INTRODUCTION TO NAND

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Abstract. An introduction to NAND and its analogue in logic, the Sheffer Stroke.

1. The Laws of Thought

> The quasi mathematical methods of Dr. Boole especially are so magical and abstruse, that they appear to pass beyond the comprehension and criticism of most other writers… (Jevons 1869)

In 1854 George Boole published "The Laws of Thought," the second of Boole's two monographs on algebraic logic. Boole's goals with the work were "to go under, over, and beyond Aristotle's logic" and in it Boole introduces the notions of algebraic symbols and equation solving to logic.

2. The Sheffer Stroke

In 1913, Henry M. Sheffer published a paper providing an axiomatization of Boolean algebras, using what is now known as the *Sheffer stroke.* He proved its equivalence to the standard formation employing the familiar operators of propositional logic (and ($\wedge$), or ($\vee$), and not ($\neg$)). The Sheffer stroke, written as $\uparrow$, denotes a logical operation that is equivalent to the negation of the conjunction operation, expressed in ordinary language as "not both". It is also called *nand* ("not and") or the *alternative denial*, since it says in effect that at least one of its operands is false.

Charles Sanders Peirce had discovered the functional completeness of NAND and NOR more than 30 years earlier, but never published the result. The Peirce arrow, corresponding to NOR, is named after him.

Problem: Construct the operators of propositional logic from the *Peirce arrow.*

Solution:

$$p \wedge q \text{ (p and q): } (p \downarrow p) \downarrow (q \downarrow q) \tag{1}$$

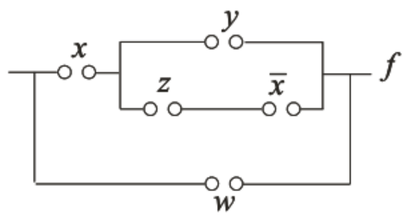
$$p \vee q \text{ (p or q): } (p \downarrow q) \downarrow (p \downarrow q) \tag{2}$$

$$\neg p \text{ (not p): } p \downarrow p \tag{3}$$

3. A Symbolic Analysis of Relay and Switching Circuits

As early as 1886 Charles Sanders Peirce saw that logical operations could be carried out by electrical switching circuits, an insight that would bear fruition with Claude E. Shannon's 1937 Master's thesis, *A Symbolic Analysis of Relay and*

FIGURE 1. An example of a circuit being used to implement Boolean logic.



Switching Circuits. In his thesis, Shannon proved that Boolean algebra could be used to simplify the arrangement of the relays that were the building blocks of the electromechanical automatic telephone exchanges of the day. Shannon went on to prove that it should also be possible to use arrangements of relays to solve Boolean algebra problems. Psychologist Howard Gardner described Shannon's thesis as "possibly the most important, and also the most famous, master's thesis of the century".

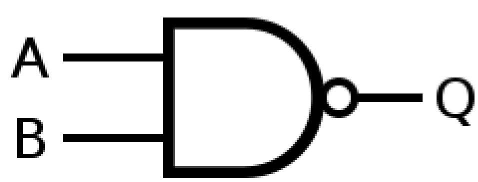
Shannon used the following correspondence between the circuits and logic symbols.

(1) If a terminal is open, it has an infinite impedance and the logic value 1 is assigned to it.
(2) For a closed terminal, the impedance is zero, and the logic value 0 is correspondingly assigned.
(3) Negation X for a terminal X is defined as the value opposite to the value assigned to X.

Problem: Find the simplified boolean expression that corresponds to the circuit in Figure 1.

Solution: The circuit can be directly converted to a boolean expression as $w \vee (x \wedge (y \vee (z \wedge \neg x)))$. The expression $x \wedge (z \wedge \neg x)$ can never evaluate to true, so the circuit has an unnecessary path and the boolean expression can be simplified to $w \vee (x \wedge y)$.

4. The NAND Gate

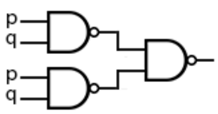


Input A	Input B	Output Q
0	0	1
0	1	1
1	0	1
1	1	1

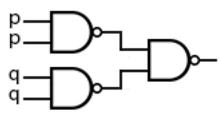
Logic gates act as *relays*, or electronically controlled switches, closing and opening circuits in response to their inputs. The NAND gate is the logic gate corresponding to logical NAND, or the Sheffer Stroke, and as such is functionally complete. An entire processor can be created using NAND gates alone and in integrated circuits using multiple-emitter transistors, it also requires fewer transistors to implement than a NOR gate.

Problem: Construct the other gates from NAND.

Solution:



And construction



Or construction

Not construction

References

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