Mathematics 2, part 2, lecture 3

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Extra optimization topics

- Nelder-Mead method
- Local search, case study
- Interior point methods, pushing the boundaries

Nelder - Mead (simplex) method

- heuristic method performing search in n-dim space
- samples n+1 points (forming a simplex) and produces a new sample of n+1 points (sometimes replacing a single point)
- zero-order method
- can crawl out of local extrema
- a.k.a. amoeba method

What is a simplex?

Simplex (n-simplex) is a convex hull of $n+1$ affinely independent points.

$x_0, x_1, x_2, \dots, x_n$ are affinely independent

How to construct a simplex with equal edge lengths?

$$x_0 \,,\; x_0 + (1, 0, \dots) \,,\; x_0 + (0, 1, 0, \dots) \,,\; \dots \,,\; x_0 + (0, \dots, 0, 1)$$

Does this work?

Nelder-Mead notation

$f \longleftarrow$ cost function minimization

$x_0, x_1, \dots, x_n \longleftarrow$ sample points

$y_0 \le y_1 \le \dots \le y_n \longleftarrow$ f-values $y_i = f(x_i)$

$x_0, x_{n-1}, x_n \longleftarrow$ best, lousy, worst points

How to produce next $n+1$ sample points?

① replace x_n with a better point x_{NEXT}, keep $x_0, \dots, x_{n-1}$

OR

② only keep x_0 and replace all the rest

Nelder-mead operations

Reflect

Expand

Contract (outside)

(inside)

Shrink

Nelder-mead termination criteria

- points $x_0, x_1, \dots, x_n$ sufficiently close
- values $y_0, y_1, \dots, y_n$ sufficiently close
- combination of above

Local search

- heuristic method for solving (hard) optimization problems

Ω ← search space / set of feasible solutions

f ← cost function minimization

$x \in \Omega$ ← feasible solution

$N(x) \subseteq \Omega$ ← neighborhood of x

$$x_0, x_1, x_2, x_3, \ldots, x_i, \ldots$$

$$\forall i \quad f(x_{i+1}) < f(x_i)$$

$$\forall i \quad x_{i+1} \in N(x_i)$$

Local search - example

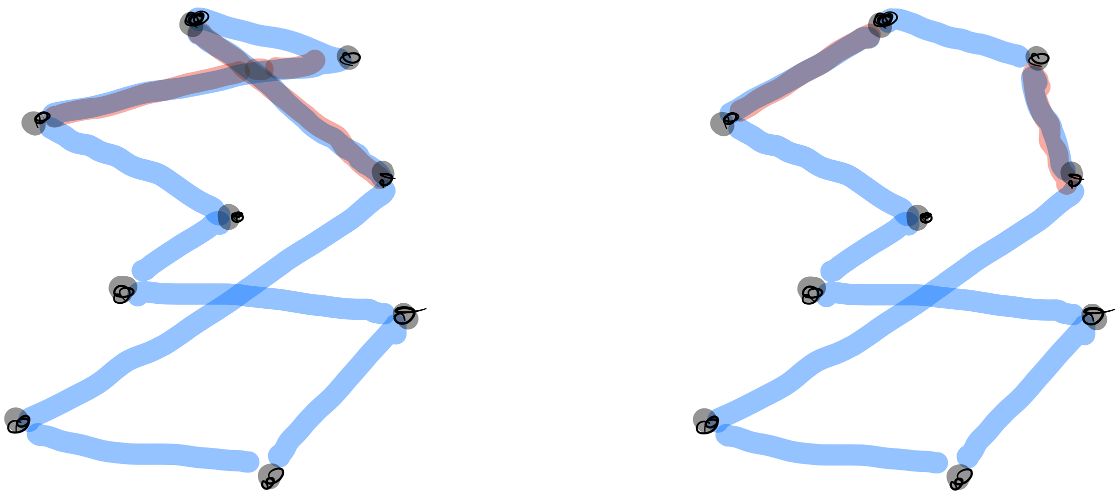
Travelling salesman problem TSP (geometric version)

input: $p_1, p_2, \dots, p_n$ points in $\mathbb{R}^2$

output: shortest closed walk containing all above points

(it should be a Hamiltonian cycle)

Neighborhood



also called a 2-opt move

Proposition If TS tour T is 2-optimal then T is without crossings.

Local search can get stuck. What now?

- simulated annealing
  allow bad steps if temperature is high
- tabu search
  do not reverse recent steps
- jump moves
  allow bad steps sometimes

Interior point methods

- not limited to LP, can be used in other convex opt. problems

Quadratic prog.

$$\min \; x^T Q x + c^T x$$

$$Ax = b$$

$$Q \succeq 0$$

Q PSD matrix

Semidefinite prog.

$$\min \; c^T x$$

$$Ax = b$$

$$x \succeq 0$$

X PSD matrix

Conic opt.

$$\min \; c^T x$$

$$Ax = b$$

$$x \in K$$

cone

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