Predicting the saturated hydraulic conductivity of sand and gravel using effective diameter and void ratio

Robert P. Chapuis

Abstract: This paper assesses methods to predict the saturated hydraulic conductivity, *k*, of clean sand and gravel. Currently, in engineering, the most widely used predictive methods are those of Hazen and the Naval Facilities Engineering Command (NAVFAC). This paper shows how the Hazen equation, which is valid only for loose packing when the porosity, *n*, is close to its maximum value, can be extended to any value of *n* the soil can take when its maximum value of *n* is known. The resulting extended Hazen equation is compared with the single equation that summarizes the NAVFAC chart. The predictive capacity of the two equations is assessed using published laboratory data for homogenized sand and gravel specimens, with an effective diameter d_{10} between 0.13 and 1.98 mm and a void ratio *e* between 0.4 and 1.5. A new equation is proposed, based on a best fit equation in a graph of the logarithm of measured *k* versus the logarithm of $d_{10}^2 e^3/(1 + e)$. The distribution curves of the differences “log(measured *k*) – log(predicted *k*)” have mean values of –0.07, –0.21, and 0.00 for the extended Hazen, NAVFAC, and new equations, respectively, with standard deviations of 0.23, 0.36, and 0.10, respectively. Using the values of d_{10} and *e*, the new equation predicts a *k* value usually between 0.5 and 2.0 times the measured *k* value for the considered data. It is shown that the predictive capacity of this new equation may be extended to natural nonplastic silty soils, but not to crushed soils or plastic silty soils. The paper discusses several factors affecting the inaccuracy of predictions and laboratory test results.

Key words: permeability, sand, prediction, porosity, gradation curve.

Résumé : Cet article examine les méthodes de prédiction de la conductivité hydraulique saturée *k* du sable et du gravier propre. Actuellement, les méthodes les plus largement utilisées en génie sont celles de Hazen et de NAVFAC (« Naval Facilities Engineering Command »). Cet article montre comment l’équation de Hazen, qui n’est valable qu’à l’état lâche, quand la porosité *n* est proche de sa valeur maximale, peut être étendue à n’importe quelle valeur de *n* que peut prendre le sol quand la valeur maximale de *n* est connue. L’équation étendue de Hazen qui en résulte est comparée à l’équation unique qui résume l’abaque de NAVFAC. La capacité prédictive des deux équations est évaluée à l’aide de plusieurs données de laboratoire publiées pour des spécimens homogénéisés de sable et gravier, de diamètre effectif d_{10} entre 0,13 et 1,98 mm et d’indice des vides *e* entre 0,4 et 1,5. Une nouvelle équation est proposée, tirée du meilleur ajustement dans un graphe du log de *k* mesurée versus le log de $d_{10}^2 e^3/(1 + e)$. Pour les trois équations, Hazen étendue, NAVFAC et nouvelle, les courbes de distribution des différences “log(*k* mesurée) – log(*k* prédite)” ont des moyennes de –0,07, –0,21 et 0,00 alors que les écarts types sont 0,23, 0,36 et 0,10. Utilisant les valeurs de d_{10} et de *e*, la nouvelle équation prédit une valeur *k* habituellement entre 0,5 et 2,0 fois la valeur *k* mesurée pour les données considérées. On montre que sa capacité prédictive peut être étendue aux sols silteux naturels non-plastiques, mais pas aux sols concassés ni aux sols silteux plastiques. L’article discute différents facteurs affectant l’imprécision des prédictions et des essais de laboratoire.

Mots clés : perméabilité, sable, prédiction, porosité, granulométrie.

Introduction

Since Seelheim (1880) wrote that permeability should be related to the squared value of some characteristic pore diameter, many equations have been proposed to predict the saturated hydraulic conductivity, *k*, of porous materials. According to several publications (Scheidegger 1953, 1954, 1974; Bear 1972; Vuković and Soro 1992; Mbonimpa et al. 2002; Aubertin et al. 2003; Chapuis and Aubertin 2003), *k* can be predicted using empirical relationships, capillary models, statistical models, and hydraulic radius theories.

The scope of this paper is limited to the prediction of the saturated hydraulic conductivity, *k*, of natural clean sand and gravel. Currently, the equation of Hazen (1911) and the Naval Facilities Engineering Command (NAVFAC 1974) chart are routinely used by engineers for such predictions, are easier to use than the more general Kozeny–Carman equation, and were retained here. The Hazen equation is only valid for

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loose packing conditions when the porosity, n, is close to its maximum value. The paper shows how the equation can be combined with the Kozeny–Carman equation (Chapuis and Aubertin 2003) and extended to any value of n the soil can take between its minimum and maximum values. The NAVFAC chart predicts k as a function of the effective diameter, d_{10}, and the void ratio, e, of the soil. It is shown here that this chart can be described by a single equation. The capacity of the extended Hazen and NAVFAC equations is assessed by comparing their predictions with published experimental k values of sand and gravel specimens that were tested in rigid-wall permeameters. Lastly, a better predictive equation based on laboratory experimental data is proposed.

Equation of Hazen

Usual equation

The k of clean sand or gravel is frequently estimated with the Hazen (1911) equation, which requires three conditions: loose compactness (n or e close to their maximum values), coefficient of uniformity C_U less than 5, and diameter d_{10} between 0.10 and 3.0 mm. According to textbooks (e.g., Terzaghi and Peck 1948; Lambe and Whitman 1969; Freeze and Cherry 1979) that refer to the Hazen equation (1911), it is usually written

$$[1] \qquad k\ (\mathrm{cm/s}) = d_{10}^2\ (\mathrm{mm}^2)$$

Original equation

The empirical (it does not satisfy units requirements) eq. [1] is not the genuine equation of Hazen (1892, p. 553). The true Hazen equation is

$$[2] \qquad v\ (\mathrm{Darcy}) = cd^2(h/L)(0.70 + 0.03t)$$

where v (Darcy) is the Darcy (1856) velocity expressed in metres per day, h is the hydraulic head loss along the distance L (h and L have the same units), d is d_{10} in millimetres, t is the water temperature in Celsius degrees, and c is a constant close to 1000 in this system of units. Thus, with respect to units,

$$[3] \qquad k = (\mathrm{m/d}) = 1000(d/1\ \mathrm{mm})^2(0.70 + 0.03t)$$

becomes

$$[4] \qquad k(\mathrm{cm/s}) = 1.157\left(\frac{d_{10}}{1\ \mathrm{mm}}\right)^2\left[0.70 + 0.03\left(\frac{t}{1\ °\mathrm{C}}\right)\right]$$

Then, the usual eq. [1] corresponds to t = 5.5 °C in the original eq. [2]. At 20 °C, for usual laboratory conditions,

$$[5] \qquad k(20\ °\mathrm{C},\ e_{\max},\ \mathrm{cm/s}) = 1.50\,d_{10}^2\ (\mathrm{mm}^2)$$

and in field conditions, for example at 10 °C,

$$[6] \qquad k(10\ °\mathrm{C},\ e_{\max},\ \mathrm{cm/s}) = 1.16\,d_{10}^2\ (\mathrm{mm}^2)$$

where $e_{\max}$ is the maximum void ratio.

Extended equation

It is proposed here to extend the Hazen equation to specimens of porosity n lower than the maximum value. The porosity n and the void ratio e are related by $e = n/(1 - n)$ or $n = e/(1 + e)$. A common relationship between k, specific surface S, and e is the Kozeny–Carman equation (e.g., Taylor 1948):

$$[7] \qquad k(e) = Ce^3/(1+e)S^2 = Cn^3/(1-n)^2S^2$$

where the constant C depends on the fluid properties and the geometry of the pore channels. According to Vuković and Soro (1992), who examined the capacities of 10 predictive formulas, the function of porosity, in the expression of hydraulic conductivity, is most realistically expressed by eq. [7]. In addition, eq. [7] relates $k(e)$ to $k(e_{\max})$ by a simple proportionality:

$$[8] \qquad k(e)/k(e_{\max}) = e^3(1+e_{\max})/[e_{\max}^3(1+e)]$$

The original Hazen equation was developed for a loose condition when the void ratio e is close to its maximum value, $e_{\max}$. Thus, from eq. [5],

$$[9] \qquad k(\mathrm{Hazen},\ e_{\max},\ 20\ °\mathrm{C}) = 1.50d_{10}^2 = Ce_{\max}^3/(1+e_{\max})S^2$$

and, by combining eqs. [8] and [9], then

$$[10] \qquad k(20\ °\mathrm{C},\ e) = 1.50\,d_{10}^2\,e^3(1+e_{\max})/[e_{\max}^3(1+e)]$$

which may be viewed as either an extended Hazen equation or a specific Kozeny–Carman equation for sand and gravel, where k is in centimetres per second and d_{10} is in millimetres.

Equation [10] requires knowing the value of $e_{\max}$ for the considered sand or gravel. This value may be determined experimentally using the American Society for Testing and Materials (ASTM) standard D4254 (ASTM 2002*a*), but frequently it is unknown. The value of $e_{\max}$ is in the range of 0.7–0.8, however, for most specimens of uniform natural sand and gravel.

NAVFAC chart

The Naval Facilities Engineering Command design manual DM7 (NAVFAC 1974) proposes a chart to estimate the saturated k value of clean sand and gravel as a function of e and d_{10}. According to the chart (sketched in Fig. 1) with the usual eq. [1] (not the original Hazen equation), there is a linear relationship between $\log(k)$ and $\log(d_{10})$ at a given e ($0.3 < e < 0.7$) for granular soils, respecting $2 < C_U < 12$, $d_{10}/d_5 < 1.4$, and $0.10 < d_{10} < 2.0$ mm, where d_5 (mm) is the 5% passing size. This may be expressed as

$$[11] \qquad k(e)/k_0(e) = [d_{10}/1\ \mathrm{mm}]^{A(e)}$$

where $k_0(e)$ and $A(e)$ are two functions of the void ratio, and d_{10} is expressed in millimetres. In eq. [11], only the nondimensional ratio k/k_0 is considered, regardless of the unit, i.e., m/s, cm/s, or ft/min.

Chapuis et al. (1989*b*) analyzed the chart to obtain the values of the coefficient $k_0(e)$ and the exponent $A(e)$. Two points were selected on each straight line (e = constant) of the chart (NAVFAC 1974) to calculate the exponent $A(e)$ and the coefficient $k_0(e)$. They appeared to be nonlinear functions of e. Their logarithms, however, were linearly related to the void ratio e through the constants a_0, a_1, b_0, and b_1:

Fig. 1. Simplified sketch of the NAVFAC (1974) chart that contains the usual eq. [1] attributed to Hazen (1911). For the original Hazen equation, see eqs. [2]–[6].

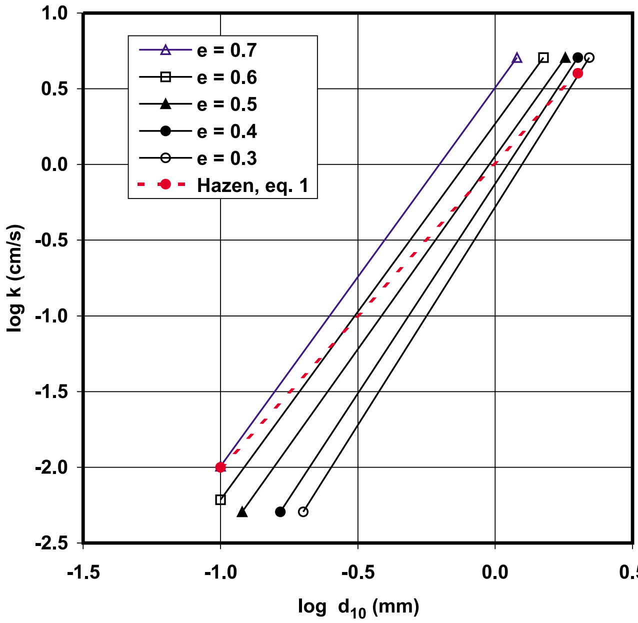
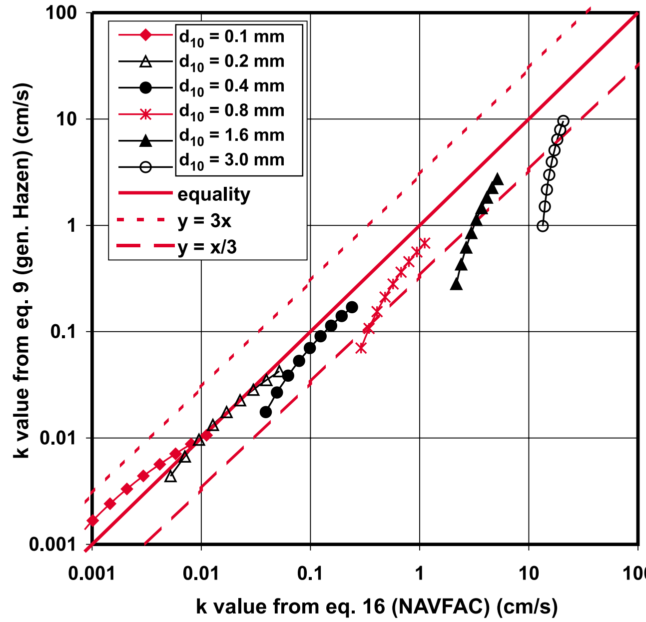


Fig. 2. Comparing the predictions of eq. [9] versus those of eq. [16] for hypothetical soils with a d_{10} of 0.1, 0.2, 0.4, 0.8, 1.6, and 3.0 mm and a void ratio e between 0.3 and 0.7.



[12] $\log A = a_0 e + a_1$

[13] $\log k_0 = b_0 e + b_1$

The coefficients of correlation were –0.9995 and 0.9994, respectively, for the curve-fitting technique that gave

[14] $A = 10^{-0.2937e+0.5504}$

[15] $k_0 = 10^{1.2921e-0.6435}$

As a result, it was found that

[16] $k(\mathrm{cm/s}) = 10^{1.291e-0.6435}[d_{10}(\mathrm{mm})]^{10^{0.5504-0.2937e}}$

which is valid for parameters respecting the four conditions of NAVFAC (1974): $0.3 < e < 0.7$, $0.10 < d_{10} < 2.0$ mm, $2 < C_U < 12$, and $d_{10}/d_5 < 1.4$.

For those interested in using eq. [16] in a spreadsheet, it is suggested that steps be taken to calculate the successive power of power functions to avoid erroneous calculations. A few k values as calculated by eq. [16] may be compared with those obtained by reading the chart. For example, when e = 0.3 and d_{10} = 0.2 mm, eq. [16] gives $k = 5.22 \times 10^{-3}$ cm/s, whereas the chart gives k = 0.01 fpm (ft per minute) = 5.08×10^{-3} cm/s; the difference is 2.7%. When e = 0.7 and d_{10} = 1.5 mm, eq. [16] gives k = 4.47 cm/s, whereas the chart gives k = 8.7 fpm = 4.42 cm/s; the difference is 1.2%. The difference between eq. [16] and data interpolated from the chart is usually less than 2%.

Comparison of predictive eqs. [9] and [16]

The k of sand and gravel may be predicted by functions of e and d_{10} using either the extended Hazen equation (eq. [9]) or the NAVFAC chart expressed as eq. [16]. The two equations are compared and then tested against laboratory permeability test data.

For the first comparison, hypothetical soils respecting the NAVFAC conditions were considered. They have a d_{10} of 0.1, 0.2, 0.4, 0.8, 1.6, and 3.0 mm and a void ratio e between 0.7 and 0.3 that are the limits of the chart. Their k values were given by eq. [16], which represents the full chart, and compared with the k values predicted by eq. [9] (extended Hazen) assuming e_{max} = 0.8.

The results of this comparison are shown in Figs. 2 and 3. It appears that eqs. [9] and [16] provide similar values (ratio between 0.6 and 2) for d_{10} between 0.1 and 0.4 mm, but they differ by a factor of 1.6–4.0 for d_{10} of 0.8 mm and a factor of up to 10 for d_{10} between 1.6 and 3.0 mm.

The results of this comparison can be viewed in light of the warning that is given in NAVFAC (1974) about its chart: "… two-thirds of the random values fall between three times the average value and one-third of the average value shown." Figures 2 and 3 confirm this warning but indicate also that the discrepancy between eqs. [9] and [16] increases with an increase in d_{10} due to their different mathematical expressions.

Using laboratory test data to assess eqs. [9] and [16]

The predictive equations must be assessed using homogenized specimens tested in controlled laboratory conditions. They cannot be assessed using field data for which nonhomogeneity is a major problem and representative sampling is another major problem. About 100 published laboratory test data (for vertical rigid-wall permeameters) were used to evaluate the capacity of eqs. [9] and [16] to predict k. The publications gave all required information, i.e., void ratio and the complete grain-size curve that respected both Hazen and NAVFAC conditions. All these tests measured the head loss directly within the specimen by using lateral piezo-

Fig. 3. Ratio of predicted values of k [k(eq. [9])/k(eq. [16])] versus k(eq. [9]) for the hypothetical soils of Fig. 2.

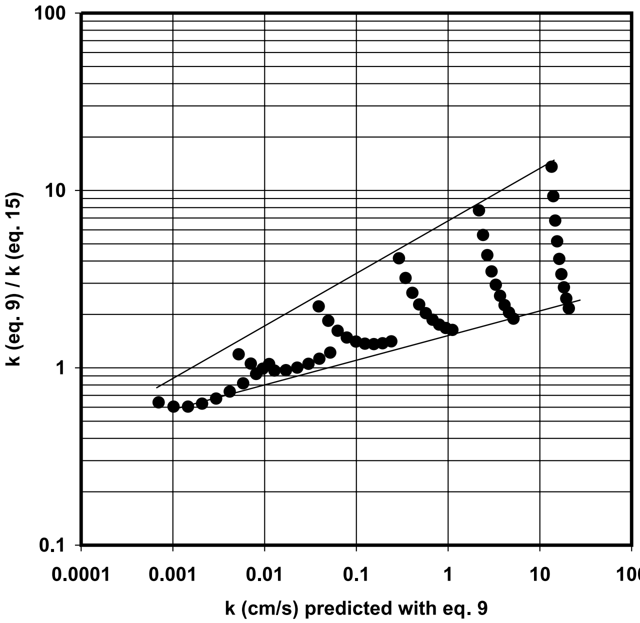
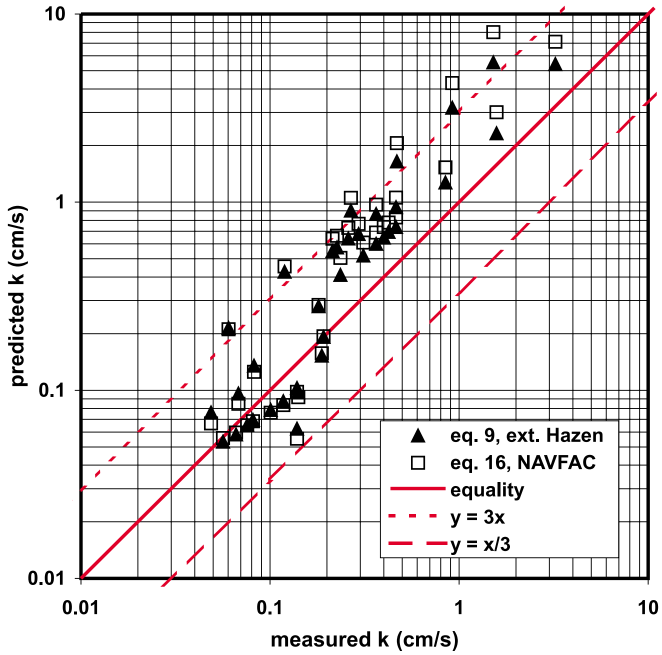


Fig. 4. Predicted versus measured k values for the tests of Mavis and Wilsey (1937).



meters to avoid errors due to head losses in pipes, valves, and porous stones or screens when the head loss is measured outside the permeameter. The importance of other features relative to the testing equipment and procedure are discussed at the end of the paper.

The test data were published by Mavis and Wilsey (1937) for specimens tested at different void ratios, Krumbein and Monk (1942) for 30 uniform soils obtained by sieving, Loudon (1952) for four types of glass beads and three samples of uniform crushed glass tested at different void ratios, and Chapuis et al. (1989*b*) for a sand tested horizontally (then $k = k_h$) and vertically (then $k = k_v$) at different e values.

Sand specimens tested by Mavis and Wilsey (1937)

The sand specimens of Mavis and Wilsey (1937) were tested at small gradients in a rigid-wall permeameter, 15 cm in diameter and 90 cm long, equipped with lateral piezometers. The tested Ottawa sand and uniform Iowa sand had grains between 0.68 and 0.93 mm. The pit run Iowa River sand had a d_{10} of 0.37 mm and a C_U of 3.2. The screened uniform Iowa sand specimens had d_{10} values of 0.39, 0.55, 0.80, 1.08, 1.50, and 1.98 mm. The screened non-uniform Iowa sand specimens had d_{10} values between 0.32 and 1.80 mm and C_U values between 1.8 and 6.0. Most predicted k values for these sand specimens are close to the measured k values (Fig. 4), usually between one third ($y = x/3$) and three ($y = 3x$) times. The predictive capacity of eqs. [9] and [16] is usually better at lower k than at higher k for this group of tests.

Sand specimens tested by Krumbein and Monk (1942)

Thirty specimens, numbered a–j and 1–20 by Krumbein and Monk (1942), were obtained by sieving glacial outwash sand. Their d_{10} value was between 0.27 and 1.40 mm, and their C_U was less than 2.8. They were tested at small gradients in a rigid-wall permeameter, 4.4 cm in diameter and 30 cm long, equipped with lateral piezometers. All specimens were tested at a porosity n = 40% (e = 0.667). Most predicted values are close to the measured k values (Fig. 5), but eqs. [9] and [16] have a tendency to overestimate the measured k value for this group of very uniform sand and gravel specimens.

Glass beads and crushed glass of Loudon (1952)

Loudon (1952) tested specimens of uniform glass beads and crushed glass at small gradients in a long, rigid-wall permeameter 6.2 cm in diameter, equipped with lateral piezometers. All specimens were tested at different n values, the maximum porosity being obtained by piping. Their d_{10} value was between 0.13 and 0.65 mm, and their C_U was less than 1.8. As the measured k values were given at 10 °C, they were converted into data at 20 °C for this study. The predicted k values for glass beads are very close to the measured k values (Fig. 6). The predicted k values for crushed glass are poor, however: the values predicted by eq. [9] are systematically underestimated by a factor of 2–3, whereas the values predicted by eq. [16] are systematically overestimated by a factor of 3–12.

Sand tested by Chapuis et al. (1989*b*)

Specimens of homogenized natural sand were tested by Chapuis et al. (1989*b*) in a vertical rigid permeameter (10.2 cm in diameter and 15 cm long) and a horizontal rigid permeameter (15 cm square section and 30 cm long), both equipped with lateral piezometers. The sand had a d_{10} of 0.16 mm and a C_U of 3.5. The maximum and minimum void ratios were e_{max} = 0.824 and e_{min} = 0.348, respectively. Two compaction modes, static and dynamic, were considered. The two modes gave different k values for the vertical and horizontal directions but the same value for the first invariant of the k tensor at a given e value, irrespective of the

Fig. 5. Predicted versus measured k values for the tests of Krumbein and Monk (1942).

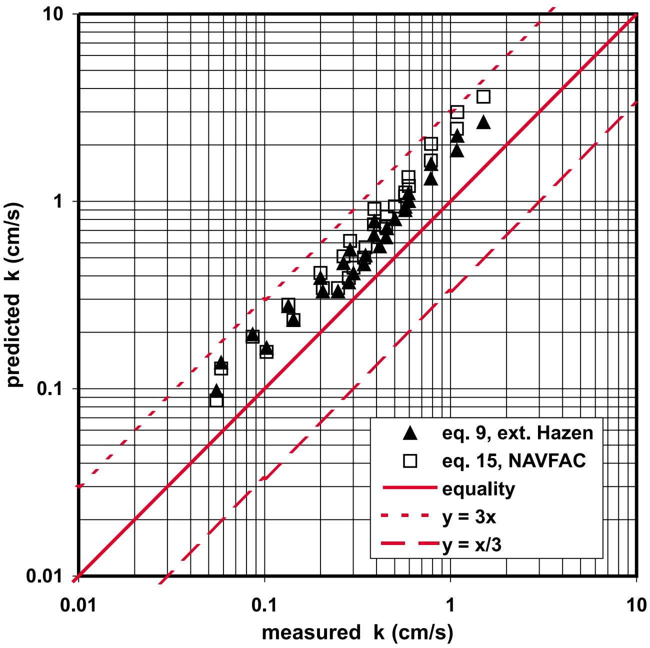
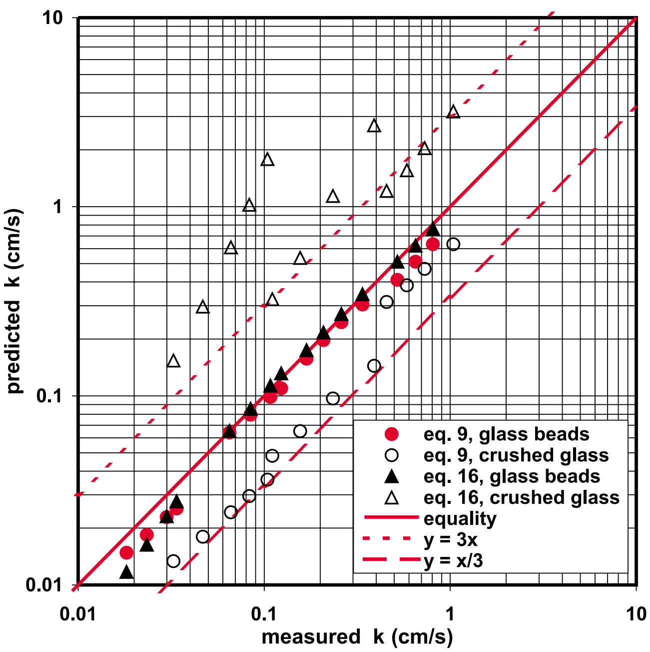


Fig. 6. Predicted versus measured k values for the tests of Loudon (1952).

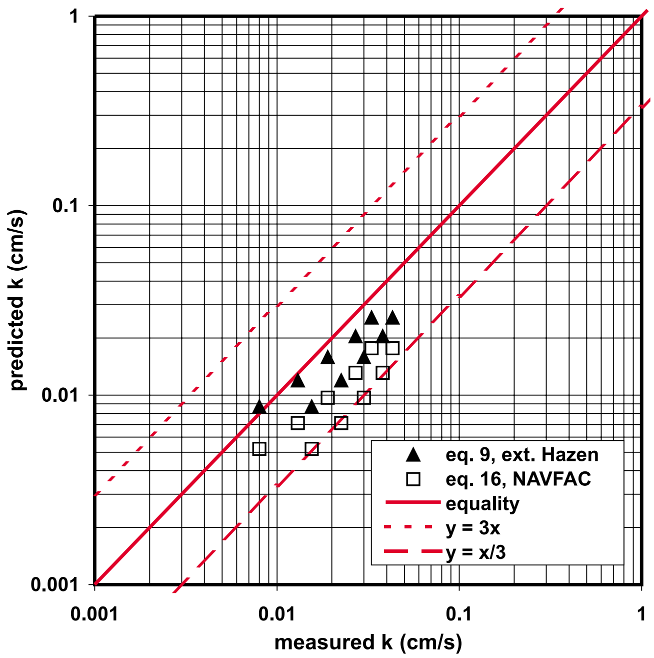


compaction mode. Equations [9] and [16] are isotropic, and thus they predict identical values for k_h and k_v, which they underestimate (Fig. 7), whereas experimentally $k_h > k_v$ for the usual (static) compaction mode (Chapuis and Gill 1989).

New predictive equation

Considering that eqs. [9] and [16] give predictions that vary from good to questionable for sand and gravel, it was

Fig. 7. Predicted versus measured k values for the tests of Chapuis et al. (1989*b*). The two sets of data correspond to k_v and k_h, with $k_h > k_v$.



decided to go back to basic considerations to try to develop a better predictive equation. According to the Kozeny–Carman equation (eq. [7]), the k value depends linearly on S^{-2}, where S is the specific surface, and on the ratio $e^3/(1 + e)$. According to eq. [9], the term d_{10}^2 in the Hazen equation corresponds to $S^{-2}e_{max}^3/(1 + e_{max})$. Thus it was considered that there should be some relationship between an experimental k value and the parameter $x = d_{10}^2 e^3/(1 + e)$. Plotting the experimental data reveals that log k is linearly related to log x (Fig. 8). This is a power-law relationship that was determined by a best-fit technique (coefficient $R^2 = 0.91$) to be

$$[17] \qquad k(\text{cm/s}) = 2.4622[d_{10}^2 e^3/(1+e)]^{0.7825}$$

where d_{10} is in mm.

Comparison of predictive capacities

The predictive capacities of eqs. [9], [16], and [17] were compared using the previously described laboratory data. The graph of log(predicted k) versus log(measured k) is shown in Fig. 9. The data corresponding to the proposed power law (eq. [17]) appear closer to the equality line and less scattered ($R^2 = 0.91$) than those corresponding to the extended Hazen equation (eq. [9]; $R^2 = 0.88$) or the NAVFAC equation (eq. [16]; $R^2 = 0.84$). The differences among the predictive capacities of the three equations are more obvious in Fig. 10, which presents the distribution curves of the differences "log(measured k) – log(predicted k)." The mean values are –0.07, –0.21, and 0.00 for eqs. [9], [16], and [17], respectively, with standard deviations of 0.23, 0.36, and 0.10, respectively. Consequently, according to the mean values and standard deviations, eq. [17] is the best predictive equation for these experimental data.

Fig. 8. Experimental k values plotted against $x = d_{10}^2 e^3/(1 + e)$.

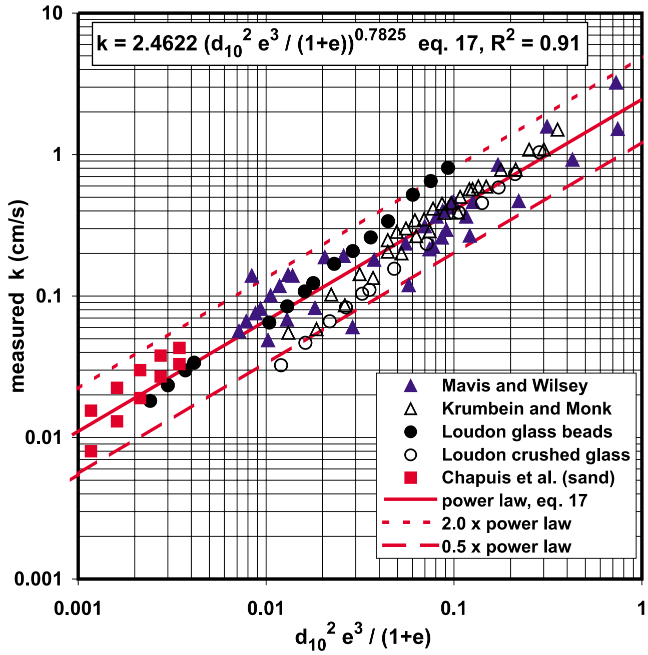


Fig. 10. Distribution curves of the difference between log(predicted k) and log(measured k) for eqs. [9], [16], and [17] using the same experimental data as for Fig. 8.

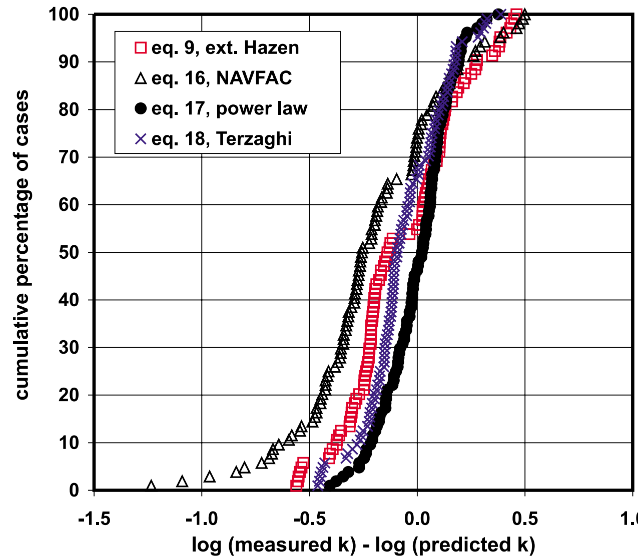
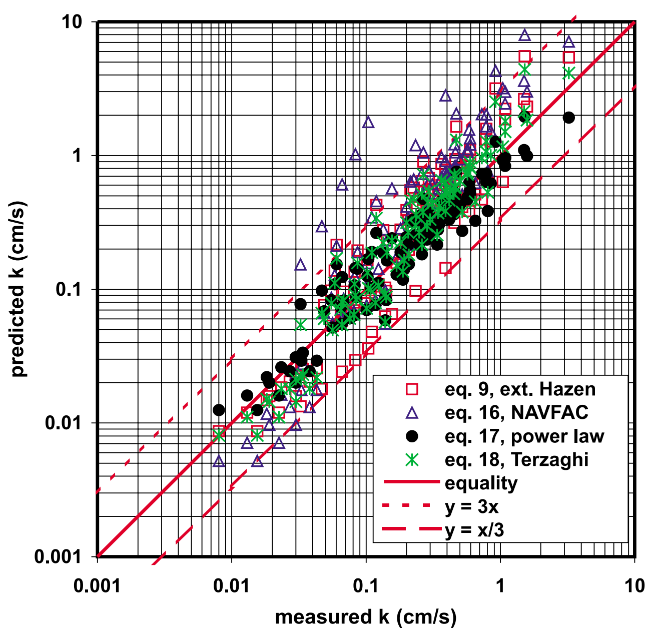


Fig. 9. Predictive capacities of eqs. [9], [16], and [17] using the same experimental data as for Fig. 8.



Other predictive methods

The predictive capacity of other equations for sand and gravel (e.g., those considered by Vuković and Soro 1992), most of them less complete than those retained here (e.g., Alyamani and Sen 1993; Shepherd 1989), was not as good as that of the proposed equation. Among the other equations, that of Terzaghi (1925) for sand is worth mentioning here:

$$[18] \qquad k(\mathrm{cm/s}) = C_0 \frac{\mu_{10}}{\mu_t}\left(\frac{n - 0.13}{\sqrt[3]{1-n}}\right)^2 d_{10}^2$$

where the constant C_0 equals 8 for smooth, rounded grains and 4.6 for grains of irregular shape (Terzaghi 1925); and μ_{10} and μ_t are the water viscosities at 10 °C and t °C, respectively. For laboratory conditions, the data are usually given at t = 20 °C, for which the ratio of viscosities is 1.30. Most specimens examined here for comparison had round to spherical particles, and thus C_0 = 8 was retained. The predictions of eq. [18] are plotted in Figs. 9 and 10, where they appear to be similar to those of the extended Hazen equation (eq. [9]), having a mean of –0.06 and a standard deviation of 0.24.

Possible extension of eq. [17]

The predictions of the proposed eq. [17] were checked against experimental results for other sand specimens (Sperry and Pierce 1995; Wiebenga et al. 1970) for which the permeameter had no lateral piezometers, or was not described, and full details of the testing procedure were not provided. According to Fig. 11, eq. [17] fairly predicts the measured k values for these sands.

The capacity of eq. [17] to predict k values for silty soils was also checked, first with data from École Polytechnique for different specimens of nonplastic silty sand ($0.0032 \le d_{10} \le 0.062$ mm and $0.44 \le e \le 0.82$). According to the results in Fig. 11, eq. [17] can still correctly predict the saturated k values of such natural soils.

The predictions of eq. [17] were also checked against the experimental results for other sand specimens, for which experimental data were not as detailed, and for crushed, sand-size materials of Loudon (1952), mine tailings (crushed rock having the gradation of silt) of Aubertin et al. (1996), and natural tills of unknown plasticity.

Fig. 11. Extending the best-fit line of Fig. 8 to the case of natural nonplastic silty soils.

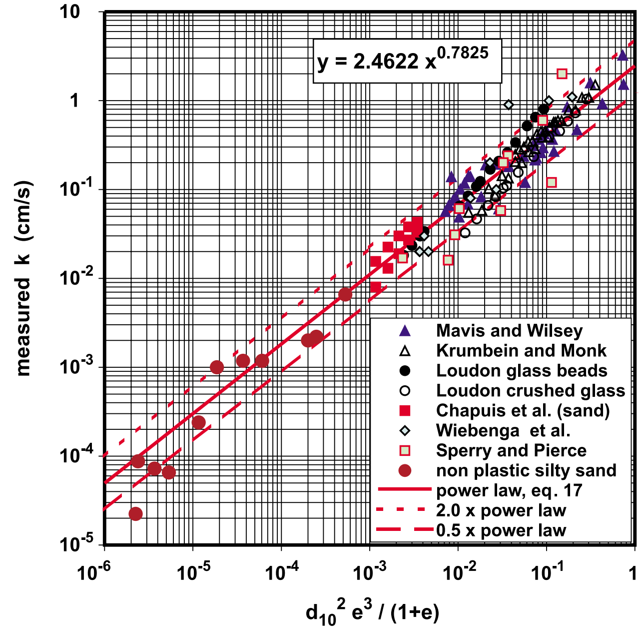
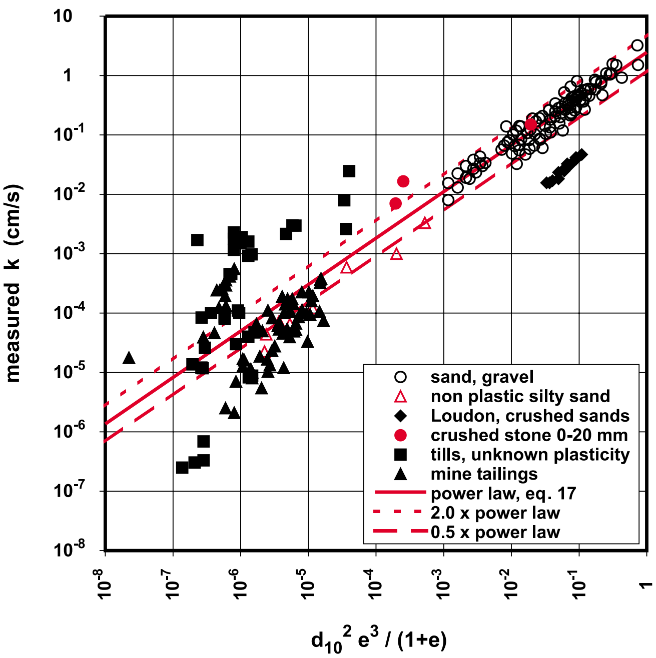


Fig. 12. Attempt to extend the power law to tills of unknown plasticity and mine tailings (crushed angular materials, silt size, may have some plasticity).



For crushed soils and rocks, the predictions of eq. [17] were poor (Fig. 12), and thus eq. [17] should not be used for these materials. Similarly, Chapuis and Aubertin (2003) found that the more general Kozeny–Carman equation gives poor predictions for crushed materials, which can be due to at least three factors. First, crushed materials have angular, sometimes acicular, fine particles, which increases the tortuosity effect. Second, as a result, the void-space geometry is not similar to that of natural soils. Third, several phenomena occur during testing of crushed materials (Bussière 1993), such as creation of new fines during compaction and migration of fines during testing (Chapuis et al. 1996), which are not considered in predictive equations.

For natural tills of unknown plasticity, the predictions of eq. [17] were also poor (Fig. 12), and thus eq. [17] should not be used for these materials. This happens because the parameters of eq. [17] do not correspond to a good evaluation of the specific surface S of these tills, which varies considerably with the liquid limit of the fine fraction. If the Brunauer–Emmett–Teller (BET) method (Chapuis and Aubertin 2003) is used to evaluate S, however, the Kozeny–Carman equation works fairly well for tills.

Discussion

Several factors must be considered when comparing predicted values of k with those measured using the rigid-wall permeameter, including inaccuracy in the tested specimen parameters, inadequate specimen preparation, hydraulic head losses (in pipes, valves, and porous stones), and possible internal erosion of fine particles.

Impact of small inaccuracies in parameters

To illustrate the effect of small inaccuracies in parameters, assume that the sand sample was first homogenized and that its gradation (sieve analysis) was obtained with one specimen taken from the homogenized sample, giving d_{10} = 0.16 mm. Another specimen of the same sample was used for permeability testing, but its gradation was not determined after testing. Assume that the sieve analysis of the tested specimen, after testing, would have given d_{10} = 0.13 mm. In addition, the void ratio was determined as 0.50 from mass and volume measurements: assume it is really 0.48 due to the various inaccuracies in measurements of masses and volumes. As a result, the predicted k using eq. [17] would be 2.00×10^{-2} cm/s with d_{10} = 0.16 mm and e = 0.50, whereas it would be 1.33×10^{-2} cm/s with the correct d_{10} = 0.13 mm and the correct e = 0.48. This variation of 1.5 or 150% in the predicted k illustrates the high sensitivity of eq. [17] to small inaccuracies in the input parameters.

Inadequate specimen preparation

Several precautions must be taken to have a good-quality test in a rigid-wall permeameter. For example, one should never place a soil core directly in the permeameter because some preferential seepage would occur along the sidewall. The soil specimen must be homogenized, placed in a few centimetre-thick layers that are compacted individually and thus pressed against the sidewall to prevent preferential seepage. Heavy compaction (e.g., using the masses of the Proctor tests) must be avoided to avoid particle breakage and generation of fines.

Another important point is the real degree of saturation within the tested specimen. In the current standard D2434 (ASTM 2002*b*), the specimen is supposed to be "saturated" after using a vacuum pump, but the real degree of saturation, S_r, is unknown. A method was proposed by Chapuis et al. (1989*a*) to determine the real S_r value of the tested specimen. This method proved that if a rigid-wall permeameter is used without several precautions that are not mandatory in D2434, S_r is frequently between 75 and 85%. Then the experimental $k(S_r)$ value is only 15–30% of $k(S_r = 100\%)$ as confirmed by usual relationships (e.g., Mualem 1976). As a result, without these precautions, the measured k value underevaluates the fully saturated k value (Chapuis 2004).

Hydraulic head losses

It is very important to use lateral piezometers to measure directly the hydraulic head at least at two points within the tested specimen. These lateral piezometers are not shown in many textbook sketches. When they are not used, the measured hydraulic head loss includes head losses in pipes, valves, and porous stones or screens that are not negligible for sand and gravel specimens. Consequently, the resulting k value is underevaluated because it is calculated using a too-high value for head loss in the specimen.

Risks of internal erosion

One of the conditions for use of the NAVFAC chart is to have a ratio d_{10}/d_5 lower than 1.4. This means that the grain-size distribution curve of the sand cannot end with a flat portion. A flat final portion is not a feature of natural soils but can be found in soil samples that result from mixing several adjacent layers. A flat final portion indicates a risk of segregation (Chapuis et al. 1996) and movement of fine particles within the tested specimen (internal erosion or suffossion). Such a risk can be evaluated using the criteria of Kezdi (1969), Sherard (1979), or Kenney and Lau (1985, 1986). These three suffossion criteria can be reduced to similar mathematical expressions involving minimum values for the secant slope of the grain-size distribution curve (Chapuis 1992). When fine particles migrate within a permeameter, they may either clog or enlarge certain channels, or they may accumulate against a porous stone and generate localized high head losses. All these phenomena result in largely different k values for different specimens of the same soil sample. The creation and migration of fines can be assessed by determining the gradation of the upper, central, and lower thirds of the specimen after testing (Chapuis et al. 1996; Chapuis 2002).

Resulting precision of rigid-wall permeameter tests

According to standard D2434 (ASTM 2002*b*) for testing sand and gravel, the real precision of this test seems unknown and therefore its bias cannot be determined. It is frequently admitted that the true k value of a soil lies between one third and three times the value given by a good laboratory test. In Fig. 8, the published experimental data for sand and gravel usually lie between one half and two times the predicted value of eq. [17]. According to the author's tests and literature review, the precision depends on testing procedures and soil intrinsic variability. For example, excellent precision (k value within ±20% for three specimens) can be reached with homogenized sand and gravel, using a rigid-wall permeameter with lateral piezometers and standard preparation procedures, when two conditions are met. First, a special procedure (using both vacuum and deaired water) must be followed with an improved permeameter for ensuring full saturation (Chapuis et al. 1989*a*), the real degree of saturation being determined using the method of Chapuis et al. (1989*a*). Second, the soil gradation must not be prone to internal erosion (Chapuis 1992). Such high-quality testing conditions are a prerequisite to assessing the capacity of equations to predict the saturated hydraulic conductivity.

Conclusion

The Hazen equation was extended as eq. [9] to be used at any void ratio, and the NAVFAC chart was shown to be equivalent to eq. [16]. By using published data for high-quality tests, eqs. [9] and [16] were used to predict the vertical hydraulic conductivity, k, of homogenized sand and gravel specimens. The distribution curves of the differences "log(measured k) – log(predicted k)" were found to have mean values of –0.07 and –0.21 for eqs. [9] and [16], respectively, with standard deviations of 0.23 and 0.36, respectively. To obtain better predictions, eq. [17] was proposed and yielded a mean value of 0.00 with a standard deviation of 0.10. For natural uniform sand and gravel, eq. [17] can be used to predict the k value that is in the range of 10^{-1} to 10^{-3} cm/s. The prediction usually falls in the range of 0.5–2.0 times the measured saturated k value for the considered data. According to experimental data, the predictive capacity of eq. [17] can be extended to natural silty sands without plasticity but not to crushed materials or silty soils with some plasticity.

The observed differences between predicted and measured k values may be because of some inaccuracy in measured soil parameters, faulty permeability testing procedures as previously discussed, and some deficiencies in the predictive equations. For example, the isotropic eq. [17] cannot correctly represent the hydraulic conductivity matrix that is anisotropic.

Nevertheless, eq. [17] is believed to represent a good predictive tool for natural uniform sand and gravel ($C_U < 12$). It can be used to estimate the k value of a series of soil specimens (after determination of d_{10} and e) and to check the quality of laboratory permeability tests. The comparison of laboratory and field tests (e.g., pumping tests) is not discussed in this paper.

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