

$$\begin{aligned}
[S_{ij}, S_{kl}] &= \begin{cases} 0, & i \neq k, j \neq l \\ \frac{i}{\sqrt{2}} J_{il}; & j = k, i \neq l \\ -\frac{i}{\sqrt{2}} J_{kj}; & i = l, j \neq k \\ \frac{i}{\sqrt{2}} J(j, l); & i = k, j \neq l \\ \frac{i}{\sqrt{2}} J(i, k); & j = l, i \neq k \\ 0; & j = l, i = k \end{cases} \\
[S_{ij}, J_{kl}] &= \begin{cases} 0, & i \neq k, j \neq l \\ -\frac{i}{\sqrt{2}} S_{il}; & i \neq l, j = k \\ \frac{i}{\sqrt{2}} S_{kj}; & i = l, j \neq k \\ -\frac{i}{\sqrt{2}} S(j, l); & i = k, j \neq l \\ \frac{i}{\sqrt{2}} S(i, k); & j = l, i \neq k \\ i(e_i e_i^T - e_j e_j^T); & i = k, j = l \end{cases} \\
[S_{ij}, D_l] &= \begin{cases} 0, & j < l + 1 \\ -i\sqrt{\frac{l+2}{l+1}} J_{ij}; & j = l + 1 \\ -\frac{i}{\sqrt{(l+1)(l+2)}} J_{ij}; & i < l + 1 < j \\ i\sqrt{\frac{l+1}{l+2}} J_{ij}; & i = l + 1 \\ 0; & i > l + 1 \end{cases} \\
[J_{ij}, S_{kl}] &= \begin{cases} 0, & i \neq k, j \neq l \\ \frac{i}{\sqrt{2}} S_{kj}; & k \neq j, l = i \\ -\frac{i}{\sqrt{2}} S_{il}; & k = j, l \neq i \\ \frac{i}{\sqrt{2}} S(l, j); & k = i, l \neq j \\ -\frac{i}{\sqrt{2}} S(k, i); & l = j, k \neq i \\ -i(e_i e_i^T - e_j e_j^T); & i = k, j = l \end{cases} \\
[J_{ij}, J_{kl}] &= \begin{cases} 0, & i \neq k, j \neq l \\ -\frac{i}{\sqrt{2}} J_{il}; & j = k, i \neq l \\ \frac{i}{\sqrt{2}} J_{kj}; & i = l, j \neq k \\ \frac{i}{\sqrt{2}} J(j, l); & i = k, j \neq l \\ \frac{i}{\sqrt{2}} J(i, k); & j = l, i \neq k \\ 0; & j = l, i = k \end{cases}
\end{aligned}$$

$$[J_{ij}, D_l] = \begin{cases} 0, & j < l+1 \\ i\sqrt{\frac{l+2}{l+1}}S_{ij}; & j = l+1 \\ \frac{i}{\sqrt{(l+1)(l+2)}}S_{ij}; & i < l+1 < j \\ -i\sqrt{\frac{l+1}{l+2}}S_{ij}; & i = l+1 \\ 0; & i > l+1 \end{cases}$$

$$[D_l, S_{ij}] = \begin{cases} 0, & j < l+1 \\ i\sqrt{\frac{l+2}{l+1}}J_{ij}; & j = l+1 \\ \frac{i}{\sqrt{(l+1)(l+2)}}J_{ij}; & i < l+1 < j \\ -i\sqrt{\frac{l+1}{l+2}}J_{ij}; & i = l+1 \\ 0; & i > l+1 \end{cases}$$

$$[D_l, J_{ij}] = \begin{cases} 0, & j < l+1 \\ -i\sqrt{\frac{l+2}{l+1}}S_{ij}; & j = l+1 \\ -\frac{i}{\sqrt{(l+1)(l+2)}}S_{ij}; & i < l+1 < j \\ i\sqrt{\frac{l+1}{l+2}}S_{ij}; & i = l+1 \\ 0; & i > l+1 \end{cases}$$

$$[D_l, D_m] = 0$$

$$\{S_{ij}, S_{kl}\} = \begin{cases} 0, & i \neq k, j \neq l \\ \frac{1}{\sqrt{2}}S_{il}; & j = k, i \neq l \\ \frac{1}{\sqrt{2}}S_{kj}; & i = l, j \neq k \\ \frac{1}{\sqrt{2}}S(j, l); & i = k, j \neq l \\ \frac{1}{\sqrt{2}}S(i, k); & j = l, i \neq k \\ e_i e_i^T + e_j e_j^T; & j = l, i = k \end{cases}$$

$$\{S_{ij}, J_{kl}\} = \begin{cases} 0, & i \neq k, j \neq l \\ \frac{1}{\sqrt{2}}J_{il}; & j = k, i \neq l \\ \frac{1}{\sqrt{2}}J_{kj}; & i = l, j \neq k \\ \frac{1}{\sqrt{2}}J(j, l); & i = k, j \neq l \\ -\frac{1}{\sqrt{2}}J(i, k); & j = l, i \neq k \\ 0; & j = l, i = k \end{cases}$$

$$\{S_{ij}, D_l\} = \begin{cases} \frac{2}{\sqrt{(l+1)(l+2)}} S_{ij}, & j < l+1 \\ -\frac{l}{\sqrt{(l+1)(l+2)}} S_{ij}; & j = l+1 \\ \frac{1}{\sqrt{(l+1)(l+2)}} S_{ij}; & i < l+1 < j \\ -\sqrt{\frac{l+1}{l+2}} S_{ij}; & i = l+1 \\ 0; & i > l+1 \end{cases}$$

$$\{J_{ij}, S_{kl}\} = \begin{cases} 0, & k \neq i, l \neq j \\ \frac{1}{\sqrt{2}} J_{kj}; & l = i, k \neq j \\ \frac{1}{\sqrt{2}} J_{il}; & k = j, l \neq i \\ \frac{1}{\sqrt{2}} J(l, j); & k = i, l \neq j \\ -\frac{1}{\sqrt{2}} J(k, i); & l = j, k \neq i \\ 0; & j = l, i = k \end{cases}$$

$$\{J_{ij}, J_{kl}\} = \begin{cases} 0, & i \neq k, j \neq l \\ -\frac{1}{\sqrt{2}} S_{il}; & j = k, i \neq l \\ -\frac{1}{\sqrt{2}} S_{kj}; & i = l, j \neq k \\ \frac{1}{\sqrt{2}} S(j, l); & i = k, j \neq l \\ \frac{1}{\sqrt{2}} S(i, k); & j = l, i \neq k \\ e_i e_i^T + e_j e_j^T; & j = l, i = k \end{cases}$$

$$\{J_{ij}, D_l\} = \begin{cases} \frac{2}{\sqrt{(l+1)(l+2)}} J_{ij}, & j < l+1 \\ -\frac{l}{\sqrt{(l+1)(l+2)}} J_{ij}; & j = l+1 \\ \frac{1}{\sqrt{(l+1)(l+2)}} J_{ij}; & i < l+1 < j \\ -\sqrt{\frac{l+1}{l+2}} J_{ij}; & i = l+1 \\ 0; & i > l+1 \end{cases}$$

$$\{D_l, S_{ij}\} = \begin{cases} \frac{2}{\sqrt{(l+1)(l+2)}} S_{ij}, & j < l+1 \\ -\frac{l}{\sqrt{(l+1)(l+2)}} S_{ij}; & j = l+1 \\ \frac{1}{\sqrt{(l+1)(l+2)}} S_{ij}; & i < l+1 < j \\ -\sqrt{\frac{l+1}{l+2}} S_{ij}; & i = l+1 \\ 0; & i > l+1 \end{cases}$$

$$\{D_l, J_{ij}\} = \begin{cases} \frac{2}{\sqrt{(l+1)(l+2)}} J_{ij}, & j < l+1 \\ -\frac{l}{\sqrt{(l+1)(l+2)}} J_{ij}; & j = l+1 \\ \frac{1}{\sqrt{(l+1)(l+2)}} J_{ij}; & i < l+1 < j \\ -\sqrt{\frac{l+1}{l+2}} J_{ij}; & i = l+1 \\ 0; & i > l+1 \end{cases}$$

$$\{D_l, D_m\} = \begin{cases} \frac{2}{\sqrt{(m+1)(m+2)}} D_l; & l < m \\ 2D_l^2; & l = m \\ \frac{2}{\sqrt{(l+1)(l+2)}} D_m; & l > m \end{cases}$$