STA 3050: TIME SERIES AND FORECASTING

ESTIMATING SEASONAL INDICES

RATIO TO TREND METHOD

As you have seen, the simple average method is used only when the time series data do not have a trend and cyclic effect but most of the economic or business time series exhibit trends. Therefore, in such cases, we cannot use the simple average method for seasonal indices. For calculating seasonal indices in such cases, we can use another method of measuring seasonal variation, i,e, the ratio to trend method. This method is based on the basic assumption that the data do not contain any cyclic components. It means that if the time series variable consists of trend, seasonal and random components then we can apply this method to compute the seasonal indices.

Therefore, it is an improved version of the single average method as it assumes that seasonal variation for a given period is a constant fraction of the trend. The measurement of the seasonal indices by this method consists of the following steps:

Step 1: In this method, we first convert the quarterly or monthly time series data into yearly. For that, we compute the average of all quarters (months) of each year and then take these averages as the yearly values of the variable.

Step 2: After converting data annually/yearly, we find trend values of this converted data by the method of least squares by fitting a suitable mathematical curve, either a trend line (straight line) or second egree polynomial (parabola).

Step 3: The yearly trend values which are calculated in Step 2 are the trend values of the mid-periods of the respective years. Based on these trend values, we determine the trend values for each quarter or month as per the case using yearly increment (slope) of the trend line (see Example 2).

Step 4: To eliminate the trend effect from the data, we express the original time series values as the percentages of the trend value assuming the multiplicative model. These percentages will contain the seasonal, and irregular components.

Step 5: After eliminating the trend effect, the series contains only seasonal and irregular effects. Therefore, we now obtain the seasonal indices free from the irregular variations by following the same procedure discussed in the simple average method in the previous section. Thus, we find the average (mean or median) of ratio to trend values (or percentages values) for each season in all years. It is suggested to prefer the median despite the mean if there are some extreme values (outliers), which are not primarily due to seasonal effects. In this way, the irregular variation is removed. If there are few abnormal values in the percentage values, then the mean should be preferred to remove the randomness. These averages are known as seasonal indices.

Step 6: If the sum of the seasonal indices is not 400 for quarterly data and 1200 for monthly data, then we find the adjusted seasonal indices by expressing each seasonal index as the percentage of the average of seasonal indices as

$$\text{Adjusted seasonal index} = \frac{\text{Seasonal index}}{\text{Average of seasonal index}} \times 100$$

For a better understanding of the procedure of ratio to trend method, let us take an example.

Example 2: Compute the seasonal indices by the ratio to trend method for the data of quarterly demand of electricity (in 1000 megawatts) given in Example 1 by assuming that there is no cyclic pattern.

Solution: We have given the quarterly time series data for 4 years. Therefore, we first convert the quarterly day into yearly by finding the average of all quarters of each year as follows:

Year	Summer	Monsoon	Winter	Spring	Average
2019	70	52	22	31	$\frac{70+52+22+31}{4} = 43.75$
2020	101	64	24	45	$\frac{101+64+24+45}{4} = 58.5$
2021	120	75	30	49	$\frac{120+75+30+49}{4} = 68.5$
2022	135	82	34	50	$\frac{135+82+34+50}{4} = 75.25$

Therefore, we get the required yearly data as given below:

Year	Average
2019	43.75
2020	58.5
2021	68.5
2022	75.25

We now determine the yearly trend by fitting a linear trend by the method of least square. The linear trend line equation is given by:

$$Y_t = \beta_0 + \beta_1 t$$

Since n = 4 (number of years) is even, therefore, we make the following transformation in time t to make calculation easy:

$$X_t = \frac{t - \text{average of two middle value}}{\text{half of interval in } t \text{ values}} = \frac{t - 2020.5}{1/2} = 2(t - 2020.5)$$

After the transformation, the normal equations for a linear trend line are:

$$\sum Y_t = n\beta_0 + \beta_1 \sum X_t$$

$$\sum X_t Y_t = \beta_0 \sum X_t + \beta_1 \sum X_t^2$$

We calculate the values of $\Sigma Y_t, \Sigma X_t, \Sigma X_t, Y_t$ and ΣX_t^2 in the following table:

Year (t)	Y_t	$X_t = 2(t-2020.5)$	X_tY_t	X_t^2
2019	43.75	–3	–131.25	9
2020	58.50	–1	–58.50	1
2021	68.50	1	68.50	1
2022	75.25	3	225.75	9
Total	**246**	**0**	**104.50**	**20**

Therefore, we find the values of β_0 and β_1 using normal equations as

$$246 = 4 \times \beta_0 + \beta_1 \times 0 \Rightarrow \beta_0 = \frac{246}{4} = 61.50$$

$$104.50 = \beta_0 \times 0 + \beta_1 \times 20 \Rightarrow \beta_1 = \frac{104.5}{20} = 5.23$$

Thus, the final linear trend line is given by

$$Y_t = 61.5 + 5.23X_t$$

We now find the trend values by putting $X_t = -3, -1, 1, 3$ in the above trend line equation as follows:

Trend value (2019) $= 61.5 + 5.23 \times (-3) = 45.81$

Trend value (2020) $= 61.5 + 5.23 \times (-1) = 56.27$

Trend value (2021) $= 61.5 + 5.23 \times 1 = 66.73$

Trend value (2022) $= 61.5 + 5.23 \times 3 = 77.19$

These trend values represent the averages of the corresponding year and are supposed to lie at the centre of the corresponding year. Therefore, we place these in the middle of the 2nd and 3rd quarters. We now determine the trend values for each quarter using the yearly increment (slope = 5.23) of the trend line.

We have the yearly increment = 5.23.

Therefore, the quarterly increment will be $= \frac{5.23}{4} = 1.31$

Similarly, the half-quarterly increment will be $= \frac{1.31}{2} = 0.66$

Thus, the trend value for the 3rd quarter of the year 2019 will be

= Trend value of 2019 + half quarterly increment $= 45.81 + 0.66 = 46.47$.

Similarly, the trend value for the 2nd quarter of the year 2019 will be

= Trend value of 2019 – half quarterly increment $= 45.81 - 0.66 = 45.15$.

Thus, the trend value for the 1st quarter of the same year will be

= Trend value of 2nd quarter – quarterly increment $= 45.15 - 1.31 = 43.84$.

In a similar way, the values for the 4th quarter of the same year will be

= Trend value of 3rd quarter + quarterly increment $= 46.47 + 1.31 = 47.78$.

Similarly, you can compute the trend values for the rest years as we have computed for all quarters of the year 2019. We calculated the same in the following table:

Quarterly trend values

Season	2019	2020	2021	2022
Summer	45.15 – 1.31 = 43.84	55.61 – 1.31 = 54.30	66.07 – 1.31 = 64.76	76.53 – 1.31 = 75.22
Monsoon	45.81 – 0.66 = 45.15	56.27 – 0.66 = 55.61	66.73 – 0.66 = 66.07	77.19 – 0.66 = 76.53
Winter	45.81 + 0.66 = 46.47	56.27 + 0.66 = 56.93	66.73 + 0.66 = 67.39	77.19 + 0.66 = 77.85
Spring	46.47 + 1.31 = 47.78	56.93 + 1.31 = 58.24	67.39 + 1.31 = 68.7	77.85 + 1.31 = 79.16

After getting the quarterly trend values, we now remove the trend effect. For that, we divide each original value into the corresponding trend value and express them in percentages as shown in the following table:

Season	2019	2020	2021	2022
Summer	$\frac{70}{43.84}\times 100 = 159.67$	$\frac{101}{54.3}\times 100 = 186$	$\frac{120}{64.76}\times 100 = 185.3$	$\frac{135}{75.22}\times 100 = 179.47$
Monsoon	$\frac{52}{45.15}\times 100 = 115.17$	$\frac{64}{55.61}\times 100 = 115.09$	$\frac{75}{66.07}\times 100 = 113.52$	$\frac{82}{76.53}\times 100 = 107.15$
Winter	$\frac{22}{46.47}\times 100 = 47.34$	$\frac{24}{56.93}\times 100 = 42.16$	$\frac{30}{67.39}\times 100 = 44.52$	$\frac{34}{77.85}\times 100 = 43.67$
Spring	$\frac{31}{47.78}\times 100 = 64.88$	$\frac{45}{58.24}\times 100 = 77.27$	$\frac{49}{68.7}\times 100 = 71.32$	$\frac{50}{79.16}\times 100 = 63.16$

Now, the above data is free from trend effect so we can apply the simple average method to calculate the seasonal indices. We now compute the average(seasonal index) $\overline{y}_i$ of each season/quarter in different years and then adjused seasonal indices as follows:

Calculations for seasonal indices

Season	2019	2020	2021	2022	Average (Seasonal Index)	Adjusted Seasonal Index
Summer	159.67	186.00	185.30	179.47	$\frac{159.67+186.00+185.30+179.47}{4} = 177.61$	$\frac{177.61}{100.98}\times 100 = 175.89$
Monsoon	115.17	115.09	113.52	107.15	$\frac{1115.17+115.09+113.52+107.15}{4} = 112.73$	$\frac{112.73}{100.98}\times 100 = 111.64$
Winter	47.34	42.16	44.52	43.67	$\frac{47.34+42.16+44.52+43.67}{4} = 44.42$	$\frac{44.42}{100.98}\times 100 = 43.99$
Spring	64.88	77.27	71.32	63.16	$\frac{64.88+77.27+71.32+63.16}{4} = 69.16$	$\frac{69.16}{100.98}\times 100 = 68.49$
Total					177.61+112.73+44.42+69.16 = 405.04	400
Average					$\frac{177.61+112.73+44.42+69.16}{4} = 100.98$	100

The average yearly seasonal indices obtained above are adjusted to a total of 400 because the total of the seasonal Indices for each quarter is 405.04 which is greater than 400. So we express each seasonal index as the percentage of the average of seasonal indices. The adjusted seasonal indices for each quarter are given in the last column of the above table.

We now discuss the merits and demerits of ratio to trend method as follows:

Merits and Demerits

- This method is undoubtedly a more rational way to measure seasonal variations than the simple average method.

- The advantage of this method over the ratio to moving average method (we will discuss in the next session) is that, unlike the ratio to moving average method, we may obtain ratio to trend values for each period for which data are available.
- The main demerit of this method is that if there are cyclical variations in the series, then the trend whether a straight line or a curve can never follow the actual data as closely as a moving average does.
- Furthermore, it is more complicated than the simple average method.

Before moving to the next method of measuring seasonal component, you can try the following Self Assessment Question.

Exercise

The marketing manager of a company that manufactures and distributes farming equipment (harvesters, ploughs and tractors) recorded the number of farming units sold quarterly for the period 2018 to 2020 which are given in the following table:

Year \ Quarter	Q_1	Q_2	Q_3	Q_4
2018	48	41	60	65
2019	58	52	68	74
2020	60	56	75	78

1. Find the quarterly seasonal indexes for farming equipment sold using the ratio to trend method by assuming that there is no cyclic effect.
2. Do seasonal forces significantly influence the sale of farming equipment? Comment.