# Roads as Economic Environment: An Agent Based Spatial Economic Model

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**Abstract**

A spatial monopolistic competition model is constructed in the spirit of the new economic geography. Firms of different type compete over a 2-D space where transport is costly for the business of households and other firms as intermediate goods. Firms enter and exit based on profitability and invest in capital. A road system alters the space of transport costs and the patterns of regional activity.

**Keywords:** transportation, new economic geography, market access

**JEL Codes:** would go here

# 1 Introduction

The goal of the model is to represent an economy distributed over a two-dimensional space where firms enter and exit based on profitability in order to analyze patterns of clustering for different sectors and how these patterns change after improving the road system. The setup is similar to the new economic geography models pioneered by Krugman (1991) and Fujita et al (1999) except for the explicit representation of space. In their models the simplifying assumptions get around the issues presented by space, effectively dealing with a fixed number of condensed 'locations' operating as open economies in perfect competition trading with each other. With their setup they can derive analytical results for two-region models and even a ring-model with an arbitrary number of locations. When extending to this two-dimensional spatial representation the primary difficulties are the market and monopsony power that stem from location relations, and that all agents face different sets of prices according to the distances between each agent and all of the other agents (the distance matrix). The discrete space model seems analytically untractable, and must be solved with a computer. The downsides to this approach are a loss of closed form solution, potentially no equilibrium, and difficulty in interpretability. However, there is increased flexibility as any functional forms can be used, and arguably an additional degree of 'reality' is gained by the explicit representation of space, both of which are important for the topic addressed here.

The model is composed of households and firms. Firms are monopolistically competitive as in Dixit-Stiglitz (1977), producing distinct varieties, $q_{ij}$, of different classes of goods that form composites, representing different industries j. $\sigma$ is the degree of substitution. For each composite there is a price index representing the cost of increasing the composite by one.

$$C_j = (\sum_i q_{ij}^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

$$P_j = (\sum_i p_{ij}^{(1-\sigma})^{1/(1-\sigma)}$$

Households are immobile and maximize utility from consuming a variety of composite goods subject to the price index they face and the wage they receive. In a two-stage optimization, consumers first choose how much of each composite to consume proportional to the price index and elasticity.

$$\max U = \prod_j C_j^{\mu_j}$$

$$\text{s.t.} \sum_j P_j C_j = w$$

$$C_j = \frac{w\mu_j}{P_j}$$

Second, given the quantity for each composite good the cost minimizing bundle is chosen, yielding the demand for each firm's good as a function of their price and the price index.[1]

$$\min \sum_i p_{ij} q_{ij}$$

$$\text{s.t.} (\sum_i q_{ij}^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)} = C_j$$

$$q_{ij} = \frac{w\mu_j p_{ij}^{-\sigma}}{P_j^{1-\sigma}}$$

The consumer faces a unique set of prices and prices indexes based on their position. I deviate from the standard ice-berg assumption of transport costs so that the transport cost is not proportional to the value of the product, and simply depends on the distance and an industry specific term to represent differences in shipping cost. The difference between the firm's price and the price consumers pay is[2]

$$p_{ijc} = p_{ijf} + e^{\tau_j d_{fc}} - 1$$

Similarly, the wage each consumer receives from the firm depends on their distance from the firm, representing the cost of commuting. Each consumer chooses to perfectly inelastically supply labor to whatever firm offers the best wage minus commute cost.

$$w_c = w_f - e^{\tau_w d_{fc}} + 1$$

The space is a discrete two-dimensional grid (100x100 in the simulation) where each position is connected to the eight positions surrounding it with a distance of 1, unless the area is designated as road in which case the distance is lower through that position[3]. Constructing the space as a network, through the Djikstra algorithm the shortest path distance between every pair of positions is calculated and stored in a matrix for reference. Among other things, the distance matrix is used to calculate the price index for every composite good at each location.

[1] Assuming $\sum_j \mu_j = 1$

[2] the value of shipping is essentially lost to the void in this framework, it is not earned as income

[3] this allows the transportation cost to vary with the distance, type of surface, and industry, while keeping the distance calculation simple for ease of computation

The firms take the prices and wages of all other firms as given and choose their own price and wage to maximize profit, utilizing labor and intermediate composite goods produced by other firms for production. Similar to consumers they first solve for how much of each composite they demand and then the cost minimizing bundle to determine their demand for every other firm's product.

$$\max \pi_{ij} = \sum_a p_{ij} q_{ija} - w_{ij} L_{ij} - \sum_k P_k C_{ijk}$$
$$\text{s.t.} \sum_a q_{ija} = Q_{ij} = L_{ij}^{\gamma} \prod_k C_{ijk}^{\beta_{jk}}$$

The index $a$ represents the quantity demanded by consumers and other firms, and $k$ represents the other composite goods used by firm $i$ in industry $j$. The output elasticities from each type of composite good can vary for each industry $j$.

It is necessary for the firms to choose both price and wage because of the spatial market power every location holds. This creates an oligopoly type framework, where prices and wages should be chosen strategically, but with a large number of firms the Nash equilibrium becomes unfeasible to calculate. Rather, this can be thought of as a dynamic ongoing process where for some amount of time firms have the opportunity to choose wage and price given everyone else, and if there is no profitable combination they leave the market. This is why an equilibrium may not exist as price and wage cycles of responses can occur, but that is part of the tradeoff. What we are interested in is the location distribution patterns that emerge, so small scale price-wage cycling is acceptable.

Because the space is discrete, labor supply response to a change in the wage cannot be calculated with a derivative, defying a closed form solution. Intuitively, as firms offer a higher wage the radius of workers they capture increases, leading to discrete jumps in labor. Given this situation, the solution is found as follows. First, the firms consider a set of prices which, given the distance matrix and prices of all other firms, maps to a total quantity demanded of their product, $Q_{ij}$. Second, the firms consider a set of wages which, given the distance matrix and wages of all other firms, maps to a set of laborers the firm would have. Third, for each combination of quantity demanded and labor supplied, the cost minimizing set of composite goods can be found.

$$\min w_{ij} L_{ij} + \sum_k P_k C_{ijk}$$
$$\text{s.t.} \sum_a q_{ija} = Q_{ij} = L_{ij}^{\gamma} \prod_k C_{ijk}^{\beta_{jk}}$$

$$C_{ijm} = \left[ \frac{Q_{ij}}{L_{ij}^{\gamma} \prod_k (\frac{\beta_{jk}}{P_k})^{\beta_{jk}}} \right]^{\frac{1}{\sum_k \beta_{jk}}} \frac{\beta_{jm}}{P_m}$$

Finally, we have a matrix of wage (labor) and price (quantity demanded) pairs where each pair dictates a cost minimizing bundle of composite goods, and therefore a total cost. This is combined with the total revenue from each price/quantity demanded to form a profit matrix, and the highest element yields the optimal wage and price combination given the price and wage of every other firm. One practical issue with this is that it is not computationally-feasible[4] to calculate the impact each firm has on the price index when they are mapping prices to quantity demanded, when they may have an effect if there are not a lot of firms nearby. This would suppress the demand for their product when raising the price. Typically this modeling problem is avoided by assuming a large number of firms at every location, which is not guaranteed in this setup. However, it is computationally feasible to calculate the effect of the firm's own wage on local demand, so that is included. The solution method is shown below.

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \rightarrow L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}$$

$$p = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \rightarrow Q = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \rightarrow TR = \begin{bmatrix} TR_1 & TR_2 & TR_3 \end{bmatrix}$$

$$\text{L , Q } \rightarrow L \underset{Q}{\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}} \rightarrow TC = \begin{bmatrix} TC_{11} & TC_{12} & TC_{13} \\ TC_{21} & TC_{22} & TC_{23} \\ TC_{31} & TC_{32} & TC_{33} \end{bmatrix}$$

$$\text{TR , TC } \rightarrow \pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix}$$

[4]it's possible I find a way to include this in a computationally efficient manner but it seems unlikely, and I think the impact is likely small. I will run some simulations to test the size

The final component to the model is how firms choose to enter. Because of the computationally intensive nature of calculating the profit maximizing location[5] I'm still experimenting with heuristic approaches, but intuitively the profitability depends on the price index at the location[6], the 'going wage', and the proximity to demand from both consumers and firms. The data for this are gathered when the price index is updated for all locations. For every location I map each category to the [0,1] scale, and construct a 'desirability index' by weighting each category. The weights are mutated to create different entry types which are tracked throughout the simulation for their survivability.

$$z = \left[\begin{pmatrix} P_1 & P_2 & P_3 \end{pmatrix} \quad w \quad demand\right]$$

$$Z = \sum_m weight_m z_m$$

A second approach is to simply randomly sample entry points and calculate the profitability of that location. If the potential profit is greater than the median firm, then a new firm will enter there. This method is computationally costly to check every point, but randomly sampling 60 points yields a high probability of finding the 95th percentile of profitable locations (with some moderate assumptions on the spatial distribution of profitability). Alternating between wage-price competition and firm entry until the space is eventually saturated.

$$P(\pi_{xy} > q_{95}) = 1 - (1 - .05)^{60} \approx .954$$

[5]there are ways to make this feasible, such as gradient descent or other grid-searching methods that I have not explored yet

[6]this has a conflicting effect, as firms want cheaper inputs but would also prefer a higher price index for their industry

Using pseudo-code, the algorithm for running the model is a bit like this.

**<u>Pseudo-code</u>**

**Initialize**

**construct space**

**calc distance matrix**

**place consumers**

**place firms**

**Loop**

**firm entry**

**price index update**

**competition**

**map wages to labor**

**map prices to demand**

**calc composite bundles**

**calc profit**

**firm exit**

**price index update**

The expected yield of the model is a statistic detailing the spatial distributions of the different industries over time as firms enter, die, or survive. Then, a road system can be experimented with to observe the impact. The simulation is operable and reasonably stable but I need more time to finish building the components.[7]

In summary, the difficulties/issues from transposing the model to the computer are the messy discrete mappings, the lack of closed form solution and ease of interpretability, the firms not considering their effect on the price index, the representation of time (events being blurred between simultaneous and sequential), the lack of strategic foresight in setting price and wage, the potential for price-wage cycles and lack of equilibrium, and the optimal entry location decision. This approach operates on an evolutionary heuristic basis.

[7]remaining: significant differences for an arbitrary number of industries potentially calibrated to real world data (so far there are 3 industries that do not differ much), improved variety of entry types (so far there are 3), spatial distribution statistic, modes for household distribution (so far it is random), a reasonably detailed potentially random road system (so far there is 1 road), safeguards to prevent economy collapse, general cleaning of structure and speed improvements