CSC 4890 Introduction.

Three kinds of Automata: power of Automata.

Finite Automata: no temporary memory
Pushdown Automata: stack.
Turing Machines: random access memory.

More flexible memory results to the solution of more computational problems.

Turing Machines cannot solve all computational problems

Language

string: a sequence of symbols from some alphabet
Language: a set of string.
Computation is translated to set membership.

String Operations:

i) Concatenation: $w = a_1 a_2 \cdots a_n$, $v = b_1 b_2 \cdots b_m$, $wv = a_1 a_2 \cdots a_n b_1 b_2 \cdots b_m$
ii). Reverse: $w^R = a_n \cdots a_2 a_1$
iii). String length: $|w| = n$

Empty string: a string with no letters. λ or ε.

$$|\lambda| = 0, \quad \lambda w = w \lambda = w$$

substring: a subsequence of consecutive characters
prefix and suffix: $w = uv$, u is prefix, v is suffix
Exponent Operation: $w^n = \underbrace{w \cdot w \cdots w}_{n}$, $w^0 = \lambda$.
The $*$ operation: Σ^*, the set of all possible strings from alphabet Σ.
i.e: $\Sigma = \{a, b\}$, then $\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, aaa, \cdots\}$
The $+$ operation: $\Sigma^+ = \Sigma^* - \lambda$, all possible strings from Σ except λ.

A language over alphabet Σ is any subset of Σ^*.

i). Addition: $\{x+y=z : x=1^n, y=1^m, z=1^k, n+m=k, n\geq 1, m\geq 1\}$.

ii). Square: $\{x\#y : x=1^n, y=1^m, m=n^2\}$.

i.e: SQURES = $\{1\#1, 11\#1111, \cdots\}$.

iii). Empty language: $\{\}$ or ϕ

Language with empty string: λ.

$\Rightarrow$ $|\{\}|=0$, $|\{\lambda\}|=1$, $\phi=\{\}\neq\{\lambda\}$

$|\lambda|=0$.

iv). Operations on languages:

$\cup$ union, $\cap$ intersection, $-$ difference,

compliment: i.e: $\overline{L} = \Sigma^* - L$.

Reverse: $L^R = \{w^R : w \in L\}$

Concatenation: $L_1 L_2 = \{xy : x \in L_1, y \in L_2\}$.

Notice the difference between Addition and Concatenation.

Exponent operation: $L^n = \underbrace{L \cdots L}_{n}$, $\Rightarrow$ $\{a,b\}^3 = \{a,b\}\{a,b\}\{a,b\}$

$= \{aaa, aab, aba, \cdots\}$.

$L^0 = \{\lambda\}$.

Star-closure: $L^* = L^0 \cup L^1 \cup L^2 \cup \cdots$

All strings that can be constructed from L.

positive-closure: $L^+ = L^* - L^0$.