Life Insurance Mathematics - Week 5 Assignment 4

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Question 1

```
A = 0.00022; B = 2.7*10^-6; c = 1.124

t_p_x <- function(t, x) {
  s <- exp(-A)
  g <- exp(-B/(log(c)))
  return(s^t * g^(c^x * (c^t - 1)))
}

age <- 0:100
px <- t_p_x(1, 0:100)
qx <- 1- px
lx <- c(10000, 10000 * cumprod(px[0:(length(px)-1)]))
dx <- c(lx * qx)
ex <- c()

for(i in 0:101){
  ex[i] <- sum(t_p_x(0:100, i))
}

life_table <- data.frame(age, qx, px, lx, dx, ex); head(life_table)
```

```
##   age           qx        px        lx       dx       ex
## 1   0 0.0002228393 0.9997772 10000.000 2.228393 85.00749
## 2   1 0.0002231944 0.9997768  9997.772 2.231446 84.05635
## 3   2 0.0002235935 0.9997764  9995.540 2.234938 83.09447
## 4   3 0.0002240421 0.9997760  9993.305 2.238921 82.12490
## 5   4 0.0002245463 0.9997755  9991.066 2.243457 81.15009
## 6   5 0.0002251130 0.9997749  9988.823 2.248614 80.17192
```

Question 2

2.1

```r
i <- 0.05

term_insurance <- function(age, n, i, life_table) {
  qx <- life_table$qx
  px <- 1 - qx
  kpx <- c(1, cumprod(px[(age+1):(age+n-1)]))
  kqx <- kpx * qx[(age+1):(age+n)]
  discount_factors <- (1 + i) ^ - (1:length(kqx))
  sum(discount_factors * kqx)
}

p1 <- 500000*term_insurance(50, 20, i, life_table); p1
```

```
## [1] 20100.41
```

2.2

```r
life_annuity_due <- function(age, n, i, life_table) {
  px <- 1 - life_table$qx #replace life table with tpx codes
  kpx <- c(1, cumprod(px[(age+1):(age+n-1)]))
  discount_factors <- (1+i)^ - (0:(n-1))
  sum(discount_factors * kpx)
}

a1 <- life_annuity_due(50, 20, i, life_table)

p2 <- p1/a1; p2
```

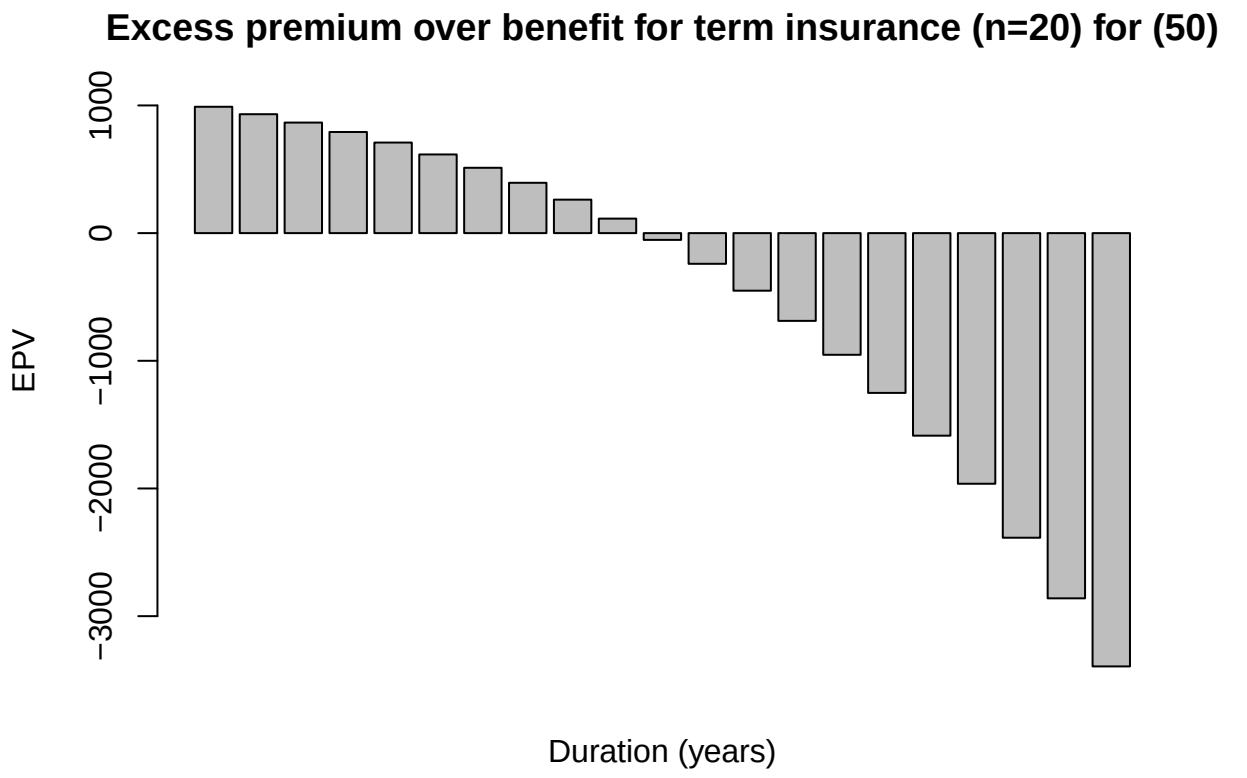
```
## [1] 1565.112
```

2.3

$$ {}_tV = 500000 \cdot A_{50+t:\overline{20+t}|} - P \cdot \ddot{a}_{50+t:\overline{20+t}|} $$

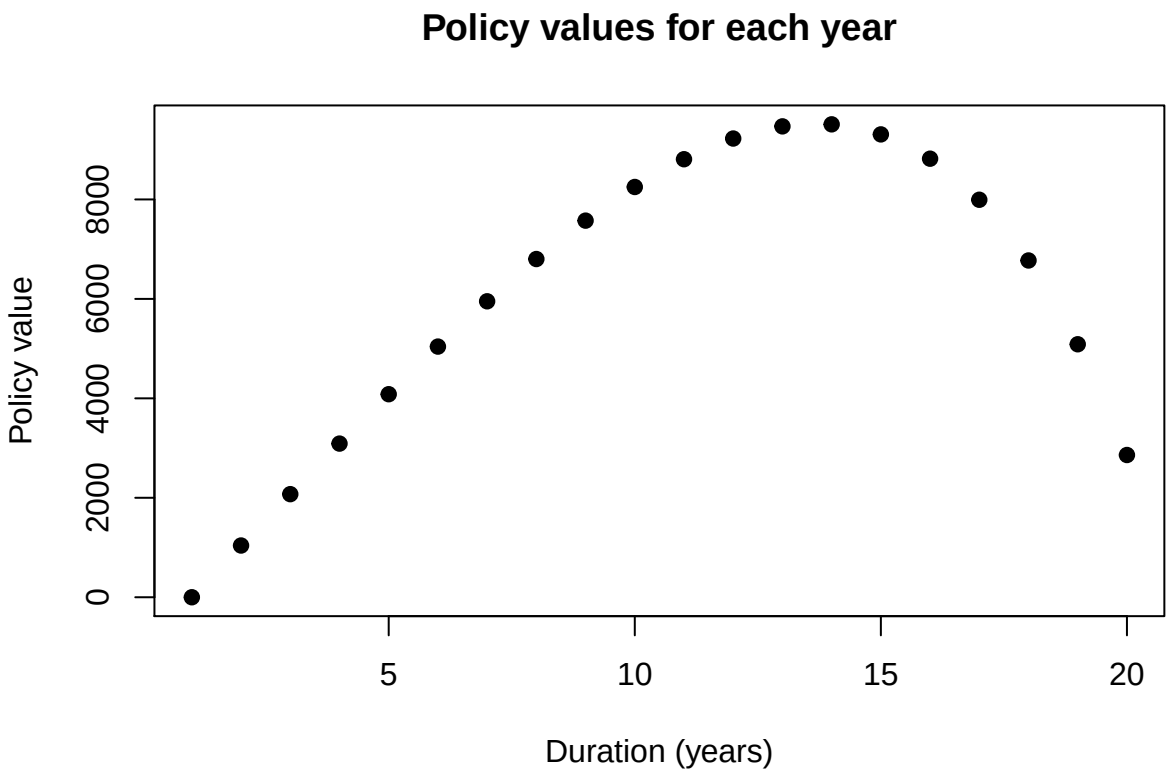
Figure 1: The expression for the policy value

```r
surplus <- c(p2 - 500000*(1/(1+0.05))*life_table$qx[(50+1):(70+1)])

barplot(surplus, ylim = c(-3000,1000),
        main = "Excess premium over benefit for term insurance (n=20) for (50)",
        xlab = "Duration (years)", ylab = "EPV")
```



2.4

```
EPV_benefits <- c(); EPV_annuities <- c()

for (k in 1:20) {
  EPV_benefits <- c(EPV_benefits, 500000*term_insurance(49 + k, 21-k, 0.05, life_table))
  EPV_annuities <- c(EPV_annuities, p2*life_annuity_due(49 + k, 21-k, 0.05, life_table))
}

EPV_benefits[20] <- 500000 * life_table$qx[70] * 1.05^-1
EPV_annuities[20] <- p2

plot(EPV_benefits - EPV_annuities,
     main="Policy values for each year", xlab = "Duration (years)", ylab = "Policy value",
     pch = 19)
```




```
EPV_benefits - EPV_annuities
```

```
##  [1]    0.000 1040.362 2072.987 3090.180 4082.955 5040.852 5951.734 6801.543
##  [9] 7574.029 8250.436 8809.135 9225.203 9469.934 9510.261 9308.088 8819.494
## [17] 7993.791 6772.408 5087.546 2860.561
```

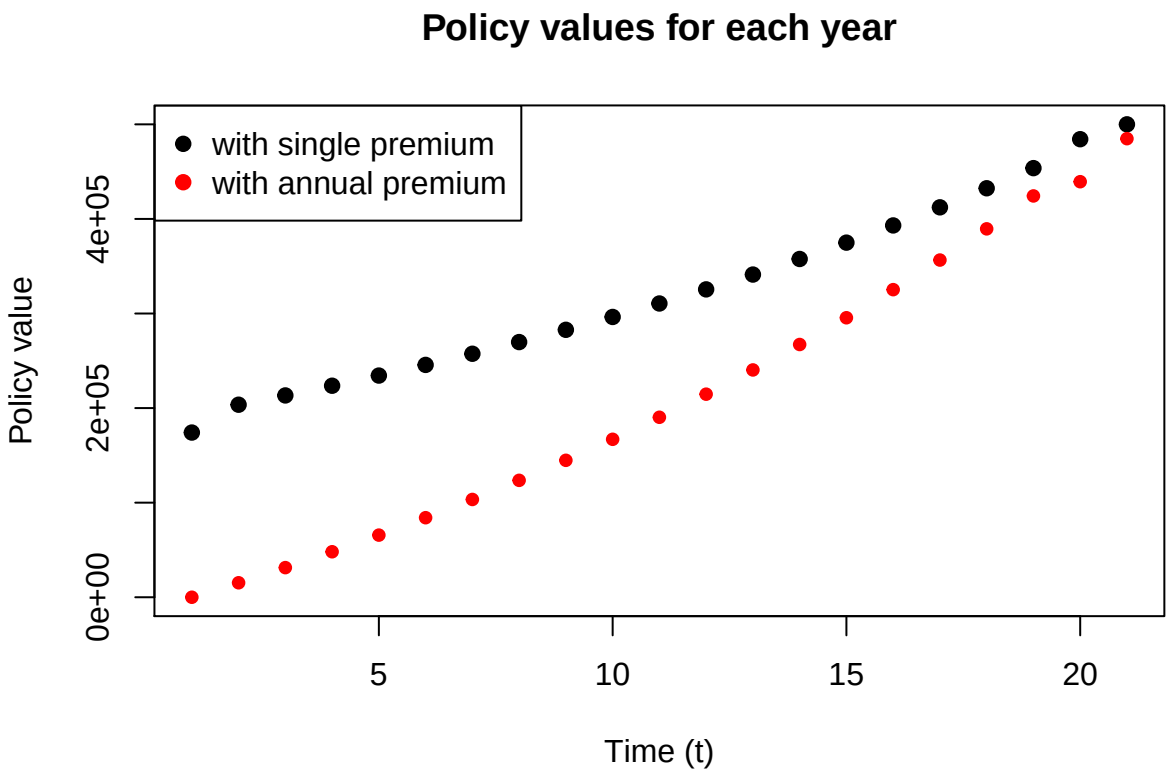
2.5

```
EPV_Endowment <- 500000*term_insurance(50, 20, 0.05, life_table)
+ 500000*(life_table$lx[71]/life_table$lx[51])*(1.05)^-20
```

```
## [1] 174118.9
```

```
annuity <- life_annuity_due(50, 20, 0.05, life_table)

Single_premium <- EPV_Endowment

annual_premium <- EPV_Endowment / annuity

EPV_premium_v <- c(Single_premium, rep(0, 20))

EPV_Endowment_v <- c()
```

```r
for (k in 1:21){
  if (k==21){
    EPV_Endowment_v<- c(EPV_Endowment_v, 500000)
  }
  else {
    EPV_Endowment_v <- c(EPV_Endowment_v, 500000*term_insurance(49+k, 21-k, 0.05, life_table)
                         + 500000*(life_table$lx[71]/life_table$lx[50+k])*(1.05)^(k-21))
  }
}

EPV_premium_v2 <- c()
EPV_Endowment_v2 <- c()
for (k in 1:21){
  if (k==21){
    EPV_Endowment_v2<- c(EPV_Endowment_v2, 500000)
    EPV_premium_v2 <- c(EPV_premium_v2, 15122.82)
  }
  else {
    EPV_Endowment_v2 <- c(EPV_Endowment_v2, 500000*term_insurance(49+k, 21-k, 0.05, life_table)
                          + 500000*(life_table$lx[71]/life_table$lx[50+k])*(1.05)^(k-21))
    EPV_premium_v2 <- c(EPV_premium_v2, 15122.82*life_annuity_due(49+k, 21-k, 0.05, life_table))
  }
}


plot(1:21, EPV_Endowment_v - EPV_premium_v,
     main="Policy values for each year",
     xlab="Time (t)", ylab="Policy value", pch = 19, ylim = c(0, 500000))
points(1:21, EPV_Endowment_v2 - EPV_premium_v2, col = "red", pch = 19, cex=0.8)
legend("topleft", legend=c("with single premium", "with annual premium"),
       col = c(1, "red"), pch=c(19, 19))
```




```
EPV_Endowment_v - EPV_premium_v
```

```
##  [1] 174118.9 203572.0 213369.1 223631.6 234382.0 245643.9 257442.7 269805.6
##  [9] 282761.6 296342.4 310582.2 325518.4 341192.2 357649.1 374940.0 393121.8
## [17] 412259.0 432424.7 453702.9 484310.1 500000.0
```

```
EPV_Endowment_v2 - EPV_premium_v2
```

```
##  [1] 5.299658e-02 1.529324e+04 3.131302e+04 4.809390e+04 6.567244e+04
##  [6] 8.408748e+04 1.033804e+05 1.235957e+05 1.447809e+05 1.669876e+05
## [11] 1.902719e+05 2.146949e+05 2.403240e+05 2.672337e+05 2.955071e+05
## [16] 3.252373e+05 3.565295e+05 3.895036e+05 4.242969e+05 4.393471e+05
## [21] 4.848772e+05
```