Programming & Numerical Analysis

Mock PC2-TEST

All figures shown in this document are created by NvG, UvA (2018)

For finding a root of the function $f(x)=0$ we can use the following algorithm:

1. Determine the functions values for x_1, x_2 and x_3 : $f(x_1), f(x_2), f(x_3)$.
2. Fit a 2-nd order degree polynomial using `pracma::polyfit()` through these 3 pairs $(x_1, f(x_1)), (x_2, f(x_2))$ and $(x_3, f(x_3))$.
3. For this polynomial, we determine the roots using the function `pracma::roots()`. Select the root that is *closest* to x_1.
4. Now x_3 is assigned the value of x_2, x_2 is assigned the value of x_1, i.e. $x_3 = x_2, x_2 = x_1$, and x_1 is the assigned value of the root selected in step 3
5. The steps 1 – 4 are repeated until a particular stop criterion is reached.

We want to determine a root of the function $f(x)=(x-1)\left(x-\frac{3}{2}\right)(x-2)$ using the algorithm described above. We therefore use a 2nd order polynomial to approximate the function using starting values $x_1 = 1.7, x_2 = 1.8$ and $x_3 = 1.9$. The iterative process stops if

(i) the distance between x_1 and x_2 is smaller than 0.001 or

(ii) when 20 iterations have been performed.

See the back for a graphical illustration.

ASSIGNMENT:

1. Write **one R script** that determines a root of $f(x)=(x-1)\left(x-\frac{3}{2}\right)(x-2)$ according to the above described algorithm. The script should show the consecutive approximations of the root of *f*; see below.
2. Example output:

```
>> source('MockPC2.R')
Iteratie 1: approx. root = 1.55179
Iteratie 2: approx. root = 1.50940
Iteratie 3: approx. root = 1.50037
Iteratie 4: approx. root = 1.50000
```

Explanation:
The user executes the red line at the command prompt. The script shows the blue text on the screen.

GOOD LUCK!

Below, you find an illustration of the first two iterations of the algorithm. In the top figure, you see the points $x_1 = 1.7, x_2 = 1.8, x_3 = 1.9$ and their function values indicated by small blue circles. The 2nd order polynomial is the red dashed line. The next x_1 is indicated by a blue star. In the bottom figure, $x_1 = 1.50940, x_2 = 1.7, x_3 = 1.8$ and the next x_1 is again indicated by a blue star.

Note: the graphs are for illustrative purposes only, you don't have to program this!