Portfolio Theory and Asset Pricing Models

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CHAPTER 1

PORTFOLIO THEORY AND ASSET PRICING MODELS

Portfolio theory, pioneered by Harry Markowitz in the 1950s, revolutionized the field of finance by providing a systematic framework for constructing optimal investment portfolios. Central to portfolio theory is the concept of diversification, which aims to minimize risk while maximizing returns by investing in a combination of assets with different risk-return profiles. The efficient frontier, a cornerstone of portfolio theory, represents the set of all possible portfolios that offer the highest expected return for a given level of risk, or the lowest risk for a given level of expected return. Efficient frontier analysis enables investors to identify the optimal allocation of assets that balances risk and return to achieve the desired investment objectives.

The Capital Asset Pricing Model (CAPM), developed by William Sharpe, John Lintner, and Jan Mossin in the 1960s, extends portfolio theory by providing a quantitative framework for pricing assets and determining expected returns. According to CAPM, the expected return of an asset is determined by its beta, a measure of its systematic risk or sensitivity to market movements, and the risk-free rate of return. The model postulates that investors are compensated for bearing systematic risk, as measured by beta, and that the risk premium is proportional to the market risk premium.

In this introduction, we explore the foundational concepts of portfolio theory, including the efficient frontier, and delve into the principles underlying the Capital Asset Pricing Model (CAPM). By understanding these concepts, investors can make informed decisions about asset allocation, risk management, and portfolio construction to optimize their investment portfolios and achieve their financial goals.

1.0.1. Basic Concepts.

Definition 1.1 (Expected Return). The average return an investor can expect to receive from an investment over time.

Definition 1.2 (Risk). The uncertainty associated with the return on an investment. Commonly measured by variance or standard deviation of returns.

Definition 1.3 (Diversification). Spreading investments across different assets to reduce risk.

Definition 1.4 (Efficient Frontier). The efficient frontier represents the set of optimal portfolios that offer the highest expected return for a given level of risk, or the lowest risk for a given level of expected return..

Definition 1.5 (Capital Allocation Line (CAL)). The Capital Allocation Line (CAL) is a tangent line drawn from the risk-free rate to the efficient frontier. It represents the best possible capital allocation between the risk-free asset and the risky portfolio.

1.1. Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is a model that describes the relationship between systematic risk and expected return for assets, particularly stocks.

1.1.1. Assumptions.

(i). Investors are rational and risk-averse.

(ii). Markets are frictionless and efficient.

(iii). All investors have the same information and can borrow or lend at the risk-free rate.

(iv). Investors have homogeneous expectations about asset returns.

(v). The market is in equilibrium.

1.1.2. CAPM Equation. The CAPM equation is given by:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{1.1}$$

where:

(i). $E(R_i)$ is the expected return of asset i.

(ii). R_f is the risk-free rate.

(iii). β_i is the beta of asset i, measuring its sensitivity to market movements.

(iv). $E(R_m) - R_f$ is the market risk premium.

Definition 1.6 (Security Market Line (SML)). The Security Market Line (SML) represents the relationship between the expected return and beta of an asset. It is a graphical representation of the CAPM.

Remark 1.7. Portfolio theory and CAPM are fundamental concepts in finance that help investors make informed decisions about asset allocation and pricing. Understanding these concepts is essential for building and managing investment portfolios.

1.2. Exercises

(1) (i). Consider a portfolio consisting of two assets: Asset A and Asset B. Asset A has an expected return of 10% and a standard deviation of 15%, while Asset B has an expected return of 8% and a standard deviation of 10%. If you allocate 60% of your portfolio to Asset A and 40% to Asset B, what is the expected return and variance of your portfolio?

(ii). Suppose you have a portfolio with three assets: Stock X, Stock Y, and Stock Z. The expected returns and standard deviations of these stocks are as follows:

Stock	**Expected Return (%)**	**Standard Deviation (%)**
X	12	18
Y	10	15
Z	8	12

If you allocate 40% of your portfolio to Stock X, 30% to Stock Y, and 30% to Stock Z, compute the expected return and variance of your portfolio.

(2) (i). Plot the efficient frontier for a portfolio composed of two risky assets. Assume that the risk-free rate is 3%, and the expected returns and standard deviations of the two assets are as follows:

Asset	**Expected Return (%)**	**Standard Deviation (%)**
A	10	15
B	8	10

Consider different combinations of asset weights.

(3) (i). Compute the beta (β) of a stock with the following information:

(a). The stock's returns have a covariance of 0.015 with the market returns.

(b). The variance of the market returns is 0.02.

(ii). Determine the beta of a portfolio with the following weights:

Stock	**Weight**
X	0.4
Y	0.3
Z	0.3

The betas of Stocks X, Y, and Z are 1.2, 0.8, and 1.5 respectively.

(4) (i). Plot the Security Market Line (SML) for a market with a risk-free rate of 4% and a market risk premium of 8%. Use different betas (from 0 to 2) on the x-axis.

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