series can be finite or infinite

2,4,8,16, ... ; 1,5,25,125, ... ; 1,3,9,27, ... are all Geometric progressions

Sum of 1st n terms of GP:

Let, $a, ar, ar^2, ar^3, \dots ar^{n-1}$ be a GP.

Then the series would be: $a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = a \frac{r^n - 1}{r - 1}$

If GP is infinite and $|r| < 1$, then, $+ar + ar^2 + ar^3 + \dots = a \frac{1}{1-r}$

CSE 2017: If there is a policy that 1/3rd of population of a community has migrated every year from one place, to some other place, what is the leftover population of that community after the sixth year, if there is no further growth in the population during this period?

- (a) 16/243rd part of the population
- (b) 32/243rd part of the population
- (c) 32/729th part of the population
- (d) 64/729th part of the population

Harmonic Sequence/Harmonic Progression

A harmonic sequence is a sequence where the sequence is formed by taking the reciprocal of each term of an arithmetic sequence. For example, $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \dots$ is a harmonic sequence as 1,4,7,10, ... is an arithmetic sequence

Fibonacci Sequence

When every term is an addition of preceding two terms, the Sequence we get is Fibonacci sequence.

1,1,2,3,5,8,13,21, ...

- It is closely related to many objects in nature
- Only important thing to remember for us is: $F_{n+1} = F_n + F_{n-1}$

- If you're asked to find 8th, 10th or 12th term, just write the series and find it – it'll be quicker

Sum of Fibonacci Series:

- Sum of first n terms = $F_1 + F_{n+1} + \dots + F_n = F_{n+2} - 1$

Other Important Series:

- **Sum of first n natural numbers:**

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$

- **Sum of squares:**

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n + 1)(2n + 1)}{6}$$

- **Sum of cubes:**

$$1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n + 1)}{2} \right]^2$$

- **Sum of first 'n' odd numbers:**

$$1 + 3 + \dots (\text{n numbers}) = n^2$$

- **Sum of first n even natural numbers:**

$$2 + 4 + \dots (\text{n numbers}) = n(n + 1)$$

- **Sum of inverses of squares of natural numbers:**

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}$$

- **Sum of inverses of powers of 2:**

$$1 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 2$$

3) IMPORTANT EXPANSIONS

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^n - 1 = (x - 1)(x^{n-1} + x^{n-2} + \dots + 1)$$

For n odd: $x^n + 1 = (x + 1)(x^{n-1} - x^{n-2} + x^{n-3} \dots - x + 1)$

For n even: $x^n + 1$ has NO real root

In addition to it, if n is even: $(x^n - 1)$ is divisible by $(x + 1)$ as well!

$$\text{As } (x^{2m} - 1) = (x^m + 1)(x^m - 1);$$

If m is odd $(x + 1)$ is factor of $(x^m + 1)$ and if m is even then it is factor of $(x^m - 1)$

NOTE: $(x^n - 1)$ is always divisible by $(x - 1)$ and

$(x^n + 1)$ is divisible by $(x + 1)$ when n is odd

In fact, $(x^n - a^n)$ is always divisible by $(x - a)$ and

$(x^n + a^n)$ is divisible by $(x + a)$ when n is odd

CSE 2021: How many pairs of natural numbers are there such that the difference of whose squares is 63?

- (a) 3
- (b) 4
- (c) 5
- (d) 2

1. What is the average of first 50 natural numbers?

- A. 25
- B. 25.5
- C. 27
- D. 30

2. What is the value of $11 + 12 + \dots + 50$?

- A. 1140
- B. 1160
- C. 1200
- D. 1220

3. If $x^4 + \frac{1}{x^4} = 47$; what is the value of $x + \frac{1}{x}$

- A. 3
- B. 4
- C. 5
- D. 7

4. If $64^2 - 36^2 = 4P$ then what is the value of P?

- A. 637

- B. 700
 - C. 600
 - D. 837
5. What will be the remainder when $67^{67} + 67$ when divided by 68?
- A. 0
 - B. 1
 - C. 66
 - D. 67

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GS FOUNDATION
BATCH FOR CSE (2023-24)
BOOKLET - 14
TARGET PRELIMS 2023: CSAT
QUANTITATIVE APTITUDE: 9 PROFIT AND LOSS

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1) IMPORTANT TERMS:

Cost price (CP): The price at which the article is purchased.

Selling price (SP): It is the price at which the article is sold

If CP > SP then there's a loss and **Loss = CP – SP**

If SP > CP then there's a profit in the transaction and **Profit = SP – CP**

Q.1 If cost price of a watch is 3000 and selling price is 5000, what is the profit?

Percentage profit = (profit/CP) *100

Percentage Loss = (loss/CP) *100

Q.2 If cost price of a watch is 3000 and selling price is 5000, what is the percentage profit?

Q.3 Anu bought a dozen bangles for Rs. 84 and sold for Rs.10 per piece. Find percentage gain for Anu?

Q.4 Sumit bought a motorcycle for 60,000 and sold it for 48,000. Find the loss per cent.

Q.5 A mobile phone is sold for 5300 at a gain of 6%. Find its cost price.

Q.6 Himanshu bought a camera for 25,000 and sold it at a loss of 10% to Pushkar. Find the price at which the camera was sold to Pushkar.

Q.7 Nishant sells his guitar at a profit of 20%. If he had bought it for 10% less and sold for 40 less, he would have gained 25%. Find the cost price of the guitar.

Q. 8 A shopkeeper gains the selling price of 20 pens by selling 100 pens. Find his gain per cent.

2) FINDING SELLING PRICE FROM GAIN % OR LOSS %

SP = (100 + percentage profit) *CP/100

SP = (100 + percentage loss) *CP/100

3) FINDING CP FROM PROFIT %

CP = (100 * SP)/ (100 +percentage profit)

Q. Find cost of a watch if a profit of 10% was made while selling it at Rs. 5000.

CSE 2020: A person bought a car and sold it for Rs. 3,00,000. If he incurred a loss of 20%, then how much did he spend to buy the car?

- (a) Rs. 3,60,000
- (b) Rs. 3,65000
- (c) Rs. 3,70,000
- (d) Rs. 3,75,000

4) ARTICLE SOLD AT TWO DIFFERENT SELLING PRICES:

Note: cost price is same for both articles.

$$\frac{\text{Sale Price 1}}{100 + \text{percentage profit 1}} = \frac{\text{Sale Price 2}}{100 + \text{percentage profit 2}}$$

Q. If a man sells his book for Rs. 720, he would have 25% loss. At what price must he sell it to gain 25%?

Here same article is sold at two selling prices leading to different profits (25 and -25)

CSE 2020: Q. As a result of 25% hike in the price of rice per kg, a person is able to purchase 6 kg less rice for Rs. 1,200. was the original price of rice per kg?

- (a) Rs. 30
- (b) Rs. 40
- (c) Rs. 50
- (d) Rs. 60

5) TWO ARTICLES SOLD AT SAME PRICE WITH PROFIT % ON FIRST = LOSS % ON SECOND

Will there be overall profit or loss in the transaction? – LOSS

$$\text{Loss} = (\text{percentage profit}/10)^2\%$$

Q. A man sold his two watches for Rs. 240 each. On one he gains 20% profit and on other he incurs loss of 20%. What is his overall gain or loss in this transaction?

- SP is same for both

$$\text{Overall loss of } 4\% - \text{total loss} = 4\% \text{ of } 480$$

CSE 2014: If Sohan, while selling two goats at the same price, makes a profit of 10% on one goat and suffers a loss of 10% on the other

- (a) he makes no profit and no loss.
- (b) he makes a profit of 1%.
- (c) he suffers a loss of 1%.
- (d) he suffers a loss of 2%.

6) MARKED PRICE AND DISCOUNT

Marked is the price a merchant marks as a price for the item and then provides certain discount on it. The discounted price is the actual selling price in such a case.

Selling Price = Marked price – Discount

If a merchant gets p% profit on item then

$$\text{Selling price} = (1 + p\%) * \text{Cost price} = \text{Marked price} - \text{Discount}$$

If discount is given as d% of marked price,

$$\text{SP} = (1+p\%) \text{ CP} = (1-d\%) \text{ MP}$$

Q. The printed price of a watch is 16,000 and it has been sold by Rohit for 13,600. Find the rate of discount at which Rohit has sold the watch.

Q. Chinmay offers a discount of 10% on the printed price of his Jacket to Abhyodaya and still is able to make 20% profit. If the printed price of the jacket is 1,200, then find the cost price of the jacket.

CSE 2020: A shop owner offers the following discount options on an article to a customer:

1. Successive discounts of 10% and 20%, and then pay a service tax of 10%.
2. Successive discounts of 20% and 10%, and then pay a service tax of 10%.
3. Pay a service tax of 10% first, then successive discounts of 20% and 10%. Which one of the following is correct?
 - (a) 1 only is the best option for the customer.
 - (b) 2 only is the best option for the customer.
 - (c) 3 only is the best option for the customer.
 - (d) All the options are equally good for the customer.

7) FALSE SCALE:

Shopkeeper uses false scale to sell his goods. Value of scale is in reality lower than true scale. Hence consumer gets less quantity of good than promised. Shopkeeper sells these goods at profit. So, in net effect shopkeeper doubly benefits – he earns profit by selling at higher price than cost price and he benefits due to false scale effect.

Let 'G' be shopkeeper's net percentage profit and let 'p%' be the percentage profit at which shopkeeper sells the item.

Then:

$$(100 + G) / (100 + p) = \text{true scale weight} / \text{false scale weight}$$

Q. A dishonest shopkeeper says he sells his products at cost price but uses 900gm for each Kg as false weight. Find his percentage gain?

8) SIMPLE AND COMPOUND INTEREST

Interest is the cost of borrowing money, where the borrower pays a fee to the lender for the loan. The interest, typically expressed as a percentage, can be either simple or compounded. Simple interest is based on the principal amount of a loan or deposit. In contrast, compound interest is based on the principal amount and the interest that accumulates on it in every period. Simple interest is calculated only on the principal amount of a loan or deposit, so it is easier to determine than compound interest.

$$\text{Simple Interest} = \frac{P \times R \times T}{100};$$

P is principal amount, R is rate of interest, T is number of years amount is invested

Compound interest accrues and is added to the accumulated interest of previous periods; it includes interest on interest, in other words. The formula for compound interest is:

$$CI = P \left(1 + \frac{R}{100}\right)^T - P$$

Where, P is principal amount; R is rate of interest in percentage; T is number of years amount is invested

$$Amount = P \left(1 + \frac{R}{100}\right)^T$$

Q. Balbir takes a loan of Rs 10000 from the SBI for a period of one year. The given rate of interest is 10% per annum. Find the interest and the amount he has to pay at the end of one year.

Q. The count of a certain breed of bacteria was found to increase at the rate of 5% per hour. What will be the growth of bacteria at the end of 3 hours if the count was initially 6000?

Q. The difference between simple and compound interests compounded annually on a certain sum of money for 2 years at 4% per annum is Re. 1. The sum (in Rs.) is

Q. What is the difference between the compound interests on Rs. 5000 for 1.5 years at 4% per annum compounded yearly and half-yearly?

- A. Rs. 2.04
- B. Rs. 3.06
- C. Rs. 4.80
- D. Rs. 8.30

NOTE: Rule of 72:

The Rule of 72 is a quick, useful formula that is popularly used to estimate the number of years required to double the invested money at a given annual rate of return.

Years To Double: 72 / Expected Rate of Return

To calculate the time period an investment will double, divide the integer 72 by the expected rate of return.

The Rule of 72 applies to cases of compound interest, not simple interest. Simple interest is determined by multiplying the daily interest rate by the principal amount and by the number of days that elapse between payments. Compound interest is calculated on both the initial principal and the accumulated interest of previous periods of a deposit.

9) PYQS

CSE 2023: A principal P becomes Q in 1 year when compounded half-yearly with R% annual rate of interest. If the same principal P becomes Q in 1 year when compounded annually with S% annual rate of interest, then which one of the following is correct?

- (a) $R = S$ (b) $R > S$ (c) $R < S$ (d) $R \leq S$

CSE 2019: Raju has Rs. 9000 with him and he wants to buy a mobile handset; but he finds that he has only 75% of the amount required to buy the handset. Therefore, he borrows 2000 from a friend. Then

- a. Raju still does not have enough amount to buy the handset.
- b. Raju has exactly the same amount as required to buy the handset.
- c. Raju has enough amount to buy the handset and he will have 500 with him after buying the handset.
- d. Raju has enough amount to buy the handset and he will have 1000 with him after buying the handset

CSE 2019: Rakesh had money to buy 8 mobile handsets of a specific company. But the retailer offered very good discount on that particular handset. Rakesh could buy 10 mobile handsets with the amount he had. What was the discount the retailer offered?

- (a) 15%
- (b) 20
- (c) 25%
- (d) 30%

CSE 2019: A shopkeeper sells an article at Rs. 40 and gets X% profit. However, when he sells it at Rs. 20, he faces same percentage of loss. What is the original cost of the article?

- (a) Rs. 10
- (b) Rs. 20
- (c) Rs. 30
- (d) Rs. 40

CSE 2017: Gopal bought a cell phone and sold it to Ram at 10% profit. Then Ram wanted to sell it back to Gopal at 10% loss. What will be Gopal's position if he agreed?

- (a) Neither loss nor gain
- (b) Loss 1%
- (c) Gain 1%
- (d) Gain 0.5%

CSE 2016: A person allows 10% discount for cash payment from the marked price of a toy and still he makes a 10% gain. What is the cost price of the toy which is marked Rs. 770?

- (a) Rs. 610
- (b) Rs. 620

- (c) Rs. 630
- (d) Rs. 640

10) COMPREHENSION

A global analysis of nitrogen use efficiency—measure of the amount of nitrogen a plant takes in to grow versus what is left behind as pollution—says that using too much fertilizers will lead to increased pollution of waterways and the air. Currently, the global average for nitrogen use efficiency is approximately 0.4, meaning 40 per cent of the total nitrogen added to cropland goes into the harvested crop while 60 per cent is lost to the environment, says a study. More than half of the world's population is nourished by food grown with fertilizers containing synthetic nitrogen, which is needed to produce high crop yields. Plants take the nitrogen they need to grow, and the excess is left in the ground, water and air. This results in significant emissions of nitrous oxide, a potent greenhouse and ozone depleting gas, and other forms of nitrogen pollution, including eutrophication of lakes and rivers and contamination of river water.

Q. Which one of the following statements best reflects the most logical, rational and crucial message implied by the passage?

- (a) An enhanced efficiency of use of nitrogen is imperative for both food production and environment.
- (b) Production of synthetic nitrogen fertilizers cannot be stopped as it will adversely affect global food security.
- (c) Alternatives to crops that require excess of nitrogen should be identified and cultivated.
- (d) Conventional agriculture using synthetic fertilizers should be replaced with agroforestry, agroecosystems and organic farming.

Benefits of good quality school education accrue only when students complete and leave school after having acquired the gateway skills. Like one learns to walk before running, similarly one picks up advanced skills only after picking the basic foundational skills. The advent of the knowledge economy poses new challenges, and one of the severe consequences of having an uneducated workforce will be our inability to keep pace with the global economy. Without a strong learning foundation at the primary level, there can be no improvement in higher education or skill development.

Q. Which one of the following statements best reflects the crux of the passage?

- (a) To become a global power, India needs to invest in universal quality education.
- (b) India is unable to become a global power because it is not focusing or promoting knowledge economy.
- (c) Our education system should focus more on imparting skills during higher education.
- (d) Parents of many school children are illiterate and are unaware of the benefits of quality education

GS FOUNDATION (2023-24) Booklet 15
&
CSAT FOUNDATION 1.0 (2023-24) Booklet 14
COMBINATORICS

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1) COUNTING: BASIS OF PERMUTATIONS AND COMBINATIONS

This topic is basically about counting. If you get the hang of how to count well, PnC is easy! Almost all questions from this topic would be in the form of ‘in how many ways can we do it’ – which is to count number of ways

For instance:

In how many ways can we pick one shape out of following 3 different circles and 2 different squares? – enlist it – 5 – so, that’s your addition principle of counting!

Now, in how many ways can we pick 1 circle and 1 square – enlist – that’s your multiplication principle!



Before answering it, we shall look at fundamental principles of counting:

1. Addition principle
2. Multiplication principle

2) FUNDAMENTAL PRINCIPLES OF COUNTING

These two principles form the basis of the entire chapter. UPSC asks very basic questions from this chapter which can be solved within seconds if you know these principles well.

Addition Principle:

The principle of addition states if a one task can be one done in ‘m’ ways and another task which is MUTUALLY EXCLUSIVE (Doing task 1 OR task 2 – we cannot do both at the same time) of the first task can be done in ‘n’ ways, then the number of possible ways in which either can be done is $(m+n)$

Ex: If I have 4 apples and 3 oranges in a basket, there are $3+4=7$ ways I can pick one fruit from the basket

Here event is picking a fruit. I can pick either apple (out of 4) OR I can pick an orange (out of 3). Picking up apple or orange is totally unrelated to each other.

(Basically, OR is addition)

In above question on circles and squares, in how many ways can we pick a shape? 1 circle out of 3 OR 1 square out of 2 – We can pick a shape in $3+2=5$ ways

Multiplication Principle:

The principle of multiplication states that if one task can be done in ‘m’ ways and another task which is INDEPENDENT of the first task can be done in ‘n’ ways, after the first task has been performed, then the number of possible ways in which both the tasks can be done is $(m \times n)$

The ***independent means*** that no matter how one task is performed, the number of ways you can perform the second task is the same.

For example, the tasks "pick one apple" and "pick one orange" are independent in the previous example, since no matter what apple I pick, I still have the same 3 oranges to pick from.

Also, in circle and square example, no matter how we pick a circle from 3, we still have 2 squares to pick from.

In How many ways can we pick 1 circle AND 1 square? $3 \times 2 = 6$

In how many ways can we pick 1 apple and 1 orange from 4 apples AND 3 oranges in a basket?

– are events independent? – YES

So, number of ways = $4 \times 3 = 12$

Basically, **AND is multiplication**

Thing to note here is that, in both above examples – events were independent.

Now, what if the events were dependent? We can simply modify our multiplication principle.

For example: Kritika wants to buy a car. The dealer says she can buy either Car A or car B. There are three models of each car viz. GXI, SXI, ZXI. There are 5 colour – red, black, green, yellow and white options for each car. How many options does she have.

What if the dealer says, car A does not come in black. How many options does she have now?

$$3 \times 5 + 3 \times 4 = 27$$

Notice that, Choice of colour is DEPENDENT on white kind of car Kritika chooses to buy.

3) FORMING NUMBERS

Q. How many 3-digit numbers can be formed from 1, 2, 3, 4, 5

Notice that, there're 3 slots to be filled – hundreds place, tens place and unit's place.

- **If repetition is not allowed**

Hundred's place can be filled with 1 OR 2 OR 3 OR 4 OR 5 – 5 options.

Now that we've chosen any one (like we chose car in above example) we've 4 options left for tens digit & 3 options left for unit's digit

We've to choose Hundred's digit AND Ten's digit AND unit's digit

Total ways = $5 \times 4 \times 3 = 60$

(Will the answer change if we've to form 3-digit numbers from 0, 1, 2, 3, 4, 5) – just remove 2-digit numbers?

- **If repetition is allowed**

In this case, whatever restrictions we had are not there. Every choice of digit is again an INDEPENDENT event. We can directly apply multiplication principle.

For each digit, there are 5 choices – Total choices = $5 \times 5 \times 5 = 125$

(Will the answer change if we've to form 3-digit numbers from 0, 1, 2, 3, 4, 5)

Here, there's no condition on how digits are related.

Let's put some conditions and see how can we count the numbers.

In case of any condition: **WE ALWAYS START WITH THE CONDITION** – we first ensure that, while counting, the condition gets satisfied and then count.

For instance, if we're to find out 3-digit numbers out of 0, 1, 2, 3, 4 – we first have to ensure that, the number in fact is a 3-digit one. And thus, we began with hundred's place – put a non-zero digit there and proceed with the counting.

Other variants of the same question as above:

- How many 3-digit numbers divisible by 2 can be formed?
- How many 3-digit numbers not divisible by 2 can be formed?
- How many 3-digit numbers divisible by 3 can be formed?
- How many 3-digit numbers divisible by 5 can be formed?
- How many 3-digit numbers greater than 300 can be formed?

Let's do all the above questions for 2-digit; 4-digit and 5-digit numbers

What we did here for 5-digit case is simply an arrangement. Basically, symbols or things (digits in this case) remain same but we only re-arrange them in various ways.

As seen above, there are exactly $n!$ ways of arranging 'n' symbols or objects

Q. How many factors of the number $28 * 36 * 54 * 105$ are multiples of 120?

- A. 540
- B. 660
- C. 594
- D. 792

4) ANAGRAMS

Anagram word you spell by rearranging the letters of another word

For example, PCUS is an anagram of UPSC. UCSP is another example.

The question is, how many anagrams are possible from the given 'n' lettered word.

Because anagram is nothing but a re-arrangement, there are exactly $n!$ anagrams possible.

Above question is same as having 'n' letters and finding out how many ways are there to arrange without repetition.

Other variants of anagram questions like

- Finding anagrams beginning or ending with vowel
- Having vowel or a consonant at particular places
- Having all vowels together etc.

Q. How many anagrams can be formed from DELHI?

Now comes the important part: