EXPECTED MEAN SQUARES

Fixed vs. Random Effects

- The choice of labeling a factor as a fixed or random effect will affect how you will make the *F*-test.

- This will become more important later in the course when we discuss interactions.

Fixed Effect

- All treatments of interest are included in your experiment.

- You cannot make inferences to a larger experiment.

Example 1: An experiment is conducted at Fargo and Grand Forks, ND. If location is considered a fixed effect, you cannot make inferences toward a larger area (e.g. the central Red River Valley).

Example 2: An experiment is conducted using four rates (e.g. ½ X, X, 1.5 X, 2 X) of a herbicide to determine its efficacy to control weeds. If rate is considered a fixed effect, you cannot make inferences about what may have occurred at any rates not used in the experiment (e.g. ¼ x, 1.25 X, etc.).

Random Effect

- Treatments are a sample of the population to which you can make inferences.

- You can make inferences toward a larger population using the information from the analyses.

Example 1: An experiment is conducted at Fargo and Grand Forks, ND. If location is considered a random effect, you can make inferences toward a larger area (e.g. you could use the results to state what might be expected to occur in the central Red River Valley).

Example 2: An experiment is conducted using four rates (e.g. ½ X, X, 1.5 X, 2 X) of an herbicide to determine its efficacy to control weeds. If rate is considered a random effect, you can make inferences about what may have occurred at rates not used in the experiment (e.g. ¼ x, 1.25 X, etc.).

Why Do We Need To Learn How to Write Expected Mean Squares?

- So far in class we have assumed that treatments are always a fixed effect.

- If some or all factors in an experiment are considered random effects, we need to be concerned about the denominator of the F-test because it may not be the Error MS.

- To determine the appropriate denominator of the F-test, we need to know how to write the Expected Mean Squares for all sources of variation.

All Random Model

Each source of variation will consist of a linear combination of σ^2 plus variance components whose subscript matches at least one letter in the source of variation.

The coefficients for the identified variance components will be the letters not found in the subscript of the variance components.

Example – RCBD with a 3x4 Factorial Arrangement

Sources of variation	σ^2	$r\sigma^2_{AB}$	$ra\sigma^2_B$	$rb\sigma^2_A$	$ab\sigma^2_R$
Rep	$\sigma^2 + ab\sigma^2_R$				
A	$\sigma^2 + r\sigma^2_{AB} + rb\sigma^2_A$				
B	$\sigma^2 + r\sigma^2_{AB} + ra\sigma^2_B$				
AxB	$\sigma^2 + r\sigma^2_{AB}$				
Error	σ^2				

Step 1. Write the list of variance components across the top of the table.
- There will be one variance component for each source of variation except Total.
- The subscript for each variance component will correspond to each source of variation.
- The variance component for error receives no subscript.

Sources of variation	σ^2	σ^2_{AB}	σ^2_B	σ^2_A	σ^2_R
Rep					
A					
B					
AxB					
Error					

Step 2. Write in the coefficients for each variance component.

- Remember that the coefficient is the letter(s) missing in the subscript.
- The coefficient for Error is the number 1.

Sources of variation	σ^2	$r\sigma^2_{AB}$	$ra\sigma^2_B$	$rb\sigma^2_A$	$ab\sigma^2_R$
Rep					
A					
B					
AxB					
Error					

Step 3. All sources of variation will have σ^2 (i.e. the expected mean square for error as a variance component).

Sources of variation	σ^2	$r\sigma^2_{AB}$	$ra\sigma^2_B$	$rb\sigma^2_A$	$ab\sigma^2_R$
Rep	σ^2				
A	σ^2				
B	σ^2				
AxB	σ^2				
Error	σ^2				

Step 4. The remaining variance components will be those whose subscript matches at least one
letter in the corresponding source of variation.

SOV	σ^2 $\quad r\sigma^2_{AB}$ $\quad ra\sigma^2_B$ $\quad rb\sigma^2_A$ $\quad ab\sigma^2_R$	
Rep	$\sigma^2 + ab\sigma^2_R$	(Those variance components that have at least the letter R)
A	$\sigma^2 + r\sigma^2_{AB} + rb\sigma^2_A$	(Those variance components that have at least the letter A)
B	$\sigma^2 + r\sigma^2_{AB} + ra\sigma^2_B$	(Those variance components that have at least the letter B)
AxB	$\sigma^2 + r\sigma^2_{AB}$	(Those variance components that have at least the letters A and B)
Error	σ^2	

Example – CRD with a 4x3x2 Factorial Arrangement

Sources of variation	σ^2	$r\sigma^2_{ABC}$	$ra\sigma^2_{BC}$	$rb\sigma^2_{AC}$	$rc\sigma^2_{AB}$	$rab\sigma^2_C$	$rac\sigma^2_B$	$rbc\sigma^2_A$
A	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB} + rbc\sigma^2_A$							
B	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rc\sigma^2_{AB} + rac\sigma^2_B$							
C	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rb\sigma^2_{AC} + rab\sigma^2_C$							
AxB	$\sigma^2 + r\sigma^2_{ABC} + rc\sigma^2_{AB}$							
AxC	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$							
BxC	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC}$							
AxBxC	$\sigma^2 + r\sigma^2_{ABC}$							
Error	σ^2							

Step 1. Write the list of variance components across the top of the table.

- There will be one variance component for each source of variation except Total.
- The subscript for each variance component will correspond to each source of variation.
- The variance component for error receives no subscript.

Sources of variation	σ^2	σ^2_{ABC}	σ^2_{BC}	σ^2_{AC}	σ^2_{AB}	σ^2_C	σ^2_B	σ^2_A
A								
B								
C								
AxB								
AxC								
BxC								
AxBxC								
Error								

Step 2. Write in the coefficients for each variance component.

- Remember that the coefficient is the letter(s) missing in the subscript.
- The coefficient for Error is the number 1.

Sources of variation	σ^2	$r\sigma^2_{ABC}$	$ra\sigma^2_{BC}$	$rb\sigma^2_{AC}$	$rc\sigma^2_{AB}$	$rab\sigma^2_{C}$	$rac\sigma^2_{B}$	$rbc\sigma^2_{A}$
A								
B								
C								
AxB								
AxC								
BxC								
AxBxC								
Error								

Step 3. All sources of variation will have σ^2 (i.e. the expected mean square for error as a variance component).

Sources of variation	σ^2	$r\sigma^2_{ABC}$	$ra\sigma^2_{BC}$	$rb\sigma^2_{AC}$	$rc\sigma^2_{AB}$	$rab\sigma^2_{C}$	$rac\sigma^2_{B}$	$rbc\sigma^2_{A}$
A	σ^2							
B	σ^2							
C	σ^2							
AxB	σ^2							
AxC	σ^2							
BxC	σ^2							
AxBxC	σ^2							
Error	σ^2							

Step 4. The remaining variance components will be those whose subscript matches at least one
letter in the corresponding source of variation.

SOV	σ^2 $r\sigma^2_{ABC}$ $ra\sigma^2_{BC}$ $rb\sigma^2_{AC}$ $rc\sigma^2_{AB}$ $rab\sigma^2_{C}$ $rac\sigma^2_{B}$ $rbc\sigma^2_{A}$	
A	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB} + rbc\sigma^2_{A}$	(Those variance components that have at least the letters A)
B	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rc\sigma^2_{AB} + rac\sigma^2_{B}$	(Those variance components that have at least the letter B)
C	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rb\sigma^2_{AC} + rab\sigma^2_{C}$	(Those variance components that have at least the letter C)
AxB	$\sigma^2 + r\sigma^2_{ABC} + rc\sigma^2_{AB}$	(Those variance components that have at least the letters A and B)
AxC	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$	(Those variance components that have at least the letters A and C)
BxC	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC}$	(Those variance components that have at least the letters B and C)
AxBxC	$\sigma^2 + r\sigma^2_{ABC}$	(Those variance components that have at least the letters A, B and C)
Error	σ^2	

All Fixed Effect Model

Step 1. Begin by writing the expected mean squares for an all random model.

Step 2. All but the first and last components will drop out for each source of variation.

Step 3. Rewrite the last term for each source of variation to reflect the fact that the factor is a
fixed effect.

Example RCBD with 3x2 Factorial

SOV	Before		After
Rep	$\sigma^2 + ab\sigma_R^2$	$\longrightarrow$	$\sigma^2 + ab\sigma_R^2$
A	$\sigma^2 + r\sigma_{AB}^2 + rb\sigma_A^2$	$\longrightarrow$	$\sigma^2 + rb\frac{\sum \alpha_i^2}{(a-1)}$
B	$\sigma^2 + r\sigma_{AB}^2 + ra\sigma_B^2$	$\longrightarrow$	$\sigma^2 + ra\frac{\sum \beta_j^2}{(b-1)}$
AxB	$\sigma^2 + r\sigma_{AB}^2$	$\longrightarrow$	$\sigma^2 + r\frac{\sum (\alpha\beta)_{ij}^2}{(a-1)(b-1)}$
Error	σ^2	$\longrightarrow$	σ^2

Rules for Writing Fixed Effect Component

Step 1. Coefficients don't change.

Step 2. Replace σ^2 with $\sum$

Step 3. The subscript of the variance component becomes the numerator of the effect.

Step 4. The denominator of the effect is the degrees of freedom.

Example 2 CRD with a Factorial Arrangement

SOV	Before	After
A	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB} + rbc\sigma^2_{A}$	$\longrightarrow \sigma^2 + rbc\frac{\sum \alpha_i^2}{(a-1)}$
B	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rc\sigma^2_{AB} + rac\sigma^2_{B}$	$\longrightarrow \sigma^2 + rac\frac{\sum \beta_j^2}{(b-1)}$
C	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rb\sigma^2_{AC} + rab\sigma^2_{C}$	$\longrightarrow \sigma^2 + rab\frac{\sum \gamma_k^2}{(c-1)}$
AxB	$\sigma^2 + r\sigma^2_{ABC} + rc\sigma^2_{AB}$	$\longrightarrow \sigma^2 + rc\frac{\sum (\alpha\beta)_{ij}^2}{(a-1)(b-1)}$
AxC	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$	$\longrightarrow \sigma^2 + rb\frac{\sum (\alpha\gamma)_{ik}^2}{(a-1)(c-1)}$
BxC	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC}$	$\longrightarrow \sigma^2 + ra\frac{\sum (\beta\gamma)_{jk}^2}{(b-1)(c-1)}$
AxBxC	$\sigma^2 + r\sigma^2_{ABC}$	$\longrightarrow \sigma^2 + r\frac{\sum (\alpha\beta\gamma)_{ijk}^2}{(a-1)(b-1)(c-1)}$
Error	σ^2	$\longrightarrow \sigma^2$

Mixed Models

For the expected mean squares for all random models, all variance components remained.

For fixed effect models, all components but the first and last are eliminated.

For mixed effect models:

1. The first and last components will remain.

2. Of the remaining components, some will be eliminated based on the following rules:

    a. Always ignore the first and last variance components.

    b. For the remaining variance components, any letter(s) in the subscript used in naming the effect is ignored.

    c. If any remaining letter(s) in the subscript corresponds to a fixed effect, that variance component drops out.

Example 1 – RCBD with a Factorial Arrangement (A fixed and B random)

SOV	Before		After
Rep	$\sigma^2 + ab\sigma_R^2$	$\longrightarrow$	$\sigma^2 + ab\sigma_R^2$
A	$\sigma^2 + r\sigma_{AB}^2 + rb\sigma_A^2$	$\longrightarrow$	$\sigma^2 + r\sigma_{AB}^2 + rb\dfrac{\sum \alpha_i^2}{(a-1)}$
B	$\sigma^2 + r\sigma_{AB}^2 + ra\sigma_B^2$	$\longrightarrow$	$\sigma^2 + ra\sigma_B^2$
AxB	$\sigma^2 + r\sigma_{AB}^2$	$\longrightarrow$	$\sigma^2 + r\sigma_{AB}^2$
Error	σ^2	$\longrightarrow$	σ^2

Steps for each Source of Variation

Error - No change for Error.

AxB - No change for AxB since only the first and last variance components are present.

B - For the middle variance component, cover up the subscript for B, only A is present.
Since A is a fixed effect this variance component drops out.

A - For the middle variance component, cover up the subscript for A, only B is present.
Since B is a random effect this variance component remains.

Rep - Replicate is always a random effect, so this expected mean square remains the same.

Example 2 CRD with a Factorial Arrangement (A fixed, B and C random)

SOV	Before		After
A	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB} + rbc\sigma^2_A$	$\longrightarrow$	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB} + rbc\dfrac{\sum \alpha_i^2}{(a-1)}$
B	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rc\sigma^2_{AB} + rac\sigma^2_B$	$\longrightarrow$	$\sigma^2 + ra\sigma^2_{BC} + rac\sigma^2_B$
C	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC} + rb\sigma^2_{AC} + rab\sigma^2_C$	$\longrightarrow$	$\sigma^2 + ra\sigma^2_{BC} + rab\sigma^2_C$
AxB	$\sigma^2 + r\sigma^2_{ABC} + rc\sigma^2_{AB}$	$\longrightarrow$	$\sigma^2 + r\sigma^2_{ABC} + rc\sigma^2_{AB}$
AxC	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$	$\longrightarrow$	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$
BxC	$\sigma^2 + r\sigma^2_{ABC} + ra\sigma^2_{BC}$	$\longrightarrow$	$\sigma^2 + ra\sigma^2_{BC}$
AxBxC	$\sigma^2 + r\sigma^2_{ABC}$	$\longrightarrow$	$\sigma^2 + r\sigma^2_{ABC}$
Error	σ^2	$\longrightarrow$	σ^2

Steps for Each Source of Variation

Error - Error remains the same.

AxBxC - The error mean square for AxBxC remains the same since there are only first and last terms.

BxC - Cover up the B and C in the subscript, A remains and corresponds to a fixed effect. Therefore the term drops out.

AxC - Cover up the A and C in the subscript, B remains and corresponds to a random effect. Therefore the term remains.

AxB - Cover up the A and B in the subscript, C remains and corresponds to a random effect. Therefore the term remains.

C - ABC term - Cover up the C term in the subscript, A and B remain. A corresponds to a fixed effect and B corresponds to a random effect. Since one of the terms corresponds to a fixed effect, the variance component drops out.

BC term - Cover up the C term in the subscript, B remains. B corresponds to a random effect. Since B is a random effect, the variance component remains.

AC term - Cover up the C term in the subscript, A remains. A corresponds to a fixed effect. Since A is a fixed effect, the variance component drops out.

B - ABC term - Cover up the B term in the subscript, A and C remain. A corresponds to a fixed effect and C corresponds to a random effect. Since one of the terms corresponds to a fixed effect, the variance component drops out.

BC term - Cover up the B term in the subscript, C remains. C corresponds to a random effect. Since B is a random effect, the variance component remains.

AB term - Cover up the B term in the subscript, A remains. A corresponds to a fixed effect. Since A is a fixed effect, the variance component drops out.

A - ABC term - Cover up the A term in the subscript, B and C remain. B and C correspond to a random effect. Since none of the terms correspond to a fixed effect, the variance component remains.

AC term - Cover up the A term in the subscript, C remains. C corresponds to a random effect. Since C is a random effect, the variance component remains.

AB term - Cover up the A term in the subscript, B remains. B corresponds to a random effect. Since B is a random effect, the variance component remains.

Deciding What to Use as the Denominator of Your F-test

For an all fixed model the Error MS is the denominator of all F-tests.

For an all random or mix model,

1. Ignore the last component of the expected mean square.

2. Look for the expected mean square that now looks this expected mean square.

3. The mean square associated with this expected mean square will be the denominator of the F-test.

4. If you can't find an expected mean square that matches the one mentioned above, then you need to develop a Synthetic Error Term.

Example 1 – RCBD with a Factorial Arrangement (A fixed and B random)

SOV	Expected mean square	MS	F-test
Rep	$\sigma^2 + ab\sigma_R^2$	1	F = MS 1/MS 5
A	$\sigma^2 + r\sigma_{AB}^2 + rb\frac{\sum \alpha_i^2}{(a-1)}$	2	F = MS 2/MS 4
B	$\sigma^2 + ra\sigma_B^2$	3	F = MS 3/MS 5
AxB	$\sigma^2 + r\sigma_{AB}^2$	4	F = MS 4/MS 5
Error	σ^2	5	

Steps for F-tests

F_{AB} - Ignore $r\sigma^2_{AB}$. The expected mean square now looks like the expected mean square for
Error. Therefore, the denominator of the F-test is the Error MS.

F_B - Ignore $ra\sigma^2_B$. The expected mean square now looks like the expected mean square for
Error. Therefore, the denominator of the F-test is the Error MS.

F_A - Ignore $rb\dfrac{\sum \alpha_i^2}{(a-1)}$. The expected mean square now looks like the expected mean square
for AxB. Therefore, the denominator of the F-test is the AxB MS.

Example 2 CRD with a Factorial Arrangement (A fixed, B and C random)

SOV	Expected Mean Square	MS	F-test
A	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB} + rbc\dfrac{\sum \alpha_i^2}{(a-1)}$	1	(MS 1 + MS 7)/(MS 4 + MS 5)
B	$\sigma^2 + ra\sigma^2_{BC} + rac\sigma^2_B$	2	MS 2/MS 6
C	$\sigma^2 + ra\sigma^2_{BC} + rab\sigma^2_C$	3	MS 3/MS 6
AxB	$\sigma^2 + r\sigma^2_{ABC} + rc\sigma^2_{AB}$	4	MS 4/MS 7
AxC	$\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$	5	MS 5/MS 7
BxC	$\sigma^2 + ra\sigma^2_{BC}$	6	MS 6/MS 8
AxBxC	$\sigma^2 + r\sigma^2_{ABC}$	7	MS 7/MS 8
Error	σ^2	8	

Steps for F-tests

F_{ABC} - Ignore $r\sigma^2_{ABC}$. The expected mean square now looks like the expected mean
square for Error. Therefore, the denominator of the F-test is the Error MS.

F_{BC} - Ignore $ra\sigma^2_{BC}$. The expected mean square now looks like the expected mean square for
Error. Therefore, the denominator of the F-test is the Error MS.

F_{AC} - Ignore $rb\sigma^2_{AC}$. The expected mean square now looks like the expected mean square
for AxBxC. Therefore, the denominator of the F-test is the AxBxC MS.

F_{AB} - Ignore $rcb\sigma^2_{AB}$. The expected mean square now looks like the expected mean square
for AxBxC. Therefore, the denominator of the F-test is the AxBxC MS.

F_C - Ignore $rab\sigma^2_C$. The expected mean square now looks like the expected mean square
for BxC. Therefore, the denominator of the F-test is the BxC MS.

F_B - Ignore $rac\sigma^2_B$. The expected mean square now looks like the expected mean square
for BxC. Therefore, the denominator of the F-test is the BxC MS.

F_A - Ignore $rbc\frac{\sum \alpha_i^2}{(a-1)}$. The expected mean square now looks like none of the expected mean
squares. Therefore, we need to develop a Synthetic Mean Square

Need an Expected Mean Square that looks like: $\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB}$.

AC = $\sigma^2 + r\sigma^2_{ABC} + rb\sigma^2_{AC}$ (missing $rc\sigma^2_{AB}$)
and

AB = $\sigma^2 + r\sigma^2_{ABC} + rc\sigma^2_{AB}$ (missing $rb\sigma^2_{AC}$)

An expected mean square can be found that includes all needed variance components if you sum the expected mean squares of AC and AB.

$$\text{AC MS} + \text{AB MS} = 2\sigma^2 + 2r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB}$$

The problem with this sum is that it is too large by $\sigma^2 + r\sigma^2_{ABC}$.

One method would be to get the needed expected mean square is by:

$$\text{AC MS} + \text{AB MS} - \text{ABC MS}$$

Thus F_A could be: $$\frac{\text{A MS}}{\text{AC MS} + \text{AB MS} - \text{ABC MS}}$$

However, this is not the preferred formula for this F-test.

The most appropriate F-test is one in which the number of MS used in the numerator and denominator are similar.

This allows for more accurate estimates of the degrees of freedom associate with the numerator and denominator.

The formula above has one mean square in the numerator and three in the denominator.

The formula for F_A that is most appropriate is

$$\frac{\text{A MS} + \text{ABC MS}}{\text{AC MS} + \text{AB MS}}$$

The numerator and the denominator then become: $2\sigma^2 + 2r\sigma^2_{ABC} + rb\sigma^2_{AC} + rc\sigma^2_{AB}$.

<u>Calculation of Estimated Degrees of Freedom</u>

Calculation of degrees of freedom for the numerator and denominator of the F-test <u>cannot</u> be calculated by adding together the degrees of freedom for the associated mean squares.

For the F-test: $F_A = \dfrac{\text{A MS} + \text{ABC MS}}{\text{AC MS} + \text{AB MS}}$

The numerator degrees of freedom = $\dfrac{(\text{A MS} + \text{ABC MS})^2}{\left[\dfrac{(\text{A MS})^2}{\text{A df}} + \dfrac{(\text{ABC MS})^2}{\text{ABC df}}\right]}$

The denominator degrees of freedom = $\dfrac{(\text{AC MS} + \text{AB MS})^2}{\left[\dfrac{(\text{AC MS})^2}{\text{AC df}} + \dfrac{(\text{AB MS})^2}{\text{AB df}}\right]}$

<u>Calculation of LSD Values – CRD with a Factorial Arrangement (A fixed, B and C Random)</u>

LSD_{ABC} (0.05) = $t_{0.05/2;\,\text{Error df}}\sqrt{\dfrac{2\text{Error MS}}{r}}$

LSD_{BC} (0.05) = $t_{0.05/2;\,\text{Error df}}\sqrt{\dfrac{2\text{Error MS}}{ra}}$

LSD_{AC} (0.05) = $t_{0.05/2;\,\text{ABC df}}\sqrt{\dfrac{2(\text{ABC MS})}{rb}}$

LSD_{AB} (0.05) = $t_{0.05/2;\,\text{ABC df}}\sqrt{\dfrac{2(\text{ABC MS})}{rc}}$

$$\text{LSD}_C\ (0.05) = t_{0.05/2;\ \text{BC df}} \sqrt{\frac{2(\text{BC MS})}{\text{rab}}}$$

$$\text{LSD}_B\ (0.05) = t_{0.05/2;\ \text{BC df}} \sqrt{\frac{2(\text{BC MS})}{\text{rac}}}$$

$$\text{LSD}_A\ (0.05) = t'_{0.05/2;\ \text{Estimated df}} \sqrt{\frac{2(\text{AC MS} + \text{AB MS - ABC MS})}{\text{rbc}}}$$

$$\text{Where Estimated df for t'} = \frac{(\text{AC MS} + \text{AB MS - ABC})^2}{\left[\frac{(\text{AC MS})^2}{\text{AC df}} + \frac{(\text{AB MS})^2}{\text{AB df}} + \frac{(\text{ABC MS})^2}{\text{ABC df}}\right]}$$

<u>Restricted vs. Unrestricted Method of Analysis of Mixed Models</u>

- The choice of using the restricted vs. the unrestricted mixed model analysis will impact what you use as the denominator of the *F*-test.

- What impacts whether an analysis is restricted or unrestricted has to do with the assumptions made on the interaction terms comprised of fixed and random effects.

- **The method of mixed model analyses taught in class is the restricted method.**

- This restricted method assumes that the sum of the interaction effects over the levels of the fixed factor equals zero.

- The unrestricted method does not assume that the sum of the interaction effects over the levels of the fixed factor equals zero.

- **The unrestricted method is method of analysis done by JMP and SAS.**

<u>Choosing Between the Unrestricted and the Restricted Methods of Analysis of Mixed Models</u>

- If you randomly choose levels of your random effect factor(s), then you should use the unrestricted model.

- You can use JMP or SAS to analyze data using the unrestricted method.

- If you are choosing the levels of the random effect factor(s), then you should use the restricted model method of analysis.

- If you wish to analyze data using the restricted method, you will either need to do the *F*-tests by hand or find a program has the option of using the restricted method.

JMP Example for an RCBD with a Factorial Arrangement (A and B both Fixed)

Data

	A	B	Rep	Yield
1	0	0	1	25.7
2	0	0	2	31.8
3	0	0	3	34.6
4	0	0	4	27.7
5	0	0	5	38
6	0	0	6	42.1
7	0	1	1	25.4
8	0	1	2	29.5
9	0	1	3	37.2
10	0	1	4	30.3
11	0	1	5	40.6
12	0	1	6	43.6
13	0	2	1	23.8
14	0	2	2	28.7
15	0	2	3	29.1
16	0	2	4	30.2
17	0	2	5	34.6
18	0	2	6	44.6
19	0	3	1	22
20	0	3	2	26.4
21	0	3	3	23.7
22	0	3	4	33.2
23	0	3	5	31
24	0	3	6	42.7
25	1	0	1	48.9
26	1	0	2	67.5
27	1	0	3	58.4
28	1	0	4	35.8
29	1	0	5	66.9
30	1	0	6	44.2
31	1	1	1	64.7
32	1	1	2	71.5
33	1	1	3	42.5
34	1	1	4	31
35	1	1	5	81.9
36	1	1	6	61.6
37	1	2	1	27.8
38	1	2	2	31
39	1	2	3	31.2
40	1	2	4	29.5
41	1	2	5	31.5
42	1	2	6	38.9
43	1	3	1	23.4
44	1	3	2	27.8
45	1	3	3	29.8
46	1	3	4	30.7
47	1	3	5	35.9
48	1	3	6	37.6

Data Filter Tabulate Graph Builder Distribution Fit Y by X Fit Model Recode

Factorial…

Columns (…

A

B

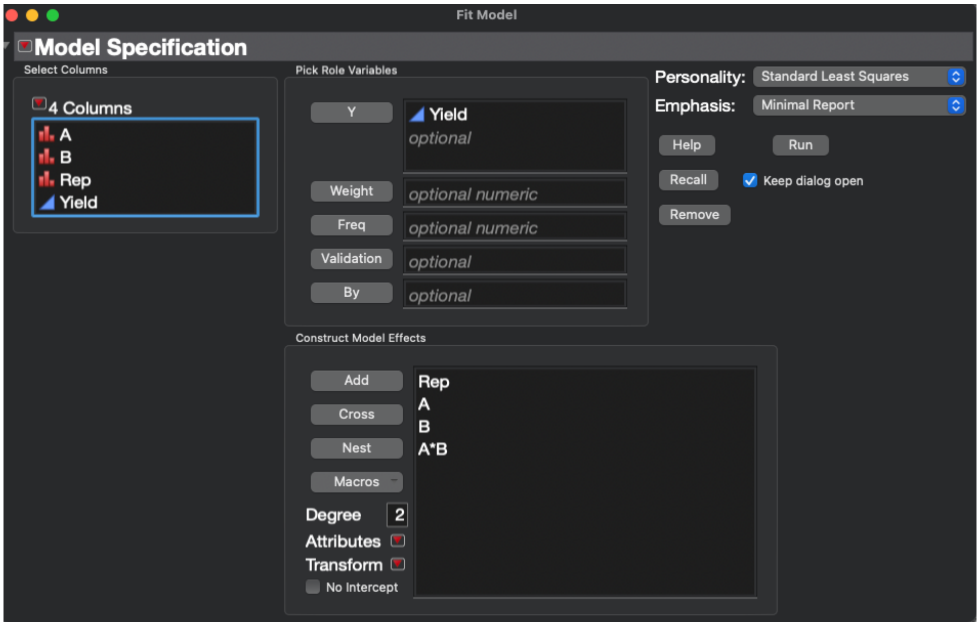
Rep

Yield

Rows

All rows	48
Selected	0
Excluded	0
Hidden	0

Model

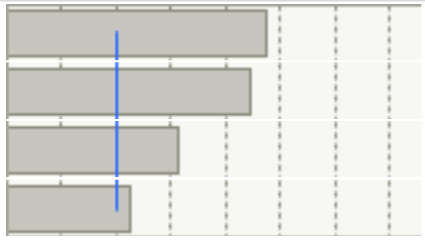

Fit Model
Model Specification
Select Columns
4 Columns
A
B
Rep
Yield
Pick Role Variables
Y
Yield
optional
Weight
optional numeric
Freq
optional numeric
Validation
optional
By
optional
Personality:
Standard Least Squares
Emphasis:
Minimal Report
Help
Run
Recall
Keep dialog open
Remove
Construct Model Effects
Add
Cross
Nest
Macros
Degree
2
Attributes
Transform
No Intercept
Rep
A
B
A*B

JMP Output from the Analysis of an RCBD with a Factorial Arrangement
A and B are Both Considered Fixed Effects

Output**Response Yield**

Effect Summary

Source	LogWorth	PValue
B	4.756	0.00002
A	4.480	0.00003
A*B	3.135	0.00073
Rep	2.269	0.00538



Summary of Fit

RSquare	0.739833
RSquare Adj	0.650633
Root Mean Square Error	8.294197
Mean of Response	38.05208
Observations (or Sum Wgts)	48

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	12	6846.9800	570.582	8.2941
Error	35	2407.7798	$\boxed{68.794}$	**Prob > F**
C. Total	47	9254.7598		<.0001*

By default, JMP uses this error MS as the denominator of all *F*-tests.

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Rep	5	5	1387.8185	4.0347	0.0054*
A	1	1	1558.3802	22.6529	<.0001*
B	3	3	2428.6240	11.7677	<.0001*
A*B	3	3	1472.1573	7.1332	0.0007*

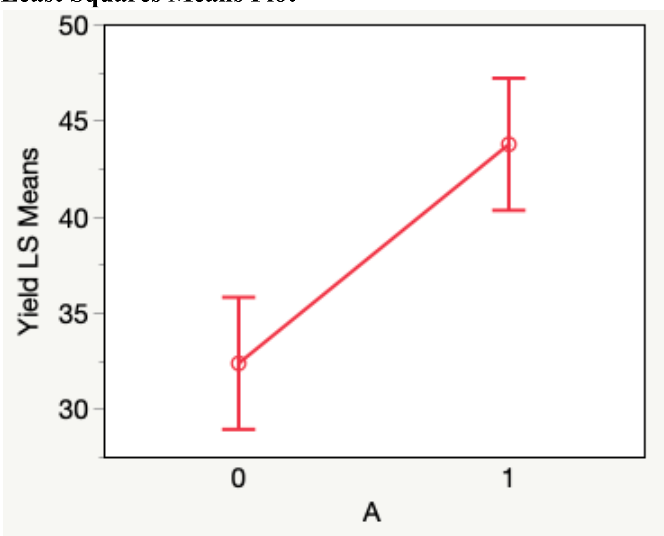
JMP Output from the Analysis of an RCBD with a Factorial Arrangement
A and B are Both Considered Fixed Effects

Effect Details

A
Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
0	32.354167	1.6930459	32.3542
1	43.750000	1.6930459	43.7500

Least Squares Means Plot



LSMeans Differences Student's t
α=0.050 t=2.03011
LSMean[i] By LSMean[j]

Mean[i]-Mean[j] Std Err Dif Lower CL Dif Upper CL Dif	0	1
0	0 0 0 0	-11.396 2.39433 -16.257 -6.5351
1	11.3958 2.39433 6.53509 16.2566	0 0 0 0

Level			Least Sq Mean
1	A		43.750000
0		B	32.354167

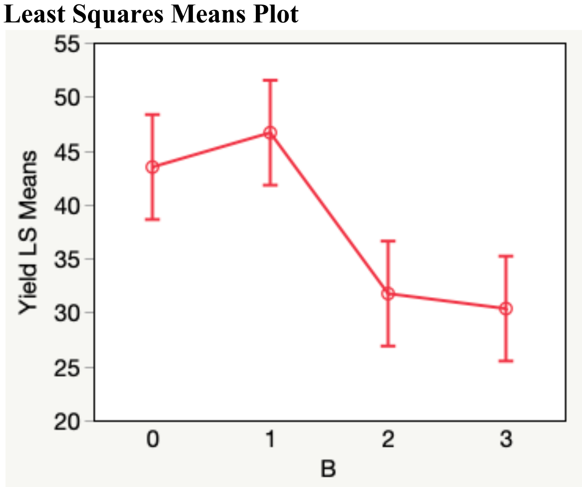
JMP Output from the Analysis of an RCBD with a Factorial Arrangement
A and B are Both Considered Fixed Effects

Levels not connected by same letter are significantly different.

B
Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
0	43.466667	2.3943285	43.4667
1	46.650000	2.3943285	46.6500
2	31.741667	2.3943285	31.7417
3	30.350000	2.3943285	30.3500

Least Squares Means Plot



LSMeans Differences Student's t
α=0.050 t=2.03011
LSMean[i] By LSMean[j]

Mean[i]-Mean[j] Std Err Dif Lower CL Dif Upper CL Dif	0	1	2	3
0	0 0 0 0	-3.1833 3.38609 -10.057 3.6908	11.725 3.38609 4.85087 18.5991	13.1167 3.38609 6.24253 19.9908
1	3.18333 3.38609 -3.6908 10.0575	0 0 0 0	14.9083 3.38609 8.0342 21.7825	16.3 3.38609 9.42587 23.1741
2	-11.725 3.38609 -18.599 -4.8509	-14.908 3.38609 -21.782 -8.0342	0 0 0 0	1.39167 3.38609 -5.4825 8.2658
3	-13.117 3.38609 -19.991 -6.2425	-16.3 3.38609 -23.174 -9.4259	-1.3917 3.38609 -8.2658 5.48247	0 0 0 0

JMP Output from the Analysis of an RCBD with a Factorial Arrangement
A and B are Both Considered Fixed Effects

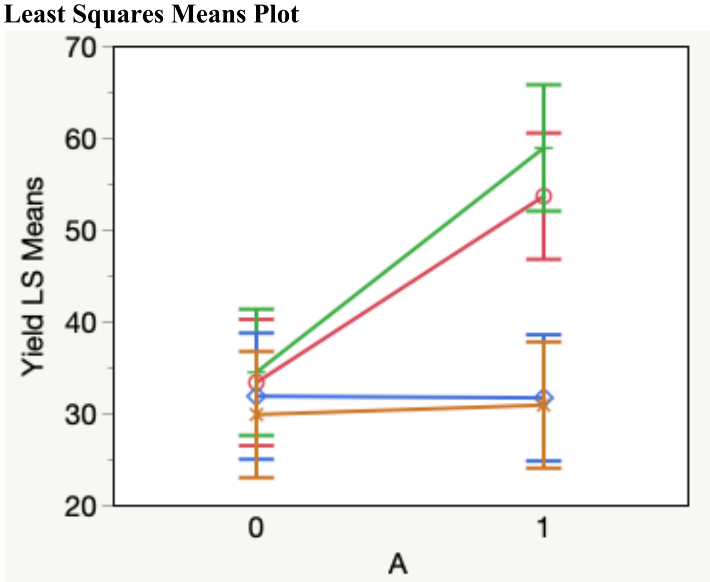
Level			Least Sq Mean
1	A		46.650000
0	A		43.466667
2		B	31.741667
3		B	30.350000

Levels not connected by same letter are significantly different.

A*B
Least Squares Means Table

Level	Least Sq Mean	Std Error
0,0	33.316667	3.3860919
0,1	34.433333	3.3860919
0,2	31.833333	3.3860919
0,3	29.833333	3.3860919
1,0	53.616667	3.3860919
1,1	58.866667	3.3860919
1,2	31.650000	3.3860919
1,3	30.866667	3.3860919

Least Squares Means Plot



JMP Output from the Analysis of an RCBD with a Factorial Arrangement
A and B are Both Considered Fixed Effects

LSMeans Differences Student's t

α=0.050 t=2.03011

LSMean[i] By LSMean[j]

Mean[i]-Mean[j] Std Err Dif Lower CL Dif Upper CL Dif	0,0	0,1	0,2	0,3	1,0	1,1	1,2
0,0	0	-1.1167	1.48333	3.48333	-20.3	-25.55	1.66667
	0	4.78866	4.78866	4.78866	4.78866	4.78866	4.78866
	0	-10.838	-8.2382	-6.2382	-30.021	-35.271	-8.0548
	0	8.60482	11.2048	13.2048	-10.579	-15.829	11.3882
0,1	1.11667	0	2.6	4.6	-19.183	-24.433	2.78333
	4.78866	0	4.78866	4.78866	4.78866	4.78866	4.78866
	-8.6048	0	-7.1215	-5.1215	-28.905	-34.155	-6.9382
	10.8382	0	12.3215	14.3215	-9.4618	-14.712	12.5048
0,2	-1.4833	-2.6	0	2	-21.783	-27.033	0.18333
	4.78866	4.78866	0	4.78866	4.78866	4.78866	4.78866
	-11.205	-12.321	0	-7.7215	-31.505	-36.755	-9.5382
	8.23816	7.12149	0	11.7215	-12.062	-17.312	9.90482
0,3	-3.4833	-4.6	-2	0	-23.783	-29.033	-1.8167
	4.78866	4.78866	4.78866	0	4.78866	4.78866	4.78866
	-13.205	-14.321	-11.721	0	-33.505	-38.755	-11.538
	6.23816	5.12149	7.72149	0	-14.062	-19.312	7.90482
1,0	20.3	19.1833	21.7833	23.7833	0	-5.25	21.9667
	4.78866	4.78866	4.78866	4.78866	0	4.78866	4.78866
	10.5785	9.46184	12.0618	14.0618	0	-14.971	12.2452
	30.0215	28.9048	31.5048	33.5048	0	4.47149	31.6882
1,1	25.55	24.4333	27.0333	29.0333	5.25	0	27.2167
	4.78866	4.78866	4.78866	4.78866	4.78866	0	4.78866
	15.8285	14.7118	17.3118	19.3118	-4.4715	0	17.4952
	35.2715	34.1548	36.7548	38.7548	14.9715	0	36.9382
1,2	-1.6667	-2.7833	-0.1833	1.81667	-21.967	-27.217	0
	4.78866	4.78866	4.78866	4.78866	4.78866	4.78866	0
	-11.388	-12.505	-9.9048	-7.9048	-31.688	-36.938	0
	8.05482	6.93816	9.53816	11.5382	-12.245	-17.495	0
1,3	-2.45	-3.5667	-0.9667	1.03333	-22.75	-28	-0.7833
	4.78866	4.78866	4.78866	4.78866	4.78866	4.78866	4.78866
	-12.171	-13.288	-10.688	-8.6882	-32.471	-37.721	-10.505
	7.27149	6.15482	8.75482	10.7548	-13.029	-18.279	8.93816

JMP Output from the Analysis of an RCBD with a Factorial Arrangement
A and B are Both Considered Fixed Effects

Level			Least Sq Mean
1,1	A		58.866667
1,0	A		53.616667
0,1		B	34.433333
0,0		B	33.316667
0,2		B	31.833333
1,2		B	31.650000
1,3		B	30.866667
0,3		B	29.833333

Levels not connected by same letter are significantly different.