# Cleveland Clinic - Medical Resource Optimization Mathematical Formulation

May 27, 2020

## Dimensions

$r \in Resources$ : set of resources

$f \in Facilities$ : set of facilities

$sl \in Service\ lines$ : set of service lines

$ss \in Sub-services$ : set of sub-services

$io \in Inpatient/Outpatient$ : set of Inpatient/Outpatient indicator

$ms \in Medical/Surgical$ : set of Medical/Surgical indicator

$d \in Days$ : set of days

$w \in Weeks$ : set of weeks

$q \in Phase\ start\ dates$ : set of phase start dates

$icu \in ICU\ Resources$ : set of ICU resources

## Data Parameters: Model Coefficients

$capacity_{f,sl,ss,r}$ is the capacity of resource $r$ at facility $f$, service line $sl$, sub-service $ss$

$utilization_{f,sl,ss,io,ms,r}$ is the usage of resource $r$ per patient per day at facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$revenue_{f,sl,ss,io,ms}$ is the revenue per patient at facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$margin_{f,sl,ss,io,ms}$ is the margin per patient at facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$demand_{f,sl,ss,io,ms,d}$ is the maximum demand of facility $f$, service line $sl$, sub-service $ss$, and day $d$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$losMean_{f,sl,ss,io,ms}$ is the mean hospitalization time of facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$minday$ is the minimum day among the set $d \in Days$, $\min_{d \in Days} d$

$maxday$ is the maximum day among the set $d \in Days$, $\max_{d \in Days} d$

$dlyRapidTest_d$ is the total daily rapid tests available across all facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$dlyNonRapidTest_d$ is the total daily non-rapid tests for COVID-19 available across all facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$week_d$ is the week number of day $d$ in set $d \in Days$

$daysBeforeSurgAdm$ is the number of days before which a surgery patient should be tested for COVID-19 (using a non-rapid test kit)

$fracInpTest$ is the fraction of inpatients tested for COVID-19 using the rapid test kit on the day of admission

$minDemRatio_{f,sl,ss}$ is the minimum proportion of demand that must be satisfied at a sub-service $ss$ if its is open at a facility $f$, service line $sl$

$newPntBfCovid_{f,sl,ss,io,ms,d}$ is the new patients admitted without COVID-19 test constraints at the facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$ on day $d$

$fixOpenFlag_{f,sl,ss}$ is a flag and is set to 1 if the facility $f$, service line $sl$, sub-service $ss$ is open, 0 otherwise.

$emerSurRatio_{f,sl,ss}$ is a proportion of emergency surgical patients at the facility $f$, service line $sl$, sub-service $ss$.

$numCancel_{f,sl,ss,io,ms}$ is the maximum number of patients cancelled at the facility $f$, service line $sl$, sub-service $ss$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$

$maxUtil_f$ is the maximum utilization allowed for ICU resource at the facility $f$.

$minDemAggFlag$ is a flag to denote if the min demand is aggregated or non-aggregated and $\in (0, 1)$

# Decision Variables

$NewPatients_{f,sl,ss,io,ms,d} \geq 0$ is the number of patients accepted in facility $f$, service line $sl$, sub-service $ss$, on day $d$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$.

$ReschedulePatients_{f,sl,ss,io,ms,d} \geq 0$ is the number of patients rescheduled in facility $f$, service line $sl$, sub-service $ss$, on day $d$, for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$.

$OpenFlg_{f,sl,ss,d} \in \{0,1\}$ is the binary variable indicating if facility $f$, service line $sl$, sub-service $ss$ is open on day $d$.

# Variables

$TotalPatients_{f,sl,ss,io,ms,d}$ is the total number of patients accepted and rescheduled in facility $f$,service line $sl$, sub-service $ss$, cumulative for day $d$ , for inpatient/outpatient indicator $io$, and medical/surgical indicator $ms$ where,

$$TotalPatients_{f,sl,ss,io,ms,d} = \sum_{\substack{d1 \in d \text{ and} \\ \max\{[d - losMean_{f,sl,ss,io,ms}+1], \\ minday\} \leq d1 \leq d}} (NewPatients_{f,sl,ss,io,ms,d1} + ReschedulePatients_{f,sl,ss,io,ms,d1})$$

$$\forall f, sl, ss, io, ms, d$$

# Objective Functions

$max \quad TotalRevenue = \sum_{f,sl,ss,io,ms,d} (NewPatients_{f,sl,ss,io,ms,d} + ReschedulePatients_{f,sl,ss,io,ms,d})\, revenue_{f,sl,ss,io,ms}$

$max \quad TotalMargin = \sum_{f,sl,ss,io,ms,d} (NewPatients_{f,sl,ss,io,ms,d} + ReschedulePatients_{f,sl,ss,io,ms,d})\, margin_{f,sl,ss,io,ms}$

# Constraints

Maximum demand constraint: Number of patients accepted for $f, sl, ss, io, ms$ should be less than the maximum demand for day $d$.

$$NewPatients_{f,sl,ss,io,ms,d} \leq demand_{f,sl,ss,io,ms,d}\, OpenFlg_{f,sl,ss,d} \quad \forall f, sl, ss, io, ms, d \tag{1}$$

Capacity constraint: Resources used for the total number of patients for $f, sl, ss$ on day $d$ should be less than or equal to available capacity of resource $r$.

$$\sum_{io,ms} utilization_{f,sl,ss,io,ms,r}\ TotalPatients_{f,sl,ss,d} \leq capacity_{f,sl,ss,r} \quad \forall f, sl, ss, d, r \tag{2}$$

Max. ICU resource utilization constraint: The utilization of ICU resources at $f$ on day $d$ should be less than or equal to the available capacity of resource $r$ times the max utilization allowed for these resource at $f$.

$$\sum_{\substack{sl,ss,io,ms,r \\ \text{and } r \in icu}} utilization_{f,sl,ss,io,ms,r}\ TotalPatients_{f,sl,ss,d} \leq \sum_{\substack{sl,ss,r \\ \text{and } r \in icu}} capacity_{f,sl,ss,r}\ maxUtil_f \quad \forall f, d \tag{3}$$

Rescheduling constraint: Rescheduling patients for $f, sl, ss, io, ms$ on day $d$ is not allowed if the sub-service is not open

$$ReschedulePatients_{f,sl,ss,io,ms,d} \leq numCancel_{f,sl,ss,io,ms}\ OpenFlg_{f,sl,ss,d} \quad \forall f, sl, ss, io, ms, d \tag{4}$$

Maximum rescheduling constraint: Total number of rescheduled patients across all days for $f, sl, ss, io, ms$ should be less than the maximum number of rescheduled patients.

$$\sum_{d} ReschedulePatients_{f,sl,ss,io,ms,d} \leq numCancel_{f,sl,ss,io,ms} \quad \forall f, sl, ss, io, ms \tag{5}$$

Aggregated minimum proportion of demand constraint: If a sub-service $ss$ is open at $f, sl$ then, we should at the least satisfy a minimum proportion of the aggregated weekly demand of $f, sl, ss$.
Note: Aggregation context is specified using $ALL$ in the indices of the $minDemRatio_f, sl, ss$.

$$\begin{aligned} &\sum_{\substack{f',sl',ss',io,ms,d \text{ and} \\ week_d = w}} (NewPatients_{f',sl',ss',io,ms,d} + ReschedulePatients_{f',sl',ss',io,ms,d}) \geq minDemRatio_{f,sl,ss} \\ &\sum_{\substack{f',sl',ss',io,ms,d \text{ and} \\ week_d = w}} newPntBfCovid_{f',sl',ss',io,ms,d}\ OpenFlg_{f',sl',ss',d} \quad \forall f, sl, ss, w \text{ and } minDemAggFlag = 1 \\ &\qquad\qquad f = f' \text{ or } f = ALL \text{ and } sl = sl' \text{or} sl = ALL \text{ and } ss = ss' \text{ or } sl = ALL \end{aligned} \tag{6}$$

Non-aggregated minimum proportion of demand constraint: If a sub-service $ss$ is open at $f, sl$ then, we should at the least satisfy a minimum proportion of the weekly demand of $f, sl, ss$.

$$\begin{aligned} &\sum_{\substack{io,ms,d \text{ and} \\ week_d = w}} (NewPatients_{f,sl,ss,io,ms,d} + ReschedulePatients_{f,sl,ss,io,ms,d}) \geq minDemRatio_{f,sl,ss} \\ &\sum_{\substack{io,ms,d \text{ and} \\ week_d = w}} newPntBfCovid_{f,sl,ss,io,ms,d}\ OpenFlg_{f,sl,ss,d} \quad \forall f, sl, ss, w\ and\ minDemAggFlag = 0 \end{aligned} \tag{7}$$

COVID-19 inpatient tests constraint: Total number of inpatients accepted (excluding surgical patients) and the emergency surgical patients admitted for $f, sl, ss, ms$ on day $d$ should be less than the total rapid test available daily rapid test available.

$$\sum_{\substack{f,sl,ss,io,ms \\ \text{and } io='I' \\ \text{and} \\ ms!='SURG'}} (NewPatients_{f,sl,ss,io,ms,d} + ReschedulePatients_{f,sl,ss,io,ms,d}) + \sum_{\substack{f,sl,ss,io,ms \\ \text{and} \\ ms='SURG'}} (NewPatients_{f,sl,ss,io,ms,d} + ReschedulePatients_{f,sl,ss,io,ms,d})\ emerSurRatio_{f,sl,ss} \leq (dlyRapidTest_d\ /fracInpTest) \quad \forall d \tag{8}$$

COVID-19 surgery patient tests constraint: Total number of surgery patients who will be admitted for surgery after '$daysBeforeSurgAdm$' days for $f, sl, ss, io$ in day $d$ should be less than the the total non-rapid test available for the day $d$.

$$\sum_{\substack{f,sl,ss,io,ms \text{ and} \\ d1=[d+daysBeforeSurgAdm] \\ \text{and } d1 \in d \\ \text{and} \\ ms='SURG'}} (NewPatients_{f,sl,ss,io,ms,d1} + ReschedulePatients_{f,sl,ss,io,ms,d1})(1 - emerSurRatio_{f,sl,ss})) \leq dlyNonRapidTest_d \quad \forall d \tag{9}$$

Sub-service open constraint: If a sub-service $ss$ is open at $f, sl$ on day $d$ then it should be open for the remainder of the horizon.

$$OpenFlg_{f,sl,ss,d+1} \geq OpenFlg_{f,sl,ss,d} \quad \forall f, sl, ss, d \text{ and } d1 \in d \tag{10}$$

Sub-service already opened constraint: If a sub-service $ss$ is already opened at $f, sl$ then we set $OpenFlg$ variable as 1 for the entire planning horizon.

$$OpenFlg_{f,sl,ss,d} = 1 \quad \forall f, sl, ss, d \text{ and } d = minday \text{ and } fixOpenFlag_{f,sl,ss} = 1 \tag{11}$$

Sub-service open on phase date constraint: The sub-service $ss$ allowed to open only on the phase dates or on the first day of the planning horizon.

$$OpenFlg_{f,sl,ss,d} = OpenFlg_{f,sl,ss,d-1} \quad \forall f, sl, ss, d \text{ and } (d-1) \in d \text{ and } d \notin q \tag{12}$$