$$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

7.7. a) $X \sim Exp(\lambda)$

$$E[X] = \int_0^\infty x \lambda e^{-\lambda x}\, dx =$$

$$= \lambda \int_0^\infty x e^{-\lambda x}\, dx$$

$$-\frac{x}{\lambda} e^{-\lambda x} - \int -\frac{e^{-\lambda x}}{\lambda}\, dx$$

$$= -\frac{x}{\cancel{\lambda}} e^{-\lambda x} - \frac{e^{-\lambda x}}{\lambda} \Big|_0^\infty =$$

$$= -\frac{x}{e^{\lambda x}}\Big|_0^\infty - \frac{e^{-\lambda x}}{\lambda}\Big|_0^\infty =$$

$$= -\frac{1}{\lambda e^{\lambda\infty}} - 0 - \frac{1}{\lambda e^{\lambda \infty}} + \frac{1}{\lambda} = \underline{\underline{\frac{1}{\lambda}}}$$

b) $Var[X] = E[X^2] - E[X]^2$

$$E[X^2] = \int_0^\infty x^2 \lambda e^{-\lambda x}\, dx =$$

$$= \lambda \int_0^\infty \frac{t^2}{\lambda^2} e^{-t} \frac{dt}{\lambda} = \frac{1}{\lambda^2}\Gamma(3) =$$

$$= \frac{2}{\lambda^2}$$

$$VAR[X] = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \underline{\underline{\frac{1}{\lambda^2}}}$$

$$= \frac{1}{n}\sum^{n} \mu = \underline{\underline{\mu}}$$

$$Var[\hat{X}] = \cancel{E[\hat{X}^2] - E[\hat{X}]^2} =$$

$$= Var\left[\frac{1}{n}\sum^{n} X_i\right] = \cancel{\frac{1}{n^2} Var[X_i]}$$

$$= \frac{1}{n^2}\sum^{n} Var[X_i] =$$

$$= \frac{1}{n^2} n \sigma^2 = \frac{\sigma^2}{n}$$

7.11

a) $E[X|Y=y]$ $\qquad f_{X|Y} = \frac{f_{XY}}{f_Y}$

$$f_{XY}(X,Y) = \frac{1}{Y+1} e^{-\frac{X}{Y+1}} 0.5^{Y+1}$$

$$f_Y(Y) = \int_{0}^{\infty} \frac{1}{Y+1} e^{-\frac{X}{Y+1}} 0.5^{Y+1}\, dx =$$

$$= \frac{1}{Y+1} 0.5^{Y+1} \int_{0}^{\infty} e^{-\frac{x}{Y+1}}\, dx =$$

$$= \frac{1}{Y+1} 0.5^{Y+1} \left. +(Y+1) e^{-\frac{X}{Y+1}} \right|_{0}^{\infty} =$$

$$= 0.5^{Y+1} = \underbrace{0.5(1-0.5)^{Y}}_{\text{geometric distribution}}$$

$$f_{X|Y} = \frac{f_{XY}}{f_Y} = \frac{1}{Y+1} e^{-\frac{X}{Y+1}} \qquad \frac{1}{\lambda}$$

$$\lambda e^{-\lambda x}$$

exponential distribution

$\text{a)}\ E[X] =$

$f(x,y) = f(x|y)f(y) = f(y|x)f(x)$

$= E[E[X|Y]] =$ law of iterated ~~expectation~~ expectation

$E[X|Y] = \frac{1}{\frac{1}{Y+1}} = Y+1$

$= E[Y+1] = 1 + E[Y] = 1 + \frac{1-0.5}{0.5} =$

$= \cancel{1.5}\ \underline{\underline{2}}$

13 ~~a) $Cov[X,Y]$~~

b) $Var[Z] = Var[2X - 3Y] =$

$= Var[2X] + Var[-3Y] + 2COV[2X, -3Y]$

$= 4Var[X] + 9Var[Y] - 2 \cdot 2 \cdot 3 COV[X,Y]$

$= 4 \cdot 5{,}357 + 9 \cdot 2{,}5 - 12(-3.175) =$

$= \cancel{81.\ldots}\ 82.0828$

14 $X \sim U(0,1)$ $Y|X = x \sim \text{Uniform}(0,x)$

a) $f_{XY} = f_{Y|X} f_X =$

$E[((\alpha X + \beta Y) - E[\alpha X + \beta Y])^2] =$

$= E[((\alpha X - E[\alpha X]) + (\beta Y - E[\beta Y]))^2] =$

$= E[(\alpha X - E[\alpha X])^2 + (\beta Y - E[\beta Y])^2 +$
$2(\alpha X - E[\alpha X])(\beta Y - E[\beta Y])] =$
$Var[\alpha X] + Var[\beta Y] + 2Cov[\alpha X, \beta Y]$