Exercises-Roots-Questions

All figures shown in this document are created by NvG, UvA (2018)

1. What will happen if the bisection method is applied to the function $f(x) = 1/(x-2)$ on the following intervals?

a. [3,7]

b. [1,7].

Suppose $f(x) = \tan(x)$. What will happen if the bisection method is applied on the intervals

c. [3,4]

d. [1,3]

2a. Which function can be used to approximate $\sqrt{2}$ using the bisection method?

b. Carry out 8 iterations of the bisection method for the interval [1.35, 1.45].

3. Determine algebraically whether the next functions have a unique fixed point for the following intervals

a. $g(x) = 1 - x^2/4$ on $[0,1]$

b. $g(x) = 2^{-x}$ on $[0, 1]$

c. $g(x) = 1/x$ on $[0.5, 2]$

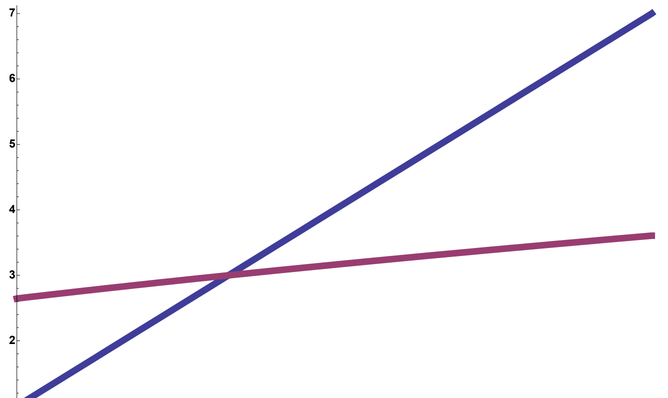
Hint: suppose $g \in C[a,b]$, then there exists a unique fixed point if

(i) $y = g(x) \in [a,b]$ for all $x \in [a,b]$

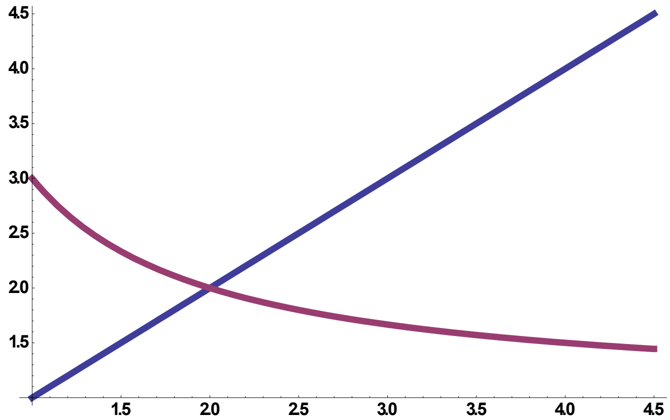
(ii) $|g'(x)| < 1$ for all $x \in [a,b]$

4. Determine graphically if the fixed point method converges for

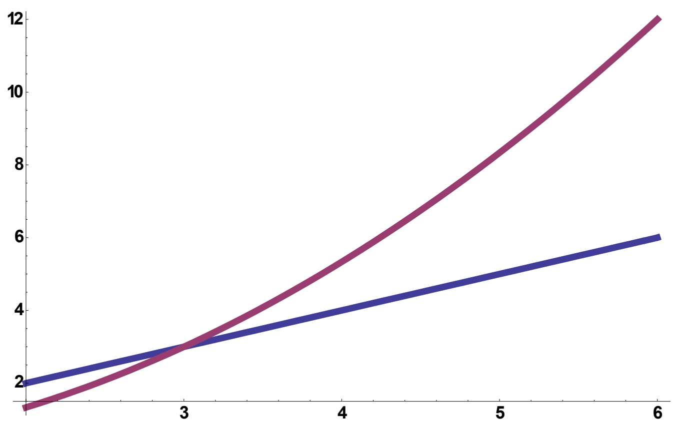
a. $f(x) = \sqrt{6+x}$ for $x_0 = 7$.



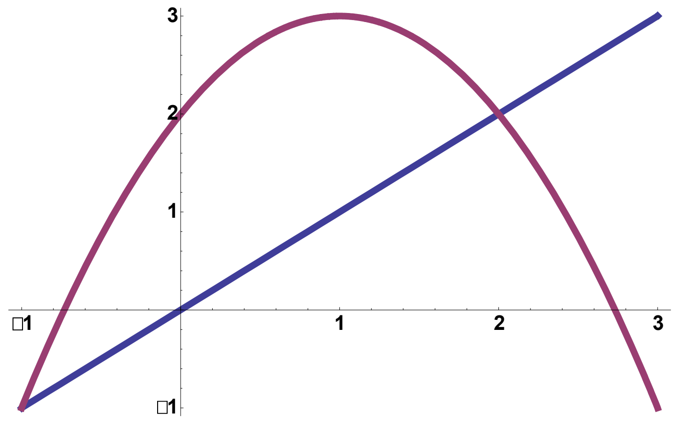
b. $g(x) = 1 + 2/x$ for $x_0 = 4$.



c. $h(x) = x^2/3$ for $x_0 = 3.5$.



d. $y(x) = -x^2 + 2x + 2$ for $x_0 = 2.5$.

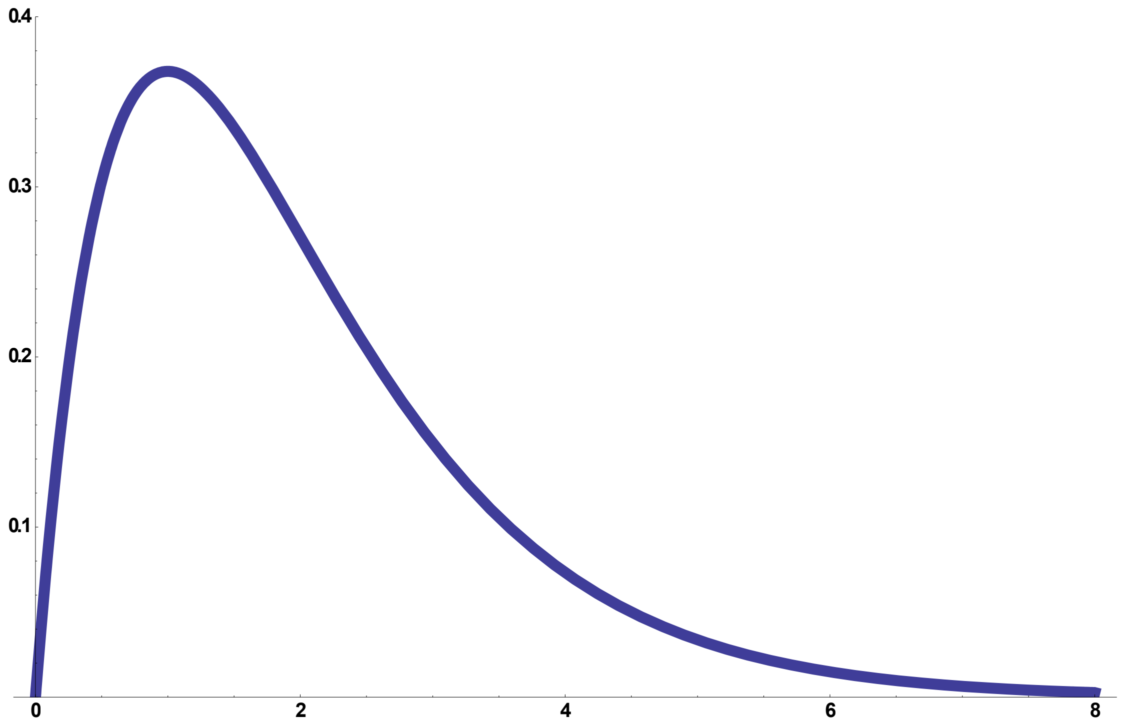


5. Suppose $f(x) = x^2 - 2$.

a. Derive the iteration equation for the Newton-Raphson algorithm.

b. Carry out 3 iterations to approximate $\sqrt{2}$ using the starting value $x_0 = 1.4$.

6. Determine graphically what happens if the Newton-Raphson algorithm is applied to $f(x) = xe^{-x}$ for $x_0 = 2$.



7. Let $f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ -\sqrt{-x} & \text{otherwise.} \end{cases}$

a. Derive the iteration equation for the Newton-Raphson algorithm.

b. What will happen for a random stating value different from zero?

8. Can Newton-Raphson be used to solve $f(x) = 0$ for $f(x) = x^{1/3}$? Motivate your answer.

9. Use the Secant method with $x_0 = -2.6$ and $x_1 = -2.4$ to approximate the root $x = -2$ for the function $f(x) = x^3 - 3x + 2$.