Exercises-Interpolation-Questions

1. Use the table below for $\sqrt{x}$ and approximate the value for $x = 0.15$ using a 3^{rd} degree polynomial and Newton forwards differences based on $x \in \{0.0, 0.1, 0.2, 0.3\}$:

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
0.0	0.000000				
0.1	0.316228				
0.2	0.447214				
0.3	0.547723				
0.4	0.632456				

2. Determine the 3^{rd} degree Lagrange polynomial based on $x_i \in \{0.0, 0.1, 0.2, 0.3\}$ shown in question 1 and approximate the value for $x = 0.15$.

3. Suppose we want to apply quadratic splines based on 3 points: (x_1, y_1), (x_2, y_2) and (x_3, y_3). Hence, we have to determine the coefficients of the following two spline polynomials ($i = 1,2$):

$$s_i(x) = a_i + (x - x_i)b_i + (x - x_i)^2 c_i.$$

a. Explain the idea behind (quadratic) splines.

b. How many unknown parameters are there in total?
State the system of equations that determine the coefficients. Suppose that the x-points are equidistant, i.e. $x_3 - x_2 = x_2 - x_1$, so $h = x_{i+1} - x_i$ can be used. Furthermore, impose the restriction that the first and second-order derivatives match in the interior points.

4. Suppose we want to estimate the parameter a according to the least squares method in the model:

$$f_L(x) = a\sin(x).$$

Based on the data points $\{(y_i, x_i), i = 1,...,n\}$, the sum of squared residuals can be defined:

$$SSR(a) = \sum_{i=1}^{N} (y_i - a\sin(x_i))^2 .$$

a. Determine the Normal equation, i.e. $\dfrac{\partial SSR(a)}{\partial a} = 0.$

b. Suppose we have the following observations:
$\{(1.26221, 1), (1.36395, 2), (0.21168, 3)\}$.
Determine an estimate for the parameter a.

5. Suppose we want to estimate the parameters a and b according to the least squares method in the model:

$$f_L(x) = a\sin(bx).$$

Based on the data points $\{(y_i, x_i), i = 1,...,n\}$, the sum of squared residuals can be defined:

$$SSR(a,b) = \sum_{i=1}^{N} (y_i - a\sin(bx_i))^2 .$$

a. Determine the Normal equations, i.e.

$$\frac{\partial SSR(a,b)}{\partial a} = 0 \quad \text{and} \quad \frac{\partial SSR(a,b)}{\partial b} = 0.$$

b. Given a sample $\{(y_i, x_i), i = 1,...,n\}$, how would you solve for a and b numerically?