

Computer Vision Sheet 07.

$$1. \int \text{Norm}_x[a|A] \cdot \text{Norm}_x[b|B] dx$$

$$= \int \frac{1}{\sqrt{2\pi}|A|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-a)A^{-1}(x-a)^T\right) \cdot \frac{1}{\sqrt{2\pi}|B|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-b)B^{-1}(x-b)^T\right) dx$$

$$= \int \frac{1}{2\pi|A|^{\frac{1}{2}}|B|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-a)A^{-1}(x-a)^T - \frac{1}{2}(x-b)B^{-1}(x-b)^T\right) dx$$

$$= \int \frac{1}{2\pi|A|^{\frac{1}{2}}|B|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-c)(A^{-1}+B^{-1})(x-c)^T + (a-b)C(a-b)^T\right) dx$$

$$= \frac{1}{\sqrt{2\pi}|A|^{\frac{1}{2}}|B|^{\frac{1}{2}}} e^{-\frac{1}{2}(a-b)C(a-b)^T} \int \frac{1}{\sqrt{2\pi|(A^{-1}+B^{-1})|}^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-c)(A^{-1}+B^{-1})(x-c)^T\right) dx$$

$$\frac{1}{\sqrt{2\pi}(|A||B|(A^{-1}+B^{-1})^{\frac{1}{2}})} \exp\left(-\frac{1}{2}(a-b)C(a-b)^T\right)$$

$$= \frac{1}{\sqrt{2\pi}(|A||B|(A^{-1}+B^{-1})^{\frac{1}{2}})} e\left(-\frac{1}{2}(a-b)(A+B)^{-1}(a-b)^T\right)$$

$$= \frac{1}{\sqrt{2\pi}|ABA^{-1}+ABB^{-1}|^{\frac{1}{2}}} e\left(-\frac{1}{2}(a-b)(A+B)^{-1}(a-b)^T\right)$$

$$= \frac{1}{\sqrt{2\pi}|A(B+A)A^{-1}|^{\frac{1}{2}}} e\left(-\frac{1}{2}(a-b)(A+B)^{-1}(a-b)^T\right)$$

$$\Rightarrow \text{Norm}_a\left(b, A+B\right) \cdot \int \text{Norm}_x\left[\frac{A^{-1}a+B^{-1}b}{A^{-1}+B^{-1}}, (A^{-1}+B^{-1})^{-1}\right]$$