Assignment 4 (Poison Solver)

(a) Scalability Study on One Node

Domain Size: 100×100 (omega = 1.1)

Ranks	STD Time (s)	RB Time (s)	STD Speedup	RB Speedup	RB Advantage
1	2.82	0.59	1.00×	1.00×	4.78×
2	1.47	0.36	1.92×	1.64×	4.08×
4	0.79	0.25	3.57×	2.36×	3.16×
8	0.47	0.20	6.00×	2.95×	2.35×
16	0.30	0.16	9.40×	3.69×	1.88×
32	0.25	0.17	11.28×	3.47×	1.47×
64	0.35	0.30	8.06×	1.97×	1.17×
72	0.45	0.43	6.27×	1.37×	1.05×

Observations: Red-Black is consistently faster. Both solvers show best performance around 16-32 processes Afterwards degradation beyond 32 ranks due to communication overhead dominates for small domains

Domain Size: 400×400 (omega = 1.1)

Ranks	STD Time (s)	RB Time (s)	STD Speedup	RB Speedup	RB Advantage
1	756.11	148.72	1.00×	1.00×	5.08×
2	378.94	75.81	2.00×	1.96×	5.00×
4	190.72	39.20	3.96×	3.79×	4.87×
8	96.66	20.97	7.82×	7.09×	4.61×
16	49.81	11.93	15.18×	12.47×	4.18×
32	27.21	7.45	27.79×	19.96×	3.65×
64	16.79	5.94	45.03×	25.04×	2.83×

Ranks	STD Time (s)	RB Time (s)	STD Speedup	RB Speedup	RB Advantage
72	15.21	5.58	49.71×	26.66×	2.73×

Observations: Near-linear scaling up to 32 ranks. RB maintains is consistent approx. 3-5 times faster. Communication overhead becomes less critical.

(b) Solver Stability Analysis

Impact of Domain Size (omega = 1.1)

Domain	Ranks	STD Iterations	RB Iterations	Iteration Increase
100	1	26,867	26,870	~0%
200	1	106,635	106,639	~0%
300	1	239,289	239,293	~0%
400	1	424,829	424,833	~0%

Observation:Iterations scale approximately as O of N squared, where N is the domain size.

Impact of Process Count (Domain = 250×250, omega = 1.1)

Ranks	STD Iterations	RB Iterations	STD Degradation	RB Degradation
1	166,352	166,355	baseline	baseline
2	166,586	166,355	+0.14%	0%
4	167,268	166,302	+0.55%	-0.03%
8	168,873	166,383	+1.52%	+0.02%
16	171,278	165,721	+2.96%	-0.38%
32	177,735	166,059	+6.84%	-0.18%
64	189,025	165,400	+13.62%	-0.57%
72	190,416	164,079	+14.46%	-1.37%

Observation: STD solver degrades significantly and has to use up to 14.5% more iterations needed at 72 ranks and RB solver remains stable: Iteration count slightly reduced with higher number of ranks

Impact of Omega (Domain = 250×250)

omega = 1.5 (Moderate Over-relaxation)

Ranks	STD Iterations	RB Iterations	STD Convergence	RB Convergence
1	70,937	70,940	Yes	Yes
16	76,454	70,688	Yes	Yes
32	82,928	70,836	Yes	Yes
64	94,959	70,553	Yes	Yes
72	97,266	70,014	Yes	YES

Observation: Both solvers are stable, but STD shows stronge degradation in comparison to RB.

omega = 1.8 (Aggressive Over-relaxation)

Ranks	STD Iterations	RB Iterations	STD Convergence	RB Convergence
1	25,741	25,741	Yes	Yes
16	31,743	25,665	Yes	Yes
32	20,916	25,726	**NaN**	Yes
64	3,437	25,615	**NaN**	Yes
72	3,039	25,435	**NaN**	Yes

Observation: STD solver becomes unstable at high rank counts and with aggressive over-relaxation.

ω = 1.95 (Near-instability threshold)

Ranks	STD Iterations	RB Iterations	STD Convergence	RB Convergence
1	7,340	7,331	Yes	Yes
4	8,585	7,333	Yes	Yes
8	18,781	7,335	**NaN**	Yes
16	7,487	7,324	**NaN**	Yes
32	3,756	7,355	**NaN**	Yes
64	1,961	7,308	**NaN**	Yes
72	1,805	7,270	**NaN**	Yes

Observation: STD becomes even more unstable at earlier ranks.

(c) Convergence Comparison to Sequential Case

Iteration Count Analysis

Do-main	omega	Sequential STD	Sequential RB	Parallel STD (64 ranks)	Parallel RB (64 ranks)
100	1.1	26,867	26,870	35,918 (+33.7%)	26,458 (-1.5%)
200	1.1	106,635	106,639	125,675 (+17.9%)	107,086 (+0.4%)
300	1.1	239,289	239,293	263,647 (+10.2%)	235,603 (-1.5%)
400	1.1	424,829	424,833	461,916 (+8.7%)	424,516 (-0.1%)
250	1.5	70,937	70,940	94,959 (+33.9%)	70,553 (-0.5%)
250	1.8	25,741	25,741	**NaN (diverged)**	25,615 (-0.5%)

Observations: RB maintains number of iterations in sequential convergence. STD degrades significantly with 8-34% more iterations needed in parallel. Degradation is worse for small domains.

(d) Response to Different Omega Values

Optimal Omega Analysis

omega	Description	STD Sequential	RB Sequential	STD Parallel (64)	RB Parallel (64)
1.1	Safe under-relaxation	166,352 iter	166,355 iter	189,025 iter	165,400 iter
1.5	Moderate SOR	70,937 iter	70,940 iter	94,959 iter	70,553 iter
1.8	Aggressive SOR	25,741 iter	25,741 iter	**Diverges**	25,615 iter
1.95	Near-optimal	7,340 iter	7,331 iter	**Diverges**	7,308 iter

The **Optimal omega** is around 1.95 for this problem.