Probability and Statistics in Finance

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Contents

CHAPTER 1

Probability and Statistics in Finance

The use of random variables, expectation, variance, correlation, and regression is fundamental in mathematical finance. Random variables represent the inherent uncertainty in financial markets, such as stock prices or interest rates. Expectation helps us estimate the average outcome we can expect from these variables. Variance measures the spread or risk associated with those outcomes. Correlation captures the degree to which two financial instruments move together, while regression allows us to model the relationship between a dependent variable (like stock price) and one or more independent variables (like economic indicators). By employing these concepts, financial professionals can analyze investments, build risk models, and make informed decisions in a complex and ever-changing market.

1.1. Random variables

A **random variable** is a variable whose possible values are numerical outcomes of a random phenomenon. There are two main types of random variables: discrete and continuous. In the context of a measure spaces, we like to think of a random variable as a measurable function to the Borel measure space. Random variables are fundamental in probability theory and are extensively used in mathematical finance to model uncertain outcomes. There are two main types of random variables:

(i). **Discrete Random Variables**: Take on a countable number of distinct values.

(ii). **Continuous Random Variables**: Take on an infinite number of possible values within a given range.

Definition 1.1 (Probability distribution). The probability distribution of a random variable describes how probabilities are distributed over the values of the random variable. For a discrete random variable X, the probability mass function (pmf) $P(X = x)$ gives the probability that X takes on the value x. For a continuous random variable X, the probability density function (pdf) $f(x)$ describes the relative likelihood of X taking on the value x. The probability that X lies within an interval $[a, b]$ is given by the integral of $f(x)$ over that interval: $P(a \leq X \leq b) = \int_a^b f(x)\,dx$.

1.2. Expectation

The **expectation** or **expected value** of a random variable is a measure of the central tendency of its distribution. It represents the average outcome if an experiment is repeated many times.

Definition 1.2 (Discrete and continuous). For a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$ and corresponding probabilities $p_1, p_2, \ldots, p_n$, the expectation $E[X]$ is defined as:

$E[X] = \sum_{i=1}^{n} x_i p_i$. For a continuous random variable X with probability density function (pdf) $f(x)$, the expectation $E[X]$ is defined as: $E[X] = \int_{-\infty}^{\infty} x f(x).\, dx$

1.2.1. Properties of Expectation.

(1) **Linearity**: For any random variables X and Y, and constants a and b:

$$E[aX + bY] = aE[X] + bE[Y]$$

(2) **Non-negativity**: If $X \geq 0$, then $E[X] \geq 0$.

(3) **Expectation of a Constant**: For any constant c, $E[c] = c$.

1.2.2. Expectation in Mathematical Finance. At this point, we give some examples in mathematical finance.

(i). **Expected Return of a Portfolio**

In portfolio theory, the expected return of a portfolio is the weighted average of the expected returns of the individual assets in the portfolio. Let R_p be the return of the portfolio, and let R_i be the return of asset i with weight w_i. Then the expected return $E[R_p]$ is given by:

$$E[R_p] = \sum_{i=1}^{n} w_i E[R_i].$$

Example 1.3. Consider a portfolio with two assets. Asset 1 has an expected return of 8% and weight of 60%, while asset 2 has an expected return of 12% and weight of 40%. The expected return of the portfolio is:

$$E[R_p] = 0.6 \times 0.08 + 0.4 \times 0.12 = 0.048 + 0.048 = 0.096 \quad \text{or} \quad 9.6\%$$

(ii). **Expected Payoff of an Option**

The expected payoff of a financial option can be calculated using its probability distribution. Consider a European call option with strike price K and maturity T. Let S_T be the stock price at maturity.

The payoff of the call option is:

$$\max(S_T - K, 0)$$

To find the expected payoff, we need the expectation of the payoff function with respect to the probability distribution of S_T.

Example 1.4. Assume that S_T follows a log-normal distribution with parameters μ and σ^2. The expected payoff of the call option is:

$$E[\max(S_T - K, 0)] = \int_K^{\infty} (S_T - K) f(S_T)\, dS_T$$

where $f(S_T)$ is the pdf of S_T. This can be evaluated using numerical methods or Monte Carlo simulation in practice.

(iii). **Expected Value of a Bond's Coupon Payment**

Bonds typically pay periodic coupon payments until maturity. The expected value of these payments can be calculated as the sum of the expected values of each individual payment.

EXAMPLE 1.5. Consider a bond with a face value of \$1,000, an annual coupon rate of 5%, and a maturity of 3 years. The annual coupon payment is:

$$C = 0.05 \times 1000 = \$50$$

The expected value of the coupon payments over the 3-year period is:

$$E[\text{Total Coupon Payments}] = 3 \times \$50 = \$150.$$

REMARK 1.6. Expectation is a fundamental concept in probability theory that is extensively used in mathematical finance. It provides a measure of the average outcome and is crucial for calculating expected returns, payoffs, and other financial metrics. Understanding how to compute and interpret expectation helps in making informed financial decisions and managing risks effectively.

1.3. Variance

The **variance** of a random variable is a measure of the spread of its values around the mean. It provides an indication of the degree of uncertainty or risk associated with the random variable.

DEFINITION 1.7 (Discrete and Continuous). For a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$ and corresponding probabilities $p_1, p_2, \ldots, p_n$, the variance $\text{Var}(X)$ is defined as:

$$\text{Var}(X) = E[(X - E[X])^2] = \sum_{i=1}^{n} (x_i - E[X])^2 p_i$$

. For a continuous random variable X with probability density function (pdf) $f(x)$, the variance $\text{Var}(X)$ is defined as:

$$\text{Var}(X) = E[(X - E[X])^2] = \int_{-\infty}^{\infty} (X - E[X])^2 f(x).\,dx$$

1.3.1. Properties of Variance.

(i). **Non-negativity**: $\text{Var}(X) \geq 0$ for any random variable X.

(ii). **Variance of a Constant**: If c is a constant, then $\text{Var}(c) = 0$.

(iii). **Scaling**: For any random variable X and constant a, $\text{Var}(aX) = a^2\text{Var}(X)$.

(iv). **Addition**: For any two independent random variables X and Y, $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$.

1.3.2. Examples in Mathematical Finance.

(1) **Variance of Portfolio Returns**

The variance of the returns of a portfolio provides a measure of the risk associated with the portfolio. For a portfolio consisting of n assets, the portfolio return R_p is a weighted sum of the individual asset returns R_i, with weights w_i.

EXAMPLE 1.8. Consider a portfolio with two assets. Asset 1 has a return R_1 with expected return $E[R_1] = 0.08$ and variance $\text{Var}(R_1) = 0.02$. Asset 2 has a return R_2 with expected return $E[R_2] = 0.12$ and variance $\text{Var}(R_2) = 0.03$. The correlation between the returns is $\rho_{12} = 0.5$, and the weights are $w_1 = 0.6$ and $w_2 = 0.4$.

The variance of the portfolio return R_p is given by:

$$\text{Var}(R_p) = w_1^2\text{Var}(R_1) + w_2^2\text{Var}(R_2) + 2w_1w_2\text{Cov}(R_1, R_2)$$

where the covariance $\text{Cov}(R_1, R_2)$ is:

$$\text{Cov}(R_1, R_2) = \rho_{12}\sigma_1\sigma_2 = 0.5 \times \sqrt{0.02} \times \sqrt{0.03}$$

Substituting the values:

$$\text{Var}(R_p) = 0.6^2 \times 0.02 + 0.4^2 \times 0.03 + 2 \times 0.6 \times 0.4 \times 0.5 \times \sqrt{0.02} \times \sqrt{0.03}$$
$$\text{Var}(R_p) = 0.0072 + 0.0048 + 0.0052 = 0.0172.$$

(2) **Variance of Option Payoffs**

The variance of the payoff of an option provides a measure of the risk associated with the option's returns. Consider a European call option with strike price K and maturity T. Let S_T be the stock price at maturity.

The payoff of the call option is:

$$\max(S_T - K, 0)$$

To find the variance of the payoff, we need the expectation of the squared payoff function with respect to the probability distribution of S_T.

EXAMPLE 1.9. Assume that S_T follows a log-normal distribution with parameters μ and σ^2. The expected squared payoff of the call option is:

$$E[(\max(S_T - K, 0))^2] = \int_K^\infty (S_T - K)^2 f(S_T)\, dS_T$$

where $f(S_T)$ is the pdf of S_T. This can be evaluated using numerical methods or Monte Carlo simulation in practice.

(3) **Variance of Bond Prices**

The variance of the price of a bond provides a measure of the interest rate risk associated with the bond. Bond prices are influenced by changes in interest rates, which can be modeled as a random variable.

EXAMPLE 1.10. Consider a bond with a face value of \$1,000, an annual coupon rate of 5%, and a maturity of 3 years. Let r be the random variable representing the interest rate. The price P of the bond is given by the present value of its future cash flows:

$$P = \sum_{t=1}^{3} \frac{50}{(1+r)^t} + \frac{1000}{(1+r)^3}$$

The variance of the bond price can be approximated using sensitivity analysis or duration and convexity measures. For small changes in interest rates, the variance can be approximated as:

$$\mathrm{Var}(P) \approx \left(\frac{\partial P}{\partial r}\right)^2 \mathrm{Var}(r)$$

where $\frac{\partial P}{\partial r}$ is the duration of the bond.

REMARK 1.11. Variance is a crucial concept in probability theory and mathematical finance. It provides a measure of the risk or uncertainty associated with a random variable. Understanding how to compute and interpret variance is essential for financial modeling, risk management, and decision-making

1.4. Covariance

The **covariance** between two random variables measures the degree to which the variables change together. It provides an indication of the direction of the linear relationship between the variables.

DEFINITION 1.12 (Discrete and continuous). For discrete random variables X and Y with possible values $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$, and joint probabilities p_{ij}, the covariance $\mathrm{Cov}(X, Y)$ is defined as:

$$\mathrm{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - E[X])(y_j - E[Y]) p_{ij}.$$

For continuous random variables X and Y with joint probability density function (pdf) $f(x, y)$, the covariance $\mathrm{Cov}(X, Y)$ is defined as:

$$\mathrm{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - E[X])(y - E[Y]) f(x, y)\, dx.\, dy$$

1.4.1. Properties of Covariance.

(i). **Symmetry**: $\mathrm{Cov}(X, Y) = \mathrm{Cov}(Y, X)$.

(ii). **Linearity**: For any random variables X, Y, and Z, and constants a and b:

$$\mathrm{Cov}(aX + bY, Z) = a\mathrm{Cov}(X, Z) + b\mathrm{Cov}(Y, Z)$$

(iii). **Covariance of a Constant**: For any constant c, $\mathrm{Cov}(X, c) = 0$.

1.4.2. Examples in Mathematical Finance.

(i). **Covariance of Asset Returns**

Covariance is widely used in finance to measure how the returns of two assets move together. It is crucial for portfolio diversification and risk management.

EXAMPLE 1.13. Consider two assets with returns R_1 and R_2. Let $E[R_1] = 0.08$, $E[R_2] = 0.12$, and the covariance between the returns be $\text{Cov}(R_1, R_2) = 0.0015$. This covariance indicates how the returns of the two assets move together.

(ii). **Covariance in Portfolio Theory**

In portfolio theory, the covariance between the returns of different assets is used to calculate the overall risk of the portfolio.

EXAMPLE 1.14. Consider a portfolio with two assets. Asset 1 has a return R_1 with variance $\text{Var}(R_1) = 0.02$, and asset 2 has a return R_2 with variance $\text{Var}(R_2) = 0.03$. The covariance between the returns is $\text{Cov}(R_1, R_2) = 0.0015$, and the weights are $w_1 = 0.6$ and $w_2 = 0.4$.

The variance of the portfolio return R_p is given by:

$$\text{Var}(R_p) = w_1^2\text{Var}(R_1) + w_2^2\text{Var}(R_2) + 2w_1w_2\text{Cov}(R_1, R_2)$$

Substituting the values:

$$\text{Var}(R_p) = 0.6^2 \times 0.02 + 0.4^2 \times 0.03 + 2 \times 0.6 \times 0.4 \times 0.0015$$

$$\text{Var}(R_p) = 0.0072 + 0.0048 + 0.00072 = 0.01272$$

(iii). **Covariance in the Capital Asset Pricing Model (CAPM)** In the CAPM, the expected return of an asset is related to its covariance with the market portfolio.

EXAMPLE 1.15. Let R_i be the return of asset i, R_m be the return of the market portfolio, R_f be the risk-free rate, and β_i be the beta of asset i. The beta is given by:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)}$$

If $\text{Cov}(R_i, R_m) = 0.0045$ and $\text{Var}(R_m) = 0.02$, then:

$$\beta_i = \frac{0.0045}{0.02} = 0.225$$

The expected return of asset i is:

$$E[R_i] = R_f + \beta_i(E[R_m] - R_f).$$

(iv). **Covariance of Bond Prices and Interest Rates**

The covariance between bond prices and interest rates is crucial for understanding interest rate risk.

EXAMPLE 1.16. Consider a bond with a face value of \$1,000, an annual coupon rate of 5%, and a maturity of 3 years. Let P be the bond price and r be the interest rate. The covariance between P and r can be estimated using historical data or a sensitivity analysis model. If the covariance is $\mathrm{Cov}(P, r) = -200$, this negative covariance indicates that bond prices decrease as interest rates increase.

REMARK 1.17. Covariance is a fundamental concept in probability theory and mathematical finance

1.5. Correlation

The **correlation** between two random variables measures the strength and direction of their linear relationship. It provides a measure of how the variables move together.

DEFINITION 1.18 (Discrete and Continuous). For discrete random variables X and Y with possible values $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$, and joint probabilities p_{ij}, the correlation ρ_{XY} is defined as:

$$\rho_{XY} = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}.$$

For continuous random variables X and Y with joint probability density function (pdf) $f(x, y)$, the correlation ρ_{XY} is defined as:

$$\rho_{XY} = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}.$$

1.5.1. Properties of Correlation.

(i). **Range**: $-1 \leq \rho_{XY} \leq 1$

(ii). **Direction**:

- $\rho_{XY} > 0$: Positive correlation (variables move in the same direction)
- $\rho_{XY} < 0$: Negative correlation (variables move in opposite directions)

(iii). **Strength**: The closer ρ_{XY} is to 1 or -1, the stronger the correlation.

1.5.2. Examples in Mathematical Finance.

(i). **Correlation of Asset Returns**

Correlation is widely used in finance to measure the relationship between the returns of two assets.

EXAMPLE 1.19. Consider two assets with returns R_1 and R_2. Let $\mathrm{Cov}(R_1, R_2) = 0.0015$, $\mathrm{Var}(R_1) = 0.02$, and $\mathrm{Var}(R_2) = 0.03$. The correlation $\rho_{R_1 R_2}$ between the returns is:

$$\rho_{R_1 R_2} = \frac{\mathrm{Cov}(R_1, R_2)}{\sqrt{\mathrm{Var}(R_1)\mathrm{Var}(R_2)}}$$

$$\rho_{R_1 R_2} = \frac{0.0015}{\sqrt{0.02 \times 0.03}} \approx 0.490$$

This positive correlation indicates that the returns of the two assets move together.

(ii). **Correlation in Portfolio Diversification**

Correlation is crucial for diversification in portfolio theory. A portfolio with assets that are negatively correlated can reduce overall risk.

EXAMPLE 1.20. Consider a portfolio with two assets. Asset 1 has a variance $\text{Var}(R_1) = 0.02$, asset 2 has a variance $\text{Var}(R_2) = 0.03$, and the correlation between the returns is $\rho_{R_1R_2} = -0.7$. The variance of the portfolio return R_p is given by:

$$\text{Var}(R_p) = w_1^2\text{Var}(R_1) + w_2^2\text{Var}(R_2) + 2w_1w_2\text{Cov}(R_1, R_2)$$

where w_1 and w_2 are the weights of the assets.

Substituting the values:

$$\text{Var}(R_p) = (w_1^2 \times 0.02) + (w_2^2 \times 0.03) + 2(w_1 \times w_2 \times -0.7 \times \sqrt{0.02 \times 0.03})$$

By adjusting the weights w_1 and w_2, an investor can create a portfolio with reduced risk due to the negative correlation between the assets.

(iii). **Correlation in Risk Management** Correlation is used in risk management to assess the potential impact of events on different assets or portfolios.

EXAMPLE 1.21. content...Consider a risk management scenario where an investor holds a portfolio of stocks and bonds. The investor wants to assess the impact of a market downturn on the portfolio. By analyzing historical data, the investor calculates the correlation between stock returns and bond returns. A high positive correlation suggests that both stocks and bonds are likely to decline in a market downturn, while a negative correlation indicates potential diversification benefits.

REMARK 1.22. Correlation is a fundamental concept in probability theory and mathematical finance. It provides insights into the relationship between random variables and is crucial for portfolio diversification, risk management, and financial modeling. Understanding how to compute and interpret correlation helps investors make informed decisions and manage risk effectively.

1.6. Regression

Regression analysis is a statistical method used to model the relationship between a dependent variable and one or more independent variables. It helps in understanding how the value of the dependent variable changes when one or more independent variables change.

DEFINITION 1.23 (Simple Linear Regression). Simple linear regression involves modeling the relationship between a single independent variable X and a dependent variable Y using a linear equation:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

where:

(1) Y is the dependent variable.

(2) X is the independent variable.

(3) β_0 is the intercept (the value of Y when $X = 0$).

(4) β_1 is the slope (the change in Y for a unit change in X).

(5) ε is the error term (captures the difference between the observed and predicted values of Y).

The goal of simple linear regression is to estimate the values of β_0 and β_1 that minimize the sum of squared errors.

Definition 1.24 (Multiple Linear Regression). Multiple linear regression extends simple linear regression to model the relationship between a dependent variable Y and multiple independent variables $X_1, X_2, \ldots, X_k$:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \varepsilon$$

where $\beta_0, \beta_1, \ldots, \beta_k$ are the regression coefficients.

1.6.1. Applications in Mathematical Finance. Regression analysis is widely used in mathematical finance for various purposes, including:

(i). **Asset Pricing Models**

Regression analysis is used to estimate parameters in asset pricing models such as the Capital Asset Pricing Model (CAPM) and the Fama-French three-factor model. These models help in understanding the relationship between asset returns and factors such as market returns, size, and value.

(ii). **Risk Management** Regression analysis is used to estimate the relationship between asset returns and risk factors. By analyzing historical data, regression models can help in identifying factors that affect asset returns and assessing portfolio risk.

(iii). **Predictive Modeling**

Regression analysis is used to build predictive models for stock prices, interest rates, and other financial variables. By analyzing historical data and identifying relevant predictors, regression models can help in forecasting future trends and making investment decisions

Example 1.25 (Capital Asset Pricing Model (CAPM)). The CAPM is a widely used asset pricing model that relates the expected return of an asset to its beta, which measures its sensitivity to market risk. The CAPM equation is:

$$E[R_i] = R_f + \beta_i(E[R_m] - R_f)$$

where:

(1) $E[R_i]$ is the expected return of asset i.

(2) R_f is the risk-free rate.

(3) β_i is the beta of asset i.

(4) $E[R_m]$ is the expected return of the market portfolio.

Regression analysis can be used to estimate the beta of an asset by regressing its historical returns against the returns of the market portfolio.

Example 1.26 (Predictive Modeling of Stock Prices). Regression analysis can be used to build predictive models for stock prices using historical data on factors such as earnings, dividends, interest rates, and market returns. By regressing stock returns against these factors, a predictive model can be developed to forecast future stock prices.

Remark 1.27. Regression analysis is a powerful statistical tool used in mathematical finance for modeling relationships between variables, estimating parameters in asset pricing models, assessing portfolio risk, and building predictive models. By understanding the principles of regression analysis and applying them to financial data, investors can make informed decisions and manage risk effectively.

1.7. Applications in Finance

In this section, we will show how the concepts we have learned can be applied in mathematical finance.

(i). Random Variables:

Stock Prices: Stock prices are inherently uncertain and can be modeled as random variables. These variables capture the randomness in future price movements.

(ii). Expectation:

Expected Return: Investors are interested in the average return they can expect from an investment. Expectation is used to calculate this by considering all possible outcomes (stock prices) and their associated probabilities.

(iii). Variance:

Risk Measurement: Variance of a stock's return reflects the volatility or risk associated with that investment. Higher variance indicates a larger spread of possible returns, signifying greater risk.

(iv). Correlation:

Portfolio Diversification: Correlation measures how closely two investments move together. It helps in portfolio diversification by selecting assets with low correlations, reducing overall portfolio risk.

(v). Regression:

Beta Calculation: Beta, a measure of an asset's volatility relative to the market, is often estimated using regression. It helps assess an asset's non-diversifiable risk. Option Pricing Models: Some option pricing models, like Black-Scholes, utilize regression to estimate the relationship between underlying asset prices and option prices.

Here's an example to illustrate how these concepts work together:

EXAMPLE 1.28. An investor is considering a portfolio with two stocks (A and B). They can model the future prices of each stock as random variables. By calculating the expected return and variance of each stock's return, they can assess the average return potential and individual risks. Furthermore, calculating the correlation between the two stocks helps them understand how the returns of A and B move together. Finally, they might employ regression to estimate the beta of each stock relative to the market. By considering all this information, the investor can make informed decisions about portfolio allocation and risk management.

In essence, these statistical tools provide a quantitative framework to analyze financial markets, measure risk, and make data-driven investment decisions

1.8. Exercises

(1) Consider a fair six-sided die. Let X be the outcome of a roll. The possible values are $1, 2, 3, 4, 5$, and 6, each with a probability of $\frac{1}{6}$. Compute the expectation and variance.

(2) Consider a continuous random variable X with a uniform distribution on the interval $[0, 1]$. The pdf is:

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute the expectation and variance.

(3) Prove the given properties of expectation in the general case.

(4) Prove the given properties of variance.

(5) Let X be a random variable with pdf $f(x) = e^{-x}$ for $x \geq 0$ (an exponential distribution with rate parameter $\lambda = 1$). Compute the expectation and variance.

(6) **Basics of Probability Theory and Statistics**

(i). Define the sample space, event space, and probability measure for the following scenarios:

(a). Tossing a fair coin.

(b). Rolling a fair six-sided die.

(c). Drawing a card from a standard deck of 52 cards.

(7) Prove the following properties of probabilities for events A and B:

(a). Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(b). Multiplication rule: $P(A \cap B) = P(A) \times P(B|A)$

Use Venn diagrams to illustrate these properties.

(8) Calculate the mean, variance, and standard deviation for the following discrete probability distributions:

(a). Binomial distribution with parameters $n = 10$ and $p = 0.3$.

(b). Poisson distribution with parameter $\lambda = 4$.

(9) **Random Variables**

(a). Define discrete and continuous random variables. Provide examples of each type.

(b). Consider the random variable X representing the outcome of rolling a fair six-sided die. Find $E[X]$, $Var(X)$, and $StdDev(X)$.

(10) **Expectation and Variance**

(a). Prove that for any random variable X, $Var(X) = E[X^2] - (E[X])^2$.

(b). Calculate the expectation and variance for the following probability distributions:

(I). Geometric distribution with parameter $p = 0.2$.

(II). Uniform distribution on the interval $[a, b]$.

(c). **Correlation Analysis**

(a). Collect stock price data for two companies over a year. Calculate the correlation coefficient between their daily returns.

(b). Interpret the correlation coefficient in terms of the relationship between the two stocks.

(d). **Linear Regression**

(a). Use historical data of a stock's returns and the market index returns to perform a simple linear regression.

(b). Interpret the slope and intercept of the regression line in the context of finance.

(e). **Portfolio Analysis**

(i). Construct a portfolio consisting of multiple stocks with different weights.

(ii). Calculate the portfolio's expected return and variance.

(iii). Analyze the trade-off between risk and return in the portfolio.

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