



School of Science and Technology
Department of Mathematics
Summer 2024 University Examinations
Tasks
Course: MTH3010A-Mathematical Finance
Date: Saturday 20th May 2024

INSTRUCTIONS:

- (i). Solve **all problems**.
- (ii). All steps must be shown while working out the problems and the solutions neat and legible.

Problem1

The dataset consists of daily returns for Stock A, Stock B, and the Market over a period of 50 days.

Day	Stock A Return	Stock B Return	Market Return
1	0.0073	0.0089	0.0058
2	0.0117	0.0102	0.0150
3	0.0054	0.0065	0.0120
4	0.0238	0.0032	0.0141
5	0.0022	0.0137	0.0078
6	0.0174	0.0161	0.0193
7	0.0139	0.0113	0.0087
8	0.0047	0.0098	0.0134
9	0.0088	0.0110	0.0046
10	0.0160	0.0123	0.0061

- (i). Calculate the expected return (mean) for Stock A, Stock B, and the Market.
- (ii). Calculate the variance of returns for Stock A, Stock B, and the Market.
- (iii). Calculate the covariance between Stock A and Stock B, and between each stock and the Market.
- (iv). Calculate the correlation coefficient between Stock A and Stock B, and between each stock and the Market.

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- (v). Perform a linear regression analysis to model the relationship between Stock A's returns and the Market returns. Find the regression coefficients (alpha and beta).
 - (vi). Perform a similar analysis for Stock B's returns and the Market returns.
 - (vii). Learn how to write a code in python that performs all these tasks. For this, you will need **numpy** and **scipy.stats** packages.

Problem 2

A stock's daily return is assumed to follow a normal distribution with a mean of 0.001 (0.1%) and a standard deviation of 0.02 (2%).

- (i). What is the probability that the stock's return on a given day will be greater than 0.03 (3%)?
- (ii). What is the probability that the stock's return will be between -0.01 (-1%) and 0.02 (2%)?
- (iii). Calculate the 95% confidence interval for the stock's return.

Problem 3: Portfolio Value

The value of a portfolio is normally distributed with a mean of \$1,000,000 and a standard deviation of \$50,000. What is the probability that the portfolio value will be less than \$950,000?

Problem 3: Trade Arrivals

The number of trades arriving at a trading platform follows a Poisson distribution with an average rate of 20 trades per hour.

- (i). What is the probability that exactly 25 trades will arrive in a given hour?
- (ii). What is the probability that at least 15 trades will arrive in a given hour?
- (iii). Calculate the expected number of trades and the variance of the number of trades in a 2-hour period.

Problem 4: Credit Card Fraud

A bank observes that the number of fraudulent credit card transactions per day follows a Poisson distribution with a mean of 1.5 transactions.

1. What is the probability that there will be no fraudulent transactions in a given day?

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2. What is the probability that there will be more than 3 fraudulent transactions in a day?
 3. If the bank observes 5 fraudulent transactions over 3 days, what is the average number of fraudulent transactions per day?

Problem 5: Stock Price Modeling

Assume a stock price follows a geometric Brownian motion, where the continuous returns are normally distributed with a mean of 0.001 and a standard deviation of 0.02. Calculate the probability that the continuously compounded return over a year (252 trading days) is greater than 5%.

Problem 6: Call Center Arrivals

The number of customer calls arriving at a call center follows a Poisson distribution with an average rate of 30 calls per hour. Additionally, the duration of each call follows an exponential distribution with a mean of 3 minutes.

- (i). What is the probability that exactly 35 calls will arrive in a given hour?
- (ii). What is the expected total duration of calls in a given hour?
- (iii). What is the probability that the total duration of calls in an hour exceeds 120 minutes?