Hydrology
CEE 440, CEE 545, GLG 471
Arizona State University

Lecture 8:

1. General Properties of the Infiltration Process

Infiltration at a point generally varies systematically with time during a precipitation event as a moisture wave is transmitted in the vertical and horizontal directions.

We define a water input event $w(t)$ (rainfall, snowmelt, run-on) over a period $t = 0$ to $t = t_w$. The infiltration rate $f(t)$ [L/T] is a flux of water into the soil profile from the surface. The infiltration capacity $f^*(t)$ (or infiltrability) is the maximum rate at which infiltration can occur, $f^*(t) = f_{max}(t)$. Infiltration can occur under ponded or non-ponded conditions. Ponding is quantified by the ponding depth of water, $H(t)$.

Three conditions are important to distinguish during the infiltration process.

(1) No ponding condition implies that water has not accumulated on the soil surface:

$$H(t) = 0,\ f(t) = w(t) \le f^*(t) \qquad (1)$$

The infiltration rate is equal to the water input which is equal or less than the maximum infiltration capacity of the soil surface.

(2) Surface ponding occurs as saturation from above (rainfall, snowmelt input):

$$H(t) > 0,\ f(t) = f^*(t) \le w(t) \qquad (2)$$

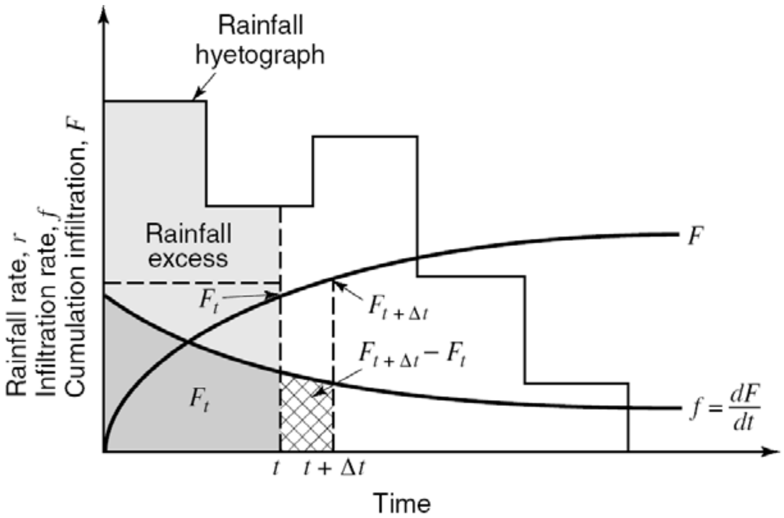
Infiltration is equal to the infiltrability which can be less than or equal to the water input rate to the soil surface.

(3) Surface ponding occurs as saturation from below (groundwater discharge):

$$H(t) \ge 0,\ f(t) = 0 \qquad (3)$$

Infiltration is by nature a complex phenomenon which is variable in space and time. It is affected by many factors:

(1) Water input from rainfall, snowmelt, irrigation $w(t)$ and ponding depth $H(t)$.
(2) Soil saturated hydraulic conductivity and its profile $K_h^*(z)$.
(3) Antecedent soil water content and its profile $\theta(z)$.
(4) Soil surface topography and roughness (runoff-runon processes).
(5) Chemical characteristics of soil surface (hydrophobicity).
(6) Physical and chemical properties of water. Soil freeze and thaw conditions.



Rainfall Hyetograph, Infiltration Rate and Cumulative Infiltration

Two basic characteristics are important in the infiltration process:

(1) A general decrease in infiltration rate as a function of time. This typically occurs as an unsaturated soil wets (high initial infiltration f_o) and progressively less storage in the soil is available for water input (lower final infiltration f_c).

(2) A progressive deepening and diffusion of an infiltration wave into a soil profile as a function of time. The wetting front (boundary between unsaturated soil profile and moisture wave) is governed by the antecedent moisture conditions and the soil profile properties.

Experimental data and modeling have allowed hydrologists to describe the infiltration process with reasonable accuracy, although many complexities are still not understood.

2. Infiltration Equations

A set of equations have been developed to predict the infiltration rate time at a point:

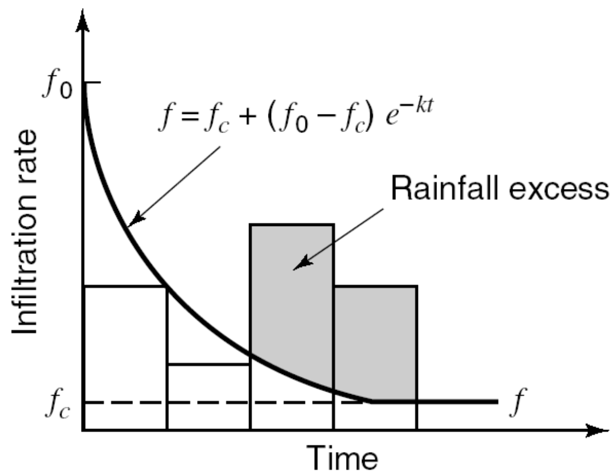
(1) Horton Infiltration Equation
(2) Green-Ampt Model

Several other infiltration equations exist (Phillip, Soil Conservation Service).

Horton Infiltration Equation:

$$f(t) = f_c + (f_o - f_c)e^{-\alpha t} \qquad (4)$$

where $f(t)$ is the infiltration rate, f_c is the ultimate infiltration capacity, f_o is the initial infiltration rate, and α is a decay parameter. The Horton Equation is represented as an exponential decay with f_c, f_o and α dependent on soil properties and antecedent wetness. It assumes a ponded soil condition throughout the infiltration process.



Horton Infiltration Equation

The cumulative infiltration $F(t)$ over a period of time t, defined as:

$$F(t) = \int_0^t f(\tau)d\tau \tag{5}$$

can be obtained for the Horton equation as:

$$F(t) = f_c t + \frac{(f_o - f_c)}{\alpha}(1 - e^{-\alpha t}) \tag{6}$$

<u>Green-Ampt Model:</u>

The Green-Ampt model (1911) utilizes Darcy's Law and conservation of mass to obtain a physically-based infiltration equation tied to soil properties and antecedent wetness. Idealized conditions include:

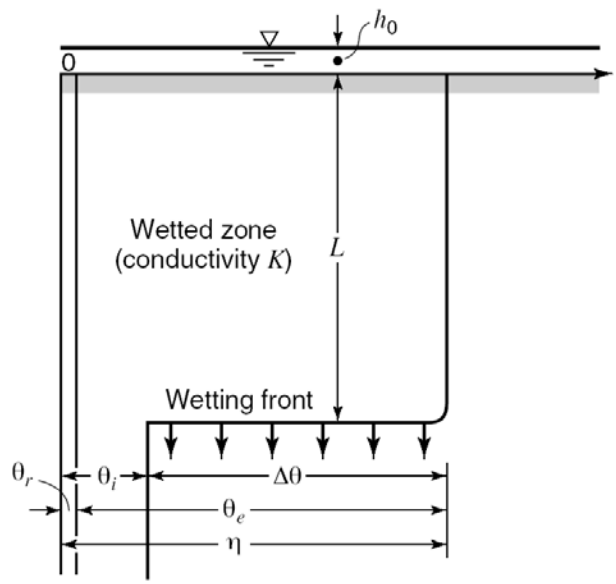
(1) Homogeneous soil profile (K_h^* and ϕ (or η) uniform with depth) of infinite depth.
(2) No water table, capillary fringe or impermeable lower boundary.
(3) Initial soil water content $\theta_i < \phi$.

The initial downward flux of water over the entire profile is given by:

$$q_z(z,0) = K_h(\theta_i) \tag{7}$$

which is considered negligible if $\theta_i << \theta_{fc}$.

Two cases in the Green-Ampt model are distinguished:
(1) Water input less than saturated hydraulic conductivity ($w < K_h^*$)
(2) Water input greater than or equal to saturated hydraulic conductivity ($w \geq K_h^*$)



Green-Ampt Model of Infiltration as Sharp Wetting Front

Case 1: If ($w < K_h^*$):

$$f(t) = w, \quad 0 < t \le t_w \tag{8}$$

$$f(t) = 0, \quad t > t_w \tag{9}$$

where t_w is the time of water input. Note that a sharp wetting front is created in the soil profile with $\theta = \phi$ in the wet soil above the front and $\theta = \theta_i$ below the wetting front.

Case 2: If ($w \ge K_h^*$): the initial infiltration follows the previous description until $\theta = \phi$ when soil saturation leads to ponding of the soil surface while the wetting front descends.

To derive the Green-Ampt Equation, consider a wetting front of depth L such that the cumulative infiltration can be defined as:

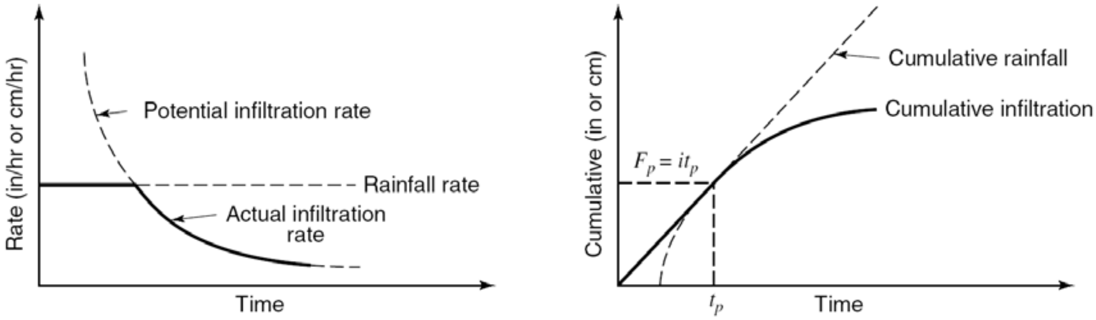
$$F(t) = L(\phi - \theta_i) = L\Delta\theta \tag{10}$$

For this same wetting depth, we can estimate the infiltration rate, $f(t)$, by applying Darcy's Law to two locations: (1) ground surface and (2) dry side of the wetting front:

$$q = -K_h^* \frac{\partial h}{\partial z} = -K_h^* \frac{\Delta h}{\Delta z} = -K_h^* \frac{h_1 - h_2}{z_1 - z_2} \tag{11}$$

The constant flux, q, through the wetting front is equal to $-f$. We can then evaluate this expression at points 1 and 2. Recall, that $z_1 - z_2 = L$, the depth of the wetting front.

$$f = K_h^* \frac{h_1 - h_2}{L} \tag{12}$$



Ponding Time for Constant Rainfall Intensity

The head at the ground surface, $h_1 = h_o$ (ponded depth), while the head at the dry side of the wetting front, $h_2 = -|\psi_f| - L$ such that:

$$f = K_h^* \frac{h_o + |\psi_f| + L}{L} \qquad (13)$$

where ψ_f is the effective tension (matric head) at the wetting front approximated from the air entry pressure (assumed to be positive in all equations used here, thus the | | symbols):

$$|\psi_f| = \frac{2b+3}{2b+6}|\psi_{ae}| \qquad (14)$$

For small ponding depth, $h_o = 0$ is a good approximation, in particular when the rainfall excess is transported downslope as runoff.

Since $L = F(t)/\Delta\theta$, and assuming $h_o = 0$, we obtain:

$$f(t) = K_h^* \frac{|\psi_f|\Delta\theta + F(t)}{F(t)} \qquad (15)$$

This can be manipulated, since $f(t) = dF(t)/dt$ to yield a single expression for the Green-Ampt equation in terms of $F(t)$:

$$F(t) = K_h^* t + |\psi_f|\Delta\theta \ln\left(1 + \frac{F(t)}{|\psi_f|\Delta\theta}\right) \qquad (16)$$

Equation (16) is a nonlinear equation in $F(t)$ that can be solved by the method of successive substitution for a given K_h^*, t, $\Delta\theta$, and $|\psi_f|$ as:

(1) Assume a value of $F(t)$. An initial good guess is $F(t) = K_h^* t$.
(2) Calculate the value of $F(t)$ using Equation (16).

Table 7.4.1 Green–Ampt Infiltration Parameters for Various Soil Classes*

Soil class	Porosity η	Effective Porosity θ_e	Wetting Front Soil Suction Head ψ (cm)	Hydraulic Conductivity K (cm/h)
Sand	0.437	0.417	4.95	11.78
	(0.374–0.500)	(0.354–0.480)	(0.97–25.36)	
Loamy sand	0.437	0.401	6.13	2.99
	(0.363–0.506)	(0.329–0.473)	(1.35–27.94)	
Sandy loam	0.453	0.412	11.01	1.09
	(0.351–0.555)	(0.283–0.541)	(2.67–45.47)	
Loam	0.463	0.434	8.89	0.34
	(0.375–0.551)	(0.334–0.534)	(1.33–59.38)	
Silt loam	0.501	0.486	16.68	0.65
	(0.420–0.582)	(0.394–0.578)	(2.92–95.39)	
Sandy clay	0.398	0.330	21.85	0.15
loam	(0.332–0.464)	(0.235–0.425)	(4.42–108.0)	
Clay loam	0.464	0.309	20.88	0.10
	(0.409–0.519)	(0.279–0.501)	(4.79–91.10)	
Silty clay	0.471	0.432	27.30	0.10
loam	(0.418–0.524)	(0.347–0.517)	(5.67–131.50)	
Sandy clay	0.430	0.321	23.90	0.06
	(0.370–0.490)	(0.207–0.435)	(4.08–140.2)	
Silty clay	0.479	0.423	29.22	0.05
	(0.425–0.533)	(0.334–0.512)	(6.13–139.4)	
Clay	0.475	0.385	31.63	0.03
	(0.427–0.523)	(0.269–0.501)	(6.39–156.5)	

*The numbers in parentheses below each parameter are one standard deviation around the parameter value given.

Soil Properties for Estimation of Green-Ampt Equation.

(3) Compare to the initial guess on $F(t)$. If different, use the new value of $F(t)$ from (2) as the next guess.

(4) Continue until the two values of $F(t)$ converge.

Once $F(t)$ is calculated, the infiltration rate is obtained as:

$$f(t) = K_h^* \left[\frac{|\psi_f| \Delta\theta}{F(t)} + 1 \right] \qquad (17)$$

The time to ponding (t_p) in the Green-Ampt Model is:

$$t_p = \frac{K_h^* |\psi_f| (\phi - \theta_i)}{w(w - K_h^*)} \qquad (18)$$

where w is the water input (rainfall) rate and θ_i is the initial water content.

The table above presents some estimated values of soil properties for use in the Green-Ampt equation. Note that the notation is slightly different. Porosity $\eta = \phi$. Effective porosity $\theta_e = \phi - \theta_r$, where θ_r is the residual soil moisture content. Wetting front soil suction head, $\psi = |\psi_f|$. Hydraulic conductivity, $K = K_h^*$.

An explicit solution to the Green-Ampt model was presented by Salvucci and Entekhabi (1994) as follows. Their approach is based on computation of three time parameters.

1) A characteristic time T^* defined as:

$$T^* = \frac{|\psi_f|(\phi - \theta_i)}{K_h^*} \qquad (19)$$

2) A compression time t_c defined as:

$$t_c = \frac{F_p}{K_h^*} - \left[\frac{|\psi_f|(\phi - \theta_i)}{K_h^*}\right] \ln\left[1 + \frac{F_p}{|\psi_f|(\phi - \theta_i)}\right] \qquad (20)$$

where $F_p = F(t_p) = wt_p$ is the cumulative infiltration up to the ponding time t_p.

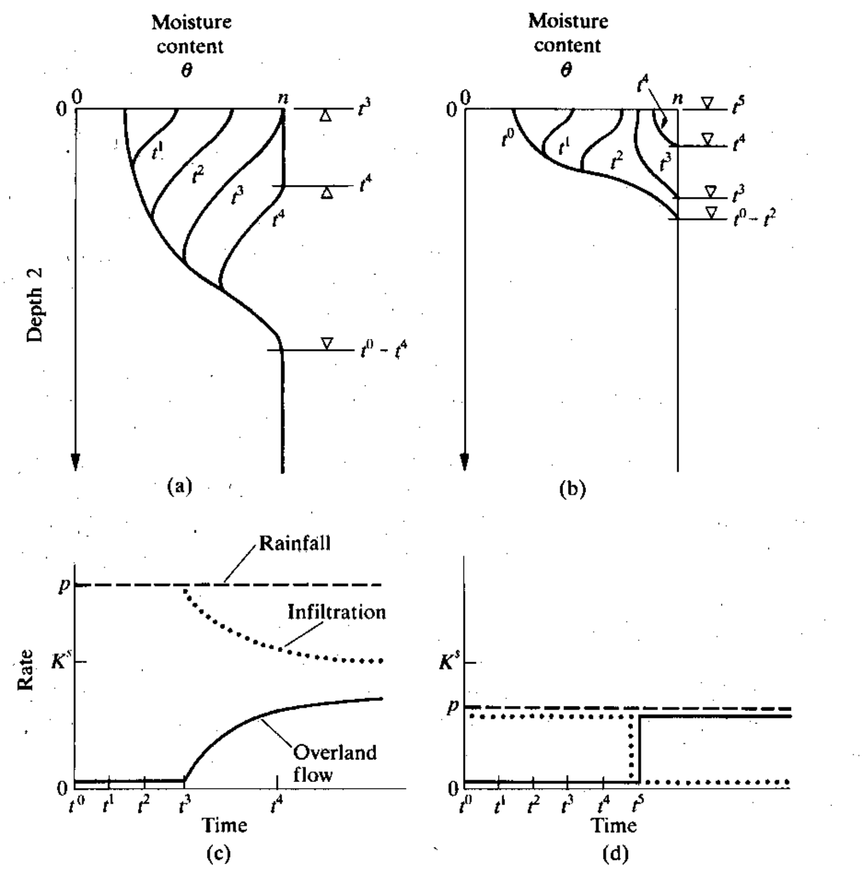
3) An effective time t_e defined as:

$$t_e = t - t_p + t_c \qquad (21)$$

With these definitions, one can compute infiltration rate and the cumulative amount without having to do the method of successive substitution.

$$f(t) = K_h^*\left[0.707\left(\frac{t_e + T^*}{t_e}\right)^{1/2} + 0.667 - 0.236\left(\frac{t_e}{t_e + T^*}\right)^{1/2} - 0.138\left(\frac{t_e}{t_e + T^*}\right)\right] \qquad (22)$$

$$F(t) = K_h^*\left\{\begin{array}{l} 0.529t_e + 0.471\left(T^*t_e + t_e^2\right)^{1/2} + 0.138T^*\left(\ln(t_e + T^*) - \ln T^*\right) \\ + 0.471T^*\left(\ln(t_e + 0.5T^* + (T^*t_e + t_e^2)^{1/2}) - \ln(0.5T^*)\right) \end{array}\right\} \qquad (23)$$



Moisture Content Versus Depth Profiles for (a) the Horton Mechanism (Infiltration-Excess Runoff) and (b) the Dunne Mechanism (Saturation from Below Runoff). Overland Flow Generation for (c) Horton Mechanism and (d) Dunne Mechanism.

3. Exfiltration and Redistribution

Infiltration into a soil is important for runoff and evapotranspiration. Following water input into the soil, water is redistributed (or exfiltrated) due to:

(1) Lateral moisture movement
(2) Evaporation and transpiration
(3) Runoff

Of the possible runoff mechanisms, the Horton and Dunne mechanisms represent important examples that differ in their water content profile and exfiltration (runoff):

Horton (Infiltration Excess): (Conditions $w > K_h^*$ and $t_w > t_p$). An infiltration wave penetrates into the soil column until the surface becomes saturated from above at ponding time t_p when infiltration is reduced and overland flow generated.

Dunne (Saturation Excess): (Conditions $w < K_h^*$). An infiltration wave can only occupy upper soil profile due to shallow groundwater zone. Once saturation is reached, overland flow is produced and infiltration is reduced to zero.