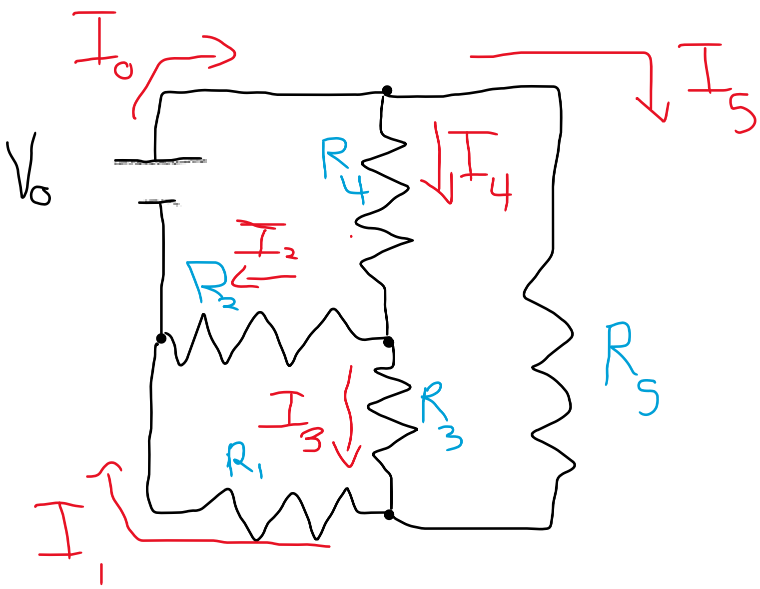

Challenge circuit: V_0 = 8.01 Volts

Resistance	V (experimental)	V (theoretical)	% error
R_1 = 72.6 kΩ	4.13 V		
R_2 = 51 kΩ	5.39 V		
R_3 = 23.5 kΩ	1.255 V		
R_4 = 16.26 kΩ	2.588 V		
R_5 = 1065 kΩ	3.83 V		

Use Kirchhoff's Laws to find the theoretical voltages and then calculate the percent error

Solution

Possible equations from junction rule:

$$I_0 = I_4 + I_5$$

$$I_0 = I_1 + I_2$$

$$I_2 + I_3 = I_4$$

$$I_1 = I_3 + I_5$$

Note that the current from the power supply never crosses a resistor until it branches, this means we don't actually need to calculate I_0, so there's no harm in elminating it.

$$I_1 + I_2 = I_4 + I_5 \tag{1}$$

And if we eliminate I_3 in the other 2 equations, we see that it gives us the same as equation 1. So we need one of the equations for I_3, and have to leave the other one out because it gives no new information. So our second equation can be this

$$I_1 = I_3 + I_5 \tag{2}$$

That gives 2 equations with 5 unknowns, so we need at least 3 loop rules to find the other 3 necessary equations.

The first loop will be the outer loop which includes the power supply, and resistors 1 and 5. That loop gives the equation

$$V_0 - R_5 I_5 - R_1 I_1 = 0 \tag{3}$$

The second loop can also use the power supply, and include resistors 2 and 4, giving the equation

$$V_0 - R_2 I_2 - R_4 I_4 = 0 \tag{4}$$

And for the last loop, we choose the loop over resistors 1, 2 and 3 which does not include the power supply.

$$R_2 I_2 - R_1 I_1 - R_3 I_3 = 0 \tag{5}$$

These 5 equations, after being appropriately rearranged, form a system of equations which can be solved using various methods from linear algebra.

Note that the R values which were measured are a bit different from the color code resistances. This means using the color code values will cause error in the theoretical voltages. However, if the math is done correctly, even if using the color code resistances, the resulting voltages and currents will still satisfy the equations above.

So let's rearrange those 5 equations and put them in a more natural form.

$$\begin{aligned}
I_1 + I_2 - I_4 - I_5 &= 0 \\
I_1 - I_3 - I_5 &= 0 \\
R_1 I_1 + R_5 I_5 &= V_0 \\
R_2 I_2 + R_4 I_4 &= V_0 \\
R_1 I_1 - R_2 I_2 + R_3 I_3 &= 0
\end{aligned}$$

For those familiar with linear algebra and matrix operations, this system forms the following matrix equation

$$\boldsymbol{R}_{mat}\boldsymbol{I} = \boldsymbol{V}$$

Where the column vector $\boldsymbol{I}$ contains as elements the currents I_1 through I_5 and the column vector $\boldsymbol{V}$ contains zeros except for elements 3 and 4 which are both V_0.

The matrix $\boldsymbol{R}_{mat}$ is a 5x5 matrix which can be combined with $\boldsymbol{V}$ and written as

$$\begin{matrix} 1 & 0 & -1 & 0 & -1 & & 0 \\ 1 & 1 & 0 & -1 & -1 & & 0 \\ R_1 & 0 & 0 & 0 & R_5 & & V_0 \\ 0 & R_2 & 0 & R_4 & 0 & & V_0 \\ R_1 & R_2 & R_3 & 0 & 0 & & 0 \end{matrix}$$

Notice that it is not necessary to write out the vector for the currents. Reducing this resulting 5x6 system via elementary row operations into a matrix where the entries on the left-hand side (the 5x5 matrix for $\boldsymbol{R}_{mat}$) contains 1s on the diagonal, and 0s off the diagonal, is one way to solve the problem. Reminder that valid elementary row operations are

1: Multiply a row by a constant

*Row -> c*Row*

2: Replace one row by adding it with another (leaving other row unchanged)

Row1 -> Row1 + Row2

3: A combination of operations 1 and 2

*Row1 -> Row1 + c*Row2*

The first step to solving this problem is to use row operation 1 on the rows which contain R values, which produces

$$\begin{matrix} 1 & 0 & -1 & 0 & -1 & & 0 \\ 1 & 1 & 0 & -1 & -1 & & 0 \\ 1 & 0 & 0 & 0 & \frac{R_5}{R_1} & & \frac{V_0}{R_1} \\ 0 & 1 & 0 & \frac{R_4}{R_2} & 0 & & \frac{V_0}{R_2} \\ 1 & \frac{R_2}{R_1} & \frac{R_3}{R_1} & 0 & 0 & & 0 \end{matrix}$$

At this point, it may be helpful to plug in numbers before proceeding. Be sure to watch unit errors. If V_0 is in units of volts, then the Rs need to be in units of ohms.

Using this method of solving the system, the objective is to first get 1s along the diagonal elements. Our first two rows both have 1s on the diagonal, but row 3 has a 0 there. It is easy to fix this by taking row 1, multiply it by -1, then adding it to row 3. This gives the following system

$$\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & \frac{R_5}{R_1}+1 & \frac{V_0}{R_1} \\ 0 & 1 & 0 & \frac{R_4}{R_2} & 0 & \frac{V_0}{R_2} \\ 1 & \frac{R_2}{R_1} & \frac{R_3}{R_1} & 0 & 0 & 0 \end{array}$$

Proceeding to use row operations will eventually produce a system which looks like

$$\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & a \\ 0 & 1 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & 0 & 0 & c \\ 0 & 0 & 0 & 1 & 0 & d \\ 0 & 0 & 0 & 0 & 1 & f \end{array}$$

This is a solution to the problem, with $I_1 = a$, $I_2 = b$, $I_3 = c$, etc…

Taking these values and multiplying by the appropriate R value gives the theoretical voltage across each resistor, e.g. $V_4 = d*R_4$.

This is a time-consuming problem, with lots of opportunities to make arithmetic or algebra errors. The solution above required a total of eighteen elementary row operations to reduce the system to the final form. Good luck!