Assignment 2

Daniel Linders

6/13/2021

The data understanding phase

A bond is a financial product that provides fixed payments for a given period. Bonds are traded on a public exchange and therefore we can observe the prices we have to pay if we want to buy a bond today. The payments of the bond are fixed and therefore known at the time we buy the bond. Therefore, we can determine a rate of return for that particular bond.

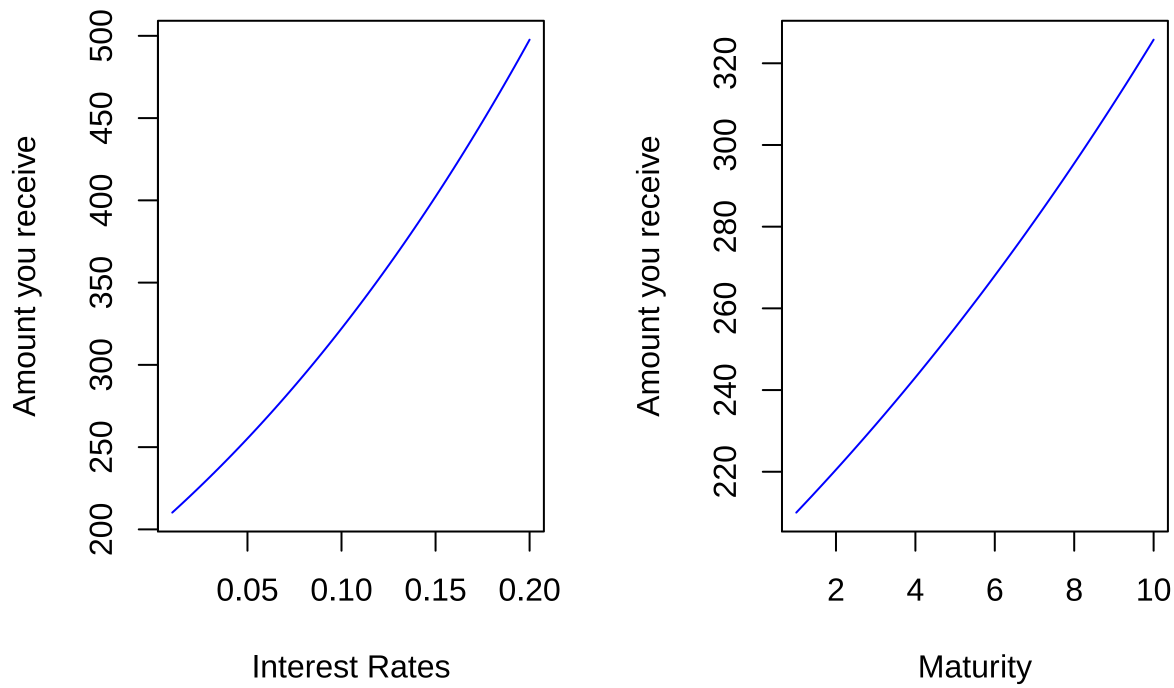
Saving at a deterministic interest rate

Assume the return you receive on an investment for 1 year is 5%. This means that if you invest today 200 Dollars, then after 1 year, your receive the amount $200(1 + 0.05) = 210$. Below, you find a plot of your payment after one year in function of the interest rate. In case you invest your money not 1, but 5 years at a yearly rate of 5% and you reinvest the interest you receive after each year, you will have the amount $200(1 + 0.05)^5 = 255.2563$ after 5 years. Note however that if you change the time you invest (also called the maturity), or the interest rate at which you can invest, the final amount you receive also changes. Indeed, if you receive a yearly constant interest rate r, you invest today the amount X for T years, and you reinvest the interest each year, then you receive after T years the following amount:

$$\text{Amount after } T \text{ years} = X\left(1 + r\right)^T.$$

Below, we determine the sensitivity of the final amount in function of r and T.

```r
InterestRate=seq(0.01,0.2,0.0001)
Maturity = seq(1,10,0.1)
par(mfrow=c(1,2))
plot(InterestRate, (1+InterestRate)^5*200, type="l", col="blue", xlab= "Interest Rates",
     ylab = "Amount you receive")
plot(Maturity, (1+0.05)^Maturity*200, type="l", col="blue", xlab = "Maturity",
     ylab = "Amount you receive")
```



Determining interest rates from market prices of bonds

Consider a bond with a maturity of T years. If you buy this product, you will have to pay the price of the bond today and you will get a fixed cashflow back after T years. For example, assume a 5 year bond which pays 250 at maturity, is trading at a price 195 dollars today. The implied rate of return of this bond can be determined as follows. Assume that we denote the return by r, then r should satisfy the following equation:

$$250 = 195(1 + r)^5.$$

Indeed, assuming the bond cannot default[1], the bond should behave like a savings account earning the rate r. We can solve this equation and find the following

$$r = \left(\frac{250}{195}\right)^{1/5} - 1 = 0.05095.$$

We conclude that, based on the ongoing market price of this bond, the market interest rate you receive for investing your money for 5 years is approximately 5.1%. The interest rate 5.1% is also called the 'yield' of the bond.

The yield curve

In reality, we have different bonds traded, each of them with different maturities. For each bond, we can use the methodology described above to extract the yield. For example, assume we see today a bond which pays at maturity T the amount A. The price of this bond today is P. Then we can determine the yield of this bond as follows:

$$r_T = \left(\frac{A}{P}\right)^{1/T} - 1.$$

Since this yield depends on the maturity, we use the notation r_T.

In Table 1, we show a table where we list bonds with different maturities. Each bond pays at its maturity the amount 100. The corresponding yields-to-maturity are listed in the last row.

[1] or at least has a tiny probability of defaulting in the coming 5 years.

Maturity	1	2	5	10
Price of the bond	93.70822	88.16747	76.25182	66.99683
Yield-to-Maturity	0.067142242	0.064990689	0.055722937	0.040865406

Table 1: The yield-to-maturity of different bonds, where each bond pays the amount 100 at the maturity.

The curve that gives the yield-to-maturity on a given trading day for different maturities is called the yield curve.

The shape of the yield curve provides information on the expectation of the market about the future economic activity. In this particular case, the yield is a decreasing function of the maturity. This means that you will receive a lower compensation if you invest your money for a longer duration. For example, if the market expects the economy will slow down, it will also expect that interest rates will go down over time, leading to a downwards sloping yield curve. An upwards sloping yield curve corresponds with a situation where the market believes the economy will grow over time.

The slope of the yield curve corresponds with the speed of the economic growth. If the market believes the economy will grow fast, investors are hesitant for putting money in a bond with a long term maturity, since one may soon be locked into a bond with a low interest rate. Therefore, the prices of the bond with large maturities will decrease and the corresponding yields will go up.

The data set

We have available a data set with the daily yield curve between January 2002 and January 2007. This data is stored in the data set `InterestRates.txt`. Each line in this data set corresponds with a given trading day on which we can observe bond prices. Each column corresponds with a given maturity. The first column is the so-called 'overnight' maturity (you can approximate this by a maturity of 1 day). The second column corresponds with a maturity of 0.5 years, the third column with a maturity of 1 year. We keep increasing the maturity by 6 months. The last column then represents a maturity of 25 years. The data set contains the yield for a given trading day for a given maturity. Therefore, by plotting a single row, we can visualize the yield curve on that given day. By plotting a column, we can observe the fluctuations in the yield for a given maturity. The bottom row contains the most recent yield curve.

We load the data and store the data in a matrix.

```r
Data=data.matrix(read.table("InterestRates.txt", header=FALSE))
head(Data)
```

```
##            V1       V2       V3       V4       V5       V6       V7       V8
## [1,] 5.773355 6.438191 6.714223 6.651193 6.499071 6.325491 6.153406 5.992542
## [2,] 5.768002 6.450607 6.750178 6.684171 6.542338 6.385233 6.230142 6.084619
## [3,] 5.775773 6.440977 6.735365 6.684521 6.557741 6.410884 6.261086 6.116426
## [4,] 5.743049 6.410330 6.694178 6.621525 6.490434 6.346198 6.200630 6.060120
## [5,] 5.741189 6.397821 6.635751 6.550174 6.416799 6.272176 6.126186 5.984857
## [6,] 5.721407 6.452250 6.695111 6.621550 6.497258 6.359611 6.219653 6.083466
##            V9      V10      V11      V12      V13      V14      V15      V16
## [1,] 5.844359 5.705781 5.572294 5.439274 5.303175 5.161842 5.014600 4.861964
## [2,] 5.949029 5.819848 5.692443 5.562261 5.425933 5.281529 5.128620 4.967978
## [3,] 5.978175 5.843796 5.709668 5.572212 5.428907 5.278456 5.120779 4.956730
## [4,] 5.925240 5.793305 5.660769 5.524238 5.381403 5.231169 5.073604 4.909691
## [5,] 5.848831 5.715661 5.582035 5.444786 5.301812 5.152152 4.995914 4.834037
## [6,] 5.951437 5.820765 5.687919 5.549626 5.403791 5.249541 5.087115 4.917606
##           V17      V18      V19      V20      V21      V22      V23      V24
## [1,] 4.705541 4.547499 4.390247 4.235959 4.086540 3.943697 3.808713 3.682811
## [2,] 4.801501 4.631677 4.461260 4.292798 4.128602 3.970780 3.821011 3.680894
```

```
## [3,] 4.787954 4.616598 4.445145 4.275916 4.111042 3.952462 3.801723 3.660300
## [4,] 4.741119 4.570033 4.398861 4.229826 4.064935 3.905998 3.754432 3.611594
## [5,] 4.668103 4.500088 4.332218 4.166494 4.004686 3.848378 3.698788 3.557097
## [6,] 4.742769 4.564771 4.386032 4.208762 4.034953 3.866416 3.704612 3.550969
##           V25      V26      V27      V28      V29      V30      V31      V32
## [1,] 3.567213 3.463220 3.372056 3.294556 3.231439 3.182822 3.148134 3.126768
## [2,] 3.551973 3.435810 3.333849 3.247042 3.176203 3.121463 3.082185 3.057694
## [3,] 3.529638 3.411238 3.306499 3.216372 3.141680 3.082581 3.038497 3.008813
## [4,] 3.478832 3.357590 3.249229 3.154709 3.074878 3.009966 2.959550 2.923176
## [5,] 3.424539 3.302488 3.192275 3.094941 3.011439 2.942132 2.886761 2.845042
## [6,] 3.406962 3.274187 3.154172 3.048107 2.957080 2.881512 2.821134 2.775648
##           V33      V34      V35      V36      V37      V38      V39      V40
## [1,] 3.118114 3.121289 3.134834 3.157218 3.187037 3.223407 3.265575 3.312792
## [2,] 3.047311 3.050065 3.064378 3.088600 3.121217 3.161257 3.207890 3.260283
## [3,] 2.992911 2.989882 2.998231 3.016390 3.042916 3.076868 3.117425 3.163771
## [4,] 2.900385 2.890439 2.892054 2.903882 2.924667 2.953493 2.989527 3.031933
## [5,] 2.816685 2.801123 2.797255 2.803922 2.820021 2.844663 2.877009 2.916222
## [6,] 2.744752 2.727828 2.723672 2.731011 2.748639 2.775586 2.810938 2.853778
##           V41      V42      V43      V44      V45      V46      V47      V48
## [1,] 3.364305 3.419364 3.477219 3.537118 3.598512 3.661238 3.725178 3.790217
## [2,] 3.317606 3.379027 3.443715 3.510840 3.579791 3.650373 3.722444 3.795859
## [3,] 3.215086 3.270552 3.329351 3.390666 3.453894 3.518837 3.585342 3.653258
## [4,] 3.079878 3.132529 3.189051 3.248612 3.310591 3.374752 3.440898 3.508833
## [5,] 2.961464 3.011895 3.066677 3.124975 3.186154 3.249938 3.316093 3.384381
## [6,] 2.903193 2.958266 3.018083 3.081731 3.148519 3.218150 3.290366 3.364909
##           V49      V50      V51
## [1,] 3.856237 3.923121 3.990752
## [2,] 3.870473 3.946143 4.022723
## [3,] 3.722432 3.792711 3.863944
## [4,] 3.578361 3.649285 3.721410
## [5,] 3.454567 3.526415 3.599688
## [6,] 3.441521 3.519944 3.599921
```

Questions

1. Make a plot with the yield curve for 5 different trading days.
2. Make a plot with the overnight yield over time. Add another maturity to this plot. What do you see?
3. Assume we call the yield at time t with maturity T by $r(t,T)$.Determine the differences $x(t,T)$ as follows:
$$x(t,T) = r(t,T) - r(t-1,T).$$
Investigate the dependencies between the yield changed for different maturities. Visualize the dependence of the change in the one year, 5 yier and 10 year yield with yield changes in the other maturities.
4. Perform a principal component analysis for the changes in the yield curve. Explain why, in this particular case, there is no need to standardize the data.
Hint: Useful plots are a biplot, a plot of the important PCs in function of the original variables. Tables showing the variance explained can help you to determine the number of principal components.
5. Use the principal component analysis to explain the changes in the yield curve over time.