Exercises-Interpolation

1a. One interval: $h = 1.96$.

$$\int_0^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz \approx \frac{h}{2}(f(0)+f(1.96)) = \frac{1.96}{2}(0.3989+0.0584) = 0.4482$$

Error: $|0.4750-0.4482| = 0.0268$

1b. Two subintervals: $h = 1.96/2 = 0.98$.

$$\int_0^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz \approx \frac{h}{2}(f(0)+f(0.98)) + \frac{h}{2}(f(0.98)+f(1.96))$$

$$= \frac{0.98}{2}(0.3989+0.2468) + \frac{0.98}{2}(0.2468+0.0584) = 0.3164+0.1496 = 0.4660$$

Error: $|0.4750-0.4660| = 0.009$; the error is $0.0268/0.009 \approx 3.0$ times smaller.

1c. Four subintervals: $h = 1.96/4 = 0.49$.

$$\int_0^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz \approx \frac{h}{2}(f(0)+f(0.49)) + \frac{h}{2}(f(0.49)+f(0.98)) + \frac{h}{2}(f(0.98)+f(1.47)) + \frac{h}{2}(f(1.47)+f(1.96))$$

$$= \frac{h}{2}(f(0)+2f(0.49)+2f(0.98)+2f(1.47)+f(1.96))$$

$$= \frac{0.49}{2}(0.3989+2\cdot 0.3538+2\cdot 0.2468+2\cdot 0.1354+0.0584) = 0.4727$$

Error: $|0.4750-0.4727| = 0.0023$; the error is $0.009/0.0023 \approx 3.9$ times smaller.

2a. Determine the Lagrange quadratic polynomial based on $x = 0, 0.98, 1.96$.

$$f_2(x) = \frac{(x-x_1)}{(x_0-x_1)}\frac{(x-x_2)}{(x_0-x_2)}f(x_0) + \frac{(x-x_0)}{(x_1-x_0)}\frac{(x-x_2)}{(x_1-x_2)}f(x_1) + \frac{(x-x_0)}{(x_2-x_0)}\frac{(x-x_1)}{(x_2-x_1)}f(x_2)$$

$$= \frac{(x-0.98)(x-1.96)}{(0-0.98)(0-1.96)}f(0) + \frac{(x-0)(x-1.96)}{(0.98-0)(0.98-1.96)}f(0.98) + \frac{(x-0)(x-0.98)}{(1.96-0)(1.96-0.98)}f(1.96)$$

$$= \frac{(x-0.98)(x-1.96)}{0.98\cdot 1.96}0.3989 + \frac{x(x-1.96)}{-0.98^2}0.2468 + \frac{x(x-0.98)}{1.96\cdot 0.98}0.0584$$

$$= 0.3989 - 0.136684x - 0.0188984x^2.$$

2b. Approximate the integral using the quadratic Lagrange polynomial.

$$\int_0^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz \approx \int_0^{1.96} 0.3989 - 0.136684z - 0.0188984z^2\,dz$$

$$= 0.3989z - \frac{0.136684}{2}z^2 - \frac{0.0188984}{3}z^3\Big|_{z=0}^{z=1.96} = 0.4719$$

2c. Verify Simpson's $\frac{1}{3}$-rule using only 1 interval.

Let op: $h = 1.96/2 = 0.98$.

$$\int_0^{1.96} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \approx \tfrac{1}{3} h(f(0) + 4f(0.98) + f(1.96))$$

$$= \tfrac{1}{3} 0.98(0.3989 + 4 \cdot 0.2468 + 0.0584) = 0.4719$$