CLIMATE SPECTRA AND DETECTING CLIMATE CHANGE

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Abstract. Part of the debate over possible climate changes centers on the possibility that the changes observed over the previous century are natural in origin. This raises the question of how large a change could be expected as a result of natural variability. If the climate measurement of interest is modelled as a stationary (or related) Gaussian time series, this question can be answered in terms of (a) the way in which change is estimated, and (b) the spectrum of the time series. These computations are illustrated for 128 years of global temperature data using some simple measures of change and for a variety of possible temperature spectra. The results highlight the time scales on which it is important to know the magnitude of natural variability. The uncertainties in estimates of trend are most sensitive to fluctuations in the temperature series with periods from approximately 50 to 500 years. For some of the temperature spectra, it was found that the standard error of the least squares trend estimate was 3 times the standard error derived under the naïve assumption that the temperature series was uncorrelated. The observed trend differs from zero by more than 3 times the largest of the calculated standard errors, however, and is therefore highly significant.

1. Introduction

Recent concerns about the possibility of climate change have focused attention on temperature series such as those constructed by Jones *et al.* (1986a, b, c) and Jones (1988). Figure 1 shows the series used by Folland *et al.* (1990, figure 7.10(c); Raper, personal communication). This series is the average of the one constructed by Jones *et al.* (1986c, updated) and another that combines the Jones *et al.* land-based data (Jones, 1988) with marine data from Bottomley *et al.* (1990).

A key question raised by these data is whether the temperature rise of around 0.5 °C is the start of a systematic warming or simply an effect of natural variability. The data show natural variations on all time scales from year-to-year up to several decades. Similar variations with a time scale of centuries, and of sufficient amplitude, could cause a trend of this magnitude. Thus the problem is to set bounds on the possible amplitude of fluctuations at the most sensitive time-scale, and hence on the possible contribution of natural variability to the observed trend.

The *power spectrum* of a phenomenon describes the way in which the probable amplitudes of fluctuations depend on time-scale. The spectrum in turn determines the *standard error* of a trend estimate, which thus provides the link between fluctuations and variability of an estimated trend. Such an analysis can focus attention on time scales and mechanisms that are the most relevant for interpreting trends in the observed data. For example, very long term fluctuations or cycles may be essentially constant over a relatively short observation period and thus will have little effect

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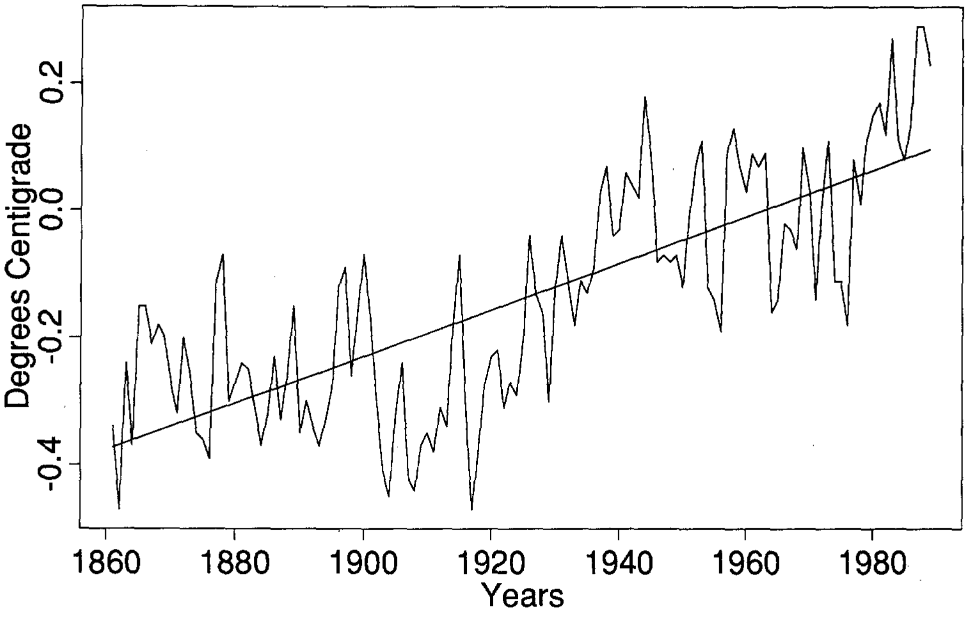


Fig. 1. Annual global temperature deviations used by Folland *et al.* (1990). Temperatures are the difference from the mean temperature over the time period. Plotted is the line estimated by least squares regression where the estimated slope is $b_{LS} = 0.367$ °C/century.

on an estimate of trend. Natural variation on a time scale similar to the length of the data record, however, may increase the variability of the estimate.

This article studies the impact of several stochastic models for variability in the global temperature series. Section 2 defines a statistical model for the temperature series and Section 3 describes some estimates of temperature trends based on this model. Section 4 contains a derivation of the standard error of a trend estimate in terms of the spectral density of the temperature series. The following section suggests several models for the spectrum of the stochastic component of global temperatures. Section 6 compares the standard errors of the trend estimates for these different models of long term dependence. The last section states some conclusions.

2. A Statistical Model for Annual Temperatures

A plausible statistical model for the annual data given in Figure 1 consists of a mean level, a possible trend plus a stationary (or perhaps a difference-stationary) time series:

$$Y_t = \mu + T_t + X_t. \tag{1}$$

Here Y_t is the annual mean temperature, T_t is the trend due to humanmade effects and X_t is a mean zero random variable expressing the natural variation in global temperature. For a stationary Gaussian process the dependence among the random components can be described by the autocovariances, $E(X_tX_{t+h}) = \gamma(h)$, or by the spectral density function, $s(f) = \Sigma_{h=-\infty}^{\infty} \gamma(h)e^{-2\pi ihf}$. It is this dependence between successive values of X_t that makes the identification of T_t difficult.

For the purposes of this discussion it will be assumed that $\{T_t\}$ can be approximated by a linear trend. Although this assumption simplifies the interpretation of variability of a trend estimate, the basic ideas are not restricted to this parametric form. A more interesting choice for $\{T_t\}$ might be the temperature increase suggested by a global climate model. However, in this case $\{T_t\}$ is likely to depend nonlinearly on parameters of interest, such as climate sensitivity, and the calculation of standard errors is therefore more difficult and less exact. Bloomfield (1991) discusses one such case. The general nature of the problem would nevertheless be expected to be similar to the linear trend case discussed below.

3. Estimates of Change

In this section some linear estimates of the rate of change are given. The simplest of these is the difference between the averages of the first and second half of the time series and will be referred to as b_{AV}. To be specific, suppose the length of the time series, T, is even and let $M = T/2$. Then

$$b_{AV} = (1/M)\left[(1/M)\sum_{t=M+1}^{T} Y_t - (1/M)\sum_{t=1}^{M} Y_t\right], \tag{2}$$

where the factor, $(1/M)$, outside the brackets converts the difference between the averages into a change per year. This type of measure is sensitive to a step-wise temperature change near $t = M$. A more appropriate measure for the gradual change assumed here is the slope estimate obtained when a straight line is fit to the series. The slope (as a change per year) obtained from a least squares regression of Y_t on t is

$$b_{LS} = \frac{\Sigma_{t=1}^{T}(t-\bar{t})Y_t}{\Sigma_{t=1}^{T}(t-\bar{t})^2}, \tag{3}$$

where $\bar{t} = (T+1)/2$ is the average of the observation times.

The two estimates mentioned above both yield unbiased estimates of the slope when $\{T_t\}$ is a linear function. However, there are unbiased estimators with smaller standard errors, though typically only slightly smaller, when the variables $\{X_t\}$ are corrrelated. One common way of adjusting the trend estimate when correlation is present is to assume that the covariances among $\{X_t\}$ can be approximated by a first order autoregressive process (see Section 5) with a correlation coefficient ρ. This model implies an autocovariance function of the form $\gamma(h) = \sigma^2\rho^{|h|}$.

Although this is only an approximation to the true covariances, for many observed time series the approximation is adequate to give an estimate that is nearly optimal with respect to the size of its standard error. The unbiased trend estimate with the smallest standard error when $\{X_t\}$ is an $AR(1)$ process can be expressed as a weighted sum of two simpler estimates. Let $b_{EP} = (Y_T - Y_1)/(T-1)$ be the estimate obtained from the endpoints of the data. Then the best estimate is

$$b_{AR1(\rho)} = \frac{w_{LS} b_{LS} + w_{EP} b_{EP}}{w_{LS} + w_{EP}}, \tag{4}$$

where

$$w_{LS} = (1-\rho)^2 T(T+1),$$
$$w_{EP} = 6\rho[(1-\rho)(T-1)+2].$$

Note that $b_{AR1(0)} = b_{LS}$ as one would expect but also that for fixed $\rho \neq 0$, $b_{AR1(\rho)}$ is close to b_{LS} for large T. Conversely, for fixed T, $b_{AR1(\rho)}$ is close to b_{EP} for ρ close to 1. If $T = 128$ and $\rho = 0.5$ to 0.7, then the weight on b_{LS} is in the range 0.95 to 0.90, so for these values, typical of annual global temperature data, $b_{AR1(\rho)}$ is essentially the same as b_{LS}.

For some series, the first order autoregressive model is not an adequate approximation. The more general pth order model may give a better fit and a corresponding trend estimate b_{ARp}. The relationship between b_{ARp} and b_{LS} is more complex than that between b_{AR1} and b_{LS} for $p > 1$, but in typical cases they are still similar. On the other hand, some series show long term dependence that is not well approximated by any autoregressive process of small enough order to be fitted successfully to the data. The fractionally integrated models (see Section 5) may give a better approximation in such cases, and trend estimates that differ more from b_{LS}.

4. Standard Errors for an Estimator Based on Correlated Observations

Each of the measures of rate of change from the previous section is a linear combination of $Y_1, Y_2, \ldots, Y_T$. That is, it may be written in the form

$$b = \sum_{t=1}^{T} b_t Y_t \tag{5}$$

for appropriate values of $\{b_t\}$. This form is convenient for expressing the standard error of the estimate in terms of the spectral density and this relationship will now be derived. Since X_t has an expected value of zero,

$$\begin{aligned}\text{Variance}(b) &= \sum_t \sum_u b_t b_u \text{Covariance}(Y_t, Y_u) \\ &= \sum_t \sum_u b_t b_u E(X_t X_u) \\ &= \sum_t \sum_u b_t b_u \gamma(t-u)\end{aligned}$$

and the standard error of b is the square root of this quantity. Now

$$\gamma(h) = \int_{-1/2}^{1/2} e^{2\pi i f h} s(f)\, df$$

and if this expression is substituted into the previous equation one obtains

$$\text{Variance}(b) = \int_{-1/2}^{1/2} w(f) s(f)\, df, \tag{6}$$

where

$$w(f) = \left| \sum_{t=1}^{T} b_t e^{-2\pi i t f} \right|^2 .$$

Finally because both s and w are the discrete Fourier transforms of real functions, both will be symmetric. Thus the integration at (6) can be reduced to:

$$\text{Variance}(b) = 2\int_{0}^{1/2} w(f) s(f)\, df \tag{7}$$

and this is the form that will be used in this article.

The variance of the trend estimate is therefore the integral of the product of two functions: a weight function, $w(f)$ that depends on the trend estimate being used and $s(f)$, the spectrum of the data, which does not depend on the particular trend estimate. Evidently, to compute the variance and hence the standard error, it is important to know $s(f)$ for frequencies where the product is large but less important where the product is small.

The weighting functions are graphed in Figure 2 for the three measures of rate of change discussed in the previous section, with a logarithmic frequency axis ($\rho = 0.5$ in the estimate b_{AR1}). The functions tend to have similar shapes with the curve associated with b_{AV} being approximately 4/3 the height of the other two. Also note that the functions for b_{LS} and b_{AR1} are nearly the same, as might be expected from the comments at the end of the previous section. In view of (7), for most choices of $s(f)$ the variances of b_{LS} and b_{AR1} will be very close to each other. For the same reason the variance of b_{AV} will tend to be larger than the other estimators. The ordinary least squares estimate of the trend has a smaller variance than b_{AV} and is easier to describe than b_{AR1}. For these reasons attention will be restricted to standard errors for b_{LS}.

The least squares estimator b_{LS} is optimal under the assumption of uncorrelated observations, and the usual (naïve) standard error reported for this estimator is based on this hypothesis. It should be emphasized that this standard error is invalid for data that are serially correlated. In Section 5 it will be shown that some reasonable models for the correlation of the temperature series imply standard errors for b_{LS} that are substantially larger than the one based on uncorrelated observations.

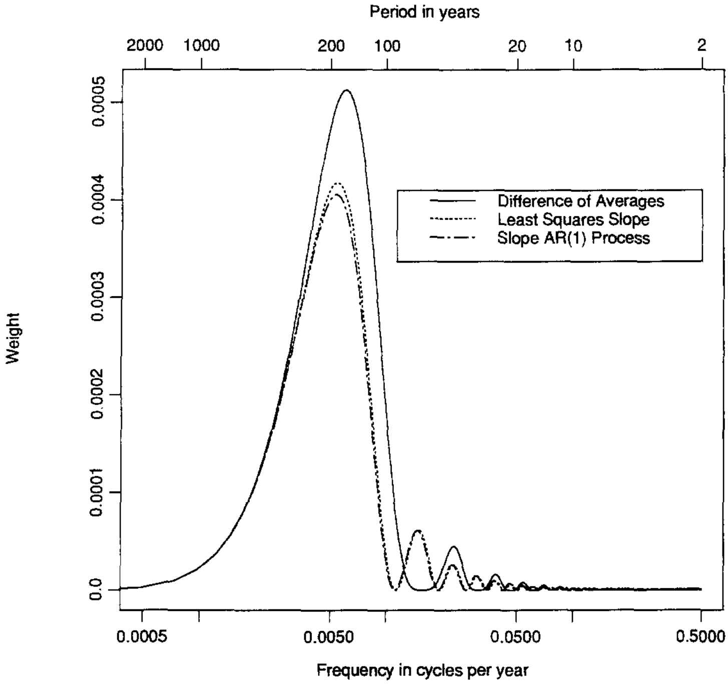


Fig. 2. Weight functions for three trend estimates based on a 128 year record.

5. Catalog of Spectra for Annual Global Temperature Series

The spectral density for global annual temperature is not known and must be estimated from the data. This section presents several different models for the correlation structure of this series and evaluates the resulting spectral densities. (The actual parameter estimates for these various spectra for the global temperature series are listed in the Appendix.) These estimated spectra are useful for investigating the sensitivity of the standard error to different degrees of correlation among the $\{X_t\}$ time series.

A useful class of models for correlated observations are *autoregressive processes.* Suppose that $\{e_t\}$ is a series of uncorrelated mean zero random variables all with variance σ^2. An autoregressive process of order p, $AR(p)$, satisfies

$$X_t = \sum_{j=1}^{p} \beta_j X_{t-j} + e_t, \tag{8}$$

where the parameters of this model are $\beta_1, \ldots, \beta_p$ and σ. The spectral density for this time series is

$$s(f) = \sigma^2 / |P(f)|^2 \tag{9}$$

where $P(f) = 1 - \Sigma_{j=1}^{p} \beta_j e^{2\pi ijf}$. Increasing the size of p in this model increases the span of the dependence between the current and past values of X_t.

The size of the low frequency components of the spectrum for autoregressive processes depends on the order p. Even so, for any choice p the spectral density at very low frequencies will be approximately equal to $\sigma^2/|P(0)|^2$. Since the standard errors will be sensitive to low frequency components of the spectral density a class of models was considered that does not force the spectral density to be bounded for small f. A *fractionally integrated white noise process* is defined as having a spectral density function:

$$s(f) = \sigma^2[2\sin(\pi f)]^{-\delta}.$$

Here $\delta < 1.0$ is required in order for s to have a finite integral. Note that as f approaches 0 this density will increase according to $f^{-\delta}$ and thus will not be bounded at $f = 0$. This process may be regarded as a simple way of modeling the $f^{-\delta}$ behavior observed in many climate spectra.

The final set of spectra considered were suggested by the one-dimensional temperature model of Wigley and Raper (1987). Here a box-upwelling-diffusion climate model is supplied with a white noise forcing function. Wigley and Raper (1990a, b) have given an analytic expression for the spectrum of the output for a one-box model. The functional form is given by

$$s(f) = S_Q \lambda^{-2} \left\{ \left(1 - \frac{\tau W}{2h} + \frac{\tau}{2h}\sqrt{\frac{v^2 + W^2}{2}}\right)^2 + \left(2\pi f \tau + \frac{\tau}{2h}\sqrt{\frac{v^2 - W^2}{2}}\right)^2 \right\}^{-1}.$$

The parameters are as follows.

$S_Q = 1$ is the variance of the white noise forcing, in $(\mathrm{Wm}^{-2})^2$.

λ is a feedback parameter with units $\mathrm{Wm}^{-2}/(^\circ\mathrm{C})$. In the model, λ^{-1} is the equilibrium temperature response to a constant 1 Wm^{-2} change in forcing. The value $(4.39\ \mathrm{Wm}^{-2})/(3\ ^\circ\mathrm{C}) = 1.4633\ \mathrm{Wm}^{-2}/(^\circ\mathrm{C})$, was used, implying a temperature rise of 3 °C for the 4.39 Wm^{-2} forcing associated with a doubling of atmospheric CO_2 concentration.

$\tau = 6.3122/\lambda$ is an average response time in years.

$W = 4\ \mathrm{m\ yr}^{-1}$ is the ocean upwelling rate.

$h = 70$ m is the depth of the mixed layer.

$v^2 = \sqrt{W^4 + (8\pi f K)^2}$ is (the square of) a characteristic velocity as a function of frequency.

$K = 1\ \mathrm{cm}^2\ \mathrm{s}^{-1} = 3155.76\ \mathrm{m}^2\ \mathrm{yr}^{-1}$ is the diffusivity of the ocean below the mixed layer.

Wigley and Raper (1990a, b) have also studied a more complicated model of this form with separate boxes for land and ocean in each hemisphere. We have estimated the spectrum of the output when this model is driven by white noise, by cross spectrum analysis of 4,096 years of input and output (Raper, personal communication).

All of these models contain parameters that have been estimated by fitting to the observed temperature data. In the case of the two Wigley and Raper spectra, only the magnitude of the white noise forcing was estimated in this way, by matching the high frequency spectrum to that of the temperature series. The statistical models were fitted by least squares or maximum likelihood (see the Appendix), simultaneously with a linear trend. The inclusion of the trend in the estimation has a similar effect to using detrended data, and necessarily leads to slight under-estimation of the power at the lowest frequencies. Leaving the trend in the data by omitting it from the fitted model gives estimated spectra with considerably more low frequency power, but gives somewhat poorer fits. Bloomfield (1991) has shown that the trend is significantly different from zero for all the noise models considered here. Thus even the long term dependence shown by the fractionally integrated models does not account for the magnitude of the observed trend.

The six spectral models for the stochastic structure in the annual temperature series are compared in Figure 3. To expand the axis near zero the abscissa is the logarithm of frequency. Although the spectra tend to have similar behavior for high frequencies their properties for lower frequencies differ dramatically. The

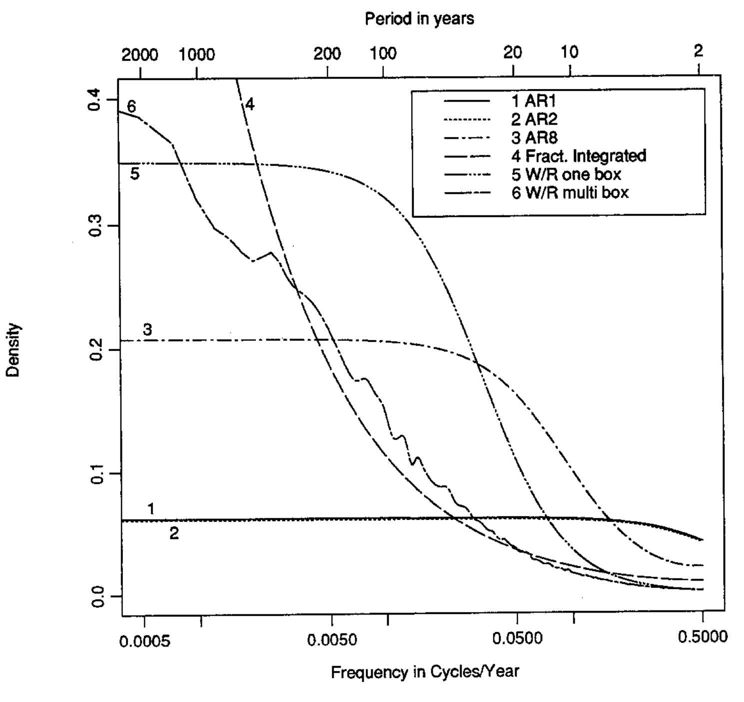


Fig. 3. Spectral densities for the global temperature series.

autoregressive models have spectra that plateau for low frequencies while the fractionally integrated model has a singularity at zero.

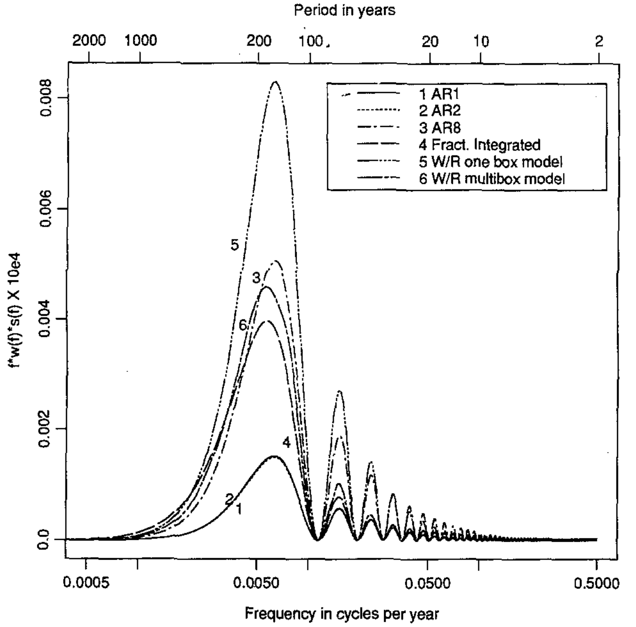
It should be noted that while the various spectra agree fairly well at high frequencies, they diverge at low frequencies. This is to be expected, as 128 years of data contain no information about fluctuations on a time scale of more than 128 years, and little for a time scale of more than say 50 years. The weighting functions of Figure 2 show however that the standard error of a trend estimate depends largely on the power spectrum at the low frequencies where the spectra diverge. We can expect therefore that the standard errors calculated for these various spectra will differ markedly. This emphasizes both the importance of careful model selection in obtaining valid standard errors, and the potential value of longer surrogate series of global temperatures.

The spectra shown in Figure 3 may be compared with other calculations of climatic spectra. Kutzbach and Bryson (1974) show spectra of a number of regional temperature series. Their spectra (displayed in the variance-conserving form of $f \times s(f)$ versus $\ln(f)$) are generally somewhat higher than those shown here, especially at the lowest frequencies. For longer time scales, Shackleton and Imbrie (1990) give δ^{18}O spectra for several deep-water cores and composites. The continuum part of their spectra (above the Milankovitch band) varies like f^{-2}, which is steeper than the behavior of any of these spectra. Shackleton and Imbrie's Figure 3 (which shows spectra in log-log form) shows a spectral density of around 10^{-2}(‰)^2/(k yr^{-1}) at $f = 0.2$ k yr^{-1} $= 2 \times 10^{-4}$ yr^{-1}. This extrapolates to 1.6×10^{-3}(‰)^2/(k yr^{-1}) at $f = 5 \times 10^{-4}$ yr^{-1}. Shackleton and Imbrie suggest that a 1 °C temperature change corresponds to a 0.25‰ change in δ^{18}O so this translates to a temperature spectrum of 2.56×10^{-2}(°C)^2/(k yr^{-1}) ≈ 25(°C)^2/(yr^{-1}). This is higher by orders of magnitude than these spectra, except for the fractionally integrated white noise spectrum which gives a density of around 5(°C)^2/(yr^{-1}) at this frequency.

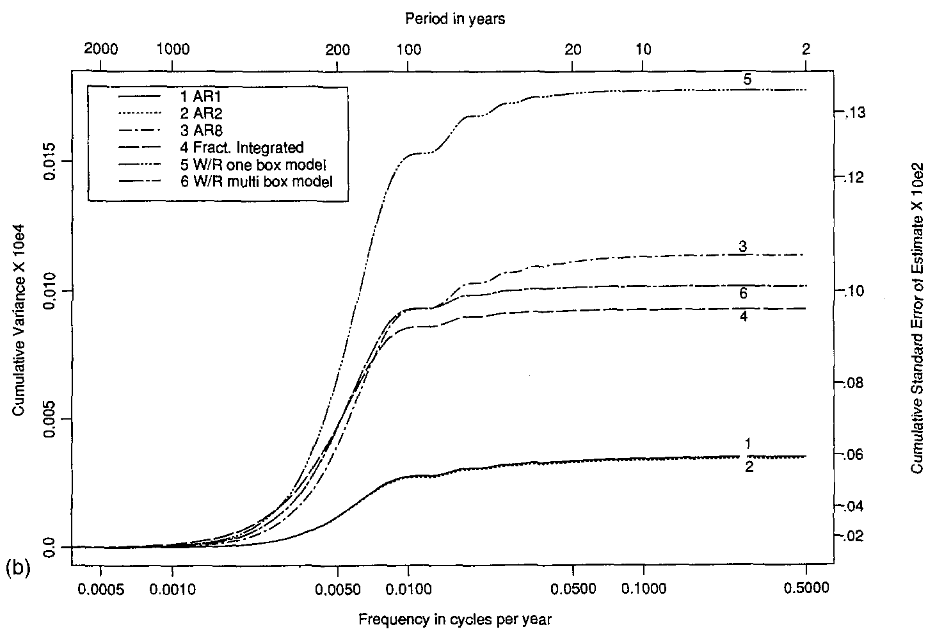
6. Sensitivity of the Standard Error

Based on the spectral representation for the variance of a trend estimate it is possible to show how different models for correlation in the temperature series influence the estimate's accuracy. As discussed in Section 3, the focus will be on the ordinary least squares estimate of a trend and the variance will be studied for the different spectra cataloged in Section 5.

Following the representation in (7), Figure 4a is a plot of the integrand $w(f) \times s(f)$ for the weight function corresponding to b_{LS} and the six spectra from Section 4. Note that the peak heights for these products are dramatically different depending on which spectral density model is considered. Also, it is interesting to see that the products for the $AR(8)$ process and the fractionally integrated process are close even though the spectra for these two series diverge sharply at very low frequencies. The reason for this similarity is that the multiplication by $w(f)$ effec-



(a)



(b)

Fig. 4. Variance functions for the least squares trend estimate. (a) Plotted are the products $f \times w(f) \times s(f)$ for the six spectral densities from Figure 3. Since the frequency axis is in a log scale the product $w(f) \times s(f)$ has been multiplied by f to preserve the areas under the plotted curves. Thus the total area under a particular curve may be interpreted as the variance of the least squares estimator for that spectrum. (b) Cumulative variance of the least squares trend estimate as a function of frequency. The functions from (a) have been integrated on a grid of 2048 equally spaced points.

tively restricts attention to a window of frequencies where the two spectra are similar. For instance, at periods of 1000 years or longer, the weighting function is always less than 10% of its maximum value.

To make some quantitative comparisons it is useful to consider the cumulative variance for each spectral model as a function of frequency:

$$V(f) = 2\int_0^f w(f')s(f')\,df' \tag{10}$$

and Figure 4b is plot of these functions. The value of this function at $f = 0.5$ cycles per year is the actual variance of b_{SS} under the hypothesis that $s(f)$ is (a good approximation to) the true spectrum of annual temperatures. For the spectra with larger variances, $V(f)$ tends to rise sharply over frequencies corresponding to periods of 50 to 500 years. This observation is another way of drawing attention to the concentration of $w(f)$ at this range of frequencies. In Figure 4b we see that the spectra that concentrate much of their power at low frequencies, such as the multibox model, tend to level off after frequencies above 0.01 cycles per year. The $AR(8)$ model, however, has a curve that continues to increase at higher frequencies. This is a reflection of the larger weight that this model puts on frequency components in the temperature series above 0.01 cycles per year.

When the cumulative variances are converted to standard errors (see the vertical axis on the right hand side of Figure 4b) we see that the resulting standard error of b_{LS} is sensitive to the choice of $s(f)$. For the temperature data plotted in Figure 1, $b_{LS} = 0.367$ °C/century, and a 95% confidence interval for this estimate based on an $AR(8)$ model is $b_{LS} \pm 1.96 \times 0.107$ °C/century.

The standard errors implied by all of these models for the three different trend estimates are listed in Table I. Also to serve as a reference, the naïve standard error assuming uncorrelated observations (white noise) has been included. This incorrect standard error corresponds to a constant spectral density at the level $\sigma^2 = 0.144$ and is smaller than the other standard errors. As expected the standard errors for b_{LS} and $b_{AR}(1)$ are very similar and both are smaller than that for b_{AV}. It should be

TABLE I: Sensitivity of the trend estimate standard errors to the spectrum

Model for the temperature series	Standard error of trend estimate in degrees C per century		
	b_{LS}	b_{AV}	b_{AR}
White noise	0.028	0.033	0.029
$AR(1)$	0.059	0.069	0.060
$AR(2)$	0.059	0.068	0.060
$AR(8)$	0.107	0.123	0.106
Fractional Int.	0.097	0.109	0.096
Wigley/Raper One Box	0.072	0.082	0.071
Wigley/Raper Multibox	0.101	0.115	0.099

noted that the standard error based on the Wigley/Raper spectrum is larger by a factor of 4.5 than the naïve one.

7. Conclusions

The variability of trend estimates is sensitive to the structure of the temperature series on a time scale in the range of 50 to 500 years. Thus in order to evaluate the observed trend in the temperature series it is necessary to understand the natural variability of global temperatures within this range. Also, these computations suggest that much longer term fluctuations in temperature will not have much impact on drawing inferences from the currently available temperature record. Although the three trend estimates have different standard errors, these differences are small relative to the effect of the choice of model for the temperature spectrum.

Despite the large increase in the standard error over a model that assumes uncorrelated annual temperatures, the standard errors are small in comparison to the estimated trend. If the spectra considered in this work are representative of the possible range of models that are reasonable for global annual temperatures, then these results indicate a significant trend in the temperatures. This trend would be difficult to explain solely from the natural variability suggested by the spectra for temperature.

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Appendix

This Appendix reports the parameter estimates for the temperature spectra from Figures 3 and 4 and in Table I.

Autoregressive Processes. Autoregressive parameters for the models used above are given in Table A.I. They were estimated for the Hansen and Lebedeff (1987, 1988) land-based global temperature record, using least squares by regressing the temperature series on a linear trend and the appropriate number of lagged versions of the series.

Fractionally Integrated Process. Parameters were found for the Hansen and Lebedeff record by maximum likelihood estimation of a linear trend plus fractionally integrated white noise. Estimates were $\delta = 0.72$ and $\sigma^2 = 0.0150$.

Wigley and Raper Parametric Spectrum. Parameter values are discussed in Section 5.

TABLE A.I: Autoregressive parameters

	Model		
	$AR(1)$	$AR(2)$	$AR(8)$
β_1	0.499	0.478	0.5090
β_2		0.014	−0.0980
β_3			0.0319
β_4			0.1307
β_5			−0.1063
β_6			0.3126
β_7			−0.2370
β_8			0.1940
σ^2	0.0155	0.0157	0.0143

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