UNIVERSITÀ DI TRENTO

Measurement of the curvature of a mirror using a Michelson interferometer

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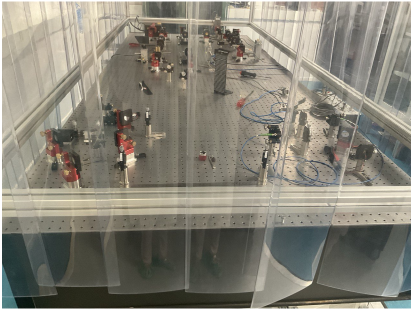
The topics covered in this presentation are:

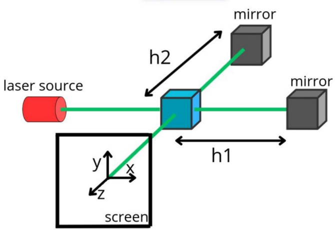
- Michelson interferometer
- Two theoretical models: geometric optics and interference of Gaussian beam
- Data analysis
- Phase shift method



The electric field can be expressed in the complex notation as:

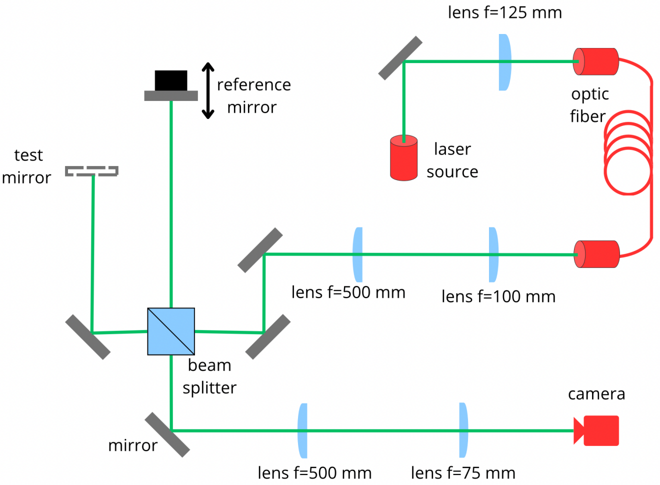
$$\vec{E} = \vec{E_0} e^{i\Phi}$$

If $\vec{E_1}$ and $\vec{E_2}$ are the electric in the two arms, the total filed is $\vec{E_{tot}} = \vec{E_1} + \vec{E_2}$, then the intensity on the screen is proportional to:

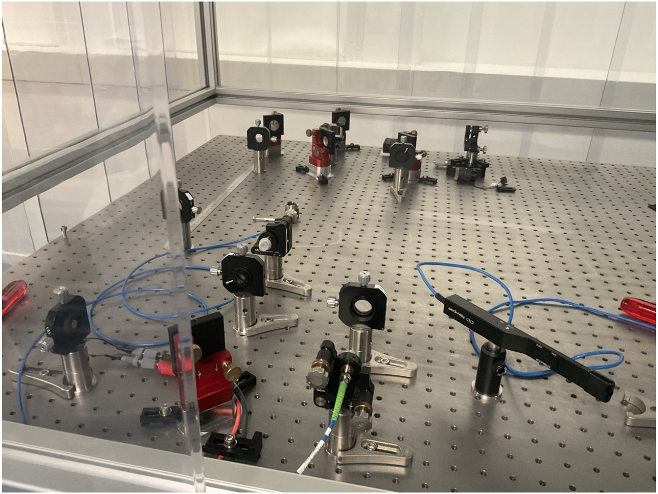


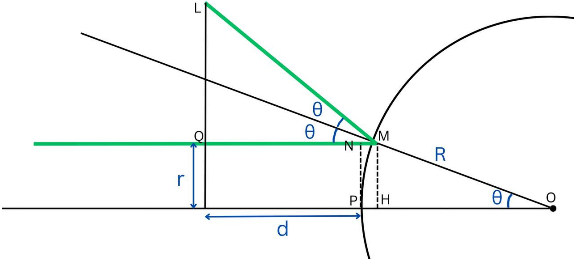
$$I \propto < |\vec{E_{tot}}|^2 > = 2I_0^2[1+\cos(\Phi_1 - \Phi_2)] = 2I_0^2[1+\cos(k\Delta z)]$$

Experimental set-up



Experimental set-up





The optical path difference in the two arms of the interferometer is:

$$\Delta z = 2d - (d + \overline{NM} + \overline{ML}) = \frac{2(1-\cos\theta)[d + d\cos\theta + R\cos^2\theta]}{2cos^2\theta - 1} \simeq$$

$$\simeq \frac{r^2}{R^2}(2d+R) \simeq \frac{r^2}{R} \quad \rightarrow \quad \boxed{I \propto \left[1 + \cos\left(\frac{kr^2}{R}\right)\right]}$$

The equation of a Gaussian beam is:

$$E = E_0 \frac{w_0}{w(z)} \exp\left(\frac{-r^2}{w^2(z)}\right) \exp\left[ik\left(z + \frac{r^2}{2R(z)}\right) - i\tan^{-1}\left(\frac{z}{z_0}\right)\right]$$

where:

$$z_0 = \frac{\pi w_0^2}{\lambda} \qquad w(z) = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2} \qquad R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

Let us call Beam_1 the beam reflected by the spherical mirror and Beam_2 the beam coming from the planar mirror:

	w_0	z_0	$w(z)$	$R(z)$
Beam_1	$\simeq 0.025$ mm	$\simeq 3.8$ mm	$w_{01} z_1 / z_{01}$	z_1
Beam_2	$\simeq 5$ mm	$\simeq 150$ m	w_{02}	z_{02}^2 / z_2

The intensity of the electric magnetic field on the screen will be:

$$I \propto |E_{tot}|^2 =$$

$$= E_0^2 \left\{ \exp\left(-\frac{2r^2}{w_{02}^2}\right) + \left(\frac{z_{01}}{z_1}\right)^2 \exp\left(-\frac{2r^2 z_{01}^2}{w_{01}^2 z_1^2}\right) + \right.$$
$$\left. + \left(\frac{z_{01}}{z_1}\right) \exp\left[-r^2\left(\frac{1}{w_{02}^2} + \left(\frac{z_{01}}{w_{01} z_1}\right)^2\right)\right] 2\cos\left[k(z_2 - z_1) + \frac{kr^2}{2}\left(\frac{z_2}{z_{02}^2} + \frac{1}{z_1}\right)\right]\right\} \simeq$$

$$\simeq E_0^2 \left[\exp\left(-\frac{2r^2}{w_{02}^2}\right) + \left(\frac{z_{01}}{z_1}\right)^2 + 2\left(\frac{z_{01}}{z_1}\right)^2 \exp\left(-\frac{r^2}{w_{02}^2}\right) \boxed{\cos\left(\frac{kr^2}{R}\right)}\right]$$

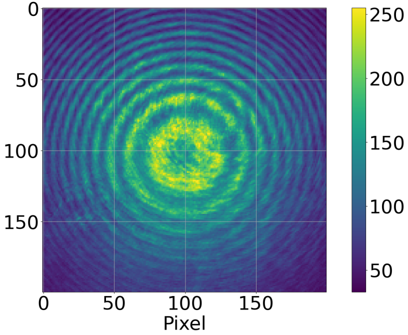
The fit function is:

$$I = A\left[B^2 + \exp\left(-2ar^2\right) + 2B\exp\left(-ar^2\right)\cos\left(br^2 + c\right)\right]$$

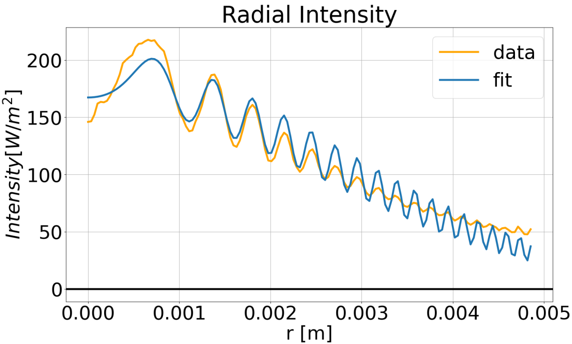
where from the previous equation we have that:

$$a = \frac{1}{w_{02}^2} \qquad b = \frac{2\pi}{\lambda(R+2d)}$$

$$B = \frac{z_{01}}{z_1}$$



Data analysis



$$R = \frac{2\pi}{b\lambda} = (2.57 \pm 0.01)\ \mathrm{m}$$

Let us consider the intensity on the camera at different position of the planar mirror ($d + \delta\Phi_z$):

$$I = A(x,y)\Big[1 + B(x,y)\cos\Big(\Phi(x,y) + \delta\Phi_z\Big)\Big] + \mathit{off}(x,y)$$

The surface of the mirror $h(x,y)$ is related to the phase $\Phi(x,y)$:

$$h(x,y) = \frac{\lambda}{4\pi}\Phi(x,y)$$

For fixed value of r, we observe that the intensity is a sinusoidal function of $\delta\Phi_z$:

$$I(\delta\Phi_z) = A\sin(\delta\Phi_z + \Phi) + \mathit{off} = A\sin(nv\Delta t + \Phi) + \mathit{off}$$

Phase shift interferometry

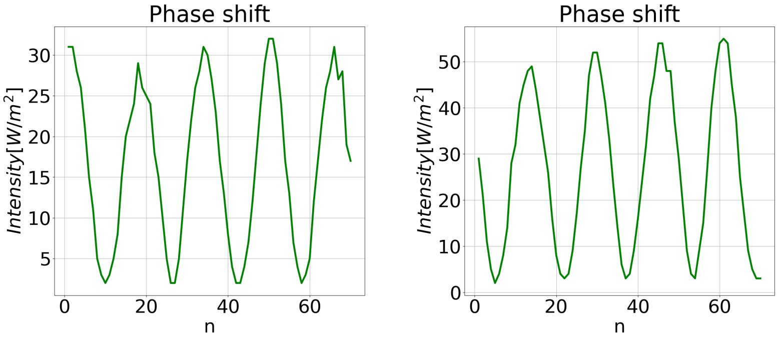


Figure: The first image is for the pixel with coordinates (0,0) and the second for the pixel with coordinates (20,50).

Analysis of the data performed by the code:

- get Φ from the data, there are two option:
  - sinusoidal fit
  - fft : fast Fourier transform
- unwrap the phase Φ
- fit to get the parameter that describe the surface, there are two option:
  - $\left[A\left((y-y_c)\cos(\alpha)+(x-x_c)\sin(\alpha)\right)^2+B\left((x-x_c)\cos(\alpha)-(y-y_c)\sin(\alpha)\right)^2\right]+o$
  - Zernike polynomials