HOMEWORK - 5

5. B.

a) $f(x,y) = 6(x-y)^2$

should not use the same variables

$$P_{x,y}(x,y) = \int_0^y \int_0^x f(x,y)\,dx\,dy =$$

$$= \int_0^y \int_0^x 6(x^2 - 2xy + y^2)\,dx\,dy =$$

$$= 6\int_0^y \frac{x^3}{3} - x^2y + xy^2 + C \Big|_0^x \,dy =$$

$$= 6\int_0^y \frac{x^3}{3} - x^2y + xy^2\,dy =$$

$$= 6\left[\frac{x^3y}{3} - \frac{x^2y^2}{2} + \frac{xy^3}{3}\right]\Bigg|_0^y =$$

$$= 6\left[\frac{x^3y}{3} - \frac{x^2y^2}{2} + \frac{xy^3}{3}\right]$$

b) $f_x(x) = \int_0^1 f(x,y)\,dy = \int_0^1 6(x-y)^2\,dy =$

$= 6\int_0^1 x^2 - 2xy + y^2\,dy = 6\left[x^2y - xy^2 + \frac{y^3}{3}\right]\Big|_0^1 =$

$= 6\left[x^2 - x + 1/3\right]$

$f_y(y) = 6\left[y^2 - y + 1/3\right]$ because of simetricity

c) $f_{x|y}(x|y) = \frac{f(x,y)}{f(y)} = \frac{6[x^2 - 2xy + y^2]}{[y^2 - y + 1/3]}$

$f_{y|x}(y|x) = \frac{f(x,y)}{f(x)} = 6\,\frac{x^2 - 2xy + y^2}{x^2 - x + 1/3} = 6\,\frac{(x-y)^2}{x^2 - x + 1/3}$

$= (x^2 - 2xy + y^2) : (x^2 - x + 1) = 1 + \frac{x - 2y}{x}$

$-x^2 + x - 1$

$-2xy + x + y^2 - 1$

$-2xy + 2y - \frac{2y}{x} + x + y^2 - 1$

$2x + 2y^2 + 2y - 2 - \frac{2y}{x}$

2.

e) Rejection sampling

uniformly generate num