Solve all problems while demonstrating each step clearly. The Quiz is worth a total of 10 points

1. Suppose an asset has the following factor sensitivities: $\beta_1 = 0.5, \beta_2 = -0.3, \beta_3 = 0.7$. If the expected returns of the three factors are $8\%,\ 6\%,$ and 10% respectively, and the risk-free rate is 4% calculate the expected return of the asset according to arbitrage pricing theory.

$$E(R_i) - \ expected\ return\ of\ an\ asset$$

$$R_f - \ risk\ free\ rate$$

$$\beta_1,\ \beta_2,\ \beta_3 - \ sensitivity\ of\ asset$$

$$E(F_1), E(F_2), E(F_3) - \ expected\ returns\ of\ the\ assets$$

$$E(R_i) = R_f + B_1 * E(F_1) + B_2 * E(F_2) + B_3 * E(F_3)$$

$$= \frac{4}{100} + 0.5 * \frac{8}{100} + (-0.3 * \frac{6}{100}) + 0.7 * \frac{10}{100}$$

In [25]:
```
(0.5 *(8/100))
```
Out[25]: 0.04

In [26]:
```
round((-0.3 * (6/100)), 4)
```
Out[26]: -0.018

In [28]:
```
round(0.7 * (10/100), 4)
```
Out[28]: 0.07

In [30]:
```
0.04 + 0.04 - 0.018 + 0.07
```
Out[30]: 0.132

$$ans\ = \ 0.132$$

2. Consider a portfolio consisting of two assets: Asset A and Asset B. Asset A has an expected

return of 10% and a standard deviation of 15%, while Asset B has an expected return of 8% and a standard deviation of 10%. If you allocate 60% of your portfolio to Asset A and 40% to Asset B, what is the expected return and variance of your portfolio?

$$E(R_A),\ E(R_B) - expected\ returns\ asset\ A\ and\ B\ respectively$$

$$w_a,\ w_b - weight\ of\ asset\ a\ and\ b$$

$$E(R_p) -\ expected\ return\ of\ portfolio$$

$$E(R_p) = w_a * E(R_A) + w_b * E(R_B)$$

$$= 0.6 * 0.1 + 0.4 * 0.08$$

In [6]:
```
0.6 * 0.1 + 0.4 * 0.08
```

Out[6]:
```
0.092
```

$E(R_p) = 0.092 = 9.2\%$

$$var(R_p) = w_a * var(A) + w_b * var(B) + w_a * w_b * cov(A, B)$$

$$cov(A, B) = 0$$

$$var(A) = sd(A)^2, var(B) = sd(B)^2$$

$$var(A) = (0.6 * 0.15)^2$$

$$var(B) = (0.4 * 0.1)^2$$

In [31]:
```
((0.6 * 0.15) **2) + ((0.4*0.1) **2) + 0
```

Out[31]:
```
0.0097
```

$var(R_p) = 0.0097$

3. Given a corporate bond with a yield of 6% and a similar maturity risk-free rate of 3%, calculate the credit spread.

$$credit\ spread = yield - risk - free\ rate$$

$$= 0.06 - 0.03$$

$credit\ spread = 3$

4. Calculate the credit valuation adjustment (CVA) for a portfolio of derivative contracts with a total notional value of $10 million, a default probability of 1%, and an LGD of 40%.

$$cVA = Exposure * Default\ probability * LGD$$

$$= 10,000,000 * 0.01 * 0.4$$

In [8]:
```
10_000_000 * 0.01 * 0.4
```

Out[8]:
```
40000.0
```

$credit\ valuation\ adjustment = 40,000$

5. Calculate the Sharpe ratio for a portfolio with an average annual return of 10% and a standard deviation of returns of 15%. The risk-free rate is 3%.

$$E(R_p) - \ is\ the\ expected\ return\ of\ the\ portfolio.$$

$$R_f - \ is\ the\ risk-free\ rate.$$

$$\sigma - is\ the\ standard\ deviation\ of\ the\ portfolio's\ returns.$$

$$sharpe\ ratio = \frac{E(R_p) - R_f}{\sigma}$$

$$= \frac{0.1 - 0.03}{0.15}$$

In [9]:
```
(0.1 - 0.03)/0.15
```

Out[9]:
```
0.46666666666666673
```

$sharpe\ ratio = 0.466667$

6. Calculate the beta of a stock with an average return of 12% and a standard deviation of returns of 18%. The average return of the market is 8%, and its standard deviation of returns is 12%.

the stock has an internal correlation of 1 with the market.

ie. The growth of the stock is directly dependent on the market

$Cor(R_i, R_m) = 1$

$\therefore$

$$\beta(sensitivity) = \frac{Cov(R_i\ (stock's\ returns) - R_m\ (marketreturns))}{var(R_M)}$$

$$\beta = Cor(R_i, R_m) * \frac{\sigma_i}{\sigma_m}$$

$$sensitivity = \frac{0.18}{0.12}$$

In [10]:
```
0.18/0.12
```

Out[10]:
```
1.5
```

$sensitivity = 1.5$

7. Consider a hypothetical portfolio with the following thirty historical returns data:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Return (%)	-1.2	0.5	1.0	-0.7	-1.5	2.3	-0.4	0.6	-2.1	1.2	0.8	-1.1	1.5	-0.3	2.0

$$Table\ 1:\ Historical\ Returns\ Data\ (Days\ 1-15)|$$

Day	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Return (%)	-1.7	0.4	-0.5	1.8	-2.2	0.9	1.1	-1.6	2.4	-0.9	1.3	-0.8	0.7	-1.9	1.4

$$Table\ 2:\ Historical\ Returns\ Data\ (Days\ 16-30)$$

Calculate the volatility of the returns and determine both the VaR and CVaR of the portfolio using the historical method on returns data, the parametric method, and Monte Carlo simulation. Comment on the results by the three methods and give a convincing justification

In [11]:
```
import numpy as np
returns = np.array([-1.2, 0.5, 1.0, -0.7, -1.5, 2.3, -0.4, 0.6, -2.1, 1.2
                    0.4, -0.5, 1.8, -2.2, 0.9, 1.1, -1.6, 2.4, -0.9, 1.3,
```

```python
returns = np.sort(returns)
returns
```

Out[11]:
```
array([-2.2, -2.1, -1.9, -1.7, -1.6, -1.5, -1.2, -1.1, -0.9, -0.8, -0.7,
       -0.5, -0.4, -0.3,  0.4,  0.5,  0.6,  0.7,  0.8,  0.9,  1. ,  1.1,
        1.2,  1.3,  1.4,  1.5,  1.8,  2. ,  2.3,  2.4])
```

In [12]:
```python
ret_elements = list(np.array(returns.shape))[0]
ret_elements
```

Out[12]:
```
30
```

In [13]:
```python
volatility = np.std(returns)
volatility
```

Out[13]:
```
1.3650396819628847
```

cVAR and VAR of the portfolio using Historical method

In [14]:
```python
# using the confidence level of 95%
import math
loss_level_hist = math.ceil(0.05 * ret_elements)
loss_level_hist
```

Out[14]:
```
2
```

In [24]:
```python
var_historical = returns[:loss_level_hist, ][-1]
var_historical
```

Out[24]:
```
-2.1
```

$VAR = -2.1$

In [16]:
```python
cvar_historical = np.mean(returns[:loss_level_hist,])
cvar_historical
```

Out[16]:
```
-2.1500000000000004
```

cVAR = -2.15

cVAR and VAR of the portfolio using Parametric Method

In [17]:
```python
mean_returns = np.mean(returns)
sd_returns = volatility
z_val = -1.65
```

$$VAR = \mu + (z_{score} * \sigma)$$

In [18]:
```python
var_parametric = mean_returns + (z_val * sd_returns)
var_parametric
```

Out[18]:
```
-2.1523154752387597
```

$$cVAR = \mu + \frac{\phi(z_{score}) * \sigma}{1 - C.L}$$

ϕ - standard normalization of the pdf

In [19]:
```
from scipy.stats import norm
phi_data = norm.pdf(z_val)

cVAR_parametric = mean_returns - ( (phi_data * sd_returns)/ (1 - 0.95))
cVAR_parametric
```

Out[19]:
```
-2.691913602055417
```

cVAR using parametric method = -2.6191

VAR using parametric method = -2.1523

cVAR and VAR of the portfolio using Monte Carlo method

In [20]:
```
generated_random_data = np.sort(np.random.normal(mean_returns, sd_returns
generated_random_data
```

Out[20]:
```
array([-5.42086659, -4.78354865, -4.77336518, ...,  4.94524895,
        4.9695628 ,  5.68176636])
```

In [21]:
```
element_at_5_percent = math.ceil(10000 * 0.05)
element_at_5_percent
```

Out[21]:
```
500
```

In [22]:
```
var_monte_carlo = generated_random_data[500,]
var_monte_carlo
```

Out[22]:
```
-2.200808787243964
```

In [23]:
```
cvar_monte_carlo =  np.mean(generated_random_data[:element_at_5_percent,]
cvar_monte_carlo
```

Out[23]:
```
-2.7230982715440586
```

cVAR using parametric method = -2.11668

VAR using parametric method = -2.702208