Life Insurance Mathematics - Week 2 Assignment 2

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Question 2

We used the Belgium data because other data available for download needed a login detail.

```
setwd("D:/UvA/BSc Actuarial Science/Year 2/Block 1/Life Insurance Mathematics/Week 2 - Life tables and s

life_table <- read.table("Belgium.txt", header = TRUE)

head(life_table)
```

```
##   Year Age      mx      qx   ax     lx    dx    Lx      Tx    ex
## 1 1841   0 0.18652 0.16580 0.33 100000 16580 88891 4028087 40.28
## 2 1841   1 0.07413 0.07148 0.50  83420  5963 80439 3939195 47.22
## 3 1841   2 0.03983 0.03905 0.50  77457  3025 75945 3858756 49.82
## 4 1841   3 0.02333 0.02306 0.50  74432  1716 73574 3782812 50.82
## 5 1841   4 0.01707 0.01693 0.50  72716  1231 72100 3709238 51.01
## 6 1841   5 0.01240 0.01233 0.50  71485   881 71044 3637137 50.88
```

2a

The force of mortality of an (x) year old in year t, versus x, for different selected time periods t is calculated under the Constant force of mortality assumption, using the formula:

$$\mu_x^* = -log(p_x)$$

```
#For year 1841

life_table_1841 <- subset(life_table, Year == 1841)

qx_1841 <- as.numeric(life_table_1841$qx)

age_1841 <- as.numeric(life_table_1841$Age)
```

```
## Warning: NAs introduced by coercion
```

```r
px_1841 <- 1 - qx_1841

mx_1841 <- -log(px_1841)

plot(age_1841, mx_1841, type = "l",
     xlab = "Age", ylab = expression(paste("mu"["x,t"])),
     main = expression(paste("mu"["x,t"], " for year 1841, 1900, 1950, 2000, 2015")),
     lwd = 2, col = 1)

#For year 1900

life_table_1900 <- subset(life_table, Year == 1900)

qx_1900 <- as.numeric(life_table_1900$qx)

age_1900 <- as.numeric(life_table_1900$Age)
```

```
## Warning: NAs introduced by coercion
```

```r
px_1900 <- 1 - qx_1900

mx_1900 <- -log(px_1900)

lines(age_1900, mx_1900, type = "l", col = 2, lwd = 2)

#For year 1950

life_table_1950 <- subset(life_table, Year == 1950)

qx_1950 <- as.numeric(life_table_1950$qx)

age_1950 <- as.numeric(life_table_1950$Age)
```

```
## Warning: NAs introduced by coercion
```

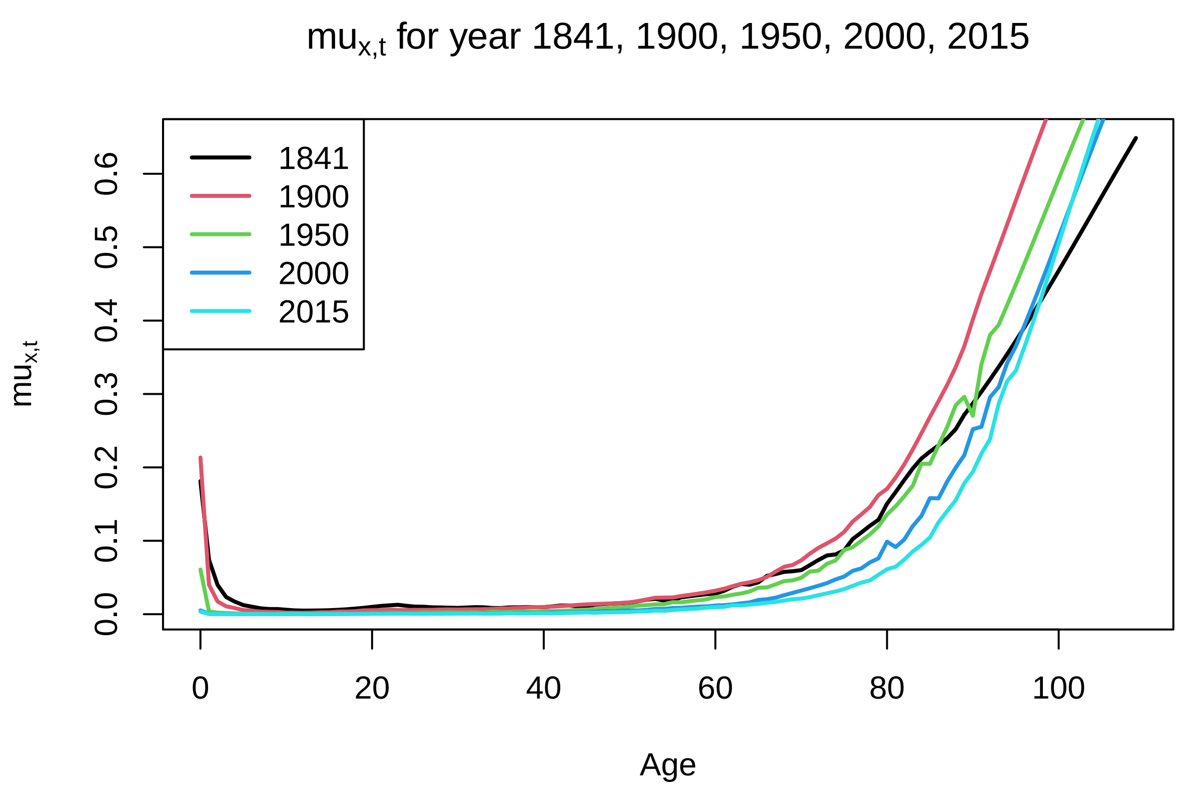
```r
px_1950 <- 1 - qx_1950

mx_1950 <- -log(px_1950)

lines(age_1950, mx_1950, type = "l", col = 3, lwd = 2)

#For year 2000

life_table_2000 <- subset(life_table, Year == 2000)

qx_2000 <- as.numeric(life_table_2000$qx)

age_2000 <- as.numeric(life_table_2000$Age)
```

```
## Warning: NAs introduced by coercion
```

```r
px_2000 <- 1 - qx_2000

mx_2000 <- -log(px_2000)

lines(age_2000, mx_2000, type = "l", col = 4, lwd = 2)

#For year 2015

life_table_2015 <- subset(life_table, Year == 2015)

qx_2015 <- as.numeric(life_table_2015$qx)

age_2015 <- as.numeric(life_table_2015$Age)
```

```
## Warning: NAs introduced by coercion
```

```r
px_2015 <- 1 - qx_2015

mx_2015 <- -log(px_2015)

lines(age_2015, mx_2015, type = "l", col = 5, lwd = 2)

legend("topleft",legend = c("1841","1900","1950", "2000", "2015"), col = 1:5, lty = 1, lwd = 2)
```



The force of mortality is as expected, as it decreases from age 0 to age 10, then subsequently increases drastically from age 70 for all 5 different time period listed.

2b

Force of mortality versus t, for different selected ages x.

```r
#For age 0
life_table_0 <- subset(life_table, Age == 0)

qx_0 <- as.numeric(life_table_0$qx)
```

```
## Warning: NAs introduced by coercion
```

```r
mx_0 <- -log(1 - qx_0)

plot(life_table_0$Year, mx_0, type = "l",
     xlab = "Year", ylab = expression(paste("mu"["x,t"])),
     ylim = c(0, 1.5), lwd = 2, col = 1,
     main = expression(paste("mu"["x,t"], " versus t, for age 0, 20, 40, 60, 80, 100")))

#For age 20

life_table_20 <- subset(life_table, Age == 20)

qx_20 <- as.numeric(life_table_20$qx)
```

```
## Warning: NAs introduced by coercion
```

```r
mx_20 <- -log(1 - qx_20)

lines(life_table_20$Year, mx_20, type = "l", col = 2, lwd = 2)

#For age 40

life_table_40 <- subset(life_table, Age == 40)

qx_40 <- as.numeric(life_table_40$qx)
```

```
## Warning: NAs introduced by coercion
```

```r
mx_40 <- -log(1 - qx_40)

lines(life_table_40$Year, mx_40, type = "l", col = 3, lwd = 2)

#For age 60

life_table_60 <- subset(life_table, Age == 60)

qx_60 <- as.numeric(life_table_60$qx)
```

```
## Warning: NAs introduced by coercion
```

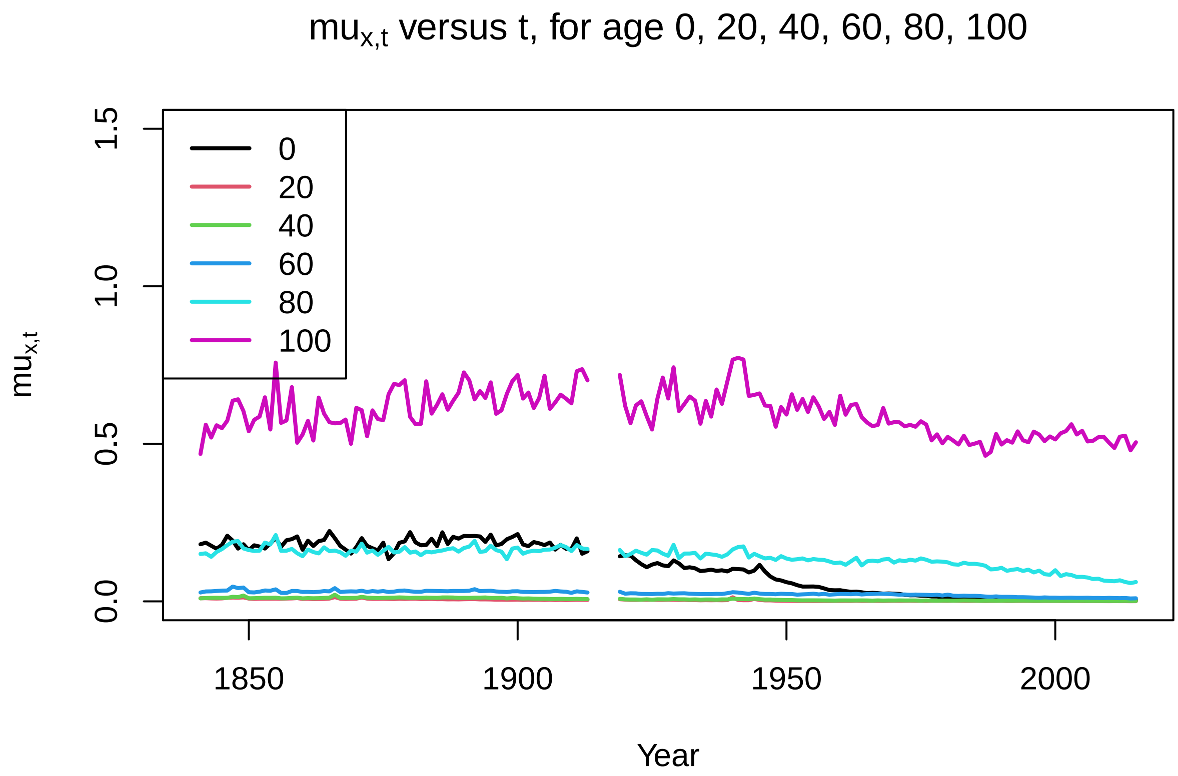
```r
mx_60 <- -log(1 - qx_60)

lines(life_table_60$Year, mx_60, type = "l", col = 4, lwd = 2)

#For age 80

life_table_80 <- subset(life_table, Age == 80)

qx_80 <- as.numeric(life_table_80$qx)
```

```
## Warning: NAs introduced by coercion
```

```r
mx_80 <- -log(1 - qx_80)

lines(life_table_80$Year, mx_80, type = "l", col = 5, lwd = 2)

#For age 100

life_table_100 <- subset(life_table, Age == 100)

qx_100 <- as.numeric(life_table_100$qx)
```

```
## Warning: NAs introduced by coercion
```

```r
mx_100 <- -log(1 - qx_100)

lines(life_table_100$Year, mx_100, type = "l", col = 6, lwd = 2)

par(xpd=TRUE)

legend("topleft", legend = c("0", "20", "40", "60", "80", "100"), col = 1:6, lty = 1, lwd = 2)
```



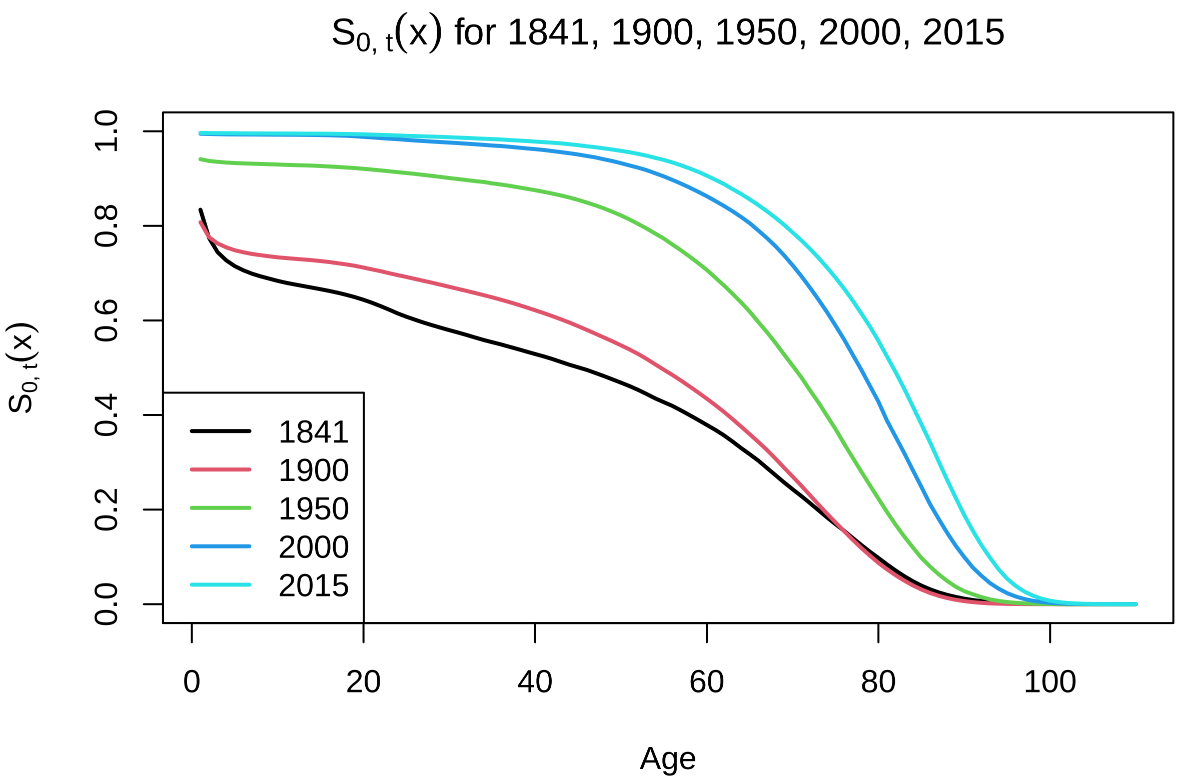
The gap between the graph is due to the missing data during the period of World War 1.

2c

The survival function $S_{0,t}(x)$ of a newborn, when using data from different selected time periods t.

```
#For year 1841

plot(1:110, cumprod(px_1841[(0+1):(109+1)]),
     type = "l", ylim = 0:1,
     xlab = "Age", ylab = expression(paste("S"["0, t"](x))), lwd = 2, col = 1,
     main = expression(paste("S"["0, t"](x), " for 1841, 1900, 1950, 2000, 2015")))

#For year 1900

lines(1:110, cumprod(px_1900[(0+1):(109+1)]),
      type = "l", col = 2, lwd = 2)

#for year 1950

lines(1:110, cumprod(px_1950[(0+1):(109+1)]),
      type = "l", col = 3, lwd = 2)

#For year 2000

lines(1:110, cumprod(px_2000[(0+1):(109+1)]),
```

```
      type = "l", col = 4, lwd = 2)

#For year 2015

lines(1:110, cumprod(px_2015[(0+1):(109+1)]),
      type = "l", col = 5, lwd = 2)

legend("bottomleft",legend = c("1841","1900","1950", "2000", "2015"), col = 1:5, lty = 1, lwd = 2)
```



As for the survival function, as time progress, the survival function of a new born increases due to advancement of medical technology hence increases the probability of survival. Therefore, explains the increase from different time period listed above.

2d

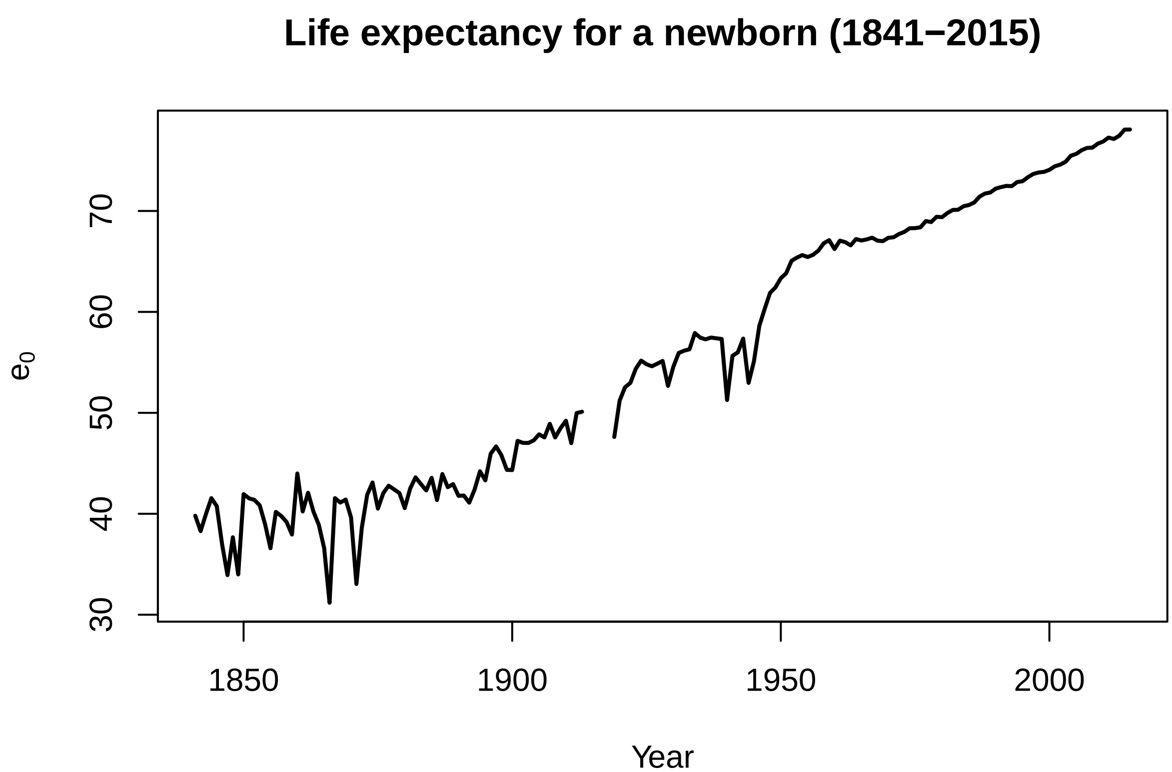
Define our own graph: The life expectancy for a newborn.

```
e_0 <- NULL

for (i in seq(1841, 2015)) {

   life_table_i <- subset(life_table, Year == i)

   px_i <- 1 - as.numeric(life_table_i$qx)
```

```
    add <- sum(cumprod(px_i[(0+1):(109+1)]))

    e_0 <- c(e_0, add)

}
```

```
## Warning: NAs introduced by coercion

## Warning: NAs introduced by coercion

## Warning: NAs introduced by coercion

## Warning: NAs introduced by coercion

## Warning: NAs introduced by coercion
```

```
plot(1841:2015, e_0, type = "l", lwd = 2,
     xlab = "Year", ylab = expression(paste("e"[0])),
     main = "Life expectancy for a newborn (1841-2015)")
```



Based on the graph, we can observe the life expectancy increased as time progress. However, there are multiple points in the time frame from 1841 to 2015, where there is a sudden drop in the life expectancy. This may be due to: 1. The abnormalities in 1860s could be caused by Third Cholera pandemic in Europe. 2. 1914-1918, where there is a gap between the graphs is due to World War 1. 3. 1940-1945 is due to World War 2.