An Extreme-Scale Implicit Solver for Complex PDEs: Highly Heterogeneous Flow in Earth's Mantle

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***Abstract*—Mantle convection is the fundamental physical process within earth's interior responsible for the thermal and geological evolution of the planet, including plate tectonics. The mantle is modeled as a viscous, incompressible, non-Newtonian fluid. The wide range of spatial scales, extreme variability and anisotropy in material properties, and severely nonlinear rheology have made global mantle convection modeling with realistic parameters prohibitive. Here we present a new implicit solver that exhibits optimal algorithmic performance and is capable of extreme scaling for hard PDE problems, such as mantle convection. To maximize accuracy and minimize runtime, the solver incorporates a number of advances, including aggressive multi-octree adaptivity, mixed continuous-discontinuous discretization, arbitrarily-high-order accuracy, hybrid spectral/geometric/algebraic multigrid, and novel Schur-complement preconditioning. These features present enormous challenges for extreme scalability. We demonstrate that—contrary to conventional wisdom—algorithmically optimal implicit solvers can be designed that scale out to 1.5 million cores for severely nonlinear, ill-conditioned, heterogeneous, and anisotropic PDEs.**

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I. Earth's Mantle Convection

Earth is a dynamic system in which mantle convection drives plate tectonics and continental drift and, in turn, controls much activity ranging from the occurrence of earthquakes and volcanoes to mountain building and long-term sea level change. Despite its central role in solid earth dynamics, we have enormous first-order gaps in our knowledge of mantle convection, with questions that are as basic as what are the principal driving and resisting forces on plate tectonics to what is the energy balance of the planet as a whole. Indeed, understanding mantle convection has been designated one of the *"10 Grand Research Questions in Earth Sciences"* in a recent National Academies report [1]. We seek to address such fundamental questions as: *(i)* What are the main drivers of plate motion—negative buoyancy forces or convective shear traction? *(ii)* What is the key process governing the occurrence of great earthquakes—the material properties between the plates or the tectonic stress?

Addressing these questions requires global models of earth's mantle convection and associated plate tectonics, with realistic parameters and high resolutions down to faulted plate boundaries. Historically, modeling at this scale has been out of the question due to the enormous computational complexity associated with numerical solution of the underlying mantle flow equations. However, with the advent of multi-petaflops supercomputers as well as significant advances in seismic tomography and space geodesy placing key observational constraints on mantle convection, we now have the opportunity to address these fundamental questions.

Instantaneous flow of the mantle is modeled by the