NAME: MTH 3010: MATHEMATICAL FINANCE COURSE NO:

Page:

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What is Mathematical Finance?

Mathematical Finance is a growing field that seeks to apply Mathematical Modelling and formulas to create financial prizing structures and Resources.

It is also known as quantitative finance or financial mathematics.

It is concerned with Mathematical modelling in the financial field.

Discrete Random variables

A random variable is a number whose value depends upon the outcome of a Random experiment.

Examples of random variables

1) Toss a coin 10 times and let X be the number of heads.

2) choose a random person in class and let X be the height of the person, in inches

Expectation and Variance of a Random Variable

Defn:

Let X be a discrete Random variable with probability function $p(x)$. Then the expected value of X, denoted $E(X)$ or μ is given by

$$E(X) = \mu = \sum_{x=-\infty}^{\infty} x P(X=x)$$

Theorem

Let X be a discrete r.v with probability function $P(X=x)$ and let $g(x)$ be a real valued function of X ~~Then~~ i.e $g: \mathbb{R} \to \mathbb{R}$, then the expected value of $g(x)$ is

Page:

NAME: COURSE NO:

Write on both sides of paper. Question:

given by

$$E[g(x)] = \sum_{x=-\infty}^{\infty} g(x)p(X=x)$$

Theorem:

Let X be a discrete r.v with probability function $p(x)$.

Then

(i) $E(c) = c$, where c is a real constant.

(ii) $E[aX+b] = a\mu + b$

(iii) $E[k\,g(x)] = k\,E[g(x)]$, where $g(x)$ is a real-valued function of X.

(iv) $E[a g_1(x) \pm b g_2(x)] = a\,E[g_1(x)] \pm b\,E[g_2(x)]$

Variance and Standard Deviation

Defn:

Let X be a r.v with mean $E(X) = \mu$, the variance of X, denoted σ^2 or $Var(X)$, is given by

$$Var(X) = \sigma^2 = E(X-\mu)^2$$

The standard deviation, denoted σ is given as

$$\sigma = \sqrt{Var(X)} = \sqrt{E(X-\mu)^2}$$

$$Var(X) = E[(X-\mu)^2] = E[X^2 - 2\mu X + \mu^2]$$

$$= E(X^2) - 2\mu E(X) + \mu^2$$

$$= E[X^2] - \mu^2 = E(X^2) - [E(X)]^2$$

Example.

Given the probability distribution of X as below, find the mean and standard deviation of X

x	0	1	2	3
$p(X=x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$

NAME: ____________________ Page: ____ COURSE NO: ____

Write on both sides of paper. Question: ____

Soln:

X	0	1	2	3	total
$P(X=x)$	$1/8$	$1/4$	$3/8$	$1/4$	1
$x P(X=x)$	0	$1/4$	$3/4$	$3/4$	$7/4$
$x^2 P(X=x)$	0	$1/4$	$3/2$	$9/4$	4

$$E(X) = \mu = \sum_{x=0}^{3} x P(X=x) = 1.75$$

$$\sigma = \sqrt{E(x^2) - \mu^2} = \sqrt{4 - 1.75^2} = 0.968246$$

Exercise

1) Suppose that X has a probability mass function given by the table below

X	2	3	4	5	6
$P(X=x)$	0.01	0.25	0.4	0.3	0.04

Find the mean and variance of X

2) Suppose that X has a probability mass function given by the table below

X	11	12	13	14	15
$P(X=x)$	0.4	0.2	0.2	0.1	0.1

Find the mean and variance of X

Continuous Random Variables:

A random variable X is continuous if there exists a nonnegative function f so that, for every interval B

$$P(X \in B) = \int_B f(x)\,dx$$

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Page:

COURSE NO:

NAME:

Write on both sides of paper. Question:

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The function $f = f(x)$ is called the density of X.

$\Rightarrow$ We will assume that a density function f is continuous

then, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$P(X \in [a,b]) = P(a \le X \le b) = \int_a^b f(x)\,dx.$$

$$P(X \le b) = P(X < b) = \int_{-\infty}^{b} f(x)\,dx.$$

By analogy with discrete random variables, we define

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) \cdot f(x)\,dx.$$

And variance is computed by the same formula

$$Var(X) = E(X^2) - [E(X)]^2$$

Example.

Let $f(x) = \begin{cases} cx & \text{if } 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$

a) Determine c

b) Compute $P(1 \le X \le 2)$

c) Determine $E(X)$ and $Var(X)$

Soln:

a) $\int_0^4 cx\,dx = 1 \quad \Rightarrow \left[\frac{cx^2}{2}\right]_0^4 = 1 \quad \Rightarrow \frac{c(16)}{2} = 1$

$\Rightarrow c = 1/8$

(b) $\int_1^2 \frac{x}{8}\,dx = \left[\frac{x^2}{16}\right]_1^2 = \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$

Page:

NAME: COURSE NO:

Write on both sides of paper. Question:

(c) $E(x) = \int_a^b x \cdot f(x)\,dx = \int_0^4 x \cdot \frac{x}{8}\,dx = \int_0^4 \frac{x^2}{8}\,dx$

$= \left[\frac{x^3}{24}\right]_0^4 = \frac{4^3}{24} = \frac{8}{3}$

$E(x^2) = \int_0^4 \frac{x^3}{8}\,dx = \left[\frac{x^4}{32}\right]_0^4 = \frac{4^4}{32} = 8$

$Var(x) = E(x^2) - \mu^2 = 8 - \left(\frac{8}{3}\right)^2 = \frac{8}{9}$

Exercise

1) A random variable x has the density function

$$f(x) = \begin{cases} c(x+\sqrt{x}) & x \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

a) Determine c

b) Compute $E[1/x]$

2) The density function of a random variable X is given by

$$f(x) = \begin{cases} a + bx & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

We also know that $E(x) = 7/6$.

a) Compute a and b

b) Compute $Var(x)$