Chen et al : More LLM calls

- x prompt, θ - LLM hyperparameters & randomness

↦ · Majority Voting : · $\theta_1 \dots \theta_K \underset{iid}{\sim} \Theta$

· $z_k = G(x, \theta_k)$, $k \in [K]$

· $\hat{y}_K = \underset{a}{\operatorname{argmax}} \sum_{k=1}^{K} \mathbb{I}(z_k = a)$, break ties arbitrarily.

· From now on, fix x (and omit from notation)

· $F(K) = P_{\theta \sim \Theta}(\hat{y}_K = y)$, where y is fixed $\in A$

↦ Def · difficulty indicator d : $\lim_{K \to \infty} F(K) = \begin{cases} 0, & \text{if } d > 0 \\ 1, & \text{if } d < 0. \end{cases}$

↦ · Lemma : $d := \max_{a \neq y} P_{\theta \sim \Theta}\{G(x,\theta) = a\} - P_{\theta \sim \Theta}\{G(x,\theta) = y\}$

is a diff. ind. (Max ~~...~~ prob. of mistake vs prob. of correctness)

↦ · Thm : if $|A| = 2$, then $F(K) = P\left\{ \mathrm{Bino}\left(K, \frac{1-d}{2}\right) > \frac{K}{2} \right\}$ for K odd

↦ Pf. $P(\hat{y}_K = y) = P\left(\underbrace{\sum_{k=1}^{K} \mathbb{I}(z_k = y)}_{} > \frac{K}{2} \right)$

$\mathrm{Ber}(p)$, $p = P_\theta\{G(x,\theta) = y\}$

& note $d = (1-p) - p = 1 - 2p$, so $p = \frac{1-d}{2}$. $\square$