Chapter 10
Sorting

Data Structures and Algorithms

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Outcomes

- **L.O.6.1** - Depict the working steps of sorting algorithms step-by-steps.
- **L.O.6.2** - Describe sorting algorithms by using pseudocode.
- **L.O.6.3** - Implement sorting algorithms using C/C++ .
- **L.O.6.4** - Analyze the complexity and develop experiment (program) to evaluate sorting algorithms.
- **L.O.6.5** - Use sorting algorithms for problems in real-life.
- **L.O.8.4** - Develop recursive implementations for methods supplied for the following structures: list, tree, heap, searching, and graphs.
- **L.O.1.2** - Analyze algorithms and use Big-O notation to characterize the computational complexity of algorithms composed by using the following control structures: sequence, branching, and iteration (not recursion).

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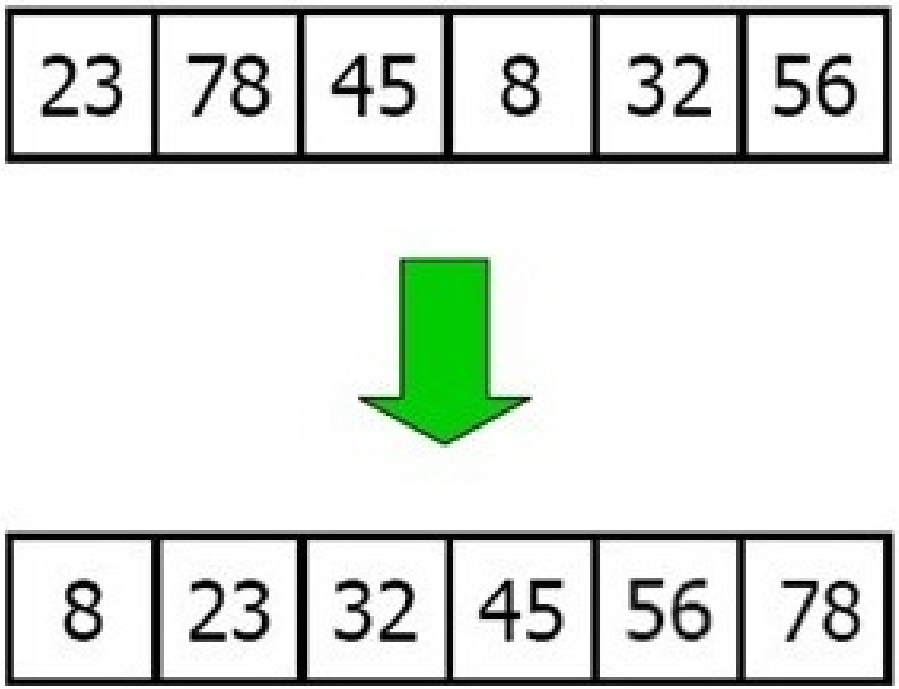
Contents

Sorting concepts

Sorting

One of the most important concepts and common applications in computing.



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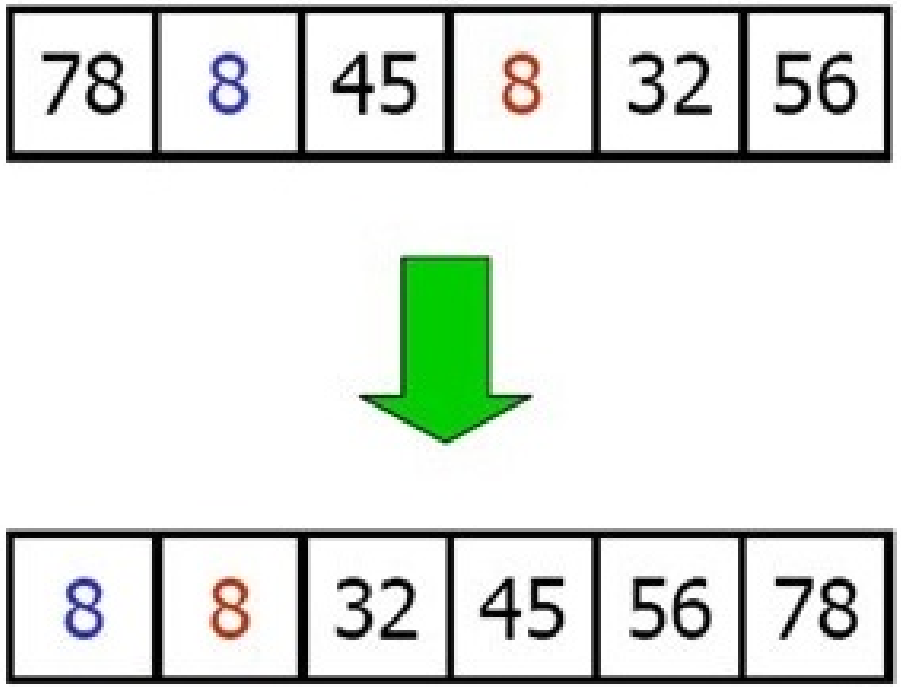
Sorting

Sort stability: data with equal keys maintain their relative input order in the output.



Sorting

Sort efficiency: a measure of the relative efficiency of a sort = number of comparisons + number of moves.

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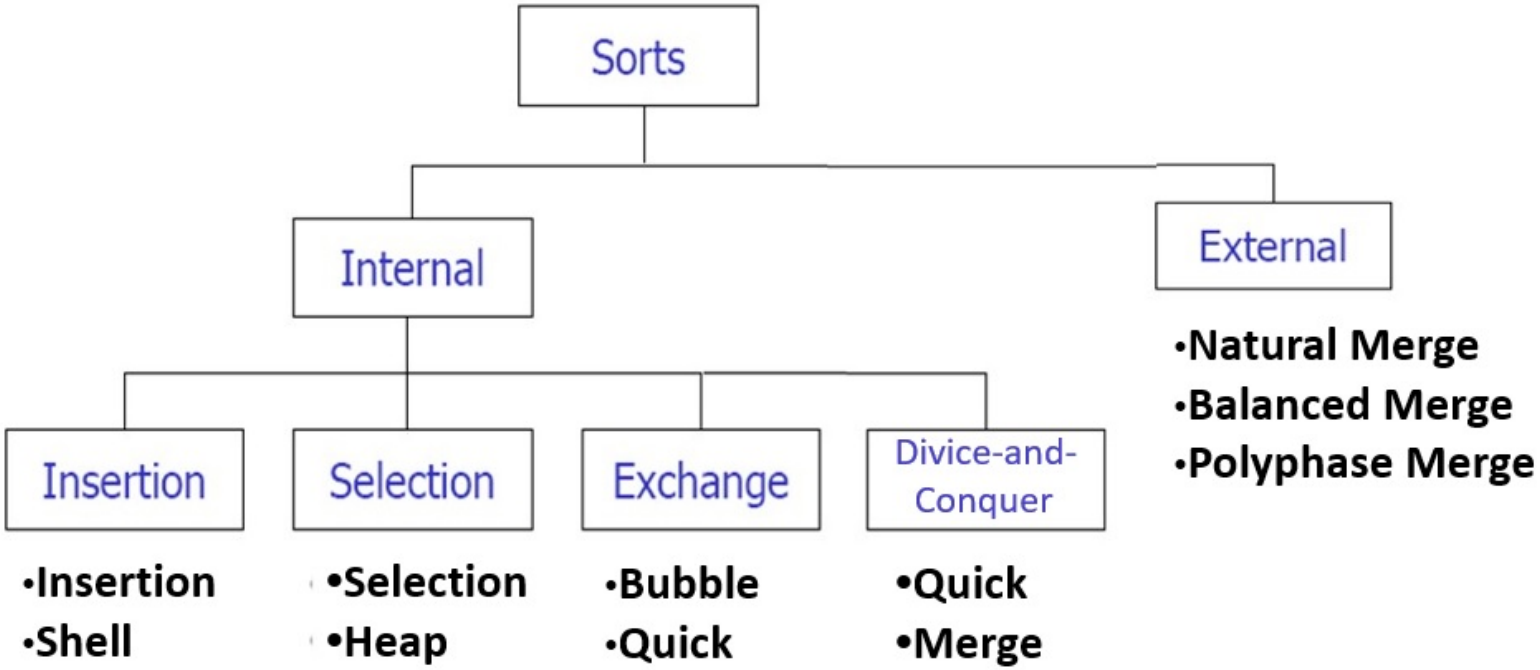
Sorting



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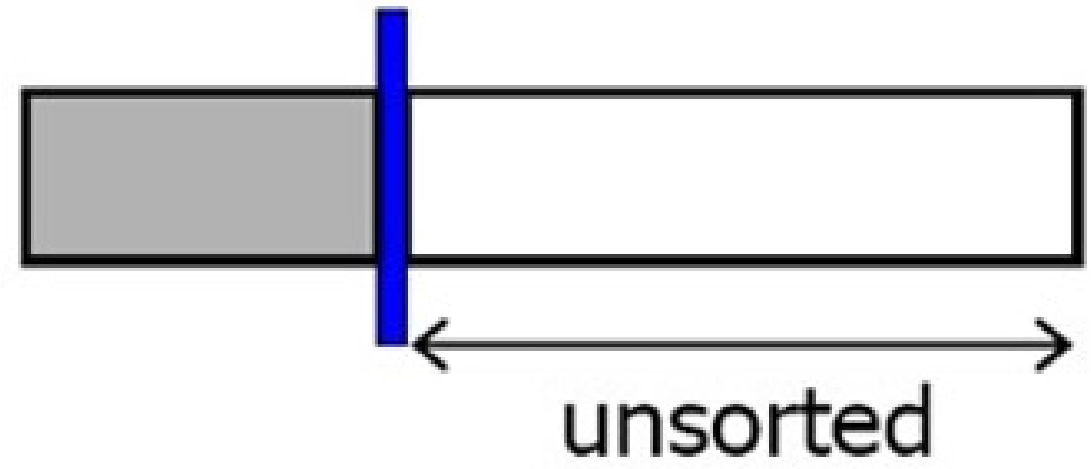
Insertion Sort

Straight Insertion Sort

- The list is divided into two parts: sorted and unsorted.
- In each pass, the first element of the unsorted sublist is inserted into the sorted sublist.



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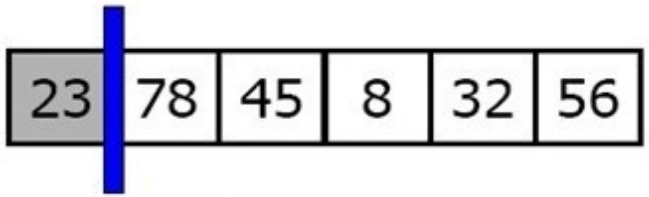
Straight Insertion Sort



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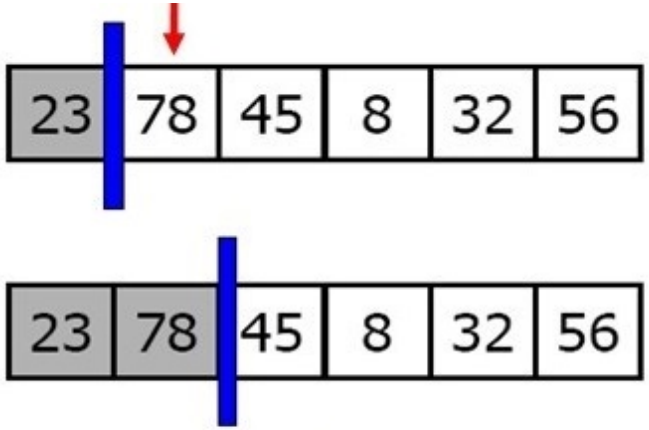
Straight Insertion Sort



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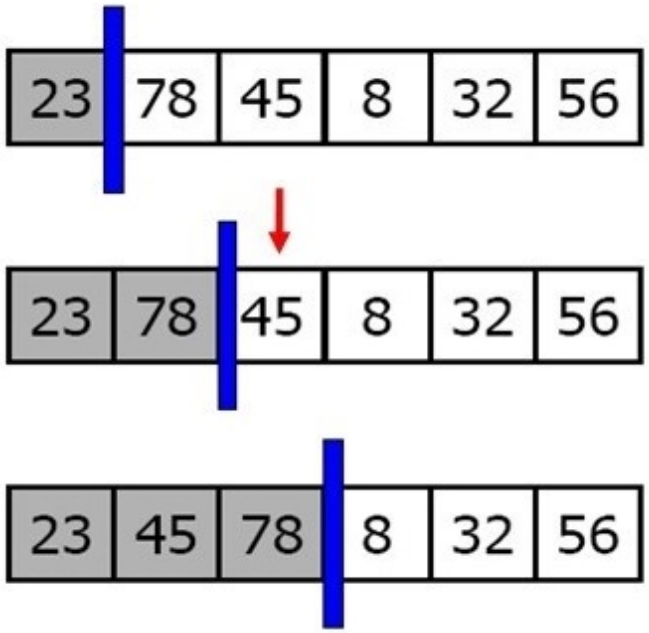
Straight Insertion Sort



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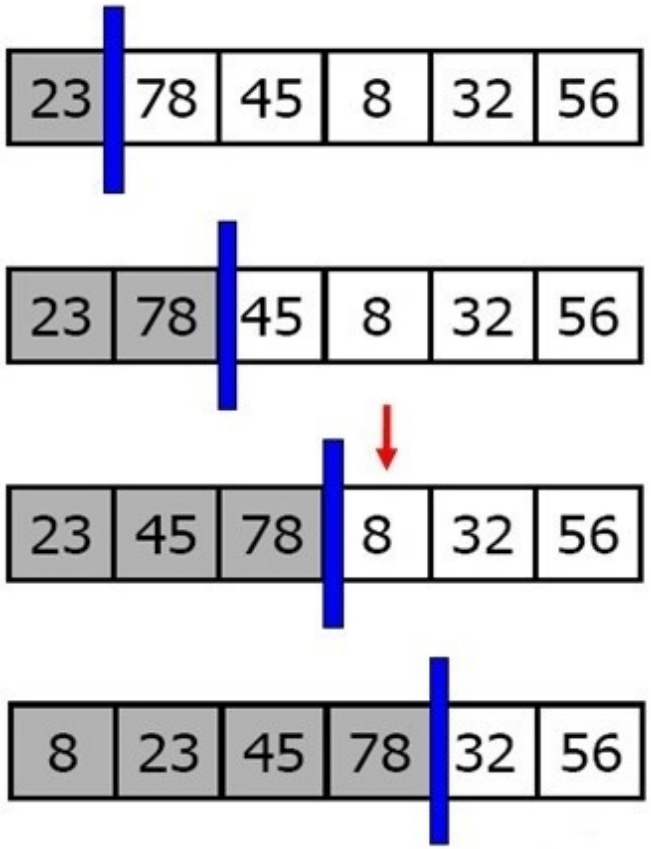
Straight Insertion Sort



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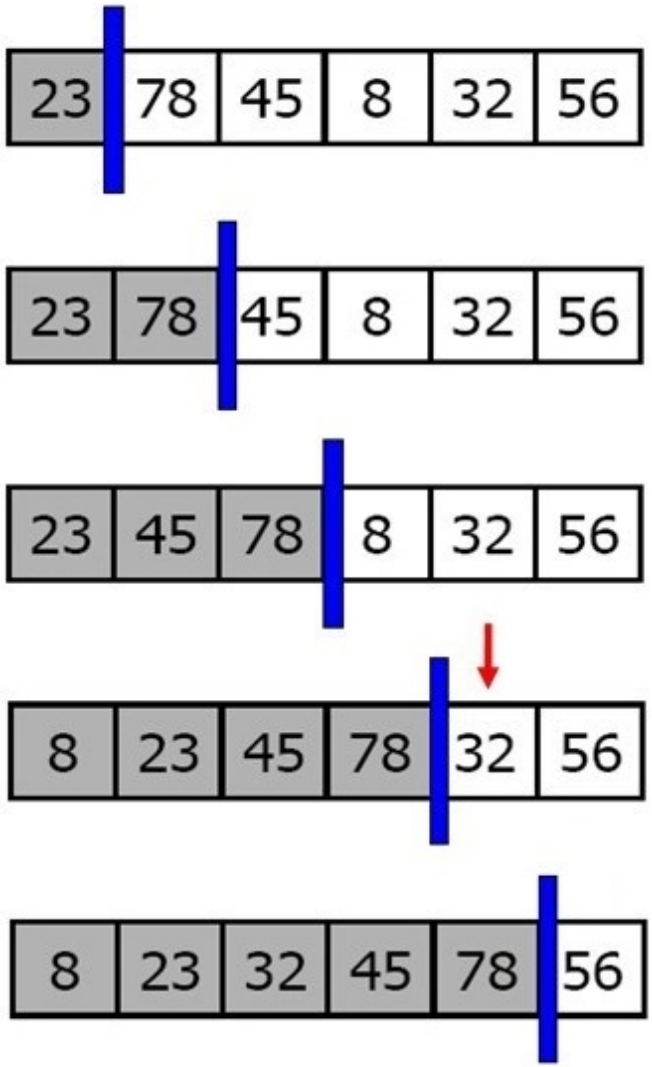
Straight Insertion Sort



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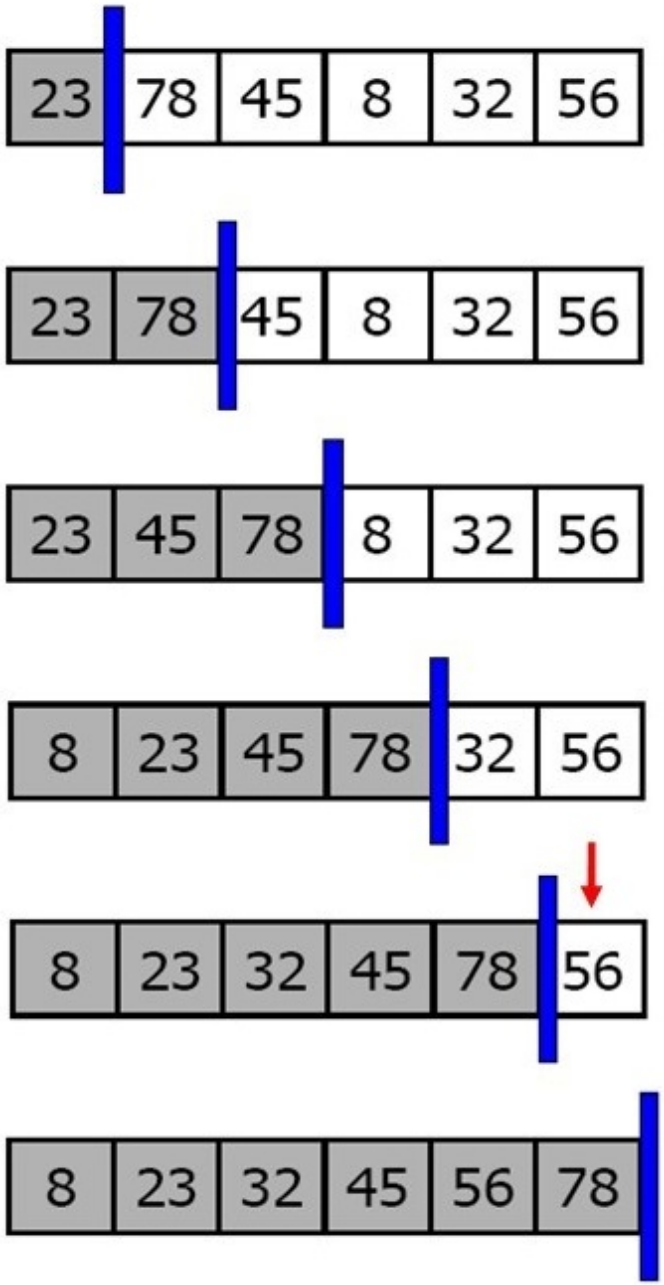
Straight Insertion Sort



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Straight Insertion Sort

Algorithm InsertionSort()
Sorts the contiguous list using straight insertion sort.

```
if count > 1 then
    current = 1
    while current < count do
        temp = data[current]
        walker = current - 1
        while walker >= 0 AND temp.key <
        data[walker].key do
            data[walker+1] = data[walker]
            walker = walker - 1
        end
        data[walker+1] = temp
        current = current + 1
    end
end
End InsertionSort
```


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Shell Sort

- Named after its creator Donald L. Shell (1959).
- Given a list of N elements, the list is divided into K segments (K is called the increment).
- Each segment contains N/K or more elements.
- Segments are dispersed throughout the list.
- Also is called diminishing-increment sort.

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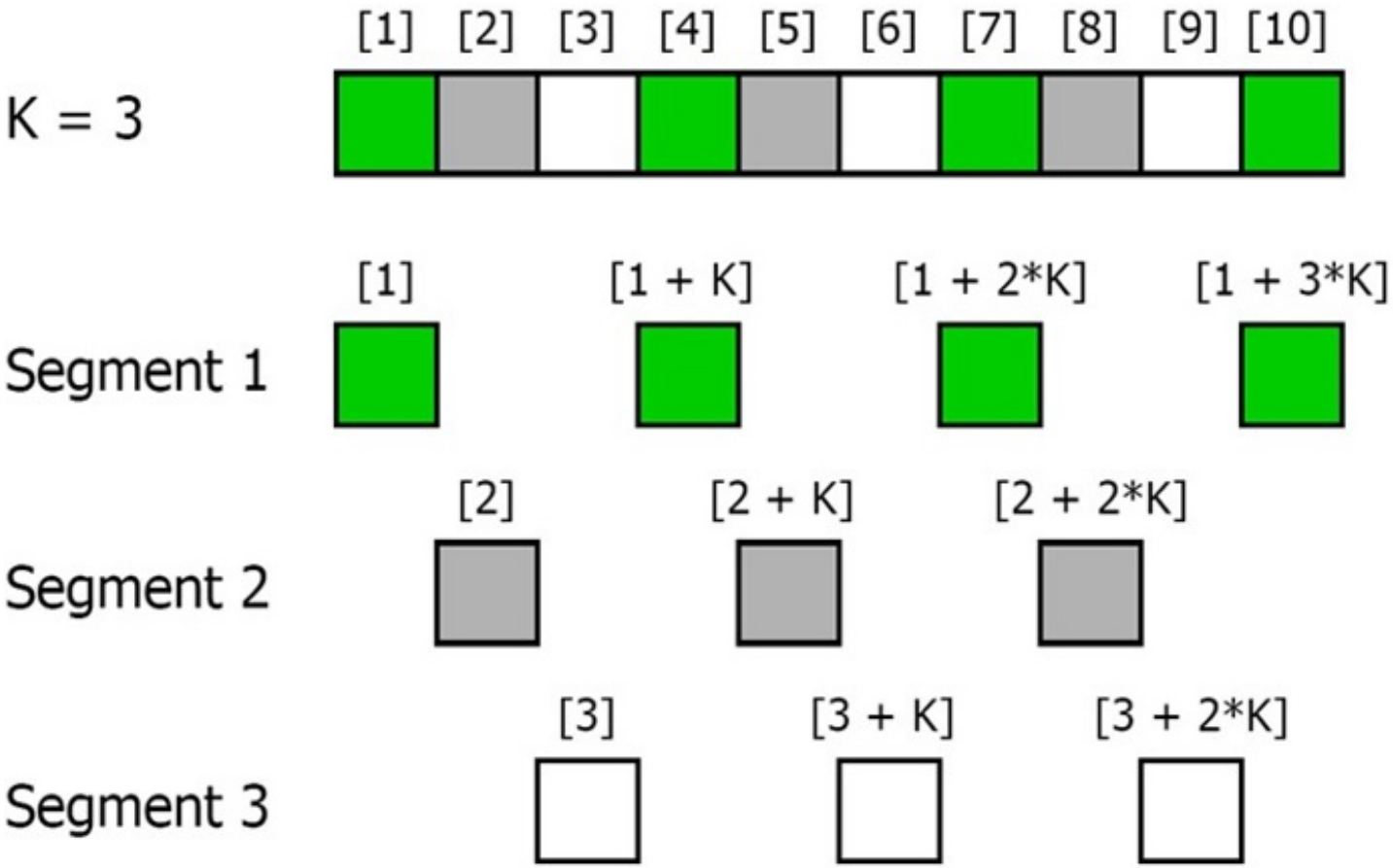
Shell Sort



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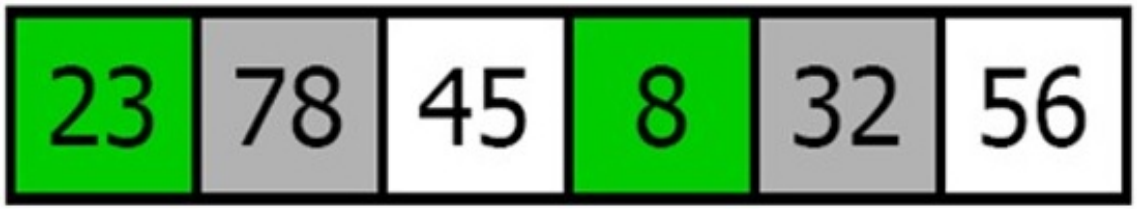
Shell Sort



- For the value of K in each iteration, sort the K segments.
- After each iteration, K is reduced until it is 1 in the final iteration.

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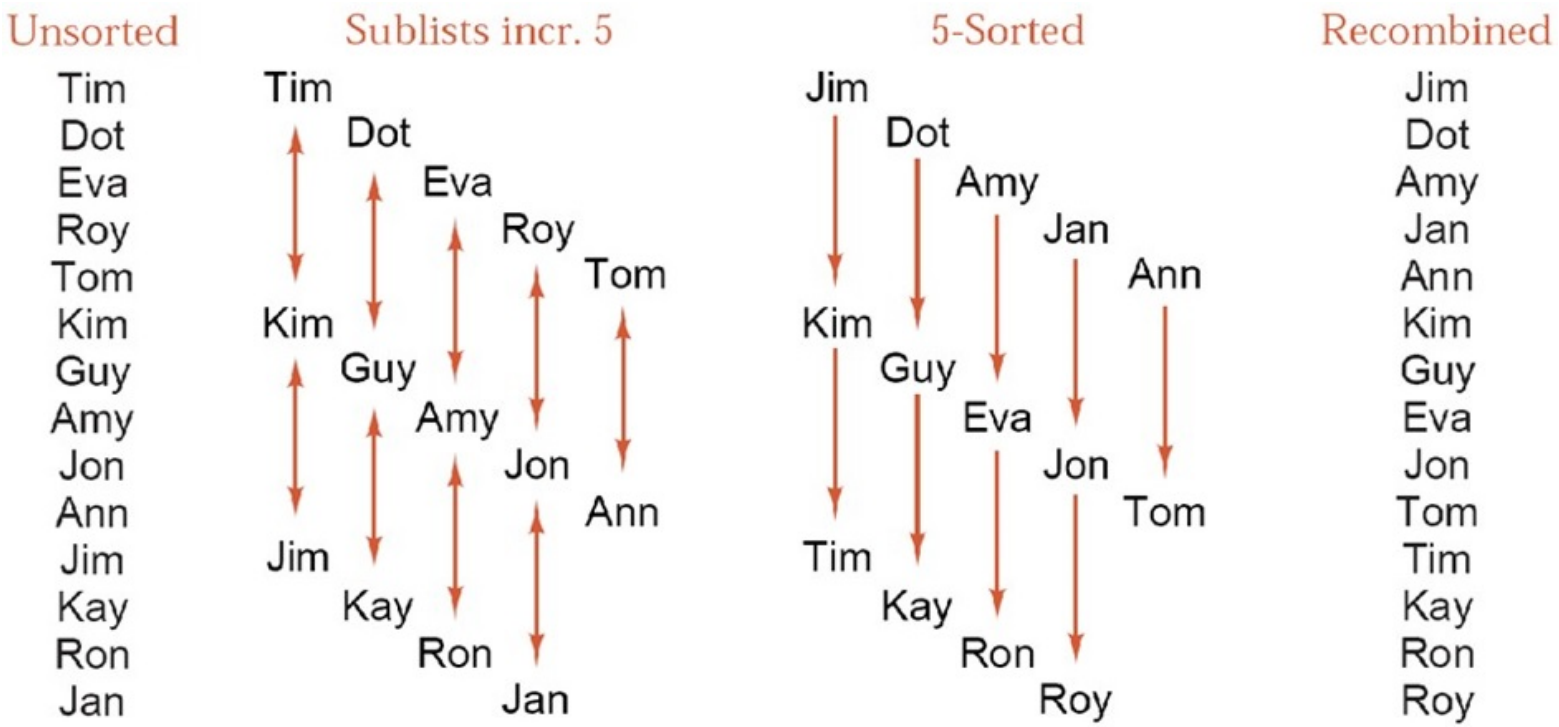
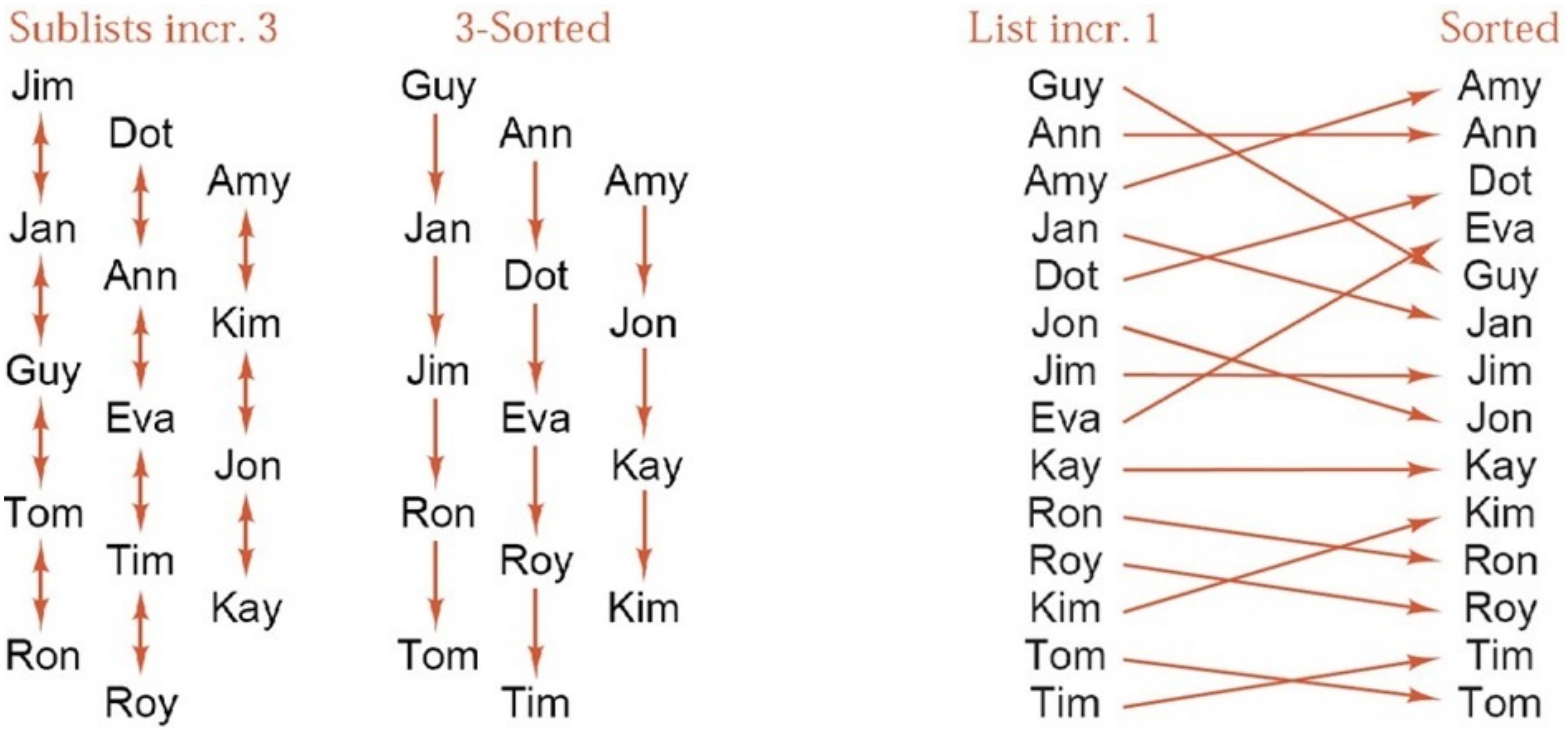
Example of Shell Sort



Example of Shell Sort



Choosing incremental values

- From more of the comparisons, it is better when we can receive more new information.
- Incremental values should not be multiples of each other, other wise, the same keys compared on one pass would be compared again at the next.
- The final incremental value must be 1.

Choosing incremental values

- Incremental values may be:
  $1, 4, 13, 40, 121, ...$
  $k_t = 1$
  $k_{i-1} = 3 * k_i + 1$
  $t = \lfloor \log_3 n \rfloor - 1$

- or:
  $1, 3, 7, 15, 31, ...$
  $k_t = 1$
  $k_{i-1} = 2 * k_i + 1$
  $t = \lfloor \log_2 n \rfloor - 1$

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Shell Sort

Algorithm ShellSort()
Sorts the contiguous list using Shell sort.

```
k = first_incremental_value
while k >= 1 do
    segment = 1
    while segment <= k do
        SortSegment(segment)
        segment = segment + 1
    end
    k = next_incremental_value
end
End ShellSort
```


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Shell Sort

Algorithm SortSegment(val segment <int>, val k <int>)
Sorts the segment beginning at segment using insertion sort, step between elements in the segment is k.

```
current = segment + k
while current < count do
    temp = data[current]
    walker = current - k
    while walker >=0 AND temp.key < data[walker].key do
        data[walker + k] = data[walker]
        walker = walker - k
    end
    data[walker + k] = temp
    current = current + k
end
End SortSegment
```


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Insertion Sort Efficiency

- Straight insertion sort:
  $f(n) = n(n+1)/2 = O(n^2)$
- Shell sort:
  $O(n^{1.25})$ (Empirical study)

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Selection Sort

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Selection Sort

In each pass, the smallest/largest item is selected and placed in a sorted list.

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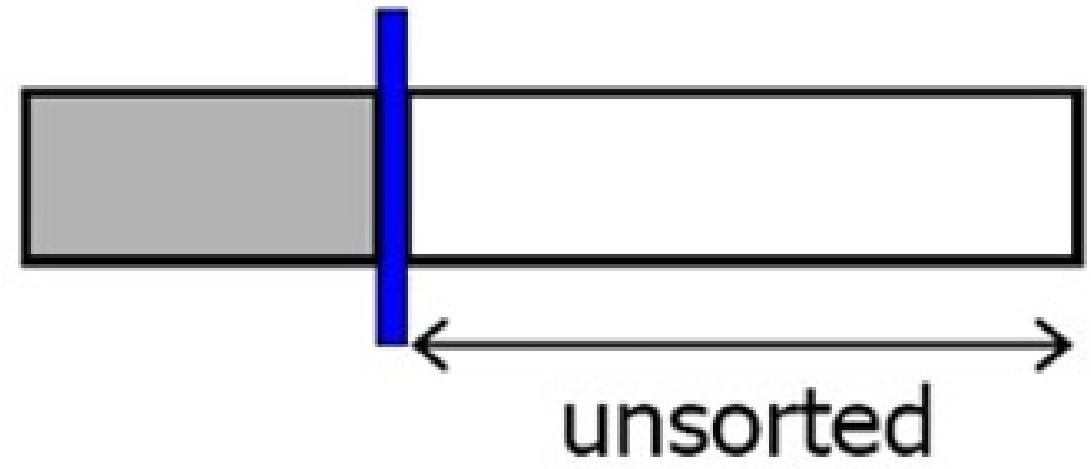
Straight Selection Sort

- The list is divided into two parts: sorted and unsorted.
- In each pass, in the unsorted sublist, the smallest element is selected and exchanged with the first element.



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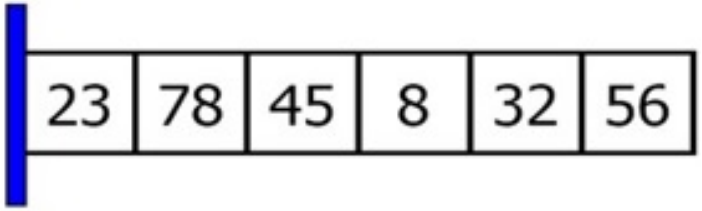
Straight Selection Sort



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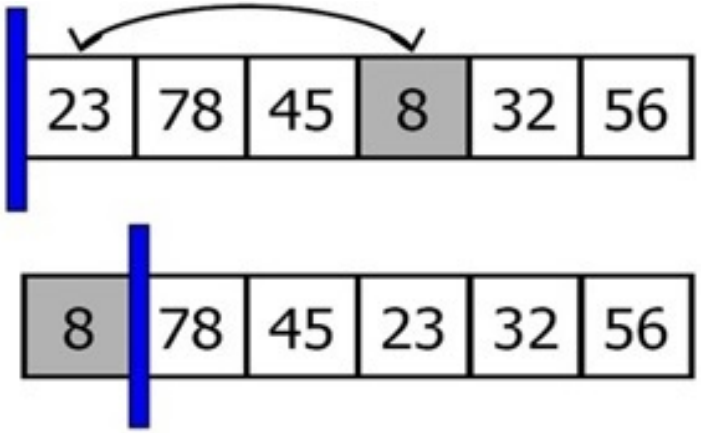
Straight Selection Sort



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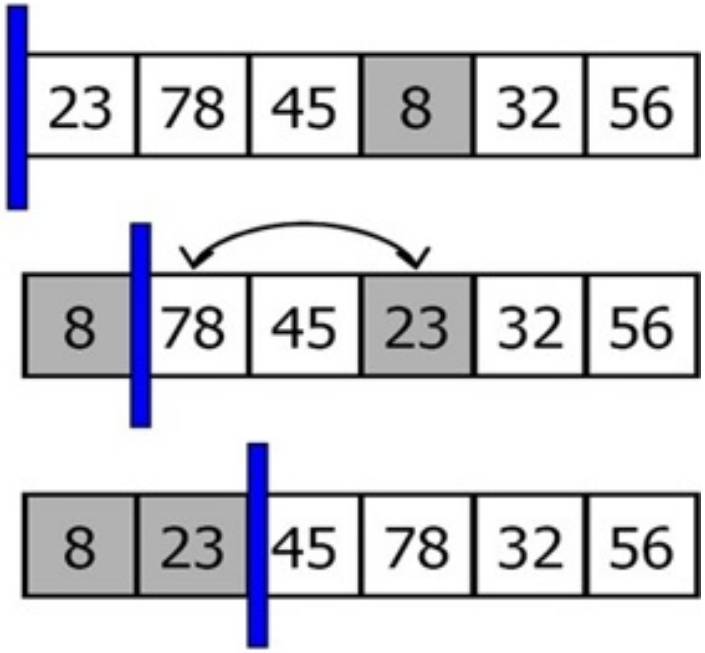
Straight Selection Sort



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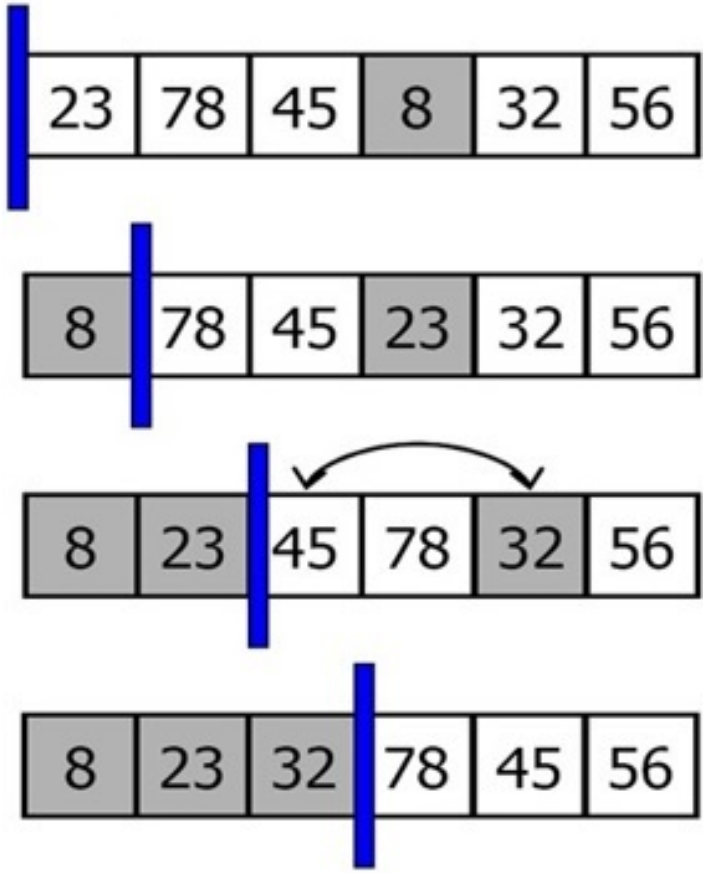
Straight Selection Sort



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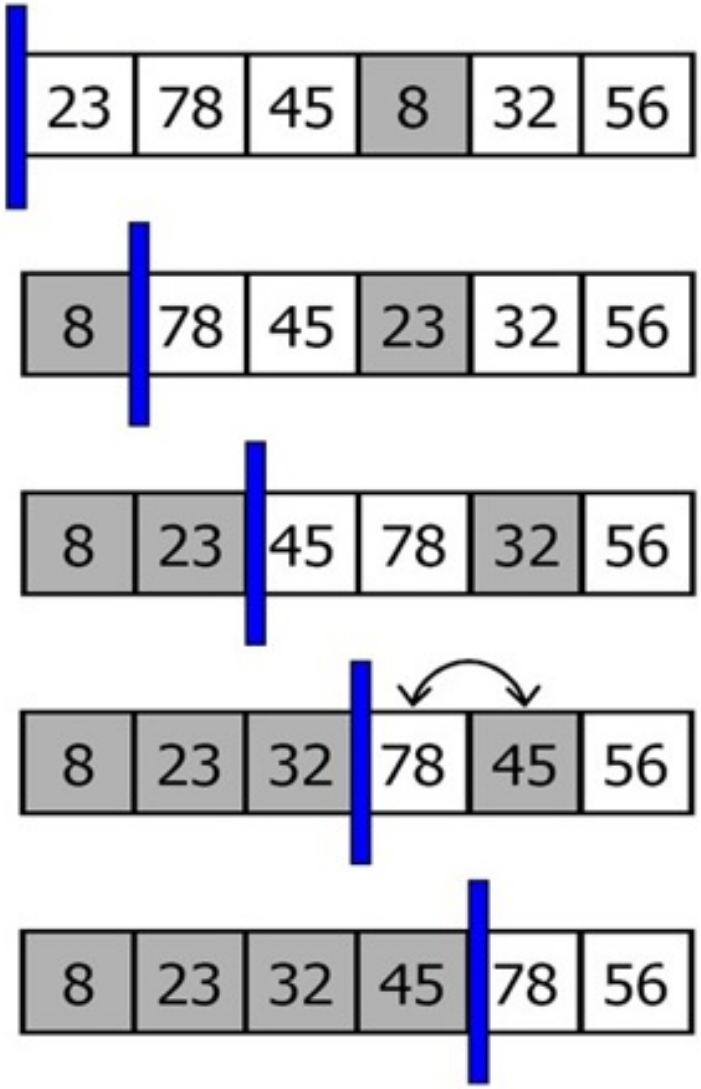
Straight Selection Sort



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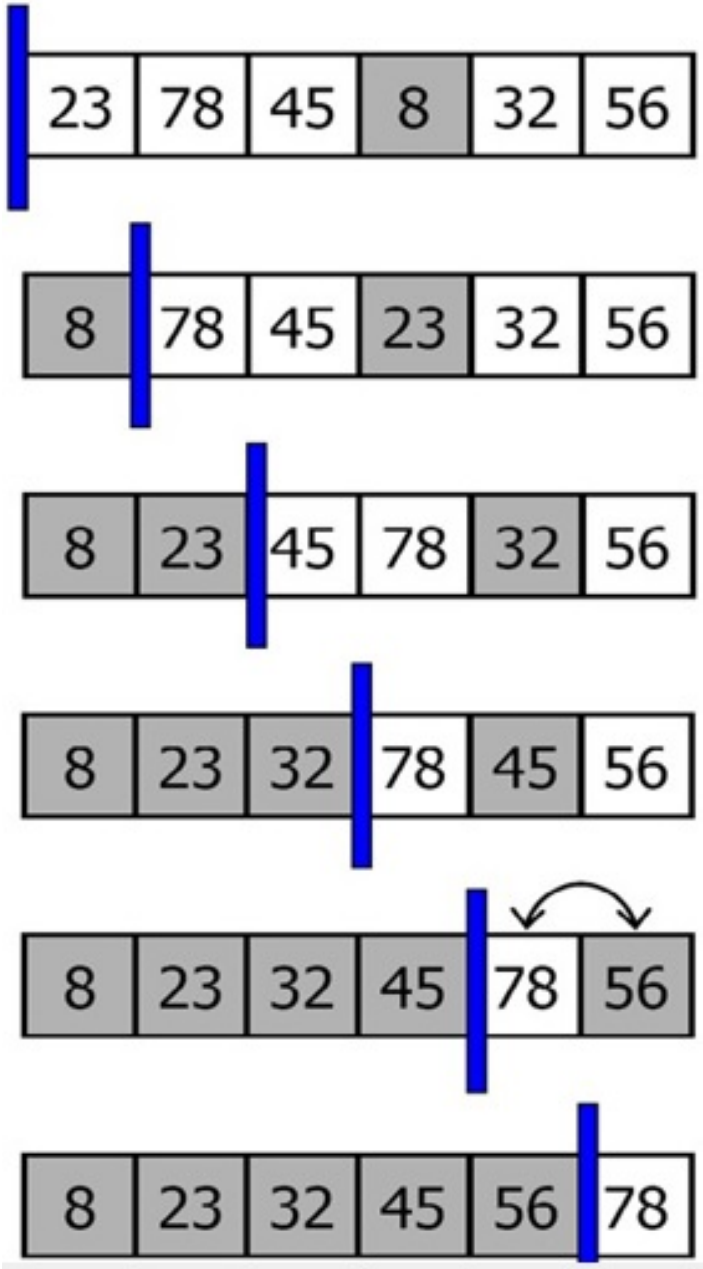
Straight Selection Sort



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Straight Selection Sort

Algorithm SelectionSort()
Sorts the contiguous list using straight selection sort.

```
current = 0
while current < count - 1 do
    smallest = current
    walker = current + 1
    while walker < count do
        if data [walker].key < data [smallest].key then
            smallest = walker
        end
        walker = walker + 1
    end
    swap(current, smallest)
    current = current + 1
end
End SelectionSort
```


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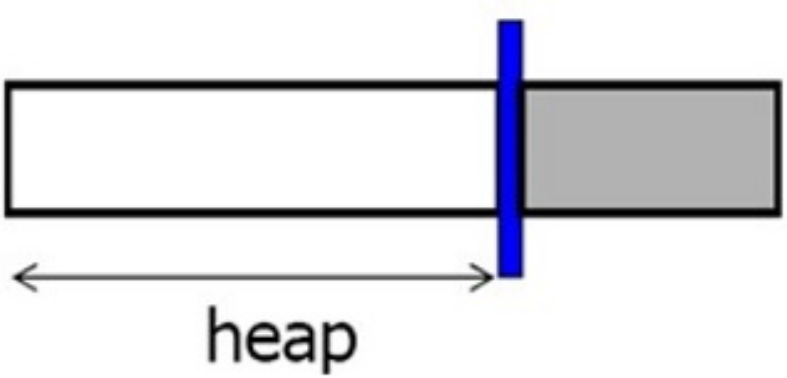
Heap Sort

- The unsorted sublist is organized into a heap.
- In each pass, in the unsorted sublist, the largest element is selected and exchanged with the last element.
- The the heap is reheaped.



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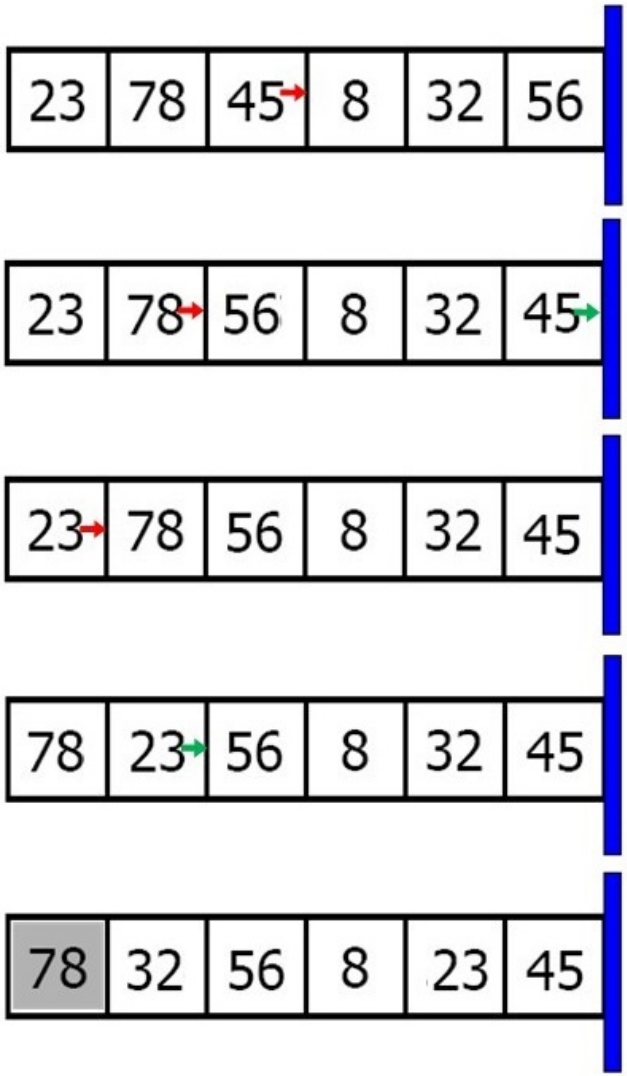
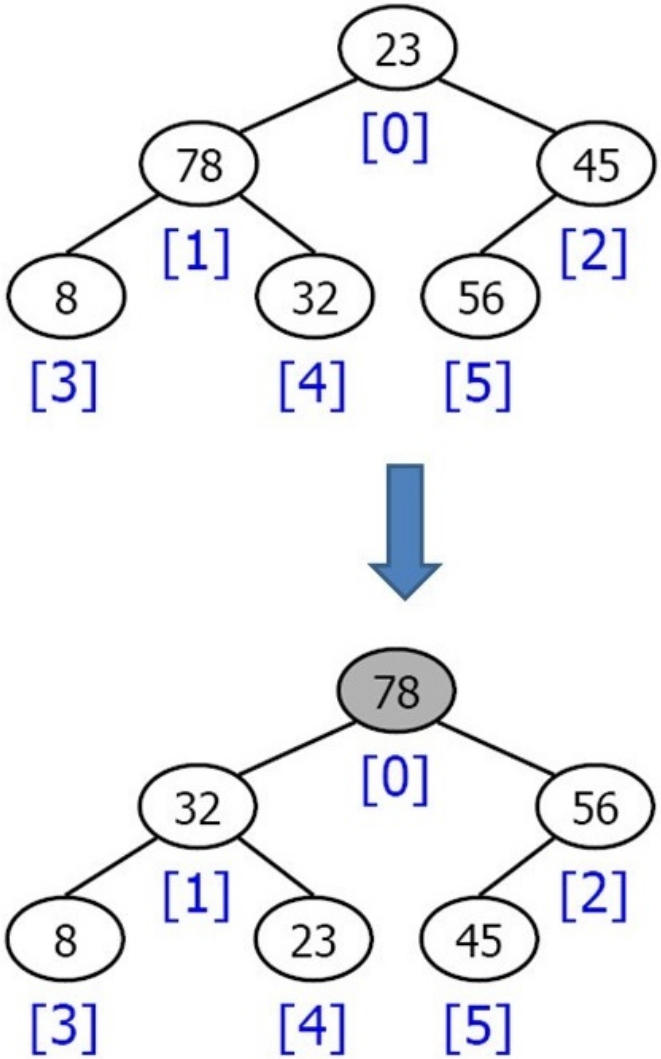
Heap Sort





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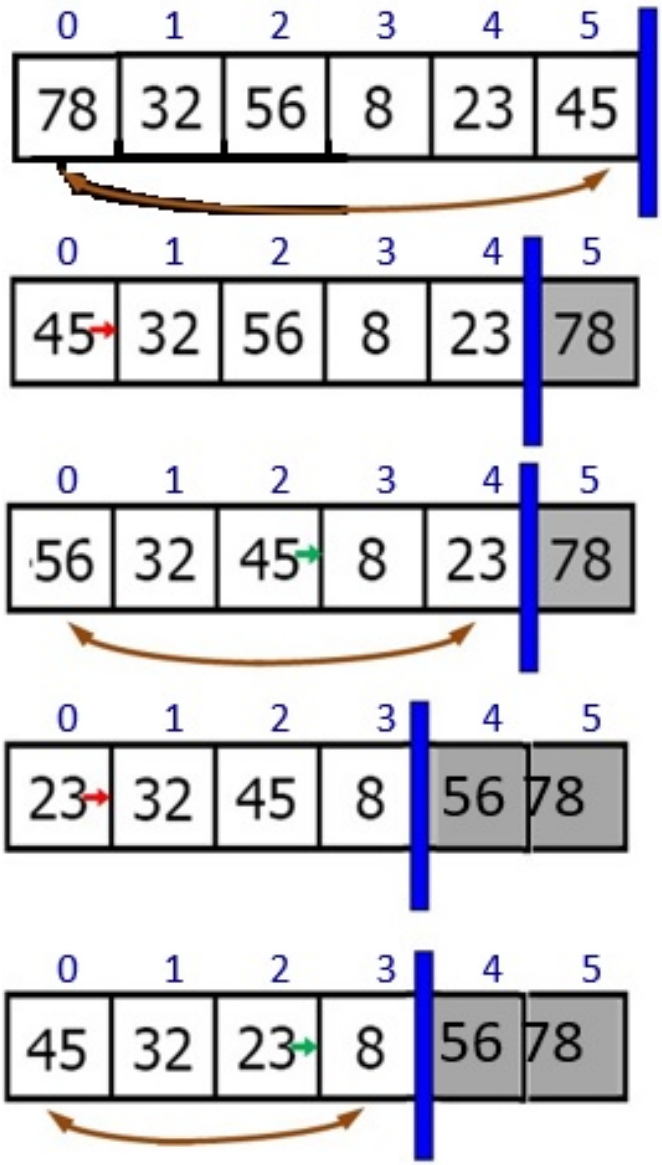
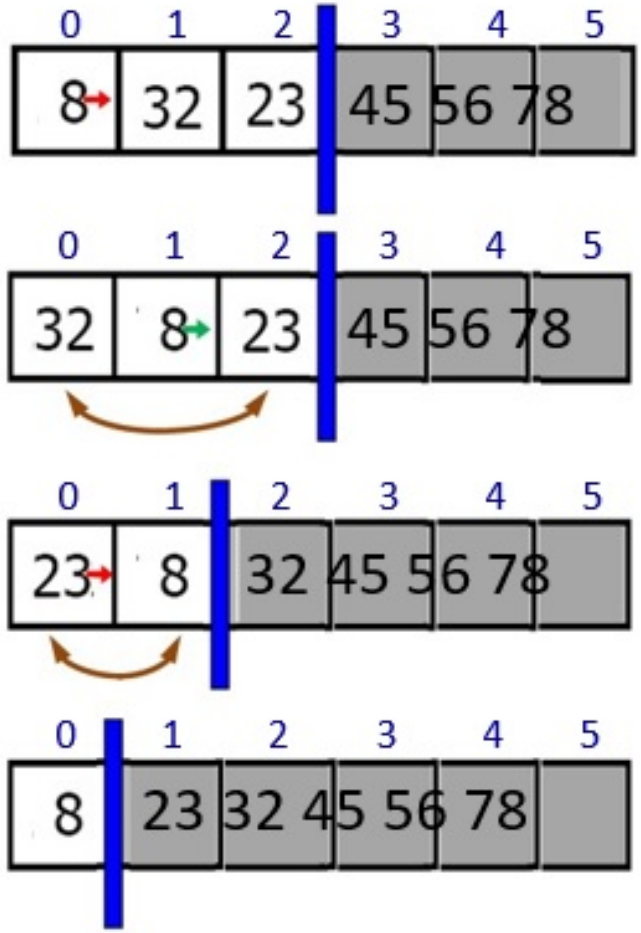
Heap Sort





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Heap Sort

Algorithm HeapSort()
Sorts the contiguous list using heap sort.

```
position = count/2 - 1
while position >= 0 do
    ReheapDown(position, count - 1)
    position = position - 1
end
last = count - 1
while last > 0 do
    swap(0, last)
    last = last - 1
    ReheapDown(0, last - 1)
end
End HeapSort
```


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Selection Sort Efficiency

- Straight selection sort: $O(n^2)$
- Heap sort: $O(nlog_2n)$

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Exchange Sort

Exchange Sort

- In each pass, elements that are out of order are exchanged, until the entire list is sorted.
- Exchange is extensively used.

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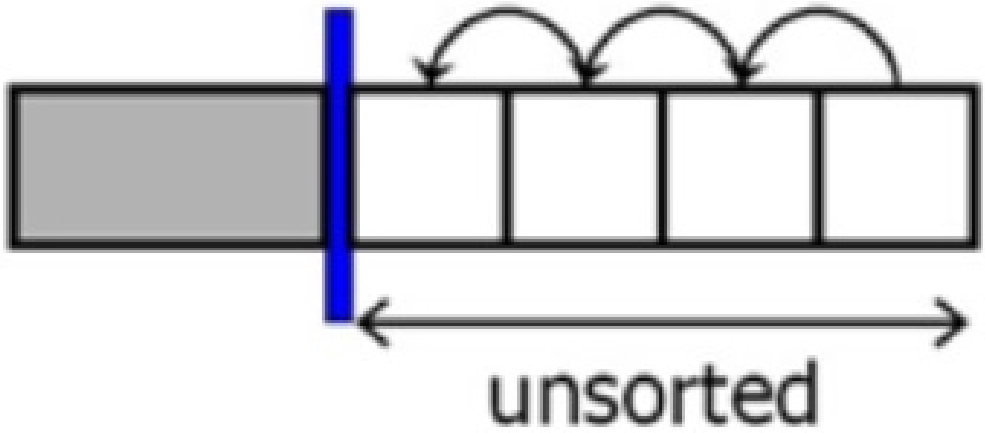
Bubble Sort

- The list is divided into two parts: sorted and unsorted.
- In each pass, the smallest element is bubbled from the unsorted sublist and moved to the sorted sublist.



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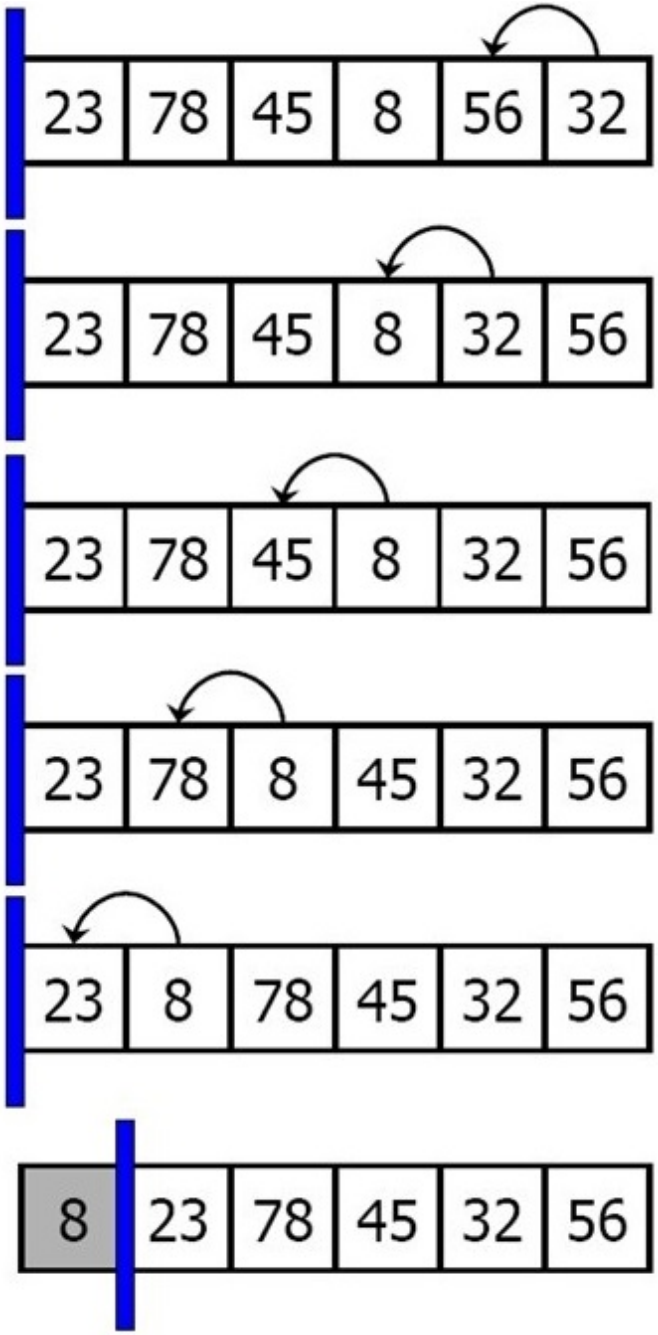
Bubble Sort



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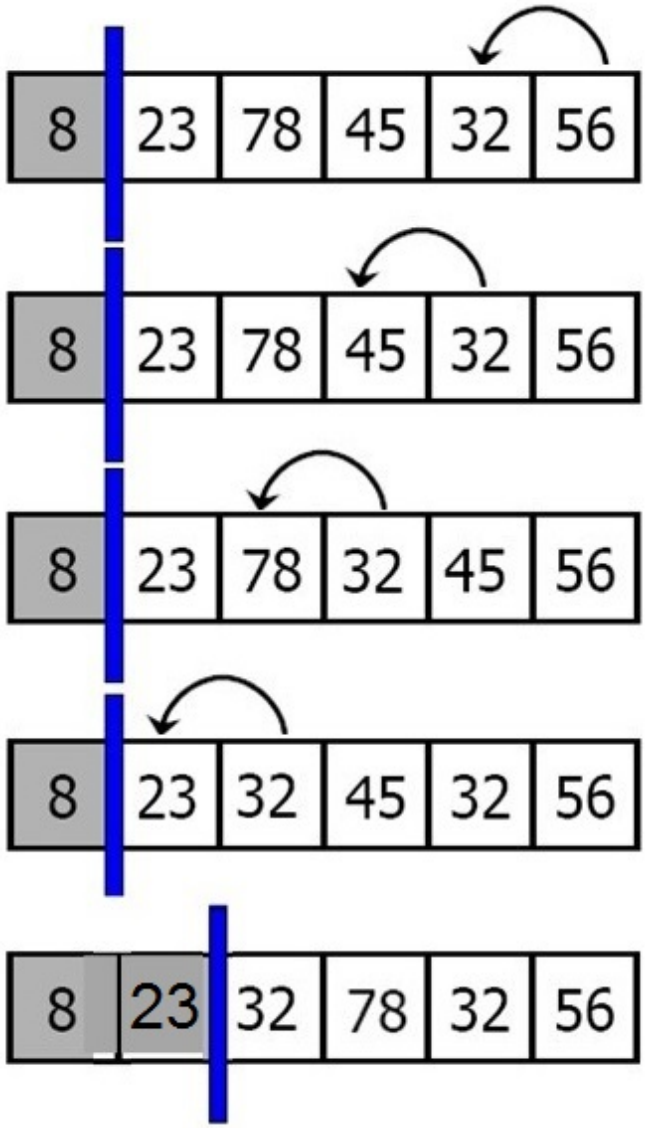
Bubble Sort



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Bubble Sort

Algorithm BubbleSort()
Sorts the contiguous list using bubble sort.

```
current = 0
flag = False
while current < count AND flag = False do
    walker = count - 1
    flag = True
    while walker > current do
        if data [walker].key < data [walker-1].key then
            flag = False
            swap(walker, walker - 1)
        end
        walker = walker - 1
    end
    current = current + 1
end
End BubbleSort
```


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Exchange Sort Efficiency

- Bubble sort:
  $f(n) = n(n+1)/2 = O(n^2)$

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Devide-and-Conquer

Devide-and-Conquer Sort

Algorithm DevideAndConquer()
if *the list has length* > 1 **then**
    partition the list into lowlist and highlist
    lowlist.DevideAndConquer()
    highlist.DevideAndConquer()
    combine(lowlist, highlist)
end
End DevideAndConquer

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Devide-and-Conquer Sort

	Partition	Combine
Merge Sort	easy	hard
Quick Sort	hard	easy

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Quick Sort

Algorithm QuickSort()
Sorts the contiguous list using quick sort.

recursiveQuickSort(0, count - 1)
End QuickSort

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Quick Sort

Algorithm recursiveQuickSort(val left <int>, val right <int>)
Sorts the contiguous list using quick sort.
Pre: `left` and `right` are valid positions in the list
Post: list sorted

if *left* < *right* **then**
    pivot_position = Partition(left, right)
    recursiveQuickSort(left, pivot_position - 1)
    recursiveQuickSort(pivot_position + 1, right)
end
End recursiveQuickSort

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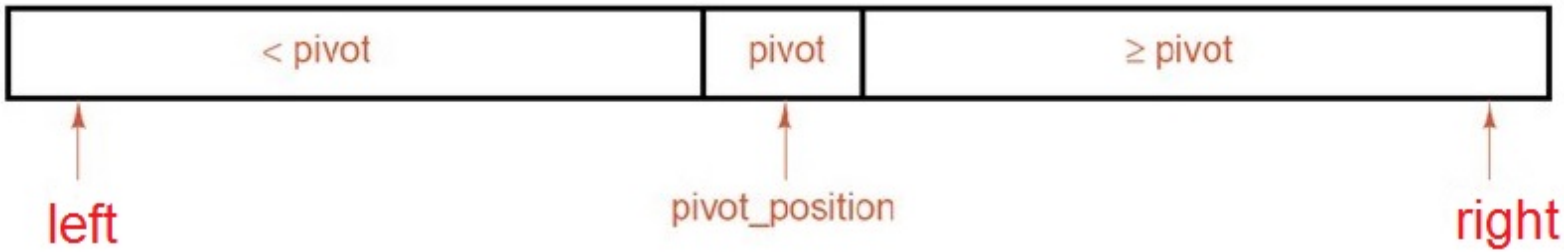
Quick Sort

Given a pivot value, the partition rearranges the entries in the list as the following figure:



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Quick Sort Efficiency

- Quick sort: $O(nlog_2n)$

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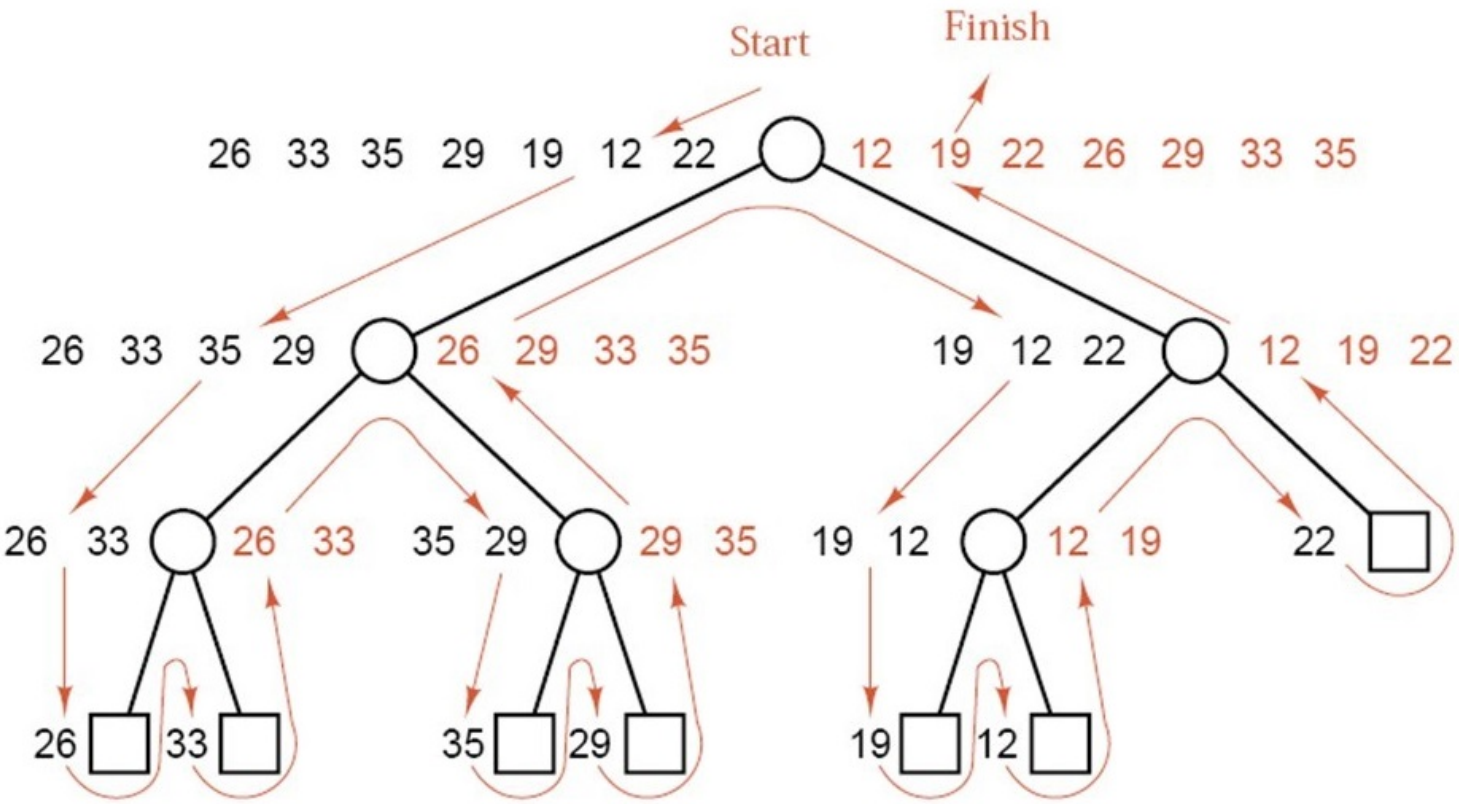
Merge Sort



Merge Sort

Algorithm MergeSort()
Sorts the linked list using merge sort.

recursiveMergeSort(head)
End MergeSort

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Merge Sort

Algorithm recursiveMergeSort(ref sublist <pointer>)
Sorts the linked list using recursive merge sort.

if *sublist is not NULL AND sublist->link is not NULL* **then**
    Divide(sublist, second_list)
    recursiveMergeSort(sublist)
    recursiveMergeSort(second_list)
    Merge(sublist, second_list)
end
End recursiveMergeSort

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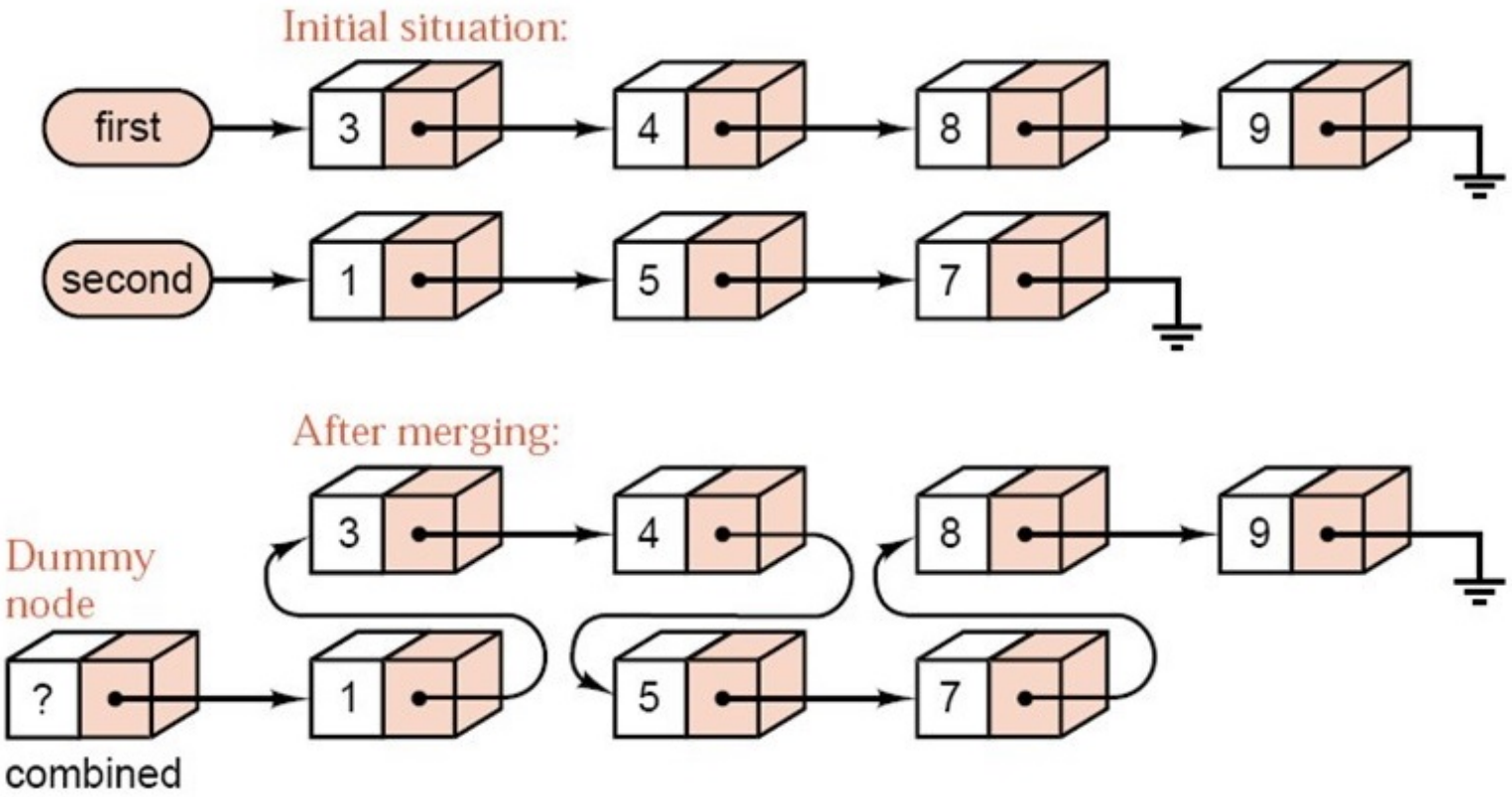
Merge Sort

Algorithm Divide(val sublist <pointer>, ref second_list <pointer>)
Divides the list into two halves.

```
midpoint = sublist
position = sublist->link    khi midpoint chạy 1 thì position chạy 2
while position is not NULL do
    position = position->link
    if position is not NULL then
        midpoint = midpoint->link
        position = position->link
    end
end
second_list = midpoint->link
midpoint->link = NULL
End Divide
```


Merge two sublists



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Merge two sublists

Algorithm Merge(ref first <pointer>, ref second <pointer>)
Merges two sorted lists into a sorted list.

```
lastSorted = address of combined
while first is not NULL AND second is not NULL do
    if first->data.key <= second->data.key then
        lastSorted->link = first
        lastSorted = first
        first = first->link
    else
        lastSorted->link = second
        lastSorted = second
        second = second->link
    end
end
```

so sánh 2 giá trị đầu của 2 mảng
lấy cái nào nhỏ hơn thì ghép vào lastSorted

// ...

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Merge two sublists

```
// ...

if first is NULL then
    lastSorted->link = second
    second = NULL
else
    lastSorted->link = first
end
first = combined.link
End Merge
```


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