Supplementary Material.
A Bayesian model for predicting the change in mortality associated with future ozone exposures under climate change

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August 11, 2014

1 Appendix A

1.1 Proof of exact integral solutions in Section 4.2

Lemma 1.

Suppose β is a scalar random variable and $\boldsymbol{X}$ is a random vector of length $N \times T$. Let β be Normally distributed, $\beta \sim N(\mu, \sigma^2)$, and assume $\boldsymbol{X}$ has some arbitrary multivariate distribution $F_{\boldsymbol{X}}$. Also assume that β and $\boldsymbol{X}$ are independent. Let $c_1, \ldots, c_N$ be constants.

Then define a new scalar random variable D such that $D = \sum_{i=1}^{N} \sum_{t=1}^{T} c_i \exp(\beta x_{it})$. It follows that

(i) the expectation of D is

$$E(D) = \int \sum_{t=1}^{T} \sum_{i=1}^{N} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x},$$

and

(ii) the variance of D is

$$\begin{aligned} Var(D) = & \int \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} c_i c_j \exp\left[\mu(x_{it} + x_{js}) + \sigma^2 (x_{it} + x_{js})^2/2\right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\ & - \left[\int \sum_{i=1}^{N} \sum_{t=1}^{T} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}\right]^2 \end{aligned}$$

Proof of Lemma 1.

Proof of (i).
Starting with the definition of expectation, we combine the exponential terms in D and in the probability distribution function for β. Then by completing the square and rearranging terms again, we isolate an integral of another Normal distribution wrt β with a different mean and variance. Integrating over the density of the pdf is 1, yielding our result.

$$
\begin{aligned}
&E(D) \\
&= \int\int D f_\beta(\beta) f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x} \\
&= \int\int \sum_{i=1}^{N}\sum_{t=1}^{T} c_i \exp\left(\beta x_{it}\right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-(\beta-\mu)^2/(2\sigma^2)\right\} f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x} \\
&= \int \sum_{i=1}^{N}\sum_{t=1}^{T} c_i \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-(\beta^2 - 2\beta\mu + \mu^2 - 2\sigma^2\beta x_{it})/(2\sigma^2)\right\} d\beta f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\
&= \int \sum_{i=1}^{N}\sum_{t=1}^{T} c_i \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big\{-(\beta^2 - 2\beta[\mu+\sigma^2 x_{it}] + \mu^2 + 2\sigma^2 x_{it}\mu + [\sigma^2 x_{it}]^2 - 2\sigma^2 x_{it}\mu - [\sigma^2 x_{it}]^2)/(2\sigma^2)\Big\} d\beta f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\
&= \int \sum_{i=1}^{N}\sum_{t=1}^{T} c_i \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-(\beta - [\mu+\sigma^2 x_{it}])^2/(2\sigma^2)\right\} \exp\left\{-(-2\sigma^2 x_{it}\mu - [\sigma^2 x_{it}]^2)/(2\sigma^2)\right\} d\beta f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\
&= \int \sum_{i=1}^{N}\sum_{t=1}^{T} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-(\beta - [\mu+\sigma^2 x_{it}])^2/(2\sigma^2)\right\} d\beta f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\
&= \int \sum_{i=1}^{N}\sum_{t=1}^{T} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}
\end{aligned}
$$

Proof of (ii).
First,

$$Var\,(D) = E\left(D^2\right) - E\,(D)^2$$

We have already derived an expression for $E\,[D]$ in part (i). Next, we derive an expression for $E\,[D^2]$. By repeatedly applying the identity $(\sum_i a_i)^2 = \sum_{i,j} a_i a_j$ for a sequence a_i, and following the steps in the proof of part (i) by completing the square and isolating an integral of another Normal distribution wrt β, we obtain,

$$\begin{aligned}
&E\left(D^2\right)\\
&= \int\int D^2 f_\beta(\beta) f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x}\\
&= \int\int \left\{\sum_{i=1}^{N}\sum_{t=1}^{T} c_i \exp\left(\beta x_{it}\right)\right\}^2 \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-(\beta-\mu)^2/(2\sigma^2)\right\} f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x}\\
&= \int\int \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} c_i \exp\left(\beta x_{it}\right) c_j \exp\left(\beta x_{js}\right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-(\beta-\mu)^2/(2\sigma^2)\right\} f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x}\\
&= \int\int \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} c_i c_j \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\{\beta^2 - 2\beta(\mu + \sigma^2 x_{it} + \sigma^2 x_{js}) + \mu^2\}/(2\sigma^2)\right] f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x}\\
&= \int\int \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} c_i c_j \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big[-\{\beta^2 - 2\beta(\mu + \sigma^2 x_{it} + \sigma^2 x_{js}) + (\mu + \sigma^2 x_{it} + \sigma^2 x_{js})^2 - (\mu + \sigma^2 x_{it} + \sigma^2 x_{js})^2 + \mu^2\}/(2\sigma^2)\Big] f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x}\\
&= \int\int \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} c_i c_j \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\{\beta - (\mu + \sigma^2 x_{it} + \sigma^2 x_{js})\}^2/(2\sigma^2)\right] \exp\left[-\left\{\mu^2 - (\mu + \sigma^2 x_{it} + \sigma^2 x_{js})^2\right\}/(2\sigma^2)\right] f_{\mathbf{X}}(\mathbf{x}) d\beta d\mathbf{x}\\
&= \int \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} c_i c_j \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\{\beta - (\mu + \sigma^2 x_{it} + \sigma^2 x_{js})\}^2/(2\sigma^2)\right] d\beta \exp\left[\mu(x_{it} + x_{js}) + \sigma^2(x_{it} + x_{js})^2/2\right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}\\
&= \int \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} c_i c_j \exp\left[\mu(x_{it} + x_{js}) + \sigma^2(x_{it} + x_{js})^2/2\right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}
\end{aligned}$$

Hence,

$$\begin{aligned}
Var\,(D) = &\int \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T} c_i c_j \exp\left[\mu(x_{it} + x_{js}) + \sigma^2(x_{it} + x_{js})^2/2\right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}\\
&- \left[\int \sum_{i=1}^{N}\sum_{t=1}^{T} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}\right]^2
\end{aligned}$$

Corollary 1.

Suppose β is a scalar random variable and $\boldsymbol{X}$ is a random vector of length $N \times T$. Let β be Normally distributed, $\beta \sim N(\mu, \sigma^2)$, and assume $\boldsymbol{X}$ and $\boldsymbol{W}$ each has some arbitrary multivariate distribution $F_{\boldsymbol{X}}$ and $F_{\boldsymbol{W}}$ respectively. Also assume that β, $\boldsymbol{X}$, $\boldsymbol{W}$ are mutually independent. Let $c_1, \ldots, c_N$ be constants.

Then define a new scalar random variables D, A such that $D = \sum_{i=1}^N \sum_{t=1}^T c_i \exp(\beta X_{it})$ and $A = \sum_{i=1}^N \sum_{t=1}^T c_i \exp(\beta W_{it})$. It follows that

(i) the expectation of $D - A$ is

$$E(D-A) = \int \sum_{t=1}^{T} \sum_{i=1}^{N} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} - \int \sum_{t=1}^{T} \sum_{i=1}^{N} c_i \exp\left\{w_{it}\mu + \sigma^2 w_{it}^2/2\right\} f_{\mathbf{W}}(\mathbf{w}) d\mathbf{w},$$

and

(ii) the variance of $D - A$ is

$$\begin{aligned}
Var\left(D-A\right) = & \int \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} c_i c_j \exp\left[\mu(x_{it} + x_{js}) + \sigma^2 (x_{it} + x_{js})^2/2\right] f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \\
& - \left[\int \sum_{i=1}^{N} \sum_{t=1}^{T} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}\right]^2 \\
& + \int \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} c_i c_j \exp\left[\mu(w_{it} + w_{js}) + \sigma^2 (w_{it} + w_{js})^2/2\right] f_{\mathbf{W}}(\mathbf{w}) d\mathbf{w} \\
& - \left[\int \sum_{i=1}^{N} \sum_{t=1}^{T} c_i \exp\left\{w_{it}\mu + \sigma^2 w_{it}^2/2\right\} f_{\mathbf{W}}(\mathbf{w}) d\mathbf{w}\right]^2 \\
& - 2\int \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} c_i c_j \exp\left[\mu(x_{it} + w_{js}) + \sigma^2 (x_{it} + w_{js})^2/2\right] f_{\mathbf{X}}(\mathbf{x}) f_{\mathbf{W}}(\mathbf{w}) d\mathbf{x} d\mathbf{w} \\
& + 2\left[\int \sum_{i=1}^{N} \sum_{t=1}^{T} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}\right] \left[\int \sum_{i=1}^{N} \sum_{t=1}^{T} c_i \exp\left\{w_{it}\mu + \sigma^2 w_{it}^2/2\right\} f_{\mathbf{W}}(\mathbf{w}) d\mathbf{w}\right]
\end{aligned}$$

Proof of Corollary 1.

Proof of (i).
By linearity of expectation and Lemma 1, we have our result.

$$\begin{aligned}
& E(D-A) \\
& = E(D) - E(A) \\
& = \int \sum_{t=1}^{T} \sum_{i=1}^{N} c_i \exp\left\{x_{it}\mu + \sigma^2 x_{it}^2/2\right\} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} - \int \sum_{t=1}^{T} \sum_{i=1}^{N} c_i \exp\left\{w_{it}\mu + \sigma^2 w_{it}^2/2\right\} f_{\mathbf{W}}(\mathbf{w}) d\mathbf{w}
\end{aligned}$$

Proof of (ii).
First,

$$Var\left(D-A\right)=Var\left(D\right)+Var\left(A\right)-2Cov(D,A)$$

From Lemma 1, we have expressions for $Var(D)$ and $Var(A)$. Next, we derive an expression for $E\left[DA\right]$ by following the same steps as the proof of Lemma 1 (ii).

$$
\begin{aligned}
&E\left(DA\right)\\
&=\int\int\int DAf_{\beta}(\beta)f_{\mathbf{X}}(\mathbf{x})d\beta d\mathbf{x}d\mathbf{w}\\
&=\int\int\left\{\sum_{i=1}^{N}\sum_{t=1}^{T}c_i\exp\left(\beta x_{it}\right)\right\}\left\{\sum_{j=1}^{N}\sum_{s=1}^{T}c_j\exp\left(\beta w_{js}\right)\right\}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-(\beta-\mu)^2/(2\sigma^2)\right\}f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\beta d\mathbf{x}d\mathbf{w}\\
&=\int\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_i\exp\left(\beta x_{it}\right)c_j\exp\left(\beta w_{js}\right)\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-(\beta-\mu)^2/(2\sigma^2)\right\}f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\beta d\mathbf{x}d\mathbf{w}\\
&=\int\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\{\beta^2-2\mu\beta+\mu^2-2\sigma^2\beta(x_{it}+w_{js})\}/(2\sigma^2)\right]f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\beta d\mathbf{x}d\mathbf{w}\\
&=\int\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\{\beta^2-2\beta(\mu+\sigma^2x_{it}+\sigma^2w_{js})+\mu^2\}/(2\sigma^2)\right]f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\beta d\mathbf{x}d\mathbf{w}\\
&=\int\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\frac{1}{\sqrt{2\pi\sigma^2}}\exp\Big[-\{\beta^2-2\beta(\mu+\sigma^2x_{it}+\sigma^2w_{js})+(\mu+\sigma^2x_{it}+\sigma^2w_{js})^2-(\mu+\sigma^2x_{it}+\sigma^2w_{js})^2+\mu^2\}/(2\sigma^2)\Big]f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\\
&=\int\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\{\beta-(\mu+\sigma^2x_{it}+\sigma^2w_{js})\}^2/(2\sigma^2)\right]\exp\left[-\left\{\mu^2-(\mu+\sigma^2x_{it}+\sigma^2w_{js})^2\right\}/(2\sigma^2)\right]f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\beta d\mathbf{x}d\mathbf{w}\\
&=\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\int\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\{\beta-(\mu+\sigma^2x_{it}+\sigma^2w_{js})\}^2/(2\sigma^2)\right]d\beta\exp\left[\mu(x_{it}+w_{js})+\sigma^2(x_{it}+w_{js})^2/2\right]f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\mathbf{x}d\mathbf{w}\\
&=\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\exp\left[\mu(x_{it}+w_{js})+\sigma^2(x_{it}+w_{js})^2/2\right]f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\mathbf{x}d\mathbf{w}
\end{aligned}
$$

Hence,

$$
\begin{aligned}
Var\left(D-A\right)=&\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\exp\left[\mu(x_{it}+x_{js})+\sigma^2(x_{it}+x_{js})^2/2\right]f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}\\
&-\left[\int\sum_{i=1}^{N}\sum_{t=1}^{T}c_i\exp\left\{x_{it}\mu+\sigma^2x_{it}^2/2\right\}f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}\right]^2\\
&+\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\exp\left[\mu(w_{it}+w_{js})+\sigma^2(w_{it}+w_{js})^2/2\right]f_{\mathbf{W}}(\mathbf{w})d\mathbf{w}\\
&-\left[\int\sum_{i=1}^{N}\sum_{t=1}^{T}c_i\exp\left\{w_{it}\mu+\sigma^2w_{it}^2/2\right\}f_{\mathbf{W}}(\mathbf{w})d\mathbf{w}\right]^2\\
&-2\int\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{T}c_ic_j\exp\left[\mu(x_{it}+w_{js})+\sigma^2(x_{it}+w_{js})^2/2\right]f_{\mathbf{X}}(\mathbf{x})f_{\mathbf{W}}(\mathbf{w})d\mathbf{x}d\mathbf{w}\\
&+2\left[\int\sum_{i=1}^{N}\sum_{t=1}^{T}c_i\exp\left\{x_{it}\mu+\sigma^2x_{it}^2/2\right\}f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}\right]\left[\int\sum_{i=1}^{N}\sum_{t=1}^{T}c_i\exp\left\{w_{it}\mu+\sigma^2w_{it}^2/2\right\}f_{\mathbf{W}}(\mathbf{w})d\mathbf{w}\right]
\end{aligned}
$$

2 Appendix B

Figure 1: *Ozone concentration projections at 12km resolution and thin plate spline interpolation of surface to 2km resolution, North Carolina.*

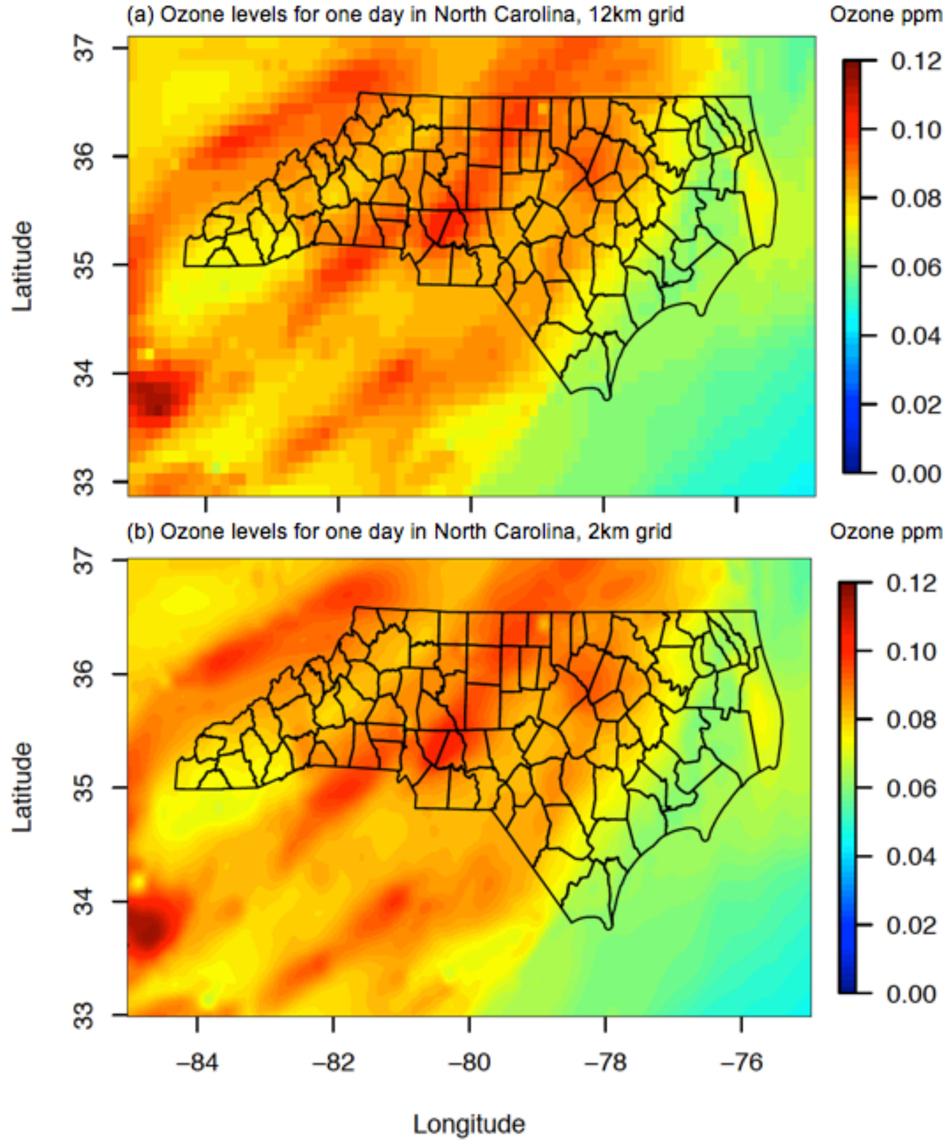


Table 1: Estimated change in ozone-related deaths in summertime and estimated percent change in total summertime mortality attributable to ozone in each state for Future A compared to the present

	Difference			Percent Change		
StateName	Mean	2.5%	97.5%	Mean	2.5%	97.5%
DELAWARE	2	-7	14	0.16	-0.45	0.90
DISTRICT OF COLUMBIA	3	-6	13	0.21	-0.35	0.83
ALABAMA	27	-45	98	0.27	-0.44	0.96
FLORIDA	37	-151	232	0.10	-0.41	0.63
GEORGIA	36	-69	143	0.26	-0.48	1.01
IDAHO	6	-1	13	0.30	-0.04	0.61
ILLINOIS	53	-98	198	0.23	-0.42	0.85
INDIANA	43	-29	126	0.35	-0.23	1.00
IOWA	21	-16	65	0.33	-0.26	1.02
KANSAS	16	-17	52	0.31	-0.32	0.97
KENTUCKY	27	-27	83	0.31	-0.31	0.95
LOUISIANA	17	-49	75	0.19	-0.53	0.80
MAINE	4	-8	17	0.14	-0.29	0.62
MARYLAND	19	-37	84	0.20	-0.37	0.84
MASSACHUSETTS	18	-58	104	0.14	-0.44	0.79
MICHIGAN	46	-74	169	0.24	-0.38	0.87
MINNESOTA	22	-10	68	0.26	-0.11	0.81
MISSISSIPPI	15	-30	56	0.24	-0.49	0.91
MISSOURI	36	-34	109	0.29	-0.27	0.87
MONTANA	4	-0	10	0.20	-0.02	0.54
NEBRASKA	11	-4	31	0.32	-0.13	0.93
NEVADA	11	3	23	0.33	0.08	0.65
NEW HAMPSHIRE	5	-5	18	0.21	-0.25	0.82
NEW JERSEY	21	-84	142	0.13	-0.50	0.85
NEW MEXICO	6	-4	19	0.20	-0.14	0.66
NEW YORK	33	-144	229	0.09	-0.40	0.65
NORTH CAROLINA	24	-75	135	0.15	-0.47	0.85
NORTH DAKOTA	3	-2	10	0.21	-0.13	0.73
OHIO	74	-71	249	0.31	-0.29	1.02
OKLAHOMA	23	-28	67	0.30	-0.36	0.87
OREGON	17	-9	46	0.25	-0.14	0.68
PENNSYLVANIA	72	-92	258	0.24	-0.31	0.86
ARIZONA	15	-13	42	0.17	-0.15	0.48
RHODE ISLAND	2	-15	21	0.09	-0.64	0.90
SOUTH CAROLINA	11	-50	73	0.14	-0.61	0.91
SOUTH DAKOTA	5	-1	12	0.29	-0.05	0.77
TENNESSEE	37	-41	116	0.29	-0.31	0.89
TEXAS	73	-152	266	0.22	-0.45	0.79
UTAH	9	2	17	0.33	0.09	0.62
VERMONT	3	-2	10	0.22	-0.20	0.84
VIRGINIA	26	-41	103	0.20	-0.33	0.82
WASHINGTON	26	-10	69	0.27	-0.10	0.69
ARKANSAS	16	-29	57	0.26	-0.47	0.91
WEST VIRGINIA	13	-16	44	0.27	-0.33	0.94
WISCONSIN	23	-35	85	0.22	-0.34	0.83
WYOMING	3	1	6	0.34	0.09	0.70
CALIFORNIA	166	12	404	0.32	0.02	0.78
COLORADO	23	8	46	0.36	0.13	0.73
CONNECTICUT	7	-37	56	0.10	-0.54	0.82

Table 2: Estimated change in ozone-related deaths in summertime and estimated percent change in total summertime mortality attributable to ozone in each state for Future B compared to the present

	Difference			Percent Change		
StateName	Mean	2.5%	97.5%	Mean	2.5%	97.5%
DELAWARE	-16	-29	-6	-1.07	-1.91	-0.38
DISTRICT OF COLUMBIA	-5	-14	2	-0.30	-0.88	0.15
ALABAMA	-99	-188	-44	-0.98	-1.84	-0.44
FLORIDA	-24	-186	133	-0.06	-0.50	0.36
GEORGIA	-128	-240	-46	-0.90	-1.67	-0.32
IDAHO	-5	-11	-1	-0.24	-0.50	-0.05
ILLINOIS	-48	-198	55	-0.21	-0.84	0.24
INDIANA	-82	-161	-25	-0.65	-1.26	-0.20
IOWA	-41	-81	-10	-0.64	-1.27	-0.15
KANSAS	-31	-68	-4	-0.58	-1.27	-0.07
KENTUCKY	-75	-136	-35	-0.86	-1.54	-0.40
LOUISIANA	-50	-117	-17	-0.54	-1.25	-0.18
MAINE	-22	-37	-10	-0.78	-1.32	-0.35
MARYLAND	-81	-156	-27	-0.81	-1.54	-0.28
MASSACHUSETTS	-75	-154	-18	-0.57	-1.15	-0.14
MICHIGAN	-67	-185	2	-0.35	-0.94	0.01
MINNESOTA	-15	-45	9	-0.18	-0.53	0.11
MISSISSIPPI	-56	-107	-22	-0.91	-1.74	-0.36
MISSOURI	-84	-165	-24	-0.67	-1.30	-0.20
MONTANA	-4	-8	0	-0.20	-0.43	0.01
NEBRASKA	-11	-27	2	-0.33	-0.82	0.07
NEVADA	-20	-33	-11	-0.59	-0.94	-0.31
NEW HAMPSHIRE	-21	-35	-9	-0.98	-1.61	-0.43
NEW JERSEY	-78	-185	9	-0.46	-1.09	0.05
NEW MEXICO	-4	-15	4	-0.16	-0.53	0.14
NEW YORK	-27	-187	136	-0.08	-0.52	0.38
NORTH CAROLINA	-159	-280	-67	-1.00	-1.75	-0.43
NORTH DAKOTA	-3	-8	2	-0.22	-0.57	0.12
OHIO	-147	-304	-48	-0.60	-1.24	-0.20
OKLAHOMA	-56	-114	-17	-0.73	-1.46	-0.23
OREGON	-7	-31	14	-0.11	-0.46	0.22
PENNSYLVANIA	-219	-424	-77	-0.73	-1.40	-0.26
ARIZONA	-32	-69	-10	-0.38	-0.79	-0.12
RHODE ISLAND	-17	-34	-4	-0.75	-1.45	-0.16
SOUTH CAROLINA	-75	-141	-25	-0.92	-1.72	-0.32
SOUTH DAKOTA	-4	-10	0	-0.28	-0.61	0.03
TENNESSEE	-121	-223	-56	-0.93	-1.69	-0.44
TEXAS	-64	-277	69	-0.19	-0.82	0.21
UTAH	-1	-7	5	-0.04	-0.25	0.17
VERMONT	-14	-24	-6	-1.20	-1.98	-0.56
VIRGINIA	-133	-229	-59	-1.05	-1.79	-0.47
WASHINGTON	-10	-44	17	-0.10	-0.44	0.17
ARKANSAS	-60	-113	-27	-0.95	-1.78	-0.43
WEST VIRGINIA	-58	-97	-32	-1.23	-2.02	-0.68
WISCONSIN	-54	-118	-12	-0.52	-1.14	-0.12
WYOMING	-2	-4	-0	-0.22	-0.46	-0.01
CALIFORNIA	-94	-263	88	-0.18	-0.51	0.17
COLORADO	-2	-13	14	-0.03	-0.21	0.22
CONNECTICUT	-43	-87	-6	-0.63	-1.27	-0.09

Figure 2: *Percent change in mortality for each state in the US using our Monte Carlo approach versus the double averaging approach, future A.*

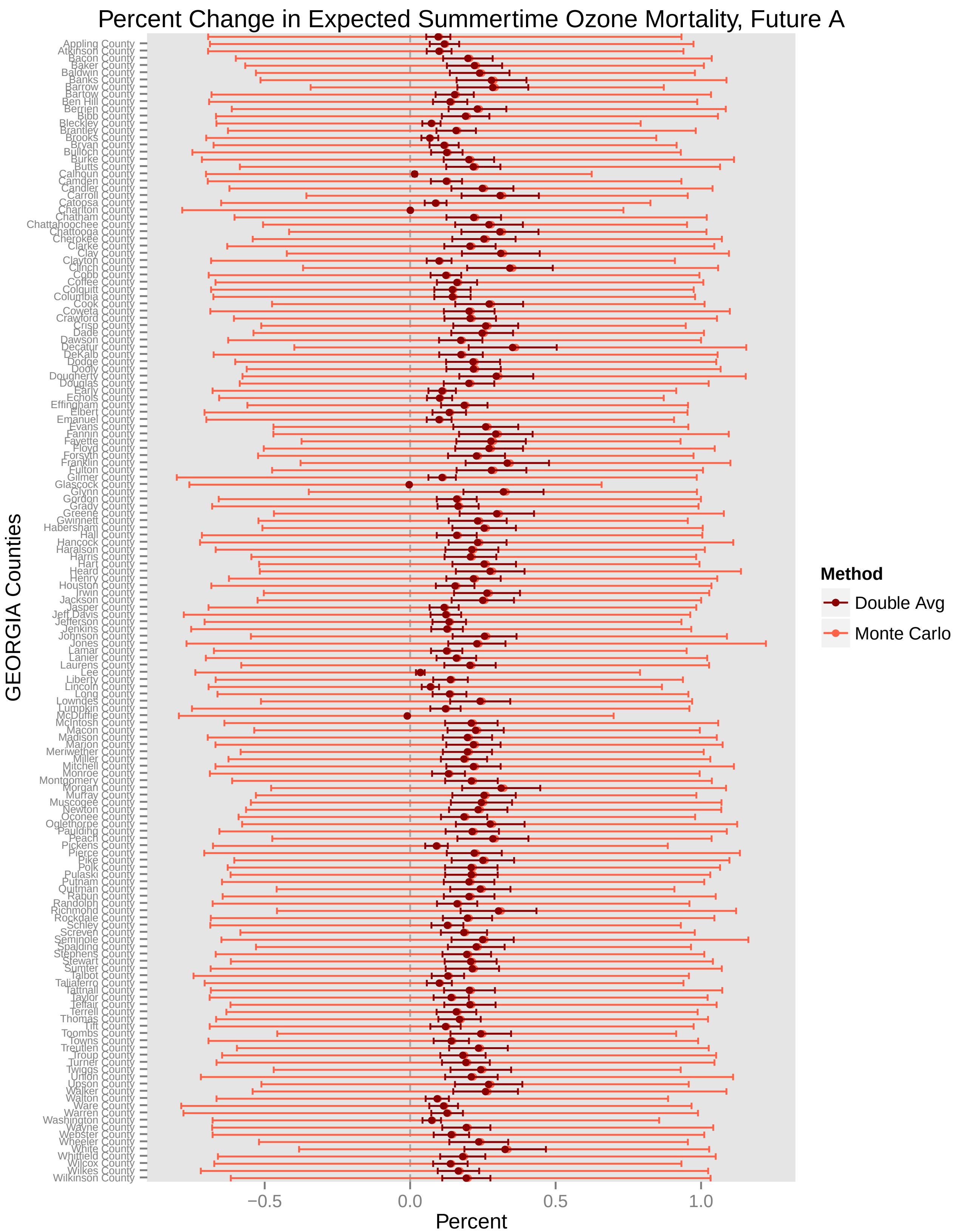


Figure 3: *Percent change in mortality for each state in the US using our Monte Carlo approach versus the double averaging approach, future B.*

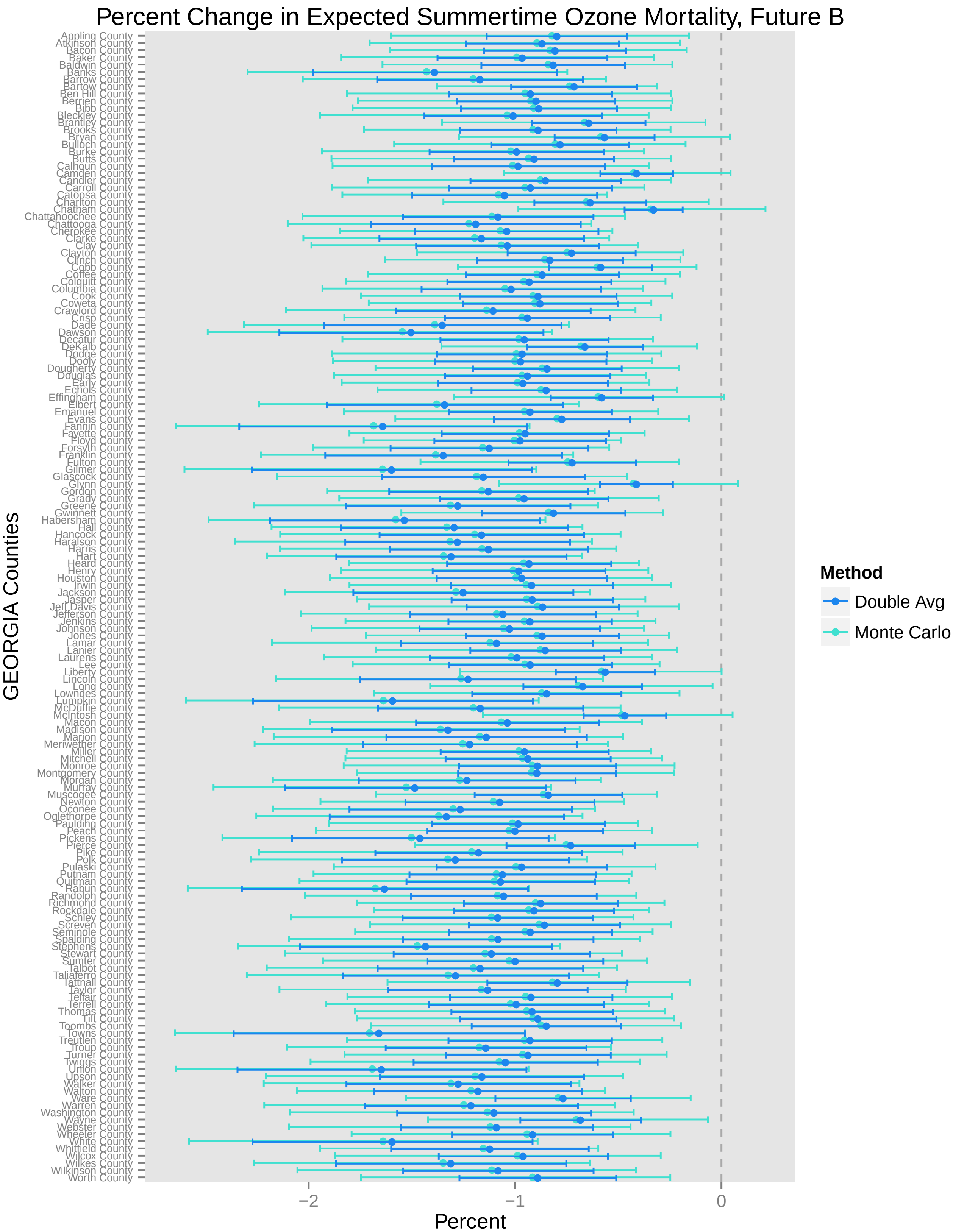
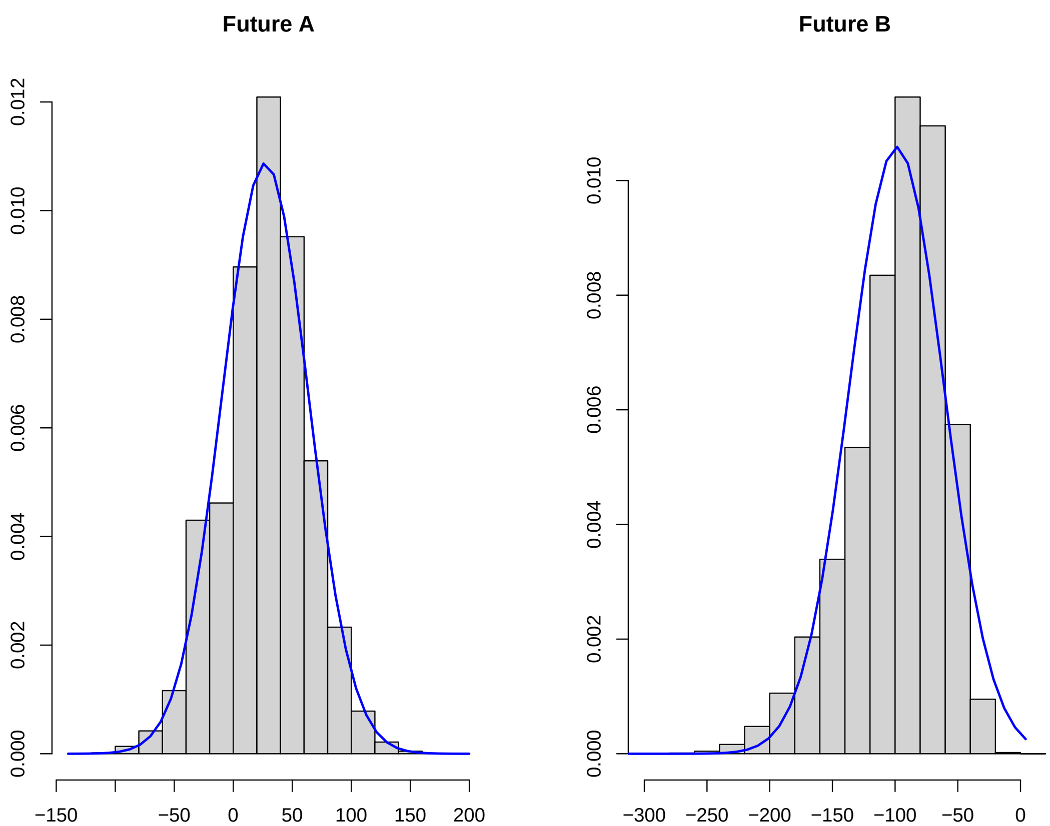


Figure 4: *Histogram of the Monte Carlo samples of ozone mortality over β and ozone, versus the Normal distribution using the mean and variance solution using the exact integral with respect to β.*



3 Appendix C

```
# Ozone and Mortality - US
# Stacey Alexeeff

# ------------------------------------------------------------- #
# Datasets used:
# Ozone simulations:
#    ~/work/Project1/O3data/daily/O3data_state##_county_YYYY_v04.nc
#    for YYYY 1996-2008, 2046-2058, 2046a-2058a
#    for ## state codes (FIPS)
# Population (US Census 2000):
#    ~/work/Project1/Censusdata/USCensus2000_PopulationData_state##_County.csv
#    for ## state codes (FIPS)
#
# Program Notes:
#     "state1" variable read in from UNIX by echo command as part of batch job

# ------------------------------------------------------------- #
library( ncdf4)
library( fields)
library( maps)

# ------------------------------------------------------------- #
# settings
 K=100
 M=13

# ------------------------------------------------------------- #
# Beta distribution from meta-analysis
# Bell et al. 2005

 # ###  US only resuls  ### #
 # 10-ppb increase in daily 24hr avg ozone associated with
 # Pooled Log-relative rate, total mortality: 0.84%
 # 95% posterior interval: 0.48% to 1.20%
 # Divide by 10 to get per 1-ppb increase
 # Divide by 100 to get beta back from percent change
 # parameters for 1-ppb increase:
  mu_beta = 0.00084          # US only results
 sigma_beta = 0.000183      # US only results
# ------------------------------------------------------------- #
# Census Data

# Population
  CensusData <- read.csv(paste("USCensus2000_PopulationData_state",state1,"_County.csv",sep="")

 # Number of counties
```

```
 Ncounty = nrow(CensusData)

# -------------------------------------------------------------- #
# CDC Data
# Daily Mortality Rate for state

source(paste("~/work/Project1/CDCdata/MortalityRate_state",state1,".r",sep=""))

# -------------------------------------------------------------- #
# Sampling - Future "a" vs. Present, Future vs. Present

# initialize
deaths.summer.diff.future <- matrix(nrow=M*M*K,ncol=Ncounty)
deaths.summer.diff.futurea <- matrix(nrow=M*M*K,ncol=Ncounty)
deaths.summer.pres <- matrix(nrow=M*M*K,ncol=Ncounty)
fun1 <- function(x,b){ sum( exp(b*x) ) }

# loop over samples of beta
for(k in 1:K){
   beta.k = rnorm(1, mean=mu_beta, sd=sigma_beta)

   # Present ozone data, loop over years
   for(m1 in 1:M){
      year.m1 = (1996:2008)[m1]
      fileName.O3data.m1 <- paste("~/work/Project1/O3data/daily/O3data_state",state1,"_county_
      NETCDF.O3data.m1 <- nc_open(fileName.O3data.m1)
      # present summer ozone, ppm, all counties
      O3_avg24hr_county.m1 <- array(NA, dim=c(Ncounty,92) )
      O3_avg24hr_county.m1[,] <- 1000*ncvar_get(NETCDF.O3data.m1, "O3_avg24hr")
      nc_close(NETCDF.O3data.m1)

   # Future, Future "a" ozone data, loop over years
   for(m2 in 1:M){
      year.m2 = (2046:2058)[m2]
      mmk = M*M*(k-1) + M*(m1-1)+m2

      # Future "a"
      fileName.O3data.m2.futurea <- paste("~/work/Project1/O3data/daily/O3data_state",state1,"
      NETCDF.O3data.m2.futurea <- nc_open(fileName.O3data.m2.futurea)
      # future summer ozone, ppm, all counties
      O3_avg24hr_county.m2.futurea <- array(NA, dim=c(Ncounty,92) )
      O3_avg24hr_county.m2.futurea[,] <- 1000*ncvar_get(NETCDF.O3data.m2.futurea, "O3_avg24hr"
      nc_close(NETCDF.O3data.m2.futurea)
      # compute deaths for each county
      # difference: future - present
      deaths.summer.diff.futurea[mmk,] <- MR*CensusData$POP100*( apply( O3_avg24hr_county.m2.f
                                          -  apply( O3_avg24hr_county.m1, 1, fun1, beta.k ) )
```

```
      # Future
      fileName.O3data.m2.future <- paste("~/work/Project1/O3data/daily/O3data_state",state1,"_c
      NETCDF.O3data.m2.future <- nc_open(fileName.O3data.m2.future)
      # future summer ozone, ppm, all counties
      O3_avg24hr_county.m2.future <- array(NA, dim=c(Ncounty,92) )
      O3_avg24hr_county.m2.future[,] <- 1000*ncvar_get(NETCDF.O3data.m2.future, "O3_avg24hr")
      nc_close(NETCDF.O3data.m2.future)
      # compute deaths for each county
      # difference: future - present
      deaths.summer.diff.future[mmk,] <- MR*CensusData$POP100*( apply( O3_avg24hr_county.m2.fu
                                       -  apply( O3_avg24hr_county.m1, 1, fun1, beta.k ) )

      # Present only (to compute percents later)
      deaths.summer.pres[mmk,] <- MR*CensusData$POP100*( apply( O3_avg24hr_county.m1, 1, fun1,
   }
   }
}


# -------------------------------------------------------------- #
# Save samples

write.csv(deaths.summer.diff.futurea, paste("Results_Ozone_Mortality_Diff_Futurea_state",state
write.csv(deaths.summer.diff.future, paste("Results_Ozone_Mortality_Diff_Future_state",state1,"
write.csv(deaths.summer.pres, paste("Results_Ozone_Mortality_Present_state",state1,"_County.csv
```