# GS FOUNDATION
# BATCH FOR CSE 2024

# CSAT - 02

## (Quantitative Aptitude_1_Basics of Arithmetic-)

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# Ace CSAT 2023

# Booklet-2: Quantitative aptitude-1

# Basics of Arithmetic

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Quantitative Aptitude Introduction

Quantitative aptitude is a surest pathway towards ensuring qualification in CSAT. And fortunately, it is also the easiest.

UPSC expects: $10^{th}$ level understanding of basic numeracy. To be specific:

- Basic numeracy (numbers and their relations, orders of magnitude, etc. (Class X level),
- Data interpretation (charts, graphs, tables, data sufficiency etc. — Class X level)

But remember that, this syllabus is indicative and not complete. For instance, percentages, averages, ratio-proportion, probability, combinatorics etc. are topics getting asked in the exam but not explicitly mentioned in the syllabus.

# 1) NUMBER SYSTEM

## 1.1 What is Number System and Number Line

Mathematics is about finding patterns, structures, regularity, rules that govern what we see and representing these patterns in a language and if there's no language, inventing a new one. A number system is such a language.

A number system is a writing system for expressing numbers; that is, a mathematical notation for representing numbers of a given set, using digits or other symbols in a consistent manner.

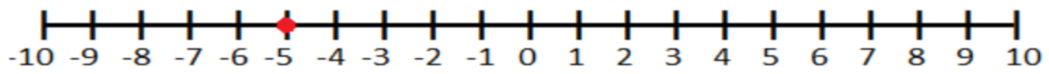
Number System is a method of representing Numbers on the Number Line with the help of a set of Symbols and rules.

**Number Line:**

A Number line is a representation of Numbers with a fixed interval in between on a straight line. A Number line contains all the types of numbers like natural numbers, Integers, etc. Numbers on the number line increase while moving Left to Right and decrease while moving from right to left.

Ends of a number line are not defined i.e., numbers on a number line range from infinity on the left side of the zero to infinity on the right side of the zero.

## 1.2 Natural Numbers




1,2,3, 4… are called natural numbers on number line. Natural numbers are called natural because they are used for counting naturally.
The set of natural numbers is the most basic system of numbers because it is intuitive, or natural, and hence the name. We use natural numbers in our everyday life, in counting discrete objects, that is, objects which can be counted like number of benches in class or number of sheep on a farm.
Every Natural number has a successor and every natural number except 1 has a predecessor.
Even Numbers: 2,4,6, …
Odd numbers: 1,3,5, …

## 1.3 Whole Numbers

All natural numbers except 1 have predecessors. So, to the collection of natural numbers we add zero as the predecessor for 1.
The resulting set is that of Whole numbers
    i.e. 0, 1, 2, 3, …
Even Numbers: 0,2,4,6, …
Odd numbers: 1,3,5, …

CSE 2023: Three of the five positive integers p, q, r, s, t are even and two of them are odd (not necessarily in order).
Consider the following:
1. p + q + r – s - t is definitely even.
2. 2p + q + 2r - 2s + t is definitely odd.
Which of the above statements is/are correct?
(a) 1 Only (b) 2 Only (c) Both 1 and 2 (d) Neither 1 nor 2

CSE 2020: Q. Consider the following sequence of numbers:
5 1 4 7 3 9 8 5 7 2 6 3 1 5 8 6 3 8 5 2 2 4 3 4 9 6
How many odd numbers are followed by the odd number in the above sequence?

(a) 5
(b) 6
(c) 7
(d) 8

### 1.4 Integers:

Integers are the collection of Whole Numbers plus the negative values of the Natural Numbers.

i.e. …, -3, -2, -1, 0, 1, 2, 3, …

Apart from this, there are rational numbers and irrational numbers on the real number line.

The sum of three consecutive integers is equal to their product. How many such possibilities are there?

*(a)* Only one

*(b)* Only two

*(c)* Only three

*(d)* No such possibility is there

## 2) PLACE VALUES IN DECIMAL SYSTEM

A decimal number is made up of an integer and a fractional part that is separated by a dot called the decimal point.

For example, 3.19 is a decimal number in which 3 is the integer part and .37 is the fractional part; in -57.3948, -57 is an integer part and .3948 is the fractional part

| Hundred Thousands | Ten Thousands | Thousands | Hundreds | Tens | Ones | . | tenths | hundredths | thousandths | ten thousandths | hundred thousandths |
|---|---|---|---|---|---|---|---|---|---|---|---|
| HTH | TTh | Th | H | T | O | . | t | h | th | tth | hth |
| 100,000 | 10,000 | 1,000 | 100 | 10 | 1 | . | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1,000}$ | $\frac{1}{10,000}$ | $\frac{1}{100,000}$ |

Whole Number Part

Decimal Point

Fractional Part

CSE 2023: For any choices of values of X, Y and Z, the 6-digit number of the form XYZXYZ is divisible by:
(a) 7 and 11 only (b) 11 and 13 only (c) 7 and 13 only (d) 7, 11 and 13

CSE 2020: Q. The difference between a 2-digit number and the number obtained by interchanging the positions of the digits is 54.
1. The sum of the two digits of a number can be determined only if the product of the two digits is known.
2. The difference between the two digits of the number can be determined.
Which of the above statements is/are correct?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

CSE 2019: Q. The ratio of a two-digit natural number to a number formed by reversing its digits is 4/7. The number of such pairs is

a. 5
b. 4
c. 3
d. 2

## 3) MATHEMATICAL OPERATIONS

### 2.1 **What is Mathematical operation**?

Mathematical operation is a **function which takes input values to a well-defined output value**.
There are five fundamental operations that are of special use to us. They're addition, subtraction, multiplication, division and power

### 2.2 **Addition on number line**:

The addition of two numbers results in the total amount or sum of those values combined. Addition operation represented as '+' takes given number to the right side on number line by unit value of number being added to it.

**Properties:**

- Addition of two even numbers is even number
- Addition of two odd numbers is an even number
- Addition of odd number and even number is odd number
- Addition of two integers is always an integer
- Addition of two natural numbers is always a natural number
- Addition is associative and commutative.

- **Addition with decimals**

For example: 0.35 + 0.5 = 0.85

## 2.3 Subtraction

Subtraction is the process of taking away a number from another. It is a primary arithmetic operation that is denoted by a subtraction symbol (-) and is the method of calculating the difference between two numbers.

Subtraction is a reverse of addition operation as here we go towards the left on number line.

**Properties:**

- Subtraction of two even numbers is even number
- Subtraction of two odd numbers is an even number
- Subtraction of odd number and even number is odd number
- Subtraction of two integers is always an integer
- Subtraction of two natural numbers is NOT always a natural number
- Subtraction is associative but NOT commutative

- **Subtraction with decimals: similar to addition**

## 2.4 Multiplication

Multiplication is an operation that represents the basic idea of repeated addition of the same number.

The numbers that are multiplied are called the factors and the result that is obtained after the multiplication of two or more numbers is known as the product of those numbers.

Multiplication is used to simplify the task of repeated addition of the same number.

**Ex:** $9 \times 6 = 9 + 9 + 9 + 9 + 9 + 9 = 54$

- I assume you're familiar with process to multiply two numbers say $569 \times 63$
- I also hope you recall your multiplication tables at least till 20!
- What is number multiplied by 1?
- What is number multiplied by 0?
- What is number multiplied by 10, 100, 1000?
- Rules for sign of **multiplication with negative numbers:**

$$negative\ number \times positive\ number = negative\ number$$
$$negative\ number \times negative\ number = positive\ number$$

So, $(-2) \times 2 = -4$ *and* $(-2) \times (-2) = 4$

**Other Properties:**

- Multiplication of two numbers if one or both are even is even number
- Multiplication of two odd numbers is an odd number

- Multiplication of two integers is always an integer
- Multiplication of two natural numbers is always a natural number
- Multiplication is associative and commutative

**Multiplication with decimals**:
$0.2 \times 3.1 = 0.62; 0.2 \times 31 = 6.2; 2 \times 0.31 = 0.62; 2 \times 3.1 = 6.2$ etc.

**Rule:**
- Write all numbers removing zeros at the end. For example: 0.30 is to be written as 0.3; 1001.00900 is to be written as 1001.009 etc.
- Multiply two or more numbers disregarding the decimal points
- Count the total digits after decimal point in all numbers. If it is 'n', put the decimal point in product after 'n' digits from right.

**2.5 Division**

The division is one of the basic arithmetic operations in math in which a larger number is broken down into smaller groups having the same number of items.
It is the inverse of the multiplication operation. While dividing numbers, we break down a larger number into smaller numbers such that the multiplication of those smaller numbers will be equal to the larger number taken.
- I assume you're familiar with long division process
- I assume you know the meaning of terms – dividend, divisor, quotient, remainder, factor

$$\boldsymbol{Dividend = Devisor \times Quotient + Remainder}$$

- Dividend is divisible by devisor if remainder is zero. In other words, it is a divisible if devisor is a factor of dividend.
- What is number divided by 1?
  (Remainder is always 0 – Hence every number is divisible by 1!)
- What is number divided by itself?
  (Remainder is always 0 – Hence every number is divisible by itself)
- What is number divided by 10, 100, 1000 etc.
- What is 0 divided by any number?
- What is number divided by 0?
  Q. Why can't we divide by 0?
- Rules for negative numbers are same as that of multiplication as division is just inverse operation of multiplication. To be specific, division by 'n' is multiplication by 1/n.

**Division with decimal point**:
Very similar to multiplication only in reverse. In multiplication, we add total digits after decimal point, here we subtract. But better way is to just remove decimal point by multiplying both numerator and denominator with appropriate power of 10 like 10, 100, 1000 etc. as it just shifts decimal point to right.

**For example:**

$$\frac{0.62}{0.2} = 3.1; \frac{0.62}{2} = 0.31; \frac{6.2}{2} = 3.1; \frac{6.2}{0.2} = 31; \frac{62}{0.2} = 310$$

**CSE 2023:** What is the sum of all 4-digit numbers less than 2000 formed by the digits 1, 2, 3 and 4, where none of the digits is repeated?
(a) 7998 (b) 8028 (c) 8878 (d) 9238

**CSE 2023:** If ABC and DEF are both 3-digit numbers such that A, B, C, D, E, and F are distinct non-zero digits such that ABC+ DEF= 1111, then what is the value of A+B+C+D+E+F?
(a) 28 (b) 29 (c) 30 (d) 31

**CSE 2023:** A 3-digit number ABC, on multiplication with D gives 37DD where A, B, C and D are different non-zero digits. What is the value of A+B+C?
(a) 18 (b) 16 (c) 15 (d) Cannot be determined due to insufficient data

**CSE 2023:** Let pp, qq and rr be 2 digit numbers where p < q < r. If pp + qq + rr =tt0, where tt0 is a 3-digit number ending with zero,
consider the following statements:
1. The number of possible values of p is 5.
2. The number of possible values of q is 6
Which of the above statements is/are correct?
(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

**CSE 2023:** Each digit of a 9-digit number is 1. It is multiplied by itself. What is the sum of the digits of the resulting number?
(a) 64 (b) 80 (c) 81 (d) 100

**CSE 2023:** AB and CD are 2-digit numbers. Multiplying AB with CD results in a 3-digit number DEF. Adding DEF to another 3-digit number GHI results in 975. Further A, B, C, D. E, F, G, H, I are distinct digits. If E= 0, F=8, then what is A+B+C equal to?
(a) 6 (b) 7 (c) 8 (d) 9

**CSE 2023:** If p, q, r and s are distinct single digit positive numbers, then what is the greatest value of (p + q) (r + s)?

(a) 230 (b) 225 (c) 224 (d) 221

**CSE 2023:** What is the sum of all digits which appear in all the integers from 10 to 100?
(a) 855 (b) 856 (c) 910 (d) 911

**CSE 2023:** D is a 3-digit number such that the ratio of the number to the sum of its digits is least. What is the difference between the digit at the hundred's place and the digit at the unit's place of D?
(a) 0 (b) 7 (c) 8 (d) 9

**CSE 2020:** Consider the following addition problem: 3P+4P+PP+PP=RQ2; where P, Q and R are different digits.
What is the arithmetic mean of all such possible sums?
(a) 102
(b) 120
(c) 202
(d) 220

**CSE 2020:** Consider the following multiplication problem:
(PQ)×3=RQQ, where P, Q and R are different digits and R≠ 0.
What is the value of (P+R) ÷Q?
(a) 1
(b) 2
(c) 5
(d) Cannot be determined due to insufficient data

**CSE 2020:** How many zeroes are there at the end of the following product?
1 x 5 x 10 x 15 x 20 x 25 x 30 X 35 x 40 x 45 x 50 x 55 x 60
(a) 10
(b) 12
(c) 14
(d) 15

**CSE 2017:** Certain 3-digit numbers following characteristics: 1. All the three digits are different. 2. The number is divisible by 7. 3. The number on reversing the digits is also divisible by 7. How many such 3-digit numbers are there?
(a) 2
(b) 4
(c) 6
(d) 8

**CSE 2015:** If ABC x DEED = ABCABC; where A, B, C, D and E are different digits, what are the values of D and E?
(a) D = 2, E = 0
(b) D = 0, E = 1
(c) D = 1, E = 0
(d) D = 1, E = 2

Is multiple of an integer an integer?
Is fraction of an integer an integer?
Is addition/ multiplication/ subtraction/ division of two integers an integer?

## 4) EXPONENTS

**The exponent or power of a number shows how many times the number is multiplied by itself**.
For example, 2×2×2×2 can be written as $2^4$, as 2 is multiplied by itself 4 times.
Here, 2 is called the "base" and 4 is called the "exponent" or "power."
We call it '2 raised to 4' or '2 to the power 4'
Similarly, $9 \times 9 \times 9 = 9^3$
In general, $x^n$ means that x is multiplied by itself for n times.

**CSE 2020:** For what value of n, the sum of digits in the number ($10^n$ + 1) is 2?
(a) For n = 0 only
(b) For any whole number n
(c) For any positive integer n only
(d) For any real number n

**SQAURE-SQUARE ROOT/ CUBE-CUBE ROOT**

- A number's 2nd power is called its square and its third power is called its cube

For example:
square of 2 = $2^2$ = 2×2 = 4 and square root of 4 is 2 or $\sqrt{4} = \sqrt[2]{4} = 2$
Cube of 2 = $2^3$ = $2 \times 2 \times 2 = 8$ and cube root of 8 is 2 or $\sqrt[3]{8} = 2$

**IMPORTANT NOTE:**
Square root of a number has 2 possible values. Square root of 4 for instance can be 2 as well as -2.
$9^{\frac{1}{2}} = \pm 3$ and so on.

**Important Squares and Cubes**

| Number | Square |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |
| 7 | 49 |
| 8 | 64 |
| 9 | 91 |
| 10 | 100 |
| 11 | 121 |
| 12 | 144 |
| 13 | 169 |
| 14 | 196 |
| 15 | 225 |
| 16 | 256 |
| 17 | 289 |
| 18 | 324 |
| 19 | 361 |
| 20 | 400 |
| 21 | 441 |
| 22 | 484 |
| 23 | 529 |
| 24 | 576 |
| 25 | 625 |
| 26 | 676 |
| 27 | 729 |
| 28 | 784 |
| 29 | 841 |

| Number | Cube |
|---|---|
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |
| 4 | 64 |
| 5 | 125 |
| 6 | 216 |
| 7 | 343 |
| 8 | 512 |
| 9 | 729 |
| 10 | 1000 |
| 11 | 1331 |
| 12 | 1728 |
| 13 | 2197 |
| 14 | 2744 |
| 15 | 3375 |

NOTE: Square of an integer is always positive. **Unit's place for any square will always be one of 0, 1, 4, 5, 6 or 9.**
In other words, numbers ending with 2, 3, 7 or 8 can NEVER be squares.

**CSE 2017:** The age of Mr. X last year was the square of a number and it would be the cube of a number next year. What the least number is of years he must wait for his age to become the cube of a number again? (CSE CSAT-2017)
(a) 42
(b) 38
(c) 25
(d) 16

**PROPERTIES OF EXPONENTS**

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $a^0 = 1 \; for \; all \; 'a'$
- $a^1 = a$
- $a^{-1} = \frac{1}{a}$
- $a^{-n} = \frac{1}{a^n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $(\frac{a}{b})^n = \frac{a^n}{b^n}$
- $a^{1/n} = \sqrt[n]{a}$

**EXPONENTS WITH FRACTIONS:**

- $a^{m/n} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$

**Unit's digit in exponent**

Regardless of powers or bases involved, all we have to look for are powers of unit's place digit of the base. It follows directly from the definition of powers.
Q. What is the unit's digit in $125^4$
Q. What is unit's digit in $21321325^{12321}$
Q. What is unit's digit in $12^{33}$
Q. What is unit's digit in $1223^{45}$
Q. What is unit's digit in $1223^{45} \times 125^{123} \times 46572^{22}$

**Exponents with decimals:**

We just have to combine multiplication with decimals with definition of exponents to deal with powers of decimals
For example: $0.2^3 = 0.008; 1.2^2 = 1.44; (1.69)^{\frac{1}{2}} = \pm 1.3;$

$Find\ (0.04)^{-\left(\frac{1}{2}\right)}. Take\ only\ positive\ value\ of\ power.$

$$(0.04)^{-\left(\frac{1}{2}\right)} = \frac{1}{0.04^{\frac{1}{2}}} = \frac{1}{\pm 0.2} = \pm 5$$

CSE 2023: What is the unit digit in the expansion of (57242)^9×7×5×3×1?
(a) 2 (b) 4 (c) 6 (d) 8

## 5) PRIME NUMBERS AND PRIME FACTORIZATION

- **Prime number** is a number that is divisible only by itself and 1.
- Numbers that have other factors than 1 and itself are called **Composite numbers.**
  For example: 2, 3, 5, 7
  Is 6 a prime number? Is 15 a prime number?
  Is 1 a prime number? Is 1 a composite number? Is 1 unique?
- A number greater than 1 with exactly two factors, i.e., 1 and the number itself is a prime number
- Enlist prime numbers from 1 to 100:
- Are there even prime numbers or do prime numbers always have to be odd?

**Q.** CSE 2023: Choose the group which is different from the others:
(a) 17, 37, 47, 97
(b) 31, 41, 53, 67
(c) 71, 73, 79, 83
(d) 83, 89, 91, 97

**CSE 2023:** Consider the following in respect of prime number p and composite number c.
1. $p+c\ /p-c$ can be even.
2. 2p+c can be odd.
3. pc can be odd.
Which of the statements given above are correct?
(a) 1 and 2 only (b) 2 and 3 only (c) 1 and 3 only (d) 1, 2 and 3

**Q.** CSE 2022: Consider the following statements in respect of two natural numbers p and q such that p is a prime number and q is a composite number:
1. p x q can be an odd number.
2. q / p can be a prime number.
3. p + q can be a prime number.
Which of the above statements are correct?
(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only

(d) 1, 2 and 3

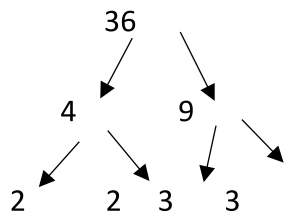
**CSE 2020:**
**Q.** Two Statements S1 and S2 are given below followed by a Question:
**S1:** n is a prime number.
**S2:** n leaves a remainder of 1 when divided by 4.
Question: If n is a unique natural number between 10 and 20, then what is n? Which one of the following is correct in respect of above Statements and the Question?
(a) S1 alone is sufficient to answer the Quest ion.
(b) S2 alone is sufficient to answer the Question.
(c) S1 and S2 together are sufficient to answer the Question, but neither S1 alone nor S2 alone is sufficient to answer the Question.
(d) S1 and S2 together are not sufficient to answer the Question.

**CSE 2019:** Consider two statements S1 and S2 followed by a question:
S1: p and q both are prime numbers.
S2: p+q is an odd integer.

**Question:** Is pq an odd integer?
Which one of the following is correct?
a. S1 alone is sufficient to answer the question
b. $2 alone is sufficient to answer the question
c. Both S1 and S2 taken together are not sufficient to answer the question
d. Both S1 and S2 are necessary to answer the question

**PRIME FACTORIZATION:**
Note that a number greater than 1 will always have a prime factor that is a number will be divisible by some prime number.
- If number is itself a prime number, it itself is a prime factor
- If number is a composite number, then it has factors other than 1 and itself, those factors can be prime or composite and thus they also will have some other factors – this way we will get some prime number as a factor of a given number

**The process of writing a number as the product of powers of prime numbers is prime factorization**.
**For example,**
- Prime factorization of 3 (prime number) is number itself
- Prime factorization of a square of prime number like 9 = $3 \times 3 = 3^2$
- the prime factorization of composite number like 36 can be done in the following way:

| Division Method (starting with the allest prime factor) | Factor Tree method |
|---|---|
| $\begin{array}{r\|l} 2 & 36 \\ \hline 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$ |  |

$$36 = 2 \times 2 \times 3 \times 3$$

Divisibility can now be better understood with this understanding.
A number N is divisible by a prime number ‘p’ if and only if ‘p’ appears in the prime factorisation of N.
This is because, when we’re writing prime factorisation, we’re basically enlisting all the prime factors of a number and writing all of them in expanded form. If prime number does not appear here, it is certainly not a factor of given number.
Further, a number N is divisible by another number M if and only if all the prime factors of M are also prime factors of N with respective powers.

**Number of Factors:**
Any number of the form $p^a q^b r^c$ will have (a + 1) (b + 1) (c + 1) factors, where $p, q, r$ are prime numbers.
For example: $6 = 2 \times 3$ will have (1+1) (1+1) = 4 factors namely: 1,2,3,6
$24 = 2^3 \times 3 \; will\; have\; (3+1)(1+1) = 8 \; factors$
- A prime number will always have only 2 factors
- If a number has exactly 3 factors, it is always of the form $p^2$ where ‘p’ is a prime number

**Q. CSE 2022**
What is the remainder when 91x92x93x94x95x96x97x98x99 is divided by 1261?
(a) 3
(b) 2
(c) 1
(d) 0

**CSE 2023:** What is the remainder when 85 × 87 × 89 × 91 × 95 × 96 is divided by 100?
(a) 0 (b) 1 (c) 2 (d) 4

**CSE 2022:** Q. 15 x 14 x13 x... x 3 x 2 x 1 = $3^m$ x n
Where m and n are positive integers, then what is the maximum value of m?
(a) 7

(b) 6
(c) 5
(d) 4

**CSE 2022:** Let p be a two-digit number and q be the number consisting of same digits written in reverse order. If p x q = 2430, then what is the difference between p and q?
(a) 45
(b) 27
(c) 18
(d) 9

## 6) DIVISIBILITY TESTS

**1. Divisibility Test of 2**

A number having, 0, 2, 4, 6, 8 at unit's place is divisible by 2. All such numbers are called even numbers.

Can any even number greater than 2 be prime?

**2. Divisibility Test of 3**

If sum of all the digits of a number is divisible by 3, number is divisible by 3.

For example: 434322 – sum of digits is 18 which is divisible by 3 hence the number is divisible by 3.

Q. Which of the following are divisible by 3?
1. 456738
2. 23234325
3. 3434738203
4. 7739202
5. 423829
6. 2938423984
7. 1000000000000000000000000000
8. 3952929929229
9. 123123123123123123123123
10. 333333333333333333333333333333333333333333333333333
11. $3^{12345}$
12. $9^{12345}$
13. $6^{12345}$
14. $2^{12345}$

Q. How many prime numbers can you form from digits 1,2,3,4,5?

**3. Divisibility Test of 4**

If a two-digit number created by the last two digits of a number is divisible by 4, the number is divisible by 4.

In short, we just have to look at last two digits and divide those by 4. If they're divisible by 4, so is the number.

For example: 234504; 234508; 23358912; 12345600; 13424340; 120 are all divisible by 4

While, 12213210; 123123; 95498330; 123142 are not divisible by 4.

Q. Which of the following numbers are divisible by 4?

1. 2389739816
2. 1781418
3. 1222222
4. 444444
5. 8888
6. 100000
7. 189898988
8. 1982187
9. 1231289123
10. 213123176

- Will any odd number be divisible by 4?
- Will every number divisible by 4, divisible by 2?
- If you know all the 2-digit numbers divisible by 4, will it suffice to test if the given number is divisible by 4 or not?

**4. Divisibility Test of 5**

If a number has 0 or 5 at unit's place, it is divisible by 5. The test for 5 is thus very simple and straightforward. You just have to examine unit's place.

For example: 12345; 231230; 1231235; 8485895; 43243200 etc. are all divisible by 5. While, 10000001; 24384234; 3432842; 3289219 etc. are not.

**5. Divisibility Test of 6**

We recall the prime factorisation of 6. We've $6 = 2 \times 3$. Hence, if a number is divisible by both 2 and 3, it is also divisible by 6. There's no separate test for 6 as such.

**NOTE:** we can check divisibility by any number using this technique. We simply write number's prime factorization or any factorization and check if all the factors with corresponding powers are factors of any other given number.

For instance, if you want to check if 864292324 is divisible by 24, we just write $24 = 2^3 \times 3$. We just check if number is divisible by $2^3 = 8$ and 3

(You can use divisibility tests or just directly divide. Since, the number is large, direct division would take more time, it is better to use tests.)

**6. Divisibility Test of 7**

We isolate last digit (units place) of given number. We double it and subtract from remaining number. If this difference is divisible by 7, the number is divisible by 7.
If the difference is too large, we repeat the above process as many times as we want.
Examples:
1234: units place is 4; its double is 8. We subtract 8 from 123. We get 115 which is not divisible by 7. So, 1234 is not divisible by 7.
2345679:

- Units place is 9.
- $9 \times 2 = 18$
- 234567 – 18 = 234549
- Now for 234549, units place is 9 and its double is 18
- 23454 – 18= 23436
- Now for 23436, units place is 6
- $6 \times 2 = 12$
- 2343 – 12 = 2331
- For 2331, units place is 1
- $1 \times 2 = 2$
- 233 – 2=231
- For units place is 1, its double is 2
- 23 – 2 = 21
- Since, 21 is divisible by 7, 2345679 is divisible by 7

**7. Divisibility Test of 8**

If the number formed by last 3 digits of a given number (hundred's, tens and unit's place) is divisible by 8, the number is divisible by 8.
Note that, for 2 we looked at last digit; for 4 we looked at last 2 digits and now for 8 we look at last 3 digits. Can you see the pattern and think what would we be looking at for checking divisibility of 16, 32, 64?
Q. which of the following are divisible by 8?
213892189333212; 21234; 83737888; 223981222; 2389000; 238914344; 29831008; 12831900; 12391777; 37817064

**8. Divisibility Test of 9**

If sum of all the digits of a number is divisible by 9, number is divisible by 9.
Thus, the test for 9 is very similar to test for 3.
Which of the following are divisible by 9?

1. 456738

2. 23234325
3. 3434738203
4. 7739202
5. 423829
6. 2938423984
7. 10000000000000000000000000008
8. 3952929929229
9. 123123123123123123123123
10. 3333333333333333333333333333333333333333333333333333
11. $3^{12345}$
12. $9^{12345}$
13. $6^{12345}$
14. $2^{12345}$

**9. Divisibility Test of 10**

If the number has 0 at unit's place, the number is divisible by 10. This is perhaps the simplest test; you just have to see if the last digit is 0 or not.

**10. Divisibility Test of 11**

For a given number, if the difference between sums of alternate digits is divisible by 11, the number is divisible by 11.

This is a very peculiar test.

Let, the number be some abcdefghijklm

We look at sums of alternate digits. So, we have (a +c + e+ g + i+ k +m) and (b+ d+ f+ h+ j+ l) as sums. If the difference between them, i.e.,

(a +c + e+ g + i+ k +m) – (b+ d+ f+ h+ j+ l) is divisible by 11 then the number abcdefghijklm is divisible by 11.

For instance,

let's take 387249254394938100 – the difference between sums is 46 – 35 = 11, which is divisible by 11. Hence the number is divisible by 11.

Let's take 333333; difference between sums is 0 – which is divisible by 11 (0 is divisible by all numbers – as remainder is always 0)

Q. Which of the following are divisible by 11?

1. 121
2. 1289319
3. 1331
4. 31298
5. 1111
6. 8573930
7. 29832139801001203

8. 3298432993
9. 2893123
10. 999999999999
11. 22222222222
12. 18181818181818
13. 1221122112211221

**11. Divisibility Test of 25**

If the last two digits of a number are 00, 25, 50 or 75, then the number

**Q. UPSC 2022:**

An Identity Card has the number ABCDEFG, not necessarily in that order, where each letter represents a distinct digit (1, 2, 4, 5, 7, 8, 9 only). The number is divisible by 9. After deleting the first digit from the right, the resulting number is divisible by 6. After deleting two digits from the right of original number, the resulting number is divisible by 5. After deleting three digits from the right of original number, the resulting number is divisible by 4. After deleting four digits from the right of original number, the resulting number is divisible by 3. After deleting five digits from the right of original number, the resulting number is divisible by 2. Which of the following is a possible value for the sum of the middle three digits of the number?

(a) 8

(b) 9

(c) 11

(d) 12

**CSE 2021:** Q. Consider all 3-digit numbers (without repetition of digits) obtained using three non-zero digits which are multiples of 3. Let S be their sum.

Which of the following is/are correct?

1. S is always divisible by 74.
2. S is always divisible by 9.

Select the correct answer using the code given below:

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

**CSE 2021:** Q. The number 3798125P369 is divisible by 7. What is the value of the digit P?

(a) 1

(b) 6

(c) 7

(d) 9

**CSE 2021: Q**. When a certain number is multiplied by 7, the product entirely comprises ones only (1111...). What is the smallest such number?

(a) 15713

(b) 15723

(c) 15783

(d) 15873

**CSE 2020:** Let XYZ be a three-digit number, where (X + Y + Z) is not a multiple of 3. Then (XYZ + YZX + ZXY) is not divisible by

(a) 3

(b) 9

(c) 37

(d) (X + Y + Z)

**CSE 2020:** Two statements are given followed by two Conclusions:

Statements:

All numbers are divisible by 2. All numbers are divisible by 3.

Conclusion-1: All numbers are divisible by 6.

Conclusion-II: All numbers are divisible by 4.

Which of the above Conclusions logically follows/follow from the two given Statements?

(a) Only Conclusion-1

(b) Only Conclusion-II

(c) Neither Conclusion-1 nor Conclusion-Il

(d) Both Conclusion-I and Conclusion-Il

**CSE 2020:** Q. How many integers are there between 1 and 100 which have 4 as a digit but are not divisible by 4?

(a) 5

(b) 11

(c) 12

(d) 13

**CSE 2020:** Q. How many five-digit prime numbers can be obtained by using all the digits 1, 2, 3, 4 and 5 without repetition of digits?

(a) Zero

(b) One

(c) Nine

(d) Ten

**Q. CSE 2020:** Q.A digit n > 3 is divisible by 3 but not divisible by 6. Which one of the following is divisible by 4?
(a) 2n
(b) 3n
(c) 2n + 4
(d) 3n + 1

**CSE 2020:** Q. An 8-digit number 4252746B leaves remainder 0 when divided by 3. How many values of B are possible?
a. 2
b. 3
c. 4
d. 6

**CSE 2019:** Q. Number 136 is added to 5B7 and the sum obtained is 7A3, where A and B are integers. It is given that 7A3 is exactly divisible by 3. The only possible value of Bis
a.2
b. 5
c. 7
d. 8

**CSE 2016:** If R and S are different integers both divisible by 5, then which of the following is not necessarily true?
(a) R - S is divisible by 5
(b) R + S is divisible by 10
(c) R x S is divisible by 25
(d) $R^2 + S^2$ is divisible by 5

## 7) BODMAS RULE

### 3.1 Order of applying mathematical operations: BODMAS rule

BODMAS stands for **Bracket, Order, Division, Multiplication, Addition, and Subtraction**. This is the fundamental rule we study in arithmetic to apply when there is more than one operation involved.

- So, first we've to resolve bracket; then we solve exponents or roots
- Thereafter, we solve division and multiplication from left to right
- And then we solve addition or subtraction from left to right

For instance, if the value of $3 + 5 \times 2 \div 2 - 6 + 2^3 \times (4 - 2)$ is asked, we use above BODMAS rule to find the value.

**Other useful properties:**

- $a - (b + c) = a - b - c$
- $a \times (b + c) = a \times b + a \times c$
- $a \times (b - c) = a \times b - a \times c$
- $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$

**Q.** CSE 2020: If $ means divided by: @ means multiplied by: # means 'minus, then the value of 10#5@1$5 is

a.0

b. 1

c. 2

d. 9

## 8) HCF/GCD AND LCM

4.1 **Factor or divisor of a number and multiple of a number**

- We've already seen what is the meaning of factor or divisor of a number: M is factor of N if N is divisible by M.
- Multiple of a number N is all such number mN where m is any natural number. So, multiples of 3 would be 6, 9, 12, 15 and so on.

**4.2 HCF (Highest Common Factor) or GCD (Greatest Common Divisor) of two numbers**

- HCF or GCD of two numbers is the highest possible number that divides both the numbers or that is factor of the both numbers.
- HCF can be defined for more than two numbers as well. In that case, HCF would be highest number among all the common factors of the given numbers.

**Example:** Consider 18 and 27.

- All the factors of 18 are 1, 2, 3, 6, 9 and 18
- All the factors of 27 are 1, 3, 9 and 27
- Highest common factor is 9 and thus HCF/GCD of 18 and 27 is 9.

**Finding HCF/GCD:**

There are various methods of finding HCF of two numbers. But most efficient and simple is the method of prime factorisation.

**1. Method of Prime-factorization:**

In this method, we write prime factorization of given numbers. We then write all the common factors with least power amongst the given numbers. Number we obtain after multiplying them is the required HCF or GCD.

Q. Find GCD of 54 and 117.

- $54 = 2 \times 3^3$
- $117 = 3^2 \times 13$
- Common factor is 3 and its least power is 2.
- GCD is $3^2 = 9$

Q. Find HCF of 512 and 288.

- $512 = 8^3 = (2^3)^3 = 2^9$
- $288 = 9 \times 32 = 2^5 \times 3^2$
- Common factor is 2 and least power is 5.
- HCF is $2^5 = 32$

**2. Method of listing Factors**

In this method, we simply list all the possible factors of both or all numbers in increasing order, the highest common factor listed is the HCF or GCD.

Q. Find GCD of 54 and 117

- Factors of 54 are 1, 2, 3, 6, 9, 18, 27, 54
- Factors of 117 are 1, 3, 9, 13, 39, 117
- HCF is 9.

***NOTE:** listing all the factors can be time consuming for larger numbers. Generally, we have to check all prime numbers till half of the number to check if they are the factor.
For smaller numbers, this method is quite simple and clear.

**3. Method of division**:

**It involves following steps:**

- Step 1: We divide larger number by smaller number and check the remainder. If the remainder is 0, smaller number is a factor of larger number and thus smaller number itself is HCF. If not, we go to step 2.
- Step 2: This remainder becomes new devisor and the earlier devisor becomes new dividend
- Step 3: We continue step 2 till we get remainder as 0. Once we get that, last divisor will be HCF of two numbers.

**Example:**

Q. Find GCD of 54 and 117

```
         2
54)117
   -108    6
       9)54
         - 54
            0
```

Hence, we have the 0 remainder. Last divisor i.e., 9 is the gcd or hcf

Q. Find HCF of 512 and 288.

```
         1
288)512
        288    1
       224)288
          -  224   3
              64)224
                 - 192   2
                      32)64
                          -64
                         0
```

Hence, we've 0 reminder. Last divisor 32 is the GCD or HCF.

**Properties of HCF:**

- HCF of two or more numbers is always a factor of all the numbers
- If any number divides all the given numbers, it also divides the HCF of the given numbers.
- HCF is always less than or equal to all the given numbers
- HCF of two prime numbers is always 1
- HCF of a prime number and the number which is not a multiple of it, is always 1. HCF of a prime number and some multiple of a prime number is that prime number.

    (Example: HCF of 2, 9 = 1; HCF of 17, 51 = 17)

**Coprime Numbers**: A pair of numbers are coprime if their HCF is 1. The numbers do not have to be primes to be coprime with each other.

**For Example:** (2, 5); (4; 9); (245; 3) are all pairs of coprime numbers

- NOTE that: If one of the two numbers is prime, then numbers are coprime if the other number is not divisible by prime. (As only factors of prime are 1 and itself, if prime is not a factor of a given number, then HCF would be automatically 1)
- Two even numbers can never be coprime
- Two consecutive numbers are always coprime

**4.3 LCM (Least Common Multiple) of two numbers**

- The LCM of two numbers or the least common multiple of two numbers is the smallest number which is a multiple of those two numbers.
- If the numbers are, n and m, then we have n, 2n, 3n, 4n, … as multiples of n and m, 2m, 3m, … as multiples of m.
- Smallest common multiple in these two lists would be our LCM.
- NOTE that, multiplication of two numbers will always yield a common multiple. In other words, mn will always be a multiple of both m and n (common multiple) – in LCM we want a smallest such common multiple. At the most LCM can be multiplication of given numbers.

**Example:** LCM of 6 and 9.

- Multiples of 6 are – 6, 12, 18, 24, 30, 36 …
- Multiples of 9 are – 9, 18, 27, 36, …
- Common multiples are – 18, 36, …
- Least common multiple thus is 18.

**Finding LCM**

Like for HCF, there are various methods to find LCM of two or more numbers. Most efficient method is using prime factorization here as well. But we shall see all the methods and you can decide which you find the easiest.

1. **Method of listing multiples**:

Here, we simply enlist multiples of both (or all) the given numbers. We then identify the smallest of the common multiples and that is our LCM.

2. **Method of prime factorization**:

We write the prime factorization of both the numbers. We then multiply each factor with the highest powers to calculate the LCM of two numbers.

Example: Find LCM of 40 and 54.

- $40 = 2^3 \times 5$
- $54 = 2 \times 3^3$
- Factors are – 2, 3 and 5. And their highest powers are 3, 3 and 1
- LCM = $2^3 \times 3^3 \times 5 = 8 \times 27 \times 5 = 1080$

**3. Finding LCM using HCF (and vice versa)**

We use the relation: $\boldsymbol{HCF \times LCM = Product\ of\ two\ numbers}$

Hence, $LCM = \frac{Product\ of\ two\ numbers}{HCF}$ Similarly, $HCF = \frac{Product\ of\ two\ numbers}{LCM}$

**Example**: Find LCM of 40 and 54

- HCF of 40 and 54 is 2
- LCM = $\frac{40 \times 54}{2} = 1080$

**Properties of LCM:**

- All the numbers are factor of their LCM
- LCM is always greater than or equal to all the given numbers
- LCM of two numbers is one of the numbers if and only if, the other number is its factor (For example: lcm of 4 and 16 is 16 as 4 is factor of 16; lcm of 8 and 56 is 56 as 8 is factor of 56 and so on)
- LCM of two coprime numbers is their product

**CSE 2023:** There are three traffic signals. Each signal changes colour from green to red and then from red to green. The first signal takes 25 seconds, the second signal takes 39 seconds and the third signal takes 60 seconds to change the colour from green to red. The durations for green and red colours are same. At 2:00 p.m, they together turn green. At what time will they change to green next, simultaneously?

(a) 4:00 p.m. (b) 4:10 p.m. (c) 4:20 p.m. (d) 4:30 p.m

**CSE 2022:** Q. What is the smallest number greater than 1000 that when divided by any one of the numbers 6, 9, 12, 15, 18 leaves a remainder of 3?

(a) 1063

(b) 1073

(c) 1083

(d) 1183

**CSE 2020:** What is the least four-digit number when divided by 3, 4, 5 and 6 leaves a remainder 2 in each case?

(a) 1012

(b) 1022

(c) 1122

(d) 1222

**CSE 2020:** Q. Joseph visits the club on every 5th day, Harsh visits on every 24th day, while Sumit visits on every 9th day. If all three of them met at the club on a Sunday, then on which day will all three of them meet again?

(a) Monday

(b) Wednesday

(c) Thursday

(d) Sunday

**CSE 2019:** Q. In a school every student is assigned a unique identification number. A student is a football player if and only if the identification number is divisible by 4, whereas a student is a cricketer if and only if the identification number is divisible by 6. If every number from 1 to 100 in assigned to a student, then how many of them play cricket as well as football?

(a) 4

(b) 8

(c) 10

(d) 12

**CSE 2019:** Seeta and Geeta go for a swim after a gap of every 2 days and every 3 days respectively. If on 1st January both of them went for a swim together, when will they go together next?

(a) 7th January

(b) 8th January

(c) 12th January

(d) 13th January

**CSE 2016:** There are five hobby clubs in a college —photography, yachting, chess, electronics and gardening. The gardening group meets every second day, the electronics group meets every third day, the chess group meets every fourth day, the yachting group meets every fifth day and the photography group meets every sixth day. How many times do all the five groups meet on the same day within 180 days?

(a) 5

(b) 18

(c) 10

(d) 3

**CSE 2015:** There are five hobby clubs in a college viz, photography, yachting, chess, electronics and gardening. The gardening group meets every second day, the electronics group meets every third day, the chess group meets every fourth day, the yachting group meets every fifth day and the photography group meets every sixth day. How many times do all the five groups meet on the same day within 180 days?

(a) 3
(b) 5
(c) 10
(d) 18

**CSE 2014:** A bell rings every 18 minutes. A second bell rings every 24 minutes. A third bell rings every 32 minutes. If all the three bells ring at the same time at 8 o'clock in the morning, at what other time will they all ring together?
(a) 12: 40 hrs
(b) 12: 48 hrs
(c) 12: 56 hrs
(d) 13: 04 hrs

**CSE 2014:** Five persons fire bullets at a target at an interval of 6, 7, 8, 9 and 12 seconds respectively. The number of times they would fire the bullets together at the target in an hour is
(a) 6
(b) 7
(c) 8
(d) 9

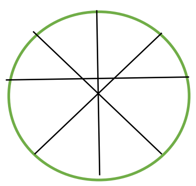
**CSE 2011:** Q. Three persons start walking together and their steps measure 40 cm, 42 cm and 45 cm
respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?
(a) 25 m 20 cm
(b) 50 m 40 cm
(c) 75 m 60 cm
(d) 100 m 80 cm

## 9) FRACTIONS

### 1.1 What is fraction

A fraction shows part of a whole. A fraction can be a portion or section of any quantity out of a whole, where the whole can be any number, a specific value, or a thing.

**For example:** If we have a cake. We cut it into say 8 equal pieces. Each piece is equivalent to 1/8 part of a whole cake.



$$Fraction : \frac{Numerator}{Denominator}$$

Here both numerator and denominator are integers.
Denominator is never 0 as fraction is essentially a division and we never divide by 0.

- **Proper Fraction**: Numerator < Denominator $Ex: \frac{2}{7}, \frac{456}{666}, \frac{123}{1222} etc.$
- **Improper fraction**: Denominator< Numerator $Ex: \frac{3}{2}, \frac{45}{23}, \frac{31233143}{12312} etc.$
- **Mixed fractions**: Whole number and a proper fraction $4\frac{1}{4}, 32\frac{2}{3}, 3\frac{3}{5}$ $etc.$

**Properties of fractions:**

- If denominator = 1, fraction reduces to an integer
- If numerator = 0, fraction is 0 (as 0 divided by anything is 0).
- Fraction doesn't change if both numerator and denominator are multiplied or divided by same number

$$\frac{a}{b} = \frac{n \times a}{n \times b} = \frac{a \div n}{b \div n}$$

Thus, $\frac{2}{3} = \frac{4}{6} = \frac{20}{30} = \frac{30}{45}$ and $\frac{30}{40} = \frac{6}{8} = \frac{3}{4} = \frac{3/5}{4/5}$ etc.

### 1.2 Equivalence of fractions:

- **When are two fractions equivalent?**

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } a \times d = b \times c$$

Example: $\frac{1}{4}$ & $\frac{48}{192}$: Since $1 \times 192 = 48 \times 4 = 192$ Thus, fractions are equal

Q. Which of the following pair of fractions are equivalent?

1) $\frac{3}{2}; \frac{45}{30}$
2) $\frac{5}{17}; \frac{4}{15}$
3) $\frac{8}{23}; \frac{16}{46}$
4) $\frac{4/7}{3/28}; \frac{16}{3}$
5) $\frac{-4}{13}; \frac{8}{-26}$

### 1.3 Terminating and Non-terminating fractions:

- Try to convert fractions $\frac{1}{2}$ $and$ $\frac{1}{3}$ into decimals by direct division.
- What is the difference?

- Now, try $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ – Which of these terminate? Which do not terminate?

**Non-terminating but Recurring/repeating and non-repeating fractions**:

- Recurring fractions are those where a group of numbers tend to repeat indefinitely after certain digits after decimal points.
- For ex: 123.12121212121212…
- 0.223456345634563456…

We show recurring fractions with small bar overhead.
In above examples, numbers would be $123.\overline{12}, 0.22\overline{3456}$

### 1.4 Converting fractions into decimals

- **Terminating fractions:**

Q. Convert 0.5 into fractions.
Q. Convert 12.123 into fraction.

- **Non-terminating but recurring fractions:**

**Type:1 – recurring starts immediately after decimal point**

Q. Convert 0.33333333… into fraction
- We write: $x = 0.3333 \ldots$ as only one digit is repeating, we multiply both sides by 10
- $10x = 3.3333 \ldots$ We now subtract x from 10x to get $9x = 3$.

Q. Convert 0.12121212… into fraction
Q. Convert 0.432432432… into fraction
Q. Convert 1.232323… into fraction
Q. Convert $0.\overline{12333}$ into fraction

(We divide by 9 or 99 or 999 etc. depending on how many digits are recurring)

**Type:2 – recurring part starts after few digits after decimal**

Q. Convert 0.1333… into fraction.

- We write $x = 0.1333 \ldots = 0.1\overline{3}$
- Like earlier, we want to eliminate recurring part
- We multiply by 10 to get only recurring part after decimal:
- $10x = 1.333 \ldots$ Now, can we eliminate recurring part by subtracting x from 10x – NO! as there’s 1 after decimal. But, like earlier, we’ve recurring part just after decimal point.
- Now, we can multiply by 10 to this equation as earlier
- $100x = 13.333 \ldots$ Now, subtracting:

- $90x = 12 \; or \; x = \frac{12}{90} = \frac{2}{15}$

**RULE:**

- Write as many numbers of 9's in the denominator as the recurring digits followed by certain number of 0's
- Number of 0's is as many digits there are between recurring part and decimal
  In above case, 9 followed by 1 zero i.e. 90 in denominator
- Numerator is difference between number after decimal taking recurring part only once and non-recurring part
  In above case: 13 – 1 = 12 in numerator
- Example: 0.33454545… = $\frac{3345-33}{9900} = \frac{3312}{9900}$

Q. Convert 0.1205050505… into fraction
Q. Convert 0.22434343… into fraction
Q. Convert 0.5123412341234… into fraction
Q. 0.35555…
Q. 0.2484848…
Q. 0.12898989…

**Type:3 – Number has non-zero number before decimal point**

- This is simple extension of earlier rule.
- We have to include number before decimal point also as a part of non-recurring part of number while subtracting

For ex: 2.3333…= $2.\bar{3} = \frac{23-2}{9} = \frac{21}{9} = \frac{7}{3}$
$3.23\overline{43} = \frac{32343-323}{9900} = \frac{32020}{9900}$

$$\boldsymbol{Fraction = \frac{Total\ Number\ taking\ recurring\ part\ only\ once - Non\ recurring\ part}{'9's\ followed\ by\ '0's}}$$

Q. Convert following decimals into fractions:

1. 1.234444…
2. 2.1232323…
3. 44.44444…
4. 1.111222222….
5. 6.00121212…

**Q. CSE 2020:** The recurring decimal representation 1.272727 … is equivalent to
(a) 13/11
(b) 14/11
(c) 127/99

(d) 137/99

**1.5 Operating on fractions: addition, subtraction, multiplication, division, powers**

- **Addition of fractions**

**Case 1) Denominator of both/all fractions is same**

- In this case, we simply add the numerators and keep denominator same to get the answer

For example:

$\frac{5}{7}+\frac{1}{7}=\frac{5+1}{7}=\frac{6}{7}$

$\frac{12}{4}+\frac{3}{4}=\frac{15}{4}=3\frac{3}{4}$

$\boldsymbol{\frac{2}{7}+\frac{3}{7}+\frac{5}{7}=\frac{10}{7}}$

**Case 2) Denominator is NOT same**

- In this case, we've to make the denominator same and use case 1.
- We recall that, fraction does not change when we multiply or divide it by the same number.
- We use this property to multiply denominators by such numbers that they become equal.

For example:

$$Consider \frac{2}{3}+\frac{3}{4};$$

$Here\ denominators\ are\ 3\ and\ 4.$

$If\ we\ multiply\ 3\ by\ 4\ and\ 4\ by\ 3, we\ shall\ get\ same\ number\ 12.$

- We multiply numerator and denominator of first ratio by 4 and
- We multiply numerator and denominator of second ratio by 3

$$\frac{2}{3}+\frac{3}{4}=\frac{2\times 4}{3\times 4}+\frac{3\times 3}{4\times 3}=\frac{8}{12}+\frac{9}{12}=\frac{17}{12}$$
$$\frac{2}{3}+\frac{3}{4}=\frac{2\times 4+3\times 3}{3\times 4}=\frac{17}{12}$$

**NOTE:** In making denominators equal, we're essentially finding a common multiple of denominators. Product of two numbers will always be their common multiple as we've seen. But, have studied anything else involving common multiples?

So, we've two options,

- We just blindly use product of denominators as common multiple and change the ratios accordingly (Works well for most small ratios. But becomes very tough in case denominators are large)
- Or we can find LCM of two numbers and change the fraction accordingly

For example: $\frac{3}{80}+\frac{5}{16}$

- $\frac{3\times16+5\times80}{80\times16}=\frac{48+400}{80\times16}=\frac{448}{80\times16}=\frac{7}{20}$
- Or note that, LCM = 80 as 16 is a factor of 80
- $\frac{3}{80}+\frac{5\times5}{80}=\frac{28}{80}=\frac{7}{20}$

- Suggestion:
    - If LCM is clearly visible like above or is easy to find (when one denominator is factor of other) OR if the numbers in denominator are too large to multiply quickly – go for LCM method
    - Otherwise just cross multiply quicky and reduce the fraction you get at the end

**Q. Find addition of following fractions:**

1) $\frac{4}{3};\frac{123}{23}$
2) $\frac{4}{5};\frac{2}{3}$
3) $\frac{4}{11};\frac{12}{29}$
4) $\frac{7}{8};\frac{9}{10}$
5) $\frac{13}{21};\frac{15}{21}$
6) $\frac{34}{102};\frac{3}{17}$
7) $-\frac{3}{5};\frac{4}{77}$
8) $-\frac{8}{13};-\frac{4}{9}$

- **Subtraction of fractions:**
  It is exactly same as addition of fractions.
- If denominators are same, we simply subtract second numerator from first

For example: $\frac{5}{12}-\frac{3}{12}=\frac{2}{12}=\frac{1}{6};\frac{4}{7}-\frac{6}{7}=-\frac{2}{7}$

- If denominators are not same, we make them same like in addition by cross multiplication or with help of LCM and do the subtraction

For example:

$$\frac{6}{7}-\frac{2}{3}=\frac{6\times3-7\times2}{7\times3}=\frac{4}{21}$$

$$\frac{5}{16}-\frac{3}{80}=\frac{5\times5}{16\times5}-\frac{3}{80}=\frac{22}{80}=\frac{11}{40}$$

$$\frac{2}{5}-\frac{6}{7}=\frac{2\times7-6\times5}{5\times7}=-\frac{16}{35}$$

**Multiplication of fractions:**

Multiplication of fractions is simply multiplication of corresponding numerators and denominators.
In other words, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

For example: $\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$; $-\frac{3}{4} \times \frac{2}{5} = -\frac{6}{20} = -\frac{3}{10}$

Rules for multiplication with negative numbers hold for fractions as well
i.e., $\boldsymbol{-ve \times -ve = +ve; -ve \times +ve = +ve \times -ve = -ve}$

- **Division of fractions**:

Division is simply reverse multiplication.
Thus, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ OR $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

For example: $\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{5}{4}$; $\frac{3/4}{2/3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$

- **Powers of fractions:**

Power of a fraction is fraction of powers of numerator and denominator. In other words,
$(\frac{\textbf{Numerator}}{\textbf{Denominator}})^{\mathbf{a}} = \frac{\textbf{Numerator}^{\mathbf{a}}}{\textbf{Denominator}^{\mathbf{a}}}$

- Rest all the properties of powers follow as it is for fractions as well

**PROPERTIES OF EXPONENTS**

- $(\frac{a}{b})^m \times (\frac{a}{b})^n = \left(\frac{a}{b}\right)^{m+n} = \frac{a^{m+n}}{b^{m+n}}$
- $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$
- $\left(\frac{a}{b}\right)^0 = 1 \; for \; all \; '\left(\frac{a}{b}\right)'$
- $\left(\frac{a}{b}\right)^1 = \left(\frac{a}{b}\right)$
- $\left(\frac{a}{b}\right)^{-1} = \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a}$
- $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \left(\frac{b}{a}\right)^n$
- $\left(\left(\frac{a}{b}\right)^m\right)^n = \left(\frac{a}{b}\right)^{mn}$
- $\left(\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)\right)^n = \left(\frac{a}{b}\right)^n \left(\frac{c}{d}\right)^n$

- $\left(\frac{a}{b}\right)^{1/n} = \sqrt[n]{\left(\frac{a}{b}\right)}$

**EXPONENTS WITH FRACTIONS:**

- $\left(\frac{a}{b}\right)^{m/n} = (\left(\frac{a}{b}\right)^{m})^{\frac{1}{n}} = \sqrt[n]{\left(\frac{a}{b}\right)^{m}}$

**Q.** What is $(\frac{a}{b})^{5p/q} \times (\frac{a}{b})^{9p/q}$?

Q. Find $(\frac{1}{3})^{15} \div (\frac{1}{6})^{15}$

Q. What is $\left(\frac{1}{4}\right)^{-5} \times 2^{-3}$

Q. What is $(37^5 \times \left(\frac{1}{3}\right)^{3.453} \div 519^{34/3})^0$

Q. Find $((\frac{1}{3})^4)^2 \div 9$

Q. Find $(0.5)^4 \times 2^5$

Q. Simplify $(\frac{1}{b})^{5p/q} \times (\frac{a}{b})^{9/q}$

### 1.6 BODMAS rule with fractions

It is exactly same as BODMAS rule for integers. No change what so ever. In the worksheet you'll find problems on this part which only require you to use operating with fractions.

### 1.7 Finding fraction of a Number

Fraction of a number is a part of a number. So, we fundamentally divide the number into smaller portion.

Fraction of a number is obtained by simply multiplying number by the fraction.
For example: 2/5 of 25 = $\frac{2}{5} \times 25 = 10; \frac{3}{8} of\ 40 = \frac{3}{8} \times 40 = 15$ and so on.

Another terminology is used sometimes. Sometimes the question may say that, 5th portion of a number is 30. It only means that, a number when divided into 5 gives 30 or $\frac{1}{5} \times number = 30$. Thus number is $5 \times 30 = 150$.

**Questions:**

1. 4/7 of a number is 84. Find the number
2. Rachel took 1/2 hour to paint a table and 1/3 hour to paint a chair. How much time did she take in all?
3. If $3\frac{1}{2}$ m of wire is cut from a piece of 10 m long wire, how much of wire is left?
4. One half of the students in a school are girls, 3/5 of these girls are studying in lower classes. What fraction of girls are studying in lower classes?

5. A herd of cows gives 4 litres of milk each day. But each cow gives one-third of total milk each day. They give 24 litres milk in six days. How many cows are there in the herd?
6. Shelly walked 1/3 km. Kelly walked 4/15 km. Who walked farther? How much farther did one walk than the other?

**Note on Irrational numbers:**

- Irrational numbers are those numbers that cannot be written as a fraction or a ratio of two integers.
- Most popular examples: $\sqrt{2}, \sqrt{3}, \pi, e$
- Square roots of all non-square rational numbers, cube-roots of all non-cube rational numbers, multiples of irrational numbers like $\pi, e$ – are all rational numbers
- Addition/subtraction of two different irrational numbers is always an irrational number
- Multiplication of two irrational numbers can be rational as well as irrational
  Ex. $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$
- Division of two irrational numbers also can be rational as well as irrational
  Ex: $\frac{3\pi}{\pi} = 3; \frac{\sqrt{54}}{\sqrt{6}} = \sqrt{9} = 3$
- Power of irrational number can be rational or irrational
  Ex: $\left(\sqrt{5}\right)^2 = 5; \left(\sqrt[4]{\frac{5}{7}}\right)^{12} = \left(\frac{5}{7}\right)^3 = \frac{125}{343}$
- Irrational and rational numbers together make real number (all numbers on the number line)