STA 3050: TIME SERIES AND FORECASTING

ESTIMATING SEASONAL INDICES

RATIO TO MOVING AVERAGE METHOD

We are now discussing the most widely used method which is known as ratio to moving average method. It is better than the previously discussed methods because of its accuracy. This method does not base any assumption regarding the presence of the components in the time series. It means that we can apply this method to calculate seasonal indices even if all components namely trend, seasonal, cyclic and irregular variations are present in the time series. The necessary steps for obtaining seasonal indices by this method are as follows:

Step 1: If the time series data is recorded monthly or quarterly of different years, then, we first arrange the data by months or quarters chronologically of different years in such a way that all months or quarters lie in a column (see Example 3).

Step 2: After arranging the data, we calculate the moving average. If monthly (quarterly) data is given, a twelve (four)-period moving average will isolate the trend/cyclical movements in the time series and produce a base set of time series values.

Step 3: The actual values include all components/effects, i.e. trend, seasonal, cyclic and irregular, and moving averages contain only the trend and cyclical effects, therefore, if we divide the actual value by the corresponding moving average then the ratio will contain only seasonal and irregular effects. This ratio is known as the seasonal ratio. Therefore, we calculate the seasonal ratio for each period (for which we have the moving average) by dividing each actual time series value by its corresponding moving average value as

$$\text{Seasona ratio} = \frac{\text{Actual value}}{\text{Moving average}} \times 100$$

A seasonal ratio is an index that measures the percentage deviation of each actual value from its moving average value, and it is a measure of the seasonal impact.

Step 4: We now obtain the seasonal indices free from the irregular variations by following the same procedure discussed in the simple average method in the previous section. For that, we prepare another two-way table consisting of the quarter/month-wise percentage values calculated in Step 3, for all years.

Let us take an example to understand this method more explicitly.

Example 3:

Apply ratio to moving average method to obtain seasonal indices for the data of quarterly demand of electricity given in Example 1.

Year	Summer	Monsoon	Winter	Spring
2019	70	52	22	31
2020	101	64	24	45
2021	120	75	30	49
2022	135	82	34	50

Solution:

As we have described in the procedure of the ratio to moving average that we first arrange the data quarterly in chronological order of different years in such a way that all quarters lie in a column and then obtain the moving average for 4 quarters as follows:

Calculation of quarterly moving average and seasonal ratio

Year	Quarter	Demand of electricity	4-quarterly MA	Centred 4-quarterly MA	Seasonal ratio
2019	Summer	70			
	Monsoon	52			
			$\frac{70+52+22+31}{4}=43.75$		
	Winter	22		$\frac{43.75+51.50}{2}=47.63$	$\frac{22}{47.63}\times100=46.19$
			$\frac{52+22+31+101}{4}=51.50$		
	Spring	31		$\frac{51.50+54.5}{2}=53$	$\frac{31}{53}\times100=59.49$
			$\frac{22+31+101+64}{4}=54.50$		
2020	Summer	101		$\frac{54.5+55}{2}=54.75$	$\frac{101}{54.75}\times100=184.47$
			$\frac{31+101+64+24}{4}=55$		
	Monsoon	64		$\frac{55+58.5}{2}=56.75$	$\frac{64}{56.75}\times100=112.78$
			$\frac{101+64+24+45}{4}=58.5$		
	Winter	24		$\frac{58.5+63.25}{2}=60.88$	$\frac{24}{60.88}\times100=39.43$
			$\frac{64+24+45+120}{4}=63.25$		
	Spring	45		$\frac{63.25+66}{2}=64.63$	$\frac{45}{64.63}\times100=69.63$
			$\frac{24+45+120+75}{4}=66$		
2021	Summer	120		$\frac{66+67.5}{2}=66.75$	$\frac{120}{66.75}\times100=179.78$
			$\frac{45+120+75+30}{4}=67.5$		
	Monsoon	75		$\frac{67.5+68.5}{2}=68$	$\frac{75}{68}\times100=110.29$
			$\frac{120+75+30+49}{4}=68.5$		
	Winter	30		$\frac{68.5+72.25}{2}=70.38$	$\frac{30}{70.38}\times100=42.63$
			$\frac{75+30+49+135}{4}=72.25$		

	Summer	120		$\frac{66+67.5}{2}=66.75$	$\frac{120}{66.75}\times 100=179.78$
			$\frac{45+120+75+30}{4}=67.5$		
	Monsoon	75		$\frac{67.5+68.5}{2}=68$	$\frac{75}{68}\times 100=110.29$
2021			$\frac{120+75+30+49}{4}=68.5$		
	Winter	30		$\frac{68.5+72.25}{2}=70.38$	$\frac{30}{70.38}\times 100=42.63$
			$\frac{75+30+49+135}{4}=72.25$		
	Spring	49		$\frac{72.25+74}{2}=73.13$	$\frac{49}{73.13}\times 100=67.01$
			$\frac{30+49+135+82}{4}=74$		
	Summer	135		$\frac{74+75}{2}=74.5$	$\frac{135}{74.5}\times 100=181.21$
			$\frac{49+135+82+34}{4}=75$		
2022	**Monsoon**	82		$\frac{75+75.25}{2}=75.13$	$\frac{82}{75.13}\times 100=109.15$
			$\frac{135+82+34+50}{4}=75.25$		
	Winter	34			
	Spring	50			

After that, we compute the seasonal ratio for each period (for which we have the moving average) by dividing each actual time series value by its corresponding moving average value as

$$\text{Seasona ratio} = \frac{\text{Actual value}}{\text{Moving average}} \times 100$$

We have computed these in the last column of the above table.

We now prepare a two-way table consisting of the seasonal ratios quarterwise for all years as shown in the following table and calculate median (seasonal indices) and adjusted seasonal indices as follows:

Season	2019	2020	2021	2022	Seasonal Index (Median)	Adjusted Seasonal Index
Summer		184.47	179.78	181.21	181.21	$\frac{181.21}{100.29} \times 100 = 180.7$
Monsoon		112.78	110.29	109.15	110.29	$\frac{110.29}{100.29} \times 100 = 109.98$
Winter	46.19	39.43	42.63		42.63	$\frac{42.63}{100.29} \times 100 = 42.51$
Spring	58.49	69.63	67.01		67.01	$\frac{67.01}{100.29} \times 100 = 66.82$
Total					401.14	400
Average					100.29	100

After understanding the ratio to moving average method, let us see the merits and demerits of this method.

Merits and Demerits

- The ratio to moving average method is the most popular method because it can also be applied when all four components of a time series are present.
- It is easier than the ratio to trend method and provides satisfactory estimates.
- It is better than the simple average method which assumes that the trend is absent. But more difficult than the simple average method.
- Its primary drawback is a loss of information when calculating trend, the moving averages (trend) of a few seasons in the beginning and an equal number of seasons in the end are not available.

ESTIMATION OF TREND FROM DESEASONALISED DATA

We have already discussed the estimation of trend. However, when the substantial seasonal component is present in the data then it is advisable to first remove the effect of seasonal effect from the data.

If we do not remove it then the trend will also be affected by seasonal effects which will make it unreliable. Hence, after the estimation of seasonal indices, the seasonal component is removed from the original data, and the reduced data are free from seasonal variations and is called deseasonalised data.

For calculating deseasonalised values, we divide the actual value by its corresponding seasonal index, that is,

$$\text{Deseasonalised value} = \frac{\text{Actual value}}{\text{Seasonal index}} \times 100$$

Once the data are free from seasonal effect, we estimate the trend by the method of least squares or moving average method.

The necessary steps for estimation of the trend from deseasonalised data are as follows:

Step 1: We first obtained seasonal indices using the ratio to trend method or ratio to moving average method.

Step 2: After that we calculate deseasonalised values by dividing the actual value by its corresponding seasonal index and multiplying by 100, that is,

$$\text{Deseasonalised value} = \frac{\text{Actual value}}{\text{Seasonal index}} \times 100$$

Step 3: We estimate trend by the method of least squares or moving average method.

Let us take an example to understand how to estimate trend from deseasonalised data more clearly.

Example 4:

Consider the seasonal indices calculated in Example 3.

1. Obtain the deseasonalised values.
2. Estimate trend values from the deseasonalised values.
3. Plot original and deseasonalised demand of electricity obtained in part (i) with trend line obtained in part (ii).

Solution:

We have calculated seasonal indices for the demand of electricity in Example 3. We now calculate the deseasonalised values. For that, we prepare a table given below and write the corresponding seasonal index in front of each actual observation and calculate deseasonalised values using the following formula:

$$\text{Deseasonalised value} = \frac{\text{Actual value}}{\text{Seasonal index}} \times 100$$

Calculations for deseasonalised demand of electricity

Year	Season	Demand of Electricity	Seasonal Index	Deseasonalised Demand of Eelectricity
2019	Summer	70	180.70	$\frac{70}{180.70}\times 100 = 38.74$
	Monsoon	52	109.98	$\frac{52}{109.98}\times 100 = 47.28$
	Winter	22	42.51	$\frac{22}{42.51}\times 100 = 51.75$
	Spring	31	66.82	$\frac{31}{66.82}\times 100 = 46.39$
2020	Summer	101	180.70	$\frac{101}{180.70}\times 100 = 55.89$
	Monsoon	64	109.98	$\frac{64}{109.98}\times 100 = 58.19$
	Winter	24	42.51	$\frac{24}{42.51}\times 100 = 56.46$
	Spring	45	66.82	$\frac{45}{66.82}\times 100 = 67.35$
2021	Summer	120	180.70	$\frac{120}{180.70}\times 100 = 66.41$
	Monsoon	75	109.98	$\frac{75}{109.98}\times 100 = 68.19$
	Winter	30	42.51	$\frac{30}{42.51}\times 100 = 70.57$
	Spring	49	66.82	$\frac{49}{66.82}\times 100 = 73.33$
2022	Summer	135	180.70	$\frac{135}{180.70}\times 100 = 74.71$
	Monsoon	82	109.98	$\frac{82}{109.98}\times 100 = 74.56$
	Winter	34	42.51	$\frac{34}{42.51}\times 100 = 79.98$
	Spring	50	66.82	$\frac{50}{66.82}\times 100 = 74.83$

We now determine the trend by fitting a linear trend line by the method of least square.

Here, time is in the form of seasons (Summer, Monsoon, Winter, Spring) which is non-numeric. Therefore, we use the sequential numbering system and take $t = 1,2,3, \ldots$ instead of the name of seasons as shown in the next table.

The linear trend line equation is given by:

$$Y_t = \beta_0 + \beta_1 t$$

Since n (= 16) is even, therefore, we make the following transformation in time t:

$$X_t = \frac{t - \text{average of two middle value}}{\text{half of interval in t values}} = \frac{t-8.5}{1/2} = 2(t-8.5)$$

After the transformation, the normal equations for linear trend line are:

$$\sum Y_t = n\beta_0 + \beta_1 \sum X_t$$

$$\sum X_t Y_t = \beta_0 \sum X_t + \beta_1 \sum X_t^2$$

Year	Season	t	Deseasonalised Demand of Electricity (Y_t)	$X_t = 2(t-8.5)$	X_tY_t	X_t^2	Trend Value
2019	Summer	1	38.74	–15	–581.1	225	44.19
	Monsoon	2	47.28	–13	–614.64	169	46.67
	Winter	3	51.75	–11	–569.25	121	49.15
	Spring	4	46.39	–9	–417.51	81	51.63
2020	Summer	5	55.89	–7	–391.23	49	54.11
	Monsoon	6	58.19	–5	–290.95	25	56.59
	Winter	7	56.46	–3	–169.38	9	59.07
	Spring	8	67.35	–1	–67.35	1	61.55
2021	Summer	9	66.41	1	66.41	1	64.03
	Monsoon	10	68.19	3	204.57	9	66.51
	Winter	11	70.57	5	352.85	25	68.99
	Spring	12	73.33	7	513.31	49	71.47
2022	Summer	13	74.71	9	672.39	81	73.95
	Monsoon	14	74.56	11	820.16	121	76.43
	Winter	15	79.98	13	1039.74	169	78.91
	Spring	16	74.83	15	1122.45	225	81.39
Total			**1004.63**	**0**	**1690.47**	**1360**	

Therefore, we find the values of β_0 and β_1 using the normal equations as:

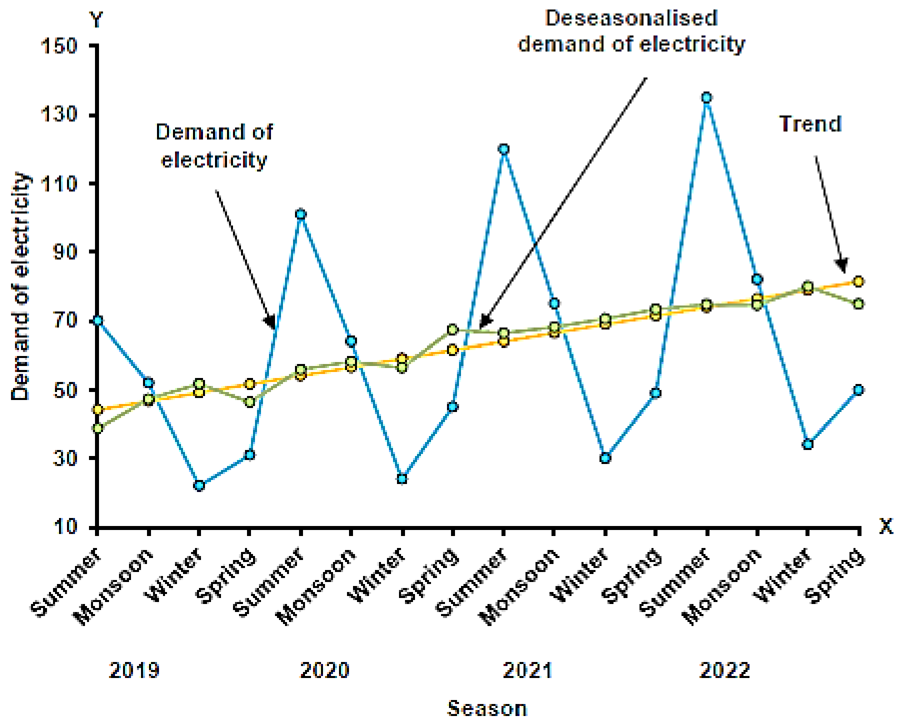
$$1004.63 = 16 \times \beta_0 + \beta_1 \times 0 \Rightarrow \beta_0 = \frac{1004.63}{16} = 62.79$$

$$1690.47 = \beta_0 \times 0 + \beta_1 \times 1360 \Rightarrow \beta_1 = \frac{1690.47}{1360} = 1.24$$

Thus, the final linear trend line is given by

$$Y_t = 62.79 + 1.24X_t$$

We calculate the trend value by putting $X_t = -15, -13, \ldots, 15$ in the above trend line. The same is calculated in the last column of the above table. We now plot the original and deseasonalised demand of electricity with the trend line



Exercise 4

A manager of a national park recorded the following data on the number of visitors (in thousands) who visited the park in each quarter of 2021 and 2022:

Season	Seasonal index	2021	2022
Summer	162	51	54
Monsoon	62	28	32
Winter	87	41	45
Spring	89	36	43

1. Calculate the deseasonalised values for each quarter.
2. Compute the trend values for the deseasonalised values.
3. Plot the quarterly number of visitors for 2021 and 2022 and also deseasonalised with trend values on the same axis.