**Please, upload your answers in Canvas: assignment PC LAB – April 21**

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**PC4a – Numerical Accuracy**

**1.** Create two R-*functions* for

and

Also write a R-*script* that plots the two composite functions

* [blue dashed line] and
* [red dotted line]

in one figure for the interval using 100 points.

A screenshot of a cell phone

Description automatically generated

**2.** Compare [using R’s build-in function] and its Taylor approximation

in the points and values . Use a vector for *x*!

Example output:

x: 0.00000000 1.00000000 2.00000000 3.00000000

cos(x): 1.00000000 0.54030231 -0.41614684 -0.98999250

N=1: 1.00000000 0.50000000 -1.00000000 -3.50000000

N=2: 1.00000000 0.54166667 -0.33333333 -0.12500000

N=3: 1.00000000 0.54027778 -0.42222222 -1.13750000

**3.** Next, we want a graphical comparison (you can reuse parts of your script of the previous question). Determine [using R’s build-in function] and its Taylor approximation

in the points and values .

Hints:

- use plot() to show cos(x) and add the Taylor approximations using line() in a for loop

- You can add a legend using

legend(2.7,1, legend=c("n=1", "n=2","n=3"),lty=c(2,3,4), col=c(2,3,4))



**4.** Determine the summations

for using nested for loops.

Example output:

Sum for p=1 is 3

Sum for p=2 is 14

Sum for p=3 is 100

Sum for p=4 is 979

**5.** A numeral represents

in the decimal system, i.e. and

in the binary system, i.e. .

Write a R-script that finds a binary representation of a decimal number with . [this is Exercise 1 of Ch.9 spuRs].

Hints:

* + treat the integer and the fractional parts separately;
  + for the integer part, , first find , the highest power of 2 that will fit, and then determine whether fits and so on; 5 translates to 101, because 21 does not fit 5 – 22 ;
  + for the fractional part, also determine whether powers, , fit the remainder.

Wat is the binary representation of :

Answer:

Example output for :

1.000110011001100110011001100110

Explanatory note: For the first 10 binary digits of the fractional part of , we can use to following table:

0.5

0.25

0.125

0.0625

0.03125

0.015625

0.0078125

0.00390625

0.001953125

0.0009765625

Hence, the first three binary digits are 0, because 0.1 is only larger for Then we are left with

.

Since , the fifth binary digit is also 1 and we are left with

Therefore, the next binary digit that is different from 0 is eight, since , etc.

setwd("D:/UvA/Year 1/Block 5/Programming and Numerical Analysis/Week 4")

rm(list = ls())

#1

f <- function(x) {

return(x/(1+x^2))

}

g <- function(x) {

return(tan(x))

}

x <- seq(-pi/2,pi/2, length.out = 100)

r1 <- f(g(x))

r2 <- g(f(x))

library(ggplot2)

ggplot() +

geom\_line(mapping = aes(x=x, y=r1), col="blue", linetype=2) +

geom\_line(mapping = aes(x=x,y=r2), col="red", linetype=3) +

ylab("")

#2

x<-0:3

Taylor <- function(x, n) {

taylor <- 0

for (i in 0:n) {

taylor <- taylor + (-1)^i \* (x^(2\*i)/factorial(2\*i))

}

return(taylor)

}

cat("x:"); cat(sprintf("%7.8f", 0:3))

cat("cos(x):") ; cat(sprintf("%7.8f %7.8f %7.8f %7.8f", cos(0), cos(1), cos(2), cos(3)))

cat("N=1:"); cat(sprintf("%7.8f %7.8f %7.8f %7.8f", Taylor(0,1), Taylor(1,1), Taylor(2,1), Taylor(3,1)))

cat("N=2:"); cat(sprintf("%7.8f %7.8f %7.8f %7.8f", Taylor(0,2), Taylor(1,2), Taylor(2,2), Taylor(3,2)))

cat("N=3:"); cat(sprintf("%7.8f %7.8f %7.8f %7.8f", Taylor(0,3), Taylor(1,3), Taylor(2,3), Taylor(3,3)))

#3

x <- seq(-4,4,by=0.05)

plot(x, cos(x), type = "l")

for (i in 1:3) {

a <- Taylor(x, i)

lines(x=x, y=a, col=i+1, lty=i+1)

}

legend(2.7,1, legend=c("n=1", "n=2","n=3"),lty=c(2,3,4), col=c(2,3,4))

#4

Sum <- function(p) {

sum <- 0

for (j in 1:(p+1)) {

sum <- sum + j^p

}

return(sum)

}

for (i in 1:4) {

cat(sprintf("Sum for p = %d is %d", i, Sum(i))); cat("\n")

}

#OR

for (i in 1:4) {

sum <- 0

for (j in 1:(i+1)) {

sum <- sum + j^i

}

cat(sprintf("Sum for p = %d is %d", i, sum)); cat("\n")

}

#5