Spacecraft Navigation (Ch. 25)

Navigation: Where am I?

There is no such Thing as an "attitude measurement"

In practice we must use available sensors to <u>estimate</u> the attitude of our spacecraft. Real Sensor measurements are corrupted by <u>noise</u> and can also be <u>biased</u>. This can make it difficult to estimate the attitude of our spacecraft.

Review of Probability (25.1)

Consider a sensor that measures the quantity x:

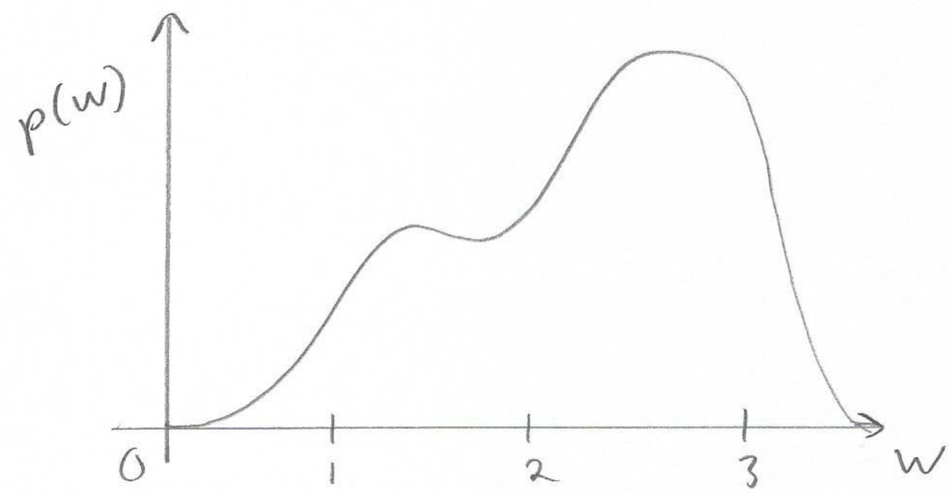
Measurement: $y = x + w$

x: true value

w: noise

The sensor noise can be described by a probability density function (pdf) $p(w)$ that quantifies the probability of what value w will take.

Ex:



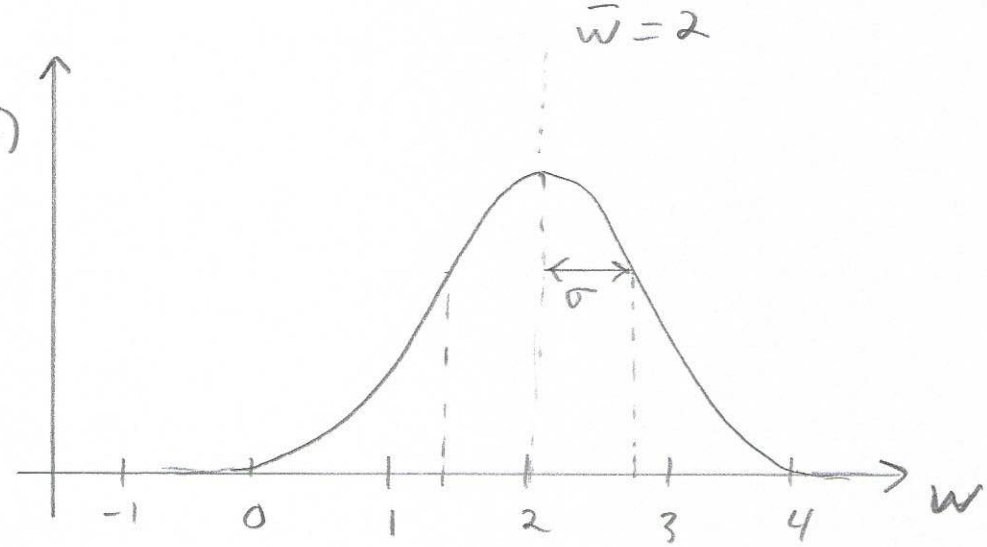
If $a \le w \le b$, then $\int_a^b p(w)\,dw = 1$.

$\Rightarrow$ Total probability of $a \le w \le b$ is one.

We will be interested in noise that has a

Gaussian probability density function.

Ex:



Mean: $\bar{w} = \mathbb{E}[w]$ (most likely value of $\bar{w}$)
(sensor bias)

↑ expected value

Variance: $\sigma^2 = \mathbb{E}[(w - \bar{w})^2]$ (How likely is w to differ from $\bar{w}$)
(sensor uncertainty)

$\sigma = \sqrt{\sigma^2}$ is the standard deviation

If $w \sim \mathcal{N}(\bar{w}, \sigma^2)$ (w is Gaussian noise with mean $\bar{w}$ and variance σ^2), Then probability that

$\bar{w} - \sigma \le w \le \bar{w} + \sigma$: 68.3%

$\bar{w} - 2\sigma \le w \le \bar{w} + 2\sigma$: 95.4%

$\bar{w} - 3\sigma \le w \le \bar{w} + 3\sigma$: 99.7%

When $\underline{w}$ is a colum matrix

Mean : $\bar{\underline{w}} = \mathbb{E}[\underline{w}]$

Covariance : $\underline{Q} = \mathbb{E}\left[(\underline{w} - \bar{\underline{w}})(\underline{w} - \bar{\underline{w}})^T\right]$

$$= \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & \\ \sigma_{n1} & \cdots & & \sigma_n^2 \end{bmatrix}$$

$\underline{w} \sim \mathcal{N}(\bar{\underline{w}}, \underline{Q})$ means $\underline{w}$ is Gaussian noise with mean $\bar{\underline{w}}$ and covariance $\underline{Q}$

Noise is often assumed to be

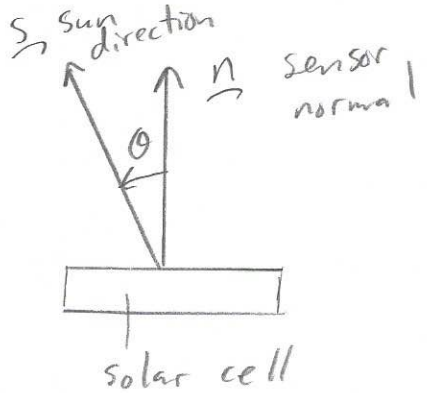
- zero mean : $\bar{\underline{w}} = \underline{0}$
- white : $\underline{Q}$ is diagonal

Spacecraft Attitude Sensors (Ch. 26.1)

No sensor directly measures attitude!
Different sensors can be used to help us estimate attitude.

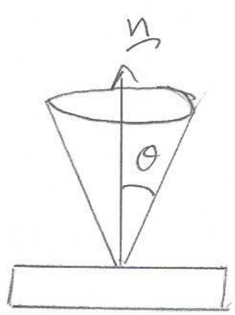
Sun Sensors

Sensor is made up of a solar cell whose electric current (i) is proportional to the intensity of light hitting it.



$$i(\theta) = i(0)\cos(\theta)$$

A single sun sensor determines a cone on which $\underset{\sim}{s}$ (sun direction vector) must lie.



Two sun sensors determine two possible sun vectors due to intersection of two cones.
A third sun sensor can narrow down to a single possible sun vector.

Pros: • Relatively accurate (high quality sun sensors ↳ $\sigma \sim 0.01° - 1°$)

Cons: • Limited field of view
• May not be useful for large angular velocities.

Magnetometers

Magnetometer measures local magnetic field vector. They are essentially a 3-D compass.

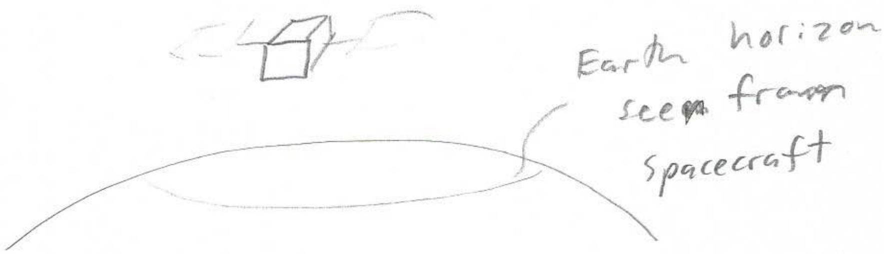
Pros:
- No field of view limitations
- No moving parts
- Inexpensive

Cons:
- Relatively inaccurate ($\sigma > 1°$)
- Only useful in low Earth orbit
- Influenced by magnetic dipole created by spacecraft's electronic components (Often placed on boom away from spacecraft's electronics).

Earth Horizon Sensors

Estimate unit vector pointing towards Earth's surface by locating Earth's horizon with infrared sensors.

Side View



Top View

2 common types

- Static: multiple fixed sensors
- Scanning: single rotating sensor (rotation could be from spin-stabilization)

Pros:
- Useful for nadir-pointing (Earth pointing) spacecraft
- Relatively accurate ($\sigma \sim 0.1°$)

Cons:
- Earth must be in field of view

Star Trackers

Star trackers are basically digital cameras that measure the direction of stars in the spacecraft body frame. The star tracker has a catalogue of Known star locations and is able estimate the spacecraft's attitude.
We will solve this attitude determination problem when we cover the TRIAD algorithm.

Pros: • Very accarate ($\sigma \sim 0.001$ deg)

Cons: • Limited field of view (FOV)
- Sun and Earth cannot enter FOV
- FOV must contain at least 2 stars.
- Need catalog of all stars you expect to use
- Very expensive

Rate Gyroscopes

Provide measurement of spacecraft's angular velocity typically using either a spinning-mass gyroscope or an optical (laser) gyroscope.

Pros: • No field of view constraints
• Relatively low noise ($\sigma \sim 10^{-10} - 10^{-7}$ rad/s)
covariance

Cons: • Biased measurement ($\bar{w} \sim 10^{-9} - 10^{-3}$ rad/s)
(can estimate bias with advanced attitude estimation techniques)
• Bias will also drift.
• High-accuracy rate gyros can be expensive

Other Possible Sensors:

- Multiple GPS receivers on spacecraft
- Accelerometer