Bayesian Learning

Bayesian Learning

- It involves direct manipulation of probabilities in order to find correct hypotheses

- The quantities of interest are governed by probability distributions

- Optimal decisions can be made by reasoning about those probabilities

Bayesian Learning

- Bayesian learning algorithms are among the most practical approaches to certain type of learning problems

- They provide a useful perspective for understanding many learning algorithms that do not explicitly manipulate probabilities

Features of Bayesian Learning

- Each training example can incrementally decrease or increase the estimated probability that a hypothesis is correct
- Prior knowledge can be combined with observed data to determine the final probability of a hypothesis
- Hypotheses with probabilities can be accommodated
- New instances can be classified by combining multiple hypotheses weighted by their probabilities

Bayes Theorem

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

- $P(h)$: prior probability of hypothesis h
- $P(D)$: prior probability of training data D
- $P(h \mid D)$: probability that h holds given D
- $P(D \mid h)$: probability that D is observed given h

Bayes Theorem

- Maximum A-posteriori hypothesis (MAP):
  (dependent on experience)

$$h_{MAP} = \text{argmax}_{h \in H} P(h \mid D) = \text{argmax}_{h \in H} P(D \mid h)P(h)$$

P(h) is not a uniform distribution over H.

$$\boxed{P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}}$$

Bayes Theorem

- Maximum Likelihood hypothesis (ML):

$$h_{ML} = \text{argmax}_{h \in H} P(h \mid D) = \text{argmax}_{h \in H} P(D \mid h)$$

P(h) is a uniform distribution over H.

$$\boxed{P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}}$$

Bayes Theorem

- 0.008 of the population have cancer
- Only 98% patients are correctly classified as positive
- Only 97% non-patients are correctly classified as negative

Would a person with a positive result have cancer or not?

Bayes Theorem

- 0.008 of the population have cancer
- Only 98% patients are correctly classified as positive
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Would a person with a positive result have cancer or not?

$$P(\text{cancer}|\oplus) >< P(\neg\text{cancer}|\oplus) \ ?$$

Bayes Theorem

- Maximum A-posteriori hypothesis (MAP):

$$h_{MAP} = \text{argmax}_{h\in\{cancer,\ \neg cancer\}} P(h \mid \oplus)$$

$$= \text{argmax}_{h\in\{cancer,\ \neg cancer\}} P(\oplus \mid h)P(h)$$

Bayes Theorem

- $P(cancer) = .008 \;\Rightarrow\; P(\neg cancer) = .992$
- $P(\oplus|cancer) = .98$
- $P(\ominus|\neg cancer) = .97 \Rightarrow P(\oplus|\neg cancer) = .03$

$$P(cancer|\oplus) \approx P(\oplus|cancer)P(cancer) = .0078$$

$$P(\neg cancer|\oplus) \approx P(\oplus|\neg cancer)P(\neg cancer) = .0298$$

Bayes Theorem

- Maximum A-posteriori hypothesis (MAP):

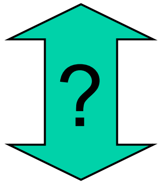
$$h_{MAP} = \text{argmax}_{h \in \{cancer,\ \neg cancer\}} P(h \mid \oplus)$$

$$= \text{argmax}_{h \in \{cancer,\ \neg cancer\}} P(\oplus \mid h)P(h)$$

$$= \neg cancer$$

Bayes Optimal Classifier

- What is the most probable hypothesis given the training data?



- What is the most probable classification of a new instance given the training data?

Bayes Optimal Classifier

- Hypothesis space = $\{h_1, h_2, h_3\}$
- Posterior probabilities = {.4, .3., .3} (h_1 is h_{MAP})
- New instance x is classified positive by h_1 and negative by h_2 and h_3

What is the most probable classification of x?

Bayes Optimal Classifier

- The most probable classification of a new instance is obtained by combining the predictions of all hypotheses weighted by their posterior probabilities:

$$\operatorname{argmax}_{c \in C} P(c \mid D)$$

$$= \operatorname{argmax}_{c \in C} \sum_{h \in H} P(c \mid h).P(h \mid D)$$

Naive Bayes Classifier

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes
5	Cloudy	Warm	High	Weak	Cool	Same	Yes
6	Cloudy	Cold	High	Weak	Cool	Same	No

7	Sunny	Warm	Normal	Strong	Warm	Same	?
8	Sunny	Warm	Low	Strong	Cool	Same	?

Naive Bayes Classifier

- Each instance x is described by a conjunction of attribute values $<a_1, a_2, ..., a_n>$
- The target function $f(x)$ can take on any value from a finite set C
- It is to assign the most probable target value to a new instance

Naive Bayes Classifier

$$c_{MAP} = \mathrm{argmax}_{c \in C}\ P(c \mid a_1, a_2, ..., a_n)$$

$$= \mathrm{argmax}_{c \in C}\ P(a_1, a_2, ..., a_n \mid c).P(c)$$

Naive Bayes Classifier

$c_{MAP} = \text{argmax}_{c \in C}\, P(c \mid a_1, a_2, \ldots, a_n)$

$= \text{argmax}_{c \in C}\, P(a_1, a_2, \ldots, a_n \mid c).P(c)$

$c_{NB} = \text{argmax}_{c \in C} \prod_{i=1,n} P(a_i \mid c).P(c)$

assuming that $a_1, a_2, \ldots, a_n$ are independent given c

Naive Bayes Classifier

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
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Naive Bayes Classifier

Estimating probabilities:

$$\frac{n_c + mp}{n + m}$$

- n: total number of training examples of a particular class
- n_c: number of training examples having a particular attribute value in that class
- m: equivalent sample size
- p: prior estimate of the probability (= 1/k where k is the number of possible values of the attribute)

Naive Bayes Classifier

Learning to classify text:

position i in the text

$$c_{NB} = \text{argmax}_{c \in C} \prod_{i=1,n} P(a_i = w_k \mid c).P(c)$$

Naive Bayes Classifier

Learning to classify text:

position i in the text

$c_{NB} = \text{argmax}_{c \in C} \prod_{i=1,n} P(a_i = w_k \mid c).P(c)$

$= \text{argmax}_{c \in C} \prod_{i=1,n} P(w_k \mid c).P(c)$

assuming that all words have equal chance occurring in every position