Principles of Probability

Homework 02

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1 3.7

a. A ... Picking a red card from a standard deck as a first card
B ... Picking a black card from a standard deck as the second card.
C ... Assuming you only pick from the black cards.

$$P(A) \times P(B|A) = \frac{\binom{26}{1}}{\binom{52}{1}} \times \frac{\binom{26}{1}}{\binom{51}{1}} \neq \frac{\binom{26}{1}}{\binom{52}{1}} \times \frac{\binom{26}{1}}{\binom{52}{1}}$$

$$P(A|C) \times P(B|A,C) = 0 \times P(B|A,C) P(A|C) \times P(B|C) = 0 \times P(B|C)$$

b. Students have 10 exams.
A ... Passing one exam
B ... Passing all your other exams
C ... Assuming we know we failed one exam

2 3.9

Let A_i be the event that the prize is behind our choosen door i, where $i \in \{1,2,3\}$. Let A_j and A_k be the probabilities that the prize is behind door j and k where $k \neq j \neq i$ and $j \in \{1,2,3\}$. Therefore if we pick door i, then the host reveals that there is nothing behind door k then the probability that the prize is behind door i is:

$$p(A_i|\overline{A_k}) = p(A_i)p(A_i|\overline{A_k}, A_i) + p(\overline{A_i})p(A_i|\overline{A_k, A_i}) = 1/3 \times 1 + 2/3 \times 0 = 1/3$$

$$p(A_j|\overline{A_k}) = p(A_i)p(A_j|\overline{A_k}, A_i) + p(A_j)p(A_j|\overline{A_k, A_i}) = 1/3 \times 0 + 2/3 \times 1 = 2/3$$

Because of the law of total probability the conditional probability $p(A_j|\overline{A_k, A_i})$ is 1. Therefore it makes sense to switch.

3 6.2

a.

$$\Sigma_{i=1}^{n} i \times \lambda(i) = 0$$

Lebeque measure of Singleton is zero.

b.

$$\int_f (w) d\lambda = w \times \lambda([0,1] \cap Q^c) = 1$$

c.

$$\int_f (w) = n \times \lambda([0,n] \cap Q^c) = n^2$$