Mathematics 2, Part 4

Homework 2

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The homework consists of three problems. The solutions are to be submitted to the appropriate mailbox on ucilnica before the exam, but preferrably in a week. The solutions should contain a clear and well explained proofs, procedures, explanations, etc.

(1) Let $f_1(p) = \int_{-1}^{1} p(x)\,dx$, $f_2(p) = \int_0^1 p(x)\,dx$ and $f_3(p) = \int_0^2 p(x)\,dx$.
   (a) You showed in HW Part4/HW1/Problem2 that $\{f_1, f_2, f_3\}$ form a basis for $(\mathbb{R}_2[x])^*$.
   (b) Find the dual basis for $\{f_1, f_2, f_3\}$.
   (c) You showed in HW Part4/HW1/Problem3 that

$$g(p,q) = \int_{-1}^{1} p(x)q(x)\,dx$$

   is an inner product on $\cancel{(\mathbb{R}_3[x])}$ $\mathbb{R}_2[x]$. Find the reciprocal basis for $\{p_1, p_2, p_3\}$ computed in HW Part4/HW1/Problem2(b).

(2) Show that $\mathrm{Bil}(V \times V)$ is isomorphic to the set of all linear transformations from V to V^* by explicitly constructing the isomorphism.

(3) Let $\mathcal{S} = \{e_1, e_2\}$ be a standard basis for $\mathbb{R}^2$ and $\mathcal{S}^* = \{\varepsilon^1, \varepsilon^2\}$ its dual basis. Let T be a $(1,2)$-tensor on $\mathbb{R}^2$ given by

$$T(\varepsilon_i, e_j, e_k) = \begin{cases} j - i, & \text{if } k = 1, \\ i - j, & \text{if } k = 2. \end{cases}$$

Let ABC be your student ID number modulo 1000. Compute $T(\alpha, v, w)$, where $\alpha = \varepsilon^1 + \varepsilon^2$, $v = \begin{bmatrix} A \\ B \end{bmatrix}$ and $w = \begin{bmatrix} C \\ 0 \end{bmatrix}$.