Disturbance Torques (Moments) on a Spacecraft (12 de Ruiter)

Gravity - Gradient Torque

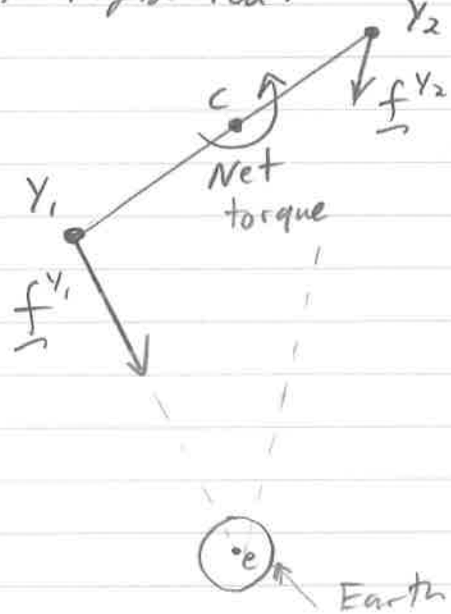
Recall that

$$\underset{\rightharpoondown}{f}^{ye} = -\frac{\mu m_y}{\|\underset{\rightharpoondown}{r}^{ye}\|_2^3}\underset{\rightharpoondown}{r}^{ye}$$

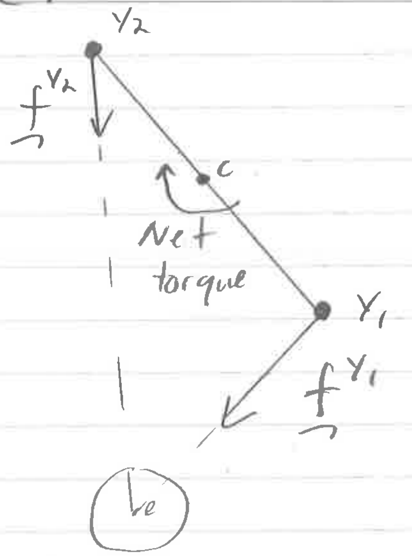
$$\|\underset{\rightharpoondown}{f}^{ye}\|_2 \sim \|\underset{\rightharpoondown}{r}^{ye}\|_2^{-2}$$

Thus, gravitational force on a mass further from Earth is smaller than force on a mass closer to Earth.

Consider two particles of equal mass connected by a massless rigid rod.

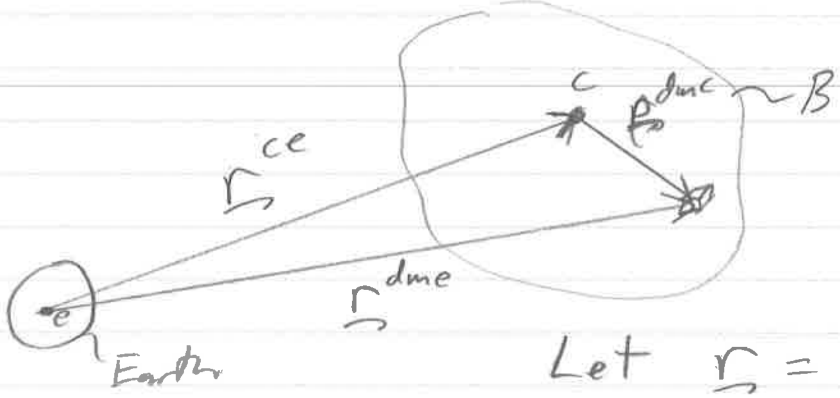


Force acting on Y_1 is larger than force acting on Y_2, thus causing a net torque (moment) about pt. c.



Gravity-gradient torque will try to align body to point toward Earth.

Consider a continuous rigid body (CRB)



Let $\underline{r} = \underline{r}^{ce}$, $\underline{\rho} = \underline{\rho}^{dmc}$

Gravitational force acting on dm is

$$d\underline{f}^{dm} = -\frac{\mu \, dm}{\|\underline{r}^{dme}\|_2^3}\underline{r}^{dme}$$

$$= \frac{-\mu}{\|\underline{r}^{ce}+\underline{\rho}^{dmc}\|_2^3}(\underline{r}^{ce}+\underline{\rho}^{dmc})\,dm$$

Note that

$$\|\underline{r}^{ce}+\underline{\rho}^{dmc}\|_2^3 = \left[(\underline{r}^{ce}+\underline{\rho}^{dmc})\cdot(\underline{r}^{ce}+\underline{\rho}^{dmc})\right]^{3/2}$$

$$\left(= \left[\underline{r}^{ce}\cdot\underline{r}^{ce} + \underline{\rho}^{dmc}\cdot\underline{\rho}^{dmc} + 2\underline{r}^{ce}\cdot\underline{\rho}^{dmc}\right]^{3/2}\right)$$

$$= \left(r^{ce^2} + \rho^{dmc^2} + 2r^{ce}\rho^{dmc}\cos\phi\right)^{3/2}$$

where $r^{ce} = \|\underline{r}^{ce}\|_2$, $\rho^{dmc} = \|\underline{\rho}^{dmc}\|_2$, ϕ is the angle between $\underline{r}^{ce}$ and $\underline{\rho}^{dmc}$.

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$$\left(\|\underline{r}^{ce} + \underline{\rho}^{dmc}\|_2^{\,3} = r^{ce3}\left(1 + \left(\frac{\rho^{dmc}}{r^{ce}}\right)^2 + 2\left(\frac{\rho^{dmc}}{r^{ce}}\right)\cos\phi\right)^{3/2} \right)$$

Assume that $\rho^{dmc} << r^{ce}$

Define $\varepsilon = \frac{\rho^{dmc}}{r^{ce}} << 1$, therefore $\varepsilon^2 \approx 0$

Therefore,

$$\frac{1}{\|\underline{r}^{ce} + \underline{\rho}^{dmc}\|_2^{\,3}} \approx \frac{1}{{r^{ce}}^3(1 + 2\varepsilon\cos\phi)^{3/2}} = f(\varepsilon)$$

Take Taylor series approximation

$$f(\varepsilon) \approx f(0) + \left.\frac{df}{d\varepsilon}\right|_{\varepsilon=0} \varepsilon$$

$$\left(\approx \frac{1}{{r^{ce}}^3} + \left.\frac{d}{d\varepsilon}\left(\frac{1}{{r^{ce}}^3(1+2\varepsilon\cos\phi)^{3/2}}\right)\right|_{\varepsilon=0} \varepsilon \right)$$

$$\left(= \frac{1}{{r^{ce}}^3} - \frac{1}{{r^{ce}}^3}(3\cos\phi)\,\varepsilon \right)$$

$$\frac{1}{\|\underline{r}^{ce} + \underline{\rho}^{dmc}\|_2^{\,3}} \approx \left(\frac{1}{{r^{ce}}^3} - \frac{3}{{r^{ce}}^5}\left(r^{ce}\rho^{dmc}\cos\phi\right) \right)$$

$$= \frac{1}{{r^{ce}}^3} - \frac{3\,\underline{r}^{ce} \cdot \underline{\rho}^{dmc}}{{r^{ce}}^5}$$

Now,

$$d\underline{f}^{dm} \approx -\frac{\mu(\underline{r}^{ce} + \underline{\rho}^{dmc})}{r^{ce^3}}\left(1 - \frac{3\underline{r}^{ce}\cdot\underline{\rho}^{dmc}}{r^{ce^2}}\right)dm$$

Total torque on B relative to c is

$$\underline{\tau}^{Bc,gg} = \int_B \underline{\rho}^{dmc} \times d\underline{f}^{dm}$$

$$= \int_B \underline{\rho}^{dmc} \times \left(-\frac{\mu(\underline{r}^{ce} + \underline{\rho}^{dmc})}{r^{ce^3}}\right) dm$$

$$+ \int_B \underline{\rho}^{dmc} \times \left(\frac{3\mu\, \underline{r}^{ce}\cdot\underline{\rho}^{dmc}}{r^{ce^5}}(\underline{r}^{ce} + \underline{\rho}^{dmc})\right) dm$$

$$= -\frac{\mu}{r^{ce^3}}\left[\left(\underbrace{\int_B \underline{\rho}^{dmc}\, dm}_{\underline{\rho}^{B/c} = \underline{0}}\right) \times \underline{r}^{ce} + \int_B \underbrace{\underline{\rho}^{dmc} \times \underline{\rho}^{dmc}}_{\underline{0}}\, dm\right]$$

$$+ \frac{3\mu}{r^{ce^5}}\left(\int_B \underline{\rho}^{dmc} \times \underline{r}^{ce}(\underline{r}^{ce}\cdot\underline{\rho}^{dmc})\, dm\right.$$

$$\left. + \int_B \underbrace{\underline{\rho}^{dmc} \times \underline{\rho}^{dmc}}_{\underline{0}}(\underline{r}^{ce}\cdot\underline{\rho}^{dmc})\, dm\right)$$

$$= \left(\frac{3\mu}{r^{ce^5}}\int_B \underline{\rho}^{dmc} \times \underline{r}^{ce}(\underline{r}^{ce}\cdot\underline{\rho}^{dmc})\, dm\right)$$

Resolve all vectors in $\mathcal{F}_b$:

$$\underline{\tau}_b^{Bc,gg} = \frac{3\mu}{r^{ce^5}} \int_B \rho_b^{dmc^\times} \underline{r}_b^{ce} \rho_b^{dmc^T} \underline{r}_b^{ce} \, dm$$

$$= -\frac{3\mu}{r^{ce^5}} \underline{r}_b^{ce^\times} \int_B \rho_b^{dmc} \rho_b^{dmc^T} \, dm \, \underline{r}_b^{ce}$$

Recall that $\rho_b^{dmc} \rho_b^{dmc^T} = \rho_b^{dmc^\times} \rho_b^{dmc^\times} + \rho_b^{dmc^T} \rho_b^{dmc} \underline{1}$

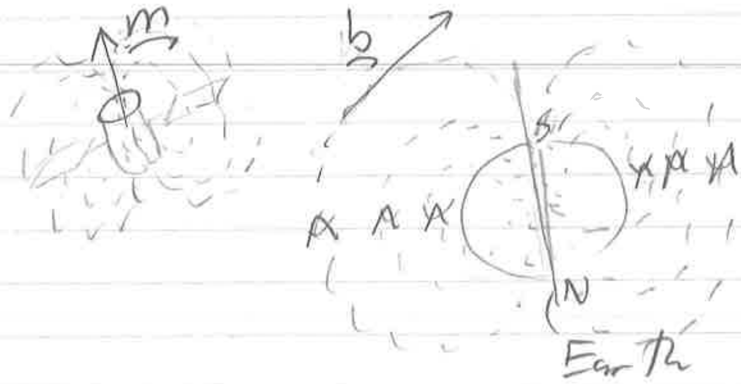
$$\underline{\tau}_b^{Bc,gg} = -\frac{3\mu}{r^{ce^5}} \underline{r}_b^{ce^\times} \int_B \rho_b^{dmc^\times} \rho_b^{dmc^\times} \, dm \, \underline{r}_b^{ce}$$

$$-\frac{3\mu}{r^{ce^5}} \underbrace{\underline{r}_b^{ce^\times} \int_B \rho_b^{dmc^T} \rho_b^{dmc} \underline{1} \, dm \, \underline{r}_b^{ce}}_{= \int_B \rho_b^{dmc^T} \rho_b^{dmc} \, dm \, \cancel{\underline{r}_b^{ce^\times} \underline{r}_b^{ce}} \, \underline{0}}$$

$$\boxed{\underline{\tau}_b^{Bc,gg} = +\frac{3\mu}{r^{ce^5}} \underline{r}_b^{ce^\times} \underline{I}_b^{B} \underline{r}_b^{ce}}$$

↳ Gravity Gradient Torque

Magnetic Disturbance Torque



Earth's magnetic field, $\underset{\rightharpoondown}{b}$, interacts with magnetization of the spacecraft due to electronic components. The spacecraft has a residual magnetic dipole moment $\underset{\rightharpoondown}{m}$. This cause disturbance torque of

$$\underset{\rightharpoondown}{\tau}^{Bc} = \underset{\rightharpoondown}{m} \times \underset{\rightharpoondown}{b}$$

By adjusting magnetic dipole of spacecraft, we can take advantage of Earth's magnetic field to produce a desired magnetic torque. This is the concept used by magnetic torque rods (magnetorquers).

area

$$\underset{\sim}{m} = N I A \underset{\sim}{n}$$

number of turns

current

Other Relevant causes of disturbance torques include

- Solar radiation pressure (SRP) ($\sim 10^{-5}$ N·m)
  (momentum transfer of photons)
- Aerodynamic forces ($\sim 10^{-7} - 10^{-1}$ depending on altitude)
  (negligible above 1000 Km alt.)
- Meteoroidal impacts
- Structural dynamics

For comparison
Gravity gradient ($\sim 10^{-4} - 10^{-3}$ N·m)

Magnetic torque ($\sim 10^{-5} - 10^{-4}$ N·m)