CS 3341 Chapter 2.

2.1.

Sample space: A collection of all elementary results, or outcomes of an experiment. (Ω)

Event: Any set of outcomes is an event, events are subsets of the sample space.

A simple space of N possible outcomes yields 2^N possible events.

Notation:

$A \cup B \rightarrow$ Union, A or B

$A \cap B \rightarrow$ intersection, A and B

$\bar{A}$ or $A^c \rightarrow$ complement, not A

$A \setminus B \rightarrow$ difference, A but not B

Disjoint: Events A and B are disjoint if their intersection is empty

$A \cap B = \emptyset$. (Mutually exclusive events will never occur at the same time).

Also call "mutually exclusive". or "pairwise disjoint".

Exhaustive: Events A. B, C, ... are exhaustive if their union equals the whole sample space.

$A \cup B \cup C \cdots = \Omega$

De Morgan's law:

$\overline{E_1 \cup \cdots \cup E_n} = \bar{E}_1 \cap \cdots \cap \bar{E}_n$;

$\overline{E_1 \cap \cdots \cap E_n} = \bar{E}_1 \cup \cdots \cup \bar{E}_n$.

2.2.

A collection M of events is a sigma-algebra on sample space Ω if:

i) $\Omega \in M$

ii) $E \in M \Rightarrow \bar{E} \in M$.

Every event in M is contained along with its complement.

iii) $E_1, E_2, \cdots \in M \Rightarrow E_1 \cup E_2 \cup \cdots \in M$.

Every finite or countable collection of events in M is contained along with its union.

Degenerate sigma-algebra:

The minimal collection

$$M = \{\Omega, \phi\}$$

forms a sigma-algebra that is called degenerate.

Power Set:

$M = 2^{\Omega} = \{E, E \in \Omega\} \Rightarrow$ power set, All possible combinations.

Probability:

(Unit Measure) ① The sample space has unit probability: $P(\Omega) = 1$

(Sigma-additivity) ② For any finite or countable collection of mutually exclusive events $E_1, E_2, \cdots \in M$.

$$P\{E_1 \cup E_2 \cup \cdots\} = P(E_1) + P(E_2) + \cdots$$

★ Only mutually exclusive events satisfy the sigma-additivity.

Complement rule:

$$P\{\bar{A}\} = 1 - P\{A\}$$

Intersection of independent events:

Occurrence of one event does not affect the probabilities of others

$$P\{E_1 \cap E_2 \cdots \cap E_n\} = P\{E_1\} \cdot P\{E_2\} \cdot \cdots \cdot P\{E_n\}$$

$\boxed{\text{in parallel vs in sequel!}}$

2.3. Combinatorics

Equally likely outcomes: $P\{E\} = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{N_F}{N_T}.$

Permutations and combinations.

i) Sampling with replacement: every sampled item is replaced into the initial set, so that any of the objects can be selected with probability $1/n$ at any time.

ii) Sampling without replacement: every sampled item is removed from further sampling.

iii) Distinguishable: different order yields a different outcome.

iv) Indistinguishable: the order is not important.

Permutations: possible selections of k distinguishable objects from a set of n are called permutations.

permutations with replacement:

$$P_r(n,k) = \underbrace{n \cdot n \cdot \cdots \cdot n}_{k \text{ items}} = n^k.$$

permutations without replacement:

$$P(n,k) = n\cdot(n-1)\cdot \cdots \cdot(n-k+1) = \frac{n!}{(n-k)!}$$

Combinations: possible selections of k indistinguishable objects from a set of n.

$c(n,k)$ or $\binom{n}{k}$.

Combinations without replacement:

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{k!(n-k!)} = \frac{n\cdot(n-1)\cdot \cdots \cdot(n-k+1)}{k\cdot(k-1)\cdot \cdots \cdot 1}$$

Combinations with replacement:

$$C_r(n,k) = \binom{k+n-1}{k} = \frac{(k+n-1)!}{k!(n-1)!}$$

r ↓ replacement.

2.4. Conditional Probability and independence.

Conditional probability of event A given event B, is the possibality that A occurs when B is known to occur.

$$P\{A \mid B\} = \text{conditional probability of A given B}$$

$$= \frac{P\{A \cap B\}}{P\{B\}}$$

intersection, general case. $\Rightarrow$ $P\{A \cap B\} = P\{B\} \cdot P\{A \mid B\}$

Independence:

Event A and B are independent if occurence of B does not affect the probability of A:

$$P\{A \mid B\} = P\{A\}$$

$$P\{A \cap B\} = P\{A\} \cdot P\{B\}.$$

9/13/2015.

In a room of 30 students, what's the probability that at least 2 students share a birthday. (365 days/year).

Solution:

Probability that no two share a birthday.

$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \cdots \cdot \frac{336}{365}$$

$$= \frac{P(365, 30)}{365^{30}}$$

$$1 - \frac{P(365, 30)}{365^{30}} = 0.71$$

9/8/2015

Refregirators are produced 25% from A, 40% from B, 35% from C.

A has a probability of 1% defect.

B → 2%

C 1.5% defective.

① The probability a refrigirator is defective.

$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$
$$= 0.25 \times 0.01 + 0.4 \times 0.02 + 0.35 \times 0.015$$
$$= 0.01575$$

② A defective refrigerator from B?

$$P\{B \mid D\} = \frac{P(B \cap D\}}{P\{D\}} = \frac{0.4 \times 0.02}{0.01575} = 0.508$$

let $B_1, B_2, \cdots B_k$ be mutually exclusive and exhaustive events in Ω.

if $E \subseteq \Omega$, then

$$P(E) = P(E \cap B_1) + P(E \cap B_2) + \cdots + P(E \cap B_k)$$
$$= P(E|B_1)P(B_1) + \cdots + P(E|B_k)P(B_k)$$
$$= \sum_{n=1}^{k} P(E|B_n)P(B_n)$$

Bayes Rule:

$$P\{B|A\} = P\{A|B\} \cdot \frac{P\{B\}}{P\{A\}}$$

Law of Total Probability:

$$P\{A\} = \sum_{j=1}^{k} P\{A|B_j\} * P\{B_j\}$$

in case of two events, $k=2$.

$$P\{A\} = P\{A|B\} \cdot P\{B\} + P\{A|\overline{B}\} \cdot P\{\overline{B}\}$$

Bayes Rule for two events:

$$P\{B|A\} = \frac{P\{A|B\} \cdot P\{B\}}{P\{A|B\} \cdot P\{B\} + P\{A|\overline{B}\} \cdot P\{\overline{B}\}} = P\{A\}$$

Question: $P\{\overline{A}|\overline{B}\} + P\{A|\overline{B}\} = 1$?

if A. B and $\{A \cap B\}$ they are not mutually exclusive or exhaustive, they cannot be used for the ~~Bg~~ Bayes Rule.

if A and B are mutually exclusive:

$$P(A|B) = \underset{\text{zero}}{0} = P(B|A).$$

9/10/2015.

A lie detector is 95% reliable at detecting a lie, and 99% reliable at not detecting a lie when truth is told. suppose people lie half the time.

Suppose that the lie detector indicates lying. what's the probability the person is lying?

T

$\frac{1}{2}$ Tell truth
- indicates truth 99% → P
- indicates lie. 1%. → N

$\frac{1}{2}$ lie
- indicates truth 5% → P
- indicates lie 95% → N

L

$P\{P|T\}=0.99$, $P\{N|T\}=0.01$, $P\{P|L\}=0.05$. $P\{N|L\}=0.95$

$P\{T\}=0.5$, $P\{L\}=0.5$

$$P\{L|P\} = \frac{P\{P|L\}\cdot P\{L\}}{P\{P\}}$$