$$\frac{c}{3}\int_0^1 x^3 y\Big|_{-1}^{\sqrt{y}} dy = \frac{c}{3}\int_0^1 -y^{\frac{5}{2}} + y\, dy = \frac{c}{3}\left[-\frac{y^{7/2}}{7/2} + \frac{y^2}{2}\right]\Bigg|_0^1 =$$

$$= \frac{c}{3}\left[-\frac{2}{7} + \frac{1}{2}\right] = \frac{c}{3}\left[\frac{-4+7}{14}\right] = c\,\frac{3}{14}$$

$$\int_0^1\int_{-\sqrt{y}}^{\sqrt{y}} c x^2 y\, dx\, dy = c\int_0^1 \frac{x^3 y}{3}\Big|_{-\sqrt{y}}^{\sqrt{y}} dy =$$

$$= \frac{c}{3}\int_0^1 y^{\frac{5}{2}} + y^{\frac{5}{2}}\, dy = \frac{2c}{3}\,\frac{y^{\frac{7}{2}}}{\frac{7}{2}}\Bigg|_0^1 = \frac{2c}{3}\,\frac{2}{7} = \frac{4c}{21}$$

$$\boxed{c = \frac{21}{4}}$$

5.5

~~ex~~ 7.3 $X \sim Pois(\lambda)$

a) $$E[X] = \sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!}\, k =$$

$$= e^{-\lambda}\sum_{k=1}^{\infty} \frac{\lambda^k}{k!}\, k = \text{(zero at } k=0)$$

$$= e^{-\lambda}\sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!} =$$

$$= \lambda e^{-\lambda}\sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} = \text{taylor expansion of } e^{\lambda}$$

$$= \lambda e^{-\lambda} e^{\lambda} = \lambda$$

b) $$Var[X] = E[X^2] - E[X]^2 =$$

$$= E[X(X-1) + X] - E[X]^2 =$$

$$= E[X(X-1)] + \lambda - \lambda^2 = \quad \text{at } k=0,1 \text{ expression zero}$$

$$= e^{-\lambda}\sum_{k=2}^{\infty} \frac{\lambda^k}{k!}\, k(k-1) + \lambda - \lambda^2 =$$

$$= \lambda^2 e^{-\lambda}\sum_{k=2}^{\infty} \frac{\lambda^{k-2}}{(k-2)!} + \lambda - \lambda^2 = \lambda$$