STAT 240 Homework 2

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March 4, 2015

Chapter 26

Review exercise 2

Part (a)

The null hypothesis is that in a box of 38 tickets, there are 18 that are red. The alternative is that in this box of 38 tickets, there are more than 18 red tickets.

Part (b)

The null says that the percentage of reds in the box is $\frac{18}{38} \approx 48\%$. The alternative says that the percentage of reds in the box is greater than 48%.

Part (c)

$$z = \frac{1890 - (\frac{18}{38})(3800)}{\sqrt{(\frac{18}{38})(\frac{20}{38})(3800)}} = \frac{1890 - 1800}{\sqrt{\frac{36000}{38}}} \approx 2.924$$

Under the null hypothesis, z has approximately a standard normal distribution. Thus the p-value for the one-sided test is $P = P(z \geq 2.924) \approx 0.0017$.

Part (d)

At the significance level 0.05, we reject the null hypothesis that there were 18 red tickets in the box. In particular, this means that there were too many reds in the 3800 roulette spins to be due to chance alone.

Review exercise 5

Part (a)

Consider a box containing 3000 tickets, from which we take a random sample of 100 without replacement. The null hypothesis is that the average of the box is 7.5 and the alternative is that the average is something other than 7.5.

Part (b)

The null hypothesis is that the average of the box is 7.5 and the alternative is that the average is something other than 7.5.

Part (c)

$$T = \frac{7.5 - 6.6}{\frac{9}{\sqrt{100}}\sqrt{\frac{100}{99}}} = \frac{0.9}{0.904534} \approx 0.9950$$

Under the null hypothesis, T has a t distribution with 99 degrees of freedom. Thus the p-value for the two-sided test is $P(t_{99} \geq T) \approx 0.16$. We fail to reject the null hypothesis that the box average is different from 7.5, at the 0.05 significance level.

Chapter 27

Review exercise 8

Part (a)

The data tell us nothing about the research questions (i) and (ii); the two proportions measure different things. The fraction 22 of 46 is the proportion of people who *predicted* that they would help whereas 2 of 46 is the proportion of people who actually agreed to help. A z-test is not applicable here.

Part (b)

This data can be used to answer question (i), can people predict how well they will behave? However, a two-sample z-test is inappropriate because the groups under comparison are comprised of the same individuals. This introduces correlation between the samples so the standard error of the difference in proportions cannot be computed with the usual formula. We should use a paired z-test instead of a two-sample z-test.

Part (c)

The proportion of volunteers in each group can be used to answer question (ii), do predicted responses influence behavior? We would conduct a two-sample z-test to compare the proportions.

The null hypothesis is that the rate of volunteers among those who are asked to predict their response is the same as the rate of volunteers among those who are not asked. The alternative hypothesis is that the rates are different. The observed proportion in the "prediction-request" group is 22/46 and the observed proportion in the "request-only" group is 14/46. The test statistic is

$$z = \frac{\frac{22}{46} - \frac{14}{46}}{\sqrt{\frac{22\times 24}{46^2}\frac{1}{46} + \frac{14\times 32}{46^2}\frac{1}{46}}} = \frac{8}{\sqrt{\frac{976}{46}}} \approx 1.7368$$

Under the null, z is approximately standard normally distributed, so the p-value for the two-sided test is

$$P(|Z| \geq |z|) = 2P(Z \geq 1.7368) \approx 0.0824$$

At the 0.05 level, we fail to reject the null hypothesis that the rates are the same.

Review exercise 9

The null hypothesis is that the rate of rejections is the same for the paper with positive results and the paper with negative results. The alternative hypothesis is that the rate is higher for the paper with negative results. The observed proportion of rejections for the positive paper is 28/53 and the observed proportion of rejections for the negative paper is 8/54. We will conduct a one-sided, two-sample z-test for the difference in proportions. The test statistic is

$$z = \frac{\frac{28}{53} - \frac{8}{54}}{\sqrt{\frac{28\times25}{53^2}\frac{1}{53} + \frac{8\times46}{54^2}\frac{1}{54}}} \approx \frac{0.5283 - 0.1481}{\sqrt{\frac{0.249199}{53} + \frac{0.1262}{54}}} = 4.5317$$

The p-value for this test is

$$P(Z \geq 4.5317) = 2.9255 \times 10^{-6}$$

There is strong evidence to reject the null hypothesis that the rejection rates are the same in favor of the alternative hypothesis that the rate of rejections is higher for the paper with negative results.

Review exercise 10

There is a natural pairing in this experiment: children from the same household are the same with respect to a variety of potential confounders for IQ, including parents' socioeconomic status, parents' education, level of parent involvement, etc. Thus, siblings in this study are highly correlated. It therefore doesn't make sense to use a test which assumes the sample of first-borns, $X_1, \ldots, X_{400}$ is independent of the sample of second-borns, $Y_1, \ldots, Y_{400}$.

In reality, the correlation between the IQ of the first-borns and the IQ of the second-borns is positive, so

$$\begin{aligned} \text{Var}(\bar{X} - \bar{Y}) &= \text{Var}\bar{X} + \text{Var}\bar{Y} - 2\text{Cov}(\bar{X}, \bar{Y}) \\ &< \text{Var}\bar{X} + \text{Var}\bar{Y} \end{aligned}$$

Therefore the estimated standard error, given by $\sqrt{0.5^2 + 0.5^2}$ overestimates the true standard error of the difference in means. Consequently, the test statistic is biased towards 0. The z-test here is inappropriate.

If we had accounted for the covariance between $\bar{X}$ and $\bar{Y}$, the estimated standard error would have been smaller, thus the test statistic z would have been larger. In other words, using a better estimator of the standard error, accounting for the correlation between siblings in the sample, would have given more power to detect a difference in sample means.