United States International University Africa,
School of Science & Technlogy, Date: 09/06/2024 Time: 1100-1240
Department of Data Science & Analytics, **Course:** MTH3010A Mathematical Finance
Semester: US2024 **Lecturer:** Joseph Owuor

Graded Quiz

Solve all problems while demonstrating each step clearly. The Quiz is worth a total of 20 points.

(3 pts) 1. Define expectation and variance in the context of probability theory. Explain their significance in financial mathematics.

(3 pts) 2. Suppose you have historical data for three stocks: A, B, and C.

Day	Stock A Return	Stock B Return	Market Return
1	0.0073	0.0089	0.0058
2	0.0117	0.0102	0.0150
3	0.0054	0.0065	0.0120
4	0.0238	0.0032	0.0141
5	0.0022	0.0137	0.0078
6	0.0174	0.0161	0.0193
7	0.0139	0.0113	0.0087
8	0.0047	0.0098	0.0134
9	0.0088	0.0110	0.0046
10	0.0160	0.0123	0.0061

Calculate the expected return and variance of a portfolio consisting of 30% of stock A, 50% of stock B, and 20% of stock C.

(3 pts) 3. Write a script in python to solve 2.

(3 pts) 4. Give the definition of a martingale and explain its role in modeling the evolution of stock prices.

(4 pts) 5. Let $X_t \sim N(\mu, \sigma^2)$ represent stock prices. Give the statement of Ito's lemma. Hence use Ito's Lemma to find the differential $df(X_t)$ if X_t follows the stochastic differential equation (SDE):

$$dX_t = \mu X_t\, dt + \sigma X_t\, dW_t,$$

where $f(X_t) = \ln(X_t)$. Furthermore, compute the mean and variance of the resulting probability distribution for $f(X_t)$.

(4 pts) 6. Prove that a Brownian motion is a martingale.