Chapter 3
A Mathematical Model for Describing a System of Primary Units of Conceptual Level Used by Applied Intelligent Systems

Abstract The first section of this chapter formulates a problem to be solved both in Chaps 3 and 4: it is the problem of describing in a mathematical way the structured meanings of a broad spectrum both of sentences and discourses in natural language. The second section states a subproblem of this problem, it is the task of constructing a mathematical model describing a system of primary units of conceptual level and the information associated with such units and needed for joining the primary units with the aim of building semantic representations of arbitrarily complicated Natural Language texts. A solution to this task forms the main content of this chapter. From the mathematical standpoint, the proposed solution is a definition of a new class of formal objects called conceptual bases.

3.1 Global Task Statement

Let's formulate a problem to be solved in Chapters 3 and 4.

In the situation when the known formal methods of studying the semantics of natural language proved to be ineffective as regards solving many significant tasks of designing NLPSs, a number of researchers in diverse countries have pointed out the necessity to search for new mathematical ways of modeling NL-communication.

Habel [113] noted the importance of creating adequate mathematical foundations of computational linguistics and underlined the necessity to model the processes of NL-communication on the basis of formal methods and theories of cognitive science.

Fenstad and Lonning ([44], p. 70) posed the task of working out adequate formal methods for Computational Semantics – "a field of study which lies at the intersection of three disciplines: linguistics, logic, and computer science." Such methods should enable us, in particular, to establish the interrelations between pictorial data and semantic content of a document.

A.P. Ershov, the prominent Russian theoretician of programming, raised in [38] the problem of developing a formal model of the Russian language. It is very

V.A. Fomichov, *Semantics-Oriented Natural Language Processing*, IFSR International Series on Systems Science and Engineering 27, DOI 10.1007/978-0-387-72926-8_3,

interesting how close the ideas of Seuren [188] about the need of ecological approaches to the formal study of NL are to the following words of A. P. Ershov published in the same 1986: “We want as deeply as it is possible to get to know the nature of language and, in particular, of Russian language. A model of Russian language should become one of manifestations of this knowledge. It is to be a formal system which should be adequate and equal-voluminous to the living organism of language, but in the same time it should be anatomically prepared, decomposed, accessible for the observation, study, and modification” ([38], p. 12).

Having analyzed the state of the researches on formalizing semantics of NL, Peregrin [168] drew the conclusion that the existing logical systems didn’t allow for formalizing all the aspects of NL-semantics being important for the design of NLPSs. He wrote that we couldn’t use the existing form of logic as such molding form that it is necessary to squeeze natural language into this form at any cost. That is why, according to this scholar, in order to create an adequate formal theory of NL-semantics, it is necessary to carry out a full-fledged linguistic analysis of all components of NL and to establish the connections between the logical approaches to the formalization of NL-semantics and linguistic models of meaning.

In essence, the same conclusion but considerably earlier, in the beginning of the 1980s (see [52, 55, 56]), was drawn by the author of this monograph. This conclusion became the starting point for the elaboration of the task statement below.

It seems that the borders of mathematical logic are too narrow for providing an adequate framework for computer-oriented formalization of NL-semantics. That is why the problem of creating mathematical foundations of designing intellectually powerful NLPSs requires not only an expansion of the first-order logic but rather the development of new mathematical systems being compatible with first-order logic and allowing for formalizing the logic of employing NL by computer-intelligent systems.

Taking this into account, let’s pose the task of developing a mathematical model of a new kind for describing structured meanings of NL sentences and complicated discourses. Such a model is to satisfy two groups of requirements: the first group consists of several very general requirements to *the form* of the model, and the second group consists of numerous requirements concerning the reflection (on semantic level) of concrete phenomena manifested in NL.

The first group of requirements to a model to be constructed is as follows:

1. A model is to define a new class of formal objects to be called *conceptual bases* and destined for (a) explicitly indicating the primary units of conceptual level used by an applied intelligent system; (b) describing in a formal way the information associated with primary units of conceptual level and employed for joining primary and compound conceptual units into complex structures interpreted as semantic representations of NL-texts.
2. For each conceptual basis B, a model is to determine a formal language (in other words, a set of formulas) $Ls(B)$ in such a way that the class of formal languages $\{ Ls(B) \mid B$ is a conceptual basis $\}$ is convenient for building semantic representations both of separate sentences and complicated discourses in NL.

3. Let $Semunits(B)$ be the set of primary units of conceptual level defined by the conceptual basis B. Then a model is to determine the set of formulas $Ls(B)$ simultaneously with a finite collection of the rules $R_1, \ldots, R_k$, where $k > 1$, allowing for constructing semantic representations both of sentences and complicated discourses step by step from the elements of the set $Semunits(B)$ and several special symbols.

For elaborating *the second group of requirements* to a model, a systemic analysis of the structure of natural language expressions and expressions of some artificial languages has been carried out; it has been aimed at distinguishing lexical and structured peculiarities of (a) the texts in Russian, English, German, and French languages; (b) a number of artificial languages used for constructing semantic representations of NL-texts by linguistic processors; (c) the expressions of artificial knowledge representation languages, in particular, of terminological (or KL-ONE-like) knowledge representation languages.

There are some important aspects of formalizing NL-semantics which were underestimated or ignored until recently by the dominant part of researchers. First, this applies to formal investigation of the structured meanings of (a) narrative texts including descriptions of sets; (b) the discourses with references to the meaning of sentences and larger fragments of texts; (c) the phrases where logical connectives "and," "or" are used in non-traditional ways and join not the fragments expressing assertions but the descriptions of objects, sets, concepts; (d) phrases with attributive clauses; (e) phrases with the lexical units "a concept," "a notion."

Besides, the major part of the most popular approaches to the mathematical study of NL-semantics (mentioned in Chap. 2) practically doesn't take into account the role of knowledge about the world in NL comprehension and generation and hence does not study the problem of formal describing knowledge fragments (definitions of concepts, etc.).

It should be added that NL-texts have authors, may be published in one or another source, may be inputted from one or another terminal, etc. The information about these external ties of a text (or, in other terms, about its metadata) may be important for its conceptual interpretation. That is why it is expedient to consider a text as a structured item having a surface structure T, a set of meanings *Senses* (most often, *Senses* consists of one meaning) corresponding to T, and some values $V_1, \ldots, V_n$ denoting the author (authors) of T, the date of writing (or of pronouncing) T, indicating the new information in T, etc. The main popular approaches to the mathematical study of NL-semantics provide no formal means to represent texts as structured items of the kind.

We'll proceed from the hypothesis that there is only one mental level for representing meanings of NL-expressions (it may be called the conceptual level) but not the semantic and the conceptual levels. This hypothesis is advocated by a number of scientists, in particular, by Meyer [157].

Let's demand that the formal means of our model allow us:

1. To build the designations of structured meanings (SMs) of both phrases expressing assertions and of narrative texts; such designations are called usually *semantic representations (SRs)* of NL-expressions.

2. To build and to distinguish the designations of items corresponding to (a) objects, situations, processes of the real world and (b) the notions (or concepts) qualifying these objects, situations, processes.
3. To build and to distinguish the designations of (a) objects and sets of objects, (b) concepts and sets of concepts, (c) SRs of texts and sets of SRs of texts.
4. To distinguish in a formal manner the concepts qualifying the objects and the concepts qualifying the sets of objects of the same kind.
5. To build compound representations of the notions (concepts), e.g., to construct formulas reflecting the surface semantic structure of NL-expressions such as "a person graduated from the Stanford University and being a biologist or a chemist".
6. To construct the explanations of more general concepts by means of less general concepts; in particular, to build the strings of the form $(a \equiv Des(b))$, where a designates a concept to be explained and $Des(b)$ designates a description of a certain concretization of the known concept b.
7. To build the designations of ordered n-tuples of objects ($n > 1$).
8. To construct: (a) formal analogues of complicated designations of the sets such as the expression "this group consisting of 12 tourists being biologists or chemists," (b) the designations of the sets of n-tuples $(n > 1)$, (c) the designations of the sets consisting of sets, etc.
9. To describe set-theoretical relationships.
10. To build the designations of SMs of phrases containing, in particular: (a) the words and word combinations "arbitrary," "every," "a certain," "some," "all," "many," etc.; (b) the expressions formed by means of applying the connectives "and", "or" to the designations of things, events, concepts, sets; (c) the expressions where the connective "not" is located just before a designation of a thing, event, etc.; (d) indirect speech; (e) the participle constructions and the attributive clauses; (f) the word combinations "a concept", "a notion".
11. To build the designations of SMs of discourses with references to the mentioned objects.
12. To explicitly indicate in SRs of discourses causal and time relationships between described situations (events).
13. To describe SMs of discourses with references to the meanings of phrases and larger fragments of a considered text.
14. To express the assertions about the identity of two entities.
15. To build formal analogues of formulas of the first-order logic with the existential and/or universal quantifiers.
16. To consider nontraditional functions (and other nontraditional relations of the kind) with arguments and/or values being: (a) sets of things, situations (events); (b) sets of concepts; (c) sets of SRs of texts.
17. To build conceptual representations of texts as informational objects reflecting not only the meaning but also the values of external characteristics of a text: the author (authors), the date, the application domains of the stated results, etc.

This task statement develops the task statements from [52, 54–56, 62, 65, 70] and coincides with the task statements in [81, 82, 85].

3.2 Local Task Statement

The analysis shows that the first step in creating a broadly applicable and domain-independent mathematical approach to represent structured meanings of NL-texts is the development of a formal model enumerating primary (i.e. not compound) units of conceptual level employed by an applied intelligent system and, besides, describing the information associated with such units and needed for combining these units into the compound units reflecting structured meanings of arbitrarily complicated NL-texts.

For constructing a formal model possessing the indicated property, first an analysis of the lexical units from Russian, English, German, and French languages was fulfilled. Second, the collections of primary informational units used in artificial knowledge representation languages were studied, in particular, the collections of units used in terminological (or KL-ONE-like) knowledge representation languages.

Proceeding from the fulfilled analysis, let's state the task of developing such a domain-independent mathematical model for describing (a) a system of primary units of conceptual level employed by an NLPS and (b) the information of semantic character associated with these units that, first, this model constructively takes into account the existence of the following phenomena of natural language:

1. A hierarchy of notions is defined in the set of all notions; the top elements in this hierarchy are most general notions. For instance, the notion "a physical object" is a concretization of the notion "a space object".
2. Very often, the same thing can be qualified with the help of several notions, where none of these notions is a particular case (a concretization) of another notion from this collection. It is possible to metaphorically say that such notions are "the coordinates of a thing" on different "semantic axes." For example, every person is a physical object being able to move in the space. On the other hand, every person is an intelligent system, because people can solve problems, read, compose verses, etc.
3. The English language contains such words and word groups as "a certain", "definite", "every", "each", "any", "arbitrary", "all", "some", "a few", "almost all", "the majority" and some other words and word combinations that these words and word groups are always combined in sentences with the words and word combinations designating the notions. For instance, we can construct the expressions "every person", "a certain person", "any car", "arbitrary car", "all people", "several books", etc. The Russian, German, and French languages contain similar words and word combinations.

Second, the model is to allow for distinguishing in a formal way the designations of the primary units of conceptual level corresponding to

- the objects, situations, processes of the real world and the notions (the concepts) qualifying these objects, situations, processes;
- the objects and the sets of objects;
- the notions qualifying objects and the notions qualifying the sets of objects of the same kind ("a ship" and "a squadron", etc.);

- the sets and the finite sequences (or the ordered n-tuples, where $n > 1$) of various entities.

Third, the model is to take into account that the set of primary units of conceptual level includes

- the units corresponding to the logical connectives "not", "and", "or" and to the logical universal and existential quantifiers;
- the names of nontraditional functions with the arguments and/or values being (a) the sets of things, situations; (b) notions; (c) the sets of notions; (d) semantic representations of NL-texts; (e) the sets of semantic representations of NL-texts;
- the unit corresponding to *the words* "a notion", "a concept" and being different from the conceptual unit "a concept"; the former unit contributes, in particular, to forming the meaning of the expression "an important notion used in physics, chemistry, and biology."

A mathematical model for describing a system of primary units of conceptual level is constructed in Sect. 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, and 3.9.

3.3 Basic Denotations and Auxiliary Definitions

3.3.1 General Mathematical Denotations

$x \in Y$ the element x belongs to the set Y

$x \neg \in Y$ the element x doesn't belong to the set Y

$X \subset Y$ the set X is a subset of the set Y

$Y \cup Z$ the union of the sets Y and Z

$Y \cap Z$ the intersection of the sets Y and Z

$Y \setminus Z$ the set-theoretical difference of the sets Y and Z, that is the collection of all such x from the set Y that x doesn't belong to the set Z

$Z_1 \times \ldots \times Z_n$ the Cartesian product of the sets $Z_1, \ldots, Z_n$, where $n > 1$

$\emptyset$ empty set

$\exists$ existential quantifier

$\forall$ universal quantifier

$\Rightarrow$ implies

$\Leftrightarrow$ if and only if

3.3.2 The Preliminary Definitions from the Theory of Formal Grammars and Languages

Definition 3.1. An arbitrary finite set of symbols is called *alphabet.* If A is an arbitrary alphabet, then A^+ is the set of all sequences $d_1, \ldots, d_n$, where $n \geq 1$, for $i = 1, \ldots, n$, $d_i \in A$.

Usually one writes $d_1 \dots d_n$ instead of $d_1, \dots, d_n$. For example, if $A = \{0, 1\}$, then the sequences of symbols 011, 11011, 0, $1 \in A^+$.

Definition 3.2. If A is an arbitrary alphabet, the elements of the set A^+ are called the *non-empty strings in the alphabet* A (or over the alphabet A).

Definition 3.3. Let A be an arbitrary alphabet, d be a symbol from A then $d^1 = d$; for $n > 1$, $d^n = d \dots d$ (n times).

Definition 3.4. Let $A^* = A^+ \cup \{e\}$, where A is an arbitrary alphabet, e is the empty string. Then the elements of the set A^* are called *the strings in the alphabet* A (or *over the alphabet* A).

Definition 3.5. For each $t \in A^*$, where A is an arbitrary alphabet, the value of the function $length(t)$ is defined as follows: (1) $length(e) = 0$; if $t = d_1 \dots d_n$, $n \geq 1$, for $i = 1, \dots, n$ the symbol d_i belongs to A, then $length(t) = n$.

Definition 3.6. Let A be an arbitrary alphabet. Then *a formal language in the alphabet* A (or over the alphabet A) is an arbitrary subset L of the set A^*, i.e. $L \subseteq A^*$.

Example 1. If $A = \{0, 1\}$, $L_1 = \{0\}$, $L_2 = \{e\}$, $L_3 = \{0^{2k}1^{2k} \mid k \geq 1\}$, then L_1, L_2, L_3 are the formal languages in the alphabet A (or over the alphabet A).

3.3.3 The Used Definitions from the Theory of Algebraic Systems

Definition 3.7. Let $n \geq 1$, Z be an arbitrary non-empty set. Then the Cartesian n-degree of the set Z is called (and denoted by Z^n) the set Z in case $n = 1$ and the set of all ordered n-tuples of the form $(x_1, x_2, \dots, x_n)$, where $x_1, x_2, \dots, x_n$ are the elements of the set Z in case $n > 1$.

Definition 3.8. Let $n \geq 1$, Z be an arbitrary non-empty set. Then an n-ary relation on the set Z is an arbitrary subset R of the set Z^n. In case $n = 1$ one says about *an unary relation* and in case $n = 2$ we have *a binary relation*. The unary relations are interpreted as the distinguished subsets of the considered set Z.

Example 2. Let Z_1 be the set of all integers, and *Odd* be the subset of all even numbers. Then *Odd* is an unary relation on Z_1. Let *Less* be the set of all ordered pairs of the form (x, y), where x, y are the arbitrary elements of Z_1, and $x < y$. Then *Less* is a binary relation on the set Z_1.

Very often one uses a shorter denotation bRc instead of the denotation $(b, c) \in R$, where R is a binary relation on the arbitrary set Z.

Definition 3.9. Let Z be an arbitrary non-empty set, R be a binary relation on Z. Then

- if for arbitrary $a \in Z$, $(a, a) \in R$, then R is *a reflexive relation* ;

- if for arbitrary $a \in Z$, $(a, a) \neg \in R$, then R is *an antireflexive relation* ;
- if for every $a, b, c \in Z$, it follows from $(a, b) \in R$, $(b, c) \in R$ that $(a, c) \in R$, then R is *a transitive relation* ;
- if for every $a, b \in Z$, it follows from $(a, b) \in R$ that (b, a) belongs to R, then R is *a symmetric relation* ;
- if for every $a, b \in Z$, it follows from $(a, b) \in R$ and $a \neq b$ that $(b, a) \neg \in R$, then R is an *antisymmetric relation* ;
- if R is a reflexive, transitive, and antisymmetric relation on Z, then R is called *a partial order on* Z [140] .

Example 3. The binary relation *Less* from the previous example is antireflexive, transitive, and antisymmetric relation on $Z1$.

Example 4. Let Z_1 be the set of all integers, and $Eqless$ be the set of all ordered pairs of the form (x, y), where x, y are arbitrary elements of Z_1, and the number x is either equal to the number y or less than y. Then $Eqless$ is a binary relation on the set Z_1. This relation is reflexive, transitive, and antisymmetric. Thus, the relation $Eqless$ is a partial order on Z_1.

Example 5. Let Z_2 be the set of all notions denoting the transport means, and *Genrel* is the the set of all ordered pairs of the form (x, y), where x, y are arbitrary elements of the set Z_2, and the notion x either coincides with the notion y or is a generalization of y. For example, the notion *a ship* is a generalization of the notion *an ice-breaker*; hence the pair (*ship*, *ice-breaker*) belongs to the set *Genrel*. Obviously, *Genrel* is a binary relation on the set Z_2. This relation is reflexive, transitive, and antisymmetric. That is why the relation *Genrel* is a partial order on Z_2.

3.4 The Basic Ideas of the Definition of a Sort System

Let us start to solve the posed task. Assuming that it is necessary to construct a formal description of an application domain, we'll consider the first steps in this direction.

Step 1. Let's consider a finite set of symbols denoting the most general notions of a selected domain: a space object, a physical object, an intelligent system, an organization, a natural number, a situation, an event (i.e. a dynamic situation), etc. Let's agree that every such notion qualifies an entity that is not being regarded as either a finite sequence (a tuple) of some other entities or as a set consisting of some other entities. Denote this set of symbols as *St* and call the elements of this set *sorts*.

Step 2. Let's distinguish in the set of sorts *St* a symbol to be associated with the semantic representations (SRs) of NL-texts either expressing the separate assertions or being the narrative texts. Denote this sort as P and will call it *the sort* "a meaning of proposition ." For instance, the string *prop* may play the role of the distinguished sort P for some applications. A part of the formulas of a new kind considered in this monograph can be represented in the form $F \,\&\, t$, where F is an SR of an NL-expression, and t is a string qualifying this expression. Then, if $t = P$, then the formula F is interpreted as an SR of a simple or compound assertion (or a statement, a proposition). In particular, the formula

$$(Weight(certain\,block1 : x1) \equiv 4/tonne)\,\&\,prop$$

can be regarded as a formula of the kind.

Step 3. A hierarchy of the concepts on the set of sorts St with the help of a binary relation Gen on St is defined, that is a certain subset Gen of the set $St \times St$ is selected. For instance, the relationships $(integer, natural)$, $(real, integer)$, $(phys.object, dynamic.phys.object)$, $(space.object, phys.object) \in Gen$ may take place.

Step 4. Many objects can be characterized from different standpoints; metaphorically speaking, these objects possess "the coordinates" on different "semantic axes."

Example 1. Every person is both a dynamic physical object (we can run, spring, etc.) and an intelligent system (we can read and write, a number of people are able to solve mathematical problems, to compose poems and music). That is why, metaphorically speaking, each person has the coordinate $dyn.phys.ob$ on one semantic axis and the coordinate $intel.system$ on another semantic axis (Fig. 3.1).

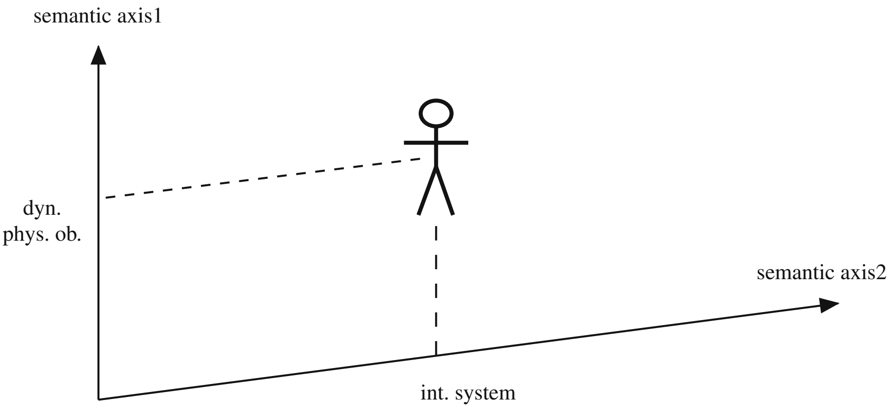

Fig. 3.1 "Semantic coordinates" of arbitrary person

Example 2. One is able to drive or to go to a certain university, that is why every university has the "semantic coordinate" $space.object$. For each university, there is a person who is the head (the rector) of this university, so the universities possess the "semantic coordinate" $organization$. Finally, a university is able to elaborate certain technology or certain device, hence it appears to be reasonable to believe that the universities have the "semantic coordinate" $intelligent.system$.

Taking into account these considerations, let's introduce a binary *tolerance relation* Tol on the set St. The interpretation of this relation is as follows: if $(s, u) \in Tol \subset St \times St$, then in the considered domain such entity x exists that it is possible to associate with x the sort s as one "semantic coordinate" and the sort u as the second

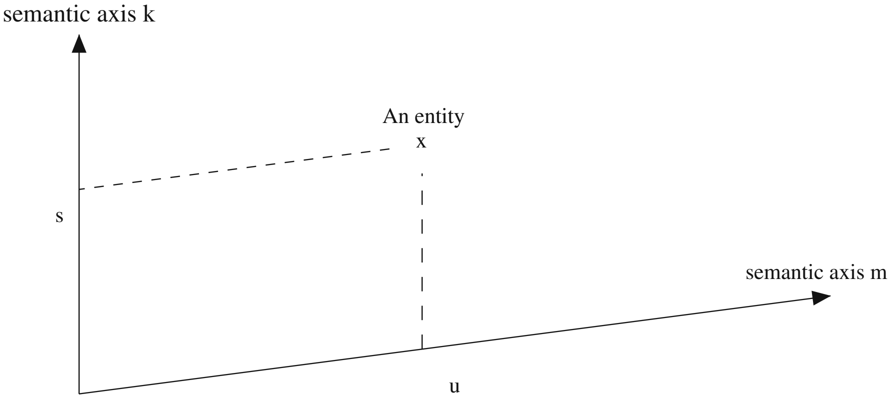

Fig. 3.2 Illustration of the metaphor of semantic axes: the case of two "semantic coordinates" of an entity x

"semantic coordinate"; besides, the sorts s and u are not comparable for the relation Gen reflecting a hierarchy of the most general concepts (see Fig. 3.2).

For instance, the sets St and Tol can be defined in such a way that Tol includes the ordered pairs

$$(space.object, organization), (space.object, intelligent.system),$$

$$(organization, intelligent.system), (organization, space.object),$$

$$(intelligent.system, space.object), (intelligent.system, organization).$$

The considered organization of the relation Tol implies the following properties: (1) $\forall u \in St$, $(u, u) \neg \in Tol$, i.e. Tol is an antireflexive relation; (2) $\forall u, t \in St$, it follows from $(u, t) \in Tol$ that $(t, u) \in Tol$, i.e. Tol is a symmetric relation.

A sort system will be defined below as an arbitrary four-tuple S of the form (St, P, Gen, Tol) with the components satisfying certain conditions.

3.5 The Formal Definition of a Sort System

Definition 3.10. A sort system is an arbitrary four-tuple S of the form

$$(St, P, Gen, Tol),$$

where St is an arbitrary finite set of symbols, $P \in St$, Gen is a non-empty binary relation on St being a partial order on St, Tol is a binary relation on St being antireflexive and symmetric, and the following conditions are satisfied:

1. St doesn't include the symbols $\uparrow$, $\{$, $\}$, (,), $[\uparrow entity]$, $[\uparrow concept]$, $[\uparrow object]$, $[entity]$, $[concept]$, $[object]$;

2. If $Concr(P)$ is the set of all such z from the set St that $(P, z) \in Gen$, then $St \setminus Concr(P) \neq \emptyset$, and for every $u \in St \setminus Concr(P)$ and every $w \in Concr(P)$, the sorts u and w are incomparable both for relation Gen and for the relation Tol;
3. for each $t, u \in St$, it follows from $(t, u) \in Gen$ or $(u, t) \in Gen$ that t, u are incomparable for the relation Tol;
4. for each $t1, u1 \in St$ and $t2, u2 \in St$, it follows from $(t1, u1) \in Tol$, $(t2, t1) \in Gen$, $(u2, u1) \in Gen$, that $(t2, u2) \in Tol$.

The elements of the set St are called sorts, P is called the sort "a meaning of proposition," the binary relations $Gen \subset St \times St$ and $Tol \subset St \times St$ are called respectively *the generality relation* and *the tolerance relation*. If $t, u \in St$, $(t, u) \in Gen$, then we often use an equivalent notation $t \rightarrow u$ and say that t *is a generalization of* u, and u *is a concretization of* t. If $(t, u) \in Tol$, we use the denotation $t \perp u$ and say that the sort t *is tolerant to the sort* u.

The symbols $\uparrow$, $\{$, $\}$, (,), $[\uparrow entity]$, $[\uparrow object]$, $[\uparrow concept]$, $[entity]$, $[object]$, $[concept]$ play special roles in constructing (from the sorts and these symbols) the strings called types and being the classifiers of the entities considered in the selected application domain.

The requirement 4 in the definition of a sort system is illustrated by Fig. 3.3 and 3.4. Suppose that the following situation takes place:

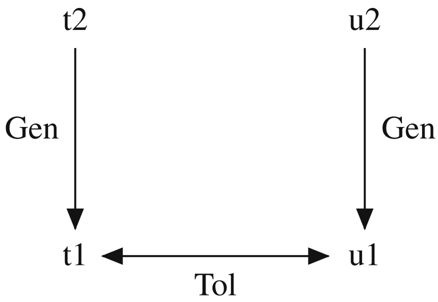

Fig. 3.3 A visual representation of the presupposition in the requirement 4 of the definition of a sort system

Then this situation implies the situation reflected in Fig. 3.4.

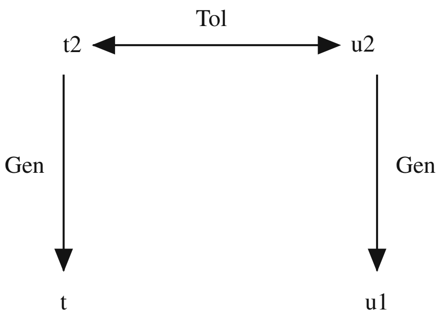

Fig. 3.4 A visual representation of the implication in the requirement 4 of the definition of a sort system

Example 1. Suppose that the sort *ints* (intelligent system) and the sort *dyn.phys.ob* (dynamic physical object) are associated by the tolerance relation *Tol*, i.e.

$$(ints,\ dyn.phys.ob) \in Tol,$$

and the sort *dyn.phys.ob* is a concretization of the sort *phys.ob* (physical object), i.e.,

$$(phys.ob,\ dyn.phys.ob) \in Gen.$$

Due to reflexivity of the generality relation *Gen*,

$$(ints,\ ints) \in Gen.$$

Using the denotation from the item 4 in the definition of a sort system, we have

$$t1 = ints,\ t2 = ints,\ (t2, t1) \in Gen,$$

$$u1 = dyn.phys.ob,\ u2 = phys.ob,\ (u2, u1) \in Gen.$$

This situation is illustrated by Fig. 3.5:

$$\begin{tikzcd}
\text{ints} \arrow[d, "\text{Gen}"'] & & \text{phys.ob} \arrow[d, "\text{Gen}"] \\
\text{ints} \arrow[rr, leftrightarrow, "\text{Tol}"'] & & \text{dyn.phys.ob}
\end{tikzcd}$$

Fig. 3.5 A particular case of the presupposition in the requirement 4 of the definition of a sort system

Then, according to the requirement 4 in the definition above, $(t2, u2) \in Tol$ (see Fig. 3.6), that is,

$$(ints,\ phys.ob) \in Tol.$$

$$\begin{tikzcd}
\text{ints} \arrow[rr, leftrightarrow, "\text{Tol}"] \arrow[d, "\text{Gen}"'] & & \text{phys.ob} \arrow[d, "\text{Gen}"] \\
\text{ints} & & \text{dyn.phys.ob}
\end{tikzcd}$$

Fig. 3.6 A particular case of the situation mentioned in the implication in the requirement 4 of the definition of a sort system

Example 2. Let's construct a sort system S_0. Let

$$St_0 = \{nat, int, real, weight.value, space.ob, phys.ob, dyn.phys.ob,$$

$$imag.ob, ints, org, mom, sit, event, prop\}.$$

The elements of St_0 designate the notions (the concepts) and are interpreted as follows: *nat* – "natural number," *int* – "integer," *real* – "real number," *weight.value* – "value of weight," *space.ob* – "space object," *phys.ob* – "physical object," *dyn.phys.ob* – "dynamic physical object," *imag.ob* – "imaginary space object," *ints* – "intelligent system," *org* – "organization," *mom* – "moment," *sit* – "situation," *event* – "event" ("dynamic situation"), *prop* – "semantic representation of an assertion or of a narrative text."

Let $P_0 = prop$, the sets Ge_1, Ge_2, Gen_0, T_1, T_2 be defined as follows:

$$Ge_1 = \{(u, u) \mid u \in St_0\},$$

$$Ge_2 = \{(int, nat), (real, nat), (real, int),$$

$$(space.ob, phys.ob), (space.ob, imag.ob), (phys.ob, dyn.phys.ob),$$

$$(space.ob, dyn.phys.ob), (sit, event)\},$$

$$Gen_0 = Ge_1 \cup Ge_2,$$

$$T_1 = \{(ints, dyn.phys.ob), (ints, phys.ob), (ints, space.ob), (org, ints),$$

$$(org, phys.ob), (org, space.ob)\},$$

$$T_2 = \{(u, s) \mid (s, u) \in T_1\},$$

$$Tol_0 = T_1 \cup T_2.$$

Let S_0 be the four-tuple $(St_0, prop, Gen_0, Tol_0)$. Then it is easy to verify that S_0 is a sort system, and the sort *prop* is its distinguished sort "a meaning of proposition."

With respect to the definition of the set Gen_0, the following relationships take place:

$$real \to nat, int \to nat, space.ob \to phys.ob, space.ob \to imag.ob,$$

$$phys.ob \to dyn.phys.ob, ints \perp phys.ob, ints \perp dyn.phys.ob,$$

$$ints \perp org, phys.ob \perp ints, dyn.phys.ob \perp ints.$$

3.6 Types Generated by a Sort System

Let us define for any sort system S a set of strings $Tp(S)$ whose elements are called *the types of the system S* and are interpreted as the characteristics of the entities which are considered while reasoning in a selected domain. Let's agree that if in a reasoning about an entity z it is important that z is not a concept (a notion), we say that z is an object.

Suppose that the strings $[\uparrow entity]$, $[\uparrow concept]$, $[\uparrow object]$ are associated with the terms "an entity," "a concept" ("a notion"), "an object," respectively, or, in other words, these strings are the types of semantic items corresponding to the expressions "an entity," "a concept" ("a notion"), "an object."

For formalizing a considered domain, let's agree to proceed from the following recommendations. If the nature of an entity z considered in a reasoning is of no importance, we associate with z the type $[entity]$ in the course of reasoning. If all that is important concerning z is that z is an object, we associate with z the type $[object]$. If, to the contrary, all that is important as concerns z is that z is a concept (a notion), we associate with z the type $[concept]$. The purpose of introducing the types $[entity]$, $[concept]$, $[object]$ can be explained also by means of the following examples.

Let E_1 and E_2 be, respectively, the expressions "the first entity mentioned on page 12 of the issue of the newspaper 'The Moscow Times' published on October 1, 1994" and "the first object mentioned on page 12 of the issue of 'The Moscow Times' published on October 1, 1994." Then we may associate the types $[entity]$ and $[object]$ with the entities referred in E_1 and E_2 respectively in case we haven't read page 12 of the indicated issue.

However, after reading page 12 we'll get to know that the first entity and the first object mentioned on this page is the city Madrid. Hence, we may associate now with the mentioned entity (object) a more informative type *popul.area* (a populated area).

Let E_3 be the expression "the notion with the mark AC060 defined in the Longman Dictionary of Scientific Usage (Moscow, Russky Yazyk Publishers, 1989)." Not seeing this dictionary, we may associate with the notion mentioned in E3 only the type $[concept]$. But after finding the definition with the mark AC060, we get to know that it is the definition of the notion "a tube" (a hollow cylinder with its length much greater than its diameter). Hence we may associate with the notion mentioned in E_3 a more informative type $\uparrow phys.ob$ (designating the notion "a physical object").

Let's consider the strings

$$[\uparrow entity], [\uparrow concept], [\uparrow object], [entity], [concept], [object]$$

as symbols in the next definitions.

Definition 3.11. Let S be a sort system of the form (St, P, Gen, Tol), and

$$Spectp = \{[\uparrow entity], [\uparrow concept], [\uparrow object]\},$$

$$Toptp = \{[entity], [concept], [object]\}.$$

Then the set of types $Tp(S)$ is the least set M satisfying the following conditions:

1. $Spectp \cup Toptp \cup St \cup \{\uparrow s \mid s \in St\} \subset M$. The elements of the sets $Spectp$ and $Toptp$ are called *special types* and *top types* respectively.
2. If $t \in M \setminus Spectp$, then the string of the form $\{t\}$ belongs to M.
3. If $n > 1$, for $i = 1, \ldots, n$, $t_i \in M \setminus Spectp$, then the string of the form $(t_1, \ldots, t_n)$ belongs to M.
4. If $t \in M$ and t has the beginning $\{$ or $($, then the string $\uparrow t$ belongs to M.

Definition 3.12. If S is a sort system, then

$$Mtp(S) = Tp(S) \setminus Spectp;$$

the elements of the set $Mtp(S)$ are called *the main types* .

Let's formulate the principles of establishing the correspondence between the entities considered in a domain with a sort system S of the form (St, P, Gen, Tol) and the types from the set $Mtp(S)$.

The types of notions (or concepts), as distinct from the types of objects, have the beginning $\uparrow$. So if a notion is denoted by a string s from St, we associate with this notion the type $\uparrow s$.

The type $\{t\}$ corresponds to any set of entities of type t. If $x_1, \ldots, x_n$ are the entities of types $t_1, \ldots, t_n$, then the type $(t_1, \ldots, t_n)$ is assigned to the n-tuple $(x_1, \ldots, x_n)$.

Example 1. We may assign the types from $Mtp(S_0)$ to some concepts and objects (including relations and other sets) with the help of the following table:

ENTITY	TYPE
The notion "a set"	$\uparrow \{[entity]\}$
The notion "a set of objects"	$\uparrow \{[object]\}$
The notion "a set of notions"	$\uparrow \{[concept]\}$
The notion "a person"	$\uparrow ints * dyn.phys.ob$
Tom Soyer	$ints * dyn.phys.ob$
The concept "an Editorial Board"	$\uparrow \{ints * dyn.phys.ob\}$
The Editorial Board of "Informatica" (Slovenia)	$\{ints * dyn.phys.ob\}$
The notion "a pair of integers"	$\uparrow (int, int)$
The pair (12, 144)	(int, int)

We can also associate with the relation "Less" on the set of integers the type $\{(int, int)\}$, with the relation "To belong to a set" the type

$$\{([entity], \{[entity]\})\},$$

with the relation "An object Y is qualified by a notion C" the type

$$\{([object], [concept])\},$$

and with the relation "A notion D is a generalization of a notion C" the type

$$\{([concept], [concept])\}.$$

3.7 The Concretization Relation on the Set of Types

The purpose of this section is to determine a transitive binary relation $\vdash$ on the set $Tp(S)$, where S is an arbitrary sort system; this relation will be called *the concretization relation* on the set $Tp(S)$. The basic ideas of introducing this relation are stated in the first subsection of this section; a formal definition of the concretization relation can be found in the second subsection.

It is worthwhile to note that the second subsection contains a rather tedious series of definitions. That is why in case you are reading this book not with the aim of modifying the described formal means but in order to apply them to the elaboration of semantic informational technologies, it is recommended to skip the second subsection of this section while reading this chapter for the first time.

3.7.1 Basic Ideas

Suppose that $S = (St, P, Gen, Tol)$ is an arbitrary sort system. The *first, simplest requirement* to the relation $\vdash$ on the set of types $Tp(S)$ is that the relation $\vdash$ coincides on the set of sorts St with the generality relation Gen.

Hence, for instance, if $phys.ob$ and $dyn.phys.ob$ are the sorts "a physical object" and "a dynamic physical object" and $(phys.ob, dyn.phys.ob) \in Gen$ (the equivalent denotation is $phys.ob \rightarrow dyn.phys.ob$), then $phys.ob \vdash dyn.phys.ob$.

The second requirement (also very simple) is as follows: Each of the basic types $[concept]$, $[object]$ is a concretization of the basic type $[entity]$, that is

$$[entity] \vdash [concept],\ [entity] \vdash [object].$$

The concretizations of the type $[concept]$ are to be, in particular, the types $\uparrow phys.ob$, $\uparrow dyn.phys.ob$, $\uparrow ints * dyn.phys.ob$, where $ints$ is the sort "intelligent system." In general, the types with the beginning $\uparrow$ (they are interpreted as the types of notions) are to be the concretizations of the type $[concept]$.

The concretizations of the type $[object]$ are to be, in particular, the types

$$phys.ob,\ dyn.phys.ob,\ ints * dyn.phys.ob,\ \{ints * dyn.phys.ob\},$$

$$\{(ints * dyn.phys.ob,\ ints * dyn.phys.ob)\}.$$

Taking this into account, the following relationships are to take place:

$$[concept] \vdash \uparrow ints,\ [concept] \vdash \uparrow ints * dyn.phys.ob,$$

$$[concept] \vdash \uparrow \{ints * dyn.phys.ob\},$$

$$[object] \vdash phys.ob,\ [object] \vdash dyn.phys.ob,$$

$$[object] \vdash ints * dyn.phys.ob,$$

$$[object] \vdash (real, real),\ [object] \vdash \{(real, real)\}.$$

Consider now a more complex requirement to the relation $\vdash$. Let $n > 1$, $C_1, \ldots, C_n$ be some classes of entities, and R be a relation on the Cartesian product $C_1 \times \ldots \times C_n$, that is, let R be a set consisting of some n-tuples with the elements from the sets $C_1, \ldots, C_n$ respectively.

If $n = 2$, one says that R is a relation from C_1 to C_2. If $C_1 = \ldots = C_n$, the set R is called an n-ary relation on the set C_1. If in the latter case $n = 2$, R is called *a binary relation on the set* C_1.

Suppose that for every $k = 1, \ldots, n$ it is possible to associate with every entity $z_k \in C_k$ a certain type $t_k \in Tp(S)$. Then we'll believe that semantic restrictions of the attributes of the relation R are given by the type $\{\,(t_1, \ldots, t_n)\,\}$.

Example 1. Let U be the class of all real numbers, and $R1$ be the binary relation "Less" on U. Then the semantic restrictions of the attributes of $R1$ can be expressed by the type $\{\,(real, real)\,\}$.

Example 2. Let C_1 be the class of all physical objects having a definite, stationary shape, and C_2 be the class of all values of the distance in the metrical measurement system. Then the function "The diameter of a physical object" can be defined as follows: if $X \in C_1$, then the diameter of X is the maximal length of a line connecting some two points of the physical object X. We can interpret this function as a relation *Diameter* from C_1 to C_2.

Then the semantic requirements to the attributes of *Diameter* can be expressed by the type $\{\,(phys.ob, length.value)\,\}$ in case the considered set of sorts St contains the sorts *phys.ob* and *length.value* denoting the notions "a physical object" and "the value of length."

Let's continue to consider the idea of introducing the concretization relation on a set of types. Suppose that for $k = 1, \ldots, n$, we've distinguished a subclass D_k in the class C_k, and the following condition is satisfied: it is possible to associate with every entity $Z \in D_k$ not only a type t_k but also a type u_k conveying more detailed information about the entity Z. Then we would like to define a binary relation $\vdash$ on the set of types $Tp(S)$ in such a way that the relationship $t_k \vdash u_k$ takes place.

Example 3. Let's expand the previous example. Suppose that D_1 is the set of all dynamic physical objects, and the type *dyn.phys.ob* is associated with every object from the set D_1. Let E_1 be the set of all people, and the type $ints * dyn.phys.ob$ is associated with each person, where *ints* is the sort "an intelligent system." Naturally, if $x \in C_1$, $y \in D_1$, $z \in E_1$, $w \in C_2$, then the following expressions are well-formed (according to our common sense):

$$Diameter(x) = w, Diameter(y) = w, Diameter(z) = w.$$

That is why let's demand that the following relationships take place:

$$phys.ob \vdash dyn.phys.ob,$$

$$dyn.phys.ob \vdash ints * dyn.phys.ob,$$

$$phys.ob \vdash ints * dyn.phys.ob.$$

These ideas can be formulated also in the following way. Let *conc* be a denotation of a notion; in particular, it is possible that $conc \in St$, where St is the considered set of sorts. Then let $Dt(conc)$ be the designation of all entities which can be qualified (in other words, characterized) by the notion with the denotation *conc*; we'll say that $Dt(conc)$ is *the denotat of the notion conc*.

Suppose that S is a sort system, Rel is the designation of a certain n-ary relation on a certain set Z, and a certain mapping tp assigns to R a description of the semantic requirements to the attributes of R of the form $(t_1, \dots, t_n)$, i.e. $tp(Rel) = \{(t_1, \dots, t_n)\}$, where $n > 1$, $t_1, \dots, t_n \in Tp(S)$.

We'll believe that $(x_1, \dots, x_n) \in Rel$ if and only if there exist such types $u_1, \dots, u_n \in Tp(S)$ that for every $k = 1, \dots, n$, the type u_k is a concretization of t_k (we'll use in this case the denotation $t_k \vdash u_k$), and x_k belongs to the denotat of u_k; this means that x_k is an entity qualified by the type u_k.

Example 4. The academic groups of university students are the particular cases of sets. The relationship

$$tp(Number - of - elem) = \{(\{[entity]\}, nat)\}$$

can be interpreted as a description of semantic requirements to the arguments and value of the function "The number of elements of a finite set" denoted by the symbol $Number - of - elem$.

Suppose that the list of all identifiers being known to an intelligent database used by the administration of a university includes the element $Mat08 - 05$, and that the mapping tp associates with this element the type $\{ints * dyn.phys.ob\}$. This means that the hypothetical intelligent database considers the object with the identifier $Mat08 - 05$ as a certain set of people (because each person is both an intelligent system and a dynamic physical object).

Let $tp(14) = nat$. Since $nat \to nat$, it follows from the relationship (if it takes place) $[entity] \vdash \{ints * dyn.phys.ob\}$ that the expression $Number - of - elem(Mat08 - 05, 14)$ is well-formed.

3.7.2 Formal Definitions

Definition 3.13. Let S be an arbitrary sort system with the set of sorts St. Then *elementary compound types* are the strings from $Tp(S)$ of the form $s_1 * s_2 * \dots * s_k$, where $k > 1$, for $i = 1, \dots, k$, $s_i \in St$.

Example 5. The string $ints * dyn.phys.ob$ is an elementary compound type for the sort system S_0.

Definition 3.14. Let S be a sort system with the set of sorts St. Then $Elt(S)$ is the union of the set of sorts St with the set of all elementary compound types. The elements of the set $Elt(S)$ will be called *elementary types* .

Definition 3.15. If S is a sort system of the form (St, P, Gen, Tol), $t \in Elt(S)$, then *the spectrum of the type t* (denoted by $Spr(t)$) is (a) the set $\{t\}$ in case $t \in St$; (b) the set $\{s_1, \ldots, s_k\}$ in case the type t is the string of the form $s_1 * \ldots * s_k$, where $k > 1$, for every $i = 1, \ldots, k$, $s_i \in St$.

Example 6. If S_0 is the sort system constructed in Sect. 3.5, then the following relationships are valid:

$$Spr(phys.ob) = \{phys.ob\},$$

$$Spr(ints * dyn.phys.ob) = \{ints, dyn.phys.ob\}.$$

Definition 3.16. Let S be any sort system of the form (St, P, Gen, Tol), $u \in St$, t be an elementary compound type from $Tp(S)$. Then the type t is called *a refinement of the sort* $u \Leftrightarrow$ the spectrum $Spr(t)$ contains such sort w that $u \to w$ (i.e. $(u, w) \in Gen$).

Example 7. Let $u = phys.ob$, $t = ints * dyn.phys.ob$. Then the spectrum $Spr(t) = \{ints, dyn.phys.ob\}$.

That is why it follows from $phys.ob \to dyn.phys.ob$ that the type t is a refinement of the sort u.

Let's remember that the sorts are considered in this book as symbols, i.e. as indivisible units.

Definition 3.17. Let S be any sort system of the form (St, P, Gen, Tol), $u \in St$, t is a type from $Tp(S)$, and t includes the symbol u. Then an occurrence of u in the string t is *free* $\Leftrightarrow$ either $t = u$ or this occurrence of u in t is not an occurrence of u in any substring of the form $s_1 * s_2 \ldots * s_k$, where $k > 1$, for every $i = 1, \ldots, k$, $s_i \in St$, and there exists such m, $1 \leq m \leq k$, that $u = s_m$.

Example 8. It is possible to associate with the function "Friends" the type

$$t1 = \{(ints * dyn.phys.ob, \{ints * dyn.phys.ob\})\}.$$

Both the first and the second occurences of the symbol (a sort) $dyn.phys.ob$ in the string $t1$ are not the free occurrences. The function "The weight of a set of physical objects" can be associated with the type

$$t2 = \{(\{phys.ob\}, (real, kg))\};$$

the occurrences of the symbol $phys.ob$ in the string $t2$ and in the string $t3 = \uparrow phys.ob$ (a possible type of the concept "a physical object") are the free occurrences.

Definition 3.18. Let S be a sort system of the form (St, P, Gen, Tol). Then

$$Tc(S) = \{t \in Tp(S) \setminus (Spectp \cup Toptp) \mid \text{the symbol} \uparrow \text{is the beginning of } t\},$$

where

$$Spectp = \{[\uparrow entity], [\uparrow concept], [\uparrow object]\},$$

$$Toptp = \{[entity], [concept], [object]\},$$

$$Tob(S) = Tp(S) \setminus (Spectp \cup Toptp \cup Tc(S)).$$

Definition 3.19. Let S be a sort system of the form (St, P, Gen, Tol). Then the transformations $tr_1, \ldots, tr_6$, being partially applicable to the types from the set $Tp(S)$, are defined in the following way:

1. If $t \in Tp(S)$, t includes the symbol $[entity]$, then the transformations tr_1 and tr_2 are applicable to the type t. Let w_1 be the result of replacing in t an arbitrary occurrence of the symbol $[entity]$ by the type $[concept]$; w_2 be the result of replacing in t an arbitrary occurrence of the symbol $[entity]$ by the type $[object]$. Then w_1 and w_2 are the possible results of applying to the type t the transformations tr_1 and tr_2, respectively.
2. If $t \in Tp(S)$, t includes the symbol $[concept]$, the type $u \in Tc(S)$, then the transformation tr_3 is applicable to the type t, and the result of replacing an arbitrary occurrence of the symbol $[concept]$ by the type u is a possible result of applying the transformation tr_3 to the type t.
3. If $t \in Tp(S)$, t includes the symbol $[object]$, the type $z \in Tob(S)$, then the transformation tr_4 is applicable to the type t, and the result of replacing an arbitrary occurrence of the symbol $[object]$ by the type z is a possible result of applying the transformation tr_4 to the type t.
4. If $t \in Tp(S)$, t includes the symbol $s \in St$, $u \in St$, $s \neq u$, $(s, u) \in Gen$, and the type w is the result of replacing in t an arbitrary free occurrence of the sort s by the sort u, then w is a possible result of applying the transformation tr_5 to the type t.
5. If $t \in Tp(S)$, $u \in St$, z is an elementary compound type from $Tp(S)$ being a refinement of the sort u, and w is obtained from t by replacing in t an arbitrary free occurrence of the sort u by the string z, then w is a possible result of applying the transformation tr_6 to the type t.

Example 9. If S_0 is the sort system built above, $t_1 = [object]$, $t_2 = space.ob$, $w_1 = ints * dyn.phys.ob$, $w_2 = dyn.phys.ob$, then w_1 and w_2 are the possible results of applying the transformations tr_4 and tr_5 to the types t_1 and t_2, respectively.

If $t_3 = \{phys.ob\}$, $w_3 = \{ints * dyn.phys.ob\}$, then w_3 is a possible result of applying the transformation tr_6 to the type t_3 (since the sort $ints * dyn.phys.ob$ is a concretization of the sort $phys.ob$, because dynamic physical objects form a subclass of the set of all physical objects). Here the element *ints* is the distinguished sort "intelligent system."

Definition 3.20. Let S be a sort system of the form (St, P, Gen, Tol), and $t, u \in Tp(S)$. Then the type u is called *a concretization of the type* t, and the type t is called *a generalization of the type* u (the designation $t \vdash u$ is used) $\Leftrightarrow$ either t coincides with u or there exist such types $x_1, x_2, \ldots, x_n \in Tp(S)$, where $n > 1$, that $x_1 = t$, $x_n = u$, and for each $j = 1, \ldots, n-1$, there exist such $k[j] \in \{1, 2, \ldots, 6\}$ that the transformation $tr_{k[j]}$ can be applied to the string x_j, and the string x_{j+1} is a possible result of applying the transformation $tr_{k[j]}$ to x_j.

Example 10. It is easy to verify for the sort system S_0 that the following relationships take place:

$$[entity] \vdash [object],\ [entity] \vdash [concept],$$

$$[object] \vdash ints,\ [object] \vdash phys.ob,\ phys.ob \vdash dyn.phys.ob,$$

$$ints \vdash ints * dyn.phys.ob,\ phys \vdash ints * dyn.phys.ob,$$

$$\{[object]\} \vdash \{phys.ob\},\ \{[object]\} \vdash \{ints * dyn.phys.ob\},$$

$$[object] \vdash real,\ [object] \vdash (nat, nat),\ [object] \vdash \{nat\},$$

$$[concept] \vdash\uparrow ints,\ [concept] \vdash\uparrow \{ints\},$$

$$[concept] \vdash\uparrow ints * dyn.phys.ob,$$

$$[concept] \vdash\uparrow \{ints * dyn.phys.ob\}.$$

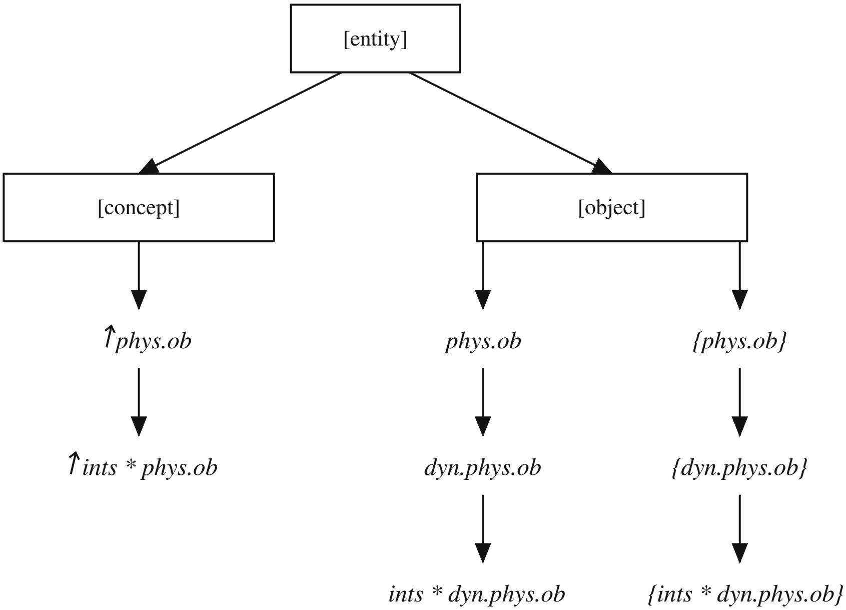

Fig. 3.7 A fragment of the hierarchy on the set of main types Mtp(S) induced by the concretization relation on the set of types Tp(S), where S is a sort system

Theorem 3.1. *Let S be an arbitrary sort system. Then the concretization relation* $\vdash$ *is a partial order on the set of types* $Tp(S)$.

Proof

The reflexivity and transitivity of the relation $\vdash$ immediately follow from its definition. Let's show that the antisymmetricity of the relation $\vdash$ follows from the properties of the transformations $tr_1, tr_2, \ldots, tr_6$.

As a result of applying the transformation tr_1 or tr_2, the number of occurrences of the symbol $[entity]$ is reduced, the difference is 1. After applying the transformations

tr_3 or tr_4, the number of the occurrences of the symbol $[concept]$ or $[object]$ is $n-1$, where n is the initial number of occurrences of the considered symbol.

If $t1, t2 \in Tp(S)$, and the type $t2$ has been obtained from the type $t1$ as a result of applying just one time the transformation tr_5, then this means that there exist such sorts $s, u \in St$, that $s \neq u$, $(s, u) \in Gen$, $t1$ includes the symbol s, and $t2$ has been obtained by means of replacing an occurrence of the symbol s in the string $t1$ by the symbol u. It follows from the antisymmetricity of the relation Gen on the set of sorts St that the reverse transformation of $t2$ into $t1$ is impossible.

If the type $t2$ has been obtained from the type $t1$ by means of applying just one time the transformation tr_6, the number of the symbols in $t2$ is greater than the number of the symbols in $t1$.

3.8 Concept-Object Systems

Let's proceed from the assumption that for describing an application domain on the conceptual level, we should choose some sets of strings X and V. The first set is to contain, in particular, the designations of notions (concepts), physical objects (people, ships, books, etc.), events, n-ary relations ($n \geq 1$). The set V should consist of variables which will play in expressions of our knowledge representation language the roles of marks of diverse entities and, besides, will be used together with the quantifiers $\exists$ and $\forall$.

We'll distinguish in X a certain subset F containing the designations of diverse functions. Each function f with n arguments will be considered as a certain set consisting of $n+1$-tuples $(x_1, \ldots, x_n, y)$, where $y = f(x_1, \ldots, x_n)$. Besides, we'll introduce a mapping tp assigning to each element $d \in X \cup V$ a certain type $tp(d)$ characterizing the entity denoted by d.

Definition 3.21. Let S be any sort system of the form $(\, St, P, Gen, Tol \,)$. Then a four-tuple Ct of the form

$$(\, X, V, tp, F \,)$$

is called *a concept-object system (c.o.s.) coordinated with the sort system S* (or *a concept-object system for S*) $\Leftrightarrow$ the following conditions are satisfied:

- X and V are countable non intersecting sets of symbols;
- tp is a mapping from $X \cup V$ to the set of types $Tp(S)$;
- F is a subset of X; for each $r \in F$, the string $tp(r)$ has the beginning $'\{\,('$ and the ending $')\,\}'$;
- St is a subset of X, and for each $s \in St$, $tp(s) = \uparrow s$;
- the set $\{u \in V \mid tp(v) = [entity]\,\}$ is countable.

The set X is called *the primary informational universe*, the elements of V and F are called *the variables* and *functional symbols*, respectively. If $d \in X \cup V$, $tp(d) = t$, then we'll say that t is *the type* of the element d.

Example. Let's construct a concept-object system Ct_0 coordinated with the sort system S_0, determined in Sect. 3.5. Let *Nat* be the set of all such strings *str* formed from the ciphers 0, 1, ..., 9 that the first symbol of *str* is distinct from 0 in case *str* contains more than one symbol. Let

$$U1 = \{person, chemist, biologist, stud.group, tour.group, J.Price,$$

$$R.Scott, N.Cope, P.Somov, Friends, Numb, Less, Knows, Isa1,$$

$$Elem, Subset, Include1, Before, \#now\#, concept\}.$$

The strings of $U1$ denote respectively the notions "a person," "a chemist," "a biologist," "a student group," "a tourist group;" some concrete persons with the initial and surname J. Price, R. Scott, N. Cope, P. Somov; the function "Friends" assigning to a person Z the set of all friends of Z; the function "Numb" assigning to a set the number of elements in it; the relations "Less" (on the set of real numbers), "Knows" ("The memory of an intelligent system Z_1 at the moment Z_2 contains a semantic representation of an assertion (in other words, of a statement, a proposition) Z_3"), "Isl" ("An object Z_1 is qualified by a notion Z_2"; an example of a phrase: "P. Somov is a chemist"), "Element" ("An entity Z_1 is an element of the set Z_2"), "Subset" ("An entity Z_1 is a subset of the set Z_2"), Include1" ("An intelligent system Z_1 includes an entity Z_2 at the moment Z_3 into a set of objects Z_4"), "Before" on the set of the moments of time.

The symbol *#now#* will be used in semantic representations of texts for denoting a current moment of time. The symbol *concept* will be interpreted as the informational unit corresponding to *the word groups* "a notion," "a concept," and, besides, the word group "a term" in the meaning "a notion," "a concept."

Let's define a mapping $t1$ from $U1$ to the set $Tp(S_0)$ by the following table:

d	t1(d)
person, chemist, biologist	$\uparrow ints * dyn.phys.ob$
stud.group, tour.group	$\uparrow \{ints * dyn.phys.ob\}$
J.Price, R.Scott	$ints * dyn.phys.ob$
N.Cope, P.Somov	$ints * dyn.phys.ob$
Friends	$\{(ints * dyn.phys.ob, \{ints * dyn.phys.ob\})\}$
Numb	$\{(\{[entity]\}, nat)\}$
Less	{(real, real) }
Knows	{(int, mom, prop)}
Isa1	$\{([object], [concept])\}$
Elem	$\{([entity], \{[entity]\})\}$
Subset	$\{(\{[entity]\}, \{[entity]\})\}$
Include1	$\{(ints, [entity], mom, \{[entity]\})\}$
Before	{(mom, mom)}
#now#	mom
concept	$[\uparrow concept]$

Let's believe that $Firm\_Ocean$, $Firm\_Rainbow$, $Firm\_Sunrise$ are the designations of the firms; $Suppliers$, $Staff$, $Director$ are the designations of the functions "The set of all suppliers of an organization," "The set of all persons working at an organization," and "The director of an organization." Let

$$U2 = \{Firm\_Ocean, Firm\_Rainbow, Firm\_Sunrise,$$

$$Suppliers, Staff, Director\},$$

and the mapping $t2$ from $U2$ to $Tp(S)$ is determined by the following relationships:

$$t2(Firm\_Ocean) = t2(Firm\_Rainbow) = t2(Firm\_Sunrise)$$

$$= org * space.ob * ints,$$

where org is the sort "organization", $space.ob$ is the sort "space object", and $ints$ is the sort "intelligent system";

$$t2(Suppliers) = \{(org, \{org\})\},$$

$$t2(Staff) = \{(org, \{ints * dyn.phys.ob\})\},$$

$$t2(Director) = \{(org, ints * dyn.phys.ob)\}.$$

Let $Vx = \{x1, x2, \ldots\}$, $Ve = \{e1, e2, \ldots\}$, $Vp = \{P1, P2, \ldots\}$,

$$Vset = \{S1, S2, \ldots\},$$

$$V_0 = Vx \cup Ve \cup Vp \cup Vset,$$

where the elements of the sets Vx, Ve, Vp, $Vset$ will be interpreted as the variables for designating respectively (a) arbitrary entities, (b) situations (in particular, events), (c) semantic representations of statements (assertions, propositions) and narrative texts, (d) sets.

Let $X_0 = St_0 \cup Nat \cup U1 \cup U2 \cup Weights$, where $Weights = \{x/y/x \in Nat,\ y \in \{kg,\ tonne\}\}$, and the mapping $tp_0 : X_0 \cup V_0 \to Tp(S_0)$ is defined by the following relationships:

$$d \in St_0 \Rightarrow tp_0(d) = \uparrow d;$$

$$d \in Nat \Rightarrow tp_0(d) = nat;$$

$$d \in Weights \Rightarrow tp_0(d) = weight.value;$$

$$d \in U1 \Rightarrow tp_0(d) = t1(d);$$

$$d \in U2 \Rightarrow tp_0(d) = t2(d);$$

$$d \in Vx \Rightarrow tp_0(d) = [entity];$$

$$d \in Ve \Rightarrow tp_0(d) = sit;$$

$$d \in Vp \Rightarrow tp_0(d) = prop;$$

$$d \in Vset \Rightarrow tp_0(d) = \{[entity]\}.$$

Let's define the set of functional symbols

$$F_0 = \{\,Friends, Numb, Suppliers, Staff, Director\,\},$$

and let

$$Ct_0 = (\,X_0, V_0, tp_0, F_0\,).$$

Then it is easy to verify that the system Ct_0 is a concept-object system coordinated with the sort system S_0.

3.9 Systems of Quantifiers and Logical Connectives: Conceptual Bases

Presume that we define a sort system S of the form $(\,St,\ P\,Gen,\ Tol\,)$ and a concept-object system Ct of the form $(\,X,\ V,\ tp,\ F\,)$ coordinated with S in order to describe an application domain. Then it is proposed to distinguish in the primary informational universe X two non intersecting and finite (hence non void) subsets Int_1 and Int_2 in the following manner: we distinguish in St two sorts int_1 and int_2 and suppose that for $m = 1, 2$,

$$Int_m = \{q \in X \mid tp(x) = int_m\}.$$

The elements of Int_1 correspond to the meanings of the expressions "every," "a certain," "any," "arbitrary," etc. (and, may be, "almost every," etc.); these expressions are used to form the word groups in singular. The elements of Int_2 are interpreted as semantic items corresponding to the expressions "all," "several," "almost all," "many," and so on; the minimal requirement is that Int_2 contains a semantic item corresponding to the word "all."

Let Int_1 contain a distinguished element ref considered as an analogue of the word combination "a certain" in the sense "quite definite" (but, possibly, unknown). If Ct is a concept-object system of the form $(X, V, tp, F\,)$, $d \in X$, d denotes a notion, and a semantic representation of a text includes a substring of the form $ref\,d$ (e.g., the substring $certain\ chemist$, where $ref = certain$, $d = chemist$), then we suppose that this substring denotes a certain concrete entity (but not an arbitrary one) that is characterized by the concept d.

Let X contain the elements $\equiv$, $\neg$, $\wedge$, $\vee$, interpreted as the connectives "is identical to," "not," "and," "or," and contain the elements $\forall$ and $\exists$, interpreted as the universal and existential quantifiers.

Figure 3.8 is intended to help grasp the basic ideas of the definition below.

Definition 3.22. Let S be any sort system of the form $(St,\ P,\ Gen,\ Tol)$, where St be the set of sorts, P be the distinguished sort "a meaning of proposition"; let $Concr(P)$ be the set of all such z from St that $(P, z) \in Gen$ (i.e. $Concr(P)$ be the set of all sorts being the concretizations of the sort P), Ct be any concept-object system of

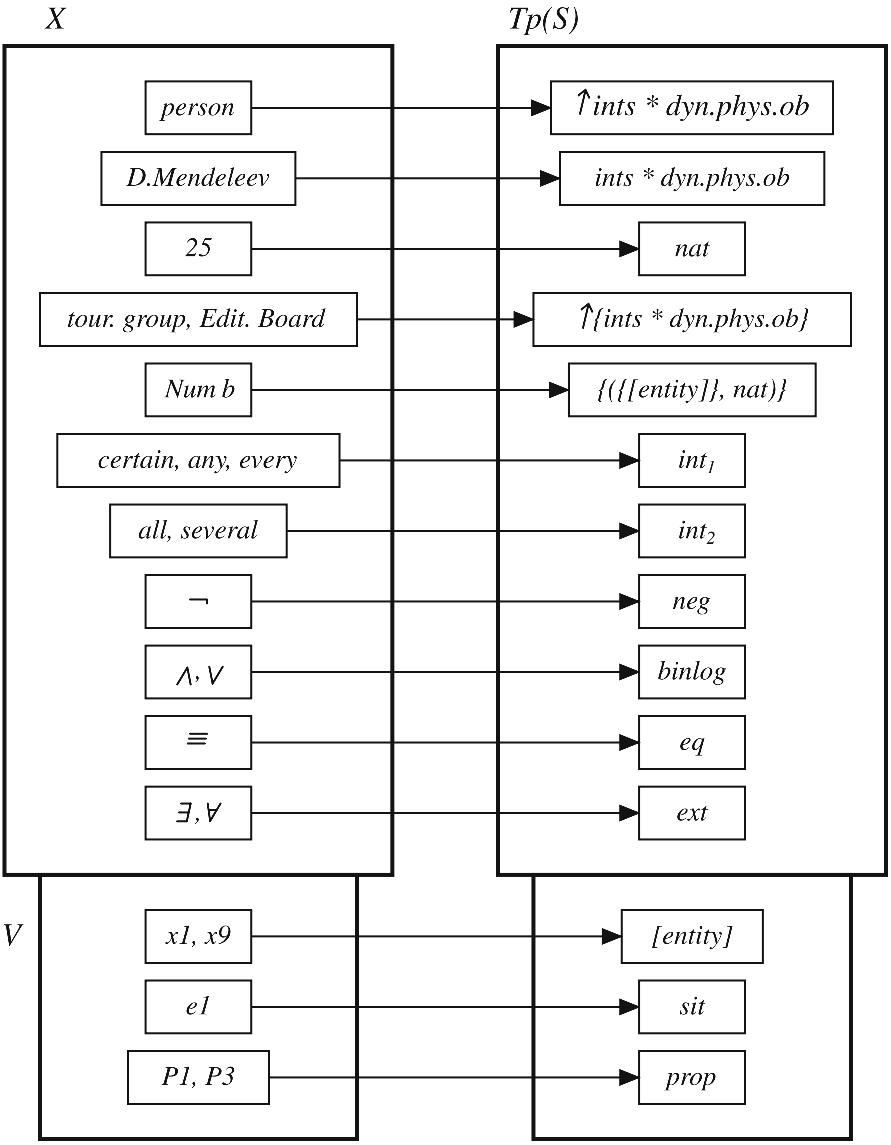

Fig. 3.8 Illustration of the basic ideas of the definitions of a concept-object system and a system of quantifiers and logical connectives

the form (X, V, tp, F) coordinated with S, ref be the intensional quantifier from X, the different elements int_1, int_2, eq, neg, $binlog$, ext be some distinguished sorts from $St \setminus Concz(P)$, and each pair of these sorts be incomparable with respect to the generality relation Gen and incomparable with respect to the tolerance relation Tol.

Then the seven-tuple Ql of the form

$$(int_1, int_2, ref, eq, neg, binlog, ext)$$

is called *a system of quantifiers and logical connectives (s.q.l.c.) coordinated with S* and *Ct* (or a s.q.l.c. for *S* and *Ct*) $\Leftrightarrow$ the following conditions are satisfied:

1. For each $m = 1, 2,$ the set $Int_m = \{ d \in X \mid tp(d) = int_m \}$ is a finite set; $ref \in Int_1$, the sets Int_1 and Int_2 don't intersect.
2. The primary informational universe X includes the subset

$$\{ \equiv, \neg, \wedge, \vee, \forall, \exists \};$$

   besides, $tp(\equiv) = eq$, $tp(\neg) = neg$, $tp(\wedge) = tp(\vee) = binlog$, $tp(\forall) = tp(\exists) = ext$.
3. There are no such $d \in X \setminus (Int_1 \cap Int_2 \cap \{ \equiv, \neg, \wedge, \vee, \forall, \exists \})$ and no such $s \in \{int_1, int_2, eq, neg, binlog, ext \}$ that $tp(d)$ and s are comparable with respect to the relation *Gen* or are comparable with respect to the relation *Tol*.
4. For each sort $u \in \{int_1, int_2, eq, neg, binlog, ext \}$ and each sort $w \in Concr(P)$, where P is the distinguished sort "a meaning of proposition," the sorts u and w are incomparable with respect to the relation *Gen* and are incomparable with respect to the relation *Tol*.

The elements of Int_1 and Int_2 are called *intensional quantifiers*, the element *ref* is called *the referential quantifier*, the symbols $\forall$ and $\exists$ are called *extensional quantifiers*.

Example 1. Let $S_0 = (St_0, prop, Gen_0, Tol_0)$ be the sort system built above, $Ct_0 = (X_0, V_0, tp_0, F_0)$ be the concept-object system coordinated with the sort system S_0; the system Ct_0 was defined in Sect. 3.8. Suppose that

$$Str = \{sort.qr.int_1, sort.qr.int_2, sort.eqvt, sort.not, sort.bin.log, sort.ext.qr\};$$

$$Gen_1 = Gen_0 \cup \{ (s, s) \mid s \in Str \};$$

$$St_1 = St_0 \cup Str;$$

$$S_1 = (St_1, prop, Gen_1, Tol_0).$$

Then, obviously, the four-tuple S_1 is a sort system.

Let's define now a concept-object system Ct_1 and a system of quantifiers and logical connectives Ql_1. Let

$$Z = \{certn, all, \equiv, \wedge, \vee, \forall, \exists\},$$

where the string *certn* is interpreted as the semantic item "a certain," and

$$X_1 = X_0 \cup Str \cup Z.$$

Let's determine a mapping tp_1 from $X_1 \cup V_0$ to the set of types $Tp(S_1)$ in the following way:

$$u \in Str \Rightarrow tp_1(u) = \uparrow u;$$

$$d \in X_0 \Rightarrow tp_1(d) = tp_0(d);$$

$$tp_1(certn) = sort.qr.int1,\ tp_1(all) = sort.qr.int2,$$

$$tp_1(\equiv) = sort.eqvt,\ tp_1(\neg) = sort.not,$$

$$tp_1(\wedge) = tp_1(\vee) = sort.bin.log,$$

$$tp_1(\forall) = tp_1(\exists) = sort.ext.qr.$$

Let the systems Ct_1 and Ql_1 be defined by the relationships

$$Ct_1 = (X_1, V_0, tp_1, F_0),$$

$$Ql_1 = (sort.qr.int_1, sort.qr.int_2, certn, sort.eqvt,$$

$$sort.not, sort.bin.log, sort.ext.qr).$$

Then it is easy to verify that Ct_1 is a concept-object system coordinated with the sort system S_1, and Ql_1 is a system of quantifiers and logical connectives coordinated with the sort system S_1 and the concept-object system Ct_1. In the system Ql_1, the element *certn* is the informational item interpreted as the *referential quantifier ref* (that is as a semantic unit corresponding to the word combination "a certain").

Definition 3.23. An ordered triple B of the form

$$(S, Ct, Ql)$$

is called *a conceptual basis (c.b.)* $\Leftrightarrow$ S is a sort system, Ct is a concept-object system of the form (X, V, tp, F) coordinated with the sort system S, Ql is a system of quantifiers and logical connectives coordinated with S and Ct, and the set $X \cup V$ doesn't include any of the symbols ′,′ (comma), ′(′, ′)′, ′ : ′, ′ ∗ ′, ′⟨′, ′⟩′, ′&′.

We'll denote by $S(B)$, $Ct(B)$, and $Ql(B)$ the components of an arbitrary conceptual basis B of the form (S, Ct, Ql). Each component with the name h of the mentioned systems of the forms

$$(St, P, Gen, Tol),$$

$$(X, V, tp, F),$$

$$(int_1, int_2, ref, eq, neg, binlog, ext)$$

will be denoted by $h(B)$.

For instance, the set of sorts, the distinguished sort "a meaning of proposition," and the primary informational universe of B will be denoted by $St(B)$, $P(B)$, $X(B)$ respectively.

We'll interpret the conceptual bases as formal enumerations of (a) primary informational units needed for building semantic representations of NL-texts and for describing knowledge about the world, (b) the information associated with these units and required for constructing the semantic representations of NL-texts and for forming knowledge fragments and representing goals of intelligent systems.

Example 2. Let S_1, Ct_1, Ql_1 be respectively the sort system, concept-object system, and the system of quantifiers and logical connectives determined above. Then, obviously, the triple $B_1 = (S_1, Ct_1, Ql_1)$ is a conceptual basis, and

$$St(B_1) = St_1,\ P(B_1) = prop,$$

$$X(B_1) = X_1,\ V(B_1) = V_0.$$

This example shows that the set of all conceptual bases is non void.

3.10 A Discussion of the Constructed Mathematical Model

3.10.1 Mathematical Peculiarities of the Model

The form of the constructed mathematical model for describing a system of primary units of conceptual level used by an applied intelligent system is original. Let's consider the distinguished features of this model seeming to be the most important both from the mathematical standpoint and from the standpoint of using the model in the design of linguistic processors.

1. The existence of the hierarchy of notions is constructively taken into account: with this aim, a partial order *Gen* on the set of sorts *St* is defined, it is called the generality relation.
2. Many entities considered in an application domain can be qualified from different points of view. For instance, people are, on one hand, intelligent systems, because they can read, solve tasks, compose music, poems, etc. But, on the other hand, people are physical objects being able to move in space. That is why we can metaphorically say that many entities have "the coordinates" on different "semantic axes."

   For taking into account this important phenomenon, we introduced a binary relation *Tol* on the set of sorts *St*, it is called the tolerance relation. The accumulated experience has shown that this original feature of the model is very important for elaborating the algorithms of semantic-syntactic analysis of natural language texts. The reason for this statement is that the use of the same word in several non similar contexts may be explained by the realization in these contexts of different "semantic coordinates" of the word.
3. The phrase "This notion is used both in physics and chemistry" (it applies, for instance, to the notion "a molecule") is very simple for a person having some command of English. However, the main popular approaches to the formalization of NL-semantics can't appropriately reflect the semantic structure of this phrase. The reason for this shortcoming is that such approaches don't offer a formal analogue of the conceptual (or informational, semantic) unit corresponding to the words "a notion," "a concept."

The situation is different for the model constructed above. First, the model introduces a special basic type $[\uparrow concept]$ interpreted as the type of the conceptual unit corresponding to the words "a notion," "a concept." Secondly, the model describes, in particular, the class of formal objects called concept-object systems. The component X of arbitrary concept-object system Ct of the form (X, V, tp, F) (this component is called the primary informational universe) can include an element (a symbol) interpreted as the conceptual unit corresponding to the words "a notion," "a concept" (see an example in Sect. 3.8).

This feature of the constructed mathematical model is important for the design of natural language processing systems dealing with scientific and scientific-technical texts and, besides, for the design of linguistic processors of applied intelligent systems extracting knowledge from encyclopedic dictionaries or updating electronic encyclopedic dictionaries by means of extracting knowledge from articles, monographs, textbooks, technical reports, etc.

4. One of the most important distinguishing features of the built model is an original definition of the set of types generated by arbitrary sort system, where the types are considered as formal characterizations of the entities belonging to considered thematic domains. In accordance with this definition, (a) the form of the types of objects from an application domain differs from the form of the notions qualifying these objects; (b) the form of the types of the objects differs from the form of the types of sets consisting of such objects; (c)) the form of the types of the notions (in other words, of the concepts) qualifying the objects differs from the form of the types of notions qualifying the sets consisting of such objects (for example, the type of the concept "a person" is different from the type of the concepts "an editorial board," "a student group").
5. The constructed model associates the types with the designations of the functions too. It may be noticed that the definition of the set of types generated by a sort system enables us to associate (in a reasonable way) the types with a number of rather non standard but practically important functions. In particular, this applies to functions with the values being (a) the set of concepts explained in an encyclopedic dictionary or in a Web-based ontology; (b) the set of concepts mentioned in the definition of the given concept in the given dictionary; (c) the set of semantic representations of the known definitions of the considered notion; (d) the number of elements of the considered set; (e) the set consisting of all suppliers of the considered firm; (f) the set consisting of all employees of the considered firm.

3.10.2 The Comparison of the Model with Related Approaches

Let's compare the constructed model with the approaches to describing primary units of conceptual level offered by first-order predicate logic, discourse representation theory, theory of conceptual graphs, and episodic logic.

In the standard first-order predicate logic, one considers the unstructured non-intersecting sets of constants, variables, functional symbols, and predicate symbols. More exactly, each functional symbol is associated with a natural number denoting the number of arguments of the corresponding function, and each predicate symbol is associated with a natural number denoting the number of attributes of the corresponding relation. In the multi-sorted first-order predicate logics, the set of constants consists of the non intersecting classes where each of them is characterized by a certain sort.

The mathematical model constructed above provides, in particular, the following additional opportunities in comparison with the multi-sorted first-order predicate logics:

- Due to the introduction of the tolerance relation as a component of a sort system, it is possible to associate with a primary unit of conceptual level not only one but, in many cases, several sorts being, metaphorically speaking, “the coordinates on orthogonal semantic axes” of the entities qualified or denoted by such a unit;
- The association of the types with the primary units of conceptual level means that the set of such units has a fine-grained structure; in particular, the types enable us to distinguish in a formal way the following: (a) the types of objects from thematic domains and the types of notions qualifying these objects; (b) the types of objects and the types of sets consisting of such objects; (c) the types of notions qualifying some objects and the types of notions qualifying the sets consisting of such objects;
- The constructed model allows for considering the primary units of conceptual level corresponding to the words and expressions “a certain,” “definite,” “any,” “all,” “several,” “the majority,” “the minority”;
- The model provides the possibility to consider the primary unit of conceptual level corresponding to the words “a notion,” “a concept.”

It should be added that the proposed mathematical model enables us to consider the functions with the arguments and/or values being semantic representations (SRS) of the assertions (propositions) and narrative texts. For instance, one of such functions can associate each notion defined in an encyclopedic dictionary with a formula being an SR of this notion. But in the first-order predicate logics, the arguments and values of the functions can be only terms but not formulas, and terms are the designations of the objects from the application domains but not the designations of the meanings of assertions (propositions) and narrative texts.

The Discourse Representation Theory (DRT) can be interpreted as one of the variants of the first-order predicate logic combining the use of logical formulas and two-dimensional diagrams for visual representation of information. That is why the enumerated advantages of the constructed mathematical model for describing the systems of primary units of conceptual level used by computer intelligent systems apply to DRT too.

Both the Theory of Generalized Quantifiers (TGQ) and the mathematical model constructed above consider the units of conceptual level corresponding to the expressions “a certain,” “definite,” “all,” “several.” However, all other enumerated advantages of the model in comparison with the first-order predicate logic are simultaneously the advantages in comparison with the approach of TGQ.

Unlike the first-order predicate logic, the notation of the Theory of Conceptual Graphs (TCG) allows for distinguishing the designations of concrete objects (concrete cities, cars, firms, etc.) and the designations of notions qualifying these objects (“a city,” “a car,” “a firm,” etc.). The other enumerated advantages of the proposed model are also the advantages in comparison with TCG.

All enumerated advantages of the constructed model can be interpreted as the advantages in comparison with the approach to structuring the collection of primary units of conceptual level provided by Episodic Logic.

The analysis carried out above allows for drawing the conclusion that the constructed model proposes a more “fine-grained” conceptual structure of application domains in comparison with the main popular approaches to the formalization of NL-semantics; the model considerably increases the resolution possibility of the spectrum of formal tools destined for investigating various application domains.

Since the end of the 1990s, the studies on the creation of ontologies for various application domains have been quickly progressing. In these studies, the term “ontology” is interpreted as a specification of conceptualization; practically this means that an ontology enumerates the notions used in a considered group of application domains and associates the notions with their definitions and knowledge pertaining to the classes of objects qualified by some notions. The first step in each computer project of the kind consists in selecting an initial (or basics) structure of the considered application domain or a group of application domains.

It can be conjectured that the constructed mathematical model for describing a system of primary units of conceptual level used by linguistic processors and for representing the information associated with such units can find applications in the projects aimed at the elaboration of more perfect ontologies in arbitrary application domains. The reason for this hypothesis is that the elaborated model proposes a formal tool with the highest “resolution possibility” in comparison with the other known approaches to the formalization of NL-semantics and, as a consequence, to the conceptual structuring of application domains.

Problems

1. What is an n-ary relation on an arbitrary non empty set Z, where $n \geq 1$?
2. What is a binary relation on an arbitrary non empty set Z?
3. What is a reflexive relation on an arbitrary non empty set Z?
4. What is an antireflexive relation on an arbitrary non empty set Z?
5. What is a symmetric relation on an arbitrary non empty set Z?
6. What is an antisymmetric relation on an arbitrary non empty set Z?
7. What is a transitive relation on an arbitrary non empty set Z?
8. What is a partial order on an arbitrary non empty set Z?
9. What is the interpretation of the generality relation being a component of a sort system?
10. What are the mathematical properties of a generality relation on the set of sorts?

11. What is the interpretation of the tolerance relation being a component of a sort system?
12. What are the mathematical properties of a tolerance relation on the set of sorts?
13. What is the difference between a generality relation and a tolerance relation?
14. What is the difference between the types of concrete things and the types of notions qualifying these things?
15. What is the difference between the types of concrete things and the types of sets consisting of these things?
16. What kind of information is given by the symbol $\uparrow$ being the beginning of a type?
17. What is the difference between the possible types of the notions "a student" and "an academic group" ("a student group")?
18. What is the difference between the possible types of a concrete ship and a concrete squadron?
19. Give an example of an elementary compound type.
20. Why is the set of elementary types broader than the set of elementary compound types?
21. What is the spectrum of an elementary type?
22. What is a refinement of a sort?
23. What are the name and interpretation of the component X of a concept-object system $Ct = (X, V, tp, F)$?
24. What is the interpretation of the referential quantifier?
25. Is it true or not that the logical connectives $\neg$, $\wedge$, $\vee$ are the elements of the primary informational universe being a component of arbitrary concept-object system?
26. What are the structure and interpretation of a conceptual basis?