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Prediction of wetting front movement during one-dimensional infiltration into soils

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Abstract. The Green-Ampt and Philip infiltration models are simplified with a capillary tube representation of porous media that can predict wetting front movement during horizontal and vertical infiltration into dry or initially moist soil. The simple model captures the fundamental physics of unsaturated fluid flow and reduces the number of parameters that must be measured. The relevant parameters are the air-liquid interfacial tension, the density and viscosity of the liquid, the initial saturation, and the permeability. Comparisons with published data show good predictions for infiltration under nonnegative source pressures but are less successful under negative source pressures. The model applies to imbibition of both water and nonaqueous phase liquids.

Introduction

Efficient and accurate methods of predicting infiltration processes in both dry and moist soils are needed because of the prevalence of contaminants placed intentionally or accidentally near the soil surface. This paper focuses on modeling one-dimensional horizontal and vertical infiltration by using a plug flow model based on a capillary tube representation of porous media to simplify the Green-Ampt equations. The predictive capability of the resulting model is tested by comparison with published data.

The fundamental equation governing unsaturated flow is Richard's equation, which, for one-dimensional horizontal infiltration, can be expressed as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K(h)\,\frac{\partial h}{\partial \theta}\,\frac{\partial \theta}{\partial x}\right) \tag{1}$$

$K(h)$ is the unsaturated hydraulic conductivity, θ is the volumetric liquid content, and h is the matric potential, or suction head. The functions $K(h)$ and $h(\theta)$ describe the hydraulic properties of the porous media and are typically represented by analytical expressions that are fitted to laboratory or field measurements. Infiltration models are usually derived by numerically or analytically solving variations of Richard's equation [*Babu*, 1976; *Brutsaert*, 1976; *Celia et al.*, 1990; *Hills et al.*, 1989; *Parlange*, 1975].

Equation (1) is a nonlinear partial differential equation that can be reduced to an ordinary differential equation by the Boltzmann transformation $\lambda = xt^{-1/2}$ [*Crank*, 1975]. This partial solution implies that the wetting front location advances proportionally to the square root of time when the value of λ corresponding to the wetting front is constant. This behavior has been observed frequently in laboratory experiments. Observed moisture profiles resembled step functions indicative of plug flow behavior, and profiles collapsed onto single curves when moisture content was plotted against the transformed variable $\lambda = xt^{-1/2}$. The majority of these experiments were performed on initially dry soil [*Bruce and Klute*, 1956; *Gardner and Mayhugh*, 1958; *Jackson*, 1963a; *Kirkham and Feng*, 1949; *Peck*, 1964; *Smiles and Philip*, 1978; *Watson and Jones*, 1982].

Vertical infiltration has usually been considered to be of greater practical importance than horizontal infiltration, because the effects of gravity significantly increase the rate of long-term liquid movement and consequently the region of liquid penetration. The two most pertinent analytical solutions are those attributed to Green and Ampt and to J. R. Philip.

Green and Ampt [1911] derived simple equations for infiltration under a constant head boundary condition by assuming plug flow behavior in which the wetted region maintains a constant moisture content and is separated from the dry region by a sharp wetting front that is pulled through the soil by soil suction forces. They began their mathematical derivation with an analogy to capillary tubes but moved early to a formulation of Darcy's law by lumping together unknown parameters into a permeability term. Their reasoning yields the following equations for horizontal and vertical infiltration, respectively:

$$x_f = \left(\frac{2K(h_o)\Delta h t}{\Delta \theta}\right)^{1/2} \tag{2}$$

$$t = \frac{\Delta \theta}{K(h_o)}\left[z_f - \Delta h \ln\left(1 + \frac{z_f}{\Delta h}\right)\right] \tag{3}$$

where x_f is the distance from the source to the wetting front during horizontal infiltration, z_f is the depth to the wetting front during vertical infiltration, Δh is equal to the head at the inlet h_o minus the suction head of the wetting front h_f, $K(h_o)$ is the unsaturated hydraulic conductivity in the wetted region, and $\Delta\theta$ is the difference between the moisture content in the wetted region and the initial moisture content. The equations compare well with numerical solutions and have been fitted closely to experimental data [*Childs and Bybordi*, 1969; *Mein and Larson*, 1971; *Mohamoud*, 1991]. Concern has focused largely on determination of the wetting front suction head h_f. *Mein and Farrell* [1974] and *Morel-Seytoux and Khanji* [1974] integrated Darcy's law and compared the result to Green and Ampt's derivation to yield

$$h_f = \int_{h_i}^{0} k_r\, dh \tag{4}$$

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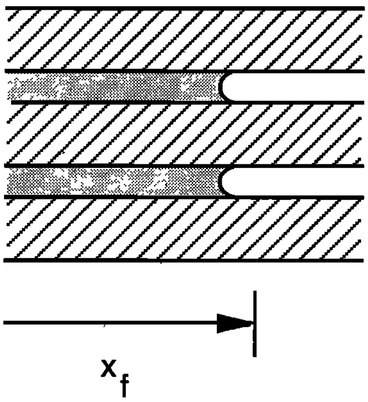

Figure 1. Capillary tube representation of infiltration into soil with complete saturation behind the wetting front.

where h_i is the suction head corresponding to the initial moisture content and k_r is the relative permeability of the wetting phase. Using this exact relationship in initially dry soils requires the complete K versus h relationship, which is not often available. *Bouwer* [1966, 1969] suggested estimating h_f with one half of the air entry pressure head, which is easier to measure. Several comparisons showed that this estimate compared well with (4) [*Brakensiek*, 1977; *Whisler and Bouwer*, 1970].

Philip [1969] defined the following relationship between the cumulative horizontal infiltration per area I and time t:

$$I = St^{1/2} \tag{5}$$

where the sorptivity S is given by

$$S = \int_{\theta_0}^{\theta} \lambda \, d\theta \tag{6}$$

He extended the solution to describe vertical infiltration with an asymptotic infinite series approximated by

$$I = St^{1/2} + At \tag{7}$$

where A is a constant determined by numerical integration. At large times the series fails to converge, and the cumulative infiltration is given by

$$I = K(h_0)t + \text{const} \tag{8}$$

There is currently no theoretically rigorous method to connect the early time solution (7) to the large time solution (8) [*Philip*, 1987]. An application of Philip's equations by *Davidson et al.* [1963] showed close agreement with experimental data. Under plug flow conditions, S reduces to $\lambda_f \Delta\theta$, where λ_f is the value of $\lambda = xt^{-1/2}$ at the wetting front.

The measurement of hydraulic parameters, which is prone to error and difficult to perform, is a major limitation in the application of both analytical and numerical models. The need for the complete $K(h)$ curve in most solution schemes has limited the ability to validate the models. Attempts at validation typically fit proposed models to cumulative infiltration or wetting front depth measurements by adjusting the parameters rather than using independently measured parameter values [*Davidoff and Selim*, 1986; *Hogarth et al.*, 1989; *Whisler and Bouwer*, 1970]. Of the models described the Green-Ampt formulation requires the least parameter characterization, which explains in large part its success: the reduction in measurement errors tends to balance the errors due to assuming a step function moisture profile. This suggests that simple equations based on easily determined fluid and porous medium properties have predictive capabilities that rival more complicated models. In the following sections we modify the Green-Ampt model for horizontal infiltration under zero source pressure into dry and moist soils, combine the Philip and modified Green-Ampt relationships to address vertical infiltration, and investigate the effects of negative source pressures. Experimental data reported in the literature are used to test the proposed relationships.

Relationships for Infiltration Under Zero Inlet Head (h_0 = 0)

Horizontal Infiltration Into Initially Dry Soil

Liquid can be modeled as moving through distinct flow channels consisting of connected pores within the porous medium. Infiltration into soils with narrow pore size distributions closely resembles plug flow, because flow channels are uniform. Less ideal soils with wider pore size distributions also exhibit step function moisture profiles when completely saturated in the wetted region, because the wetting front is sharpened by lateral transfer of liquid from larger flow channels to smaller ones. The assumption of complete saturation is approximate, because trapped air reduces the saturation levels, but it is a reasonable estimate under zero and positive inlet heads. The medium can then be modeled as parallel capillary tubes of uniform radius R such that the location of the wetting front x_f is described, as shown in Figure 1. The velocity of the front can be obtained from Poiseuille's Law [*Scheidegger*, 1974],

$$\frac{dx_f}{dt} = \frac{\rho g R^2}{8\mu} \frac{\Delta h}{x_f} \tag{9}$$

where ρ is the liquid density, μ the liquid viscosity, and Δh the difference between the imposed inlet head h_0 and the capillary suction head h_f at the meniscus. The capillary suction is given by

$$h_f = -\frac{2\sigma}{\rho g R} \tag{10}$$

where σ is the interfacial tension between the wetting liquid and air. The infiltrating liquid is assumed to completely wet the solid such that the liquid-solid contact angle is zero. During infiltration under zero inlet head, Δh is equal to $-h_f$, and (9) then integrates to yield

$$x_f = (1/\sqrt{2})(\sigma/\mu)^{1/2} R^{1/2} \sqrt{t} \tag{11}$$

This result was first obtained by *Washburn* [1921].

Numerous studies show that the permeability k of soil is proportional to the square of a characteristic length representing an average pore radius or an effective grain diameter [*Scheidegger*, 1974]. This implies

$$k = \beta R^2 \tag{12}$$

where β is a function of the geometry of the flow channels and the porosity of the porous media. Using this relationship to express R in terms of k allows (11) to be modified for soils to obtain

$$x_f = B(\sigma/\mu)^{1/2} k^{1/4} \sqrt{t} \tag{13}$$

where

$$B = \frac{1}{\sqrt{2}\beta^{1/4}} \tag{14}$$

Equation (13) clearly follows the form

$$x_f = \lambda_f \sqrt{t} \tag{15}$$

as seen repeatedly in past analyses. The coefficient B reflects the porous medium geometry and should be constant for geometrically similar porous media. The concept of geometric similitude was used by *Miller and Miller* [1956] to describe porous media that differed in scale by a magnifying factor while retaining essentially the same geometrical structure. Scaling theory has been used frequently in the literature, and geometric similitude in coarse soils has been supported by the experimental work of *Klute and Wilkinson* [1958], *Miller and Miller* [1955], and *Wilkinson and Klute* [1959].

The well-known Kozeny-Carmen equation [*Scheidegger*, 1974]

$$k = \left[\frac{p^3}{180(1-p)^2}\right] d_m^2 \tag{16}$$

supports the contention that $\beta^{1/4}$, and therefore B, is approximately constant among coarse-grained soils. Equation (16) differs from (12) in that an effective grain diameter d_m is used instead of pore radius, and the proportionality parameter, in brackets, is a function of porosity p. The ratio of grain diameter to pore radius should itself be constant among geometrically similar soils. In the typical range of soil porosity (0.35–0.4) the proportionality parameter in (16) raised to the one-quarter power varies from 0.15 to 0.18, indicating little variability of B with porosity.

Data from a number of horizontal infiltration experiments are available in the literature for checking the applicability of (13). Table 1 lists experiments where wetted lengths or sorptivities were measured with time during horizontal infiltration into initially dry porous media characterized sufficiently for determining the permeability. Medium characteristics ranged from glass beads to sandy clays. The inlet heads were zero in these experiments, and the fluid viscosity and interfacial tension were evaluated at the experimental temperatures. In the 19 experimental results listed in Table 1 the average value of B is approximately 0.5 with a narrow spread. This is significant, considering that the permeability varied by nearly 4 orders of magnitude and both water and an alcohol were used as pore fluids. The consistency in B values, despite the wide variation in porous media, suggests that the condition of geometric similarity need not be rigorously satisfied.

Table 1. Calculation of B From Reported Horizontal Infiltration Data

Media, μm	k, 10^{-12} m^2	B	Source
Glass beads (210–325)	12	0.4	*Youngs and Price* [1981][a]
Glass beads (115–180)	10	0.4	
Glass beads (60–95)	6.7	0.5	
Sand (350–500)	58	0.4	
Sand (250–350)	55	0.4	
Sand (180–250)	20	0.45	
Graded sand (180–250)	19	0.5	
Slate dust (40–125)	0.28	0.4	
Sandy loam (60–350)	0.29	0.3	
Silt loam (2–60)	0.018	0.3	
Glass beads (75)	4.5	0.6	*Bruce and Klute* [1956][b]
Coarse sand (50–250)	11	0.84	
Mason county fine sand	3.3	0.57	
Bloomfield sand	68	0.57	
Columbia silt loam	0.77	0.45	*Davidson et al.* [1963][c]
Hesperia sandy loam	0.50	0.35	
Slate dust	0.21	0.45	*Peck* [1964][c]
Glass beads (105–125) with 20% ethyl alcohol in water	13	0.5	*Malik et al.* [1979][b]
Glass beads (105–125) with methyl alcohol	13	0.7	
Average		0.5	

All experiments used water as the infiltrating liquid unless otherwise indicated.

[a] B calculated with measured sorptivities divided by $\Delta\theta$.

[b] Values of k calculated with the Kozeny-Carmen and Fair-Hatch equations from reported particle size distributions [*Freeze and Cherry*, 1979].

[c] Measured permeability.

Horizontal Infiltration Into Initially Moist Soil

Initially moist soil can be modeled as a set of capillary tubes with stagnant liquid coating the tube walls, as illustrated in Figure 2a, and infiltration into these tubes can be treated as a combination of viscous displacement and capillary suction. Contact between the infiltrating and initially stagnant liquids creates viscous forces that pull the initially stagnant liquid forward and into the center of the tube behind the meniscus. The velocity profile behind the meniscus is parabolic, as indicated in Figure 2b, while the stagnant coating ahead of the meniscus remains immobile until contact with the meniscus. The model is similar to the case of liquid moving in an initially dry tube, with the exception of a larger capillary suction (or a more negative head) at the meniscus corresponding to the smaller radius of the air-filled central tube. Application of Poiseuille's Law results in a wetting front velocity given by

$$v_f = \frac{dx_f}{dt} = \frac{\rho g R^4}{8\mu r^2}\frac{\Delta h}{x_f} \tag{17}$$

where R is the radius of the capillary and r is the radius of the air tube. Assuming that the capillary suction at the meniscus can be described by (10) with r replacing R, (17) can be integrated for the case of zero inlet head to obtain

$$x_f^2 = \tfrac{1}{2}(\sigma/\mu)(R^4/r^3)t \tag{18}$$

The air tube radius r can be expressed in terms of R and initial saturation S_i, as follows:

$$r = R\sqrt{1 - S_i} \tag{19}$$

Substituting (19) into (18) gives

$$x_f = \frac{1}{\sqrt{2}}\left(\frac{\sigma}{\mu}\right)^{1/2} \frac{R^{1/2}}{(1 - S_i)^{3/4}} \sqrt{t} \tag{20}$$

This relationship is almost identical to (11), except for the saturation factor in the denominator. Again, using the proportionality between permeability and R^2, we obtain

$$x_f = B\left(\frac{\sigma}{\mu}\right)^{1/2} \frac{k^{1/4}}{(1 - S_i)^{3/4}} \sqrt{t} \tag{21}$$

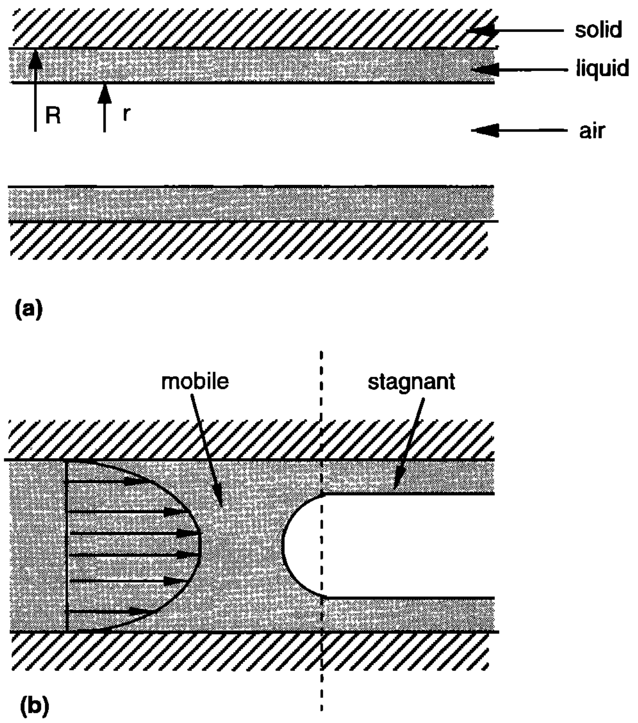

Figure 2. Model of liquid movement during infiltration into initially moist soil: (a) initially moist soil; (b) liquid movement during infiltration showing the parabolic velocity profile.

Equation (21) assumes that the saturation of each soil pore is identical and equal to the overall initial saturation of the soil sample. This is clearly unrealistic, since pore saturation varies with pore size, but it serves as a working approximation. Equation (21) can be expressed in the form

$$x_f = \lambda_f \sqrt{t} \tag{22}$$

where

$$\lambda_f = B\left(\frac{\sigma}{\mu}\right)^{1/2} \frac{k^{1/4}}{(1 - S_i)^{3/4}} \tag{23}$$

Smiles and Philip [1978] performed a series of horizontal infiltration experiments on a clay-sand mixture (30% kaolinite in fine sand, $k = 1 \times 10^{-14}$ m^2) moistened to three different initial moisture contents prior to packing. Data were presented as plots of θ versus $xt^{-1/2}$, with profiles varying significantly from plug flow behavior at higher initial saturations. We estimated initial saturations by assuming 2.65 for the specific gravity of solids and using reported values of dry bulk density and initial moisture content. Values of B calculated from (21) using their results are shown in Table 2. As can be seen, the calculated values of B are close to the value of 0.5 obtained in the case of initially dry soil. The experiments of Smiles and Philip also show that the initial liquid is pushed in front of the infiltrating liquid.

Table 2. Calculated Values of B From Horizontal Infiltration Experiments Performed by *Smiles and Philip* [1978] on an Initially Moist Mixture of 30% Kaolinite in Fine Sand

θ_i	S_i	$x_f t^{-1/2}$, 10^{-4} m/s$^{1/2}$	B
0.04	0.13	14.5	0.49
0.11	0.37	17.7	0.47
0.16	0.53	17.0	0.36

Values of $x_f t^{-1/2}$ were obtained from reported θ versus $xt^{-1/2}$ plots.

The Green-Ampt relationship given by (2) can be equated with the moist capillary tube prediction in (21) to arrive at an estimate for the wetting front suction head:

$$h_f = -\frac{\sigma B^2 \Delta\theta}{2\rho g \sqrt{k}(1 - S_i)^{3/2}} \tag{24}$$

Table 3 compares predicted values of h_f for initially dry soil ($S_i = 0$) with values presented by *Childs and Bybordi* [1969] and *Mein and Larson* [1973]. Childs and Bybordi did not measure wetting front suction heads directly but rather estimated them by using values that best fit the Green and Ampt vertical infiltration equation to data obtained from experiments on sand columns. The predicted values from (24) are about a third of the reported values, which in itself is not a bad comparison, considering the uncertainties and estimations involved; however, errors of this magnitude may result in unreasonably erroneous infiltration estimates under negative inlet heads, as will be discussed later. Mein and Larson calculated h_f from measured characteristic curves using (4). Suction head values predicted from (24) are reasonably close to the reported values for three of the soils, but large discrepancies in suction head values exist for the Ida silt loam and Yolo light clay. An examination of the capillary suction versus relative conductivity curves presented in *Mein and Larson* [1973] shows unex-

Table 3. Comparison of Wetting Front Suction Heads $h_{f,p}$ Predicted With Equation (24) With Reported Wetting Front Suction Heads $h_{f,r}$

Soil	Porosity	$\Delta\theta$, cm^3/cm^3	k, 10^{-12} m^2	$h_{f,p}$, m	$h_{f,r}$, m	$h_{f,p}/h_{f,r}$	Source
Sand	0.37	0.37	15	−0.09	−0.18	0.50	*Childs and Bybordi* [1969]
	0.37	0.37	48	−0.05	−0.16	0.31	
	0.36	0.36	80	−0.04	−0.13	0.31	
	0.36	0.36	120	−0.03	−0.10	0.30	
	0.36	0.36	210	−0.025	−0.08	0.31	
	0.35	0.35	370	−0.02	−0.06	0.33	
Plainfield sand	0.48	0.48	3.4	−0.23	−0.12	1.9	*Mein and Larson* [1973]
Columbia silt loam	0.52	0.52	1.4	−0.39	−0.24	1.6	
Guelph loam	0.52	0.52	0.37	−0.78	−0.31	2.5	
Ida silt loam	0.53	0.53	0.029	−2.8	−0.07	40	
Yolo light clay	0.50	0.50	0.012	−4.1	−0.22	18	

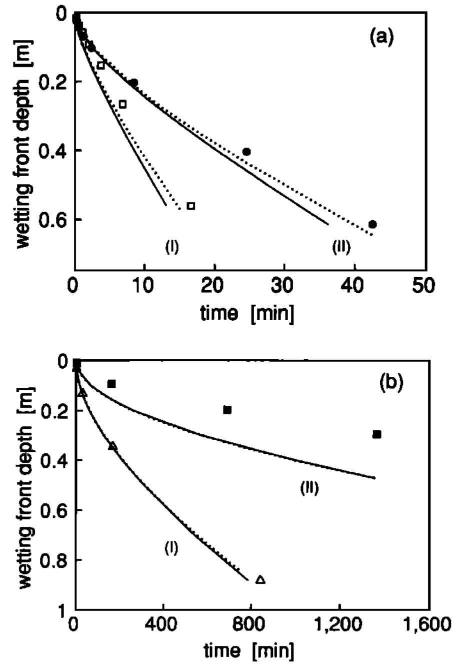

Figure 3. Data (symbols) measured by *Youngs and Price* [1981] and predictions with the modified Green-Ampt (solid curves) and Philip (dotted curves) formulations for vertical infiltration of water into (a) (set I) graded beach sand and (set II) 60–95 micron glass beads, and (b) (set I) slate dust and (set II) Rothamsted silt loam.

pectedly high values of relative permeability for these two soils. This makes it unclear whether the poor comparison is due to inaccurate measurements or unsuccessful predictions.

Vertical Infiltration

Vertical infiltration is governed by a combination of capillary and gravity effects. At small times after infiltration begins, capillary suction dominates, while at large times gravity forces dominate, and the rate of infiltration approaches the value of the hydraulic conductivity. The process can be described with the expression for h_f derived above and the Green-Ampt vertical infiltration equation, which, for zero inlet head, simplifies to

$$t = \Delta\theta/K_s[z_f + h_f \ln(1 - z_f/h_f)] \qquad (25)$$

Alternately, a piecewise representation analogous to the formulation of *Philip* [1957, 1987] can be applied:

$$z_f(t) = \lambda_f t^{1/2} + \tfrac{2}{3}(K_s/\Delta\theta)t \qquad t \le t_g \qquad (26a)$$

$$z_f(t) = (K_s/\Delta\theta)(t - t_g) + z_f(t_g) \qquad t > t_g \qquad (26b)$$

where λ_f is $S/\Delta\theta$ and z_f is $I/\Delta\theta$ under plug flow conditions and K_s is the saturated hydraulic conductivity. This approach divides vertical infiltration into two parts: times less than t_g, in which both capillarity and gravity are significant, and times greater than t_g, in which gravity dominates [*Philip*, 1969]. The constant t_g is the time at which the infiltration rate is equal to the hydraulic conductivity and is given by

$$t_g = \left(\frac{3}{2}\frac{\lambda_f \Delta\theta}{K_s}\right)^2 \qquad (27)$$

The coefficient λ_f represents the contribution of capillarity and can be quantified with (23). The following discussion compares data from infiltration experiments described in the literature with predictions from the modified Green-Ampt and Philip models.

Youngs and Price [1981] measured cumulative infiltration versus time during vertical infiltration of water into 10 initially dry soils that varied significantly in grain size, grain shape, and gradation. The soils are the same as those used in their horizontal experiments (see Table 1). Because of the large physical differences among the soils, the condition of geometric similitude was only weakly satisfied. A pressure head of 0.003 m was applied to the surface, and saturated moisture contents and hydraulic conductivities were measured. For the purposes of this comparison we divided the cumulative infiltration measurements by the saturated moisture contents to obtain approximate wetting front depths. Predictions with the modified Green-Ampt and Philip models were generally within 50% of measured results. Typical results are shown in Figure 3. Predictions of the two solutions may differ, because the modified Philip solution is mathematically less exact than the modified Green-Ampt model; however, the differences are small.

Parlange et al. [1985] infiltrated water under a constant inlet head into a 0.94-m high column filled with Grenoble sand ($k = 4.3 \times 10^{-12}$ m^2). The sand was moistened to an initial water content of 0.08 cm^3/cm^3 ($S_i = 0.21$). A constant pressure head of 0.023 m was maintained at the soil surface during infiltration, and gamma ray attenuation was used to measure water contents at several times. The pressure and conductivity were determined as functions of the water content from transient flow data. Measured wetting front depths and the depths predicted from the modified Green-Ampt and Philip models closely agree, as shown in Figure 4.

Davidoff and Selim [1986] performed field infiltration experiments on Norwood fine to silty sandy loam with an initial saturation of about 0.37. The soil was treated with three different winter crop covers that used combinations of wheat, nitrogen, and common vetch. Water was held in 0.45-m diameter infiltrometer rings at a pressure head of 0.1 m, the volume of water entering the soil was monitored as a function of time, and saturated hydraulic conductivities were measured in the laboratory. We calculated average porosities (~0.44) and changes in moisture content (~0.27) in the wetted region from measured values of bulk density and initial water content, assuming a solid density of 2.65 g/cm^3. The actual depth to the wetting front was approximated with the cumulative infiltration divided by the difference between initial and saturated mois-

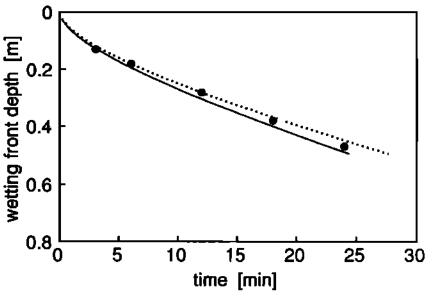

Figure 4. Data (symbols) measured by *Parlange et al.* [1985] and predictions with the modified Green-Ampt (solid curve) and modified Philip (dotted curve) models for vertical infiltration of water into uniform Grenoble sand.

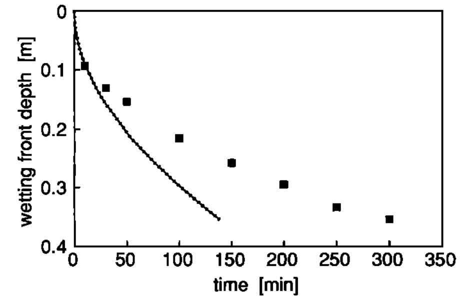

Figure 5. Data (symbols) measured by *Davidoff and Selim* [1986] and predictions with the modified Green-Ampt (solid curve) and modified Philip (dotted curve) models for vertical infiltration of water into Norwood sandy loam using a wheat and nitrogen crop cover.

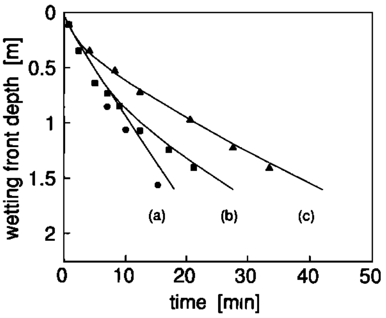

Figure 6. Data (symbols) measured by *Childs and Bybordi* [1969] and modified Green-Ampt predictions (curves) for vertical infiltration of water into (a) coarse sand, (b) 0.64 m of coarse sand over fine sand, and (c) 0.21 m of coarse sand over fine sand.

ture contents. Model predictions were generally within ±50% of the approximate experimental depths, with no significant trend with type of crop cover. Comparisons for the soil with a wheat and nitrogen crop cover ($k = 0.11 \times 10^{-12}$ m^2) showed the best agreement between the data and predictions and are shown in Figure 5.

The previously described experiments were generally not performed long enough for gravity to dominate infiltration. Only in Figure 3a(I) did the infiltration time exceed t_g.

Infiltration into layered soil was investigated by *Childs and Bybordi* [1969]. They visually recorded the movement of the wetting front during water infiltration into initially air-dried columns packed with a coarser sand fraction ($k = 48 \times 10^{-12}$ m^2) overlying a finer fraction ($k = 15.0 \times 10^{-12}$ m^2), and they modified the Green-Ampt relationship for the general case of multiple layers. For two layers their result is

$$t - t_1 = \frac{\Delta\theta_2}{K_2}\left[z - z_1 + \left(\frac{z_1 K_2}{K_1} - z_1 - \Delta h\right)\ln\left(\frac{z + \Delta h}{z_1 + \Delta h}\right)\right] \tag{28}$$

where parameters of the upper and lower layers are denoted with the subscripts 1 and 2, respectively. The constant z_1 is the thickness of the upper layer, t_1 is the time for the wetting front to reach z_1, z is the depth from the surface to the wetting front, and $\Delta h = h_o - h_f$. Equation (28) holds when z is greater than z_1 and can be derived by transforming the upper layer into a hydraulically equivalent layer of conductivity K_2. Childs and Bybordi obtained hydraulic conductivity values by fitting the Green-Ampt equation to preliminary infiltration experiments performed on each sand fraction. Values of wetting front depth predicted from (28) using a wetting front suction given by (24) are compared with measured depths in Figure 6. All three curves follow the pattern of an initially rapid descent that decreases with depth to a velocity approaching the value of the saturated hydraulic conductivity.

Relationships for Infiltration Under Negative Inlet Heads (h_o < 0)

The preceding analysis can be extended to the case of negative source pressures, which act in opposition to the suction forces that try to draw liquid into the soil. When the source pressure is more negative than the air entry pressure of the soil, the liquid will only move through selected flow paths, resulting in a partially saturated region. The following analysis begins with horizontal infiltration into initially dry soil.

The effect of a negative inlet head on the capillary tube model is to limit liquid movement to selected channels. Only tubes with capillary suction heads more negative than the inlet head will fill and transmit water, while larger tubes with less capillary suction will remain unsaturated, as illustrated in Figure 7. The connectors between the model capillary tubes help maintain a steep wetting front by allowing liquid to distribute from larger to smaller conducting tubes. A negative inlet head of h_o will allow capillary tubes of radii R_o and smaller to fill, where R_o is given by

$$R_o = -\frac{2\sigma}{\rho g h_o} \tag{29}$$

Liquid then flows through a matrix of smaller tubes that forms a saturated porous medium within the unsaturated system. As in the previous case of zero inlet head, the saturated microfabric can be modeled as a set of parallel capillary tubes of uniform radius R with permeability k_m corresponding to the permeability of the microfabric. The resulting relationship de-

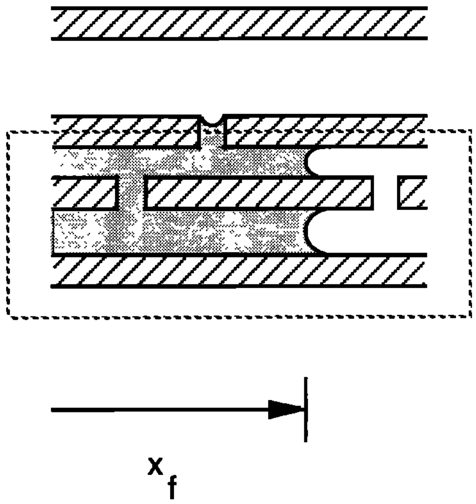

Figure 7. Capillary tube representation of infiltration into soil under negative inlet head. The boxed area corresponds to a microfabric that is completely saturated behind the wetting front.

Table 4. Comparison of Predicted $\lambda_p = x_f t^{-1/2}$ and Measured $\lambda_m = x_f t^{-1/2}$ for Horizontal Infiltration of Water at Various Inlet Heads Into Two Initially Air-Dried Soils

Soil	h_o, m	$\Delta\theta$, cm^3/cm^3	kk_r, 10^{-12} m^2	h_f, m	λ_p/λ_m	Source
Slate dust	0.01	0.40	0.22	−0.8	1.1	*Peck* [1964]
	−0.10	0.40	0.22	−0.8	1.2	
	−0.19	0.40	0.22	−0.8	1.2	
	−0.31	0.40	0.22	−0.8	1.3	
	−0.39	0.37	0.16	−0.9	1.5	
	−0.50	0.32	0.07	−1.2	1.8	
	−0.60	0.28	0.04	−1.5	2.5	
Columbia silt loam	−0.02	0.42	0.77	−0.4	1.2	*Davidson et al.* [1963]
	−0.50	0.39	0.25	−0.7	1.0	
	−1.00	0.29	0.02	−2.4	3.2	

The variable h_f is the predicted suction head at the wetting front.

scribing the length of the wetted region during horizontal infiltration is then

$$x_f = B\left(\frac{h_f - h_o}{h_f}\right)^{1/2}\left(\frac{\sigma}{\mu}\right)^{1/2} k_m^{1/4}\sqrt{t} \tag{30}$$

The wetting phase permeability kk_r of the overall medium behind the wetting front is given by

$$kk_r = \frac{\mu K(h_o)}{\rho g} \tag{31}$$

where k is the intrinsic permeability and k_r is the relative permeability. Darcy's law relates k_m to kk_r:

$$k_m = \frac{a_T}{a_m} kk_r \simeq \frac{1}{S_w} kk_r \tag{32}$$

where a_m is the cross sectional area of the saturated microfabric, a_T is the total cross sectional area of the soil element, and S_w is the saturation, or the fraction of pore space filled with liquid, in the wetted region. Equation (30) then becomes

$$x_f = B\left(\frac{1}{S_w}\right)^{1/4}\left(\frac{h_f - h_o}{h_f}\right)^{1/2}\left(\frac{\sigma}{\mu}\right)^{1/2}(kk_r)^{1/4}\sqrt{t} \tag{33}$$

The wetting front suction head h_f can be obtained by equating (33) with (2), giving

$$h_f = -\frac{\sigma B^2 \Delta\theta}{2\rho g\sqrt{S_w kk_r}} \tag{34}$$

For initially moist soil the relationships become

$$x_f = \lambda_f \sqrt{t} \tag{35}$$

where

$$\lambda_f = \frac{B}{(1 - S_i)^{3/4}}\left(\frac{1}{S_w}\right)^{1/4}\left(\frac{h_f - h_o}{h_f}\right)^{1/2}\left(\frac{\sigma}{\mu}\right)^{1/2}(kk_r)^{1/4} \tag{36}$$

$$h_f = -\frac{\sigma B^2 \Delta\theta}{2\rho g\sqrt{S_w kk_r}(1 - S_i)^{3/2}} \tag{37}$$

Vertical infiltration can also be modelled with the Green-Ampt equation (3) or the piecewise formulation modified from Philip, as in the case of zero inlet head, with λ_f and h_f obtained from (36) and (37).

The model was assessed by comparison with measured data from horizontal infiltration experiments performed by *Peck* [1964] and *Davidson et al.* [1963]. The relevant parameters needed to evaluate the model were measured in both investigations. *Peck* [1964] performed horizontal infiltration experiments at effective inlet pressure heads ranging from +0.01 to −0.60 m by changing the air pressure in the column. For each inlet head the value of $\lambda_f = x_f t^{-1/2}$ corresponding to the wetting front was measured from the data and compared with the corresponding predicted values, as shown in Table 4. According to measurements presented by Peck, the slate dust should be saturated at pressure heads ranging from 0 to about −0.33 m; predicted and experimental values compared well over this range. Discrepancies tended to increase with increasingly negative inlet heads.

Davidson et al. [1963] performed horizontal and vertical infiltration experiments on Columbia silt loam with a fritted glass bead plate connected to a constant head buret to maintain negative inlet heads. The loam was initially air dried at a moisture content of 0.031 cm^3/cm^3. Horizontal experiments were performed at h_o = −0.02 m, −0.5 m, and −1.0 m, while vertical experiments were performed at these inlet heads and at −0.75 m (kk_r = 0.04 × 10^{-12} m^2). Measured data and predictions for Columbia silt loam are shown in Figure 8 and Table 4. The low permeability of the loam prevented gravity from dominating over the timescale of the vertical infiltration experiments; the linear trend of the data in Figure 8 illustrates the influence of capillarity in these experiments.

For both slate dust and Columbia silt loam, model predictions are less accurate as the inlet head h_o decreases, because lower inlet heads result in lower liquid saturations within the wetted region. The lower the moisture content, the greater the air saturation in the wetted region blocking water movement between small pores. The model representation in Figure 7 assumed that there was a continuous microfabric.

Infiltration of Nonaqueous Phase Liquids

The formulation of the capillary tube model for wetting fronts is directly applicable to nonaqueous phase wetting liquids (NAPLs), as indicated by the experiments performed with alcohol included in Table 1. Our discussion of NAPL infiltration will be limited to initially dry soil because of the difficulty in quantifying water displacement in initially water-moist soil. In this section the model is compared with data from infiltration experiments that used oil as the infiltrating liquid.

Weaver et al. [1994] used a falling head boundary condition during vertical infiltration of Soltrol into sand. Approximately 126 cm^3 of dyed Soltrol 220 (σ = 0.025 N/m; μ = 4.8 × 10^{-3}

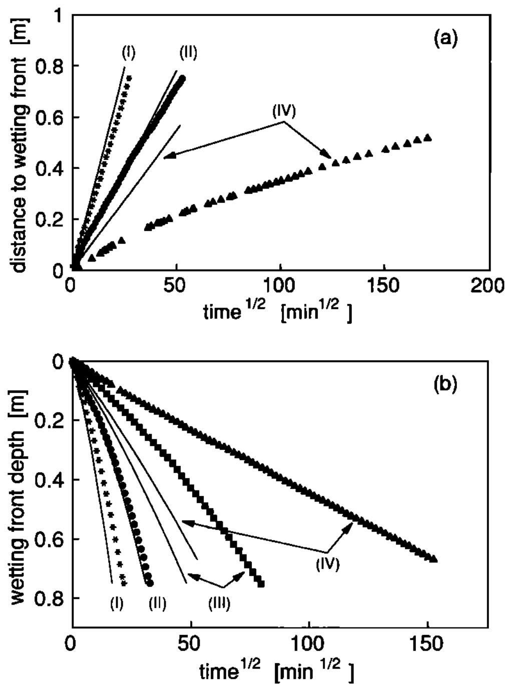

Figure 8. Data (symbols) measured by *Davidson et al.* [1963] and modified Green-Ampt predictions (curves) for (a) horizontal and (b) vertical infiltration of water into Columbia silt loam. The inlet head was held at (I) −0.02 m, (II) −0.5 m, (III) −0.75 m, and (IV) −1.0 m.

Pa s; ρ = 790 kg/m^3) were released at the surface of a column packed with uniform sand having a permeability of 90×10^{-12} m^2. The sand had a porosity of 0.41 and was water wet to a saturation of 0.06. The investigators visually observed the front position and the ponded depth, which decreased from a maximum of 6 cm. *Reible et al.* [1990] modified the Green-Ampt solution for a falling head boundary condition by setting the ponded height h equal to the initial ponded height h_o minus the quantity infiltrated after the head began to fall:

$$h = h_o - \theta_{oil} z_f \tag{38}$$

θ_{oil} is the quantity of oil behind the wetting front in cm^3 of oil per cm^3 total soil volume, and z_f is the depth of the wetting front. Combining this relationship with Darcy's law gives

$$t = \frac{\theta_{oil}\mu}{\rho g k k_r}\left[\frac{z_f - z_1}{1 - \theta_{oil}} - \frac{h_o - h_f}{(1 - \theta_{oil})^2}\ln\left(\frac{(1 - \theta_{oil})z_f - h_f + h_o}{(1 - \theta_{oil})z_1 - h_f + h_o}\right)\right] + t_1 \tag{39}$$

where z_1 and t_1 are the depth and time at which the boundary condition changes from constant to falling head. The value of h_f is then obtained from (34) with S_w replaced by the saturation of oil and $\Delta\theta$ equal to θ_{oil}. Figure 9 compares the observed locations of the front with those calculated from (39). The oil saturation and the relative oil permeability were set equal to 1 in the model calculations. All ponded Soltrol had infiltrated into the soil after about 7 min. Because the ponded depth was so small, predictions assuming a constant head boundary condition over the 7-min period yield almost identical results.

An analysis of data measured by *King* [1964] illustrates the sensitivity of predictions to errors in estimates of relative per-

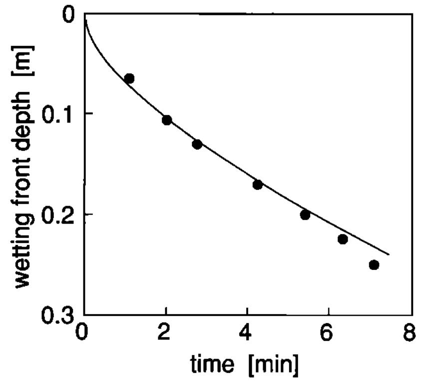

Figure 9. Data (symbols) measured by *Weaver et al.* [1994] and modified Green-Ampt predictions (curve) for vertical infiltration of Soltrol into uniform sand under a falling head boundary condition.

meability and wetting front suction head. A test of our proposed model against reported data for vertical infiltration of Soltrol C, a light oil, into air-dried Touchet silt loam at an inlet head of −0.735 m of oil shows significant deviations in predictions when h_f is decreased by a factor of 2 or k_r is increased by a factor of 2 while holding all other variables constant, as seen in Figure 10. King performed a number of additional horizontal and vertical infiltration experiments under negative inlet heads that were poorly predicted by the modified Green-Ampt model. In half the cases the imposed inlet head was more negative than the value of h_f calculated from (34), which resulted in the erroneous prediction that no liquid should have entered the soils. Our analysis of King's data set indicates that the model is more sensitive to errors in h_f than in k_r.

Discussion

This paper presents a wetting front model for fluid infiltration into porous media that captures the fundamental physics of wetting fluid flow by capillary suction. The extension from the initial capillary tube model to porous media requires evaluation of a dimensionless constant B from horizontal infiltra-

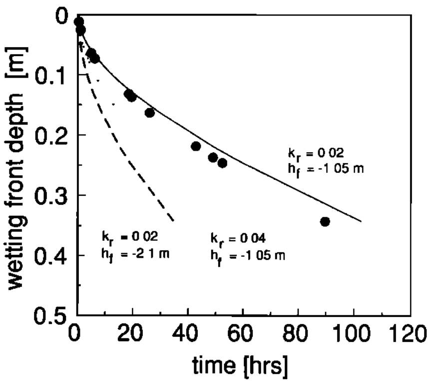

Figure 10. Data (symbols) measured by *King* [1964] and model predictions (curves) for vertical infiltration of Soltrol C into Touchet silt loam at an inlet head of −0.735 m of oil. The solid curve is the prediction using the k_r value measured by King and the h_f value obtained from (37); the dotted and dashed curves are predictions obtained by changing the values of k_r or h_f by a factor of 2.

tion experiments performed under ideal conditions of zero inlet head and initially dry media. There is some confidence in the value of the parameter, given the range of media, permeability, and fluids used in the reported experiments. A further extension to horizontal infiltration for partially wet porous media and for negative inlet heads also finds support in experimental data available in the literature. The results of the horizontal infiltration analysis are used to modify the Green-Ampt and Philip vertical infiltration models with some predictive success, again with minimal parameter requirements.

The idea that allows the current analysis to extrapolate from capillary tubes to porous media is that most soils, particularly granular soils, have pore structures similar enough to transport fluids in the same manner. This means that the difference in transport properties between capillary tubes and soils can be addressed through the coefficient B for a wide variety of soils. This geometric similarity is supported by the observation that permeability and soil suction have been modeled in very different soil types using the same mathematical relationships with few fitting parameters [*Brooks and Corey*, 1966; *van Genuchten*, 1980].

The separation of liquid and structural influences in the proposed model allows nonideal conditions to be addressed with relative ease, unlike alternate models proposed in the literature. Wetting nonaqueous phase liquids and varying environmental conditions, such as temperature changes, are taken into account by inserting the appropriate values of interfacial tension, density, and viscosity of the infiltrating liquid. The mathematical simplicity of the formulation allows for relatively painless adaptations to infiltration into initially moist soil, falling head boundary conditions, and layered media, as was previously shown.

In contrast to the case of nonnegative source pressure, the proposed model does not consistently predict infiltration under negative inlet heads. The expectation is that the model will work well at inlet heads between zero and the bubbling pressure head, because the wetted region should be close to liquid saturation. This has been confirmed by *Peck*'s [1964] data on slate dust, as shown in Table 4. When the inlet head is more negative, errors increase because air bubbles block liquid movement, causing the unsaturated hydraulic conductivity behind the wetting front to decrease as the length of the wetted region increases. In addition, the model cannot consistently predict functional values of wetting front suction head when the imposed head is negative, because relatively small errors (a factor of 2) can result in unsuccessful predictions, as indicated by the analysis of *King*'s [1964] data. The model fails to adequately capture the physical processes for these cases, and a more refined method is necessary. During infiltration under zero or positive inlet heads, however, calculation of the wetting front suction head is only necessary when using the Green-Ampt vertical infiltration equation, and, in this case, predictions are relatively insensitive to errors in the calculated wetting front suction head and agree well with measured data.

The relevance of the $(\sigma/\mu)^{1/2}$ factor in describing wetting front movement has been noted in previous discussions on infiltration [*Swartzendruber*, 1954; *Jackson*, 1963b], but the complete connection to (13) had not been made. *Zimmerman and Bodvarsson* [1989, 1991], for example, use a nondimensionalized form of the Boltzmann transform to derive approximate closed form solutions for the length of the wetted region during horizontal infiltration under positive or zero inlet heads, assuming the potential and hydraulic conductivity relationships can be described by *van Genuchten* [1980] or *Brooks and Corey* [1966] formulas. Both analyses show the length of the wetted region to increase with the square root of time as a function of moisture content, permeability, and empirical parameters. If both the inlet head and the initial moisture content are zero, the solution using the van Genuchten formulas becomes

$$x_f = \left(\frac{2k \dfrac{n+1}{n}}{\alpha \mu \theta_s^m m^{1/n} (\theta_s - \theta_r)^{1/n}} \right)^{1/2} t^{1/2} \tag{40}$$

and the solution using the Brooks and Corey formulas reduces to

$$x_f = \left[\frac{2k}{\alpha\mu} \left(\frac{1}{\theta_s} + \frac{1}{2\eta(\theta_s - \theta_r)} \right) \right]^{1/2} t^{1/2} \tag{41}$$

where θ_s and θ_r are the saturated and residual moisture contents, α, n, and η are empirical parameters, and $m = 1 - 1/n$. Comparison of the van Genuchten and Brooks-Corey formulas shows that α is inversely proportional to the bubbling pressure, which is in turn proportional to $\sigma/k^{1/2}$. These observations reduce both (40) and (41) to the proportionality

$$x_f \propto (\sigma/\mu)^{1/2} k^{1/4} t^{1/2} \tag{42}$$

Again, the same combination of interfacial tension, liquid viscosity, and permeability is seen to be the key factor in predicting the length of the wetted region during infiltration. The *Zimmerman and Bodvarsson* [1989, 1991] approaches were not extended to negative inlet heads.

The proposed model shows potential for being a reliable and convenient method of estimating the extent of horizontal and vertical infiltration under nonnegative inlet heads. Finer details, such as the variation in moisture content in the wetted region, cannot be predicted and must be obtained by actual measurements. However, measurements are necessary to check even the most precise numerical models, simply because subsurface uncertainties preclude highly accurate modeling results.

The proposed model does not provide new information or predict what cannot already be estimated by current models and methods, but rather it synthesizes established ideas and selectively emphasizes the most pertinent concepts to obtain predictions more easily and quickly than other models, without necessarily sacrificing accuracy. Current laboratory investigations are further exploring the accuracy of the model.

Using a plug flow model of infiltration, the Green-Ampt equations and the piecewise formulation of Philip can be modified and combined to minimize the number of parameters needed to model one-dimensional horizontal and vertical infiltration. Minimizing parameter requirements in turn minimizes inaccuracies due to measurement errors. For infiltration under zero inlet head the necessary parameters are the air-liquid interfacial tension, the viscosity of the liquid, the initial saturation, and the permeability of the soil. A simple relationship for wetting front suction head is derived and is needed to predict infiltration under negative inlet heads. The model can be directly applied to nonaqueous phase wetting liquids. Comparisons of predictions with data reported in the literature are good when the inlet head is zero but are less successful for infiltration under negative inlet heads because of the sensitivity of the model to errors in estimates of wetting front suction head and relative permeability.

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