Fractal Dynamics of Spectral Density Calculations

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Contents

1   Introduction

This folder contains supplementary information on the complex dynamics of methods for solving the Silverstein-Marchenko-Pastur equation.

We provide scripts to compute the Julia sets of iterative methods for this problem. These scripts are written in the language UltraFractal.

- Version: 0.0.1

- Requirements: UltraFractal (tested on 5.04)

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- License: `GPL-3`

2   Contents

There are two folders, corresponding to the Fixed Point and Newton Methods.

2.1   ./Fixed Point Method

- `./MP_Iter_Sol.ufm` - This contains an UltraFractal formula. It programs the fixed point iteration method for a population spectrum $H = \frac{1}{2}(\delta_1 + \delta_{10})$.

- `./MP_Iter_Julia_Set.png` - This is a picture showing the Julia set of the dynamical system. The Julia set (white) belongs to the negative complex half-lane $\{z : im(z) < 0\}$, which confirms empirically that the method converges for z with positive imaginary part.

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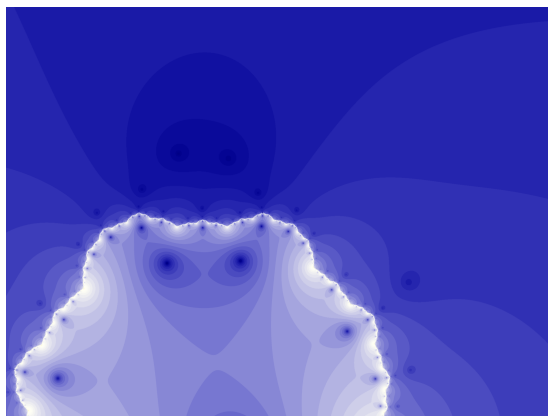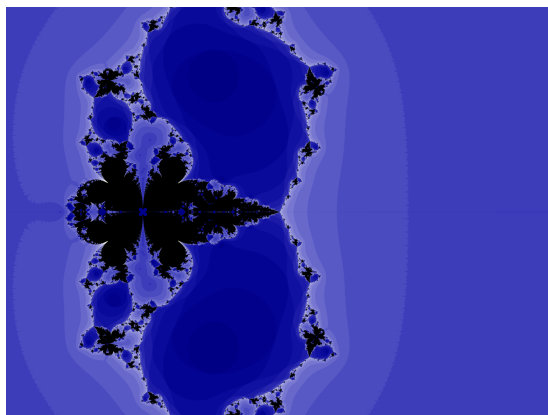
Figure 1: Julia set of Fixed Point Method.

Figure 2: Julia set of Newton Method.

2.2 ./Newton Method

- ./MP_newton.ufm - This contains an UltraFractal formula. It programs Newton's method for a population spectrum $H = \delta_1$.

- ./high_resolution_MP_null.png - This is a picture showing the Julia set of the dynamical system. The Julia set is now a complicated-looking fractal. This provides empirical support for our claims in the accompanying paper that Newton's method is numerically sensitive.