Quick Guide to Vectrix Notation

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Vectrix notation was originally developed by Peter C. Hughes, the author of [1], and is used in [1, 2]. The notation used in AEM 2012, when compared to [1, 2], is slightly modified.

This document summarizes all notation and the language used when discussing physical vectors, reference frames, components of physical vectors, etc.

1 The Three "*r*'s"

The three "r's": *relative*, with *respect* to, *resolved*.

relative: used to describe position and attitude (orientation). The position of point q is defined *relative* to point p. Similarly, the attitude (orientation) of reference frame b is described *relative* to reference frame a.

with *respect* to (w.r.t): used to describe the time-rate-of-change of an entity, such as a physical vector, with respect to a reference frame.

resolved: describing a frame invariant entity, such as a physical vector, in terms of the physical basis vectors of a reference frame.

2 Vectrix Nomenclature

$\underrightarrow{r} = \underrightarrow{\boldsymbol{\mathcal{F}}}_a^{\mathsf{T}} \mathbf{r}_a = \mathbf{r}_a^{\mathsf{T}} \underrightarrow{\boldsymbol{\mathcal{F}}}_a$: physical vector r.

$\underrightarrow{0}$: zero physical vector with zero magnitude and no direction.

$\mathcal{F}_a$: reference frame a, defined by orthonormal dextral physical basis vectors $\underrightarrow{a}^1$, $\underrightarrow{a}^2$, and $\underrightarrow{a}^3$.

$\mathbf{r}_a = \begin{bmatrix} r_{a1} \\ r_{a2} \\ r_{a3} \end{bmatrix}$: physical vector r resolved (or expressed) in frame $\mathcal{F}_a$.

$\underrightarrow{\boldsymbol{\mathcal{F}}}_a = \begin{bmatrix} \underrightarrow{a}^1 \\ \underrightarrow{a}^2 \\ \underrightarrow{a}^3 \end{bmatrix}$: vectrix associated with frame $\mathcal{F}_a$.

$\mathbf{r}_a^{\times} = \begin{bmatrix} 0 & r_{a3} & -r_{a2} \\ -r_{a3} & 0 & r_{a1} \\ r_{a2} & -r_{a1} & 0 \end{bmatrix}$: the cross product matrix of the column matrix $\mathbf{r}_a$.

$\underrightarrow{u} \cdot \underrightarrow{v} = \mathbf{u}_a^{\mathsf{T}} \mathbf{v}_a$: dot product of physical vectors $\underrightarrow{u}$ and $\underrightarrow{v}$.

$\underrightarrow{u} \times \underrightarrow{v} = \underrightarrow{\boldsymbol{\mathcal{F}}}_a^{\mathsf{T}} \mathbf{u}_a^{\times} \mathbf{v}_a$: cross product of physical vectors $\underrightarrow{u}$ and $\underrightarrow{v}$.

$\left\| \underrightarrow{u} \right\|_2 = \sqrt{\underrightarrow{u} \cdot \underrightarrow{u}} = \sqrt{\mathbf{u}_a^{\mathsf{T}} \mathbf{u}_a}$: Euclidean norm of the physical vector $\underrightarrow{u}$.

$\mathbf{C}_{ba} = \underrightarrow{\boldsymbol{\mathcal{F}}}_b \cdot \underrightarrow{\boldsymbol{\mathcal{F}}}_a^{\mathsf{T}}$: direction cosine matrix (DCM) describing the orientation of frame $\mathcal{F}_b$ relative to frame $\mathcal{F}_a$.

$\underrightarrow{r}^{zw}$: the position of point z relative to point w.

$\mathbf{r}_a^{zw}$: the position of point z relative to point w resolved in frame $\mathcal{F}_a$.

$\underrightarrow{\omega}^{ba}$: the angular velocity of frame $\mathcal{F}_b$ relative to frame $\mathcal{F}_a$.

$\boldsymbol{\omega}_c^{ba}$: the angular velocity of frame $\mathcal{F}_b$ relative to $\mathcal{F}_a$ resolved in frame $\mathcal{F}_c$.

$\underset{\rightarrow}{v}^{zw/a} = \underset{\rightarrow}{r}^{zw^{\bullet a}}$: the velocity of point z relative to point w with respect to $\mathcal{F}_a$.

$\mathbf{v}_b^{zw/a}$: the velocity of point z relative to point w with respect to frame $\mathcal{F}_a$ resolved in frame $\mathcal{F}_b$.

$\underset{\rightarrow}{a}^{zw/a/a} = \underset{\rightarrow}{v}^{zw/a^{\bullet a}} = \underset{\rightarrow}{r}^{zw^{\bullet a \bullet a}}$: the acceleration of point z relative to point w with respect to $\mathcal{F}_a$.

$\mathbf{a}_b^{zw/a/a}$: the acceleration of point z relative to point w with respect to frame $\mathcal{F}_a$ resolved in frame $\mathcal{F}_b$.

$\underset{\rightarrow}{f}^{p}$: the force $\underset{\rightarrow}{f}$ applied at point p.

$\underset{\rightarrow}{\tau}^{\mathcal{B}p}$: the torque applied to the rigid body $\mathcal{B}$ relative to point p.

$\underset{\rightarrow}{m}^{yw} = \underset{\rightarrow}{r}^{yw} \times \underset{\rightarrow}{f}^{y}$: the moment on particle y relative to point w due to force $\underset{\rightarrow}{f}^{y}$.

$\underset{\rightarrow}{p}^{ip/a} = m_i \underset{\rightarrow}{r}^{ip^{\bullet a}}$: the momentum of particle i relative to point p with respect to frame $\mathcal{F}_a$.

$\underset{\rightarrow}{h}^{ip/a} = \underset{\rightarrow}{r}^{ip} \times (m_i \underset{\rightarrow}{r}^{ip^{\bullet a}})$, the momentum of particle i relative to point p with respect to frame $\mathcal{F}_a$.

$\mathbf{J}_b^{\mathcal{B}p}$: the second moment of mass of the rigid body $\mathcal{B}$ relative to point p resolved in frame $\mathcal{F}_b$, a 3×3 matrix.

$\mathbf{c}_b^{\mathcal{B}p}$: the first moment of mass of the rigid body $\mathcal{B}$ relative to point p resolved in frame $\mathcal{F}_b$, a column matrix.

3 Mathematical Nomenclature

$\mathbf{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$: 3×3 identity matrix.

$\mathbf{1}_i$: 3×1 column matrix with 1 in the i^{th} row and zeros everywhere else (e.g., $\mathbf{1}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{1}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{1}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$).

$\mathbb{R}$: the set of all real numbers.

$\mathbb{R}^n$: the set of all n by 1 matrices composed of real numbers.

$\mathbb{R}^{n \times m}$: the set of all n by m matrices composed of real numbers.

$\mathbb{P}$: the set of all physical vectors.

$\in$: "is an element of" (e.g., $\mathbf{r}_a^{zw} \in \mathbb{R}^3$).

$\exists$: "there exists" (e.g., $\exists\, \underset{\rightarrow}{0} \in \mathbb{R}$ such that $\underset{\rightarrow}{u} + \underset{\rightarrow}{0} = \underset{\rightarrow}{u}$ for all $\underset{\rightarrow}{u} \in \mathbb{P}$).

$\forall$: "for all" (e.g., there exists $\underset{\rightarrow}{0} \in \mathbb{R}$ such that $\underset{\rightarrow}{u} + \underset{\rightarrow}{0} = \underset{\rightarrow}{u}\ \forall\, \underset{\rightarrow}{u} \in \mathbb{P}$).

References

[1] P. C. Hughes, *Spacecraft Attitude Dynamics*. Mineola, New York: Dover, 2nd ed., 2004.

[2] A. H. J. de Ruiter, C. J. Damaren, and J. R. Forbes, *Spacecraft Dynamics and Control: An Introduction*. West Sussex, UK: John Wiley & Sons, Ltd., 2013.