*Lighting Res. Technol.* 2008; **40**: 347–358

# A physical model of the atmospheric aerosol turbidity for estimating the illuminance of direct sunlight

**AD Brown** BSc, MSc, PhD[a]
[a]School of Architecture, The University of Sheffield, Western Bank, Sheffield, S10 2TN, UK

Received 6 July 2005; Revised 14 August 2008; Accepted 16 September 2008

The aerosol turbidity index is the key variable in simple parameterisation models of direct normal solar radiation on a plane perpendicular to the direction of the sun. Techniques that use fixed design standard values of the turbidity index do not adequately account for variations in aerosol attenuation. This study presents a simple physical model to estimate the Unsworth–Monteith illuminance turbidity index $T_{Ui}$, using aerosol mass concentration ($PM_{10}$) data that are routinely available from air pollution monitoring stations. $T_{Ui}$ is parameterised in terms of the aerosol scale height and the ground level aerosol extinction coefficient. A measured set of hourly average 'clear sun' direct normal illuminances, and $PM_{10}$ data for the urban location of Sheffield UK was used to validate the proposed model.

## Nomenclature

| | |
|---|---|
| CDA | Clean dry atmosphere |
| $E_{e0}$ | Extraterrestrial direct normal solar irradiance ($Wm^{-2}$) |
| $E_{e0\lambda}$ | Spectral extraterrestrial direct normal solar irradiance ($Wm^{-2}$) |
| $E_{ed}$ | Diffuse solar irradiance ($Wm^{-2}$) |
| $E_{eg}$ | Global solar irradiance ($Wm^{-2}$) |
| $E_{es}$ | Direct normal solar irradiance ($Wm^{-2}$) |
| $E_{v0}$ | Extraterrestrial direct normal solar illuminance (lux) |
| $\overline{E_{v0}}$ | Mean value of extraterrestrial direct normal solar illuminance (lux) |
| $E_{vs}$ | Direct normal solar illuminance (lux) |
| $H$ | Aerosol scale height (km) |
| $J$ | Julian day |
| $m$ | Relative optical mass (air mass), dimensionless |
| $m_R$ | Rayleigh optical mass, dimensionless |
| $m_a$ | Aerosol optical mass, dimensionless |
| $m_g$ | Uniformly mixed gases optical mass, dimensionless |
| $m_o$ | Stratospheric ozone optical mass, dimensionless |
| $m_w$ | Water vapour optical mass, dimensionless |
| $m_n$ | Stratospheric nitrogen oxide optical mass, dimensionless |
| mc | Aerosol mass concentration ($\mu g\, m^{-3}$) |
| $p$ | Pressure (mb) |
| $p_o$ | Pressure at sea level (mb) |
| $p_s$ | Partial pressure of water vapour (mb) |
| $PM_{10}$ | Aerosol mass concentration (10 micron) ($\mu g\, m^{-3}$) |
| $PM_{2.5}$ | Aerosol mass concentration (2.5 micron), ($\mu g\, m^{-3}$) |
| RH% | Relative humidity |
| $T$ | Ambient temperature (K) |
| $T_L$ | Linke irradiance turbidity index, dimensionless |
| $T_{Li}$ | Linke illuminance turbidity index, dimensionless |
| $T_{um}$ | Unsworth-Monteith irradiance turbidity index, dimensionless |
| $T_{ui}$ | Unsworth-Monteith illuminance turbidity index, dimensionless |
| $u_n$ | Stratospheric nitrogen dioxide amount at normal temperature and pressure (see text for units) |

Address for correspondence: AD Brown, 29 Hazeldene Avenue, Bridgend, CF31 2JW, UK.
E-mail: A.D.Brown@uwic.ac.uk

© The Chartered Institution of Building Services Engineers 2008 10.1177/1477153508098686

| | |
|---|---|
| $u_o$ | Stratospheric ozone amount at normal temperature and pressure (see text for units) |
| $V$ | Visibility (km) |
| $V_\lambda$ | Photopic luminous efficiency function |
| $w$ | Precipitable water vapour content of the atmosphere (see text for units) |
| $\alpha$ | Angstrom exponent |
| $\alpha_R$ | Optical depth of CDA, dimensionless |
| $\alpha_{vR}$ | Illuminance optical depth of CDA, dimensionless |
| $\beta$ | Angstrom turbidity coefficient |
| $\varepsilon$ | Sky clearness index |
| $\lambda$ | Wavelength (nm). |
| $\sigma$ | Total atmospheric extinction coefficient ($km^{-1}$) |
| $\sigma_{0.555}$ | Aerosol extinction coefficient at 0.555 μm ($km^{-1}$) |
| $\tau_{0.555}$ | Aerosol optical depth at 0.555 μm, dimensionless |
| $\tau_{ai}$ | Illuminance aerosol optical depth, dimensionless |
| $\tau_{gi}$ | Illuminance mixed gases optical depth, dimensionless |
| $\tau_{ni}$ | Illuminance nitrogen dioxide optical depth, dimensionless |
| $\tau_{oi}$ | Illuminance ozone optical depth, dimensionless |
| $\tau_{Ri}$ | Illuminance Rayleigh optical depth, dimensionless |
| $\tau_{wi}$ | Illuminance water vapour optical depth, dimensionless |
| $\tau_a$ | Irradiance aerosol optical depth, dimensionless |
| $\tau_g$ | Irradiance mixed gases optical depth, dimensionless |
| $\tau_R$ | Irradiance Rayleigh optical depth, dimensionless |
| $\tau_w$ | Irradiance water vapour optical depth, dimensionless |
| $\theta_z$ | Solar zenith angle (degrees) |

## 1. Background

### 1.1. Introduction

Above the Earth's atmosphere the strength of the direct solar beam has a practically constant value, the so-called extraterrestrial solar constant. On traversing the earth's atmosphere, before reaching the surface, the solar beam is attenuated by the constituents of the atmosphere. When cloud is present in the direction of the sun this dominates the attenuation of direct sunlight and far outweighs the effects of other atmospheric constituents. When the sky is cloudless in the direction of the sun, the attenuating processes are in order of importance: scattering and absorption by aerosol particles; Rayleigh scattering by air molecules; absorption by air molecules (the so-called mixed gases and water vapour); and absorption by stratospheric nitrogen dioxide and ozone.[1]

In addition to its predominant attenuating effect, aerosol attenuation is also the most variable of the atmospheric attenuating processes. For example, adjacent rural and urban locations can be subject to differing turbidities, and levels of turbidity at a given location fluctuate on a daily and seasonal basis. Appropriate estimates of the turbidity are needed for use in the simple parameterisation models used for estimating direct normal solar radiation (i.e. that received by a surface at right angles to the solar beam). The accuracy of the parameterisation models is largely determined by the accuracy with which the turbidity can be estimated. The current work presents a simple physical model of the Unsworth-Montieth[2] illuminance turbidity index that uses inputs of routinely measured $PM_{10}$ atmospheric aerosol mass concentrations.

### 1.2. Current techniques for estimating turbidity

Existing techniques can be divided into two main approaches, namely a design standards approach based on empirical observation, and a theoretical approach linking vertical turbidity to horizontal visible range.

According to visibility theory[3] the total extinction coefficient along a ground level horizontal path and the visibility are related by

$$V = \frac{3.912}{\sigma} \tag{1}$$

The total extinction coefficient is made up of contributions due to water vapour absorption, gaseous absorption, Rayliegh scattering, and aerosol scattering and absorption. The aerosol extinction coefficient can be isolated after using empirical algorithms to account for the contribution of the other extinction mechanisms.[4]

Algorithms relating vertical turbidity to ground level horizontal visibility have relied on standard atmosphere-type models of the vertical optical structure of the atmosphere. An example of this approach is due to King and Buckius,[5] who used the model atmospheres of McClatchey *et al.*[6] to produce the following algorithm for estimating Angstrom's turbidity index at 550 nm from visibility measurements,

$$\beta = (0.55)^{\alpha}\left(\frac{3.91}{V} - 0.01162\right) \times \left[\frac{(1.577 - 1.132)(V - 5)}{(23 - 5) + 1.132}\right] \tag{2}$$

The possibility of using visibility observations to make practical turbidity estimates was revisited in a paper describing a spectral irradiance code, SMARTS2,[7]which presents what is in effect an up-dated version of Equation (2) but using more recent model atmospheres in its development.

The importance of accounting for the attenuating effects of aerosols was demonstrated in a study that developed an empirical model of luminous efficacy for the urban location of Geneva.[8] The model included the Angstrom turbidity index estimated from local airport visibility via Equation (2), as an input parameter. The model was tested against published direct luminous efficacy models (mostly simple empirical correlations to solar altitude) and found to perform significantly better.

Several workers have proposed empirical design standard values of atmospheric turbidity that can be used when no other information is available. Design standard values of the turbidity are specified for land-use categories, for example industrial, urban, and rural sites, and are intended to reflect the typical atmospheric aerosol loading for such sites. A well-known example of the above approach for estimating Linke irradiance turbidity was due to Dogniaux (in Molineaux *et al.*[8]). In the case of daylighting studies, Page (in Tregenza and Sharples[9]) suggests the following design standards values for the Linke illuminance turbidity: clean dry air 1.0; dry mountain areas 1.5; rural 2.5; urban 3.0; and industrial 5.0.

Navvab *et al.*[10] conducted a study at San Francisco (USA) to compare measured values of the Linke turbidity index with values estimated via design standards approaches. The study revealed short-comings in the ability of the design standards approach to model the observed seasonal and diurnal variations in the atmospheric turbidity of the site.

The foregoing review allows certain conclusions to be drawn. First, as atmospheric turbidity is a highly variable quantity, the design standard approach with constant estimated values of turbidity at a given site, has obvious weaknesses in terms of its temporal and spatial resolution.[10] Verbal descriptions of land-use type could also be ambiguous.

The approach whereby turbidity is estimated from visibility observations is physically appealing in that it offers a simple parameterisation model of turbidity in terms of routine observations of an optical quantity, which can be related to atmospheric aerosol loading. This allows some possibility of accommodating the spatial and temporal variations in contrast to a design standards approach.

The algorithm of King and Buckius[5] is consistent with the convention used in simple parameterisation models for dealing with attenuation by the various atmospheric constituents.[11] The approach of the latter authors employs the concept of a reduced or scale height, together with a 'measure' of the attenuation at ground level, in a simple algorithm which can be used to estimate turbidity. The study by Molineaux *et al.*[8] shows that even a gross estimate of the variation in aerosol attenuation on an hour-to-hour basis and using local visibility observations brings significant improvements to solar radiation modelling. However, there are problems with using visibility data related to their subjective nature and availability at a site.[12]

A combination of the above-mentioned concerns as to the adequacy of existing approaches, together with the availability of an algorithm for measuring illuminance turbidity, immune to solar altitude dependence,[13] and the increased availability of hourly automatic atmospheric dust measurements have prompted this study.

### 1.3. Theoretical background

Simple parameterisation models can be used to estimate the amount of broadband direct solar radiation incident at the Earth's surface. These models are based on the well-known Bouguer–Lambert law and are of two distinct forms depending on how the atmospheric turbidity index is defined. In their commonly used forms, the models can be exemplified by,

$$E_{es} = E_{e0} \exp -[m(\alpha_R T_L)] \quad \text{(after Navaab } et\ al.^{10}\text{)} \tag{3}$$

$$E_{es} = E_{e0} \exp -m\left[\tau_w + \tau_g + \tau_a + \tau_g\right] \quad \text{(after Leckner}^{11}\text{)} \tag{4}$$

In Equation (3) $\alpha_R$ denotes the optical depth of a clean dry atmosphere (CDA),[9] which for broadband applications has been averaged over all wavelengths. $T_L$ is the Linke turbidity index,[10] which for many years been the means by which atmospheric turbidity has been quantified. Linke's approach was to treat the atmospheric attenuation as consisting of a practically invariant contribution from dry air molecules, and a more variable contribution from water vapour molecules and aerosol particles. The Linke turbidity index, which is a dimensionless number, can be thought of as the number of clean dry atmospheres required to produce the observed attenuation due to molecules of dry air, plus associated water vapour and aerosol.[10]

A spectral model in the form of Equation (4) (here integrated over the extraterrestrial solar spectrum) was originally due to Leckner.[11] The $\tau$ terms are the optical depths of the individual attenuating processes, again in this case, averaged over all wavelengths. Leckner[11] provided algorithms for estimating each of the optical depths based on convenient ground level measurements or reasonable assumptions about the relevant atmospheric constituents, and the concept of scale heights. An update of Leckner's model[11] with a more complete parameterisation and optical depth algorithms incorporating more recent knowledge, was provided by Gueymard[7] (see Equation (7) below).

It can be seen that compared to Equation (3), Equation (4) offers a more complete physical parameterisation by considering the individual optical depths of each constituent of the atmosphere. In addition, because Leckner's parameterisation avoids convolving water vapour and aerosol attenuation it allows a specific treatment of what might be considered the 'true' turbidity i.e. that due to atmospheric aerosols.[13] In a climatological study of aerosols and solar radiation, which presaged the work of Leckner,[11] Unsworth and Monteith[2] used a similar parameterisation approach to measure the aerosol optical depth. In recognition of their priority the aerosol optical depth $\tau_a$ in Equation (4) is called the Unsworth–Monteith turbidity index, denoted as $T_{UM}$.

The current work concerns the modelling of solar direct illuminances from the point of view of day-lighting in buildings. In Equation (4) the value of the solar constant $E_{e0}$ is obtained by integrating the extraterrestrial spectral irradiance over the whole solar spectrum. In daylighting studies, however it is the visible portion of the solar spectrum (380–770 nm) which is of interest, so the integration is performed over this wavelength band, with a weighting function, called the photopic luminous efficiency function $V_\lambda$ to account for the variation in the sensitivity of the human eye to different wavelengths.[14] The function $V_\lambda$ has a peak value of 1 at 555 nm and a value of 0 at 380 nm and 770 nm. Application of the photopic luminous efficiency function converts the broadband irradiance into illuminance. Equations analogous to 3 and 4 can be written for direct normal illuminance,

$$E_{vs} = E_{v0} \exp -[m(\alpha_{vR} T_{Li})] \tag{5}$$

$$E_{vs} = E_{v0} \exp -\left[m\left(\tau_{Ri} + \tau_{wi} + \tau_{ai} + \tau_{gi}\right)\right] \tag{6}$$

where, $E_{v0} = \int_{380\,\mathrm{nm}}^{770\,\mathrm{nm}} E_{e0\lambda} V_\lambda \mathrm{d}\lambda$, and $\tau_{Ri}$ etc. are the average illuminance optical depths for each attenuating process calculated by integrating the wavelength dependency of each process over the visible band. In Equation (6) the illuminance aerosol optical depth $\tau_{ai}$ can be replaced by $T_{ui}$ the illuminance Unsworth–Monteith turbidity index. Gueymard and Kambezidis[13] have presented a more complete parameterisation and algorithms for estimating the illuminance optical depths, and the individual air masses,

$$E_{vs} = E_{v0} \exp -\big(m_R \tau_{Ri} + m_g \tau_{gi} + m_o \tau_{oi} + m_n \tau_{ni} + m_w \tau_{wi} + m_a T_{Ui}\big) \tag{7}$$

Gueymard and Kambezidis[13] have demonstrated that as a consequence of the application of the photopic luminous efficiency function $V_\lambda$, $T_{Ui}$ approximates to a spectral aerosol optical depth at the peak wavelength of the photopic curve, i.e. 555 nm. A useful consequence of the pseudospectral nature of illuminance quantities in this context is the minimisation of parasitic virtual diurnal variation of the turbidity index as a result of changing air mass, this being a problem in the case of broadband irradiance turbidity.[13]

Consistent with his scheme for estimating the optical depths of other atmospheric constituents, Leckner[11] attempted to estimate the spectral aerosol optical depth at 555 nm, $\tau_{0.555}$, from ground based observations of visibility (via Equation (1)), and using the concept of an aerosol scale height, by means of the following equation,

$$\tau_{0.555} = \sigma_{0.555} H. \tag{8}$$

In Equation (8) above, the aerosol scale height $H$ (km) can be defined as the vertical height of an equivalent homogeneous atmosphere required to give the observed optical depth for a given ground level extinction coefficient.[15]

However, Leckner[10] did not pursue this scheme for estimating aerosol optical depths because of concerns about the subjective quality of visibility data.

The current work draws together the model form of Equation (8) with the insight of Gueymard and Kambezidis[13] that the spectral aerosol optical depth at 555 nm and $T_{ui}$ can be considered practically equivalent, to produce the model,

$$T_{ui} = \sigma_{0.555} H \tag{9}$$

### 1.4. Model development

A literature search was made to find simple practical means for estimating $\sigma_{0.555}$ and $H$ for use in the model (Equation (9)). A relation between the ground level spectral aerosol extinction coefficient and ground level aerosol mass concentration is given by an empirical parameter called the aerosol mass extinction coefficient, $\sigma_{0.555}$/mc where mc is the aerosol mass concentration.[16] Trier and Horvath[16] reviewed values of aerosol mass extinction coefficients at 550 nm for $PM_{10}$ particles obtained at urban sites worldwide, and noted that a value of about 5 $m^2$/g appeared to be representative of urban atmospheres. In the units used in the current study this value becomes $5 \times 10^{-3}\,km^{-1}/\mu gm^{-3}$. The latter value is assumed in the current study as being an independent estimate of the aerosol mass extinction coefficient representative of urban conditions.

For the purposes of the current model the aerosol scale height, $H$ was assumed to have a fixed standard value of 1 km. This assumption for the value of $H$ has been made in previous studies where measurements of the vertical spectral aerosol optical depth have been used to estimate the ground level aerosol extinction coefficient via Equation (8).[16] With the coefficients inserted in Equation (9) the current model becomes,

$$T_{ui} = 0.005.\text{mc}.1 \tag{10}$$

where the last term on the right-hand side refers to the assumed constant aerosol scale height of 1 km, and mc is in units of $\mu g/m^3$.

Gueymard and Kambezidis[13] noted that $T_{Ui}$ in Equation (7) should strictly include attenuation by tropospheric nitrogen dioxide, $NO_2$, as well as aerosol attenuation. In the current study any attenuation due to tropospheric $NO_2$ has been neglected because the relevant hourly data were not available. It has been shown that even for very polluted sites the contribution of $NO_2$ to overall $T_{Ui}$ was relatively small compared to the aerosol contribution.[13] Published summary data for ground level concentrations of $NO_2$ in Sheffield[17] indicated levels that would have a negligible effect on illuminance turbidity.

## 2. Data collection

All solar radiation, meteorological and atmospheric aerosol measurements were made at the urban location of Sheffield (UK) (52 23 N, 128 W), for the period March 1997 to June 1998. The solar radiation measurements relevant to this study, were of direct normal irradiance, horizontal global irradiance and direct normal illuminance. The measurement site was a city centre flat roof near to ground level, chosen so as to have as unrestricted a view of the horizon as possible, and to be free of overshadowing and reflection from adjacent buildings. A Licor photometer was used for direct illuminance measurements, and an Eppley pyrheliometer for direct irradiance measurements. Both instruments were mounted on an Eppley Sun tracker. Global irradiance measurements were made with a Kipp and Zonen pyranometer. According to the manufacturers the uncertainties in the measurements are ±5% for the Licor photometer. The solar radiation data was automatically logged at 1-min intervals using a Delta-T data logger. The solar tracker was adjusted for changing solar declination at least weekly, and more frequently near the equinoxes. The raw data was subjected to simple quality control checks[18]: $0 < E_{eg} < 1.2E_{e0}$; $0 < E_{ed} < 0.8E_{e0}$. Data preprocessing also involved restricting the data to instances where solar altitude was $>10^\circ$ in order to eliminate any residual air mass effects.[13]

To eliminate cloud contamination, and to isolate the effects of aerosols, sky clearness selection criteria were applied to the data. Nominal threshold values of direct irradiance >200 $\mathrm{Wm}^{-2}$,[10] and sky clearness $\varepsilon \geq 6.2$,[19] were used to indicate conditions where the sun was not obscured by cloud (i.e. 'clear sun' conditions). The selected 1-min data were then processed into hourly averages for hours where the above criteria were satisfied for at least 95% of the time.[12]

Hourly average atmospheric $PM_{10}$ aerosol mass concentration at ground level was measured in units of $\mu g/m^3$ using a tapered element oscillating microbalance instrument (TEOM) manufactured by Ruprecht and Pataschnik. The TEOM instrument was located at a city-centre site ~1.5 km from the solar radiation measurement site and was operated as part of the Department of the Environment's Automatic Urban Network. There were periods during the measurement campaign when the TEOM instrument was out of action for maintenance, and no aerosol data were available.

Ambient relative humidity and air temperature were measured as hourly averages at the same site as the solar radiation measurements. The latter parameters were measured with a Sky Instruments 'Data Hog' unit fitted with a Stevenson screen.

It is believed that the sky clearness criterion applied to exclude cloud contamination, may have excluded instances where the sky was clear in the direction of the sun, but the rest of the sky contained large quantities of white cloud. Under such conditions (common in the UK) the sky clearness index might be depressed below the nominal threshold value used to indicate a clear sky, by the high-sky brightness. The data selection software would then have excluded such cases even though in reality 'clear sun' conditions prevailed. The circumstances described above may have contributed to the database being smaller than expected. In all 158 simultaneous hourly average measurements were available for model development. The data are considered to be representative of the range of seasonal, diurnal, and air pollution conditions at the site.

## 3. Model validation and results

The proposed model (Equation 10) was evaluated by plotting measured against modelled values of direct solar illuminance. Evaluation was also conducted for an existing technique to allow comparisons to be made. In accordance with practice in the field of solar energy engineering the test statistics employed in the model evaluations were the coefficient of determination, $R^2$ (at the 99% confidence level), the mean bias error (MBE), and the root mean squared error (RMSE).[20] In the current work MBE and RMSE are given in units of klx, and also as percentages. Percentage RMSE and MBE are calculated by expressing the difference between modelled and measured $E_{vs}$ as a percentage of the measured value, in the formulae used to calculate MBE and RMSE.

The existing technique selected for evaluation was due to Page (in Tregenza and Sharples[9]). Page suggested empirical design standard values for the Linke illuminance turbidity factor for various land-use types (see sec.1.2 above). The design standard values of the Linke illuminance turbidity, together with estimates of the mean extinction coefficient of clean dry air, the air mass, and the extraterrestrial illuminance can be inserted into Equation (5) to yield the direct illuminance.

All the measured data was for solar altitudes above 10°, so the relative optical mass (air mass) $m$ can be approximated from the following algorithm,[9]

$$m = \sec \theta_Z \tag{11}$$

The mean extinction coefficient of clean dry air can be approximated as,

$$\alpha_{vR} = \frac{0.1}{(1 + 0.0045\,\mathrm{m})} \quad (12)$$

after Navaab *et al.*[10] (in Tregenza and Sharples[9]). The following algorithm was used to account for seasonal variations in $E_{v0}$ due to the ellipticity of the earth's orbit,[9]

$$E_{v0} = \overline{E_{v0}}\left[1 + 0.034\cos\left[\frac{2\pi(J-2)}{365}\right]\right] \quad (13)$$

The question of which value of $\overline{E_{v0}}$ to use in the current analysis was problematic. The value of $\overline{E_{v0}}$ used by Page in 1982 was 127.5 klx. This was the value used to obtain the empirical design standard values of the Linke illuminance turbidity quoted in Section 1.2 above. A more recent value of $\overline{E_{v0}}$ used by Molineaux *et al.*[7] for example, and based on the extraterrestrial spectrum presented by Gueymard,[8] is 133 klx. To compare the performance of the proposed model with the design standards model, the earlier value of 127.5 klx was used with the latter model, as this is appropriate to the design standard values of the Linke illuminance turbidity given by Page.

If we use the above algorithms and approximations, together with the design standard value of $T_{Li} = 3.0$ as representative of the urban conditions of Sheffield, Page's technique can be tested against the measured direct illuminance recorded for the current study. Figure 1 shows modelled versus measured direct illuminance. The evaluation statistics for the plot gave, $R^2 = 0.67$, MBE $= -7.60$ klx ($-10.2\%$) and RMSE $=$ 11.82 klx (16.7%).

The value of the design standard Linke illuminance turbidity is fixed, so practically all of the variation in modelled illuminance is due to changing air mass. Direct illuminance is strongly negatively correlated to air mass, therefore any model using this as a predictor will give a good correlation. However, systematic bias in the predicted illuminances, that is values consistently too high or too low, depends on choosing a representative design standard value of the illuminance turbidity for the location.

The plot shows that there is a tendency to under prediction of illuminances in Sheffield, when a fixed design standard value for the Linke illuminance turbidity is used. The model under-prediction is revealed by the negative MBE, and is graphically shown in Figure 1. Figure 1 shows a sharp cut-off of modelled illuminances at about 90 klx even though measured values go up to about 100 klx.

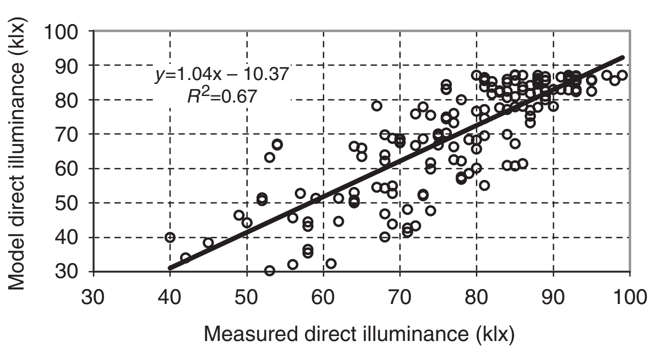


**Figure 1** Model vs measured direct illuminance using the design standards approach

The maximum possible modelled illuminances obtained with Page's model depend on the latitude of the location. In Sheffield the highest mid-summer solar altitudes (and hence the lowest air masses) do not exceed about 60° (air mass = 1.15), which results in a maximum modelled value of about 90 klx.

In terms of the Unsworth–Monteith illuminance turbidity index the parameterisation of the direct illuminance is given by Equation (7). Gueymard and Kambezidis provide algorithms for estimating the optical depths of each atmospheric constituent, (apart from $T_{\mathrm{Ui}}$), and the individual air masses.[13] In the current work, because all illuminance measurements were made at a solar altitude greater than 10°, all air masses can be considered identical to the Rayleigh optical mass and the usual approximation can be made using Equation (11). In Equation (7), the optical depths are given by the following algorithms,[13]

$$\tau_{\mathrm{Ri}} = 0.0966\frac{p}{p_0} - \left(4.606 \times 10^{-4}\frac{p}{p_0}\right)m_{\mathrm{R}} \quad (14)$$

$$\tau_{\mathrm{oi}} = 0.0768u_{\mathrm{o}} \quad (15)$$

$$\tau_{\mathrm{gi}} = \frac{2.569 \times 10^{-4}\,(p/p_0)^{0.3}}{(1 + 0.32245m_{\mathrm{R}})} \quad (16)$$

$$\tau_{\mathrm{ni}} = 2.48u_{\mathrm{n}} \quad (17)$$

$$\tau_{\mathrm{wi}} = \frac{(1.506 \times 10^{-3}(p/p_0))w^{0.97}}{(1 + 0.065m_{\mathrm{w}})} \quad (18)$$

In the current work, small variations in $\tau_{\mathrm{Ri}}$, $\tau_{\mathrm{gi}}$ and $\tau_{\mathrm{wi}}$ due to changes in atmospheric pressure were neglected because the latter data were not available. In Equations (15) and (17), $u_{\mathrm{o}}$ and $u_{\mathrm{n}}$ refer to the total amounts of stratospheric ozone and nitrogen dioxide, respectively. In the engineering literature the amounts of these gases are usually given in units of atm cm or cm (NTP) which are used interchangeably. Both the latter units refer to the height of a column of gas at normal temperature and surface pressure, (NTP).[21] In the current study values of $u_{\mathrm{o}}$ and $u_{\mathrm{n}}$ of 0.3434 cm (NTP) and 0.0003 cm (NTP), respectively, were taken from Gueymard and Kambezidis as representative of middle and high-latitude sites. In a similar way to the stratospheric gases, the precipitable water content of the atmosphere is given in the literature in units of cm and Dogniaux (quoted in Navaab *et al.*[10]) suggests values between 2 and 4 cm for temperate locations. In the current study, hourly average precipitable water was calculated from measured values of the hourly average air temperature and relative humidity using the following algorithm,[21]

$$w = 0.493\mathrm{RH}\%\frac{p_{\mathrm{s}}}{T} \quad (19)$$

Values of the precipitable water estimated using the above algorithm were inserted in Equation (18). $T_{\mathrm{ui}}$ was estimated using the currently proposed model, Equation (10). A value of 133 klx as used by Molineaux *et al.*[8] (see above) was assumed for $\overline{E_{\mathrm{v0}}}$. The latter value of $\overline{E_{\mathrm{v0}}}$ was used in Equation (13) to account for the seasonal variations in the extraterrestrial illuminances and the resulting value of $E_{\mathrm{v0}}$ was inserted in Equation (7).

The plot of measured against modelled direct illuminance is shown in Figure 2. The evaluation statistics for the plot were $R^2 = 0.66$, MBE = −0.04 klx (−0.4%) and RMSE = 9.74 klx (14.6%). It can be seen from the plot that in contrast to Figure 1 there is no sharp cut-off of the maximum-modelled value, and the model is able to represent the full range of illuminance measured at the site. The latter fact is reflected in the small negative value of the MBE for the current plot. This situation can be compared to the larger negative MBE in the case of

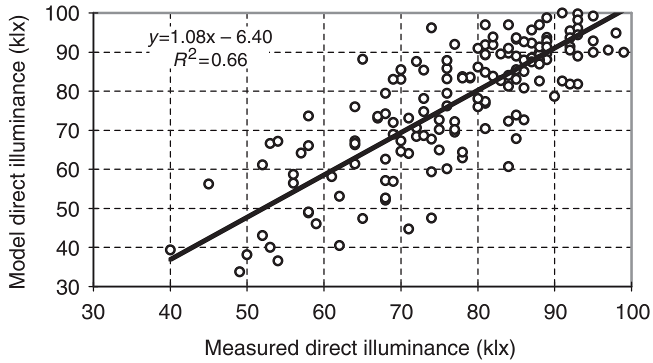


**Figure 2** Model vs measured direct illuminance using the proposed model

Figure 1 that revealed a marked model underprediction. In addition, the plot revealed a smaller value of the RMSE compared to Figure 1. The values of the $R^2$ statistic are comparable for both plots, which is a consequence of the strong air mass dependency.

## 4. Conclusions

This work shows that estimates of illuminance turbidity made using the currently proposed model allow more realistic estimates of direct normal solar illuminance than design standards techniques. This is because the input to the current model is measured atmospheric aerosol mass concentrations, which represent the state of the atmosphere at any given time.

Possible sources of uncertainty in the current model can be attributed to the model coefficients used. The assumed value for the aerosol mass extinction coefficient used in the current model, taken from Trier and Horvath,[16] appears to be representative of average conditions in terms of the chemical composition and size distribution of the Sheffield aerosol. However, the value of the extinction coefficient at any instant will depend on both the aerosol chemical composition and the proportion of the more optically important $PM_{2.5}$ present in the $PM_{10}$ fraction. The greater the proportion of $PM_{2.5}$, the larger the value of the mass extinction coefficient. Harrison *et al.*[22] showed how meteorological conditions at a given location influence the relative proportions of the two aerosol size fractions. Use of measured $PM_{2.5}$ data, where available, (together with an appropriate value of the aerosol mass extinction coefficient) would reduce this source of uncertainty.

The assumption of a fixed aerosol scale height is reasonable, but variations from this value are likely to produce uncertainty in the current model. Studies have shown that measured scale heights can range in value from a few hundred metres under cold stagnant meteorological conditions, to several kilometres under buoyant well-mixed conditions.[4]

Future work might look at evaluating the current parameterisation model at other locations, both urban and rural, using locally appropriate coefficients if available. The model should also be evaluated over a wider range of illuminances. The current parameterisation model form and coefficients can however, be used to provide estimates of the 'clear sun' direct normal solar illuminance in urban locations where $PM_{10}$ data are available.

## Acknowledgments

The author would like to thank to the following for their assistance: Professor Peter R. Tregenza (University of Sheffield) for project supervision and guidance. Professor Steve Sharples (University of Sheffield) for helpful comments and suggestions. Andy Elleker (Environmental Protection Unit of Sheffield City Council), for supplying air pollution data. This work was supported by a scholarship awarded by the University of Sheffield.

## References

1 Molineaux B, Ineichen P. On the broadband transmittance of direct irradiance in a cloudless sky and its application to the parameterisation of atmospheric turbidity. *Solar Energy* 1996; 6: 553–556.
2 Unsworth MH, Monteith JL. Aerosol and solar radiation in Britain. *Quarterly Journal of the Royal Meteorological Society* 1972; 98: 778–797.
3 Middleton WEK. *Vision through the atmosphere*. Toronto, University of Toronto Press, 1952.
4 Tomasi C. Features of the aerosol scale height for particulate extinction in hazy atmospheres. *Journal of Applied Meteorology* 1982; 21: 931–944.
5 King R, Buckius RO. Direct solar transmittance for a clear sky. *Solar Energy* 1979; 22: 297–301.
6 McClatchey RA, Fenn RW, Selby EA, Volz FE, Garing SJ. *Optical properties of the atmosphere*. AFCRL-72-0497, Air Force Cambridge Research Laboratories, 1972.
7 Gueymard C. *SMARTS2, a Simple Model of the Atmospheric Radiative Transfer of Sunshine: Algorithms and performance assessment*. FSEC-PF-270-95, Florida Solar Energy Centre, 1995.
8 Molineaux B, Ineichen P, Delaunay JJ. Direct luminous efficacy and atmospheric turbidity-improving model performance. *Solar Energy* 1995; 55: 125–37.
9 Tregenza PR, Sharples S. *Daylighting algorithms*. Energy Technology Support Unit, (ETSU) S/1350, UK Department of Trade and Industry, 1993.
10 Navvab M, Karayel M, Ne'eman E, Selkowitz S. Analysis of atmospheric turbidity for daylight calculations. *Energy and Buildings* 1984; 6: 293–303.
11 Leckner B. The spectral distribution of solar radiation at the Earth's surface-elements of a model. *Solar Energy* 1978; 20: 143–150.
12 Littlefair PJ. Measurements of the luminous efficacy of daylight. *Lighting Research and Technology* 1988; 20(4): 177–188.
13 Gueymard C, Kambezidis HD. Illuminance turbidity parameters and atmospheric extinction in the visible spectrum. *Quarterly Journal of the Royal Meteorological Society* 1997; 123: 679–697.
14 IES Calculation Procedures Committee Recommend Practice for the Calculation of Daylight Availability. *Journal of the Illuminating Engineering Society of North America* 1984; 13(4): 381–392.
15 Elterman L. Relationships between vertical attenuation and surface meteorological range. *Applied Optics* 1970; 9: 1804–1810.
16 Trier A, Horvath H. A study of the aerosol of Santiago de Chile- II. Mass extinction coefficients, visibilities and Angstrom exponents. *Atmospheric Environment* 1993; 27A(3): 385–389.
17 *Air Pollution in the UK: 1994*. Abingdon: National Environment Technology Centre, 1995.
18 CIE. *Guide to recommended practice of daylight measurement*. Vienna, Commission International de l'Eclairage, 1994.
19 Perez R, Ineichen P, Seals R. Modelling daylight availability and irradiance components from direct and global irradiance. *Solar Energy* 1990; 44(5): 271–289.

20 Muneer T. *Solar radiation and daylight models for energy efficient design of buildings*. Oxford, Butterworths-Heinemann, 1997.
21 Iqbal M. *An introduction to solar radiation*. Toronto, Academic Press, 1983.
22 Harrison RM, Deacon AR, Jones MR, Appleby RS. Sources and processes affecting concentrations of $PM_{10}$ and $PM_{2.5}$ particulate matter in Birmingham (UK). *Atmospheric Environment* 1997; 31(24): 4103–4117.