

NIGEM TECHNICAL DOCUMENTATION

The National institute's Global Econometric model, NiGEM is a global model, and most countries in the OECD are modelled individually. There are also separate models of Bulgaria, Brazil, China, Estonia, Hong Kong, Indonesia, India, Latvia, Lithuania, Romania, Russia, Singapore, South Africa, Taiwan and Vietnam while the rest of the world is modelled through regional blocks. All country models contain the determinants of domestic demand, export and import volumes, prices, current accounts and net assets. Economies are linked through trade, competitiveness and financial markets and are fully simultaneous.

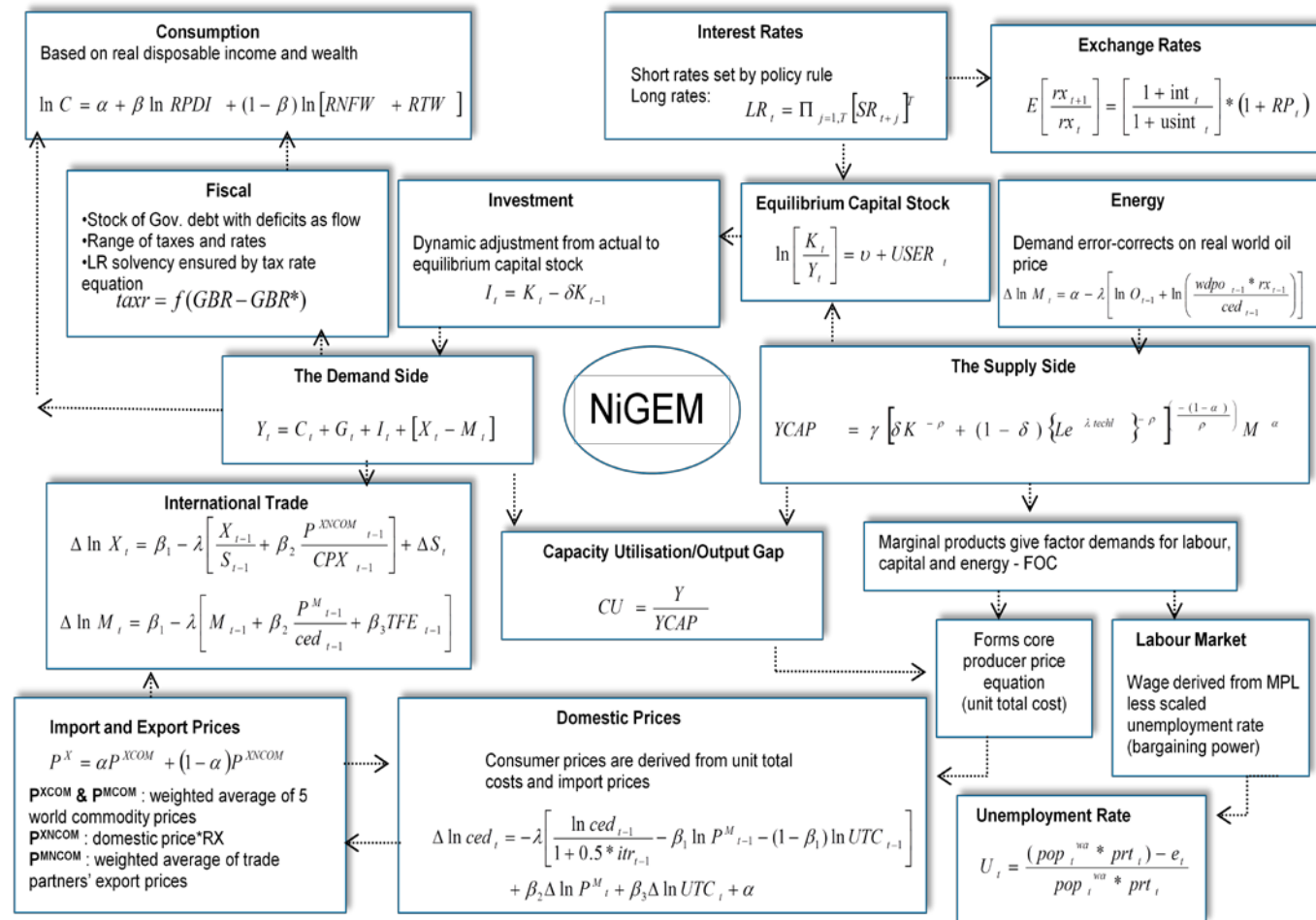
A major use of the model is in policy analysis. In policy analyses the model can be switched between forward looking, rational expectations mode and adaptive learning for consumers, firms, labour and financial markets. Policy environments are very flexible, allowing a number of monetary and fiscal policy responses. The model framework can be used by any user to build a bespoke model or to change the existing structure.

For a macro-econometric model to be useful for policy analyses, particular attention must be paid to its long-term equilibrium properties. At the same time, we need to ensure that short-term dynamic properties and underlying estimated properties are consistent with data and well-determined. Output is tied down in the long run by factor inputs and technical progress interacting through production functions. As far as possible, the same long run theoretical structure of NiGEM has been adopted for each of the major industrial countries, except where clear institutional, or other factors, prevent this. As a result, variations in the properties of each country model reflect genuine differences in data and estimated parameters, rather than different theoretical approaches.

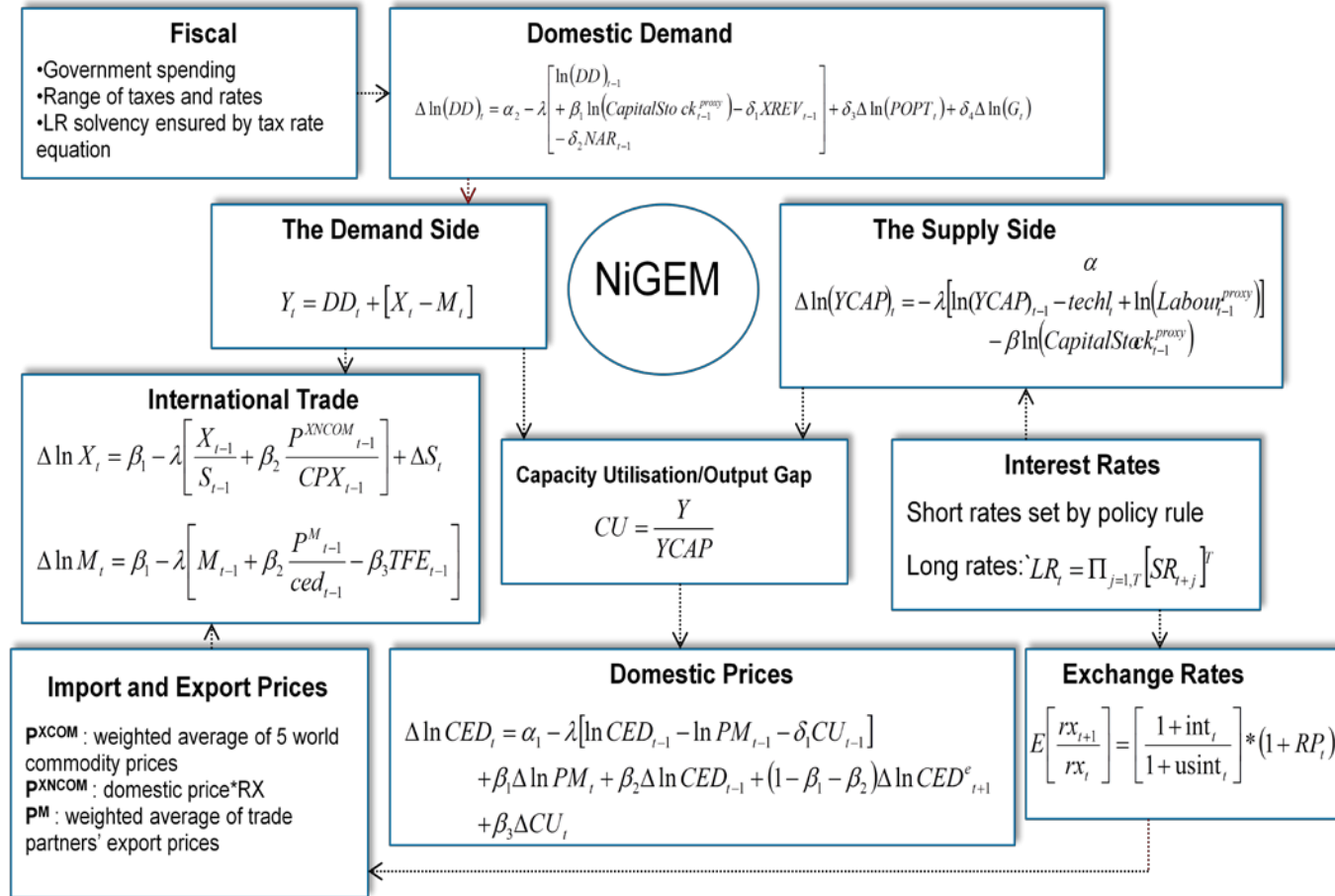
NIGEM TECHNICAL DOCUMENTATION	1
NIGEM TECHNICAL OUTLINE.....	3
Major country model structure	3
Reduced country model structure	4
Structure.....	5
Monetary policy options in NiGEM.....	12
Exchange rates options in NiGEM.....	13
Fiscal policy options in NiGEM.....	14
References	14

NIGEM TECHNICAL OUTLINE

MAJOR COUNTRY MODEL STRUCTURE



REDUCED COUNTRY MODEL STRUCTURE



STRUCTURE

Production and price setting

In major countries, GDP (Y) is driven by demand in the short-run, and is modelled as an identity relationship, summing the components of demand: C (consumption), PSI (private sector investment), GC (government consumption), GI (government investment), DS (stock building), $XVOL$ (exports), $MVOL$ (imports), RES (chain basing residual)

$$Y = C + PSI + GI + GC + DS + XVOL - MVOL + RES \quad (1.1)$$

These models rely on an underlying constant-returns-to-scale CES production function with labour-augmenting technical progress. This is embedded within a Cobb-Douglas relationship to allow the factors of production to interact with oil usage:

$$Q = \gamma \left\{ s(K)^{-\rho} + (1-s)(Le^{\lambda t})^{-\rho} \right\}^{-1/\rho} M^{1-\alpha} \quad (1.2)$$

where Q is real output, K is the total capital stock, L is total hours worked, t is an index of labour-augmenting technical progress and M is oil input. This constitutes the theoretical background for the specifications of the factor demand equations, forms the basis for unit total costs and provides a measure of capacity utilization, which then feeds into the price system. Barrell and Pain (1997) show that the elasticity of substitution is estimated from the labour demand equation, and in general it is around 0.5. Demand for labour and capital are determined by profit maximisation of firms, implying that the long-run labour-output ratio depends on real wage costs and technical progress, while the long-run capital output ratio depends on the real user cost of capital

$$\ln(L) = c_1 + \ln(Q) - (1 - \sigma)\lambda t - \sigma \ln(w/p) \quad (1.3)$$

$$\ln(K) = c_2 + \ln(Q) - \sigma \ln(c/p) \quad (1.4)$$

where c_1 and c_2 are constant terms related to the other parameters in the model, w/p is the real wage and c/p is the real user cost of capital. The user cost of capital is influenced by corporate taxes, depreciation and risk premia and is a weighted average of the cost of equity finance and the margin adjusted long real rate, with weights that vary with the size of equity markets as compared to the private sector capital stock. Business investment is determined by the error correction based relationship between actual and equilibrium capital stocks. Government investment depends upon trend output and the real interest rate in the long run. Prices are determined as a constant mark-up over marginal costs in the long term.

In reduced models, the short to medium term, GDP is driven by the demand side:

$$Y = DD + XVOL - MVOL \quad (1.5)$$

In the longer term, GDP is driven by the supply side:

$$\Delta \ln(Q) = \alpha - \lambda [\ln(Q)_{-1} - techl + \ln(POPT)] - \beta \ln(USER) \quad (1.6)$$

where *USER* (long real interest rate plus a risk premium) acts as a proxy for capital stock, *techl* is trend productivity growth, *POPT* is population and acts as a proxy for labour input.

Domestic demand depends on the user cost of capital (*USER*), export income (*XREV*) (especially important for commodity exporters), wealth (proxied by net foreign asset ratio) (*NAR*), population developments (*POPT*) and government spending (*G*).

$$\Delta \ln(DD)_t = \alpha_2 - \lambda \left[\begin{array}{l} \ln(DD)_{-1} + \beta_1 \ln(USER)_{-1} - \delta_1 XREV \\ - \delta_2 NAR \end{array} \right] + \delta_3 \Delta \ln(POPT) + \delta_4 \Delta \ln(G) \quad (1.7)$$

Labour market

NiGEM assumes that employers have a right to manage, and hence the bargain in the labour market is over the real wage. Real wages, therefore, depend on the level of trend labour productivity as well as the rate of unemployment. Labour markets embody rational expectations and wage bargainers use model consistent expectations. The dynamics of the wage market depend upon the error correction term in the equation and on the split between lagged inflation and forward inflation as well as on the impact of unemployment on the wage bargain (Anderton and Barrell 1995). There is no explicit equation for sustainable employment in the model, but as the wage and price system is complete, the model delivers equilibrium levels of employment and unemployment. An estimate of the NAIRU can be obtained by substituting the mark-up adjusted unit total cost equation into the wage equation and solving for the unemployment rate. Labour supply is determined by demographics, migration and the participation rate.

Derived from the underlying production function, the profit maximizing condition from the labour side sets the real wage equal to the marginal product of labour. This forms the core long-run solution to 3 equations: labour demand (*EE*), nominal wages (*WAGE*), and unit or marginal costs (*UTC*).

$$\Delta \ln(ee * hours) = \varepsilon_1 + \varepsilon_2 \left\{ \ln(ee_{-1} * hours_{-1} / ycap_{-1}) + 0.5 \ln \left(\frac{wage_{-1}}{py_{-1}} \right) + 0.5 * techl_{-1} \right\} + \sum \varepsilon_{3i} (\Delta \ln(y) - \varepsilon_{6i} \Delta techl)_{-i} + \sum \varepsilon_{4i} \Delta \ln(ee_{-i} * hours_{-i}) + \sum \varepsilon_{5i} \Delta \ln \left(\frac{wage}{py} \right)_{-i} \quad (2.1)$$

$$\Delta \ln(wage) = \omega_1 + \omega_2 \left\{ \ln \left(\frac{wage_{-1}}{py_{-1}} \right) + techl_{-1} - 2 \ln(ycap_{-1} / (e_{-1} * hours_{-1})) \right\} + \omega_{3a} \Delta \ln(CED)_t + (1 - \omega_{3a}) \Delta \ln(CED^e)_t + \omega_4 U_{-1} \quad (2.2)$$

$$\Delta \ln(utc) = \pi_1 + \pi_2 \{ \ln(utc)_{-1} - \ln(wage)_{-1} - 2 \ln(e_{-1} * hours_{-1} / ycap_{-1}) - techl_{-1} \} + \sum \pi_{3i} \Delta \ln(wage)_{t-i} + \pi_4 \ln(cu_{-1}) \quad (2.3)$$

Where PY is the GDP deflator and CED is the consumer expenditure deflator, U is the unemployment rate and CU is capacity utilisation (output gap).

A number of cross equation restrictions are imposed on the parameters in the three equations, to ensure that the model delivers a unique NAIRU. Allowing ε_{6i} to differ from 1 allows the growth rate of productivity to affect the NAIRU. This is consistent with finding by and Mankiw (2002) and others.

Consumption, personal income and wealth

Consumption decisions are presumed to depend on real disposable income and real wealth in the long run, and follow the pattern discussed in Barrell and Davis (2007). Total wealth is composed of both financial wealth and tangible (housing) wealth where the latter data is available.

$$\ln(C) = \alpha + \beta \ln(RPDI) + (1 - \beta) \ln(RFN + RTW) \quad (3.1)$$

where C is real consumption, $RPDI$ is real personal disposable income, RFN is real net financial wealth and RTW is real tangible wealth. The dynamics of adjustment to the long run are largely data based, and differ between countries to take account of differences in the relative importance of types of wealth and of liquidity constraints.

Each country on the model has a stock of foreign assets and a stock of liabilities. These are linked to the stock of domestic financial assets and the stock of domestic private sector and public sector liabilities. A proportion of government debt is owned abroad, as are proportions of the national stock of equities and the stock of banking assets. Some national financial wealth is held in foreign equities and bonds as well as banks. Income flows from asset stocks are allocated in relation to ownership, and hence net property income from abroad depends on income receipts and payments on bonds, equity holdings and bank. Once model and judgement based forecasts for asset prices, exchange rates and interest rates have been made the forecast for wealth follows automatically. The wealth and accumulation system allows for flows of saving onto wealth and for revaluations of existing stocks of assets in line with their prices determined as above. When foreign equity and bond prices change, domestically held assets change in value.

Financial markets

We generally assume that exchange rates are forward looking, and ‘jump’ when there is news. The size of the jump depends on the expected future path of interest rates and exchange rate risk premia, solving an uncovered interest parity condition, so that the expected change in the exchange rate is given by the difference in the interest earned on assets held in local and foreign currencies.

$$e_t = e_{t+1} \left(\frac{1 + r_t^*}{1 + r_t} \right) (1 + rp_t) + w_t \quad (4.1)$$

where e_t is the bilateral exchange rate at time t (defined as domestic currency per unit of foreign currency), r_t is the short-term nominal interest rate at home set in line with a policy rule, r_t^* is the interest rate abroad and rp_t is the exchange rate risk premium.

Interest rates are determined by policy rules adopted by monetary authorities as discussed in Barrell, Hall and Hurst (2006). Nominal short term interest rates are set in relation to a standard forward looking feedback rule. Our default rule follows a ‘two-pillar’ strategy, targeting a combination of inflation and a nominal aggregate. Forward looking long-term interest rates (LR) are a forward convolution of expected short-term interest rates:

$$(1 + LR_t) = \prod_{j=1}^T (1 + r_{t+j})^{1/T} \quad (4.2)$$

We assume that equity markets are also forward looking, with equity prices determined by the discounted present value of expected profits, adjusted by an equity risk premium.

Public sector

We model corporate (*CTAX*) and personal (*TAX*) direct taxes and indirect taxes (*ITAX*) on spending, along with government spending on investment and on current consumption, and separately identify transfers and government interest payments. Each source of taxes has an equation applying a tax rate ($?TAXR$) to a tax base (profits, personal incomes or consumption). As a default we have government spending on investment (*GI*) and consumption (*GC*) rising in line with trend output in the long run, with delayed adjustment to changes in the trend. They are re-valued in line with the consumers’ expenditure deflator (*CED*). Government interest payments (*GIP*) are driven by a perpetual inventory of accumulated debts. Transfers (*TRAN*) to individual are composed of three elements, with those for the inactive of working age and the retired, depending upon observed replacement rates. Spending minus receipts give us the budget deficit (*BUD*):

$$BUD = CED * (GI + GC) + TRAN + GIP - TAX - CTAX - MTAX \quad (5.1)$$

We have to consider how the government deficit (*BUD*) is financed. We allow either money (*M*) or bond finance (*DEBT*), so that the debt stock is related to historical deficits:

$$BUD = \Delta M + \Delta DEBT \quad (5.2)$$

rearranging gives:

$$DEBT_t = DEBT_{t-1} - BUD_t - \Delta M_t \quad (5.3)$$

In all policy analyses we use a tax rule to ensure that Governments remain solvent in the long run (Barrell and Sefton, 1997). This ensures that the deficit and debt stock return to sustainable levels after any shock. A debt stock target can also be implemented. The tax rate equation is of the form:

$$TAXR = f(\text{target deficit ratio} - \text{actual deficit ratio}) \quad (5.4)$$

If the Government budget deficit is greater than the target, (e.g. -3 % of GDP and target is -1% of GDP) then the income tax rate is increased.

External trade

NiGEM models the national accounts definition of exports and imports of goods and services. Historical trade patterns play an important role in the model equations, with current parameters for a set of key variables derived from a bilateral matrix for goods and services trade in 2010. Other key short-term and long-term elasticities are estimated through econometric techniques. At the global level, trade is driven by the demand side (imports), as no country can export a good unless there is another country willing to import it.

At the country/regional level, import demand is modelled as a function of demand (total final expenditure) and competitiveness (import prices relative to domestic prices). In the long-run we impose a demand elasticity that is greater than one, to allow world trade to continue rising as a share of world GDP throughout the forecast horizon (globalisation). Import prices are modelled as a weighted average of export prices in the rest of the world, where the weights are derived from the trade patterns underlying the bilateral trade matrix. This ensures consistency at the global level between export and import prices. Long-run price sensitivities differ across countries, as they are based on econometric estimation. The long-run is estimated within a dynamic error correction equation, so that short-term responses and the speed of adjustment towards the long-run reflect past behaviour in each country.

Exports are also modelled as a function of demand and competitiveness. External demand is modelled as a weighted average of import demand in all other countries and regions in the model, where, again, the weights are derived from the trade patterns underlying the bilateral trade matrix. We impose a unit elasticity on demand in all countries, which ensures approximate global consistency in export and import volumes, and also implies that the global trade share for each country is a function of its competitiveness. The competitiveness indicator that we use is the export price of the home country relative to a weighted average of export prices in the rest of the world. Again, the weights are derived from the bilateral trade matrix, and it is assumed that exporters compete against others who export to the same markets.

While the model is designed to deliver approximate global consistency in export and import volumes, values and prices, there is still a possibility, particularly over the longer term, of drift away from this balance. In order to correct for any such drift, additional adjustment terms are included in the model to reallocate any discrepancy between export and import values or volumes proportionately across countries. Any historical discrepancy between export and import data is not corrected, so it is the ratio of the two that is held constant over the forecast horizon.

The role of commodities in NiGEM

Oil price equation in NiGEM

$$\Delta \ln(wdpo)_t = \alpha \left[-\beta * \left[\ln \left(\frac{wdpo}{usced} \right)_t - \gamma * \ln(wdoi)_t \right] + \Delta \ln(wdpxncom)_t + \delta * [\Delta \ln(wdpo)_{t-1} - \Delta \ln(wdpxncom)_{t-1}] \right] \quad (6.1)$$

wdpo: price of oil (energy)

wdoi: world gas, oil and coal intensity

wdpxncom: world export price of manufacture

usced: proxy for world consumer expenditure deflator

- Real price rises in line with global oil demand
- Forecast baseline projections taken from the EIA
 - Incorporates futures markets, so consistent with expectations
 - Most price rises assumed to be permanent

Factors of production

$$Q = \gamma \left\{ s(K)^{-\rho} + (1-s)(Le^{\lambda t})^{-\rho} \right\}^{-1/\rho} M^{1-\alpha} \quad (6.2)$$

Supply-side based on CES relationship between capital (K) and labour (L), embedded in a Cobb-Douglas framework that allows the introduction of energy (M) as a factor input. Based on this relationship, energy accounts for a constant share of production costs over the long run, but deviates from long-run share in the short- to medium-term. The demand for energy error corrects gradually on the real oil price, with unit elasticity in long-run so a permanent rise in the oil price will reduce oil usage, and lead to a decline in potential output

$$\Delta \ln(oi)_t = \varepsilon - 0.25 * \left[\ln(oi)_{t-1} + 0.0025 + \ln \left(\frac{wdpo_{t-1} * rx_{t-1}}{ced_{t-1}} \right) \right] \quad (6.3)$$

oi: energy intensity

rx: nominal exchange rate

ced: consumer expenditure deflator

Terms of trade

Export and import prices are modelled as a weighted average of commodity and non-commodity prices

$$px = \alpha_1 * pxcom + (1 - \alpha_1) * pxncom \quad (6.4)$$

$$pm = \alpha_2 * pmcom + (1 - \alpha_2) * pmncom \quad (6.5)$$

Where α_1 is the share of commodities in goods and services exports and α_2 is the share commodities in imports (2010 trade patterns)

Commodity import and export prices are a weighted average of 5 commodities

$$pxcom = \beta_1 wdp_o + \beta_2 wdp_{fdv} + \beta_3 wdp_{fld} + \beta_4 wdp_{anf} + \beta_5 wdp_{mm} \quad (6.6)$$

wdp_{fdv} : global food prices

wdp_{fld} : global beverage prices

wdp_{anf} : agricultural raw materials

wdp_{mm} : global price of metals

$$pmcom = [\delta_1 wdp_o + \delta_2 wdp_{fdv} + \delta_3 wdp_{fld} + \delta_4 wdp_{anf} + \delta_5 wdp_{mm}] * rx \quad (6.7)$$

So a 1% rise in the oil price raises export prices by $\alpha_1\beta_1\%$ and import prices $\alpha_2\delta_1$ (holding RX fixed).

This leads to the following:

- Direct role through import prices
- indirect role through capacity utilisation in marginal cost/producer prices

Disaggregated energy model

When the disaggregated model is used, wdp_o is replaced by an average price of energy weighted by the country-specific commodity production/consumption figures, wdp_o^* .

$$wdp_o^* = \frac{\alpha_1 wdp_o + \alpha_2 wdp_g + \alpha_3 wdp_f}{\alpha_1 + \alpha_2 + \alpha_3} \quad (6.8)$$

wdp_g : world price of gas

wdp_f : world price of coal

Monetary policy options in NiGEM

Two-pillar rule

This policy brings current nominal GDP back to its target level. Nominal GDP is determined by the GDP deflator by default, but it is also possible to use a consumer expenditure deflator.

$$sr_t = \gamma * sr_{t-1} + (1 - \gamma) * \left[\alpha * \ln \left(\frac{nom_t^*}{nom_t} \right) + \beta * (inf_{t+1} - inf_{t+1}^*) \right] \quad (11.1)$$

sr: nominal short-rate
 nom: nominal GDP
 nom*: nominal GDP target
 inf: inflation expectations
 inf*: inflation expectations target
 α: money (or nominal GDP) coefficient
 β: inflation coefficient

Taylor rule

Short-term interest rates to respond to variations in GDP growth rate and inflation.

$$sr_t = \gamma * sr_{t-1} + (1 - \gamma) * \left[\alpha + \beta * (inf_{t+1} - inf_{t+1}^*) + \delta * \ln \left(\frac{y}{ycap} \right)_t \right] \quad (11.2)$$

y: GDP
 ycap: trend output for capacity utilisation
 α: long run equilibrium
 β: inflation coefficient
 δ: output levels

Nominal GDP targeting

Monetary policy responds to bring the monetary base back to its target level. Nominal GDP is determined by the GDP deflator by default, but it is also possible to use a consumer expenditure deflator.

$$sr_t = \gamma * sr_{t-1} + (1 - \gamma) * \alpha * \ln \left(\frac{nom_t^*}{nom_t} \right) \quad (11.3)$$

Orphanides rule

Essentially a modified Taylor rule with 3-period look ahead and the change in capacity utilisation included.

$$sr_t = \gamma * sr_{t-1} + (1 - \gamma) * \left[\alpha + inf_{t+1} + \beta * (inf_{t+3} - inf_t^*) + \delta * \ln \left(\frac{y}{ycap} \right)_t + \mu * \left(\frac{y_{t+3}/ycap_{t+3}}{y_{t-1}/ycap_{t-1}} \right) \right] \quad (11.4)$$

μ: change in capacity utilisation

Price level targeting

Rates set by variations in Price level and inflation.

$$sr_t = \gamma * sr_{t-1} + (1 - \gamma) * \left[\alpha * \ln \left(\frac{ced}{ced^*} \right)_t + \beta * (inf_{t+1} - inf_{t+1}^*) \right] \quad (11.5)$$

ced: consumer expenditure deflator

ced*: consumer expenditure deflator target

Within/shadow EMU

Rates set equal to EMU short rates

$$sr = sr^{emu} \quad (11.6)$$

Exchange rates options in NiGEM

Forward exchange rates

Exchange rates will be determined by the current interest rate differential and contemporaneous expectations of the exchange rate next period. Given that foreign exchange markets are rational and efficient, then any previously unanticipated change in, say government policy, will cause agents to revise expectations of the trajectory of the exchange rate in the future. This revision to expectations is likely to result in the exchange rate 'jumping' today. The model is constructed so that the jump in the exchange rate is sufficient to ensure that the economy jumps to a new saddle path with a private sector wealth equilibrium.

$$rx_t = \ln(rx_{t+1}) - 0.25 * \ln \left(\frac{100+sr}{100+ussr} \right)_t \quad (11.7)$$

rx: nominal exchange rate

ussr: US nominal short-rate

Open arbitrage

Open arbitrage implies that the contemporaneous change in the exchange rate is identically equal to the interest rate differential lagged one period. Therefore the current exchange rate is determined by both yesterday's exchange rate and yesterday's interest differential

$$rx_t = \ln(rx_{t-1}) - 0.25 * \ln \left(\frac{100+sr}{100+ussr} \right)_{t-1} \quad (11.8)$$

Fixed real

$$rx_t = rx_{t-1} * \frac{(ced*usced)_t}{(ced*usced)_{t-1}} \quad (11.9)$$

ced: consumer expenditure deflator

used: US consumer expenditure deflator

Within/shadow EMU

$$rx_t = rx_{t-1} * \frac{elrx_t}{elrx_{t-1}} \quad (11.10)$$

Exchanges rates can also be set to exogenous

Fiscal policy options in NiGEM

The budget rule ensures that the government stays solvent in the long run, i.e. that the deficit and debt stock return to sustainable levels in all scenarios. The budget rule adjusts the aggregate tax rate when the public debt ratio (*GBR*) deviates from its target (*GBRT*).

$$taxr_t = taxr_{t-1} * \left[\frac{0.01 * y_{t-1} * p_{yt-1} / 100 * (\beta * (gbrt_{t-1} - gbr_{t-1}))}{\pi} \right] \quad (11.11)$$

In simulation, the target is exogenous by default, so solvency is in place. The target can be endogenised for all or part of the scenario period to turn solvency off for a desired period of time.

References

- Anderton, R., and Barrell, R. (1995) 'The ERM and Structural Change in European Labour Markets: A study of 10 countries', *Weltwirtschaftliches Archiv*, Band 131, Heft 1.
- Ball, L. and Mankiw, N. G. (2002) 'The NAIRU in Theory and Practice', *Journal of Economic Perspectives*, Vol. 16, No. 4, pp. 115–136.
- Barrell, R., Hall, S. G. and Hurst, A. I. (2006) 'Evaluating Policy Feedback Rules using the joint density function of a stochastic model', *Economics Letters*, Vol. 93, No. 1, pp. 1-5.
- Barrell, R., and N. Pain (1997) 'Foreign Direct Investment, Technological Change, and Economic Growth Within Europe', *Economic Journal*, 107, pp. 1770-1776.
- Barrell, R., and Sefton, J. (1997) 'Fiscal Policy and the Maastricht Solvency Criteria', *Manchester School*, June