目录

Logistic Regression

build a logistic regression model to predict whether a student gets admitted into a university

1.1 Visualizing the data

The first two columns contains the exam scores and the third column contains the label.

```
data = load('./ex2data1.txt')
```

```
data = 100×3
   34.6237   78.0247         0
   30.2867   43.8950         0
   35.8474   72.9022         0
   60.1826   86.3086    1.0000
   79.0327   75.3444    1.0000
   45.0833   56.3164         0
   61.1067   96.5114    1.0000
   75.0247   46.5540    1.0000
   76.0988   87.4206    1.0000
   84.4328   43.5334    1.0000
      ⋮
```

```
x = data(:, [1, 2]); y = data(:, 3);
x, y
```

```
x = 100×2
   34.6237   78.0247
   30.2867   43.8950
```

```
   35.8474   72.9022
   60.1826   86.3086
   79.0327   75.3444
   45.0833   56.3164
   61.1067   96.5114
   75.0247   46.5540
   76.0988   87.4206
   84.4328   43.5334
      ⋮
```

```
y = 100×1
     0
     0
     0
     1
     1
     0
     1
     1
     1
     1
     ⋮
```

找到正类的行

```
pos = find(y == 1)
```

```
pos = 60×1
     4
     5
     7
     8
     9
    10
    13
    14
    16
    17
     ⋮
```

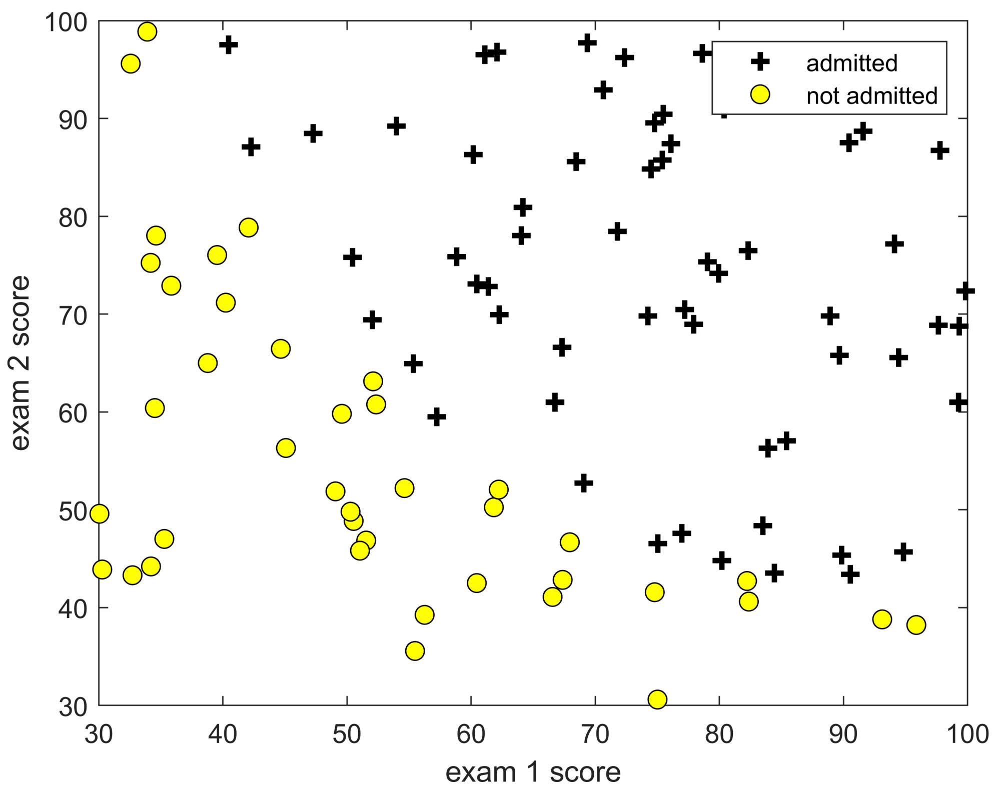
```
neg = find(y == 0)
```

```
neg = 40×1
     1
     2
     3
     6
    11
    12
    15
    18
    21
    23
     ⋮
```

要画两个分数，而不是 X 和 y。

同时要区分正类负类

```
figure;
% x 轴 exam 1 的分数, y 轴 exam 2 的分数
plot(x(pos, 1), x(pos, 2), 'k+', 'LineWidth', 2, 'MarkerSize', 7);
xlabel('exam 1 score'); ylabel('exam 2 score');
hold on;
plot(x(neg, 1), x(neg, 2), 'ko', 'MarkerFaceColor', 'y', 'MarkerSize', 7); legend('admitted',
```



1.2 implementation

1.2.1 Warmup exercise: sigmoid function

$h_\theta(x) = g(\theta^T x)$

$g(z) = \dfrac{1}{1+e^{-z}}$

```
sigmoid(0)
```

```
ans = 0.5000
```

1.2.2 Cost function and gradient

cost 如下:

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}\log\left(h_\theta(x^{(i)})\right) - (1-y^{(i)})\log\left(1-h_\theta(x^{(i)})\right)\right]$$

这个是化简后的，原本的思想就是一个分段判断。同时 gradient 如下：

$$\theta_j := \theta_j - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)x_j^{(i)}$$

和线性回归的梯度下降公式长得一样，但 h 不一样。

```
initial_theta = zeros(size(x, 2) + 1, 1); % 给 \theta_0
X = [ones(length(y), 1), x] % \theta_0 的
```

```
X = 100×3
    1.0000   34.6237   78.0247
    1.0000   30.2867   43.8950
    1.0000   35.8474   72.9022
    1.0000   60.1826   86.3086
    1.0000   79.0327   75.3444
    1.0000   45.0833   56.3164
    1.0000   61.1067   96.5114
    1.0000   75.0247   46.5540
    1.0000   76.0988   87.4206
    1.0000   84.4328   43.5334
      ⋮
```

```
costFunction(initial_theta, X, y)
```

```
ans = 0.6931
```

1.2.3 Learning parameters using *fminunc*

```
options = optimset('GradObj', 'on', 'MaxIter', 400);

[theta, cost] = fminunc(@(t)(costFunction(t, X, y)), initial_theta, options);
```

```
Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the optimality tolerance.

<stopping criteria details>
```

```
cost,theta
```

```
cost = 0.2035
theta = 3×1
  -25.1613
    0.2062
    0.2015
```

用更合适的初始参数，能够得到更小的损失

```
test_theta = [-24; 0.2; 0.2];
```

```
[cost_test, grad] = costFunction(test_theta, X, y);
cost_test
```

```
cost_test = 0.2183
```

```
X(:, 2:3)
```

```
ans = 100×2
   34.6237   78.0247
   30.2867   43.8950
   35.8474   72.9022
   60.1826   86.3086
   79.0327   75.3444
   45.0833   56.3164
   61.1067   96.5114
   75.0247   46.5540
   76.0988   87.4206
   84.4328   43.5334
      ⋮
```

```
plot_x = [min(X(:,2))-2,  max(X(:,2))+2]
```

```
plot_x = 1×2
   28.0588  101.8279
```

```
plot_y = (-1 ./ theta(3)).* (theta(2) .* plot_x + theta(1))
```

```
plot_y = 1×2
   96.1660   20.6540
```

这里要求画决策边界，我还是看不懂卡这里了，故先跳过

$\theta_0 + \theta_1 x_1 + \theta_1 x_2 = 0$

用这个去变形，就能理解。

```
plot_x = [min(X(:,2))-2,  max(X(:,2))+2];
plot_y = [];
```

1.2.4 Evaluating logistic regression

```
sigmoid([1 45, 85] * theta)
```

```
ans = 0.7763
```

```
p = predict(theta, X)
```

```
p = 100×1
     0
     0
     0
     1
     1
     0
     1
```

```
      0
      1
      1
      ⋮
```

```
fprintf('Train Accuracy: %f\n', mean(double(p == y)) * 100);
```

```
Train Accuracy: 89.000000
```

2 Regularized logistic regression

2.1 Visualizing the data

```
data = load('ex2data2.txt')
```

```
data = 118×3
    0.0513    0.6996    1.0000
   -0.0927    0.6849    1.0000
   -0.2137    0.6923    1.0000
   -0.3750    0.5022    1.0000
   -0.5132    0.4656    1.0000
   -0.5248    0.2098    1.0000
   -0.3980    0.0344    1.0000
   -0.3059   -0.1923    1.0000
    0.0167   -0.4042    1.0000
    0.1319   -0.5139    1.0000
      ⋮
```

```
x = data(:, 1:2)
```

```
x = 118×2
    0.0513    0.6996
   -0.0927    0.6849
   -0.2137    0.6923
   -0.3750    0.5022
   -0.5132    0.4656
   -0.5248    0.2098
   -0.3980    0.0344
   -0.3059   -0.1923
    0.0167   -0.4042
    0.1319   -0.5139
      ⋮
```

```
y = data(:, 3)
```

```
y = 118×1
      1
      1
      1
      1
      1
      1
      1
      1
      1
```

```
    1
    ⋮
```

```
m = length(y);
X = [ones(m, 1), x]
```

```
X = 118×3
    1.0000    0.0513    0.6996
    1.0000   -0.0927    0.6849
    1.0000   -0.2137    0.6923
    1.0000   -0.3750    0.5022
    1.0000   -0.5132    0.4656
    1.0000   -0.5248    0.2098
    1.0000   -0.3980    0.0344
    1.0000   -0.3059   -0.1923
    1.0000    0.0167   -0.4042
    1.0000    0.1319   -0.5139
      ⋮
```

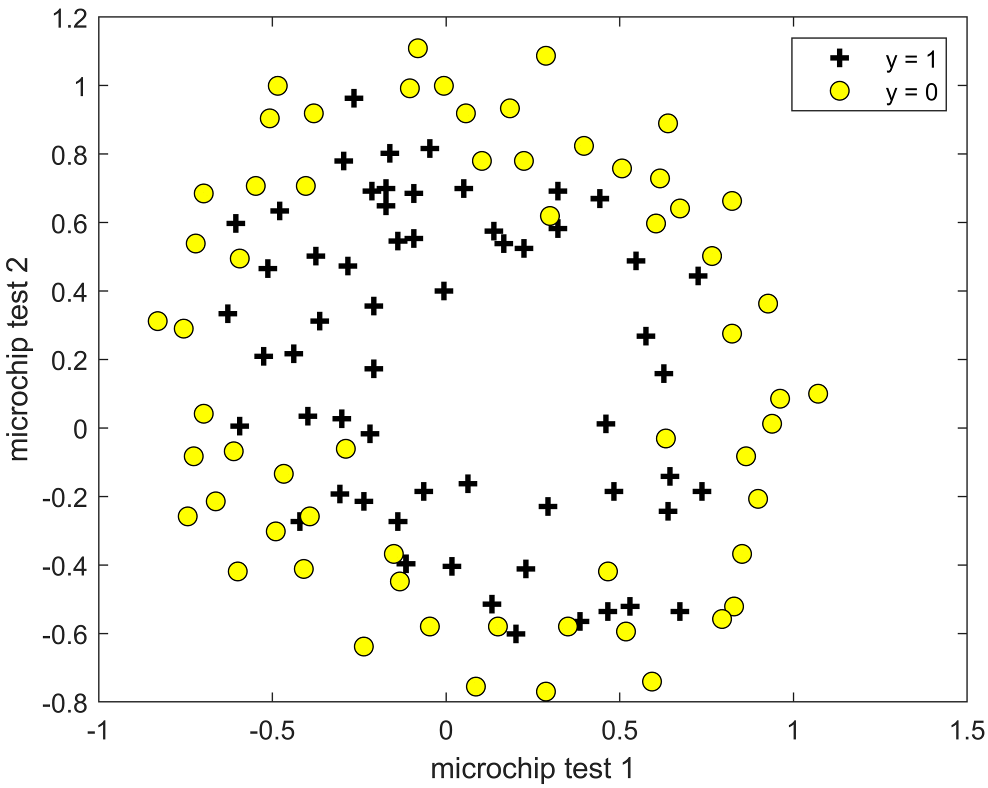
```
pos = (y == 1)
```

```
pos = 118×1 logical ##
   1
   1
   1
   1
   1
   1
   1
   1
   1
   1
     ⋮
```

```
neg = (y == 0)
```

```
neg = 118×1 logical ##
   0
   0
   0
   0
   0
   0
   0
   0
   0
   0
     ⋮
```

```
figure;
% x 轴 exam 1 的分数, y 轴 exam 2 的分数
plot(x(pos, 1), x(pos, 2), 'k+', 'LineWidth', 2, 'MarkerSize', 7);
xlabel('microchip test 1'); ylabel('microchip test 2');
hold on;
plot(x(neg, 1), x(neg, 2), 'ko', 'MarkerFaceColor', 'y', 'MarkerSize', 7); legend('y = 1', 'y =
```



2.2 Feature mapping

利用此函数可以建立非线性的代价边界。

$$mapFeature(x) = [1, x_1, x_2, x_1^2, x_1x_2, x_2^2, x_1^3, ..., x_1x_2^5, x_1^6]^T$$

这里的 x1, x2 是两个 feature 的对应变量取值，在本次实验中就是 microchip test 1 的分数，和 Micochip test 2 的分数。

下文会有使用

2.3 Cost function and gradient

引入正则的逻辑回归代价函数如下：

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}[-y^{(i)}\log\left(h_\theta(x^{(i)})\right) - (1-y^{(i)})\log\left(1-h_\theta(x^{(i)})\right)] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

注意，不惩罚 θ_0，所以关于它的梯度下降要分开来看。

$$\theta_0 := \theta_0 - a\frac{1}{m}\sum_{i=1}^{m}((h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)})$$

$$\theta_j := \theta_j - a[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} + \frac{\lambda}{m}\theta_j]$$

```
% Add Polynomial Features
% Note that mapFeature also adds a column of ones for us, so the intercept
% term is handled
X = mapFeature(X(:,2), X(:,3))
```

```
X = 118×28
    1.0000    0.0513    0.6996    0.0026    0.0359    0.4894    0.0001    0.0018 ···
    1.0000   -0.0927    0.6849    0.0086   -0.0635    0.4691   -0.0008    0.0059
    1.0000   -0.2137    0.6923    0.0457   -0.1479    0.4792   -0.0098    0.0316
    1.0000   -0.3750    0.5022    0.1406   -0.1883    0.2522   -0.0527    0.0706
    1.0000   -0.5132    0.4656    0.2634   -0.2390    0.2168   -0.1352    0.1227
    1.0000   -0.5248    0.2098    0.2754   -0.1101    0.0440   -0.1445    0.0578
    1.0000   -0.3980    0.0344    0.1584   -0.0137    0.0012   -0.0631    0.0054
    1.0000   -0.3059   -0.1923    0.0936    0.0588    0.0370   -0.0286   -0.0180
    1.0000    0.0167   -0.4042    0.0003   -0.0068    0.1634    0.0000   -0.0001
    1.0000    0.1319   -0.5139    0.0174   -0.0678    0.2641    0.0023   -0.0089
      ⋮
```

```
% Initialize fitting parameters
initial_theta = zeros(size(X, 2), 1);

% Set regularization parameter lambda to 1
lambda = 1;
[cost, grad] = costFunctionReg(initial_theta, X, y, lambda);
cost, grad
```

```
cost = 0.6931
grad = 28×1
    0.0085
    0.0188
    0.0001
    0.0503
    0.0115
    0.0377
    0.0184
    0.0073
    0.0082
    0.0235
      ⋮
```

```
fprintf('Cost at initial theta (zeros): %f\n', cost);
```

```
Cost at initial theta (zeros): 0.693147
```

```
fprintf('Expected cost (approx): 0.693\n');
```

```
Expected cost (approx): 0.693
```

```
fprintf('Gradient at initial theta (zeros) - first five values only:\n');
```

```
Gradient at initial theta (zeros) - first five values only:
```

```
fprintf(' %f \n', grad(1:5));
```

```
0.008475
0.018788
0.000078
0.050345
0.011501
```

```
fprintf('Expected gradients (approx) - first five values only:\n');
```

```
Expected gradients (approx) - first five values only:
```

```
fprintf(' 0.0085\n 0.0188\n 0.0001\n 0.0503\n 0.0115\n');
```

```
0.0085
0.0188
0.0001
0.0503
0.0115
```

2.3.1 Learning parameters using fminunc

```
options = optimset('GradObj', 'on', 'MaxIter', 400);
[theta, J, exit_flag] = fminunc(@(t)(costFunctionReg(t, X, y, lambda)), initial_theta, options)
```

```
Local minimum found.

Optimization completed because the size of the gradient is less than
the default value of the optimality tolerance.

<stopping criteria details>
```

```
theta, J, exit_flag
```

```
theta = 28×1
    1.2727
    0.6252
    1.1811
   -2.0200
   -0.9174
   -1.4316
    0.1240
   -0.3655
   -0.3572
   -0.1751
      ⋮
J = 0.5290
exit_flag = 1
```

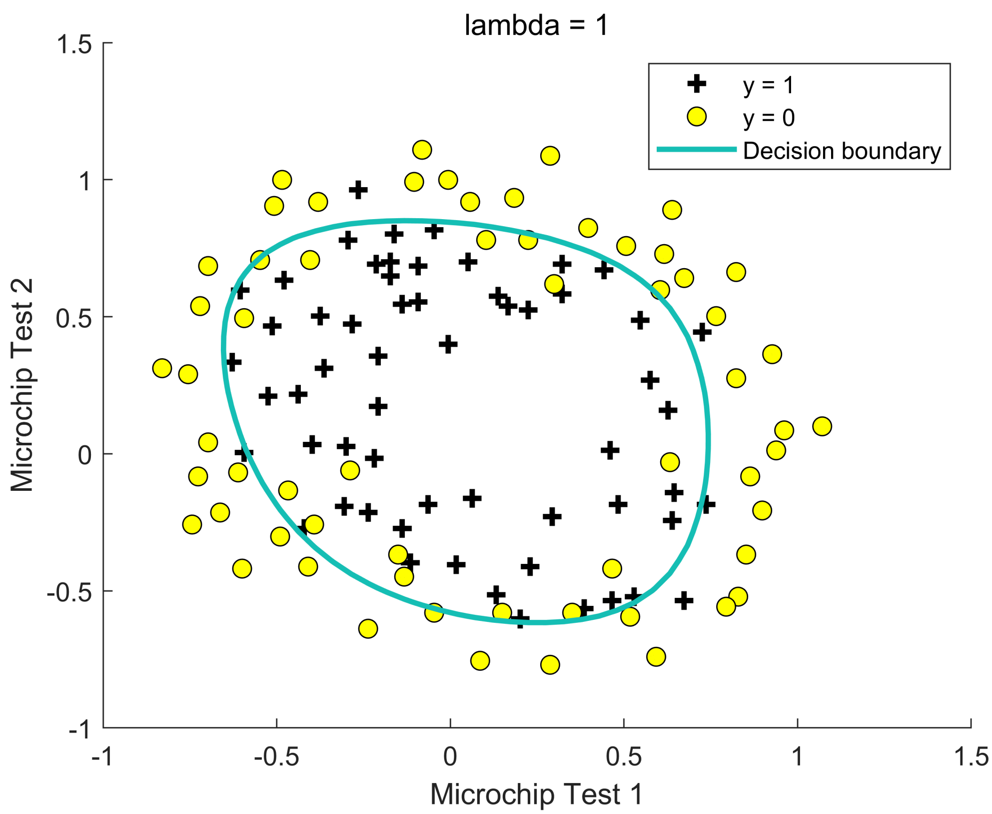
2.4 Plotting the decision boundary

```
plotDecisionBoundary(theta, X, y);
hold on;
title(sprintf('lambda = %g', lambda))

% Labels and Legend
```

```
xlabel('Microchip Test 1')
ylabel('Microchip Test 2')

legend('y = 1', 'y = 0', 'Decision boundary')
hold off;
```



Compute accuracy on our training set

```
p = predict(theta, X);

fprintf('Train Accuracy: %f\n', mean(double(p == y)) * 100);
```

```
Train Accuracy: 83.050847
```

```
fprintf('Expected accuracy (with lambda = 1): 83.1 (approx)\n');
```

```
Expected accuracy (with lambda = 1): 83.1 (approx)
```

2.5 Optional exercises

改变 λ 来观察决策边界

减少 λ 过多，过拟合。

```
lambda = 0;
[theta, J, exit_flag] = fminunc(@(t)(costFunctionReg(t, X, y, lambda)), initial_theta, options
```

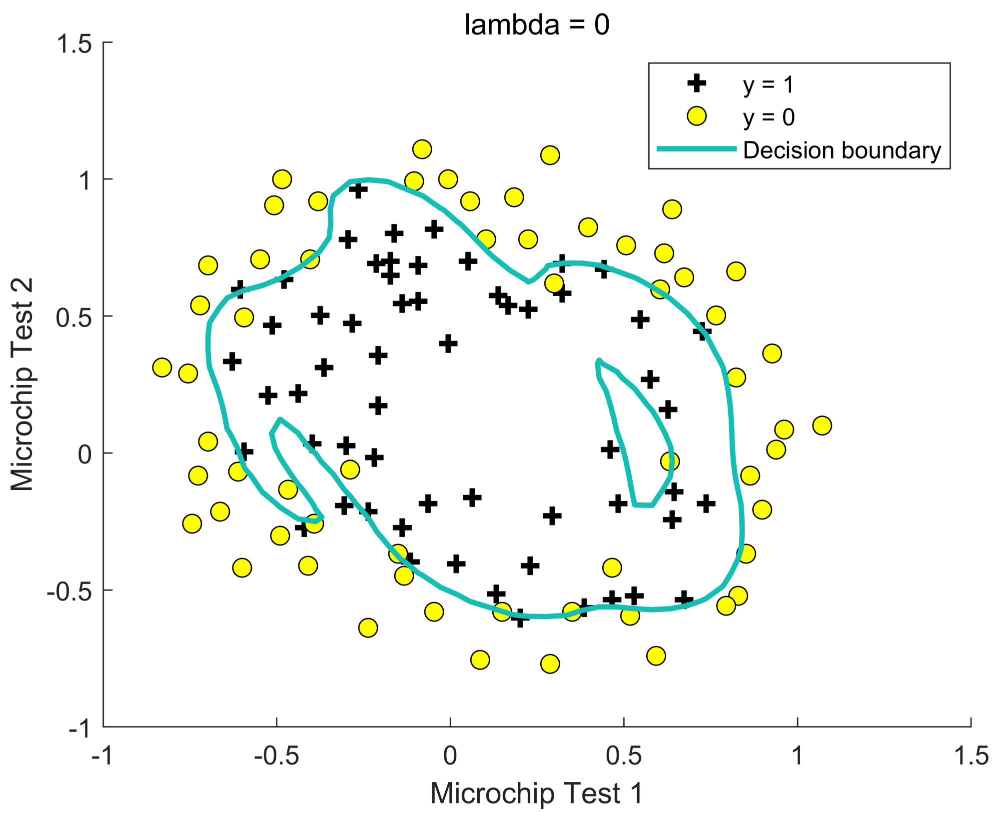
```
Solver stopped prematurely.

fminunc stopped because it exceeded the iteration limit,
options.MaxIterations = 400 (the selected value).
```

```
plotDecisionBoundary(theta, X, y);
hold on;
title(sprintf('lambda = %g', lambda))

% Labels and Legend
xlabel('Microchip Test 1')
ylabel('Microchip Test 2')

legend('y = 1', 'y = 0', 'Decision boundary')
hold off;
```




```
p = predict(theta, X);
fprintf('Train Accuracy: %f\n', mean(double(p == y)) * 100);
```

```
Train Accuracy: 88.983051
```

增加 λ 过多，欠拟合

```
lambda = 100;
[theta, J, exit_flag] = fminunc(@(t)(costFunctionReg(t, X, y, lambda)), initial_theta, options)
```

```
Local minimum found.
```

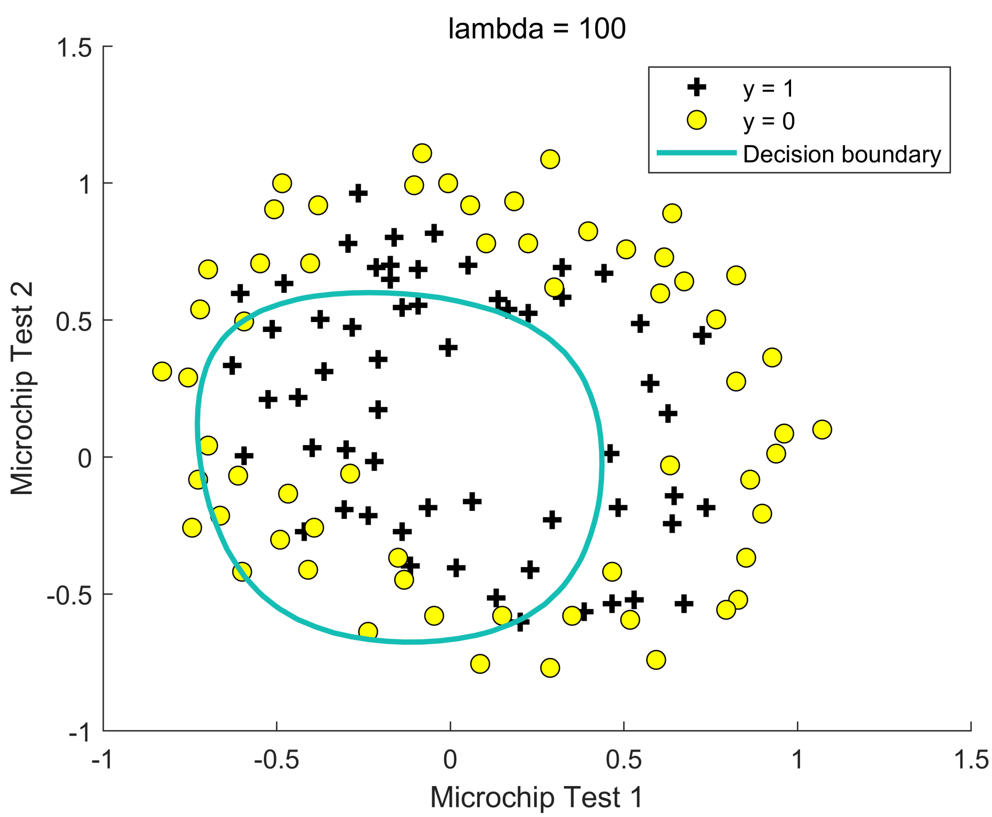
```
Optimization completed because the size of the gradient is less than
the default value of the optimality tolerance.

<stopping criteria details>
```

```
plotDecisionBoundary(theta, X, y);
hold on;
title(sprintf('lambda = %g', lambda))

% Labels and Legend
xlabel('Microchip Test 1')
ylabel('Microchip Test 2')

legend('y = 1', 'y = 0', 'Decision boundary')
hold off;
```




```
p = predict(theta, X);
fprintf('Train Accuracy: %f\n', mean(double(p == y)) * 100);
```

```
Train Accuracy: 61.016949
```

掌握不好的地方

1. plotDecisionBoundary 中，关于直线边界和非线性边界

2. predict 的编写中，如何把概率转为 0 或 1 的分类。

函数自定义

$$g(z) = \frac{1}{1 + e^{-z}}$$

```
function g = sigmoid(z)
    g = 1 ./ (1 + exp(-z));
end
```

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left[-y^{(i)} \log \left(h_\theta(x^{(i)}) \right) - (1 - y^{(i)}) \log \left(1 - h_\theta(x^{(i)}) \right) \right]$$

$$\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

```
function [J, grad] = costFunction_my(theta, X, y) % 结果 ok
    m = length(y);
    h_theta = sigmoid(X * theta);
    grad = sum((h_theta - y) .* X, 1) ;
    J = sum(-y .* log(h_theta) - (1 - y) .* log(1 - h_theta), 1) ./ m;
end

function [J, grad] = costFunction(theta, X, y)
    m = length(y);
    J = 0;
    grad = zeros(size(theta));

    h_theta = sigmoid(X * theta);
    J       = sum(-y .* log(h_theta) - (1 - y) .* log(1 - h_theta), 1) ./ m;

    grad = (X' * (h_theta - y)) ./ m; % 这个真的要有思维
    %grad = sum((h_theta - y) .* X, 1) ;
end
```

```
function p = predict(theta, X)
    m = size(X, 1); % Number of training examples

    % You need to return the following variables correctly
    p = zeros(m, 1);

    % 我想的还是用 if，要用如下方式， >= 0.5 就是正类
    k = sigmoid(X * theta) >= 0.5;
    p(k) = 1;
end
```

```

```

```
function out = mapFeature(X1, X2)
    % MAPFEATURE Feature mapping function to polynomial features
    %
    %   MAPFEATURE(X1, X2) maps the two input features
    %   to quadratic features used in the regularization exercise.
    %
    %   Returns a new feature array with more features, comprising of
    %   X1, X2, X1.^2, X2.^2, X1*X2, X1*X2.^2, etc..
    %
    %   Inputs X1, X2 must be the same size
    %

    degree = 6;
    out = ones(size(X1(:,1)));
    for i = 1:degree
        for j = 0:i
            out(:, end+1) = (X1.^(i-j)).*(X2.^j);
        end
    end

end
```

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}[-y^{(i)}\log\left(h_\theta(x^{(i)})\right) - (1-y^{(i)})\log\left(1-h_\theta(x^{(i)})\right)] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

$$\theta_0 := \theta_0 - a\frac{1}{m}\sum_{i=1}^{m}((h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)})$$

$$\theta_j := \theta_j - a[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} + \frac{\lambda}{m}\theta_j]$$

```
function [J, grad] = costFunctionReg(theta, X, y, lambda)
    m = length(y);
    grad = zeros(size(theta));
    J = costFunction(theta, X, y) + theta(2:end, :)' * theta(2:end, :) * lambda / (2 * m);
    h_theta = sigmoid(X * theta);
    reg_theta = theta; % 赋值是深拷贝
    reg_theta(1) = 0;
%     theta_1=[0;theta(2:end)];     % 先把theta(1)拿掉，不参与正则化
    grad = X' * (h_theta - y) / m + lambda / m * reg_theta; % 太棒了跟我想的一样
end
```

```
function plotDecisionBoundary(theta, X, y)
```

```
    %PLOTDECISIONBOUNDARY Plots the data points X and y into a new figure with
    %the decision boundary defined by theta
    %   PLOTDECISIONBOUNDARY(theta, X,y) plots the data points with + for the
    %   positive examples and o for the negative examples. X is assumed to be
    %   a either
    %   1) Mx3 matrix, where the first column is an all-ones column for the
    %      intercept.
    %   2) MxN, N>3 matrix, where the first column is all-ones

    % Plot Data
    plotData(X(:,2:3), y);
    hold on

    if size(X, 2) <= 3
        % Only need 2 points to define a line, so choose two endpoints
        plot_x = [min(X(:,2))-2,  max(X(:,2))+2];

        % Calculate the decision boundary line
        plot_y = (-1./theta(3)).*(theta(2).*plot_x + theta(1));

        % Plot, and adjust axes for better viewing
        plot(plot_x, plot_y)

        % Legend, specific for the exercise
        legend('Admitted', 'Not admitted', 'Decision Boundary')
        axis([30, 100, 30, 100])
    else
        % Here is the grid range
        u = linspace(-1, 1.5, 50);
        v = linspace(-1, 1.5, 50);

        z = zeros(length(u), length(v));
        % Evaluate z = theta*x over the grid
        for i = 1:length(u)
            for j = 1:length(v)
                z(i,j) = mapFeature(u(i), v(j))*theta;
            end
        end
        z = z'; % important to transpose z before calling contour

        % Plot z = 0
        % Notice you need to specify the range [0, 0]
        contour(u, v, z, [0, 0], 'LineWidth', 2)
    end
    hold off

end

function plotData(X, y)
    %PLOTDATA Plots the data points X and y into a new figure
    %   PLOTDATA(x,y) plots the data points with + for the positive examples
    %   and o for the negative examples. X is assumed to be a Mx2 matrix.
```

```matlab
    % Create New Figure
    figure; hold on;

    % ===================== YOUR CODE HERE ======================
    % Instructions: Plot the positive and negative examples on a
    %               2D plot, using the option 'k+' for the positive
    %               examples and 'ko' for the negative examples.
    %
    pos = y == 1; neg = y == 0;
    % Plot Examples
    plot(X(pos, 1), X(pos, 2), 'k+','LineWidth', 2, 'MarkerSize', 7);
    plot(X(neg, 1), X(neg, 2), 'ko', 'MarkerFaceColor', 'y','MarkerSize', 7);


    % =========================================================================

    hold off;

end
```