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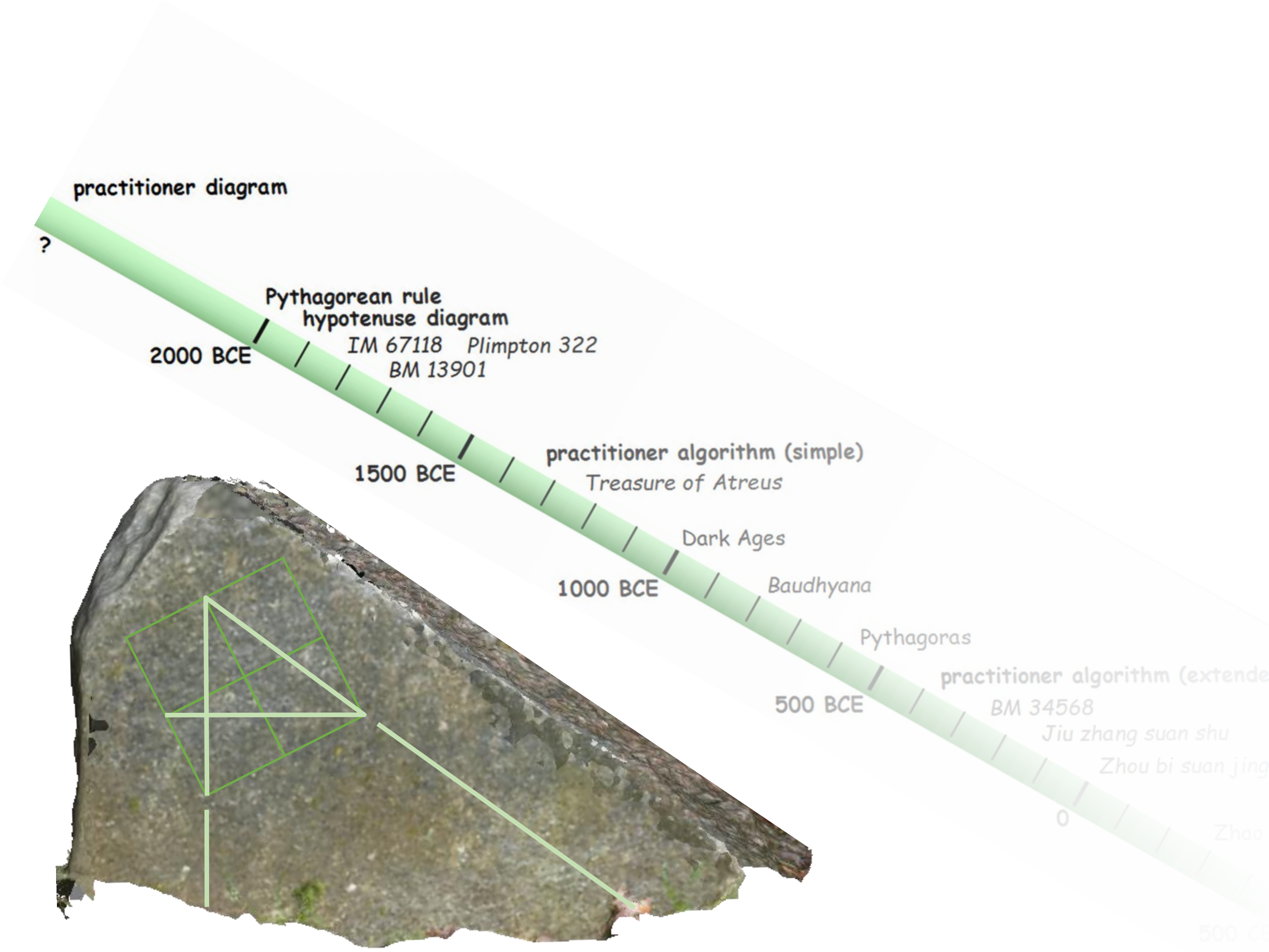
Prehistoric tomb surveying



Prehistoric tomb surveying

Dirk Kruithof, September 2019

Introduction

Prehistoric tomb surveying (PTS) is a series of papers in which Neolithic and Bronze Age surveying is investigated in the light of geometry. Have the megalithic tombs simply been fitted together according to a habitual style like for example the pottery they contain or have they been designed according to some geometrical practise? The latter seems to be true, since the design of the tombs involved can be grasped by what is called a *grid-of-squares geometry* in the PTS-papers. The main result of this practise is known as the 'Pythagorean triangle' nowadays, since the Pythagorean Theorem lies at the basis. Neolithic surveyors were not used to theorems. Nevertheless, with the grid-of-squares geometry they knew how to construct 'Pythagorean' triangles 3000 years before Pythagoras lived. In the PTS-papers this construction is called *practitioner diagram* as displayed on the stone of the front page .

The PTS-papers concern both the geometrical and the archaeological aspects of the tombs. First a general overview of the subject is given (PTS-1), then the practitioner diagram is associated with the earliest written math problems (PTS-2). There even seems to exist an early 'guide' for the construction of the practitioner diagram (PTS-3). Finally, the geometrical features of the tombs themselves get through. PTS-4 relates the geometrical setups of some small Dutch hunebedden to the archaeological understanding of their construction. Since the application of the practitioner diagram gives the small hunebedden some dolmen-like characteristics, PTS-5 proposes for the new term 'pseudo-dolmen'. The other papers show a carrousel of geometrical setups, ever by the same rules of the grid-of-squares geometry: PTS-6 about the entrances of Dutch hunebedden, PTS-7 about some dolmens along the Baltic coasts in Germany, PTS-8 about some Causse-type dolmens in the French Ardèche, PTS-9 about the entrances (stomions) of the Mycenaean tholos tombs and PTS-10 about the mounds of German megalithic tombs. The grid-of-squares geometry happens to be applied to all aspects of megalithic building for a long time.

Contents *Page in the PDF*

Prehistoric tomb surveying (1)
Between geometry and happen chance

Dirk Kruithof, September 2019

This is the first paper of a study regarding the geometrical layout of megalithic tombs of the Neolithic and Bronze Age. The study covers two main subjects: (1) The geometrical properties of the tombs themselves and (2) the underlying framework of a prehistoric surveying practise. The first subject heavily depends on archaeology since it reveals what changes to a tomb have taken place over time - think of decay, vandalism, restoration and excavation. The second subject is embedded in the history of math. It presents the Neolithic and Bronze Age surveying practise as the elementary know how that preceded a lot of geometrical solutions found in Old Babylonian, Seleucid, Demotic and Han math texts. After discussing the surveying practise, itself in this paper, the geometrical properties of the tombs are treated in separate papers, and also the link to the ancient math problems is investigated apart.

Surveyors as 'identity builders'

Prior to the discussion of the actual subject of this paper - the geometry behind tomb orientation - a more general view on the phenomenon of prehistoric megalithic tombs must be presented. There exist many opinions about the function and architecture of the megalithic monuments and the reader should be aware of what ideas form the background for the current study.

Although the container term *megalithic tomb* suggests the existence of some general type of tomb, this is at least the case. There exist a lot of eye-catching differences amongst megalithic tombs where it comes down to for example the building material (depending on what was found in the surroundings), the position of the entrance (in literature a distinguishing feature) or if the supports of a capstone are placed upright or recumbent. Some tombs have a portal or a corridor, others have an antechamber or a court, but many simply have a threshold. From the beginning it has been discussed by archaeologists if initially tombs were covered by a mound or cairn completely, partly or not at all. Yet, despite of all the differences, we recognize the style of the tombs undoubtedly as being megalithic. Of course, people had smaller boulders at their disposal, and it would have cost much less effort to pile them up instead of cutting out large slabs or dragging and splitting a boulder that weighted tons. Many dolmens have large oversized capstones. The fondness on 'huge-notwithstanding-the-effort', warrants the question what the surveyors made to devote themselves to megaliths so universally.

Kim [1] reserves a special place for technology in the exchange of cultural elements. Where other elements spread by contact between groups solely, technology must be adopted by the decision-makers of a group. Therefore, the embracement of megalithic surveying seems to be based in socio-political considerations by family heads or village chiefs. Kim [2] places the

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[1] Kim 2003, pg.285
[2] Kim 2014, pg.270

dolmen-building of South Korea in the context of the ability of the elite to mobilize labour and (thereby) to create group identity. This explains the egalitarian grave gifts in the ten thousand of dolmens on the peninsula. Also, Brindley's extensive study of pottery found in Dutch hunebedden [3] reveals that the quality of the tomb pottery reflects that of the contemporary pottery found in ordinary settlements and that it is in no way special. Concluded from the differences in characteristics and quality, the pottery belonged to several individuals or small groups. The frequency of the interments makes it plausible that it was not the ordinary people being buried in the hunebedden [4].

The architecture of the tombs agrees with such an egalitarian custom. There are no signs that the elite tried to create a unique identity for themselves by means of for example iconography or an atypical tomb setup. Megalithic monuments display a very homogenous building technique. Huge capstones are placed by means of roof upon often smaller supports. The supports are placed in foundation pits filled with flake stones. Gaps between the supports are filled by flakes too that are pushed in place from the outer. Inside the floor is paved by cobbles. Sometimes the chamber got subdivided by walls of smaller slabs or was surrounded by a mound that was kept in place by a ring of slabs. From the current study it will become clear that also the form of a tomb was subject to sudden rules. Such a common setup speaks in favour of the existence of a conceptual idea, which was maintained and exchanged amongst larger populations. As a consequence, there must have been 'carriers' of these ideas - people who were experienced in stone dressing and surveying. We might call them specialists. They must have showed an exceedingly interest in the applied techniques and thereby were able to communicate about tomb building on a higher level than ordinary people. If we compare the setup of ritual places to the layout of farmer houses in the middle Bronze Age in the Netherlands [5], it appears that the position of the post holes of the ritual places are determined

Figure 1
Dolmens of various regions have a similar look nevertheless:
Jukrim-ri (Gochang, South Korea) [6], Izvorovo (Rodopi, Bulgaria) , Langholz (Schleswig-Holstein, Germany), Les Oeilantes (Gard, France), Eext (Drenthe, the Netherlands), Kerkadoref (Brittany, France)

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[3] Brindley 2003, pg.43
[4] Van Ginkel, Jager, van der Sanden 2005, pg.85/87
[5] Waterbolk 2009, pg.152-155
[6] Young Suk 2006, pg.25

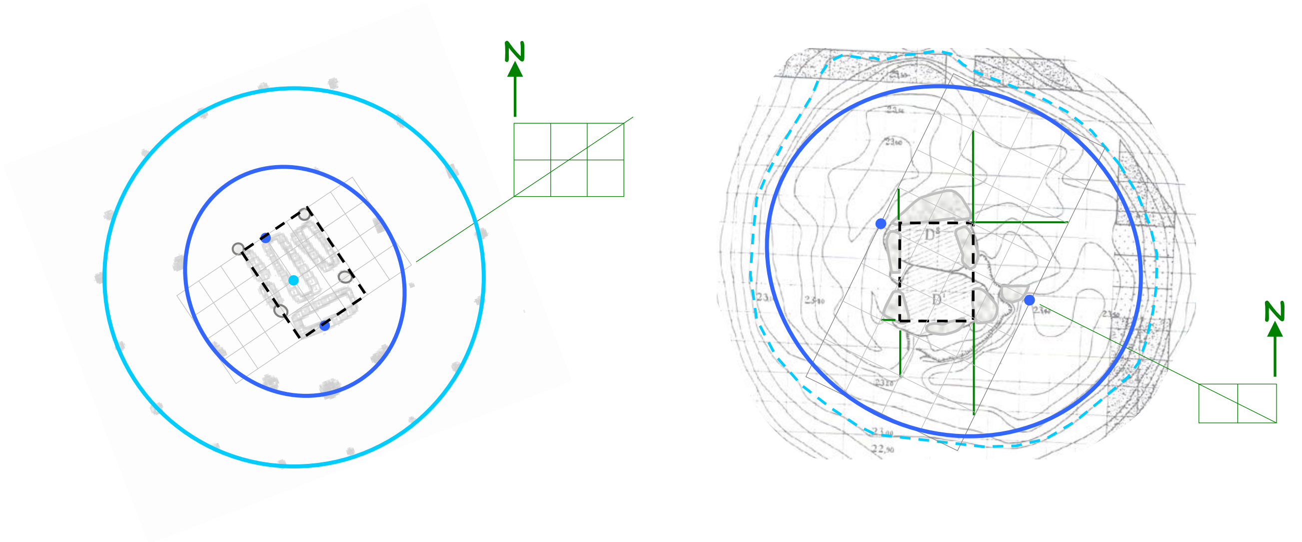

Figure 2
Left: Clarification of the geometrical properties of the Bronze Age tumulus 75 near Zeijen. The grey dots represent post holes. Four of them form a square in the middle. Additionally, there are an ellipse and a circle created by post holes. (Excavation plan by Van Giffen. [7])
Right: Clarification of some geometrical properties of the Neolithic hunebed D40 near Emmen. The light blue dashed curve follows one of the contour lines of the hunebed mound. (Mound plan by Van Giffen [8], filled up with the ground plan of D40.)

One of the graves in the tumulus is found outside the post hole square, but still is situated within a 3×4 'Pythagorean' rectangle of which the dimensions are determined by the centre points of the ellipse. Also, the chamber of the hunebed can be described as a 3×4 rectangle (black dashed lines). In both plans the ellipses have the same geometrical properties.

by the same surveying rules as utilised for the smaller hunebedden. Conversely the post holes of farmer houses are arranged in a more or less regular pattern too, but do not show such a geometrical interest. Maybe we must think of tomb building as ritualised by geometrical rules.

We know of two ancient societies where surveying was embedded in a sacral social environment since it was documented like this by themselves. From the Early Kingdoms to the Roman period in Egypt, it was the pharaoh who started the building of a temple. There are depictions and descriptions of how the pharaoh participated in the 'stretching of the cord ceremony' with the goddess Seshat in order to orientate the bedrock of a sacred building [9]. In Egypt society was organised hierarchical under the control of the pharaoh. On the contrary, during the middle and late Vedic period in India the priestly Brahmana families operated much less centralised. This is the time when the well known Sulba-Sutras saw the light. The Brahmana families were involved in building altars and lighting the holy fires. The forms and places were determined by geometrical rules strictly. There are even rules for preserving the occupied area when transforming one altar form into another [10]. We can imagine Neolithic

[7] Waterbolk 2009, pg.155
[8] Van Giffen 1925/27, Atlas, plate 127
[9] Miranda et al. 2008, pg.57
[10] Plofker 2009, pg.18

surveying to have functioned somewhere in this spectrum. Quarrying and surveying techniques can have been maintained and improved within families who stood in close interregional contact - even maybe via marriage. Once a village chief embraced the idea of megalithic building, the family would grow.

In Neolithic times tomb orientation was not chosen randomly and seems to have played a role in a group's cultural heritage. For example, at a regional level megalithic tombs in the French Ardèche can be distinguished not only by their construction but also by their orientation pattern. While Causses type dolmens are made of slabs only, the Bas-Rhone type applies a mix of slabs and dry masonry. Additionally, both types mainly face in different directions - western from due south for the Bas-Rhone type and eastern from due south for the Causses type [11]. An interesting case here is the small necropolis of Les Géandes. This group is located at the frontier of both dolmen types. Amongst the dolmens of the Causses type the only Bas-Rhone type dolmen shares a southern orientation with the others. In the Netherlands the overall orientation of Dutch hunebedden ranges from about azimuth 40º to 170º [12]. Yet pairs of hunebedden on the same field share an easterly orientation that deviates no more than 30º. In some cases, the orientations of the individual tombs are placed in a mutual setup, expressing unity. In paper PTS-4 [13] of this study the surveying practise behind such a group will be investigated. There are signs that these tombs were reconfigured in the late Neolithic or early Bronze Age [14]. The renewal incorporated a different orientation setup as well. Yet both the earlier and later orientations are based on the same geometrical practise and do not reflect an improvement. This seems to approve on the idea that the new people regarded tomb orientation as an important aspect of their identity - either because they regarded the process of orientation as a requirement for proper tomb ownership or because the tomb orientation itself expressed some social or religious code.

The idea of surveying as the occupation of specialists, who exchanged their know how in interregional contacts legitimates the expectation that common orientation strategies have been applied for a rather long time in a wide area. It will become clear that these strategies are found all over Europe from the Neolithic dolmens along the Baltic Sea to the Mycenaean tholos tombs in Bronze Age times. Tholos tombs were built until the 12th century BCE and thus coincide with the earliest written accounts on mathematics in Old Babylonia and Egypt. The expectation that links can be found between tomb surveying and these early math texts is affirmed by the current study too.

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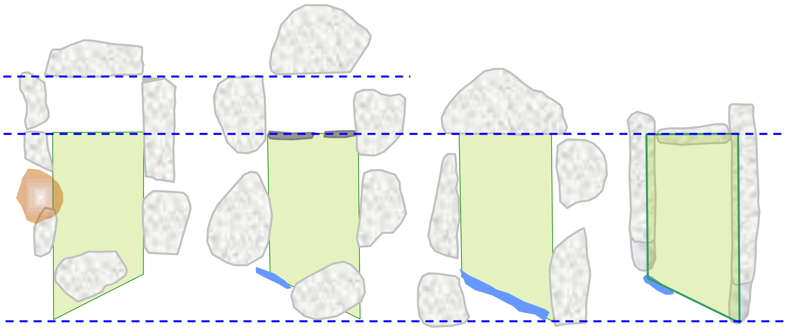
[11] Chevelier 1999, fig.4. Chevalier distinguishes the Langedoc-type type, with or without antechamber (2600-2200 BCE) (pg.49), the Bas-Rhône-type, with or without antechamber (±2200 BCE) (pg.54) and the Causses-type. The latest being subdivided in plain dolmens, dolmens with a vestibule and dolmens with a coudé (corridor at an angle) (pg.56/57).

[12] Gonzalez-Garcia, 2003, table 1

[13] Reverences to the other papers are abbreviated 'PTS-n', where PTS stands for the study title 'Prehistoric tomb surveying' and 'n' for the serial number of the paper.

[14] For example, two small hunebedden (D13 and D40) have their latest entry dug into an existing mound. All yet existing hunebed mounds consist of a primary and secondary layer.



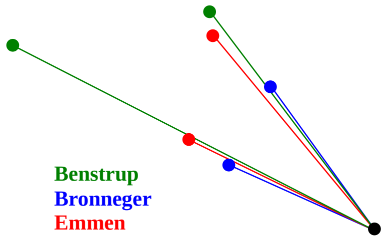


Figure 3
From left to right: Hunebed D22 of Bronneger (Netherland), dolmens 385b and 384 [15] of Serrahn (Germany, destroyed) and dolmen C of la Devèse (France). These Neolithic tombs happen to have the same chamber form. Especially the end stones and entry stones make the same angle to the orientation of the chamber.
Most right: A systematic survey of a few groups of three tombs on a field reveals that they are organised in one central and two satellite tombs. The black dot represents the central tombs: Benstrup 967, Bronneger D25 and Emmen D40. The coloured dots represent the nearby satellite tombs.

What to think of geometrical tomb properties?

The current study started off with a comparison of the mutual orientation of three small groups of tombs (called hunebed in the Netherlands). They are the only remaining groups with exactly three tombs on the same field, belonging to the western funnel beaker culture. The tombs are arranged as a central tomb accompanied by two satellite ones. Within each group the satellites and the central tomb share the same mutual orientation - the first more or less at 143° and the second more or less at 117°. Later on, the ground plans of some German dolmens and Dutch hunebedden were compared. Within a rectangular chamber they appear to have an end stone or door slab characteristically placed at an angle of more or less 117° once more. The isomorphic setups can be mirrored or extended, but the conformity of the tombs is undeniable. Most of them have different orientations.

In the description above the scientifically suspicious wording 'more or less' turns up deliberately. The orientations or angles are *more or less* the same. But what is that 'more or less' - how much deviation is meant by it? Before continuing something needs to be said about the *goodness of fit* of a supposed geometrical setup. How can it be decided that two tombs have a similar ground plan? What are the limits of deviation and similarity here? Obviously, a pile of slabs simply leaning upon each other can form a shelter, but they are not called a dolmen because of that. Somehow the notion of upright supports and lying capstones are within the province of the concept 'dolmen' and a pile of slabs needs to conform itself to that. In other words, it is our ability to recognise the surveying features upright and flat, that (amongst others) distinguishes a dolmen from its environment. Even if it has collapsed, this mental re-surveying enables us to recognize the dolmen. Thereby it is

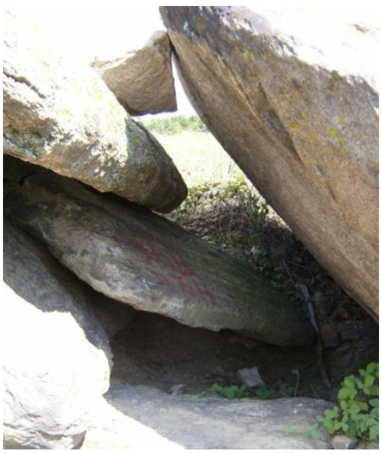


Figure 4
A collapsed dolmen near Ostar Kamlak in the Rodopi mountains of Bulgaria.

[15] The numbering of Dutch and German tombs follows the system of respectively Van Giffen and Sprockhoff.

said, that the *goodness of fit* should not necessarily be confined to the situation as we encounter (and thus can measure) it but must be extended to the surveying features that could have been applied. Our expectation of what could have been achieved in Neolithic times determines how we think about which features are needed to fit and which are not.

Hoskin for example cannot believe that Neolithic surveyors were able to bisect angles, for the reason that "*the Greek geometers arrived at the concept of bisecting an angle only after a long process of abstraction, and it seems unlikely that prehistoric peoples could have matched this*" [16]. When discussing stone squares with Pythagorean dimensions, Ruggles points at the ratio 5:12 of the so-called station stones of Stonehenge. Like the Crucuno stone square these stones are more or less astronomically aligned. He doubts that we have to "*postulate that the builders conceptualized Pythagorean geometry to appreciate this fact."* [17]. The arguments of Hoskin and Ruggles are valid in themselves but start off with sudden expectations that can be questioned. There is no reason to assume that a process of abstraction is the only way to reach the know how of bisection and since the geometry of both mentioned stone squares is more exact than the astronomical alignment it should not be the geometrical setup that is discredited. The Euclidian mathematics (6th century BCE) depended heavily on the earlier Old Babylonian geometrical algebra (2nd millennium BCE). Here algebraic problems were solved in a geometrical kind of wording. Even 3rd degree equations were solved like that. These high standard early math texts were recorded in a scribal environment, which was not interested in geometer subjects like bisecting angles. But this geometrical algebra in itself was founded in the practise of a prior geometer's lay tradition [18]. This tradition must have been in a rather developed stage already when it was adopted by the school environment.

When trying to explain the similarities of the chamber setups and the mutual tomb orientations (figure 3), initially these setups were tested against celestial phenomena. Indeed, the mutual orientations between the satellite and central tombs come close to the minor and major lunar stand stills. But it will be demonstrated in paper PTS-5 that also here the geometrical properties are more precise than the astronomical alignments. Moreover, the excavation and restoration photographs of the group near Emmen approve on a sudden surveyor's practise in favour of a geometrical explanation. This does not defeat the possibility of an intended lunar alignment, but if it was intended then the surveyors used geometry to approximate the alignments [19]. Also, the similar chamber forms cannot be explained astronomically. Most of the involved tombs have distinct orientations, so that comparable sightlines cannot be associated with a single alignment. Yet some tombs occur with identical orientation, but they are located at different latitudes. This contradicts with the fact that the rising and setting spots on the horizon for a sudden celestial body hold true for only one latitude. Another problem with the astronomical explanation counts for those oriented to stars. While the sun and the moon rise and set near the same spot on the horizon during the millennia, stars do not. Therefore, these alignments must match the time of tomb building. For example, González-García and Costa-Ferrer find that the alignment of many portals of Dutch hunebedden agree with stars that have a declination of about -35º [20]. But here tombs are included that have been built during Brindley’s horizon 1 and 3, which creates a time span of

[16] Hoskin 2001, pg.18
[17] Ruggles 2005, pg.121
[18] Hoyrup 2001 (all over the book)
[19] In an environment where clouds and mist often hide the sky, it would be a logical step to construct alignments instead of creating them by sighting. This can explain the mis-alignments by a few degrees too.
[20] González-García and Costa-Ferrer 2003, pg. 116-117

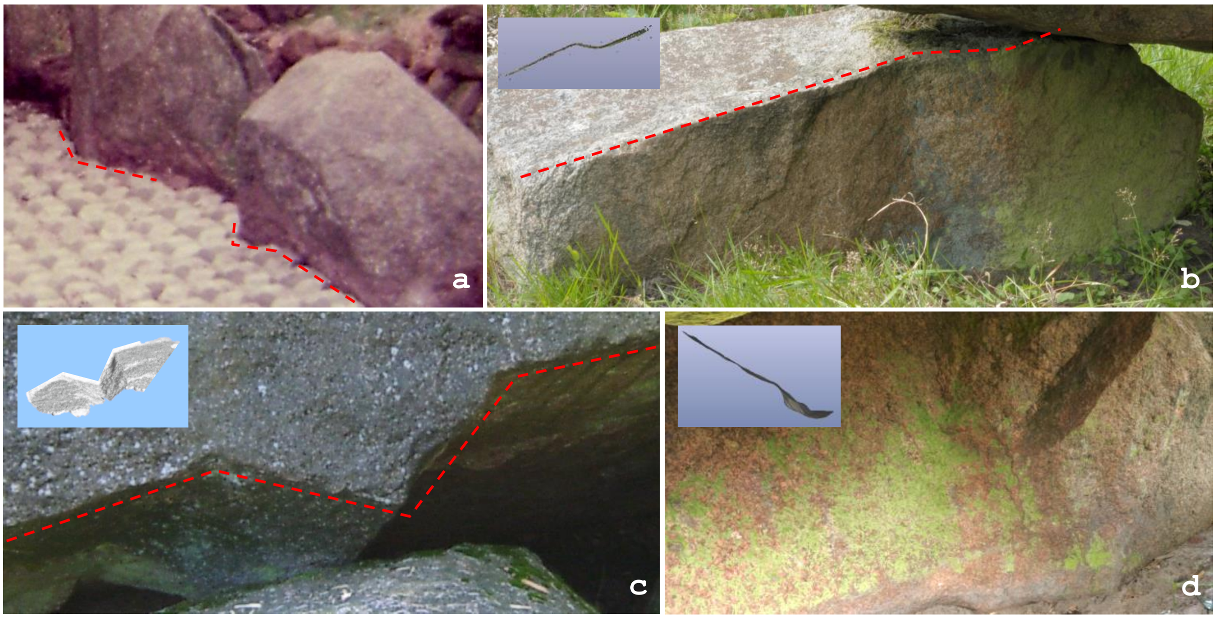


Figure 5
Marks at stones. The red dotted lines highlight a mark. The insets show a 3D model of the stone surfaces.
(a) Side stones Z3 and Z4 of hunebed D38. Photo of the sealing of the chamber content. (© ROB, 1984)
(b) Side stone Z2 of hunebed D39 having a double surface.
(c) A stone in the vicinity of the group near Emmen. Probably the stone comes from a dismantled hunebed. The surfaces at both sides of the mark twist by only 1° compared to each other. Two horizontal stripes left of the mark seem to be the remains of stone carving.
(d) Side stone Z2' of hunebed D40. The inset shows the flatness of the stone in contrast to the bulb near its right side. Because of recent erosion the bulb has vanished partly.

200 years (±3350 – 3150 BCE). During horizon 1 a star with declination -35° ascended only a few degrees above the horizon and was hardly visible by atmospheric extinction. During horizon 3 such a star even stayed below the horizon. Thereby an astronomical alignment is refuted. On the other hand, orientations related to declination -35° can be explained by the surveyor's practise too. They include the most applied orientation of 193° and 167° so that we should not be surprised to find many portals being orientated towards this direction. The orientation of the portals will be the subject of paper PTS-6 of the current study.

It was brought up already that the group of satellite and central hunebedden near Emmen showed a surveyor's practise that made the geometrical setup plausible. Some hunebed side stones happen to have bulbs on a further flat chamber side. Such bulbs should not be regarded as bad masonry but as purposely carved features. Neolithic stonemasons were able to create perfectly flat surfaces and the bulbs must have made their work more troublesome. Heggie [21] points out that it should be the remarkable features that make a geometrical setup convincing. "*Equally important, however, is our demand that there should be features of the sites which would be surprising were the proposed geometry wrong.*" The bulb at side stone Z4 of hunebed D38 plays a special role in the mutual orientations of the group since it serves as starting point for the orientation towards D39 and D40. The sightline towards D40 runs from the mark at Z4 through the entrance where it is guided by side stone Z4'. As a combined entrance and side stone this support stands perpendicular to the chamber and shows a unique feature: it protrudes a few decimetres into the chamber. This is discussed in detail in paper PTS-4. Near

[21] Heggie 1982, pg.67

Emmen the nicest example of a stone mark comes from a dismantled hunebed (figure 5c). The stone is piled up a few hundred metres to the northeast of the Emmen group. Its flat surfaces at both sides of the mark must have been carved separately since they mutually twist by almost a degree - a proof of great craftsmanship knowing that the stonemason had to work by eye. Outside the Netherlands another nice prism has been carved to form a mark. The erweiterde Dolmen 409 near Frauenmark has such a prism not as a bulb, but instead chiselled away from the end stone.

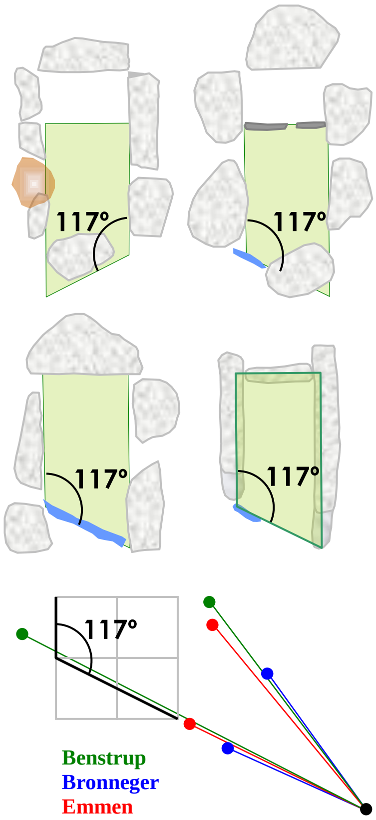


Figure 6
The angle of 117° in the tomb layouts and mutual orientations of figure 2. It can be constructed in a grid of squares.

When looking at the ground plans of figures 3 and 6, it comes to mind that marks are not the only special features. Here it is the placement of the door slab or end stone together with the constant ratios of the walls that are special. It demonstrates that even geometrical properties themselves can be remarkable. Within the ground plans the door slab (Türplatte) stands at an angle of 117° to the side walls of the chamber. This reminds of one of the mutual orientations of the groups of tombs, and the correspondence was a decisive moment in the current study. Apparently, the mutual orientations had to be explained geometrically. Literature did not deliver a punch line here, but finally the 117° angle was recognized as a right angle plus a diagonal through two adjacent squares, ⧄⧄ [22]. Herewith the discovery of a surveying practise had started that can be typified best as a *grid-of-squares geometry*. After some playing around with diagonals in such a grid of squares, it came to light that all Pythagorean triangles could easily be constructed by a simple trick (figure 7) without any knowledge of the Pythagorean Rule [23] nor the Pythagorean method [24]. Moreover, the constant ratios of the sides of the compared dolmens happened to result from the simplest Pythagorean triangle with sides 3, 4 and 5. This was a breakthrough in understanding the underlying geometry of a lot of megalithic tombs. They have the integral right triangle 3×4×5 inscribed in the chamber layout by means of the trick in an orientated grid of squares. Such an easy remembered trick - from now on called the *practitioner diagram* [25]- could have circulated amongst the surveyors very well. And thus, after all, there is no necessity that Neolithic surveyors encapsulated the Pythagorean geometry nor that they had an abstract understanding of bisection. It seems more like the opposite. Probably the practitioner diagram laid the base for the earliest math texts (see PTS-2).

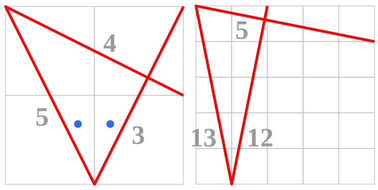


Figure 7
The practitioner diagram in a grid of squares as meant in the text. Depending on the ratio of the diagonals it produces different integral right triangles. Bisection makes part of the figure auto-matically. The blue dots divide the 53¼° angle of triangle into two equal angles of 26½°

[22] Because of the ease of reading and understanding, in this paper the wording '(a) diagonal(s) through two/three adjacent squares' will be replaced by the icon ⧄⧄/⧄⧄⧄.
[23] The square of the diagonal equals the sum of the squares of the width and the height ($d^2 = w^2 + h^2$).
[24] Integral right triangles can be found by the formula $w = n$, $d = n^2 + 1$ and $h = n^2 - 1$.
[25] The word *practitioner* is chosen to contradict with words like *schoolish*, *official* or *professional*. The emphasis lies on the practical application of the diagram and not on the theoretical basis or consequences of it.

Can Neolithic surveyors have created the practitioner diagram?

Apart from the need to find remarkable features in a tomb's layout that approve on a specific geometry, Neolithic people must have had the tools to create it. Heggie calls them equally important arguments to support a geometry [26]. The practitioner diagram requires that Neolithic men were able to (1) setup a levelled surface, (2) find the cardinals points, (3) could create a grid of squares and (4) bisect an angle. How can these four requirements be met in Neolithic times? The archaeological record does not deliver any prove, but there are some techniques described in the earliest written or depicted sources that could have been applied by a Neolithic surveyor.

Ad (1). At least in some of the Dutch hunebedden neither the soil beneath the chamber nor beneath the mound was levelled. The first has been settled by the investigation of the mounds of D39 and D40 by Lanting [27] and the second is established by the vertical cross-sections of the chamber of hunebed D30 by Van Giffen [28]. Yet this does not prove if there has been a temporary levelled surface raised upon the ground or not. Afterwards the chamber itself was built in a foundation pit and the mound was raised to as least the middle of the supports so that a temporary surface would not have left any traces. Anyway, a levelled soil is only one of the possibilities to fix the cardinal points and setup a grid of squares.

The easiest way of levelling can be performed by rope and stick. A rope is attached in the middle of the stick so that it will hang in balance more or less (figure 8a). Then position the stick between two upright poles and mark the height at one pole. Turn the stick around and mark at the other pole. The marks on both poles are levelled and so is a rope, which is stretched between both marks. In order to level out the ground between the poles, the height of the marks above the ground should be made even. A similar technique is suggested by Lehner for the construction of the platforms for the Egyptian pyramids. Instead by means of a hanging stick, the leveling was realized by a more advanced A-shaped tool with a plumb line. Lehner thinks that the railings or latticework that divided the platform in a grid of squares, was used to level the platform as well [29]. Much later in the Chinese Han text *Zhou bi suan jing* we find the phrase: "*Levelled a trysquare is used to stretch a rope*" [30]. It makes part of a short hymn on the applications of a trysquare in section #A5. Two section before, #A3 contains a text that can be titled as 'the lore of the folded trysquare'. This folded trysquare seems to describe the practitioner diagram (see paper PTS-3).

Ad (2). Also, the cardinal points can be found by pole and rope solely as it was done by the ancient Egyptians. The extreme eastern and western positions of a circumpolar star (the star is called Meskhet by them) are fixed by means of a pole (figure 8c2). True north is found via bisection (figure 8b) [31]. This method requires some knowledge of the movement of the stars

[26] Heggie 1982, pg.67
[27] D39: Lanting 1984, pg. 2, D40: Brindley and Lanting 1991/1992, pg. 116
[28] Van Giffen 1925/27, Atlas, plate 137
[29] Lehner 1997, pg.212
[30] ctext.org/dictionary.pl?if=en&id=59433 (Chinese text project, 2017)
Zhou bi section #A, 'The book of Shang Gao' gives the impression of a collection of riddles about the trysquare, which are brought together within the framework of a discussion between some unknown Shang Gao and the duke of Zhou. The latter must give the riddles the cachet of importance. Cullen (1996, pg. 153-155) ascribes the compilation to the period of intermediate ruler Wang, when the know how of laymen was esteemed very high.
[31] Depicted in the temple of Horus at Edfu. Shaltout and Belmonte 2005, pg.18.
Again, this demonstrates how the concept of bisection was established by surveying long before the Greek were able to work out their abstraction of it.

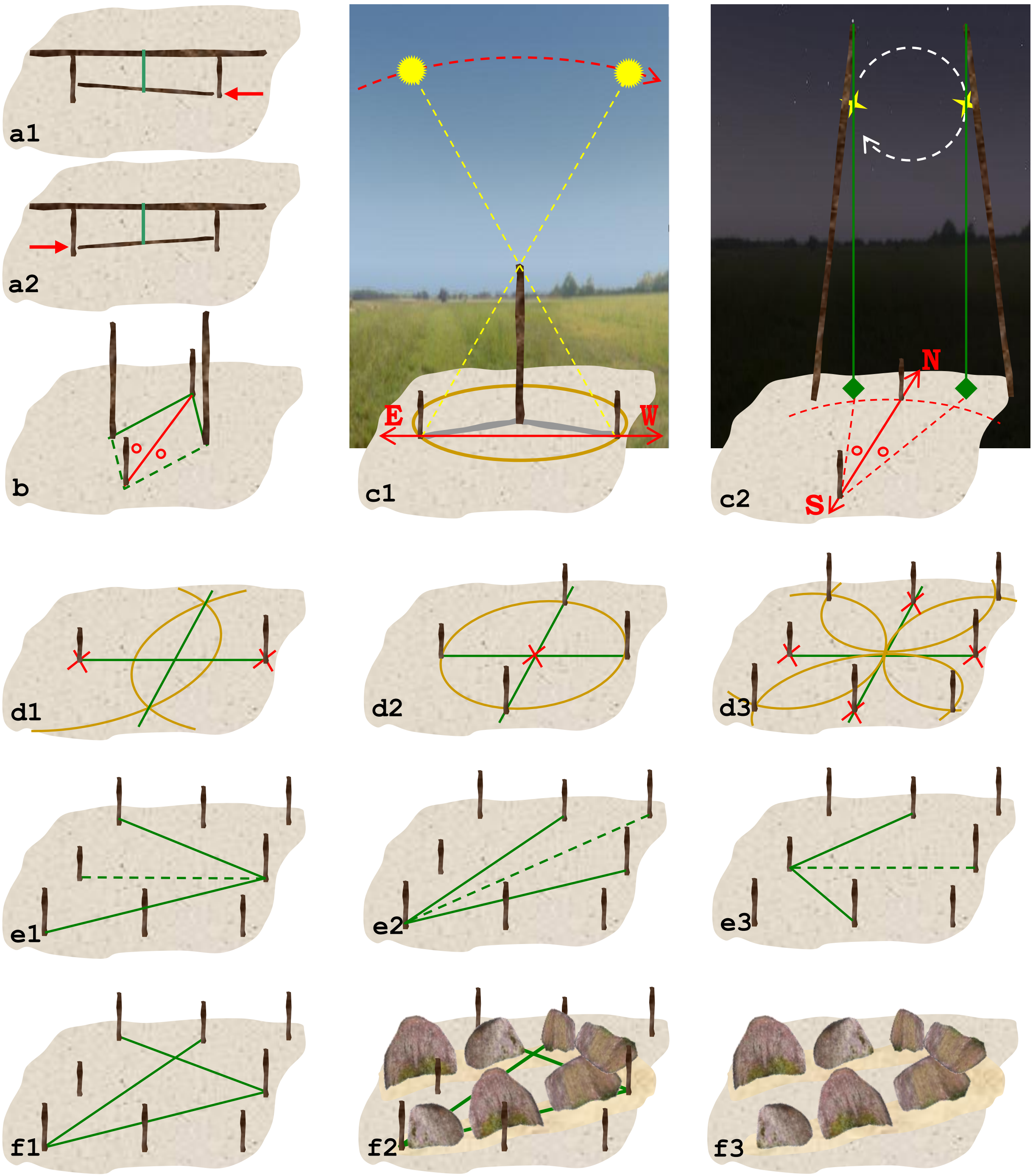
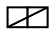


Figure 8
The surveyor's toolbox of the grid-of-squares geometry
(a) Levelling can be performed by turning a balanced stick between to poles. The positions of the red arrows are marked. These positions are levelled.
(b) A rhombus figure is applied to bisect any angle, depending on the length of the rope in use.
(c) North and south are established by means of sighting the extreme positions of a circumpolar star. East and west are established by the intersection of the shadow of a pole top with a circle drawn around it.
(d) The construction of a grid of squares. First a perpendicular line is setup in which a grid of squares is created. The red crosses show the centre points of the circles that need to be drawn.
(e) Different ways of bisecting an angle in a grid of squares. The first two agree with the construction of the practitioner diagram.
(f) The practitioner diagram is stretched between the poles. Then the positions of the stones are marked and finally, the tomb is built up.

since it should be achieved with stars that reach their extreme positions in the same night during the winter half year. Another method is described in the Katyayana Sulba Sutra (± 350 BCE, India). A rope is used to draw a circle around a gnomon (figure 8c1). Both points where the morning and afternoon shadow of the gnomon's top touches the circle, are marked. The marks establish true east and west [32]. The method must have been much older since an earlier part of the Sulba Sutra, the Baudhayana (± 800 BC), uses it without further explanation - as if it was the most common thing to do. In order to get a good result, one needs to work in a levelled plane. Once the line north-south or east-west is established, one of the most applied surveyor's methods can be used to setup the complementary line. Draw circles around two spots on the line that will intersect at each side of the line (figure 8d1). A line through the intersections aligns at the complementary cardinal points.

Ad (3). The earliest description to create such a perpendicular line is found in the Baudhayana. There the instructions proceed with the setup of a square as the reader is told, but in fact the framework of a grid of squares is produced. First the cardinally oriented lines are setup having four poles at equal distance from the intersection (figure 8d2). By circling around these poles, one finds the position of the corner poles at the intersection of the circles (figure 8d3) [33].

Ad (4). Ancient surveyors were not able to distinguish angles as we do. For example, an angle of 41.6º coincides with a specific value on a protractor but not with any straightforward geometrical construction [34]. In fact, the concept 'angle' did not exist at all - probably not even the term right angle. Angles were linked to the figures they were created with. The right angle is related to the rectangle and not to a trapezium. While on a protractor the bisection of an angle is simply performed by division (*41.6º ÷ 2 = 20.8º*), ancient surveyors needed a figure that could help them construct the bisection. The only figure that can provide bisection of whatever angle is the rhombus. After the midst of a rope has been marked, both ends are attached to a pole (figure 8b). The rope needs to be much longer than the distance between the poles. After fastening, the rope is stretched by pulling the mark and a stick is placed at the extreme position. Then the rope is stretched the opposite way and another stick is placed. The line between both sticks bisects the angle of the rope.

Besides the tools to construct the bisection, an ancient surveyor should have gained a mental perception of the objective: point at the midst of two directions. The practitioner diagram in a grid of squares contains an easy understood bisection [35]. The biggest sharp angle of the Pythagorean triangle evolves from a double angle produced by two combined  (compare figure 8e1 with 8f1). For an ancient surveyor this must have meant that there should exist a reverse construction too, which in fact is a method of bisection. The way the practitioner diagram is created in the grid of squares almost invites to such a method since the double angle consists of half a rhombus. So, the mental completion of the bisection via the rhombus was just one step away, if they used the practitioner diagram.

[32] Plofker 2009, pg.19
[33] Even nowadays this construction is found in student's math books. The figure is often called a flower.
[34] Noteworthy: the opposite is as much valid. It is not possible to express the sharp angles of a Pythagorean triangle in a decimal notation. Rounded they are 53.3º and 36.7º, but the number of decimal positions is infinite.
[35] Hoyrup calls this type of geometry 'naive' (directly seen) as opposed to 'critical' (scrutinised). (Hoyrup 1998, p.14, note 30). The naive geometry is the domain of surveyors and the critical geometry that of the school tradition.

The method of exploring the geometrical tomb properties

Even if an excavation plan is available, it is desirable to create a ground plan by modern means. In some cases, it appeared that north in an excavation plan diverges a few degrees from true north. The exploration of a tomb's geometrical properties starts off by setting up a cross-section.

In order to have a cross-section of a tomb correctly orientated, the *NOAA Magnetic Declination Calculator* [36] is consulted for the actual magnetic declination for the tomb's longitude and latitude - this is the difference between magnetic north and true north. Initially the cross-sections were produced by a sighting compass, an electronic total-station like tool (angle, distance and inclination measurer) and a laser-leveller projecting a horizontal line on the supports [37]. The measured data was supplied to plotting software that produced a 2D ground plan (figure 9). In 2013 this procedure was replaced by the less laborious and more advanced technique of photogrammetry. The cardinal directions were determined by four targets. First, they were orientated in the field by a sighting compass and a laser crossbeam leveller. Later on, they were attached to a 3D printed platform that was orientated by an electronic leveller compass [38]. The complete setup was recorded by about hundred photographs on average. Photogrammetry software was used to produce a 3D model of the tomb (figure 10). This method has great advantages. The extensive photo reportage makes it possible to check on unforeseen features and because of the 3D model, one can extrapolate the situation at floor level and even correct subsided stones in a tentative virtual restoration.

Once a cross-section is produced it is ready to be explored. Known setups are tested and new ones tried. In fact, this is an ongoing process for all tomb plans simultaneously. For example, though the comparability of hunebed D13 and D22 was recognized in an early stage, the complete understanding of how the surveyor might have proceeded to shape D13 came only after altering the construction order of the 43º grid construction. The alternative order was suggested by the

Figure 9
Plotting a 2D cross-section. A laser-leveller is used to project a cross-section on the stones. With an electronic angle and distance measurer the 2D data of the curves is taken. The data is inputted to the application SiteMsm.

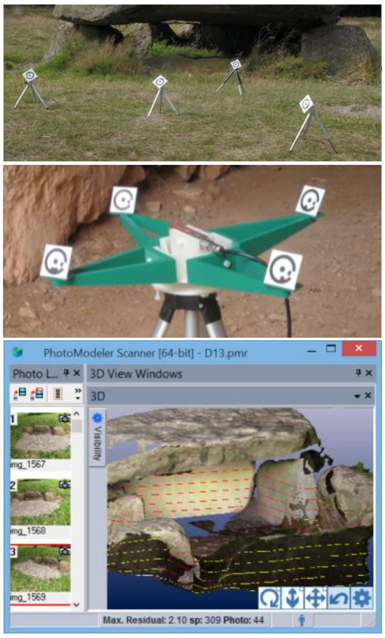


Figure 10
Creating an orientated 3D model. Four targets in a levelled plane indicate the cardinal points. A 3D model was created by means of photogrammetry.

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[36] ngdc.noaa.gov/geomag-web/calculators/mobileDeclination (2004-2018)
[37] Compass: Recta DP10 (±0.3º), angle measurer: Stabila (angle ±0.1º), distance measurer: Leica D5 (±0.5mm, inclination ±0.1º), laser leveller: KBW-0629 (±1mm/m), plotting software: SiteMsm
[38] Photogrammetry software: PhotoModeler Scanner (2013-2016), compass: Recta DP10 (±0.3º), laser crossbeam: Leica LINO L2P5 (leveller ±0.2mm/m, dot ±0.2mm/m), leveller compass: Rion HCM505b (±0.3º horizontal, ±0.1º vertical)

mutual setup of two dolmens in the Ardèche. That a setup of a hunebed in the Netherlands could be completed by a geometrical approach taken from dolmens in the Ardèche, affirms the possibility of contact amongst a group of specialists.

In the current study the expectation of what geometrical features could have been achieved by Neolithic surveyors is derived from the grid-of-squares geometry. The more features align with diagonals and grid lines the better the fit, meanwhile counting the inscribed triangle as the crown on the setup. It is not argued that this type of geometry can cover all tomb setups or even all elements of a single tomb setup, but it is the chosen context for the *goodness of fit* in the current study - a reference point for the expectation of what can be fit. If a tomb setup fits to the grid-of-squares geometry, then the applied surveying is in the province of an ongoing development towards the geometrical algebra (see PTS-2).

The method of fitting a tomb's ground plan to a supposed geometrical setup covers the complete layout of the tomb. It is rather easily guessed if the ground plan can fit to the practitioner diagram. The figure is used as an overlay over the tomb layout (figure 11). Only in rare cases the figure's grid is orientated towards the cardinal points. Most often its orientation can be produced via a corresponding geometry to that of the practitioner diagram. If the figure is constructed by , then angles of 26½º or the bisection 13¼º are expected. A construction by agrees with angles of 18½º or 9¼º. On average a tomb layout is expected to match the geometry with no more deviation than one or two degrees for its sightlines, but rather often this is hard to decide on. Most of the time the supports and slabs do not have perfectly flat faces and do not stand in a nice row. However, this is not regarded the most important aspect of the goodness of fit.

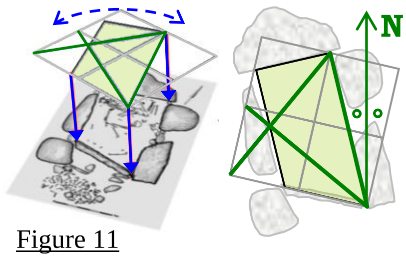


Figure 11
Often it is easy guessed how the practitioner diagram fits to the ground plan of a tomb. Here the orientation of the grid of squares is found through bisection.
(Plan of Serrahn 384, taken from: Schuldt 1972, pg. 76).

If a layout fits to the geometry depends on the tomb's peculiarities like marks and special geometrical properties in the first place. The studied dolmens of the French Ardèche often have a right angle between one of the side slabs and the end stone. It may be obvious that this corner should fit to the right corner of the triangle in the practitioner diagram. The guard stones near the entrance of the Causses type dolmens follow the orientation of the grid-of-squares as do the door slabs of the ‘erweiterde Dolmens’ along the Baltic Sea. Such tomb features are given a higher priority in the *goodness of fit* than the exact sizes of, angles between and orientations of the lines that make up the figure. Of course, the deviations may not be excessive, but due to weathering, vandalism, restorations and excavations many supports can no longer be assumed to stand on their original position exactly. This is disastrous to know the orientation of a single line, for it is not possible to distinguish a deliberate orientation from an accidental one. But in order to recognize a surveyor's setup this is much less harmful. As a rule of thumb, one may expect such influences to disturb a setup instead of accidentally

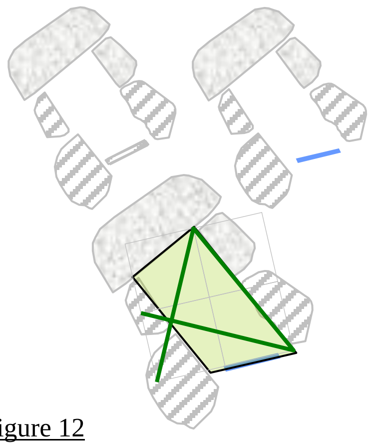


Figure 12
In 1977 dolmen Langholz 68 in Schleswig-Holstein (Germany) got restored. The dashed side stones lay scattered and were replaced.
Top left: Ground plan as restored.
Top right and bottom: Virtual ground plan when the door slab is positioned along the grid of squares like in dolmen 384. Here the door slab was resting upon the cobbled floor (Schuldt 1972, pg. 22) and thus formed a secondary feature.

create one (as demonstrated in figure 12). Yet it stresses the need to uncover as much influences as possible in order to know where in the figure the worst deviations are to be expected.

Geometry or happen chance?

The studied tombs cover only a very small subset of all megalithic tombs. Thereby it becomes hard to draw definite conclusions. Yet some important questions have been answered positively supporting the idea of a Neolithic geometry. Most important is the existence of a 'carrying device'. It is to be expected that Neolithic society has brought forward a group of people that had geometry and masonry as their profession. They probably maintained long distance contacts and exchanged geometrical know how, maybe in the form of challenges. The practitioner diagram can have been such a challenge very well (figure 15). Furthermore, the involved geometry has been in reach for those people since it can be performed by pole and rope solely. This is supported by ancient documents. Not discussed yet is the ongoing development of the practitioner diagram. If one compares the applied figures in tombs along the Baltic Sea, in the Netherlands and in the Ardèche it seems that initially diagonals through *two* adjacent squares was most popular. This shifted to diagonals through *three* adjacent squares in later times. Both diagonals result in a practitioner diagram that produces a 3×4×5 triangle, but only the latter found its way into ancient math text. When the Neolithic turns into the Bronze Age a diagonal through *five* adjacent squares turns up. It introduces another Pythagorean triangle, namely 5×12×13. We find it in the stationary stones of Stonehenge and in the stomions of the Mycenaean tholos tombs. By then the Old Babylonians demonstrated a thorough understanding of how to use geometry for their algebraic problems. Paper PTS-2 will describe how the practitioner diagram can have formed the basis for math problems in Old Babylonian, ancient Egyptian, Demotic, Seleucid and Chinese Han texts. Again, this stresses the widespread application of the practitioner diagram, both in terms of space and time. The 'strength' of the figure lies in its simplicity. The figure includes just two geometrical practises: the grid of squares and a few similar diagonals. Nevertheless, one can use it to find for example the hypothenuse diagram, the Pythagorean rule or many integral triples. And because of this simple geometry, similarities in the layout of the tombs and sometimes their mutual positions are to be expected. All this is demonstrated in the papers to follow.

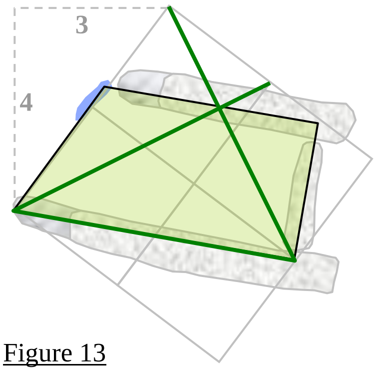


Figure 13
Dolmen la Devèse C in the Gard (France). Both the inscribed triangle and the grid fit the common chamber form of figure 2. The grid of this dolmen has the same orientation as that of figure 5.

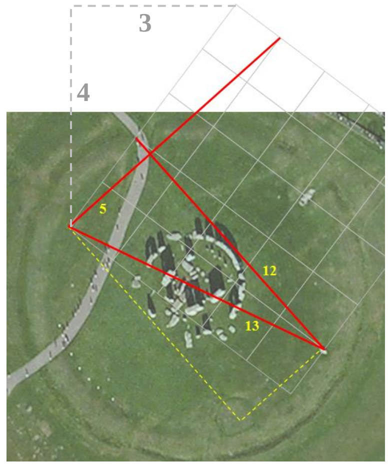


Figure 14
The station stone rectangle of Stonehenge can be orientated and constructed by a practitioner diagram that produces a Pythagorean triangle in a grid of squares.

Map: © 2018 Google, DigitalGlobe, Getmapping plc, Infoterra ltd & Bluesky.

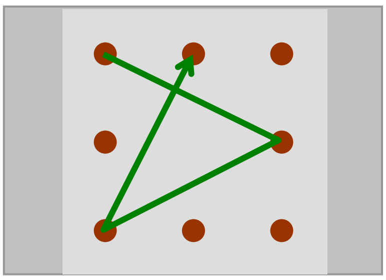


Figure 15
The practitioner diagram as a surveyor's challenge. You get handed over eight poles and a rope. Can you create the numbers one to five?
(Answer in paper PTS-2.)

Prehistoric tomb surveying (2)
Links to the earliest mathematical texts

Dirk Kruithof, September 2019

This is the second paper of a study regarding the geometrical layout of megalithic tombs of the Neolithic and Bronze Age. Grid lines and diagonals through adjacent squares determine the layout of both the tomb plan and the orientation of the tomb. This type of geometry can be linked to the algorithms of finding integral right triangles as ascribed to Pythagoras and to a main problem type in Babylonian Seleucid, Egyptian Demotic and Chinese Han mathematical texts. The current paper attends to the underpinning of such claims in terms of the transmission and development of the related geometry.

In the PTS-papers [1] that follow, many ground plans of Neolithic and Bronze Age megalithic tombs pass by, which have a Pythagorean triangle inscribed. The triangle is drawn in a grid of squares by means of a threesome of diagonals. The complete figure is called the *practitioner diagram* in this study. Both the tombs and the practitioner diagrams are orientated based on the cardinal points and the grid of squares. In this paper the mathematics of the practitioner diagram will be compared to the way the earliest math texts work with diagonals. But first something must be said about the different geometrical mindset in ancient times. We are used to look at geometrical figures like circles, triangles or rectangles as a composition of lines. Based on that composition it is possible to find entities like the perimeter or the area inside. Ancient geometers looked at these figures almost the other way around. A rectangle was seen as a field in the first place. Its boundaries were not lines since length and breadth were characteristics of the area itself. They were often represented by a trysquare - the geometers tool par excellence. The size of a cord stretched between both legs (the diagonal) was a derivative feature, only of interest when it was needed in a construction. A triangle [2] existed of half the area. In the same way we should look at the practitioner diagram. Two of its diagonals form a trysquare and between them the other diagonal is stretched like a cord. Together they *are* half the area of a trysquare.

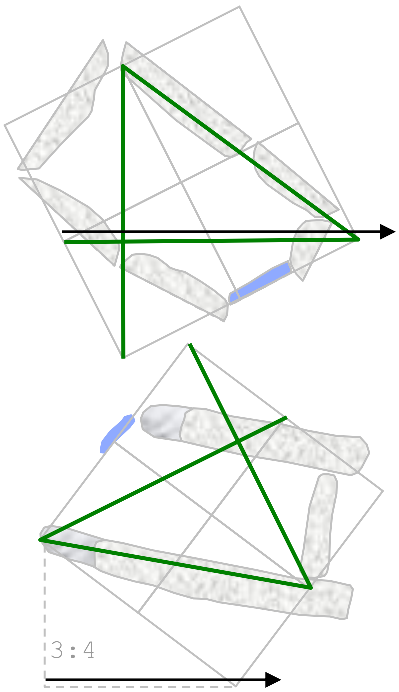


Figure 1
The ground plan and orientation of these dolmens correspond to the practitioner diagram.
Top: The 'erweiterde Dolmen' of Ruserberg (Schleswig Holstein, Germany), ± 3400 BC.
Bottom: 'Causse dolmen' La Dévèse C (Gard, France), ± 2800-2200 BC.

[1] PTS stands for the title of the current study *Prehistoric tomb surveying*. The study is divided in several papers. Each of them covers another subject. Other papers are referred to as PTS-n, where n stands for the serial number.
[2] By lack of goniometrical functions non-right triangles could not be worked out unless they could be compartmentalised with right triangles. A nice example comes from the Old Babylonian corpus: clay tablet *YBC 8633*. Here an isosceles triangle is divided into bundles of three semi-right triangles. The author probably was aware of the fact that he calculated an approximation. (see Hoyrup 2002, pg.254-256)

The practitioner diagram and the Pythagorean Rule

Within the tombs two types of the practitioner diagram are applied. The first was constructed by diagonals through two adjacent squares [3] (figure 1), the other by diagonals through three adjacent squares (figure 2). Both diagrams yield the well-known Pythagorean triangle with sides 3×4×5. It cannot be said if the surveyors knew that they constructed this triangle, but it is likely so. If one draws utility diagonals as in figure 3 the sizes of all line segments are known by counting solely. By the same method the sizes in a diagram with diagonals through three and even more adjacent squares become known. Each diagram yields a triangle with sizes of an integral Pythagorean triple. The oldest enumeration of such triples [4] is annotated in the earliest part of the Sulba Sutra, the Baudhyana (± 800 BCE), with the words:

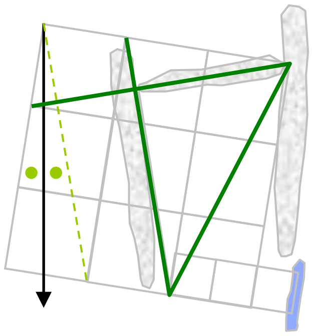


Figure 2
The ground plan and orientation of 'Causse dolmen' Les Géandes 2 (Gard, France), ± 2800-2200 BC.

> (1.12) The areas of the squares produced separately by the length and the breadth of a rectangle together equal the area [of the square1 produced by the diagonal.
>
> (1.13) This is observed in rectangles having sides 3 and 4, 12 and 5, 15 and 8, 7 and 24, 12 and 35, 15 and 36.

Verse 1.13 says that the rule of verse 1.12 (known as the Pythagorean Rule [5]) is *observed*, not found or calculated. By playing around with the practitioner diagram and some additional utility diagonals one can see the truth of the rule indeed (figure 3). The number of adjacent squares for the diagonals is in the order of the enumeration: 3, 5, 4*, 7, 6* and again 3. The diagonals with an asterisk need to be drawn twice as large in order to get integral triples.

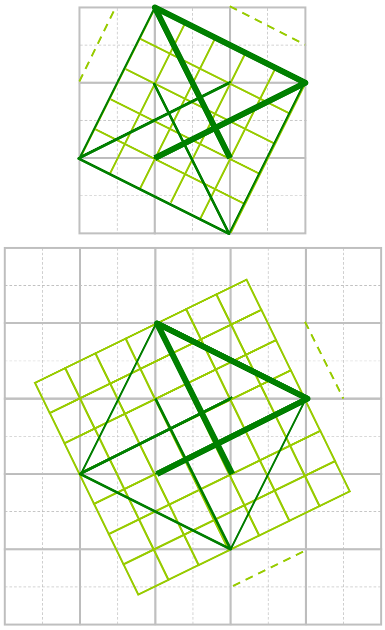


Figure 3
Utility diagonals in the practitioner diagram show the sizes of a Pythagorean triangle. When a set of diagrams are placed together and the upper and lower green grids are merged, the hypotenuse diagram evolves (see figure 4).

It has been discussed widely what culture should be honoured to have invented the Pythagorean Rule. The earliest application of the rule is found in the Old Babylonian clay tablet *IM 67118* [6], probably written around 1850 BCE when the kingdom of Eshnunna had separated itself from the central power of Ur. Hoyrup argues that the Pythagorean Rule was *not* a school invention, but something taken from

[3] Because of the ease of reading and understanding, in this paper the wording '(a) diagonal(s) through two/three adjacent squares' will be replaced by the icon /.
[4] Although Plimpton 322 is older, its triples follow an algorithm based on reciprocals and are not integral.
[5] Following Hoyrup (2002) the wording *Pythagorean Rule* stands for a method of solving problems and contrasts with the term *Pythagorean Theorem* which is a geometrical proof by means of line segments. Note that within the Pythagorean rule the square of side *s* is a physical square and therefore is written in the form □*s*. Conversely the algebraic square of some number is written as s^2 in this paper.
[6] Often quoted after the excavation number Db_2-146 from Tell Dhibai.

the lay environment in the Old Babylonian periphery around the 19th century BCE [7]. Assuming that surveying as a specialist's occupation is linked to megalithic tomb building (see PTS-1), the 'invention' will have taken place somewhere between the earliest megalithic tombs (4000 BCE) and the earliest passed down text (1850 BCE).

The practitioner diagram and the hypotenuse diagram

Clay tablet *IM 67118* uses the Pythagorean Rule as a check on the solution of the preceding problem. The real problem introduces the relation of a diagonal of a rectangle to its surface encompassed by its sides. The diagonal and the surface of a rectangle are given, and the sides are asked for. Such a setup cannot be solved by the Pythagorean Rule. The solution is found via the properties of what is known as the *hypotenuse diagram* (figure 4 and 5-top). First the little square in the middle of the diagram ($\square\delta$) is calculated as the difference of the square of the diagonal and twice the area of the rectangle. Subsequently the text finds the height and base via a standard cut-and-paste practise, which is applied in several clay tablets and must be seen as the core procedure of the Old Babylonian algebra (figure 5-bottom).

Although *IM 67118* has the earliest application of the hypotenuse diagram, the diagram itself is best known from the *xian tu* drawing in the *Zhou bi suan jing* (compiled in the early first century CE, China). The drawing became famous owing to the now outdated claim that it represents the oldest proof of the Pythagorean Rule. In fact, it demonstrates that the square of the diagonal of a rectangle equals to the area of two rectangles plus the square of the difference of the height and the base: $\square d = 2\square hb + \square(h - b)$ (figure 5-top). Most peculiar is the drawing's place in the text: right behind the cryptic hymn about the trysquare, section #A3 of the *Zhou bi* [8]. The hymn reads as a guide of how to create a hypotenuse diagram out of a folded trysquare [9]. It is said to be passed on from 'the times of Zhou' or in other words from 'the good old

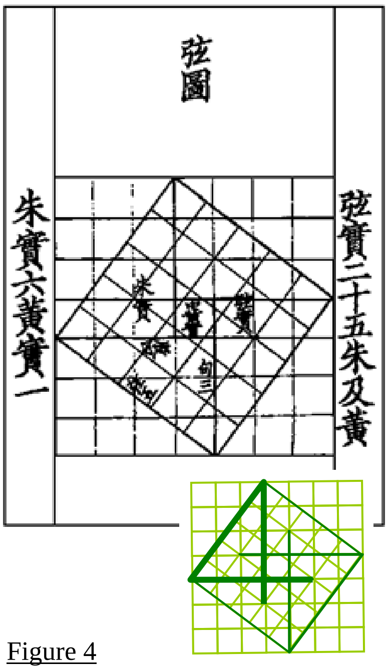


Figure 4
Top: The hypotenuse diagram (*xian tu*) as depicted in Song edition of the Han mathematical text *Zhou bi suan jing*. (1st century CE).
Bottom: The hypotenuse diagram created from practitioner diagrams.

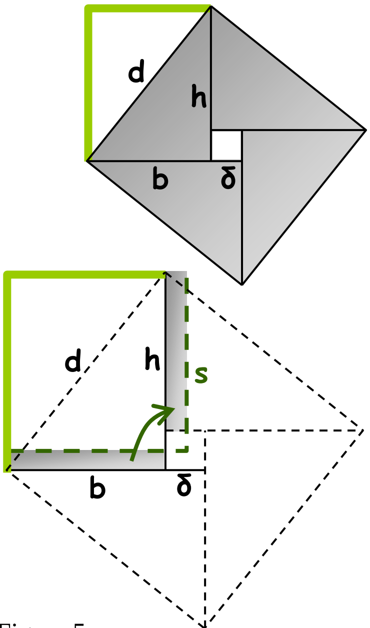


Figure 5
The method of the Old Babylonian clay tablet IM 67118 obv.
Top: The hypotenuse diagram.
$\square\delta=\square d\text{-}2\square hb$ and $\delta=\sqrt{\square\delta}$.
Bottom: Standard cut-and-paste.
$\square s$ equals to $\square hb+\frac{1}{4}\square\delta$.
Side $s=\sqrt{\square s}$ and thus side $h=s+\frac{1}{2}\delta$ and side $b=s\text{-}\frac{1}{2}\delta$.

---

[7] Hoyrup 2002, pg.386-387.
[8] Numbering of the sections follows that of Cullen (1996).
[9] Cullen (1996, pg.88) argues against the hypotenuse diagram making part of the original *Zhou bi* because the first commentator Zhao writes that he constructed diagrams in accordance with the text. Furthermore, he thinks of section #A3 as a hodgepodge of some general knowledge about the Pythagorean Rule. I am inclined to think of section #A3 as part of a framework containing riddles from the lay environment - compact but meaningful. It does not matter so much who added the diagram. More important is how the person could have known that section #A3 could be related to the diagram. This is discussed further in PTS-3.

times'. Of a trysquare having legs 3×9, the long leg is folded into parts of 4 and 5 and thus forming a Pythagorean triangle. The folded trysquare is copied to form a field of 25. Apparently, their diagonals are placed in a square. The composition of four practitioner diagrams in figure 3 can stand in for the construction. The hypotenuse diagram comes about when the utility diagonals of both figures are merged.

Like with the Pythagorean Rule it is hard to say when and where the hypotenuse diagram was discovered. Both applications are found in one of the oldest clay tablets (*IM 67118*) and for both a link to the practitioner diagram can be pointed out. Maybe the coming to existence of the Pythagorean Rule was entwined with that of the hypotenuse diagram. Although there is no historical indication, it would have been easy to prove the Pythagorean Rule by reshaping the hypotenuse diagram according to the cut-and-paste practise (figure 6). When both were linked indeed, then the hypotenuse diagram evolved roughly between 4000-2000 BCE too.

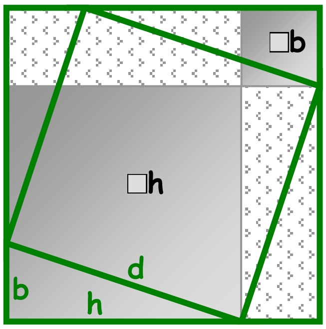


Figure 6
Proof of the Pythagorean rule via hypotenuse diagram and cut-and-paste practise.

Within the hypotenuse diagram we see □*(h + b)* = □*d* + 2□*hb*. The dotted surfaces amount to 2□*hb* also, and thus the rest (grey surfaces) to □*d*.
Therefore: □*d* = □*h* + □*b*.

The extended practitioner diagram

Two ancient texts seem to be linked to the practitioner diagram directly. The first, *Zhou bi* section #A3, was written to demonstrate the construction of the hypotenuse diagram. The other, the Seleucid theme text *BM 34568* (2nd half of the 1st millennium BCE), presents an overview of methods that are applied to work with diagonals. While the *Zhou bi* depends on the practitioner diagram by only, the first problem of *BM 34568* presents two basic formulas that seem to refer to diagrams with both and [10].

In a diagram created from the crossing of the diagonals creates two similar trysquares of which the legs have a ratio 1:2 (figure 7, top). The ratio stems from the number of adjacent squares for the diagonal. (If desirable this can be checked by the utility diagonals like in figure 3 and 4 once more.) By putting protruding line *p* as the unit of the figure (size 1) the other line outside the trysquare (*r*) has size 2 and defines the ratio. According to this ratio, height *h* has size 4. By merging the height and the protruding line, the total length of the diagonal becomes $d = h + p = 4 + 1 = 5$. Subsequently the base can be calculated by subtracting line *r* from the diagonal as $b = d – r = 5 – 2 = 3$. Since line *r* can be found reversely by applying the ratio to the height ($r = 1{:}2 \times h$), in fact calculation $b = d – r$ gives the relation between the diagonal, the base and the height. The diagonal evolves

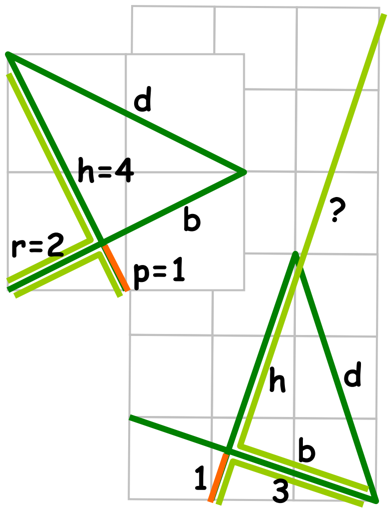


Figure 7
Top: The practitioner diagram with ratio 1:2. The sizes of the line segments are proved by two similar trysquares (light green).
Bottom: In the diagram with ratio 1:3 the similar trysquares are absent. Elongation of the height would complete it after all.

[10] Here I disagree with Hoyrup that problem #1 is nothing else as "an early instance of a mathematical trivial interest in the rectangle or right triangle where *w*, *l* and *d* form an arithmetical progression" (Hoyrup 2002, pg.396). Thereby it would be separated from the rest of the contents of the tablet.

when the ratio is applied to the height and added to the base: $d = 1{:}2 \cdot h + b$. In the other diagram with , the trysquare ratio becomes 1:3. Here the diagonal evolves when the ratio 1:3 is applied to base b in order to find the protruding line that is added to the height: $d = 1{:}3 \cdot b + h$. In a standard application with the protruding line having size 1, this translates to $d = 1 + h$ always. Both formulas $d = 1{:}2 \cdot h + b$ and $d = 1{:}3 \cdot b + h$ seem to be the starting point of *BM 34568*. Problem #1 states:

```
4 is the height and 3 the base. What is the diagonal? Since you do not know.

Join 1/2 of your height to your base. That is it.
4 -the height- steps of 1/2: 2. Join 2 to 3: 5. The diagonal is 5.

Join the third of your base to your height. That is the diagonal.
3 -the base- steps of 1/3: 1. Join 1 to 4: 5. The diagonal is 5.
```

Although the formula for works out correctly, it cannot be derived from the practitioner diagram directly. The diagram lacks the two similar trysquares. By looking for a parallel situation as with the , the elongation of the triangle's height is an obvious step (figure 7, bottom). Thereby the similar trysquares are created after all and the diagonal can be calculated via the procedure of the *lore of the folded trysquare* presented in *Zhou bi* section #A3.

The text of *Zhou bi* section #A3 runs as follows:

```
Thus, fold a trysquare resulting in:
a base broad 3,
an upright high 4,
a diameter aslant 5.

When squared - its outside,
halve its one trysquare 11.
Put round as to occupy a tray
you can fix 3, 4 and 5.
```

Right before this text in section #A2 the trysquare is said to stem from the square tables and thus we must not be surprised to find the phrase 'when squared'. Although we do not have an explanative description of how a trysquare can be squared, it seems obvious (the only thing that can be thought of) that the squaring determines the ratio of the trysquare's legs, in this case *base:upright* $:: 3{:}3^2$. Then we come to the most cryptic part. The squaring is followed by *its outside*. Apparently a trysquare must be thought of as having an inside and an outside. Maybe we must think of the trysquare legs as broad lines. In ancient times often a line was represented as a broad line. The broadness of the line fixed the measuring unit and was taken to be 1 by default. In that case, if we let the inside match the ratio $3{:}3^2$, the outside of the upright becomes 3^2+1. This makes sense, since the outcome is halved so that the size of the diagonal becomes 5 as required by the text. The reason for the halving can be clarified by the extended practitioner diagram (figure 8). Because of the construction by means of diagonals through

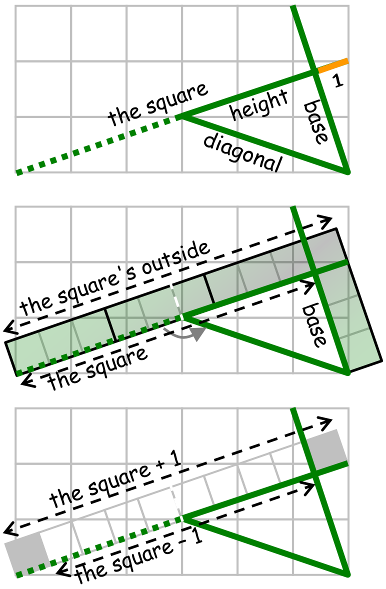


Figure 8
Top: The extended practitioner diagram.
Middle: A perception of the folded trysquare of *Zhou bi* section #A3.
Bottom: The sizes of the folded trysquare are calculated as:
$base = 3$
$diameter = ½(3^2 + 1) = 5$
$height = ½(3^2 – 1) = 4$

[11] Song version. The Qian edition says: "`halve that - one trysquare`".

fields of 1×m squares (a row of adjacent squares), the small segment that protrudes the triangle (orange) will always have size 1 and thus can determine the broadness of the trysquare. The elongated height will always equal to the square of the base. (If desirable this can be checked by the utility diagonals.) A double diagonal is compiled of the segments 'the square' and '1' (thus 3^2+1), which equals to 'the square's outside'. Since the outside makes up a double diagonal it can be folded halfway to get the triangle's diagonal. This method holds true for all Pythagorean triples that are created via the practitioner diagram.

The practitioner diagram and the practitioner algorithm

By the lore of the folded trysquare the simplest form of what will be called the *practitioner algorithm* is established. When the number of adjacent squares for the diagonals is represented by *m*, the sizes of the triangle can be calculated as $b = m$, $h = ½(m^2 – 1)$ and $d = ½(m^2 + 1)$. According to Proclus (5th century CE), the algorithm must be accredited to Pythagoras [12]. Nevertheless, nothing of what we know about the mathematics of the Pythagoreans approves that they have elaborated on the algorithm, so this seems to have been a popular tale in Proclus' time [13].

Concerning the practitioner algorithm an interesting problem type turns up in the early Old Babylonian record. Problem #9 of clay tablet *BM 85196* (2000-1800 BCE) reads: "`A pole 30 [?] a reed from [?]. From the top 6 it descended. At the bottom what did it move?`". Although the text is a little damaged, the essence of it is clear: This is one of the many 'pole-against-the-wall' problems. Since the problem translates to a right triangle of which two sides are known, the solution runs along the Pythagorean Rule. We have to wait more than one and a half millennium before the problem turns up again in a Mesopotamian clay tablet: the Seleucid theme text *BM 34568*, problem #12. "`A reed and a wall I erected. 3 kus I went down. 9 kus it moved away. What is the reed? What is the wall?`". Here the Pythagorean Rule can not help, since only one of the givens translates to a triangle side. The other given is the lowering, which looks like the segment that protrudes from a practitioner diagram, but not being '1'. Consequently, the solution of the folded

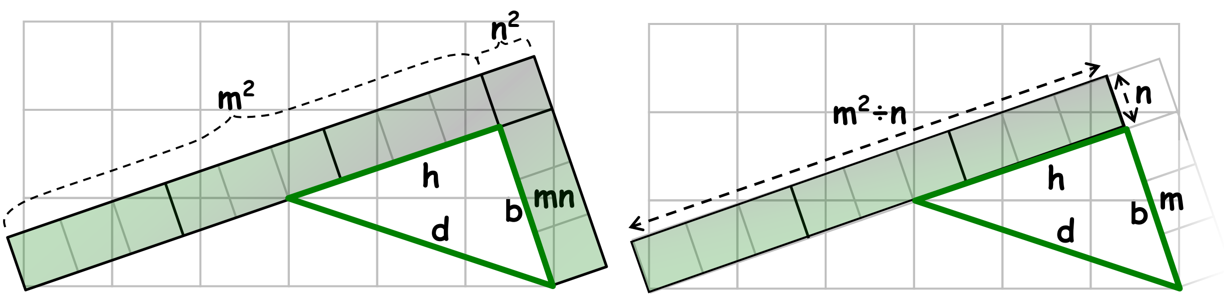


Figure 9
Two different approaches of the practitioner algorithm based on the trysquare ratio *m:n*. With the left diagram the focus is on the broadness of the trysquare and with the right diagram it is on the trysquare area (as it is called by Liu) as in most of the problems of *Jiu zhang suan shu* chapter 9.

Problem	Diagram	Practitioner algorithm		
Jiu zhang 9.6:	Left	$b = mn÷n$,	$h = ½(m^2 – n^2)÷n$,	$d = ½(m^2 + n^2)÷n$
Jiu zhang 9.7:	Right	$b = m$,	$h = ½(m^2÷n – n)$,	$d = ½(m^2÷n + n)$

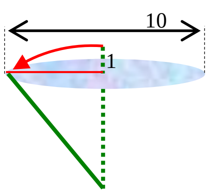
[12] From Proclus' *Commentary on the elements*, proposition 47 - www.pandd.demon.nl/proclus.htm#par (2016).
[13] Sidoli 2010, pg.21

trysquare must fail, as the legs can no longer represent broad lines. Probably it lasted until halfway the first millennium BCE before it was invented how to generalise the simple practitioner algorithm so that problems like *BM 34568* #12 could be tackled. The rumour about the solution must have spread like a wildfire, for suddenly the problem type turns up in Seleucid (Mesopotamia), Demotic (Egypt) and Han (China) texts of around 350-100 BCE.

There exists an eye-catching difference in approach that has led to two different sets of formulas to calculate the sizes of a triangle. We find both approaches in one and the same chapter of the *Jiu zhang suan shu* [14].

Jiu zhang 9.6

A reed standing in a pond

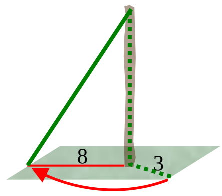



```
Square half the side of the pond.
From it we subtract
the square of 1 chi [that is]
the height above the water
to obtain the depth of the water.
The sum of the result and
the height above the water
is the length of the reed.
```

Formula [15]: $h = ½(m^2–n^2)÷n$

Jiu zhang 9.7

A rope hanging from a pole




```
Square the distance from the foot.
Divide by the length of the end
lying on the ground.
Add the result to the length of the end.
Halve it, giving the length of the rope.
```

Formula: $d = ½(m^2÷n+n)$

Although algebraically the algorithms are no more than a different notation, the deviation in the proceedings demonstrates a different look at the diagram of the folded trysquare (figure 9). Within problem 9.6 initially the trysquare has a sudden broadness n, but it is scaled down to a broad line by the final division '÷n'. On the other hand, Liu, the first commentator on the *Jiu zhang* (± 200 BCE), treats the broadness of the trysquare as a rectangle [16] that fits the formula of problem 9.7. On the whole Liu approaches the concerning problems of chapter nine by the manipulation of a gnomon and its area. In his comments he writes: "`Square the base for the area of the gnomon`". Afterwards this area is divided by the broadness to find the sum of the diagonal and the height, which agrees with the long leg of the trysquare: "`Take it to divide the area of the gnomon to obtain the sum of the diameter and the height`". Apparently, we should regard the area of the long leg to be the area of the trysquare. Thereby the focus shifts from the broadness of the trysquare to the area of the long leg, which reflects another perspective in the approach of the practitioner algorithm. Nevertheless, the author of the problems seems to have been aware of the broadness of the trysquare still. Right after the division of the trysquare area he persistently states: "`then one`" followed by "`the size obtained`" most of the time. Within the proceedings this statement does not have any function (and is not translated in the problems above). It can very

[14] Kangshen 1999, pg.469-472

[15] The final part ÷n is left out in the text since it would yield a division by one. Yet we find 'the square of 1 chi', also being superfluous as a calculation, but decisive when it comes down to the distinction of the two algorithms.

[16] In a comment on *Zhou bi* section #A3 Cullen (2002, pg.768, note 9) puts forward that the legs of an ancient Chinese trysquare should be regarded as rectangular strips. Interestingly Cullen reserves the word 'gnomon' for the shadow pole. Yet Liu talks about the 'area of the gnomon', which is taken from its base.

Source	Period	Data	Result	Method
(PR = Pythagorean Rule, PA = practitioner algorithm)				
BM 85196 #9	2000-1800 BCE	d, v	h, b	PR
BM 34568 #12	300-100 BCE	*v, b*	*d*	*PA*
		b, d	h	PR
pCairo #24, #25, #26	300-250 BCE	d, b	h, v	PR
pCairo #27, #28, #29	300-250 BCE	d, v	h, b	PR
pCairo #30, #31	300-250 BCE	*b, v*	*d*	*PA*
Jiu zhang #9.8	100 BCE-100 CE	*h, u*	*d*	*PA*

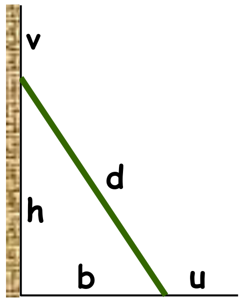


Table 1
Overview of problems of the 'pole-against-the-wall' type. The BM numbers are from Old Babylonia, pCairo from ancient Egypt and the *Jiu zhang suan shu* from the Han dynasty in China. Two ways of solving are applied. The Pythagorean Rule is used when the size of the pole d and the height h or base b are given ($h = \sqrt{(\Box d - \Box b)}$ or $b = \sqrt{(\Box d - \Box h)}$). If either the vertical displacement v and base b or the horizontal displacement w and height h are given, then the problem is solved via the practitioner algorithm ($d = ½(b^2 + v^2) \div v$ or $d = ½(h^2 + u^2) \div u$).

well be a declaration, that the broadness of the long leg has been reduced to a broad line so that the calculation resulted in the actual length of the long leg (and thus in the diagonal plus the height) as intended.

Problems of the type 'a reed (or lotus) in the pond' have a long-lasting tradition that reaches from the *Jiu zhang* till the 16th century CE. It turns up with Bhaskara I (7th century CE) [17], in the Lilavati (11th century CE) and Kangshen reports an even later occurrence from Europe (16th century CE) [18]. More of such widespread problems occur in ancient times like that of the 'the pole against the wall' or that of the 'broken bamboo (or reed)'. If in those problems the givens translate to two sides of a triangle, then the Pythagorean Rule is applied and else when the givens translate to one triangle side together with a lowering or displacement the practitioner algorithm is used (table 1). Probably this reflects the state of the geometer's affairs of the second half of the first millennium BCE although it must be said, that the texts at our disposal were used in a school tradition and can be the elaboration of an older lay tradition.

Into the heart of the oldest 'algebra'

Considering the long time, the practitioner diagram was applied before the earliest math texts saw the light, it would have been rather late if the second half of the first millennium was the first time that the ideas behind the practitioner diagram had influenced the official math tradition. Already we have seen the hypotenuse diagram and the Pythagorean Rule in the Old Babylonian record, but still they were restricted to the periphery. The following will demonstrate that the material can very well have been incorporated in the heart of the Old Babylonian geometrical algebra right away.

Let us return to the lore of the folded trysquare. It is a rather unique fortitude that we find the hypotenuse diagram right after the lore. Most of the time we have to guess what kind of drawing is used in an ancient geometrical solution. Accompanying drawings and calculations were made in the sand or on temporary devices like a dust board. Only typical wordings of a text remind of the figures in use. In *Zhou bi* section #A3 this is the case with the usage of the

[17] Gupta 2011, pg.40
[18] Kangshen 1999, pg.470

word ‘diagonal’ instead of the general term ‘bowstring’ [19] and with the cryptic phrase ‘its outside’. Especially this phrase is interesting as it refers to the little line segment outside the triangle of the practitioner diagram (orange in figures 8 and 10). During the process of scholastisation in the core region of Old Babylonia this protruding little line possibly got frozen in a standard vocabulary about the broad line too. Several times clay tablet *BM 13901* (Ur, 1800 BCE) posits a 'wasitum' with value 1. A 'wasitum' is something that sticks out of something, and in its plain meaning within the given context it is almost as cryptically as 'the outside' of the square in the *Zhou bi*. The 'wasitum' creates a broad line along one of the sides of a square area. *BM 13901* problem #1 starts with "`The field and my base I have accumulated: ¾. You posit the wasitum: 1`". Looking for the origin of this configuration, there is no clear hint from the Old Babylonian times. Much later when Liu commented upon *Jiu zhang* problem 9.6 (a reed in a pond), he states: "`Obtain the height and the hypotenuse from the width and the difference between the width and the hypotenuse. Square the base for the area of the gnomon`" [20].
This reveals that Liu must have had an ambivalent idea of the representation of a square base. In the first sentence he treats it as the long leg of the gnomon (trysquare) that comes from the square tables, according to the *Zhou bi* section #A3. In the second sentence he must have thought of a square field like in *BM 13901* problem #1. Since his viewpoint is not inherent to the problem, Liu must have used a standard and well-known representation of equivalent components (figure 10). In the same way *BM 13901* #1 presents *the field and my base* as a configuration that needs no further explanation.

In cases like *BM 13901* the Old Babylonian cut-and-paste practise seems to function as a schoolish extension to a known configuration from the lay tradition. We have seen this also with problem *IM 67118*. This problem starts off with the hypotenuse diagram (figure 11) and then also continues with the cut-and-paste practise (figure12). That the cut-and-paste practise should be regarded as tool on its own, is demonstrated by tablet *YBC 6967* (Larsa, ±1800 BCE). It is of the so called 'igum-igibum' (reciprocal) type problems, another standard Old Babylonian practise. *YBC 6967* does not give any context and thus is a pure algebraic problem to which the cut-and-paste practise is applied (figure 13).

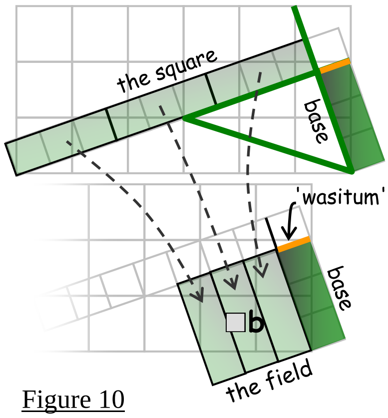


Figure 10
BM 13901. Equivalent components build up the field from the long leg (the square) of a trysquare.

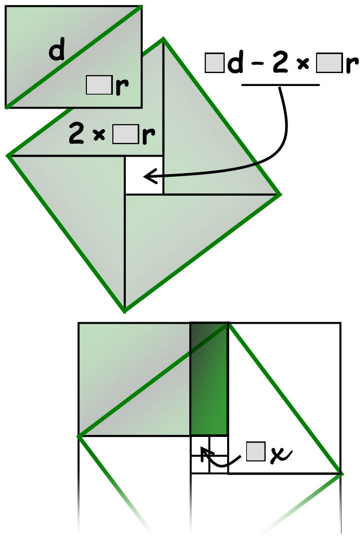


Figure 11
IM 67118. Preparation of a rectangle (field + base) via the hypotenuse diagram. (See fig. 5)

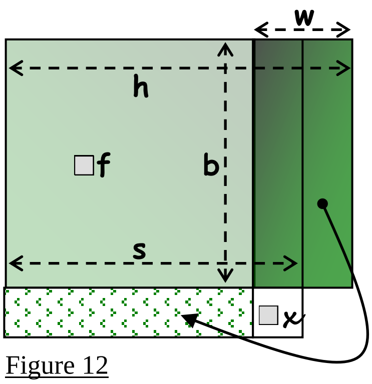


Figure 12
The standard cut-and-paste practise. A rectangle is represented by a square field plus its broad base.

$\square x = (\tfrac{1}{2}w)^2$
$\square s = \square(\square f \cup \square bw) + \square x$
$h = \sqrt{\square s} + \tfrac{1}{2}w$
$b = \sqrt{\square s} - \tfrac{1}{2}w$

[19] A cord stretched between the legs of a trysquare is called 'bowstring' (Cullen 1996, note 90)
[20] Comment of Liu to problem 9.6 of the *Jiu zhang*. - Kangshen 1999, pg.469

Also, from Larsa and contemporary with *YBC 6967* is the famous table type tablet *Plimpton 322*. It lists combinations of widths and diagonals of rectangles that have a length of 1. It does not use the Pythagorean Rule like the earlier discussed list of triples in the Baudhyana, neither does it present integral triples. Instead, the list can be explained by increasing/decreasing values of regular reciprocal pairs. Bruins [21] was the first who recognised the structure of the table and Robson [22] linked it to the standard Old Babylonian cut-and-paste practise (figure 13). Although the method of finding the triples has been demystified thereby, a direct relationship between the square of the diagonal and a real diagonal is missing. Only by introducing the Pythagorean Rule one knows (not *sees*) that a Pythagorean triple is involved. Robson concludes: "*... we will have a composite large square that is the sum of 1 (itself a square) and an imaginary small square. This set of three squares, all generated by a pair of reciprocals, obeys the Pythagorean rule*". (With the underlined phrase is meant: □*h* = *1* = □*h*, compare to figure 13) Some contradiction hides in the fact that the logic of the cut-and-paste method is practical while that of the underpinning Pythagorean Rule is theoretical. Instead of using the Pythagorean Rule as a kind of tailpiece to know that the method produces Pythagorean triples, the author of *Plimpton 322* can have known beforehand that it would. Regarding the algorithm that leads to *h*, *b*, and *d* in figure 13, the formulas are only a factor *r* away from the algorithm based on the lore of the broken trysquare. Via a simple scaling down by *r* (figure 14) the author can have adjusted the algorithm of the lore to the needs of the schoolish 'igum-igibum' and cut-and-paste practises. From the table itself it becomes clear that he was familiar with scaling anyway. (Some listed diagonals and widths appear to be scaled up or down by their common factors.) Additionally, he did not have to guess where to start the reciprocal list. In figure 15 one can easily see that the difference of the reciprocal pairs may not exceed 2 [23]. Width and height will switch position in the diagram if it does, so that it would no longer be the width that 'comes up', as the first column header of the table requires.

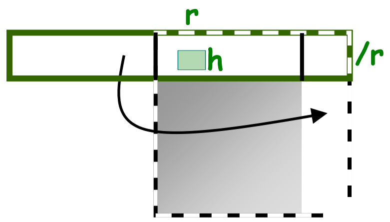


Figure 13
Cut-and-paste diagram via the reciprocal method of *YBC 6967*.
$h = r \cdot /r$
$b = ½(r – /r)$
$d = ½(r + /r)$

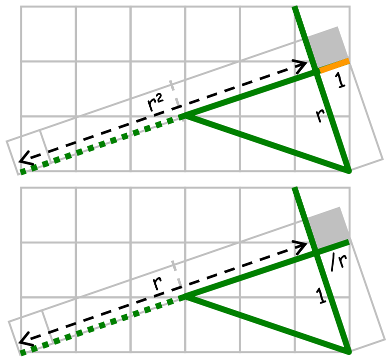


Figure 14
Downscaling by factor *r* gives the proportion:
1:r:r² :: /r: 1:r.

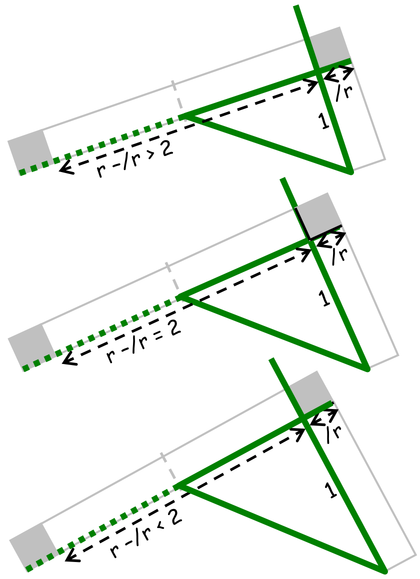
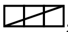


Figure 15
Depending if *r–/r* is greater than, equals to or is less than 2, the diagram results in different triangle shapes. This is easily seen, since *r–/r* needs to be halved to get the corresponding triangle side. Only the bottom diagram fits to *Plimpton 322*.

[21] Bruins 1949
[22] Robson 2001, pg.116
[23] Abdulaziz 2010, pg.8. He writes: "*Observe that as α varies between 1 and 1 +√2, (α−1/α)/2 varies between 0 and 1; and when α increases beyond 1 +√2, (α−1/α)/2 increases beyond 1. In the former case, the longer side of the triangle must be 1; while in the latter the reverse is true. The case α < 1 will be ignored since it leads to negative values of α−1/α.*" Here α agrees with *r* in figure 15.

Increasing surveyor skills

The preceding demonstrates how the practitioner diagram possibly left its traces in the official math tradition. These influences did not only touch the periphery but also the administrative centres. An interesting question is to what extent this was a one-way influence or that (tomb) surveying changed because of new requirements in the official circles. Building dolmens and passage graves seems to have followed a uniform tradition for millennia without any improvement. Many divergent layouts can be ascribed to variations within the feasibility of a grid-of-squares geometry. Maybe the lack of development has to do with the rather egalitarian society (see PTS-1). Later, when the elite comes into prominence during the Bronze Age, the geometry of dolmens is not affected but bigger structures like Stonehenge are built, which do show a new approach. Although Stonehenge may be a rather isolated example during the late Bronze Age, the Mycenaean culture brought forth a lot of tholos tombs that feature the same geometry. In addition to the usual practitioner diagram based on  or , these structures apply diagonals through five adjacent squares producing the 5×12×13 Pythagorean triangle. In Stonehenge the position of the station stones and in the tholos tombs the dimensions and orientation of the stomions (entrances, see PTS-8) follow this pattern. Although large dwellings are found near Stonehenge, it seems that there was no administrative centre and thereby the society lacked the supporting institution for an official math tradition. From the Mycenaean culture such administrative centres are known, but the clay tablets do not show any arithmetic inquisitiveness. Therefore, the new geometry seems to be a surveyor's improvement in a changing world - maybe an adaptation to the larger scale of the new structures.

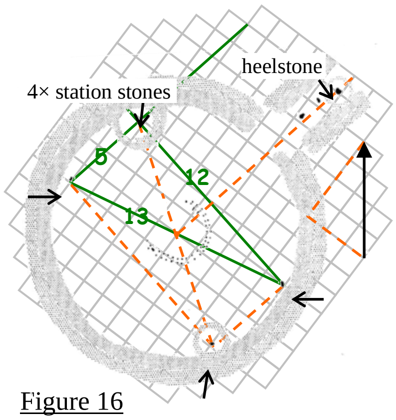


Figure 16
Stonehenge phase two. The station stone rectangle can be orientated and constructed by the practitioner diagram.

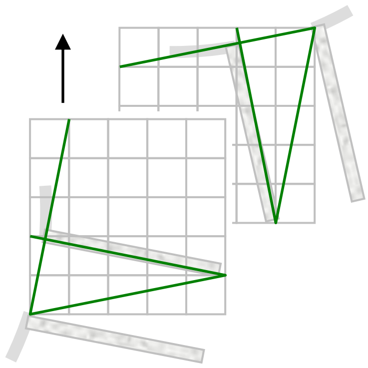


Figure 17
The stomions of the Cyclopean tholos tomb (left) and the tholos tomb of Clytaemnestra (right). Both are formed by the practitioner diagram for triangle 5×12×13.

During the Mycenaean culture gradually the dome size of the tholos tombs increased and finally culminated in the so-called Treasury of Atreus (±1250 BCE). It has not been a treasury, and neither was it the tomb of Atreus, but it was impressive enough. Only after almost a millennium the dome size was surpassed by the Romans. The weakest point of a dome is the connection to the stomion. Both the horizontal and vertical curves of the dome should be fitted to the rectangular setup of the stomion. Yet the surveyors wanted to adhere to a setup via the practitioner geometry [24]. Mostly they followed a strategy by which one connection point fitted the geometry while the other stomion wall was lengthened or shortened to make it connect to the dome nicely. Often this resulted in stomion walls not running parallel or in one wall standing perpendicular to the dome and the other not. But this does not hold true for the well-preserved Treasury of Atreus. The architect figured out how to

[24] PTS-1 calls the setup of tomb ground plans a grid-of-squares geometry. This grid-of-squares geometry is not goal-minded although there can have been some trial and error tactics. The geometry of the Mycenaean stomions goes one step further, because the surveyor had to purposely create a setup that fitted dome and stomion.

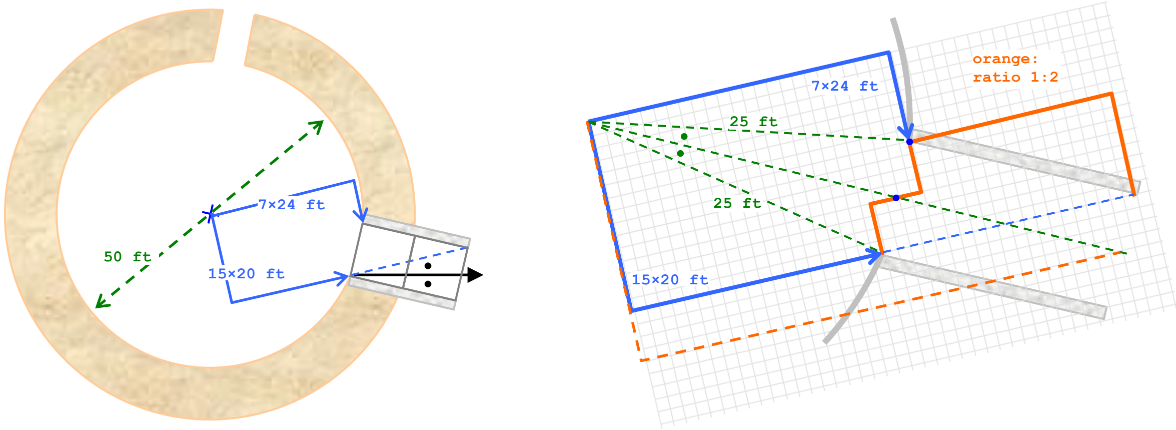


Figure 18
Left: Ground plan of the dome and stomion of the Treasury of Atreus.
Right: Geometry of the connection between the dome and the stomion.
The architect used a radius of 25 feet and recognised in it the diagonals of trysquares with legs 7×24 and 15×20 (a five times enlargement of 3×4). The mutual distance between the connection points can be described by a diagonal of a trysquare with legs 4×8 so that their midpoint is found at 2×4 from each connection point. On its turn this midpoint can be defined by a trysquare with legs 11×22 reckoned from the dome centre, which in fact is an 11 times enlargement of a trysquare with legs 1×2. Thereby the architect could adhere to the traditional orientation via the bisection of a by one of the cardinal orientations (stomion of the left figure).

connect both stomion walls to the dome exactly but needed a larger diameter than ever tried before. Maybe he stumbled into the solution because of a challenge to build the largest tholos tomb. So, he advanced to the practitioner diagram based on a diagonal through 7 adjacent squares and found 25 (in triangle 7×24×25), which is a multiple of 5 (in triangle 3×4×5). The setup via a diagonal through 7 adjacent squares is not seen elsewhere in the Mycenaean tombs. In fact, it is not in the Treasury of Atreus either, for the sides are directly counted in a grid of squares instead of created via the practitioner diagram (figure 18, right). This really reflects a different mindset. Maybe the architect even did not *draw* the practitioner diagram but *calculated* the sizes by the practitioner algorithm. Regrettably, this heyday lasted for just a short moment. The Middle East began to suffer from political instability, the Mycenaean kingdoms collapsed, and Greece sank into its Dark Ages (1100 - 800 BCE). If the late Mycenaeans were the first to apply the practitioner algorithm just before they sank into remembrance, then it was Pythagoras who got the credits after Greece woke up again.

The invention of the practitioner algorithm

Is it reasonable to assume that the architect of the Treasury of Atreus found the 7×24×25 triangle by means of the algorithm linked to the lore of the folded trysquare? In order to frame the time of the invention of the algorithm two moments fix the limits. Around 1800 BCE, when the Old Babylonian geometric algebra culminated, the hypotenuse diagram and the Pythagorean Rule show up already but not the simple practitioner algorithm by which Pythagorean triples are found. All three can be regarded to be forks of the practitioner diagram. At the other end of the time frame, around 300 BCE the extended practitioner algorithm has spread from Egypt to China already. Since the extended form evolved around 300 BCE, the simple version must have been older, and it is not unthinkable that it turned up when Greece scrambled up from the Dark Ages again. If the architect of the Treasury of Atreus knew the algorithm (only a single century before the Dark Ages) it must have been preserved during the Dark Ages somehow, apparently in a lay environment. This is not unthinkable, since the algorithm proved to be very sustainable after all. In the fifth century CE Proclus still remembered it, but wrongly accredited it to Pythagoras.

We see the same story in China, where the preamble of the *Zhou bi* places the lore of the folded trysquare in the times of Zhou (first half of the first millennium BCE). Although discussable from the viewpoint of the text, there can hide some truth in it [25]. The latest dolmens were built in eastern China shortly after 1000 BCE and thus more or less contemporary with the times of Zhou [26]. Also, here it can have been the lay environment that kept the algorithm going on. Moreover, such an early arrival of the simple form in China can explain why the extended practitioner algorithm showed up in two variants in the *Jiu zang*. Only one occurrence (the reed in the pond) happens to apply the same method as the solution in the Seleucid problems of *BM 34568*, while the others have a slightly different approach. Since the 'eccentric' problem of the-reed-in-a-pond belongs to the core riddles that circulated the world (it also shows up in Baskara I for example), it can have taken a deviant route into the *Jiu zhang*. The typical approach of the *Jiu zhang* (figure 9, problem 9.7) can be regarded as a kind of Chinese 'dialect' of the algorithm. When Hoyrup compared the contents of *BM 34568* and *Jiu zhang* chapter 9 [27], he concluded that the Chinese approach must have been a local development. But he restricted his comparison to a possible exchange via merchants. If the practitioner algorithm has been an invention of the surveyor's environment indeed, then the time from the Zhou dynasty until the compilation of the *Zhou Bi* and the *Jiu zhang* is long enough to develop a local approach of it. Yet the similar solution style of the western and eastern problems warrants a common offspring in the form of the algorithm based on the lore of the folded trysquare.

When we accept the course of history above, then it is most likely that the practitioner algorithm in its simple form was invented somewhere during the second half of the second millennium BCE. It must have existed before megalithic tomb surveying reached the eastern coasts of Asia. This makes it plausible that the architect of the Treasury of Atreus belonged to one of the first who made use of it.

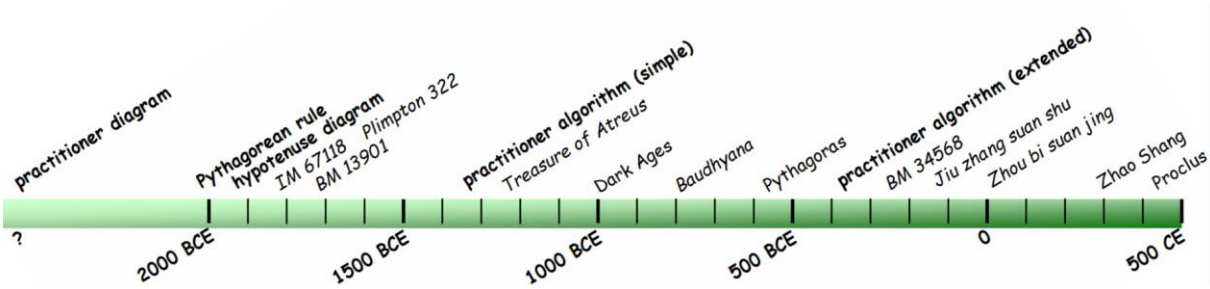


Figure 19
Timeline with the development of algorithms based on the practitioner geometry.
Ancient texts and persons in this paper are added to it.

[25] Cullen (1996, pg.153-155) thinks of the preamble as an adaptation to the political situation during the compilation of the *Zhou bi*. At that moment, during the reign of Wang Mang, it was politically correct to refer to the times of Zhou. Wang Mang called on to an assembly of many knowledgeable men amongst the lay environment. It was the right time for a surveyor's lore to become known in official circles.
[26] Regrettably it was impossible to get hold of some excavation plans of dolmens in eastern China or Korea. It cannot be said if there exist dolmens that show a setup by means of the practitioner diagram. Although very well possible, a surveyor's link between east and west Eurasia must stay hypothetical until these ground plans have been studied too.
[27] Hoyrup 2016, pg.35

Prehistoric tomb surveying (3)
The 'Zhou bi suan jing' section A3

Dirk Kruithof, September 2019

From an early stage onward the text of Zhou Bi section #A3 is commented upon frequently. Mostly the comments compare its mathematics with the Pythagorean Theorem, but we should not clamp down our a posterior knowledge of the theorem on the text. This paper follows a different path and tries to follow it as if it were a cookery recipe: how to fold (or break) a trysquare in order to 'fit' the sides of the Pythagorean triangle. Odd wordings appear to be chosen aptly. And it is exactly as the preceding section #A2, the introduction of the text, says: all depends on the square tables and the right proportions.

The *Zhou bi suan jing* is an ancient Chinese mathematical text consisting of eleven different sections, each handling about a specific math or astronomical subject. There is no narrative about the author, but the first commentator is Zhao Shuang (3rd century CE). In his preface Zhao puts forward that he tried to understand the text and therefore drew diagrams in accordance with it. One of those drawings is called the hypotenuse diagram. This diagram together with a reference to the Duke of Zhou, has often made people think of the text as the earliest prove of the Pythagorean Theorem. Although Zhao presents the text as very old and authoritative, in the modern view of Cullen [1] it is nothing more than a socio-political statement in order to gain respect amongst the contemporaries. He does not accept the drawings of the hypotenuse diagram as appropriate for section #A3 either. The next paragraph is a summary of the reasoning behind.

In ancient China astronomy was closely related to the official calendrical system and the political powers. Astronomy was to establish the calendars firmly in order to support those powers. Now and then the system went out of line with the sky and had to be fine tuned. But slow and sure the cracks in the walls of the official calendrical system became apparent and the old model of gai-tian (the heavenly canopy over the earthly chariot) was to be replaced by a new one: the hun-tian (the spherical model). During the Western Han there was a lot of astronomical activity amongst non officials and it is mainly there that the invention of the new system took place. The *Zhou bi* is most probably a compilation of the ideas of one of those non-official groups. Still the text stands in the tradition of the gai-tian but contains some hun-tian adoptions. Section #A on the other hand stands on its own. It seems to be a preface, added in the times of Wang Mang. This high official established a short dynasty (9-23 CE), interrupting that of the Han. Wang Mang compared himself to the old Duke of Zhou. He assembled thousands of knowledgeable men from throughout the empire, aiming at a revival of the 'lost classical records'. This explains the beginning of section #A: "Long ago, the Duke of Zhou ...".

According to Cullen [2] the Zhou Bi must be regarded as a collection of texts that have been appended in different times, the last one being section #A. Yet it is possible to pursue one step further on the path of diffusion in the materials, since section #A in itself can be treated as a compilation too. The short conversations between the Duke of Zhou and Shang Gao are used

[1] Cullen 1996, pg. 172-173
[2] Cullen 1996, pg. 148-156

as a framework that brings together some independent riddles. Part #A2 (without 'Shang Gao replied'), #A3 (without 'Therefore'), #A5 (without 'Sang Gao said'), #A6 and #A7 (without 'Thus') are texts that can stand on their own, handling different subjects. Although the compilation of section #A probably took place in a short well traceable period, the contents maybe did not. In the time of Wang Mang, the riddles of section #A can have looked extremely old - or politically correct: from the old times of the Duke of Zhou.

The riddles of section #A can be summarized as follows:
#A2 - The numbers of the circle, the square and the trysquare depend on the square tables.
#A3 - Placing two folded trysquares in a ring, creates the numbers 3, 4 and 5.
#A5 - The various applications of a trysquare.
#A6 - The colours of heaven and earth, the circle and the square.
#A7 - True knowledge comes from the trysquare.

Section #A1 introduces the discussion between the Duke and Sang Gao and section #A4 is a short interlude. Generally spoken section #A is a hymn of praise on the trysquare. Yet another text of the *Zhou Bi* could be added to this hymn and it functions as a comparable riddle about the trysquare. It is the very short section #C about circling the square and squaring the circle. The text just mentions them without telling how to perform, but it does tell us the attributes needed - the compasses and the trysquare - and the people who use them: those who do the work of the Great Artificer. This does not really surprise, since the trysquare is a typical surveyor tool. It is tellingly that the usage of the trysquare comes to us in riddle format, while the application of the gnomon and its shadow in the other sections stands in the tradition of the (official) astronomers. Here we probably meet the almost universal pattern of the practisers' *applied* mathematics paralleling the *theoretical* mathematics of the astronomers. Cullen points out that the Zhou Bi probably was used in some kind of school, where the master interpreted the texts and delivered them to his pupils [3]. Furthermore, he finds the mathematics of section #A too elementary to be able to connect it to an epoch [4]. Probably *school* isn't exactly the right wording for section #A. More likely it was the *surveyor's environment* where riddles are passed on to the apprentices. The text of section #A and #C could have come to being as follows. Their riddles survived the succession of the epochs as an oral tradition and finally got collected and canonized within a particular school. Such a view does justice to both Zhao (the content of section #A comes from ancient times) and Cullen (the compilation is a socio-political statement).

The riddles of section #A are rather shallow on the side of their mathematics. A riddle must be easily remembered and does not contain much information in itself. Often it is used to expose one's knowledge and serves as the kick-off for a broader approach of the subject. Zhao labels the mathematics of the *Zhou Bi* as brief, far-reaching and accurate. According to him the text is hard to penetrate, but he (as we) lag behind the initial audience. The apprentices of an ancient master would have become his accompanying demonstrations to show the verity of the riddle. Section #A of the *Zhou Bi* offers the riddles just as they are, and we may guess how far the attached know how once reached. In this paper an attempt is made to reconstruct one such demonstration, attached to paragraph #A3.

[3] Cullen 1996, pg. 149-150
[4] Cullen 1996, pg. 140. Further he states that the cosmological statements are common places from the late Warren States onwards. Those statements are found in one of the riddles of section #A only - #A6. So, the other riddles cannot be dated at all.

The riddle of section #A2 and #A3 runs as follows [5]:

The numbers of the pattern are created from the circle and the square
The circle is created from the square
The square is created from the trysquare
The trysquare is created from the square tables [i]

Thus, break [ii] *a trysquare resulting in*
a base [iii] *broad 3*
an upright [iv] *high 4*
a diagonal aslant 5

Already squared [using] its outside
halve its one trysquare /or:/ *halve that: [making] one trysquare* [v]
Put round as to occupy a tray. [Now] you can fix 3, 4 and 5

Two trysquares occupy an area [vi]*) of 25*
This is meant by 'accumulating trysquares'

[i] Literally: nine nines make eighty-one.
[ii] Cullen translates: fold. He associates it with the idea of 'folding to the right', which means something like 'make a trysquare's angle'. This is found elsewhere in contemporary texts about the trysquare. But there the trysquare results from the folding or must check the rightness of the folding. Here it is the trysquare itself that becomes folded, while the resulting angle is not right. To distinguish I use 'break' instead.
[iii] Literally: sentence.
[iv] Literally: thigh (bone).
[v] The first rendering follows the Song text, the second one follows the emendation of the Qian’s edition. There is little difference in meaning: In the Song text *one trysquare* is the object that is halved and in Qian’s emendation *one trysquare* is the resulting halved trysquare.
[vi] Literally: length. This agrees with the ideas presented in this paper. An area is seen as a grid of squares that can be rearranged in a row and then be counted.

After a short introduction the text states that a trysquare is created from the square tables. This is important information since we are told to fold such a trysquare. What can it mean? Although commentator Zhao Shuang does not make much of it, Liu Hui seems to understand it better in his comments on the *Jiu zang suan su*. In his explanation of problem 9.6 (a reed in a pond) he states: "Square the base for the area of the gnomon." In the preceding passage he has explained what is meant by 'the area of the gnomon', but in rather vague wording. He is more clear about it when he explains problem 9.7 (a rope and a pole): "The end of the string lying on the ground is the difference between the hypotenuse and the height. Take it to divide the area of the gnomon to obtain the sum of the hypotenuse and the height." Apparently, the area of a gnomon equals the sum of a triangle's hypotenuse and height. For our modern ears this is an incorrect statement, because it is not possible to divide an area in line segments.

Suppose somebody draws a horizontal rectangle and tells us that he drew a wall. We can imagine that. But we would have rejected if he had drawn a vertical line and told us it is a wall seen from aside. Then we expect it to be represented by a small vertical rectangle since a wall has some thickness. In the same way ancient people expected a drawing of a trysquare to have thickness. While we would draw two perpendicular lines, for them a trysquare should be represented by so called *broad lines*. Broad lines do have a surface and thus one can cut an area in strips to form such a broad line. This makes it possible to divide the 'area of the gnomon' into two strips representing the height and diagonal of a triangle.

[5] On web page 'http://ctext.org/dictionary.pl?if=en&id=59430#char32780' the full text is found with detailed remarks on the meaning and usage of the Chinese tokens.

By the explanation of Liu, it becomes clear that, if a trysquare stems from the square tables, the square of the base must equal the sum of a triangle's hypotenuse and height. When section #A3 asks us to fold a trysquare, we need to find a way to fold 'the area of the gnomon' in order to separate the hypotenuse and the height. How that is done can be demonstrated by Liu's comment to problem 9.13 (the broken bamboo). Again, he paraphrases the area of the gnomon and then says: "`The total height of the bamboo is the sum of the height and the hypotenuse.`" Here the total height of the bamboo represents the upright of the gnomon and the distance where the tip is folded to the ground is its base (figure 1, top). Accordingly, 'the area of the gnomon' can be interpreted in three ways: (1) as the square of the gnomon's base, (2) as the upright of the gnomon and (3) as the sum of the hypotenuse and diagonal of the resulting triangle. This is the key to understand what is going on in *Zhou bi* section #A3. The folding of the trysquare can simply be performed in the way the bamboo is broken: fold the upright of the trysquare so that the height and the hypotenuse of the required triangle evolve.

Now it is possible to interpret the first phrase of the solution of #A3. It says, "`already squared`", which results in the upright of the gnomon: $u = b^2 = 3^2 = 9$. In the comments above, Liu makes a difference between the 'total height' and just the 'height'. The total height refers to the unbroken bamboo and thus to the upright of an unfolded trysquare. The distance between the ends of such a trysquare is called the 'bowstring' since a rope can be stretched to obtain the size. Using the Pythagorean Rule the size is $s = \sqrt{(3^2+9^2)} = 9.5$. Section #A3 does not talk about the bowstring but about a diagonal of size 5. The word 'diagonal' is correct in this place since it refers to the hypotenuse that together with the plain height forms the upright of the trysquare. This plain height corresponds to the height of the broken bamboo and thus to the height where the upright must be folded (figure 1, top).

After the squaring the text continues with "`its outside`". Since the trysquare consists of broad lines it has an 'inside' and an 'outside'. The standard broadness of an ancient broad line equals to 1. Thus, if the long leg has size 9 on the inside (the square of base 3), it must have size 9 + 1 = 10 on the outside (figure 1, bottom). The next phrase tells us "`halve that`" giving 10 ÷ 2 = 5, exactly the required size for the diagonal. So, if one knows the size of base b of a folded trysquare, the diagonal can be found by the algorithm $d = ½(b^2+1)$. The remaining part of the 'area of the gnomon' is the height with size 9 – 5 = 4. Thereby we got a triangle

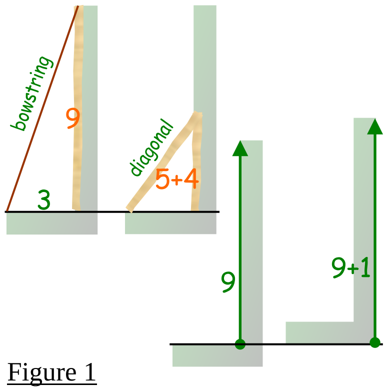


Figure 1
Top: Problem 9.13 of the *Jiu zang*. A bamboo of length 9 is broken, so that a Pythagorean triangle evolves
Bottom: Two typical usages of a trysquare demonstrating the inside and the outside of a trysquare.

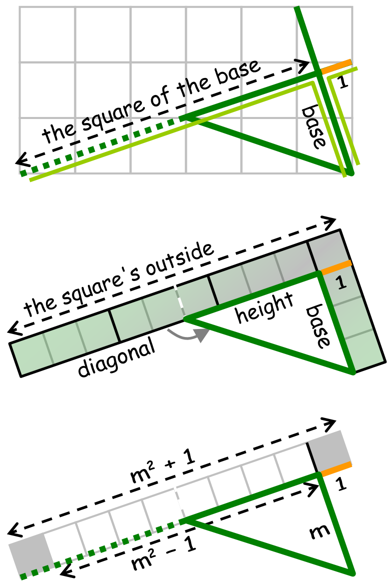


Figure 2
Top: The practitioner diagram. Two similar trysquares (light green) prove the square of the base via the proportion *1:base :: base:base²*.
Middle: The folded trysquare with the positions of the base, the height and the diagonal. The trysquare is folded at the white dashed line.
Bottom: The sizes of the triangle are calculated as:

$base = m$
$diameter = ½(m^2 + 1)$
$height = ½(m^2 – 1)$

Here *m* stands for the number of adjacent squares that produce the diagonal of the practitioner diagram.

with sides 3×4×5 as required. A folded trysquare seems to be the equivalent to what we call a right triangle today. When the text carries on with "`one trysquare`" it seems to declare the newly formed right triangle. Although section #A3 appears to be a very cryptic text, it gives an insight in how Pythagorean triples were found in ancient times.

Obviously, the algorithm $d = ½(b^2+1)$ works out correctly for the trysquare of section #A3. But what about the other Pythagorean triangles? If it would have been a lucky strike to use a base size of 3, while other sizes did not work, probably the algorithm would never have come into circulation. How could both the author of section #A3 and Liu Hui be so sure about the fact that the square of the trysquare's base equals to the sum of the height and diagonal of the folded trysquare? The most likely answer is that they knew about a diagram wherein this can be observed. Paper PTS-2 [6] shows how the algorithm can have evolved from the so-called *practitioner diagram* applied in the ground plans of several Neolithic and Bronze Age tombs. From the diagram not only the algorithm of section #A3 can be elaborated, but also that of the above-mentioned problems of the *Jiu zang*. Probably the process described in *Zhou bi* section #A3 stems from a lore that once circulated amongst tomb surveyors [7]. The lore of the folded trysquare is easily remembered and so is the practitioner diagram, which can prove the trueness of the lore for every integral right triangle. Within the practitioner diagram two similar trysquares can be found of which one is the folded trysquare while the other always has the ratio *1:base* (or better *broadness:base)*. Because of the similarity the folded trysquare must have ratio *base:upright* = *base:base*^{2}. Moreover, within the diagram it is crystal clear where to fold the trysquare to get the diagonal (figure 2). Thus, *Zhou bi* section #A3 reads as a manual on how to find the sizes of the right triangle produced by a practitioner diagram.

Once a folded trysquare has been prepared the text of section #A3 continues with the creation of the *hypotenuse diagram*. It tells us to put round a set of prepared trysquares so that they "`occupy a tray`". If we bring in the practitioner diagram for this operation it becomes obvious why the hypotenuse diagram is placed right behind this section [8]. Within the practitioner diagram's grid of squares "`you can fix 3, 4 and 5`" indeed. The trick hides in two auxiliary grids drawn with the same proportion 1:3 as the diagonals of the folded trysquare (figure 3). Indeed, one can simply count the sizes of the triangle's sides then.

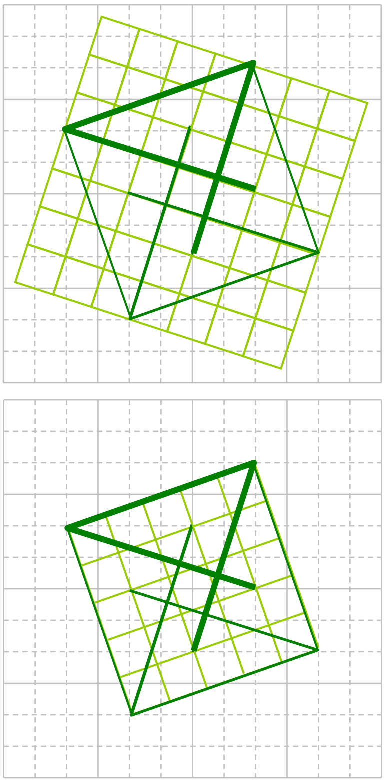


Figure 3
Two utility grids produced by the same diagonals through three adjacent squares as that of the practitioner diagram. By these utility grids the sizes of the Pythagorean triangles are 'fixed'.

[6] PTS stands for the title of the current study *Prehistoric tomb surveying*. The study is divided in several papers. Each of them covers another subject. Other papers are referred to as PTS-n, where n stands for the serial number.

[7] Historically this makes sense too. The latest dolmens in eastern China were built in the early times of the Zhou dynasty. The lore of the folded trysquare can have been conserved in the lay environment. This puts the introduction of section #A in a different light, when it says: "`Long ago, the Duke of Zhou asked Shang Gao`". Maybe there was a vague remembrance of the Zhou dynasty linked to riddles about the trysquare, which were brought together in section #A.

[8] Cullen (1996, pg.88) argues against the hypotenuse diagram making part of the original *Zhou bi* since the first commentator Zhao writes that he constructed diagrams in accordance with the text. On the other hand, Zhao did not understand the contents of section #A3 and therefore was unable to link it to the hypotenuse diagram. Whoever added the diagram, he must have known that section #A3 was related to it.

Finally, section #A3 ends with a message, which seems to be an appendage by the person who recorded the original lore. It does not fit to the guidebook style of the rest of the section and the task to fold a trysquare having sizes 3, 4 and 5 has been closed by the preceding statement "`you can fix 3, 4, and 5`". Moreover, the configuration switches from 'a tray with folded trysquares' to a set of 'accumulated trysquares' and where initially the number of folded trysquares was left open ("`put round as to occupy a tray`") now the number is set to precisely two ("`two trysquares occupy an area of 25`"). Apparently, the phrase "`accumulating trysquares`" had a technical meaning to the author who must have thought that it affiliated to the lore. Without further explanation we are left in the dark of what can be meant by it.

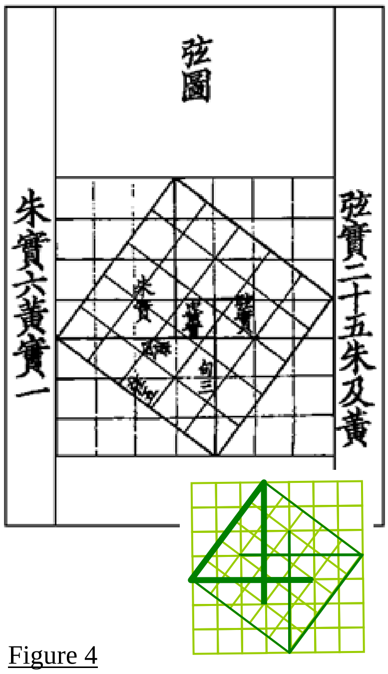


Figure 4
When both utility grids of figure 3 are merged, the hypotenuse diagram evolves.

To conclude, once again the question must be raised of how to judge upon *Zhou Bi* section #A3. Where initially it was presented as a proof of the Pythagorean Theorem, finally it became qualified as possibly a 'garbled and pretentious statement of the fairly obvious' [9]. Probably the truth lies in between. It does not proof the theorem and neither garbles it. The content of section #A3 with its seemingly odd phrases can be linked to later assertions by Liu Hui in his comments on the *Jiu zang*. Because of the emphasis on the trysquare (the surveyor's tool par excellence) an origin in the lay environment is likely, which is affirmed by the historical context [10]. As a lore, the contents can be very old. It can be linked to the practitioner diagram, which appears to be the modelling principle of a lot of megalithic tombs during the Neolithic and the Bronze Age. Consistent with this background, section #A3 does nothing but passing on the know how of folding a right triangle.

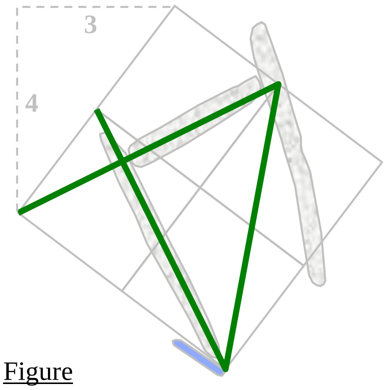


Figure
Ground plan of dolmen Les Géandes 6 in the Ardèche (France). Both the orientation and form can be constructed by a practitioner diagram.

[9] Cullen 1996, pg.87

[10] Important to note: Regrettably I have not succeeded to get hold of some dolmen excavation plans from eastern China or Korea. The structure of these so-called Table type dolmens is comparable to those found in Europe. The carriers of this typical building style are likely to have transported the practitioner diagram too. For now, this is the weakest link of the hypothesis.

Prehistoric practitioner geometry (4)
Mutual orientations between megalithic tombs

Dirk Kruithof, September 2019

The current study of mutual orientation between megalithic tombs in each other's vicinity must be seen as the initial study through which the existence of a prehistoric geometry was tracked down. Since the mutual orientations of the involved hunebedden repeated in dolmen setups at different latitudes, the initial quest for an astronomical interpretation abandoned the field to a geometrical interpretation. Different stages of tomb setups found at consecutive times in various places in Europe form the steppingstones towards the chronicled geometry found in ancient mathematical texts. At first the setups were not esteemed fully yet as if they were separate utilizations of a few geometrical features. Bit by bit these features tied up and paved the way for the idea of an evolving grid-of-squares geometry. Grid lines and diagonals through adjacent squares determine the layout of the ground plan and the mutual orientations of the tombs involved.

Earlier studies of hunebed orientation

In the Netherlands scientific hunebed investigation started in 1878 with the visit of the so-called English Committee in person of *Lukis* and *Dryden* to Drenthe. The British Society of Antiquaries funded this committee because it was alarmed by the unsubstantiated restorations that had taken place under the patronage of the Dutch authorities. When the committee made its tour through Drenthe the tide had turned already. In 1918 *Van Giffen* was instructed to document the condition of all hunebedden. Apart from documenting and photographing, he conducted several scientific excavations, restored many supports [1] based on ground marks and urged the authorities to take the preservation of the hunebedden very seriously.

Both the English committee and Van Giffen have taken the orientation of the hunebedden, but for most of the hunebedden the azimuth values differ by several degrees. In more recent years the work has been repeated by Bom (1978), Reijs (1997), González-García and Costa-Ferrer (2003) and Langbroek (2004). Yet this has not improved the consensus. Apart from the plans of Lukis and Druyden and of Van Giffen, these studies take an undefined longitudinal centre line through the chamber as 'the' hunebed orientation. Only *González-García and Costa-Ferrer* measured the orientation of the portal too. The centre line approach makes it difficult to compare the data and to check on well-defined regularities. Based on these studies one must conclude that it is not possible to hard link hunebed orientation to celestial phenomena. Maybe the sun (Van Giffen), maybe the moon (Reijs and Langbroek) and maybe some star (González-García and Costa-Ferrer) has played a role in hunebed orientation.

[1] There seems to exist no naming convention for the stones that are used to form the walls of a tomb. In literature they are called: support, side stone, end stone or upright. Since *upright* suggest that a stone stands on its short side this term is solely used accordingly in the current study. The term *side stone* is reserved for stones in walls of at least two stones wide while *end stone* identifies a stone that closes off a series of trilithons. Sometimes a stone is called *entrance stone* which means that a side stone flanks the entrance. End stones and entrance stones are subdivided in *left* and *right* now and then - looking from outside in the entrance. Stones that make up the walls of a passage are referred to by *portal stone*. When no further connotation is suggested, wall stones are called *support*. Stones lying upon supports are called *capstone* always.

The mutual orientation of central and satellite hunebedden

The current study started off with a casual observation in 2004 regarding a group of threesome hunebedden in a field near Emmen. Two of them (D38 and D39) have a longitudinal axis more or less perpendicular to their orientation towards the third (D40). The setup gives the impression of a close architectural unity. Most of the Dutch hunebedden stand apart, eight pairs are found at sight distance from each other and only two groups of threesome hunebedden exist. After visiting the other group (D23, D24 and D25) it appeared that the perpendicularity should be refuted as a benchmark for the orientations. Yet there was a striking correspondence in the mutual orientations as if they were two satellite hunebedden and a central one. Actually, another group was found near Benstrup (Germany) having the same setup (965, 966 and 967). Both the Dutch hunebedden and those near Benstrup belong to the TRB west group.

The orientations between the hunebedden were measured using a handmade instrument, which combined a surveying compass (± ¾º), an electronic level (±½º) and a precision gauge for sighting over distances (±¼º). The measurements for Emmen and Bronneger were taken in September 2004 and for Benstrup in May 2005. The magnetic deviation is calculated by the NOAA Deviation Calculator being -0.1º for Emmen (Sep 2004) and +0.3º for Benstrup (April 2005) [2].

In the hunebedden the corner to the right of the entrance was appointed as the reference point for the measurements (figure 2). In D23 the position of the eastern end stone has been filled by Van Giffen and the reference point was chosen near it. For hunebed 965 the eastern chamber of the Langbett was chosen as satellite hunebed. The Langbett is in a bad condition so that the original configuration and the reference point had to be guessed [3]. The eastern end stone is missing and probably the accompanying capstone lies outside the hunebed.

All mutual orientations within the Dutch groups were taken back and forth. Additionally, the angles between the reference points and their distances were measured as a check for the setups. Near Benstrup the surroundings of the hunebedden was overgrown by bushes. Central hunebed 967 was hardly accessible straight ahead from the satellites 965 and 966. Finally, by pushing aside branches and by using big brightly coloured targets it was possible to take the mutual orientations, but no better than within a fault of 3º. Because of time constraints the orientations were taken in one direction and the measurement of the distances was skipped. The results of the measurements are listed in table 1 and 2.

Figure 1
Plotting a 2D cross-section. A laser-leveller is used to project a cross-section on the stones. With an electronic angle and distance measurer the 2D data of the curves is taken. The data is inputted to the application SiteMsm (developed for this purpose specifically).

[2] http://www.ngdc.noaa.gov/geomag-web/?useFullSite (January 2017)
[3] Since the point could be guessed within a meter while the distance between 965 and central tomb 967 amounts to about 100 meters, this has disturbed the results no more than ± 1°.

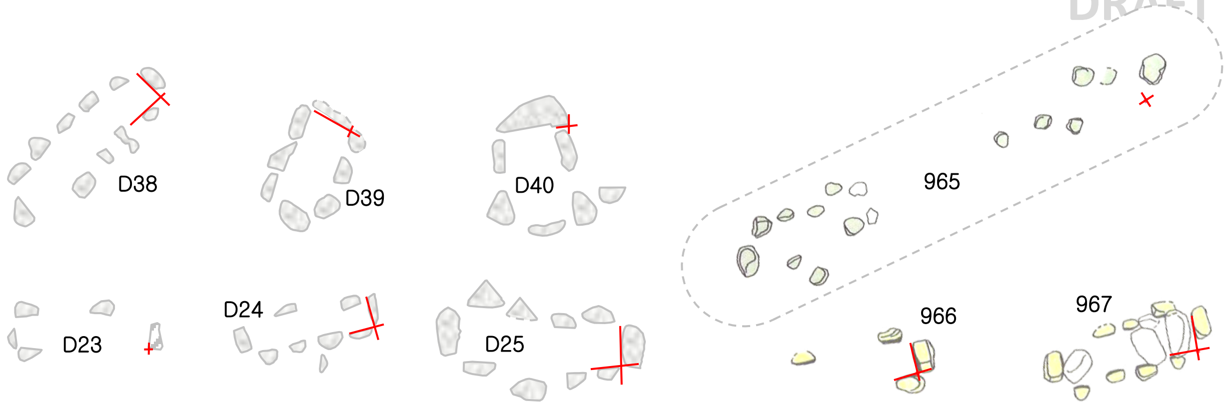


Figure 2
Reference points for the measurement of the mutual orientations and distances.

Orientation	Distance	Inclin.	Azimuth		
			Forward	Backward	Average
Emmen					
D38 → D39	28½ m	+ 1º	194¼º	14½º	194¼º
D38 → D40	66¼ m	+ ½º	140½º	321º	140¾º
D39 → D40	55 m	– ½º	116¼º	296¾º	116½º
Bronneger					
D23 → D24	23 m	– 2º	210º	30½º	210¼º
D23 → D25	47 m	– 1¼º	145¼º	325º	145º
D24 → D25	42 m	+ ¼º	115º	295º	115º
Benstrup					
965 → 966	n.a.	– ½º	79¼º	n.a.	79¼º
965 → 967	n.a.	– 3º	116¼º	n.a.	116¼º
966 → 967	n.a.	n.a.	143¼º	n.a.	143¼º

Table 1
Measurements between the reference points in the hunebed groups. Declination correction: for Emmen and Bronneger -0.1º (September 2004) and for Benstrup +0.3º (April 2005).

Angle	Measured	Calcutated from azimuth	Deviation
Emmen			
D38 : ∠ D39/D40	53¼º	194¼º – 140¾º = 53½º	¼º
D39 : ∠ D38/D40	102º	297º – 194¼º = 102¼º	¼º
D40 : ∠ D38/D39	25º	140¾º – 116½º = 24¼º	¾º
Bronneger			
D23 : ∠ D24/D25	64¾º	210¼º – 145º = 65º	¼º
D24 : ∠ D23/D25	84¾º	115º – 30¼º = 84¾º	0º
D25 : ∠ D23/D24	30º	145º – 115º = 30º	0º
Benstrup			
965 : ∠ 966/967	n.a.	116¼º – 79¼º = 37º	n.a.
966 : ∠ 965/967	114º	259¼º – 143¼º = 116º	2º
967 : ∠ 965/966	n.a.	143¼º – 116¼º = 27º	n.a.

Table 2
Measurements of the angles between the reference points in the hunebed groups.
The angles between the azimuth values of table 1 are checked against the measured angles.

First conclusions

Amongst the three groups, the orientations of the satellite hunebedden towards the central hunebed match within 5º. Such a correspondence can never have been arranged without some kind of measuring technique. The measuring technique can have been twofold: a matter of astronomical alignment or of a geometrical practise. Firstly, the astronomical interpretation was tested. No bright stars rise in the proximity of the azimuth values, but the orientations do resemble the major and minor lunar standstills at 144º and 123º more or less [4]. It was investigated if there existed some architectural features that could support these azimuth values, but they did not. Especially a sightline at 123º is problematic, since it runs a few meters south of hunebed D40 when looking from D39. Accounting for the sloping ground makes things even worse. Moreover, it appeared that the orientations of about 143º and 116º repeat in megalithic tombs at different latitudes (see PTS-5 to PTS-7) and therefore should be attributed to different celestial phenomena. This weakens the astronomical hypothesis drastically and finally it was given up.

On the other hand, the (on average) 116º and 143º orientations fit well to a geometrical interpretation via a grid of squares. A diagonal through two adjacent squares [5] produces an angle of 27º. When a second one is created as an offset of the first, the angle becomes twice as big: 53º. The same angle is obtained by a Pythagorean triangle with sides 3×4×5 (figure 6). When the grid is aligned east-west, the corresponding orientations become 117º and 143º respectively.

Since the mutual orientations only deviate 5º amongst the groups it was expected that the results would be even better when the survey was repeated but then anticipating on the layout of the hunebedden. Closer study revealed that in the groups near Bronneger and Emmen the 143º orientation could be pinned at 140º by some peculiarities. When a diagonal through the central hunebed D40 (Emmen) is elongated towards D38 it happens to run through this hunebed's entrance. By the distance of about 70 meters between the hunebedden, the orientation of the sightline can be determined exactly. Within central hunebed D25 (Bronneger) side stones Z1 and Z2 [6,7] have flattened sides at 140º too.

Figure 3
Top: Systematic orientations of the group near Bronneger.
Middle: Systematic orientations of the group near Emmen.
Bottom: Systematic orientations of the groups of Benstrup, Bronneger and Emmen compared.
(Field plans: Map data © 2015 Google)

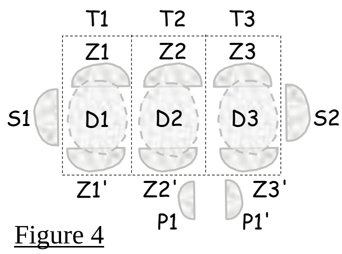


Figure 4
Numbering of tomb stones.

[4] Applications: "Horizon", A.G.K Smith, 2014 (version 0.11b) / "CyberSky", S.M. Schimpf, 2011 (version 5). The accuracy of the programs is within an arc second as far back to 3000 BCE.
[5] Because of the ease of reading and understanding, in this paper the wording '(a) diagonal(s) through two/three adjacent squares' will be replaced by the icon /.
[6] The numbering of the stones follows the system of Van Giffen (figure 4). *D25-Z2'* will be used as short-hand for *side stone Z2' in hunebed D25*.

Figure 5
The mutual orientation of hunebedden D23 and D25 is supported by the interstices between the supports.
Left: View on D25 via a sightline running diagonally through the first trilithon of D23.
Right: Sightlines from D23 run diagonally through the eastern trilithons of D25.

With these first positive results it was decided to undertake a more thorough study of both Dutch groups. Thanks to the surveys of the English committee in 1878 and of Van Giffen in 1918 the early condition of the Dutch hunebedden is well documented per site. Additionally, hunebed D40 has been excavated completely and the mound of D39 was investigated by digging three trenches. For the current study this brought in a lot of valuable information.

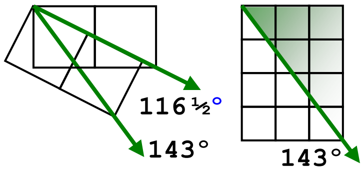


Figure 6
A geometrical interpretation of the mutual orientations of the satellite and central hunebedden.

The satellite and central hunebedden D23, D24 and D25 near Bronneger

Architectonics

Hunebedden D23, D24 and D25 are comparable of size and consist of four trilithons. While D25 is complete, the others lack a lot of stones. Especially D23, which exists of only one trilithon and the accompanying end stone (nevertheless in situ). In D24 the end stones together with the southern side stones are complete. Of the northern wall only side stone Z1 is in situ. Hunebed D25 does not have a passage and that of D24 has been restored partly by Van Giffen in 1961. If D23 once had an entrance is unsure.

Recent history

Almost all of the Dutch hunebedden have suffered from treasury hunting, stone robbing, tourism, vandalism or overzealous 'restoration', and thus a main issue for the current study concerns the reliability of the stone configuration as we find it nowadays. Fortunately, Lukis and Dryden made accurate plans in 1878 (van der Sanden 2015), Van Giffen (1925/27) created an Atlas of all hunebedden and described the hunebed condition per stone and Bakker and Waterbolk (1980) collected the reports of later restorations. These works are of inestimable value for the current study.

None of the hunebedden D23, D24 or D25 have been excavated. There has been treasury hunting in hunebed D25 from before 1918, the moment that Van Giffen started his atlas.

[7] Side stone Z2 has subsided a little into the chamber by the weights of the capstone. Since the capstone slid of towards the first trilithon the subsiding occurred perpendicular to the side at 140º. It is unlikely that this affected the orientation of the side.

Lukis of the English committee wrote in 1878: "*I found much pottery around and within the chamber which has been recently dug into, also a fragment of a burnt bone. In searching for the floor level at the west end I met with fragments of 36 different urns.*" He also found some fragments close to D23. The hunebed chambers got sealed against digging in 1984. Although Van Giffen labels each side stone of D25 (except Z3) as 'more or less in situ', it seems that both middle trilithons (including Z3) have subsided downhill somewhat - probably due to the digging. Meanwhile capstone D2 slid off partly. In his atlas Van Giffen portrayed the hunebed more rectangular than it appears to be. From Van Giffen's comments and drawings it can be concluded that D24-Z4 was visible in his time but according to Lukis it was still there in Janssen's time, suggesting that it was not when he himself visited the hunebed. Later Van Giffen writes that this stone had subsided into the chamber [8]. In 1960 both hunebedden D23 and D24 were on Van Giffen's wish list of restorations. Compared to his plan in the atlas, he raised up side stone D24-Z4 underneath capstone D24-D4 and placed back one of the portal stones. Furthermore, he raised up D23-Z3 and put in place capstone D23-D1, which had slipped from the side stones before the visit of Dryden and Lukis in 1878.

Discussion

In order to get a clear image of D25 a new ground plan was produced by photogrammetry. Both outer trilithons stand in situ while the inner trilithons have subsided some 20 cm downhill indeed. After this was corrected in a virtual restoration, the hunebed appeared to exist of two compartments: a wider western (210 cm) and narrower eastern (165 cm) part [9] (figure 7). Both consist of two trilithons so that the whole gives the impression of a double dolmen with the entrances connected. If both parts were built in one go or if one of the parts was built first cannot be said without an excavation. If we take Schuldt's widths for the dolmens of Mecklenburg as a reference, then the western part could be a *Groβdolmen* and the eastern part an *erweiterde Dolmen* [10]. In chronological order the erweiterde Dolmen evolved before the Groβdolmen. Furthermore, as will be discussed with the geometry, some sightlines between hunebed D24 and D25 are blocked by the western part. In itself, this does not necessarily prove that this part is of later date, but it would agree with such a conclusion.

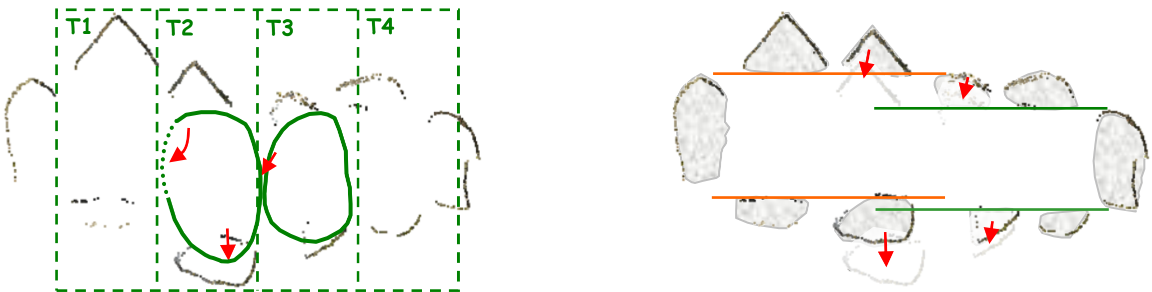


Figure 7
Virtual restoration of hunebed D25 near Bronneger. The plan at current ground level was produced using photogrammetry.
Left: Trilithon T1 and T4 are supported by an end stone and therefore still stand in situ. Trilithon T2 and T3 have subsided downhill a little, probably because of some digging into the chamber. It seems that capstone D3 caused capstone D2 to slide underneath capstone D1 partly (the dotted part).
Right: By 'undoing' the subsiding in a virtual restoration it becomes clear that the hunebed exists of a western and eastern part, each having a different width.

[8] Van Giffen 1925/27, Part I, pg.70
[9] Sizes taken from the atlas of Van Giffen.
[10] Schuldt 1972, pg.22/24

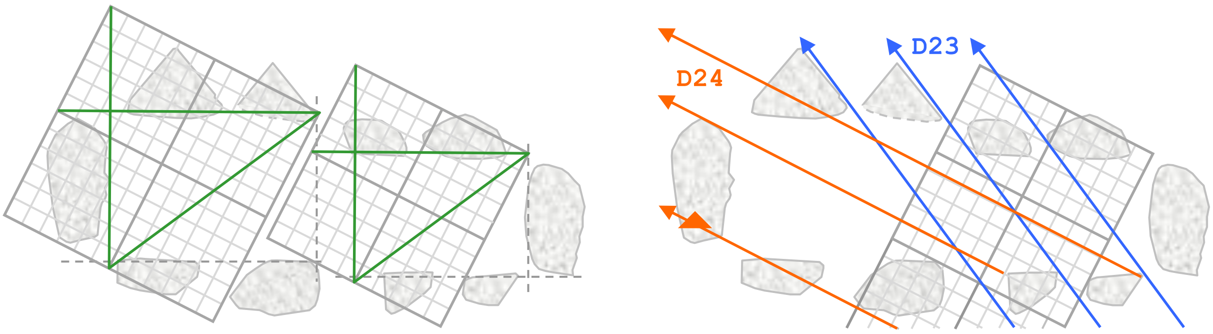


Figure 8
Ground plan of the virtually restored hunebed D25 near Bronneger. Both the dimensions of the wider western and narrower eastern part fit to a practitioner diagram [11] based on respectively 6×6 or 5×5 squares. The grid has an orientation of 27º (in itself produced by ⧄) and is used for the orientation of 117º towards D24 and143º towards D23. The orange prism represents a prismatic bulge on top of capstone D1 (figure 12).

Geometry

Both parts in the virtually restored ground plan of hunebed D25 can be created via a rather straight forward geometrical approach. Mainly it comes down to the application of ⧄. Such a diagonal produces an angle of 26½º. The geometry starts off by the orientation of the grid itself by means of ⧄ on due east (90º + 26½º ≈ 117º). Within this grid a Pythagorean triangle 3×4×5 is produced via three ⧄. When the grid size is taken to be a foot (27.5 cm, extrapolated to floor level [12]) then the main grid for the triangle counts 6 feet for the western and 5 feet for the eastern part (figure 8).

Within the grid a sightline of 143º can be created by yet another ⧄ (116½º + 26½º = 143º). Since such 143º sightlines affiliate to the setup of D25 seamlessly, it conveys the idea that they should be regarded as the primary orientation and those of 140º as the secondary. The same situation we will meet in the group near Emmen. The 140º sightlines cannot be derived from the grid but should be setup via bisection [13] of ⧄ from due south (figure 9).

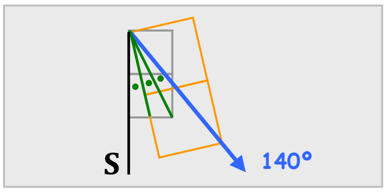


Figure 9
Geometrical construction of 140º. Two bisections give a trisection.

Figure 10
Side stones Z1 (right) and Z2 (left) of hunebed D25. The inset shows the almost perfectly flattened left side of Z2 in a top view of its 3D model. The stone has subsided a little but nevertheless kept its orientation within 1º.

[11] The practitioner diagram is extensively discussed in PTS-2. It can be the link between surveying as it appears in megalithic structures and the official mathematics in its earliest written form.
[12] At pg.19 (figure 30) a foot size of 26.5 cm is found for the nearby hunebedden D21 and D22.
[13] For a discussion about the possibility that Neolithic surveyors could have performed a bisection see PTS-1, pg.10 ad(4).

Since the remains of hunebed D23 exist of one trilithon, only a single sightline of the mutual orientation with D25 can be measured. At a distance of 50 meters a fault of ⅓º (the fault of the compass) will result in 30 cm deviation, and thus one cannot know from viewing in the field alone through which interstice between the supports of D25 the sightline runs. Luckily the field conditions are so that a 3D model could be created via photogrammetry [14]. Thereby it could be established which interstices of D25 interconnect with the sightlines at 140º and 143º through the trilithon of D23. In both hunebedden the 143º sightline happens to run between the supports S1 and Z1. Both hunebedden have their centre line along the east-west line, which makes it likely that the consecutive sightlines at 143º would have run through the consecutive interstices of both hunebedden when D23 would have been complete. This seems to contradict the earlier suggestion that hunebed D25 comprised a western and eastern part of different date. A completely different situation occurs with the 140º sightline. This one runs between D25-Z2 and D25-Z3. Other sightlines at 140º are problematic and do not connect through interstices.

Three sightlines at 117º run through the interstices of hunebed D24 towards D25 of which two lines are blocked by the western half of the hunebed (supports S1 and Z1). Maybe the surveyor tried to compensate by a prismatic bulge upon capstone D25-D1 (figure 11). We will come across more bulges that mark the beginning of a sightline in the group near Emmen but a mark on top of a capstone is not seen elsewhere and should be treated with some reticence.

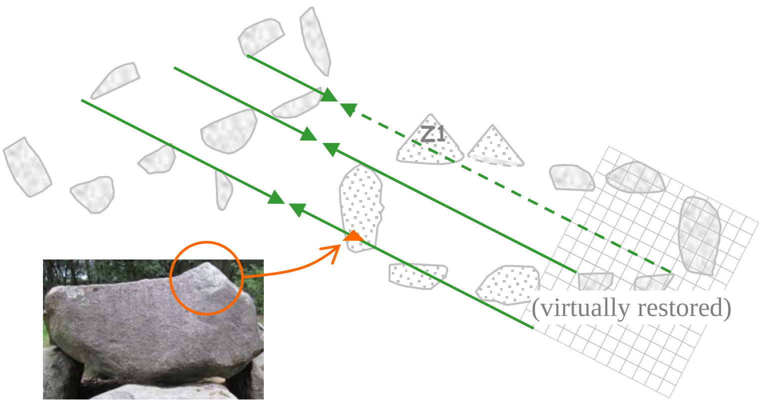


Figure 12
Plan: Mutual orientation between hunebedden D24 and D25. The sightlines fit to the grid of the eastern part of D25. Side stone Z1 of hunebed D25 blocks the northern most sightline.
Inset: Seen from the entrance of D24 the capstone D1 of hunebed D25 shows a prismatic bulge on top (orange prism in the plan).

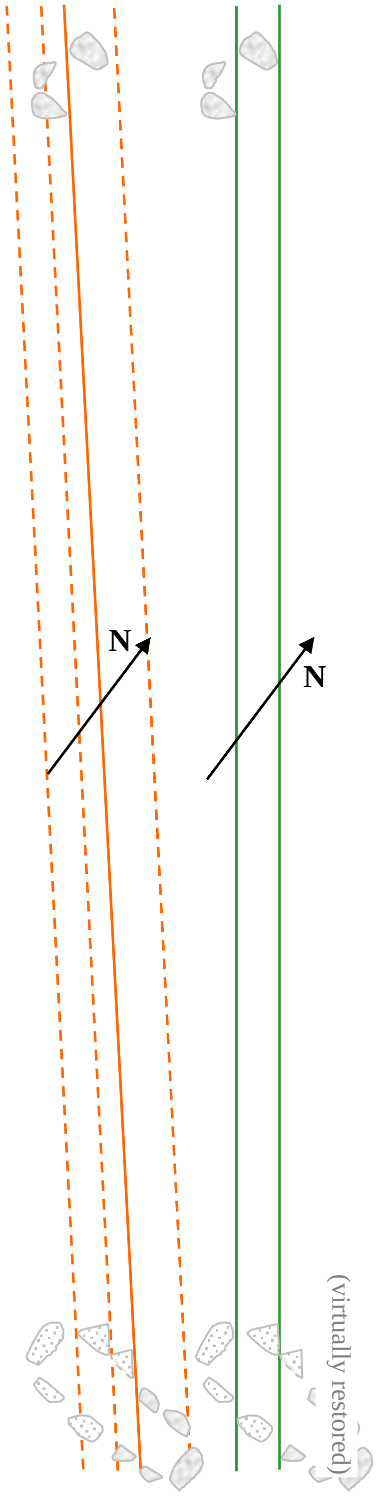


Figure 11
Mutual sightlines between hunebedden D23 and D25.
Left: Sightlines at 140º. Most of the sightlines do not fit well (dashed lines).
Right: Sightlines at 143º. Only two can be established in D23 but they do fit well.

[14] Generally, photogrammetry is unable to produce 3D information from a natural environment.

Finally, there seems to fail a mutual orientation between D23 and D24. The angle between the centre lines of both hunebedden is too sharp to let sightlines run through interstices. Hunebed D23 requires sightlines at about 35º. D24 does so at about 15º. If there have been marks to fix sudden sightlines cannot be said because of the incompleteness of both hunebedden.

Conclusion

From the above it appears that the three hunebedden D23, D24 and D25 partially behave like a geometrical complex of two satellite tombs (D23 and D24) and one central tomb (D25). Sightlines at 143º can have run between the interstices of both side walls of D23 and D25 in the same way. This also holds true for the sightlines that run at 117º through the interstices of D24, but the connection in D25 is problematic, except for the one that starts at a corner of entrance stone D25-Z3’. From there another sightline starts at 140º along the flattened side of D25-Z1 towards D23, which seems to be a secondary feature. Sightlines at 140º do not connect nicely between D23 and D25. All suggested sightlines can be constructed via ⧄ in a grid-of-squares geometry. None of the tombs have been scientifically excavated, and thus it cannot be said if the tombs are contemporary or not.

The satellite and central hunebedden D38, D39 and D40 near Emmen

Architectonics

Hunebedden D40 and D22 are the only ones in the Netherlands existing of two trilithons. D40 has its entrance in one of the long sides, but it is reached from over the mound [15]. In this sense it is comparable to the small hunebed D13, the only Dutch hunebed with a staircase. Furthermore, the southern end stone of D40 does not have the required height in order to bring the hunebed to a close. All in all, hunebed D40 looks like a weird mix between an *erweiterde Dolmen* and a *passage grave*.

D40 is found together with two other ones, D38 and D39, in a field near Emmen. At first sight both other tombs are regular. D38 has five and D39 has three trilithons and in both tombs a passage fails. The entrance of D38 displays an anomaly in that side stone Z4' lies on its long side simulating a passage but still being a side stone. It protrudes a little into the chamber.

Recent history

Van Given edited his atlas in 1918 and no changes worth mentioning took place ever since. Fortunately, hunebed D39 and D40 are surrounded by their mounds, still reaching halfway the side stones and keeping them in place [16]. When Van Giffen documented them, all side stones were in situ, although end stone S1 had subsided outward. Two capstones of D39 had been robbed. In hunebed D38 one side stone (Z1') and three capstones were robbed. Except for D38-Z3 and D38-Z4' the others stood in situ. According to Van Giffen D38-Z3 had subside into the chamber, while D38-Z4' had 'fallen' outward. His opinion about D38-Z4' can be rejected for none of the subsided side stones in the Dutch hunebedden lie neatly flat <u>in</u> the ground as D38-Z4' does and neither do they slide out of their foundation pit into the chamber meanwhile. This would have violated the natural forces [17]. Thus, it was positioned so deliberately.

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[15] Van Giffen, 1925/27, Part II, pg.191. According to Bakker (1992, pg.22) many hunebedden without a portal are likely to have had stairs once.
[16] On the photographs of Van Giffen (1918) both mounds reach to the top of the supports.

Shortly after his survey Van Giffen excavated D40 completely in 1918, producing a great amount of valuable information. While writing his report he ran into opacities and in 1921 he reopened some trenches in the mound. Probably in 1925, since he became interested in the phases of hunebed mounds, Van Giffen dug three trenches in the mound of D39 too. This seems to be documented by drawings only [18]. In 1960 Van Giffen reports to the ministry that he had restored twelve hunebedden "conclusively" [19], amongst them the group near Emmen. At that time Van Giffen focussed on the aggregation of the fields around the hunebedden more than on the restoration of the hunebedden themselves. He even no longer reported the restorations per stone as he did before. Apparently, Van Giffen did not restore if a restoration would take too much effort and would not generate increasing value in terms of heritage [20]. Compared to the plans in the atlas, subsided D38-Z3 and D39-S1 were the only supports that got restored. Until 1984 no other activity took place than the increasing interest of the public. In that year the chamber floors of the three hunebedden got sealed (paved by open tiles 20 cm beneath the ground) in order to protect them against illegal digging. By the end of the year Lanting re-excavated the trenches in the mound of D39. Differences in the situation near D39-S2 confirm that Van Giffen had restored this end stone [21]. The final activity in one of the three hunebedden took place two years later in autumn 1986, when Lanting reopened some trenches in the mound of D40. He wanted to establish the soil on which the mound was raised. The profiles of Van Giffen are not clear about it. Conclusively it can be said that the configuration of the three hunebedden is rather well preserved and reflects that of prehistoric times to a high degree. Especially the stones bearing marks that support some geometry, seem to be authentic.

Discussion

Some interesting features came to light during the excavation of hunebed D40 by Van Giffen in 1918. Van Giffen created detailed excavation plans, drew profiles of the mound and took a lot of photographs. On one of his plans he tried to give a 3D indication of the excavation, having the contours of the supports both at mound and floor level (figure 13). Here we can follow the earlier mentioned diagonal at 140º towards hunebed D38 but also see that this fits the situation near the top of the supports only. At floor level the diagonal has an orientation of 143º. The double orientation evolves from a narrow interstice between D40- Z2 and D40- S2. One of the

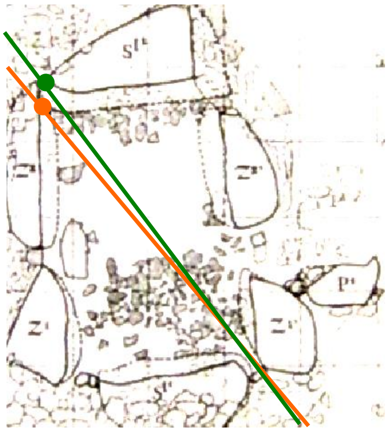


Figure 13
Composite plan of hunebed D40 produced by van Giffen when he excavated the hunebed in 1918 (Van Giffen 1925/27, Atlas, plate 129). Dotted contour lines show the stones at top level, normal contour lines at floor level. The coloured dots compare to the ones in figure 14. The orange line runs at 140º and the green one at 143º.

[17] A comparable situation is found with Van Giffen's excavation report of hunebed D21, where he judges that side stone Z1 had 'fallen' into the chamber. More on it later.
[18] Lanting 1985 (his letter), pg.2
[19] van Giffen, Oct. 1960 31th. Although Van Giffen reports to the minister that he restored the hunebedden "conclusively", in the magazine *Nieuws-Bulletin KNOB* number 11 of 1960 (pg. 242) van Giffen writes that the hunebedden D38, D39 and D40 were restored "more or less".
[20] For example, the subsided side stones that support a capstone (Z4 of D24 -already restored before- and Z3 of D25) still slant as they do in the Atlas, but other subsided side stones (Z3 of D23 and Z3 of D38) were restored.
[21] Lanting 1984, drawings kept at GIA (Groninger Institute of Archaeology).

excavation photographs displays how neatly the interstice has been carved (figure 14). This is a unique feature not seen elsewhere in a hunebed. End stone D40-S2 has another characteristic in the form of a bulge at its further flat chamber side. In the current group three such bulges have been carved: at D38-Z4, D40-Z2' and D40-S2. The bulges are most pronounced near the top of the stones (figure 15). (Later more on the geometrical function of such bulges in the current group of hunebedden.) Since we talk about secondary features, any redressing of the stones should have left traces to be found during an excavation. In the documentation of Van Giffen's excavation of D40 we find peculiarities that hint in this direction, indeed.

Figure 14
The northerly end stone D40-S2, photographed during the excavation by van Giffen in 1918. Its contours follow the form of side stone D40-Z2. The result is a narrow interstice that determines the orientation of the diagonal. The coloured dots agree with those of figure 13.

Many remarks in the report demonstrate that Van Giffen had troubles in interpreting the oddities of this hunebed. There is the path of grit in the northwestern edge of the mound, there are cobbles spread everywhere in the mound except for the 'cobble free zone', the entrance that must be entered from over the mound, the humus layers that flow into each other, and what to think of the northern portal stone found upright in the middle of the chamber. Moreover, a lot of digging had disturbed some important locations in the mound, not only recent but also prehistoric. Van Giffen concluded that the hunebed must have been ruined twice in prehistoric times already. Maybe 'ruined' is not the right word. It can have been reoccupations or renovations.

The first adjustment took place when the mound got raised to its secondary level. When Lanting requests for a re-excavation of some trenches in the mound [22], he states that the cobbles in the mound lie on the primary level and come from the pavement in the chamber. Together with the artefacts, also parts of the floor can have been cleared out. The re-excavation was granted, and it clarified, which layers of the profiles should be regarded as primary and secondary mound levels. According to Lanting the secondary mound should be dated no earlier than the Bell Beaker Culture. Potsherds of this culture rested upon the primary mound [23]. Additionally, this fixes a terminus post quem for a path of grit recorded by Van Giffen.

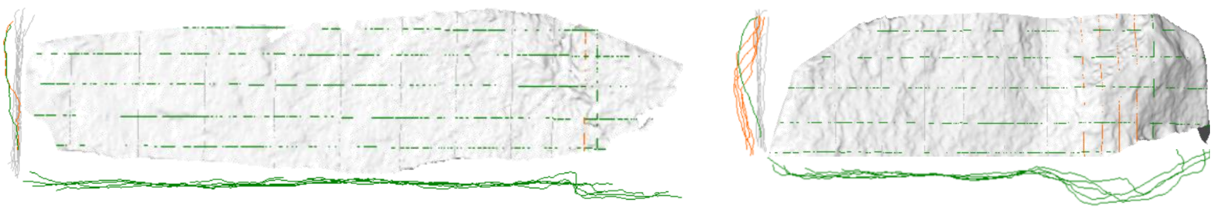


Figure 15
Horizontal and vertical cross-sections through the chamber face of the end stone S2 (left) and side stone Z2' (right) of hunebed D40. The vertical cross-sections display that the bulges at the left are most pronounced near the top, although the one of Z2' has been distorted (orange lines). The small depression directly to the left of the bulges is common feature. It emphasises the bulge - deliberately dressed or not.

[22] Lanting 1986 (letter)
[23] Lanting 2008, pg.273

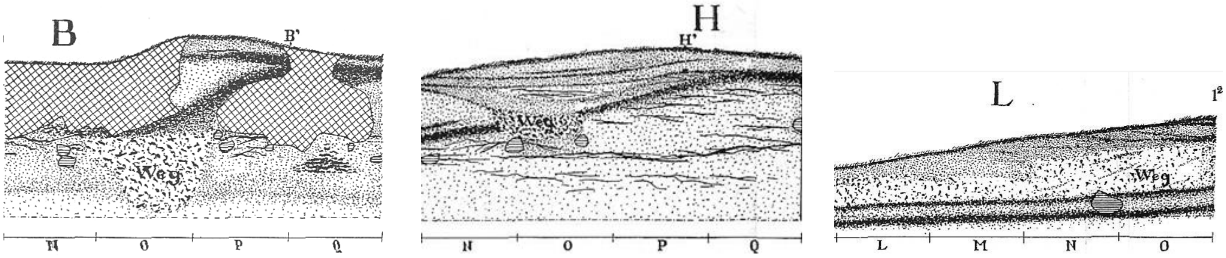


Figure 16
Profiles B, H and L (Van Giffen 1925/27, Atlas, plate 130). Profile B is taken near the chamber and profile L near the foot of the mound. Following the path (called 'weg' in the drawings) on the profiles (B→H→L) it becomes clear that near the chamber the path cuts through the earliest layers, while near the foot it flares out upon a later layer.

The grit path in the mound of D40 is one of the must peculiar features amongst the Dutch hunebedden. Van Giffen reports a compact thick path near the chamber (D40-Z2 and D40-S2), which is found beneath, in and above the secondary mound [24]. He was able to follow the path about 3½ meters towards the mound foot, and we can follow him by means of the vertical profiles he drew. In the consecutive profiles B, H and F we can see how the path narrows and becomes less thick. Then suddenly in profile I it is lying much higher and widely flared out, and the package becomes even thicker (but less massive) and wider in profile L. Apparently, the packages near the chamber and near the foot are of different origine.

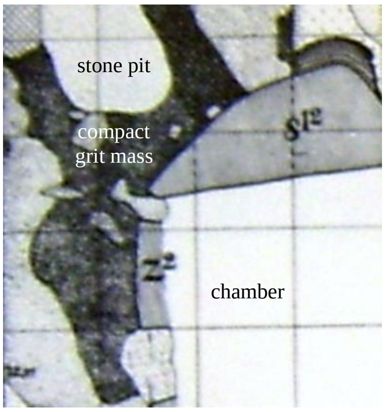


Figure 17
The black-grey mass that forms a path of grit. (Van Giffen 1925/27, Atlas, plate 128).

Furthermore, Van Giffen reports an oval pit cutting through the grit deposition, as if a large stone had lied there (figure 17), and he found cobbles piled up against end stone D40-S2. One can imagine the following process to have happened. D40-S2 was dragged out of the chamber in order to get dressed anew, and the oval pit can have been the place where it was left behind. This explains why the path cuts into the primary mound near the chamber and lies upon the secondary mound a few metres away. Capstone D40-D2 had to be taken off before. Both D40-S2 and D40-Z1 were redressed and the residual grit was deposited around. When the capstone and D40-S2 were replaced the grit got compressed to the massive package that Van Giffen found near the chamber. A ramp had to be created in order to drag the capstone back up. Cobbles were piled up, and sand mixed with grit was carried around to fix them. Some of it got spread near the foot of the mound. Thus, Van Giffen had not been completely wrong, when he concluded that the path was used to place the capstone. However, it was not when the hunebed got built, but after it was reshaped.

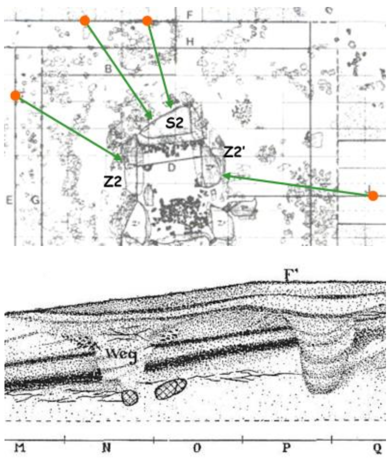


Figure 18
Top: Excavation plan of D40 by Van Giffen (1925/27, Atlas, plate 129). The red dots mark the position of the holes underneath the tertiary layer.
Bottom: Profile F (by Van Giffen). Box N shows the grit path (‘weg’) through which a hole cuts. Half-way box P and Q another hole is found.

[24] Van Giffen 1925/27, Part II, pg.187. Van Giffen writes *primary mound* instead of *secondary mound*. The re-examination of Lanting showed that Van Giffen missed the faint humus layer of the primary mound.

Yet another feature adds to this picture. Profiles E, F and B display a few holes in the mound. As Raemakers pointed out [25], they are covered by the same humus layer as the grit path and can have been places where poles once stood. The creation of the interstice between D40-Z2 and D40-S2 must have been a precision task, and it is very well thinkable that they used pole and rope to keep the stones in place in order to found them. The holes are in the right places for it (figure 18). One of them cuts through the grit path (profile F, box N), which proves that at least this supposed pole was used after the stone dressing. Like D40-S2, D40-Z2' was redressed with a bulge, and probably the precise positioning counts for this stone too. A mounting pole can have stood in the hole of profile J. Although it is likely that there were more poles in use, those are the ones that show up in Van Giffen's profiles.

A redressing of the stones after the raising of the mound to secondary level, coincides well with the bulges near the top of the supports but only if these tops were left visible. Apparently, the surveyors wanted to leave the orientation at floor level intact, while creating a new one. Whether they reused the tomb for burial or simply reconfigured the tomb according to new views, is unsure. Nevertheless, because of the markings they left behind in the form of bulges and flat surfaces, it is possible to grasp the geometrical setup behind the new configuration. On its turn in yet a later stage, this secondary orientation can have been undone too. Although normally the secondary mound level reaches to the capstones, it can be that with D40 it was an intermediate level. Lanting recovered even a tertial layer in the mound.

With the reorientation comes a fundamental question. Did the surveyors modify the existing situation, or can they have started all over again? In 1982 a formerly unknown dolmen (numbered G5) was found under a 2 meters thick layer of clay near Heveskesklooster. It was a threefold surprise: the tomb was built far more northerly than the core region in Drenthe, it clearly had not been a hunebed but an 'erweiterde Dolmen' and it had been disassembled in the late Neolithic already. Some large pieces were hewn from its stones [26]. Apparently, those people did not hesitate to renovate drastically. Ten years later, by the end of 1992 another interesting find came to light in the field of the group near Emmen. Noordam explored a little depression (called D39a) next to hunebed D39 and concluded that it must be the remains of a dismantled stone cist or small hunebed. Maybe in prehistoric times already? A close look at D38 reveals a small bend in the middle of the hunebed. The orientation of the western half follows the mutual orientation of the end stones, but the eastern half has an orientation more or less perpendicular to the initial 143º orientation towards hunebed D40 (figure 19). If now the strange position of D38-Z4' is explained by a simple rotation, then the eastern half of D38 will look like a former dolmen-like tomb (figure 19). In that case, the field would have been occupied by dolmen-like tombs only, plus the not yet dismantled D39a. Maybe its stones were needed to build the western half of D38.

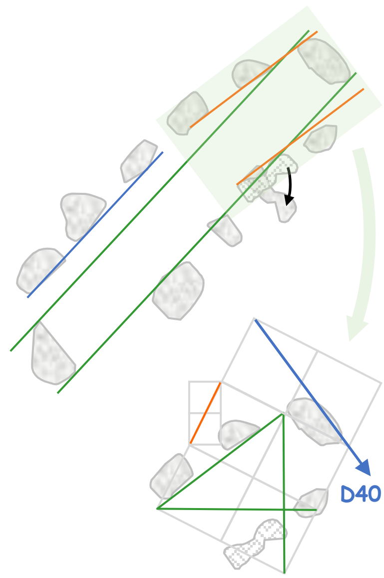


Figure 19
Top: Ground plan of D38. Stone Z4' can have been turned ±90º to its current position.
Bottom: The northeastern part of D38 configured as if it is an *erweiterde Dolmen*. It has a straightforward geometrical setup based upon ⧄. Also, the sightline towards D40 can be constructed like that.

[25] Personal communication (October 2018, 5th)
[26] Bakker 1994b, pg.75

Geometry

Because of the discussion above, we must reckon with two different setups, one at floor level and one near the top of the supports (which almost coincides with current mound level).

The geometry at floor level

Basically, a hunebed consists of two parts: the chamber and the surrounding mound. Many Dutch hunebedden lost their mound in the 19th century because of an overzealous ‘restoration’ with the unwarranted view that the mound was erected in later times to hide the tomb. Apparently, hunebedden D39 and D40 were overlooked, since in the photographs of Van Giffen (1918) their mounds reach to the capstones still. Van Giffen produced a map with contour lines of the mound of D40. Since the primary phase of each mound is raised together with the building of the hunebed, the mound contours must be reckoned to the initial setup. It appears that the contours can be described by an ellipse drawn via ⧄. The diameter has an orientation of 117º, also created via ⧄ (figure 20) [27]. This coincides with the mutual orientation between hunebed D39 and D40.

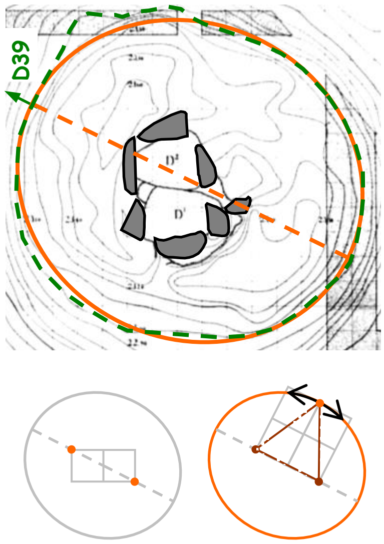


Figure 20
The mound of hunebed D40 forms an ellipse. Ground plan by Van Giffen (1925/27, Atlas, plate 127). Geometrically the ellipse axis can be constructed by a ⧄. The bottom diagrams demonstrate how this is performed by pole (brown dots) and rope (brown triangle).

Probably the initial grid for the geometrical setup of D40 had an orientation of 117º too. With the practitioner diagram [28] drawn in it, the right sides of the triangle define the outline of the chamber (figure 21). Furthermore, D40-Z1 can have been a kind of anchor for the grid, since its two outer sides neatly fit to it. Yet its chamber side does not align with the triangle (and neither with D40-Z2). A grid of 35 cm will fit to both this stone and the practitioner diagram. (The grid size agrees with the grid size of 34½ cm in D38 as suggested later on. Maybe they used a large foot size.)

The geometrical setup of hunebed D39 is ambivalent. Since a ⧄ produces an angle of 27º and two such angles equal to one of the angles in a Pythagorean triangle, it cannot be distinguished if the grid should be orientated towards the cardinal points or via a trysquare 3×4. Nevertheless, the geometrical ingredients of the setup stay the same. Like D40, hunebed D39 has an inscribed right triangle created by means of the practitioner diagram. The dimensions of the chamber are defined by the base (the long side) and the height of the triangle (figure 22). The hunebed orientation is obtained from the grid by a ⧄ and so is the sightline that runs from the southwestern corner of D39 towards the southeastern corner of D40.

[27] In PTS-9 the tomb mounds, drawn up in Sprockhoff's atlases of Schleswig-Holstein and Mecklenburg, are classified based on the grid-of-squares geometry. Most of them have a comparable setup as that of hunebed D40.
[28] The practitioner diagram is extensively discussed in PTS-2. It can be the link between surveying as it appears in megalithic structures and the official mathematics in its earliest written form.

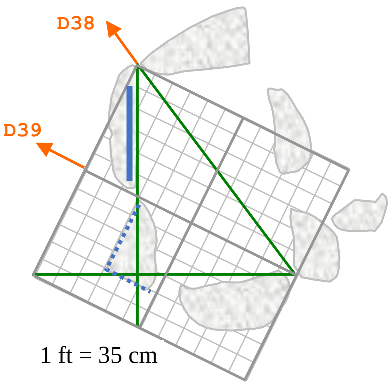


Figure 21
Floor level setup of hunebed D40. The inscribed right triangle is produced by a ⧄. The main grid measures 6×6 squares of a smaller grid with a (foot ?) size of 35 cm.

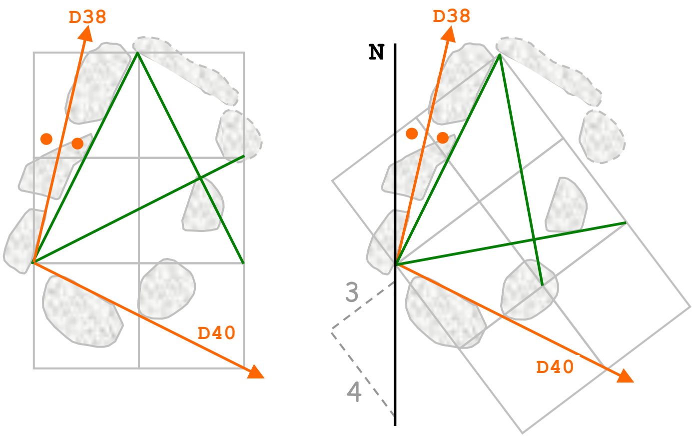


Figure 22
The ground plan of hunebed D39 is ambivalent. Although the basic principle stays the same, the grid orientation can be either true north or created by a trysquare 3×4. The hunebed orientation is derived from a ⧄. The mutual orientation with hunebed D38 is created via bisection of ⧄.

The earlier mentioned diagonal at 143º of D40 in its primary phase aligns with D38-S2. Again, the sightline runs along the outside but here there is no ‘guiding’ side. No other features of D38 can be linked to an 143º orientation but they can have been there before the hunebed was turned into the secondary orientation. Without excavation we cannot be sure about the evolvements in D38.

The geometry at mound level

First, it was checked if a setup by parallel sightlines, like in the group near Bronneger, supported the orientations at 117º and 140º in the group near Emmen too. This appeared to be more troublesome than expected. While on one hand orientations over long distances can be measured more exactly, on the other hand such distances make it hard to distinguish between parallel sightlines [29 (pg.18)]. Therefore, per hunebed the distances between the supposed sightlines were measured and compared to the distances in the target hunebed (figure 23). The comparison demonstrated that such parallel sightlines were there but, since they run at regular distances, still a shift between those sightlines was possible. Only when the aerial photographs of Google Maps became detailed enough, the real interconnection between D39 and D40 could be clarified (figure 24).

Figure 23
Parallel sightlines through hunebed D40. The mutual distances match those in hunebed D39

While measuring the distances between the sightlines, there was need to locate them in a systematic way. In the group near Bronneger already the eye-catching flat faces of D25-Z1 and D25-Z2 were recognized as ‘guides’ for the 140º sightlines where they crossed the hunebed through the interstices between the side stones. Additionally, in the group near Emmen there seemed to be more or less prismatic bulges on some flat stone faces.

Figure 24
Secondary orientation of the group near Emmen displayed on Google Maps (© Google, Kaartgegevens, 2019).

Inset:
Top: The mutual orientation of hunebedden D38 and D40 by parallel sightlines at 140º.
Bottom: The mutual orientation of hunebedden D39 and D40 by parallel sightlines at 116½º.
Line endings with diamonds reflect a bulge mark on a stone. Short yellow lines reflect a flat stone side. Dots are placed in important hunebed corners. Dashed lines show the expected position of some sightlines.

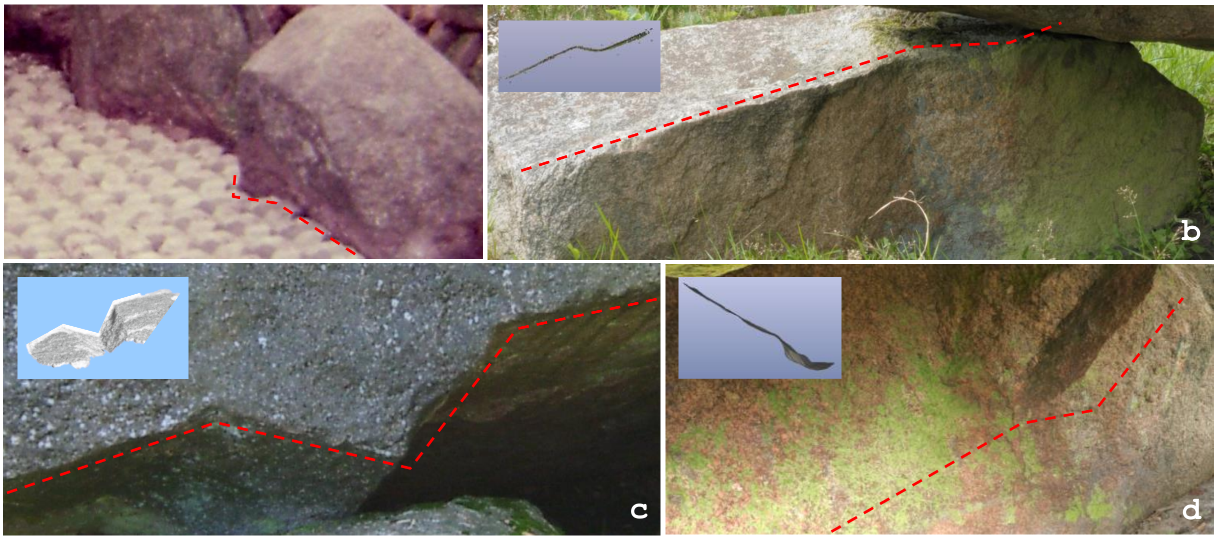


Figure 25
Marks at stones. The red dotted lines highlight a mark. The insets show a 3D model of the stone surfaces.
(a) Side stones Z3 and Z4 of hunebed D38. Photo of the sealing of the chamber content. (© ROB, 1984)
(b) Side stone Z2 of hunebed D39 having a double surface. Inset: top view of the chamber side in a 3D model.
(c) A stone in the vicinity of the group near Emmen. Probably, the stone comes from a dismantled hunebed. The surfaces at both sides of the mark twist by only 1º compared to each other. Two horizontal stripes to the left of the mark look like the remains of stone carving.
(d) Side stone Z2' of hunebed D40. Because of weathering the bulb has vanished near the top. Inset: top view of the chamber side in a 3D model. The flatness of the stone contrasts to the bulge at the right.

The idea rose that there existed some kind of functional distinction for the types of marks. The bulges seem to pinpoint the begin or the end of a sightline, while flat sides indicate the orientation of it. Indeed, the orientation pattern of the group can be ‘read’ like that.

We find marks at:

- D38 side stone Z4: A bulge on a further flat surface
- D38 side stone Z2': Two flat sides at 140º.
- D39 side stone Z2: A shift between two flat surfaces
- D40 side stone Z1: Perpendicular flat sides
- D40 side stone Z2': A distorted bulge on a further flat surface.
- D40 side stone Z2: A flat side marking north-south.
- D40 side stone S2: A bulge on a further flat surface.

The marks of D40-Z1 and D40-Z2 have been discussed already (figure 21) while that of D40-S2 plays a role in the hunebed setup and not in the orientation (figure 27). The role of the other marks is clarified in figure 24, inset.

At the time of the secondary orientation, the 117º orientation was kept as it was. This can be concluded from some marks: A sightline connects the mark of D40-Z2’ with the mark of D39-Z2, running between D40-Z2 and D40-S2 (figure 25b,d and 23, inset). Furthermore, the chamber side of the end stones of D39 can have had an orientation of 117º, since the hunebed stands perpendicular to the mutual orientation of D39 and D40. Both end stones display a 1½º-2º divergence from 117º. As argued before, D39-S1 was restored by Van Giffen, and the reopening by Lanting of one of Van Giffen’s trenches in the mound of D39 revealed that the northeastern part got raised in prehistoric times. Thereby the anomalies can be explained.

From hunebed D39 a sightline is created to D38-Z4 by the bisection of the hunebed orientation (again via ⧄) and true north. By lack of stone marks or parallel lines between D38 and D39, the exact origin of the sightline can range from the chamber side to the outside of D39-Z1 by compass measurement alone [29]. Again, the detailed map of Google Maps clarified the setup (figure 24). The sightline comes from D38-Z4 and runs along the chamber side of D39-Z1. It also runs along the chamber side of D39-Z2 but this a peculiar stone.

D39-Z2 has an extraordinary feature as it has *two* perfect flat faces at its chamber side (figure 25b). Reckoned from the chamber corner S1/Z1, these surfaces are rotated by 3º from 27º back to 24º. (The 27º face guides the before discussed sightline between D40-Z4 and D39-Z1.) This 3º rotation reminds of the reoriented sightline from 143º back to 140º between hunebedden D38 and D40. Although it is a *surprising feature* [30] in the light of Heggie's demands for an accepted geometry, no additional geometrical features seem to exist in D39, which are supported by it.

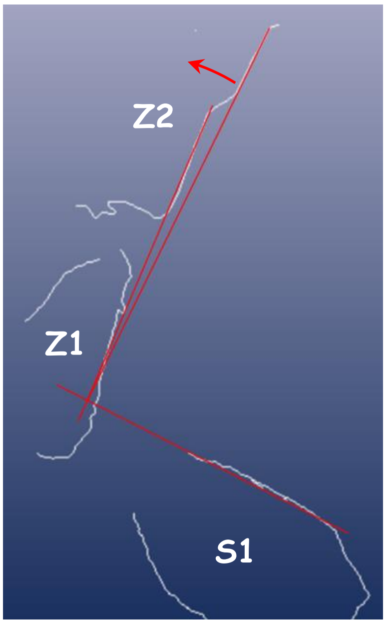


Figure 26
The southeastern part of D39. Side stone Z2 has a double face – at 27º and 24º. They are rotated around the chamber corner.

The secondary orientation of 140º for the diagonal through hunebed D40 is created from the combination of a bisection of ⧄ and the ⧄ itself (13º + 27º = 40º), which is taken as an offset from due south (180º - 40º = 140º). In fact, such a setup results in a trisection. We must doubt that the surveyor was aware of it. Probably, he regarded the creation of ⧄ and its bisection as tools instead of geometrical figures.

In the western part of D38 a grid can have been orientated by the same trisection as that of the 140º diagonal in D40. Here the grid size can have measured 34½ cm. (Maybe it reflects a large foot size, since the same grid size is found in the initial setup of D40). Side stone Z2' has two flat sides along the grid lines. With its width of 91 cm it is slightly smaller than three grid distances (figure 28). Another observation in D38 concerns a 193º sightline towards hunebed D39. Within a grid at 140º it is easily created by means of a trysquare with legs 3×4. Again, this sightline starts at the bulge of side stone Z4. It runs freely alongside stone Z2', but only after the edge of Z2' had been cut off. This was checked by means of a stretched cord (figure 29).

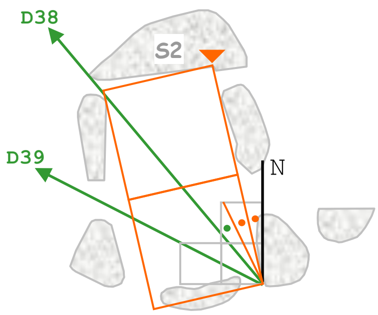


Figure 27
Geometrical setup of the secondary orientation of D40. Note that the bulge at end stone S2 (orange arrow) fixes the orientation of the large double square.

[29] Measurements taken by a Recto DP10, having a 0.3º fault. In natural circumstance the fault easily increases to half a degree. Over a distance of 30 to 70 meters (D38-D39 or D38-D40), this translates to 25 to 60 cm.
[30] Heggie, 1982, pg.67 (see PTS 1, pg. 7)

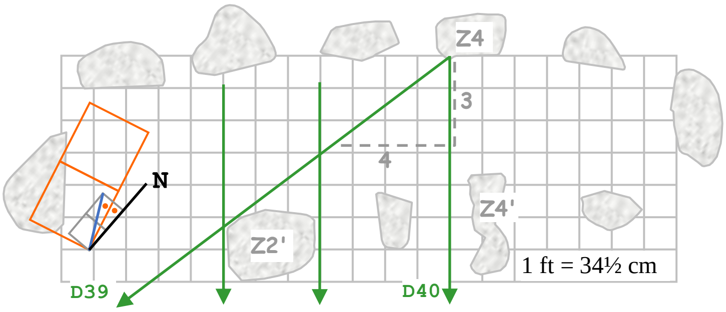


Figure 29
The reduced edge of D38-Z2'. The cord begins at the bulge of D38-Z4 and is stretched at 193¼°.

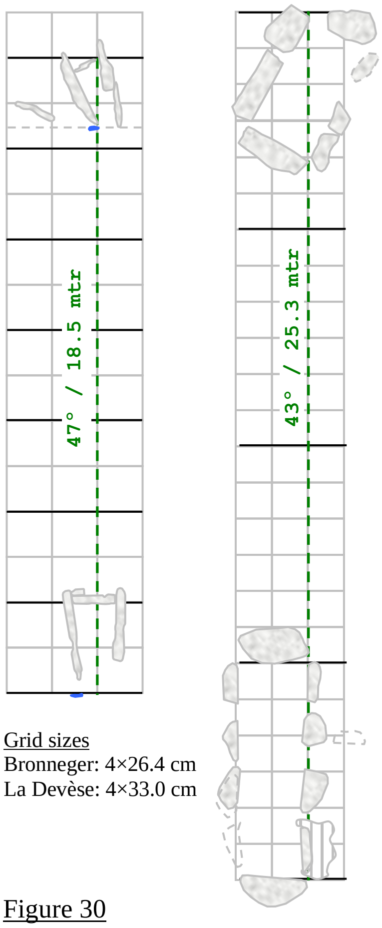
Figure 28
The geometrical setup of the secondary phase of hunebed D38. The orientation of the grid equals that of the 140° diagonal through hunebed D40 (figure 27). Two sightlines start at the bulge of side stone Z4: one towards hunebed D39 and the other towards D40. The angle between both lines can be described by a Pythagorean triangle. The width of side stone Z2' equals to a little less than three grid steps.

Conclusion

Like the group near Bronneger, the group near Emmen can be regarded as geometrical complex – at least for its secondary orientation. Most features of the primary orientation in D38 got lost because of the secondary orientation. In the secondary orientation we have three points (the southeast corner of D40, the bulge at Z4' in D38 and the southwest corner of D39) that are interconnected by sightlines. The setup depends upon ⧄, its bisection and the 3×4 trysquare. The secondary orientation can be 'read' via marks like bulges and flat sides at the stones. Because of the excavation of D40 we can reproduce the primary orientation too. For some reason the mutual orientation between D38 and D40 was rotated counterclockwise by 3°. Thereby a rather straight forward hunebed orientation was replaced by more a complex trisection. By means of marks at the supports the relation between the three group members became tighter. The secondary orientation seems to have taken place during the Bell Beaker Culture or the early Bronze Age. It must have been a culture that regarded the layout as an important characteristic for a tomb.

Neighbouring tombs utilising the same grid orientation

While studying the orientations of the small necropolis of La Devèse in the Ardèche, two dolmens at about 15 meters distance of each other advanced a setup that was already known from the hunebedden D21 and D22 in Drenthe. The longitudinal axis of the southern tomb and a diagonal of the northern tomb have a shared orientation: 43° in Drenthe and 90° – 43° = 47° in the Ardèche. Although we find the same geometrical ingredients at both locations, the configurations are not a copy of one another. In hunebed D21 the sightline at



Grid sizes
Bronneger: 4×26.4 cm
La Devèse: 4×33.0 cm

Figure 30
Interrelated orientations of tombs based on a mutual sightline.
Left: The ground plans of dolmens La Devèse A and B in the Ardèche based on a combined 3D model.
Right: The ground plans of hunebedden D21 and D22 based on the excavation plan of Van Giffen (1925/27, Atlas, plate 146).

43º runs along one of its walls, while in La Devèse A the 47º sightline is guided by half a cupule [31] at the top of the end stone. The diagonal in La Devèse B runs from southwest to northeast and in hunebed D22 from southeast to northwest. Thereby, at both locations a close unity is suggested.

Hunebed D21 and D22 near Bronneger

Architectonics

Hunebed D21 is a very generic tomb with four trilithons and a little passage of which only one support remains (that was restored by Van Giffen). It is a well-ordered tomb in that the side stones stand perfectly in a row and thereby really give the impression of a wall. Probably this has been the case in D22 - the smallest hunebed of the Netherlands - too, but a large oak has disturbed the entrance part. D22 is composed of only two trilithons of which the first shows a weird solution: Support Z1' consists of two small stones Z1'a and Z1'b supporting the same capstone. Such a feature is not seen elsewhere.

Recent history

When Van Giffen started the excavation of D21 and D22 in 1918, it appeared that the chambers of the hunebedden were not touched in recent history. Only the mounds and the passage of D21 seem to have suffered from stone robbing. D21-Z1 was found lying in the chamber floor. This situation must have endured from the times that the tomb was in use. The oldest artefacts come from the lowest chamber floor of D21. They are appointed to horizon 1 (± 3350-3300 BC) of Brindley's time scale [32]. That of D22 belongs to horizon 3 (± 3250-3150 BC) and is about hundred years younger [33]. In 1960 both hunbedden were patched up a little. Hunebed D21 was stabilised by raising the ground in and around the tomb.

Discussion

The finds of the primary and secondary layer of hunebed D21 appeared to be in a jumble, except for the southwestern part [34]. This is an interesting observation when compared to the photographs of the excavated chamber (figure 31). In the southwestern part a white line has attached to the supports

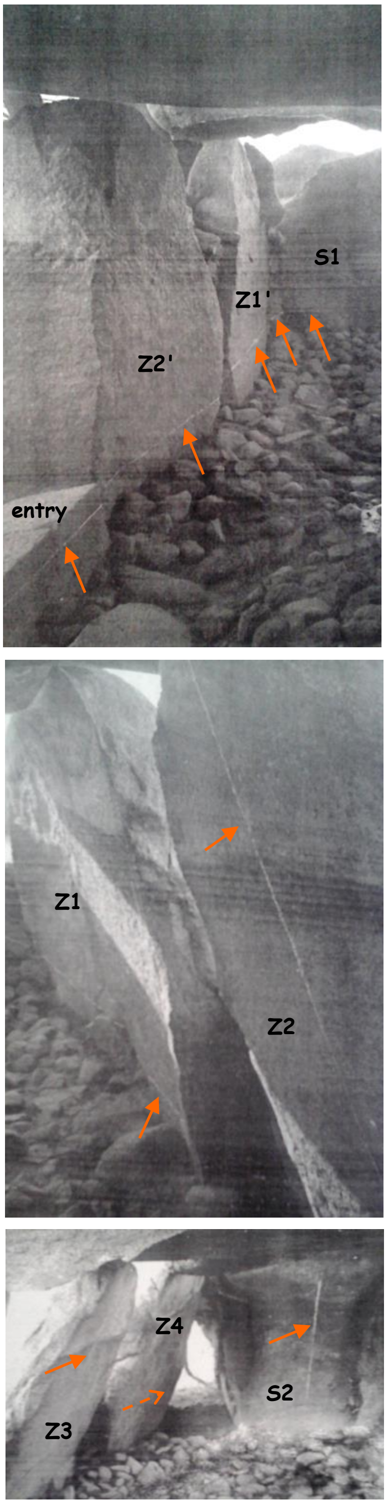


Figure 31
The interior of hunebed D21 - photographed by Van Giffen during the excavation in 1918. Orange arrows point at a white calcite line on the supports.
S1, Z1, Z1', Z2': The line runs horizontally at second floor level.
Z2, Z3, Z4?, S2: The line runs vertically.

[31] Cupules are holes of about 10 to 20 cm in diameter. They are rather common in the supports of Ardèchoise dolmens.
[32] van Ginkel et al. 1999, p.174.
[33] van Ginkel et al. 1999, p.176.
[34] Van Giffen 1925/27, Part II, pg.243

at the height of the second floor. This line cannot exist of veins of quartzite since they continue over at least five supports at the same height. Strangely side stone Z1 (having the line too) was found in the secondary floor. Since a tertiary floor was paved over it directly [35] probably the secondary floor was removed to make the stone fit. We cannot but guess how and why the white line came into being. The next is just a tentative course of affairs.

In the mixed filling of the primary and secondary phase Van Giffen found a few little pieces of burned bone and stone, and the remains of a fire in the chamber [36]. Van Giffen did not find baked pottery, and thus the bones were selected to be put into the fire. Probably the capstones of the northeastern part were removed before the fire was lit. When the fire was extinguished a solution of calcite (burned bone) in water can have flooded over the secondary floor and be absorbed by the supports. Most peculiar some supports (Z2, Z3, Z4 and S2) have the white line running almost vertical, and thus were turned over at the moment that the fire was extinguished. Afterwards the supports Z2 to S2 were turned upright again (which explains the vertical white lines), and the capstones were replaced. With all this activity the chamber filling of the first two periods got mixed.

Finally, for some unsure reason side stone Z1 was laid down. But there is a hint. Van Giffen found a large stone upon the primary sill of which he was uncertain about its function. Was it a large secondary sill or was it a small portal capstone or side stone [37]? The stone fitted perfectly between the entrance stones Z2' and Z3', being flat at that side and rounded at the other side. Maybe the stone was the capstone or side stone indeed but got another function and therefore was flattened at the bottom side. It can have been a tumbled barrier, which once closed the primary entrance. In that case the reason for lying down side stone Z1 will have been to create another entrance for the tertiary layer. Regrettably the mound around D21 was completely disturbed when Van Giffen started his excavation, so that as regards this there is no way of verification.

If the white line is connected to the reuse of the chamber, then this has not simply been a matter of clearance but also of renovation. Maybe it was the same people who renovated D21 and built D22. Not only did they use the orientation of D21 for the grid of D22, but also applied the same sightlines within the grid producing an orientation of 167º. This reorientation can have needed a relocation of the interstices, what can explain that some stones of D21 were turned over.

When looking at the sky around the time of the supposed renovation (± 3150 BCE), we find α-Crux rising at 167º [38]. This star is mentioned rather often as a target of orientation in ancient cultures [39]. The primary entrance opens to azimuth 133º (perpendicular to 43º), which is the orientation of the winter solstice [40]. Now the idea imposes itself, that the orientation towards the rising sun abandoned the field to an orientation at α-Crux, but such a claim is not problem free. Because of a low inclination, atmospheric extinction would have rendered the star invisible. People needed to extrapolate the location of the star’s rising on the horizon.

[35] Van Giffen 1925/27, Atlas, plate 147
[36] Van Giffen 1925/27, Part II, pg.243
[37] Van Giffen 1925/27, Part II, pg.246
[38] Corrected for –½º inclination in the direction of azimuth 167º. The hunebed lies on the sloping grounds of the small river the Hunze.
[39] González-García, Costa-Ferrer 2003, pg.117.
[40] Corrected for +½º inclination of azimuth 133º.

Since these azimuth values are geometrically constructed, we cannot simply accept the alignments in themselves, but must allow for the possibility that surveyors were eager to establish them by geometry – or that they applied geometric rules for some other means.

Geometry

Both tombs utilise a grid at 43º. The orientation of 43º belongs to one of the most applied setups in Neolithic tombs. It is likely that hunebed D22 (with the earliest artefacts in horizon 3) got the grid from D21 (having artefacts in horizon 1). The chamber length of D21 seems to have been measured off three times and then end stone D22-S2 was placed (figure 30). From there an 'inscribed' Pythagorean triangle fits the chamber of D22 so that the interstice in double stone D22-Z1 marks its beginning (figure 33, top) [41]. Furthermore, in hunebed D21 as well as D22 the grid at 43º causes the same sightlines. They are formed by trysquares with legs 2×3 (figure 32, inset). In all known situations this trysquare is used to establish the geometrical relation between 43º and 167º orientations. Since one of the sightlines at 167º runs through the entrance of D22, 167º seems to be its orientation. Nowadays a huge oak stands in the middle of the entrance. Within hunebed D21 the entrance stands perpendicular to 43º at 133º.

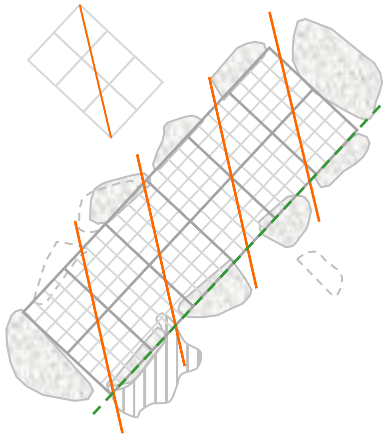


Figure 32
Geometrical plan of hunebed D21. Sightlines at 167º.
The hashed item refers to a big oak growing against the hunebed.

Conclusion

The current geometrical setup of the hunebedden pair D21 and D22 seems to date from the tertiary phase of D21. It cannot be said if D22 already existed or was built new when D21 got renovated. Quite sure the surveyor grasped the orientation of D21 as a guide for the grid of squares in D22. Maybe the renovation took place under influence of altered standards for hunebed orientation, for example the alignment to different celestial objects. Furthermore, the plan of D22 can be linked to the setup of dolmen Sprockhoff 384 near Serrahn in Mecklenburg while the mutual orientation between D21 and D22 compares to the setup of the dolmens A and B of La Devèse near Barjac in the Ardèche (next paragraph). Apparently, the surveyors utilised sudden geometrical practises on a super-regional level and for a long time.

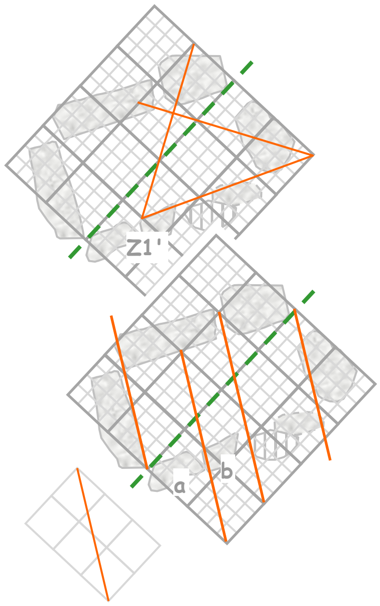


Figure 33
Geometrical plan of hunebed D22.
The hashed item refers to a big oak growing in the entrance part.
Top: The inscribed 3×4×5 triangle (see PTS-1, figure 2)
Bottom: Sightlines at 167º.

[41] If hunebed D22 is compared to other dolmens having an inscribed triangle (especially Sprockhoff 384 near Serrahn) then it becomes clear that end stone S2 should be reckoned to the chamber geometry (PTS-1, figure 2).

Dolmens La Devèse A and B near Barjac

About these dolmens we can be short for most has been said now. Both dolmens belong to a small necropolis of five tombs spread through the forest of La Devèse. No information is found about their excavation or restoration. The group resides on the little causse (limestone tableland) of the Ardèche near Barjac. Because of the uniform building style Chevalier [42] called these dolmens the Causse-type dolmens. The chamber is formed by three limestone slabs. An end slab is clamped between two side slabs. Often a dolmen is closed by a front slab, which seems to be a secondary feature. Dolmen La Devèse B actually has such a slab. Other smaller slabs get vague names like 'pillar' or 'stèle' (plaque) in literature. They appear on different places in the tomb plan and have different functions. Some are used to reinforce a chamber slab and others seem to narrow the entrance. When such stones belong to the entrance of a dolmen, in the current study they happen to have the additional function of pointing out the orientation of the grid of squares used in the dolmen setup (see PTS-8). Therefore, these stones are called 'guide stones' in this study.

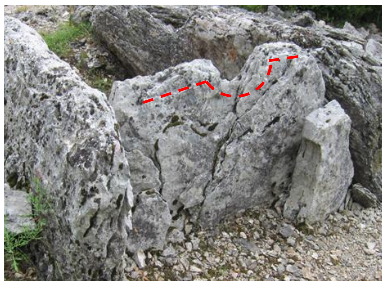


Figure 34
Dolmen La Devèse A near Barjac. A half cupule serves as mark for the 47° sightline towards dolmen La Devèse B.

Geometry

Although both dolmens La Devèse A and B share a heart line at 47° and have their grid of squares orientated along it, it is discussable if these dolmens truly share one and the same grid as the hunebedden D21 and D22 do. Because of the sloping terrain it is hardly doable to measure off the exact sizes. On the other hand, the half cupule of La Devèse B has the same height as the chamber floor of dolmen A, so that a levelled elongation of the 47° sightline will have been rather easy - either from A to B or visa versa. Both dolmens have a guide stone at 137° (the small blue stones in figure 30).

In fact, the geometrical setup is mirrored compared to that of hunebedden D21 and D22. First, the shared sightline is constructed by means of a grid of 2×3 squares combined with a bisected ⧄ (figure 35, top). Once the sightline and grid at 47° is established, the left side is placed via ⧄ in relation to the sightline. (In D22 the right side is placed via ⧄ at a sightline of 43° – see figure 35.) Because of the typical Causse-type placement of the opposite wall in dolmen B, the tomb plan does not allow for an inscribed 3×4×5 triangle like in D22.

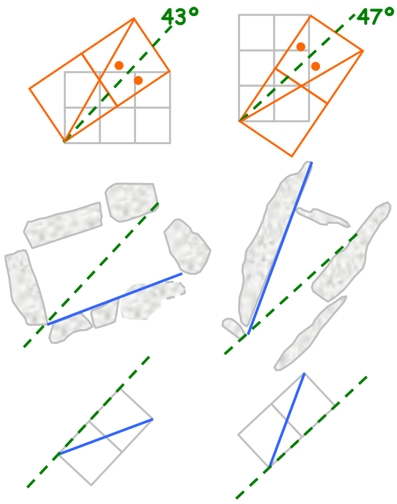


Figure 35
Mirrored plans of dolmen La Devèse B and hunebed D22. The shared sightlines with respectively La Devèse A and hunebed D21 are dashed.

[42] Chevalier 1999, pg.s56

Final conclusions

Maybe the most important conclusion is that the Neolithic tombs discussed in this paper were planned and not just fitted together. Peculiarities like marks and curious placed stones emphasize the two phases some hunebedden went through (which can be recognized in their mounds too) and demonstrate that those phases were meaningful to the owners. When excavating Neolithic tombs, it would be a worthful practise to examine and document such features. This in remembrance to Lukis of the English committee, who wrote: "*what is really needed when treating of rude stone monuments is perfect accuracy of description, and no omission of any detail or feature which may reasonably be supposed to be connected with the structures*" [43].

The studied tombs of this paper display a setup by means of a grid-of-squares geometry. The most common ingredient is a ⧄: either plain, bisected or combined into a Pythagorean triangle (the practitioner diagram). Additionally, a field of 2×3 squares is used for the orientation of sightlines. Generally spoken it seems that the surveyors applied the diagonals of trysquares with legs 1×2, 2×3 and 3×4 to setup a required plan or orientation. If they used geometry for astronomical alignments is unsure. Regarding the subsequent numbers for the trysquare legs, even some kind of numerology can be expected.

Finally, surveying must have been a profession in Neolithic times already. The mutually orientated tombs were not built in one go. Formerly applied geometrics were adjusted or used in new setups. This means that the surveyors had generally accepted methods at their disposal. Apparently, there existed a means of exchange of their know how - from generation to generation (master-apprentice relationship) and from region to region (interregional contacts). In PTS-1 (pg.3) it has been suggested that surveying was maintained and exchanged along family-lines.

[43] Van Giffen 1925/27, Part II, pg.296

DRAFT

Prehistoric tomb surveying (5)
Small hunebedden or pseudo-dolmens

Dirk Kruithof, September 2019

While studying the geometry of Neolithic tombs along the Baltic coasts and in the Netherlands, the similarities between the early forms of the Baltic dolmens and the later smallest Dutch hunebedden became more and more conspicuous. Not only their tomb plans followed the rules of the grid-of-squares geometry (see for example PTS-4 [1]*), but some tombs did have identical setups. The idea came off that those small hunebedden should be regarded as a hybrid form - dolmens that were turned into a hunebed by adding an end stone in the place where real dolmens have their entrance. In this paper the geometrical characteristics that make them comparable are highlighted.*

Hunebed D22, a mix of dolmen and hunebed

If one expects to find regularities one also has to expect exceptions. This is what Van Giffen experienced when he excavated hunebed D22 in 1918. The hunebed lies hidden in a small mound - only two supports are visible. For his atlas of hunebedden, Van Giffen located the hidden ones by means of a dipstick. Thus, he found side stone Z1' [2]. Later on, when he excavated the hunebed, side stone Z1' appeared to exist of two small supports instead of the expected single stone. The expensive plates for the atlas had been created already and except for a textual rectification Van Giffen did not pay more attention to it. Generally, such peculiarities are imputed to the availability of stones or the caprices of the tomb builders but probably this one should not. During the preparations of the study of geometrical features of dolmens along the Baltic coasts, a rather similar ground plan to that of D22 was found in the work of Schuldt. His plan of the *erweiterde Dolmen* 385b [3] of Serrahn can be used as an overlay over hunebed D22, so that two striking features come to light (figure 1). Firstly, in the position of the double stone Z1' dolmen 385b has a small stone fence so that the two half supports of D22 seem to be there on purpose. Secondly, dolmen 385b has both an end stone

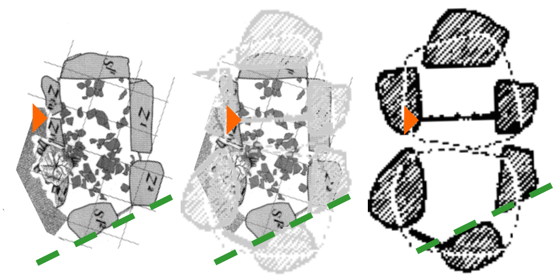


Figure 1
Comparison of hunebed D22 (left, Bronneger, the Netherlands) with Erweiterde Dolmen 384 (right, Serrahn, Germany).
D22 according to Van Giffen (1925/27), Atlas, plate 145.
Dolmen 385b according to Schuldt (1972), pg.165, table 11b.

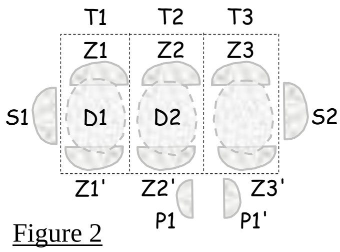


Figure 2
Numbering of tomb stones.

[1] Reverences to the other papers of this study are abbreviated 'PTS-n', where PTS stands for the study title 'Prehistoric tomb surveying' and 'n' for the serial number of the paper.
[2] The numbering of the stones follows the system of Van Giffen (figure 2). Additionally, the trilithons (consisting of two side stones and a capstone) have been numbered too.
[3] Numbering according to Sprockhoff. Dolmen 385b no longer exists.

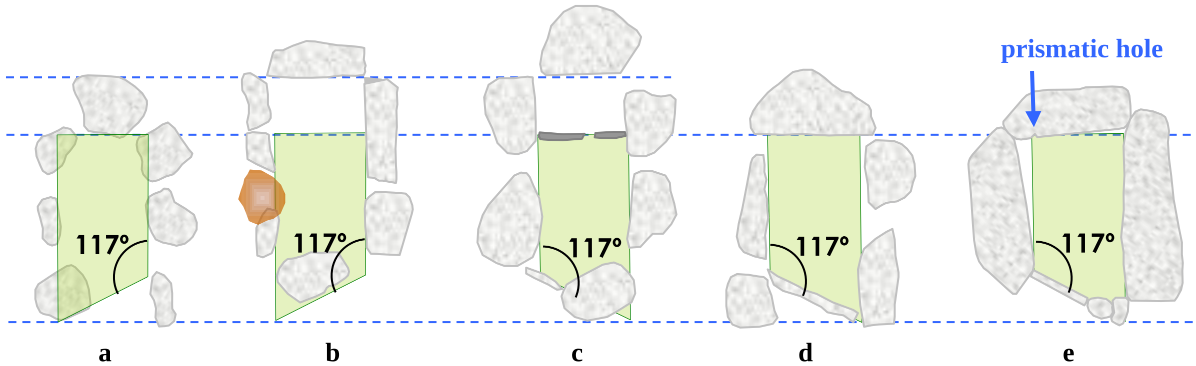


Figure 3
These dolmens have the same proportions, but do not have the same dimensions or orientations.
(a) G5, Raemaekers and Jansen 2013, p.7
(b) D22, Van Giffen 1925/27, Atlas, plate 145
(c) 385b, Schuldt 1972, p.165
(d) 384, Schuldt 1972, p.76
(e) 409, own field plan

and a door slab (*Türplatte*) in the position where other erweiterde Dolmens have a clear entrance. As if it is half dolmen and half passage grave [4]. Most peculiar, in the overlay the inner side of this end stone touches the outer side of the corresponding end stone of D22. Furthermore, the area encompassed by these touching faces and the double stone (D22) / fence (385b) has the same shape as the ground plan of some real dolmens. There a door slab standing with a characteristic 117° angle to the chamber orientation produces an encompassed area that unlikely arises by happenchance (figure 3). This same form is encompassed in D22 and 385b. All in all, these are too much oddities to be neglected and the question rises if there exist other hybrid hunebedden.

Hunebedden D22 and D13, different ground plans but identical setups

In PTS-4 it is argued that the grid of squares for the setup of hunebed D22 is a derivative of the nearby hunebed D21, which seems to be older because of the findings. The grid is orientated at 43°, which is a common feature amongst Neolithic tombs and can be generated in two steps using the grid-of-squares geometry. A nice example comes from hunebed D13, where the setup can be followed in the ground plan actually (figure 4). It starts off with a grid that is orientated at the cardinal points. In the grid a diagonal through a field of 2×3 squares is 'drawn' [5]. Side stones Z1 and Z2 were placed along it. Upon this diagonal the bisection [6] of a diagonal through two adjacent squares ⍁ [7] is used to get the final orientation of 43°. This grid is used to produce an inscribed Pythagorean triangle of sides 3×4×5. Both the bisection of ⍁ and the inscribed right triangle are common features of Neolithic tomb plans. The surveyor of D13 must have planned the layout in advance and put the stones into place according to the plan. The right angle of the triangle fits to the interstice between side stone Z1 and Z2, while these stones stand along the initial diagonal. Triangle tops fitting to interstices is another often seen feature, so that we may assume it was designed deliberately.

[4] In the strict classification of Schuldt dolmen 384 should be regarded as an intermediary between the early pure form of an erweiterde Dolmen and a derivative form having the entrance between the short and long side of the dolmen. Schuldt (1971, pg .23)
[5] Most likely this was reached by stretching a rope between poles (see PTS-2).
[6] For a discussion about the possibility that Neolithic surveyors could have performed a bisection see PTS-1, pg.10 ad(4).
[7] Because of the ease of reading and understanding, in this paper the wording '(a) diagonal(s) through two adjacent squares' will be replaced by the icon ⍁, which shows exactly what is meant by it.

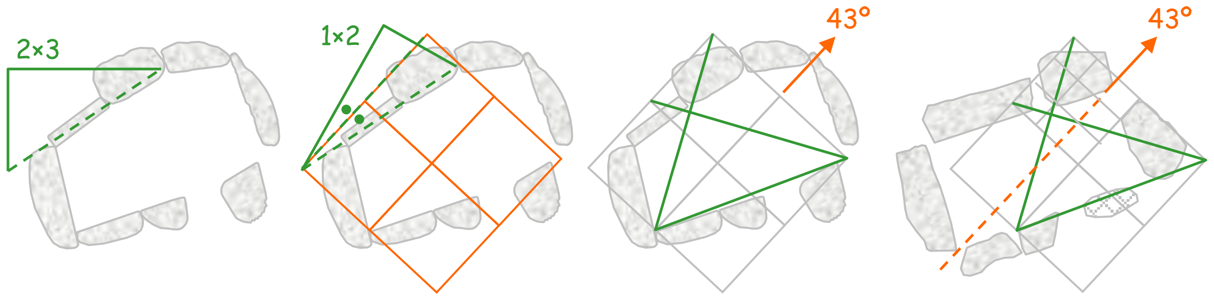


Figure 4
The three consecutive stages of the orientation of hunebed D13. Most right: comparison to the plan of D22. The dotted line comes from hunebed D21, from where the orientation was taken. See PTS-4, figure 30.

At first glance the ground plans of D13 and D22 are rather different. D22 has a rectangular chamber while the chamber of D13 widens to the northeast and has a kink between side stones Z2 and Z3 in the northern wall. Only the orientation of the southern wall seems to be a commonality. A closer look reveals a similar tomb diagonal at 43º, which forms the jumping-off place for the setup of both tomb plans. Within the grid the inscribed Pythagorean triangle is formed, although its position in D22 conforms to dolmen 385b more than to D13. Additionally, one finds parallel sightlines at 167º and 193º. Parallel sightlines are known as a characteristic phenomenon of orientation from the threesome hunebedden on a field near D22 (Bronneger) and from a comparable group near Emmen (see PTS-4). Because of the sightlines it becomes clear that the surveyors of D13 and D22 had a similar setup in mind but implemented it by a different stone configuration. Since the applied grid at 43º depends on the bisection of ▭ upon a diagonal through a field of 2×3 squares, the sightline at 167º could be created via reverse engineering easily. Azimuth values of 167º and 193º are the most applied orientations amongst Neolithic tombs. Their mutual angle is constructed by ▭ while true south forms its bisection (figure 5, lowest inset).

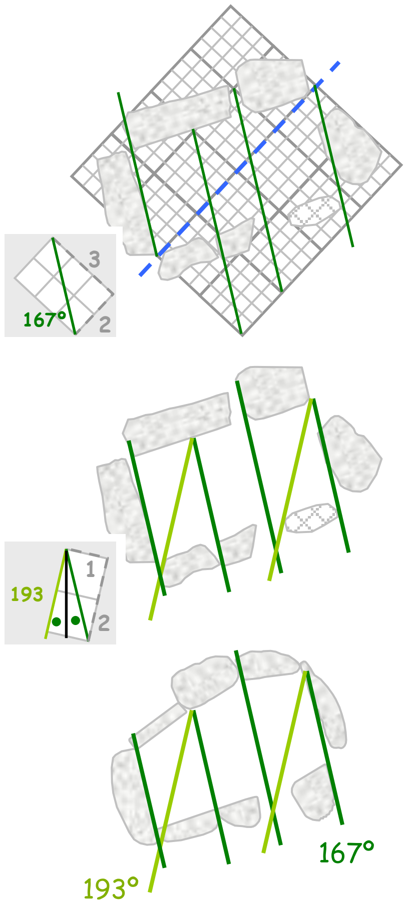


Figure 5
The construction of sightlines at 167º and 193º in the hunebedden D22 and D13. The grid of D22 (top) is based on foot size of 26½ cm (see PTS-4, figure 30).

Hunebed D22 compared to D12 and G5

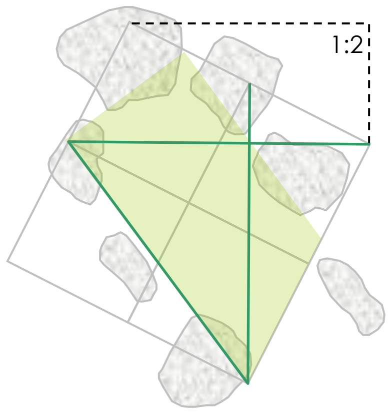
Currently the remains of hunebed D12 exists of one end stone and three side stones in a row. When Van Giffen restored the hunebed in 1952, he also performed a cursorily excavation in order to judge upon the dimensions of the chamber. On several places in the extension of the remains he found pits filled with sand, which he called extraction pits. Van Giffen assumed that the hunebed consisted of five trilithons initially. Later Raemaekers and Jansen [8] pointed at the fact that Van Giffen did not find any pieces of chamber filling in the

[8] Raemaekers and Jansen 2013, pg.4,5

extension and that there were no remains of a chamber floor. According to them the hunebed never did have more than three trilithons. Since D12 has much in common with the true dolmen G5 - similar dimensions, number of supports and orientation - they suggested that D12 could have been a true dolmen too.

Dolmen G5 lay hidden under a 2-meter-thick clayey soil until it was dug up during the development of an industrial park near Delfzijl (the Netherlands) in 1982. Its discovery surprised the archaeologists in two ways. G5 had its entrance at the short side and was labelled as the only Dutch dolmen therefore, and secondly it had been dismantled in prehistoric times already. Its capstones lied scattered around. Although nothing can be said about the motive behind the configuration, it proves that prehistoric people altered a tomb after its initial usage already.

Tomb D12 and G5 have a longitudinal azimuth of about 143º. The comparable orientation should not be ascribed to the possibility that they are of the same tomb type but to the identical geometrical setup for the ground plan. The azimuth can be interpreted as an offset by a Pythagorean triangle on due east (90º + 53º). In fact, the idea of the ground plan of D12 is more similar to that of D22 in the way that both tombs have the outside of an end stone along the grid of squares that produces the inscribed triangle. If D12 should be compared to D22, then it likely had a second end stone too (S1 in figure 6). At the northwestern end, where this end stone should be expected, the mound and soil had been disturbed heavily, which leaves us with the impression of stone digging.



Although the setup of D12, D13 and D22 is comparable to that of true dolmen G5, there is a big difference. Where G5 has the entrance at the short southern side, D12, D13 and D22 have a blocking end stone in that position. The entrance of D13 and D22 is found between the uprights Z2' and Z3' as it is in the other small hunebedden and this can have been the case in D12 too. Unfortunately, the entrance position of D12 remains indecisive, since the side stones of the southwestern wall have been robbed and no excavation has taken place. Furthermore, considering the disintegrated condition of dolmen G5, this dolmen could have been destined for a reconfiguration. Only the placement of an additional end stone would have sufficed to adjust it to the habit of the small Dutch hunebedden. But this can be no more than an unsubstantiated guess since the prehistoric renovation or dismantlement of G5 was never completed.

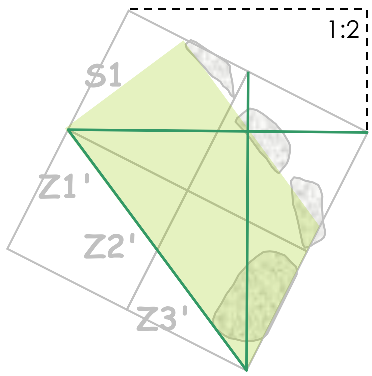


Figure 6
Dolmen G5 (top) and hunebed D12 (bottom) get their comparable orientation from the same geometrical setup.

Yet another five comparable hunebedden

Setups with an inscribed triangle orientated to the cardinal points seem to have been popular for the small Dutch hunebedden. Apart from G5 and D12 we find them again in the hunebedden D25 and D40. In PTS-4 these hunebedden are presented as a central hunebed having two satellite hunebedden each. Furthermore, the ground plan of D25 was virtually restored concerning the subsided second and third trilithons. Thereby it occurred that D25 exists of two halves as if it is a combination of two dolmens facing each other. Each half consists of two trilithons and thereby has the same configuration as D40. The eastern 'dolmen' of D25 has the dimensions of an *erweiterde Dolmen* while the dimensions of the western part is comparable to a *Großdolmen* [9]. Chronically the erweiterde Dolmen precedes the Großdolmen.

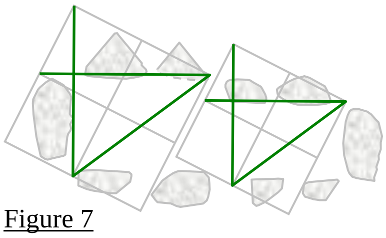


Figure 7
The geometrical setup of tomb D25. (Virtually restored - see PTS-4)

In the discussion of how the tomb type *Ganggrab* (passage grave) could have evolved from the *Großdolmen*, it is put forward that some dolmens got elongated per trilithon initially. Hunebed D25 sheds another light on the subject. Where three-trilithon-hunebedden display a 'single dolmen' geometrical setup for the complete hunebed, D25 being a four-trilithon-hunebed is divided into two partitions by means of a 'double dolmen' setup. Because of the different dimensions for both parts, the hunebed can have been built in two phases actually [10]. Both 'dolmens' do not fit to each other exactly either. The southern wall of the eastern part stands a little to the outside of the western part although it is the smallest of the two. The configuration of a double dolmen does not exclude other developments that can have led to the passage grave type. Probably Neolithic surveyors did not apply a standard method of elongation but were pioneering. Yet, while the form may have been subject to the experiment, in D25 the surveying method using the grid-of-squares geometry remained the continuous factor.

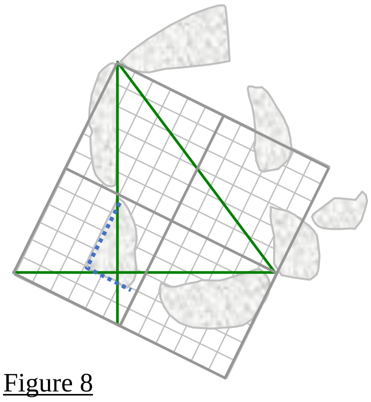


Figure 8
The geometrical setup of tomb D39.

D38 is another hunebed that can have started as a dolmen. In PTS-4 the strange position of D38-Z4 is discussed – at the moment being an entrance stone but maybe initially a side stone. In that case the northeastern part of the hunebed looks like an ordinary erweiterde Dolmen. It follows a straight-forward application of ⧄ for its orientation, and the dimensions of the chamber are defined by an inscribed Pythagorean triangle with sides 3×4×5. The hunebed has not been excavated.

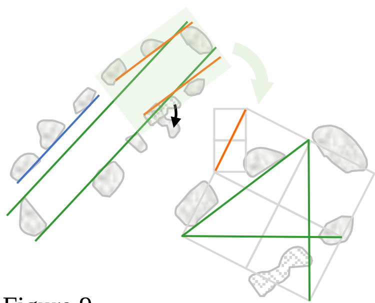


Figure 9
Top: Ground plan of D38. Stone Z4' can have been turned ±90º to its current position.
Bottom: A possible initial stage as an erweiterde Dolmen.

The list of dolmen-like hunebedden is completed by another two small hunebedden, D6 and D39. Thereby all small hunebedden seem to have been subject to a geometrical setup comparable to that of true dolmens. Both D6 and D39 have the entrance in the same position as the other small hunebedden. The tomb plan of D39 near Emmen is ambiguous regarding the pure geometrical setup (see PTS-4).

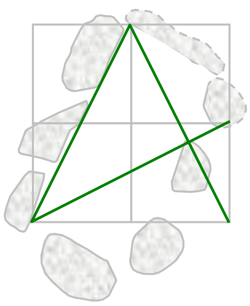


Figure 10
The geometrical setup of tomb D39.

[9] Schuldt 1972, pg.22/24
[10] This is extensively discussed in PTS-4.

Compared to the other tombs in the list, the most straight-forward setup is preferable having the top of the inscribed triangle fitting to the entrance. D39 makes part of a group of three hunebedden of which the geometrical properties are interrelated (see PTS-4).

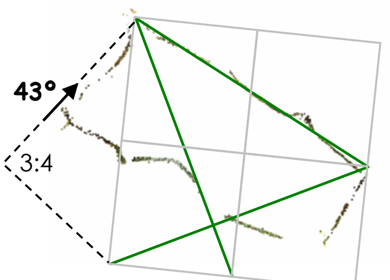


Figure 11
Horizontal cross-section of a 3D model of the chamber of D6, showing the geometrical setup.

D6 near Tynaarlo makes an exception amongst the Dutch hunebedden in that it has not been touched in modern times – neither by excavation nor by vandalism. Additionally, the original configuration has been conserved very well. The northern wall forms the basis of an inscribed right triangle having sides 3×4×5. Its top is found between the side stones – a situation found in the Urdolmen of Barkvieren too [11].

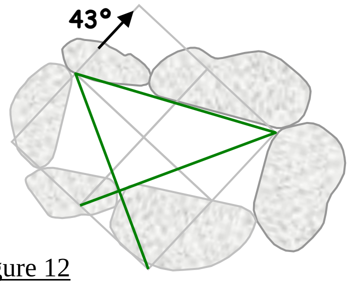


Figure 12
The geometrical setup the Urdolmen of Barkvieren.

Although the Urdolmen is much smaller, yet both tombs show the same ratio between height and width because of this position of the triangle top. They utilise azimuth 43º as their base orientation, although in D6 the grid-of-squares is orientated via an offset by means of a 3×4×5 triangle.

Pseudo-dolmens in the Netherlands

The small tombs of two or three trilithons seem to take up their own position in the spectrum of the Dutch hunebedden [12]. Both their geometrical setup and their dimensions can be linked to the properties of true dolmens. Based on the dolmen like geometrical setup, such small hunebedden could be called pseudo-dolmens, therefore. Such a pseudo-dolmen can be typified as a hunebed of two or three trilithons of which the geometrical setup produces an inscribed right triangle constructed in an orientated grid of squares. The pseudo-dolmen cannot be called a true dolmen since it does not have its entrance in its short side. Yet there is a 'hard' linkage to the dolmens in the area of the Baltic coasts in the form of hunebed D22 and dolmen 385b. Moreover, the idea of mutual influences is supported by the ceramic finds of some hunebedden. Kuyavish elements are present in the finds of Brindley's horizon 4 and 5 [13]. Yet they are of later date than the earliest finds of pseudo-dolmen D22, which come from horizon 3 (a difference of about 50 years). Apparently, the influencing of geometrical setups predated that of cultural elements. This agrees with the view of Kim, who states that the spread of techniques follows the path of decision makers amongst the elite, while other cultural utterances spread by means of contact between ordinary group members (PTS-1) [14]. Probably the surveying techniques came by a different carrier as the adoption of the Kuyavish ceramic style. The people who created the dolmen like setups in the Dutch regions, can have maintained interregional contacts sharing their technical state of the art. Only a few decennia later, broader contacts followed in the wake of them, exchanging other cultural elements.

[11] This interesting dolmen is discussed widely in PTS-7. Although one of the earliest dolmens, it has a clever setup, which must have been produced by a skilful surveyor.
[12] Maybe the larger hunebedden lost their dolmen like structure or the dimensions were not suitable for a setup that incorporated the right triangle. Any way it is hard to find benchmarks that connect the majority of their chambers to the grid of squares geometry. Nevertheless, it seems that their entrances can be linked, which is the subject of PTS-6.
[13] Bakker 1973, pg.244/246. The Kuyavish elements appear in Havelte E, which corresponds to Brindley's horizons 4 and 5. The earliest ceramics from D22 comes from horizon 3.
[14] Kim 2003, pg.285

Prehistoric tomb surveying (6)
The entrances of Dutch hunebedden

Dirk Kruithof, September 2019

In their study of hunebed orientation González-García and Costa-Ferrer included the orientation of the entrances also. A histogram of the entrance orientations displayed two peaks that mirrored around due south [1] *(azimuth 180º ± 15º). Although problematic, González-García and Costa-Ferrer associated the values with the rising and setting of a star with a declination of about -35º. Since the azimuth values agree with a bisection of a diagonal through two adjacent squares by due south more or less, maybe an explanation could be given in the context of the grid-of-squares geometry as described in PTS-1* [2]*. It was decided to have a systematic study of the entrances, therefore. Since quite a few left entrance stones happen to have rather flat surfaces, they were chosen to be the subject of study.*

The selection of entrance stones

Hunebed entrances are found midst the southeastern chamber wall. Some entrances are extended by a portal of one or two trilithons. From a usage point of view, the entrance orientation could be quite important. The idea of an orientated entrance is emphasized by a lot of entrance stones that have flattened surfaces for both the chamber and entrance side. Often, they stand in combination with other portal stones that have vaulted surfaces. Sometimes the placement of the other stones completely fails to support the idea of an entrance wall. Such setups seem to witness for some kind of intrinsic value of the flattened surfaces. The surfaces of a select group of left entrance stones were investigated on the possibility of a constructed orientation. The field work took place from autumn 2013 till the end of 2014.

A first selection was made via the comparison of earlier researches of tomb orientation [3]. Tombs were included when the reported orientations of their longitudinal axis did not deviate more then 3º amongst the different studies. It was assumed that the chamber sides of the entrance stones were related to the orientation of the longitudinal axis. Large deviations could

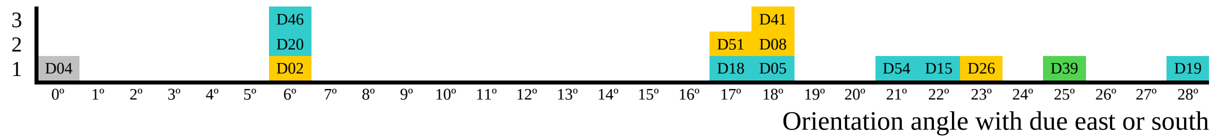


Figure 1
Histogram of the orientation of the longitudinal axes of some hunebedden, selected on the accuracy of the measurements (see text). The angles can be clockwise or anti-clockwise compared to due east or south. Yellow fields are to the north of east and blue fields are to the south of east. Green (D39) is to the west of south. Tomb D34 has not been rendered - it has an angle of 41½º to the east of south. The clear peaks come close to 6¾º from the bisection row of 53¼º and to 18½º from the row of 36¾º. Tomb D34 approximates 43º and near the half of it at 21½º there is a small cluster. Finally, D39 approaches 26½º, again from the bisection row of 53¼º.

[1] González-García and Costa-Ferrer 2003, pg.116
[2] PTS stands for the title of the current study *Prehistoric tomb surveying*. The study is divided in several papers. Each of them covers another subject. Other papers are referred to as PTS-n, where n stands for the serial number.
[3] The next researches have been consulted: Van Giffen 1925/27, Reijs 1997, González-García and Costa-Ferrer 2003 and Langbroek 2004

give problems then. This first selection was narrowed down by the demand that the left entrance stone should have well preserved flattened chamber and entrance sides and a traceable restoration history [4]. Only the left entrance stones of D2, D4, D5, D15 and D20 remained. From these five stones it could be concluded that there existed no correlation between the longitudinal axis and chamber side of the left entrance stone. The chamber and the left entrance stone have an independent orientation. Therefore, the criterion of less then 3º deviation amongst the studies was dropped and accordingly the selection was renewed. The final selection consisted of eight left entrance stones that stand in situ and have not been restored (D4, D17, D25, D30, D36, D37, D42, D43n) and six that have undergone a well documented restoration (D1, D2, D5, D15, D20, D43s).

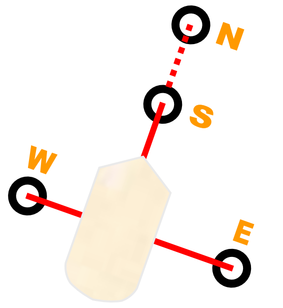


Figure 2
Establishing the cardinal points in a horizontal plane by means of a laser leveller and four targets.

Building the 3D model

Of all fourteen selected stones a 3D model was produced using photogrammetry. Before taking the photographs, the cardinal points were measured off in a horizontal plane and marked by special targets. Afterwards, the photos were processed in Photomodeler Scanner (the photogrammetry application) and a 3D model was produced. Five cross-sections at different levels in the 3D model (from top to ground) show the flatness of the surfaces. In a cross-section the accuracy of the angle between the chamber and entrance sides of a stone depends on the flatness purely and not on an instrumental fault. The correctness of a cross-section's orientation depends on the exact positioning of the targets for the cardinal points. Four targets were positioned with their central dot in the laser beams of a cross-laser-leveller [5]. One of these beams was directed to due north via a sighting compass [6]. For the six restored stones in the

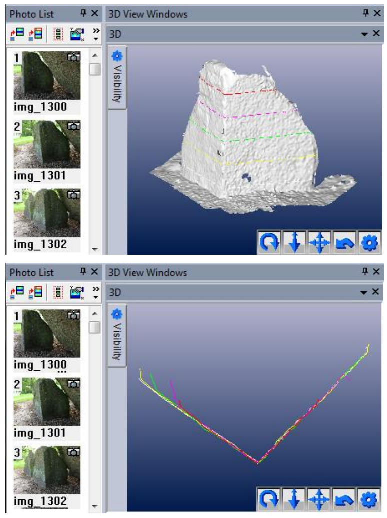


Figure 3
Screenshots of Photomodeler Scanner.
Top: Horizontal contour lines are created in a triangulated surface.
Bottom: Top view of the contour lines without displaying the surfaces.

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[4] Van Giffen (1925/27), Bakker and Waterbolk (1980) and Bakker (1992).
- *Tomb no longer exists:* D33, D48, G2, G3, G4.
- *Left entrance stone is missing or replaced:* D9, D10, D12, D23, D26, D31, D34, D44, D45, D49, D51, D52, D53, G1, G5.
- *Van Giffen reported a serious disturbance:* D2, D3, D7, D16, D29 and D54.
- *At least one side is not visible, curved, wrenched or decrepit:* D3, D6, D8, D11, D13, D14, D19, D21, D22, D24, D27, D28, D29, D31, D32, D34, D35, D38, D39, D40, D41, D46, D47 and D51.
- *In the restoration report other supports are mentioned, but not the left entrance stone (indication that it was not touched):* D7, D14, D15, D16, D35, D36, D37.
- *Van Giffen found the left entrance stone 'in situ' for:* D1, D4, D6, D8, D11, D13, D17, D19, D21, D28, D34, D39, D40, D42, D50. In 1936 hunebed D50 got heavily damaged.
- *Van Giffen noted 'almost in situ':* D5, D18, D20, D24, D25, D27, D30, D32, D38, D41, D43n, D46. Of those D24, D27, D32, D41 and D46 are deselected in one of the lists before. The restoration of D5 (1960) has been described in detail. In 1873 Gregory 'restored' hunebed D18 but left behind no documentation. From the excavation reports of D20, D25, D30 and D43n the impression arises that the left entrance stones stand in situ (resp. Holwerda 1912, Lukis and Dryden 1878, Van Giffen 1918 and Holwerda 1913 - see Bakker and Waterbolk (1980) p. 14, 37, 62 / Bakker (1992) p.161 / Van Giffen (1925/27, Part I)).

[5] Leica LINO L2P5, ± ¼º

[6] Recta DP10, ± ⅓º

selection (except D15) and one non-restored stone (D42) the field model was used to produce an additional virtual restoration. For example, in the case a restored stone of which both the entrance and chamber side mismatch an ideal orientation by +1º, so that a rotation of -1º would fit geometrically with their mutual angle (which cannot be disturbed by restoration), then the stone is virtually restored. Generally, archaeologists do not take notion of geometrical features, and thus we must grant them a misplacement by a few degrees. In the section *List of studied stones* per stone the adjustment is described and justified. Both the original and virtual model are presented.

The processing and analysis of the data

The angle between the chamber and entrance side of an entrance stone is used as a guide to the orientation of the stone. It is called the *constructing angle* (α) here. All angles concerning the stone sides can be described in factors of the constructing angle. They can be used as the angle itself (α), its bisection (α÷2) or as a factor (α•2, α•3 or α•4). Three of those constructing angles showed up amongst the stones: as bisection of a diagonal through two or three adjacent squares (α = 13¼º or α = 9¼º) and as the bisection of the special angle 43º (α = 21½º) – see figure 4. The constructing angle is used as an offset to the cardinal points, to the longitudinal axis of the chamber or to the entrance or chamber side of the stone. As a rule, all relations between chamber side, entrance side, cardinal points and the longitudinal axis should be described in order to get the left entrance stone successfully orientated.

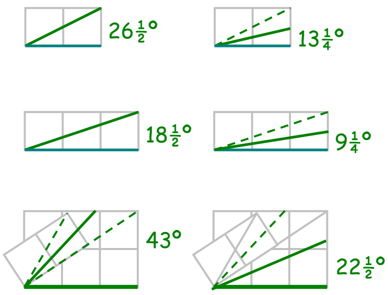

Figure 4
The constructing angles are produced via bisection of diagonals in a field of squares. Left the diagonal and right the bisection.

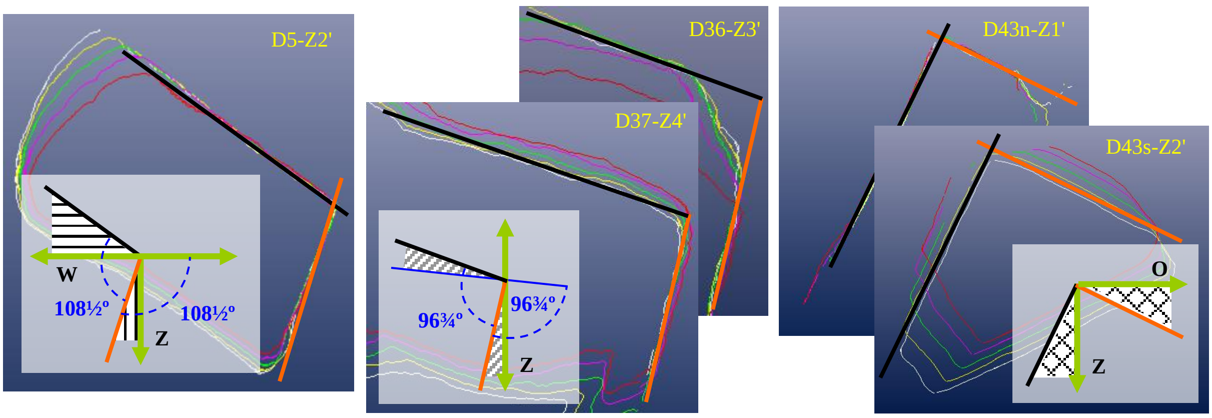

Figure 5
The ideal geometrical orientation of some left entrance stones (chamber side: black, entrance side: orange). Hunebedden D36 and D37 stand at a distance of about 15 meters. Their left entrance stones show the same setup. The displayed angles (blue) are based on the constructing angles as *90º+α•2=108½º* (D5-Z2') and *90º+α÷2= 96¾º* (D36-Z3' and D37-Z4'), where α equals to the bisection of a diagonal through three and two adjacent squares respectively. Tombs D43n and D43s make part of a so called *Langbett*. If the proposed adjustment in the description of D43z-Z2' is correct, then their left entrance stones have identical orientations too. The chamber sides have an offset of α to the longitudinal axis (orientated at α too), and the entrance sides stand perpendicular.

Three models are listed in table 1 and 3: a field model, a virtually restored model and a geometrical model. The field model represents the current situation in the field. The virtually restorations are discussed in the *List of studied stones*, and the geometrical model reflects the ideal values based on the constructing angles.

Table 1 lists the measured and ideal angles between the entrance and chamber side of a stone. Since the angle is measured in the horizontal plane (a cross-section), it depends on the slope of a stone [7]. When a stone that stands upright is pushed to slope 80º, the angle between its sides will increase no more the 1º. Higher slopes will result in a rather big deviation between the angle in the field and the ideal angle. The biggest deviations are found with D2-Z2' and D43s-Z2'. They are justified in the *List of studied stones*.

Table 2 depends on table 3. It gives average values for the orientation of the chamber and entrance side of a stone, and for their mutual angle. It does so for the virtually restored and non-restored stones, so that the impact of the virtual restoration can be esteemed.

Table 3 is the main table and is divided in section a) that lists the non-restored stones, and section b) that lists the virtually restored stones. Here you find all measured values and how they relate to the ideal values as well as to the constructing angle.

Reflection and conclusion

In the field model of the stones in situ, the angles deviate 1.1º on average from the geometrical model and for the other stones 2.0º. After the virtual restoration of the latest stones the overall deviation amounts to 0.7º which is comparable to the deviation of the stones in situ. A similar picture emerges from the azimuth values of the chamber and entrance sides.

With the stones in situ, the azimuth values in the field deviate no more than 0.6º on average from the geometrical expectation. With the virtually restored stones, the deviation of field values amounts to 2.3º on average, since some stones needed a few degrees rotation in order to fit them to the geometrical setup. This is diminished to 0.9º after the rotation. On average the overall deviation between restored setup and geometrical setup amounts to 0.4º, which is comparable to the deviation of 0.6º of the non-restored stones.

As a conclusion it occurs that the real angles in the field fit the ideal geometrical ones rather well. Taken together the average deviation of the angles and azimuth values, it is warranted to allege that the involved left entrance stones have been orientated according to a few geometric principles. The bisection of a diagonal through two adjacent squares ($\alpha = 13\frac{1}{4}$º) is used as the constructing angle for 11 setups out of 13. Thereby it is by far the most preferred setup, which agrees with the overall picture in the other papers of the current study.

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[7] The angle between the sides in a cross-section equals to $\alpha_{cross\text{-}section} = 2 \times \tan^{-1}(\ \tan(\frac{1}{2}\alpha_{sides}) \div \sin(\alpha_{slope})\ )$, where an upright stone has $\alpha_{slope} = 90^o$ and a lying stone has $\alpha_{slope} = 0^o$. With a slope of 90º, $\alpha_{cross\text{-}section}$ equals α_{sides}. A slope of 0º results in a dividing by zero but $\alpha_{cross\text{-}section} = 180^o$ in that case. The next table shows how the angle increases when the slope diminishes (backward or forward gives the same result). We see that from 90º to 80º the angle increases no more than 1º.

Slope:	90º	80º	70º	60º	50º	40º	30º	20º	10º	0º
Angle:	**110º**	111º	113º	118º	124º	132º	141º	153º	166º	180º
	90º	91º	94º	98º	105º	115º	127º	142º	160º	180º
	70º	71º	73º	78º	85º	95º	109º	128º	152º	180º

List of studied stones

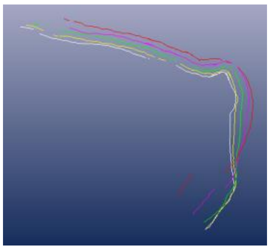
A detailed description of the history of the left entrances stones follows now. First a status report is given according to Van Giffen (1925/27). Then the recent history (excavations, restorations and vandalism) is reported on. This is a compilation of the information in Bakker and Waterbolk (1980) and Bakker (1992). Third the encountered status in the field is given. Finally, it is described if, why and how the 3D field model is virtually restored.

The cross-sections of the stones are produced via photogrammetry. Within the 3D model five cross-section are displayed in a horizontal plane. Bottom-up the colours are: white, yellow, green, purple and red. The distances between the cross-sections are spread equally and depend on the vertical size of the stones above the ground.

The left entrance stones that are in situ

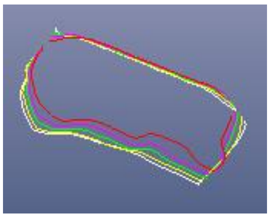
D4-Z3' [8]

Van Giffen: In situ.
Recent: No restoration has taken place.
Status: Unchanged



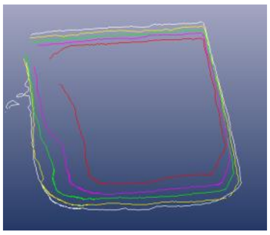
D15-Z3'

Van Giffen: More or less in situ.
Recent: The hunebed has been restored quite extensively in 1952. The stone was pushed inward a little.
Status: The stone stands according to the restoration in 1952.



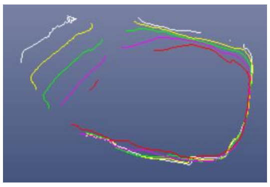
D17-Z4'

Van Giffen: In situ.
Recent: Probably the earliest report comes from 1547. A lot of diggings, small excavations and restorations have taken place.
Status: At the moment the stone leans a little outward and the capstone of the trilithon has slid off. This has affected the orientation probably, while the disturbances foil a substantiated virtual restoration. The stone is excluded from the selection.



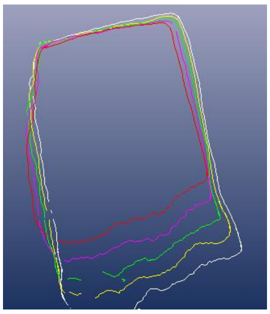
D25-Z2'

Van Giffen: More or less in situ.
Recent: Lukis and Dryden dug into the hunebed in 1878. A small restoration took place in 1960, but this stone stayed untouched probably.
Status: Unchanged.



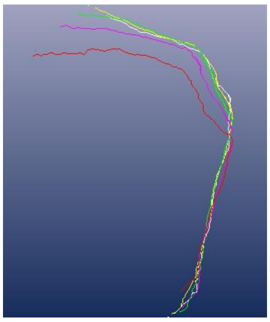
[8] Numbering of the stones according to Van Giffen.

D30-Z2'



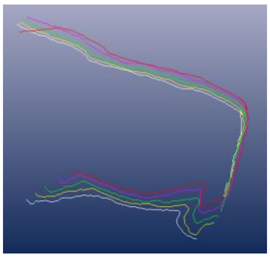
Van Giffen: More or less in situ.
Recent: Ongoing with the excavation of 1918 the hunebed has been restored. Details have not been found.
Status: A photograph from 1918 shows the stone from top to toe. The current placement has not changed since then.

D36-Z3'



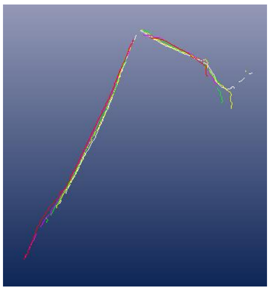
Van Giffen: Practically in situ.
Recent: In 1952 there has been a minor restoration, but the stones have not been touched. In 1960 Van Giffen thinks of a reorientation but this has not been actuated probably [9].
Status: Unchanged.

D37-Z4'



Van Giffen: Practically in situ.
Recent: In 1952 there has been a minor restoration, but the stones have not been touched. In 1960 Van Giffen thinks of a reorientation but this has not been actuated probably [9].
Status: Unchanged.

D43n-Z1'



Van Giffen: Practically in situ.
Recent: Hunebed D43 has been excavated by Holwerda in 1913. In 1961 another excavation took place, this time concentrating upon the fencing mainly.
Status: Unchanged.

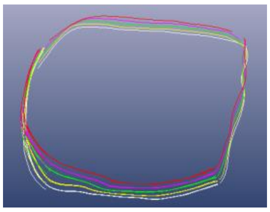
The left entrance stones that have been virtually restored

D1-Z3'

Van Giffen: At its original place.
Recent: In 1954 the hunebed got restored. The stone has been pushed inward a little to support the replaced capstone.
Status: Still the stone slopes outward a little.
Virtual restoration: The sides are restored to slope a general 80º and the stone is rotated -3º meanwhile. The southern chamber wall aligns at azimuth 77º (90º-α, own

[9] Bakker and Waterbolk 1980, p.37. In the report hunebedden D36 and D37 have been mixed up. The authors get their information from the Nieuwe Drentsche Volksalmanak of 1964 and deduced from the descriptions, the numbering of the hunebedden was exchanged in the article.

measurement), which agrees with the angle between the chamber and entrance side of the stone. So, we have a constructing angle of α = 13¼º. A stone orientation of 90º-α for the chamber side and of due south for the entrance side would have made the stone align with the chamber wall, but this requires a rotation of -16º. Instead a placement of due west for the chamber side and of 180º+α for the entrance side gives the most applied orientation for the entrance (193º) and requires a rotation of -3º only. This is more acceptable but, in both cases, the multiple agreement with the constructing angle makes a virtual restoration trustworthy.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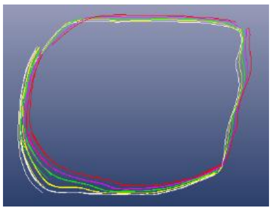
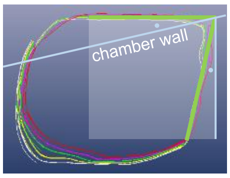
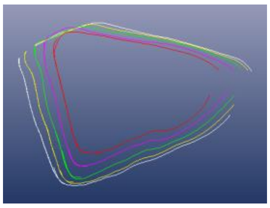



(virtually restored)

D2-Z2'

Van Giffen: Leaning inside.
Recent: With the restoration of 1952 the stone has been pushed outward.
Status: Now the stone slopes outward. Towards the top it distorts quite a lot.
Virtual restoration: The chamber side is adjusted to slope a general 80º.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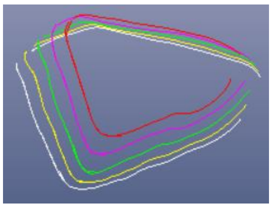
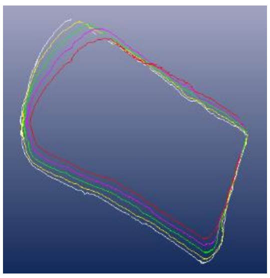

(virtually restored)

D5-Z2'

Van Giffen: Almost in situ.
Recent: During the restoration in 1962 the orientation of the stone has been adjusted a little.
Status: Maybe due to the restoration the entrance side of the stone slopes outward now.
Virtual restoration: The orientation is altered by +4º and the chamber and entrance side are placed to slope 80º. A constructing angle α of 18½º can be applied to the geometrical setup of the northern wall (90º+α, own measurement), the entrance side (180º+α), the chamber side (270º+2α), and the angle between chamber and entrance side (90º+α). Because of the multiple agreement with the constructing angle, the virtual restoration becomes trustworthy.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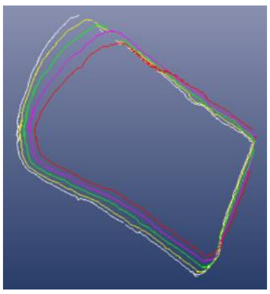
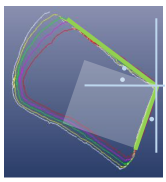
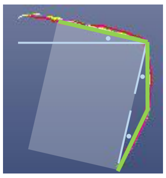


(virtually restored)

D20-Z3'

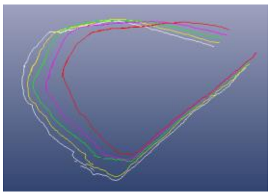
Van Giffen: Almost in situ.
Recent: Holwerda has excavated the tomb in 1912 and in 1961 a minor restoration took place.
Status: Maybe due to the excavation, the stone has rotated a little.
Virtual restoration: This stone has a double entrance side. When a rotation of +2º is assumed during the excavation in 1912, turning the stone back will bring out the ideal azimuth values for all three sides, based on a constructing angle α of 13¼. The chamber side gets an orientation of 90º+α, the 'real' entrance side of due south and the additional side an orientation of 180º+2α. Because of the multiple agreement with the constructing angle, the virtual restoration becomes trustworthy.


(field)



(virtually restored)

D42-Z5'

Van Giffen: In situ.
Recent: The hunebed has been restored in 1960, but the left entrance stone stayed untouched probably.
Status: The left entrance stone of D42 is sloping backward for the entrance side nowadays.
Virtual restoration: Adjusted to have the entrance and the chamber side slope 80º.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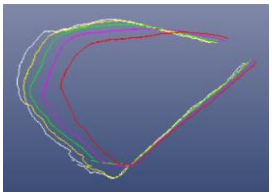
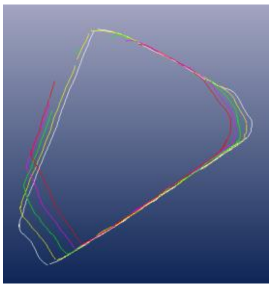
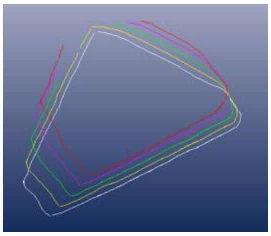

(virtually restored)

D43s-Z2'

Van Giffen: In situ.
Recent: The tomb has a sad restoration history. It starts in 1869, when the mound has been dug away. The Dutch government acquired the hunebed in 1870 and according to Pleyte in short time some capstones had disappeared. In 1913 a first scientific excavation took place by Holwerda. He produced a realistic ground plan. The trilithon with the entrance stone seems to have subsided towards the entrance since then. In Van Giffen's atlas (1925/27) we can see the reason why. Here the capstone of the next trilithon lies slantwise over the mentioned one. This was not the case in 1913. The trilithon of Z2' must have subsided by the weight of this capstone and by lack of buttress of the mound. Van Giffen restored the capstone again in 1960. In the ground plan of Holwerda the side stone stands explicitly not aligned with the longitudinal orientation of the tomb. Nowadays it does.

Status: As said, the trilithon of Z2 and Z2' has subsided eastward and the capstone slants about 20º now. Z2 leans 25 cm too far inward. The capstone fits barely and thus both supports must have undergone the same subsiding.
Virtual restoration: By virtually pushing the trilithon back so that its capstone gets levelled, Z2' will slope 10º more [10]. Meanwhile the position and orientation of the stone adjusts to the situation on the ground plan of Holwerda (figure 6). Moreover, the orientation of the chamber and entrance side become equal to that of the left entrance stone of twin tomb D43n that stands in situ.


(field)


(virtually restored)

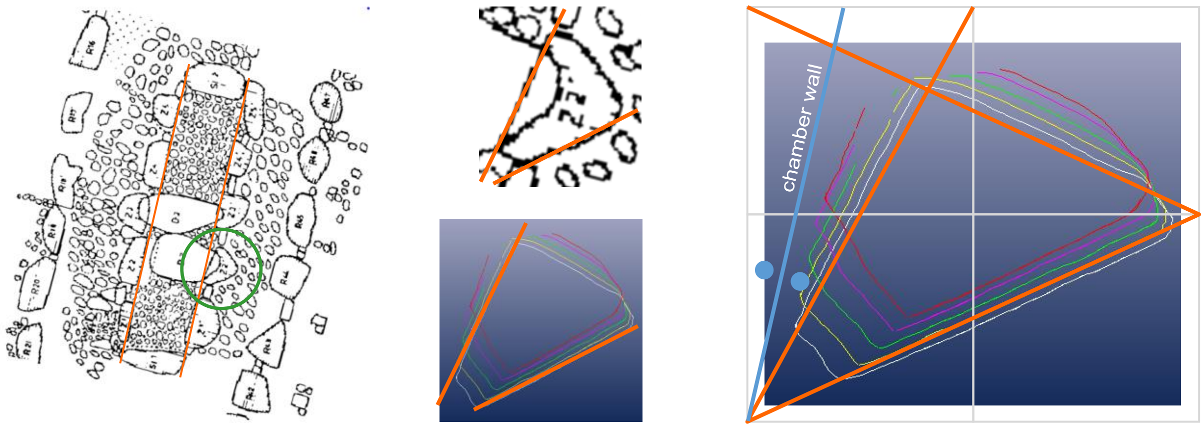


Figure 6
Left: The southern chamber of hunebed D43 as drawn by Holwerda. Its longitudinal axis (orange) is oriented towards 13¼º, which equals to the construction angle α.
Middle: Except for the entrance side of D43s-Z2', there is a good agreement between the plan of Holwerda and the virtually restored model of the stone. Apparently Holwerda had not noticed that chamber and entrance side stand perpendicular to each other.
Right: Entrance stone D43s-Z2' has been dressed to fit to the practitioner diagram (see PTS-2). All sides show the typical lightly rugged surface that seems to result from rubbing. The chamber orientation is constructed via bisection of the angle between true north and the chamber side of D43s-Z2'.

[10] One might doubt if a side stone that slopes extraordinary already, should be pushed to slope even more. If this is regarded unacceptable, then the current capstone should be rejected as the original one for it barely fits between the side stones of the concerning trilithon. Considering the size of the capstone (± 1.6 meter) it may well have been a former side stone. It is very unlikely that it was dragged into its current place during the destructive activities in 1870, when the tomb was acquired by the state. There is no presumable place where it could have come from. According to the excavation of Holwerda in 1913 the ostensible gap between S1 and Z2 is filled by half a side stone (Van Giffen 1925/27, Part II, p.116/120). If this was the former place, then the exchange must have taken place in prehistoric times already. In that case the current situation must be regarded as 'intended' again.

Table 1
The angles of the flattened chamber and entrance sides and their deviation from the ideal geometrical values of table 3. The first eight entrance stones stand in situ and have not been virtually restored.

Entrance stone	**D4 Z3'**	**D25 Z2'**	**D30 Z2'**	**D36 Z3'**	**D37 Z4'**	**D42 Z5'**	**D43n Z1'**	**D1 Z3'**	**D2 Z2'**	**D5 Z2'**	**D15 Z3'**	**D20 Z3'a**	**D20 Z3'b**	**D43s Z2'**
Geometrical values	103¼	96¾	90	96¾	96¾	53¼	90	76¾	36¾	108½	68½	103¼	76¾	90
Field model	102½	98	89½	98½	96	52	89½	78½	34½	110	68½	103½	75½	84
Deviation	¾	1¼	½	1¾	¾	1¼	½	1¾	2¼	1½	0	¼	1¼	6
Virtual restoration								76	35½	108	68½	103½	76½	90
Deviation								¾	1¼	½	0	¼	¼	0

Table 2
The average deviation of the angles and orientations of the left entrance stones in table 3.

FieldAvg() gives the average deviation of the field models compared to the geometrical model.
RestAvg() gives the average deviation of the virtually restored stones compared to the geometrical model.

Orientations of the chamber sides:
FieldAvg(in situ) = 0.6º FieldAvg(others) = 2.6º FieldAvg(all) = 1.5º
RestAvg(others) = 0.3º RestAvg(all) = 0.5º

Orientations of the entrance sides:
FieldAvg(in situ) = 0.6º FieldAvg(others) = 2.0º FieldAvg(all) = 1.2º
RestAvg(others) = 0.4º RestAvg(all) = 0.5º

Angles between chamber and entrance sides:
FieldAvg(in situ) = 1.1º FieldAvg(others) = 2.0º FieldAvg(all) = 1.3º
RestAvg(others) = 0.3º RestAvg(all) = 0.7º

Table 3
All orientations and angles of the surfaces are measured in the cross-sections of the 3D model. If one of the cross-sections clearly deviates from the others, it is disregarded as a local disturbance. Most of the time this counts for the cross-sections near the top. Therefore, table 3a,b lists the average azimuths of the three lowest levels per stone. The maximum deviation amongst the cross-sections from their average is written behind the azimuth values. Sometimes a disturbance shows up in all cross-sections, but none of them happen to cover the whole stone side. Still the azimuth of these surfaces can be taken then. In the end, the photographs were decisive about the correct interpretation of the cross-sections.

Table 3a
The left entrance stones that stand in situ. The orientations and angles are listed per entrance stone.
When the azimuth of the longitudinal axis is placed between brackets, its value diverges over 3º amongst different studies.
'Diag. n×m' stands for the angle produced by a diagonal through a field of n×m squares.

Entrance stone	**D4-Z3'**	**D25-Z2'**	**D30-Z2'**	**D36-Z3'**	**D37-Z4'**	**D42-Z5'**	**D43n-Z1'**
Field model							
Longitudinal axis	270º	{273º}	{168º}	{285º}	{281º}	{308º}	{197º}
Chamber side	283º ±0º	284º ±0º	167º ±1º	291º ±0º	290½º ±½º	282º ±0º	206½º ±½º
Entrance side	180½º ±½º	186º ±0º	77½º ±½º	192½º ±½º	194½º ±½º	230º ±0º	117º ±0º
Angle between sides	102½º	98º	89½º	98½º	96º	52º	89½º
Geometric model							
Longitudinal axis	270º	{---}	{166¾º}	{283¼º}	{283¼º}	{306¾º}	{198½º}
Chamber side	283¼º	283¼º	166¾º	290º	290º	283¼º	206½º
Entrance side	180º	186¾º	76¾º	193¼º	193¼º	230º	116½º
Angle between sides	103¼º	96½º	100º	96¾º	96¾º	53¼º	90º
Constructing angle	**α = 13¼º**	**α = 13¼º**	**α = 13¼º**	**α = 13¼º**	**α = 13¼º**	**α = 13¼º**	**α = 13¼º**
Entrance vs. chamber side	90º + α	90º + α ÷ 2	90º	90º + α ÷ 2	90º + α ÷ 2	α • 4	90º
Chamber side vs. cardinal	α	α	α	α + α ÷ 2	α + α ÷ 2	α	α • 2
Entrances side vs. cardinal	0º	α ÷ 2	α	α	α	α • 3	α • 2
Long. axis vs. cardinal	0º	{---}	{α}	{α}	{α}	{diag. 3×4}	{diag. 1×3}
Long. axis vs. chamber side	α	{---}	{0º}	{α÷2}	{0º}	{---}	{---}

Table 3b
The left entrance stones that have been virtually restored (see the description per stone above). The orientations and angles are listed per entrance stone.
When the azimuth of the longitudinal axis is placed between brackets, its value diverges over 3º amongst different studies.
'Diag. n:m' stands for the angle produced by a diagonal through a field of n×m squares.

Entrance stone	**D1-Z3'**	**D2-Z2'**	**D5-Z2'**	**D15-Z3'**	**D20-Z3'**	**D43s-Z2'**
Field model						
Longitudinal axis	{263º}	263½º	288º	292½º	276½º	{197º}
Chamber side	274º ±1º	282º ±1º	304º ±0º	291½º ±½º	285½º ±½º	201º ±0º
Entrance side	195½º ±½º	247½º ±½º	194º ±0º	223º ±0º	182º / 210º ±0º	117º ±0º
Angle between sides	78½º	34½º	110º	68½º	103½º / 75½º	84º
Virtual restoration	Slope	Slope		Slope		Slope
	Rotate -3¼º		Rotate +4º		Rotate -2º	
Restored model						
Longitudinal axis	{263º}	263½º	288º	291½º	276½º	{197º}
Chamber side	270º ±0º	282½º ±1½º	307º ±0º	291½º ±½º	283½º ±½º	206º ±0º
Entrance side	194º ±1º	247º ±0º	199º ±0º	223º ±0º	180º / 207º ±0º	116º ±0º
Angle between sides	76º	35½º	108º	68½º	103½º / 76½º	90º
Geometric model						
Longitudinal axis	{263¼º}	263¼º	288½º	291½º	276¾	{198½º}
Chamber side	270º	283¼º	306¾º	291½º	283¼º	206½º
Entrance side	193¼º	246½º	198½º	223º	180º / 206½º	116½º
Angle between sides	76¼º	36¾º	108¼º	68½º	103¼º / 76¾º	90º
Constructing angle	**α = 13¼º**	**α = 13¼º**	**α = 9¼º**	**α = 21½º**	**α = 13¼º**	**α = 13¼º**
Entrance vs. chamber side	90º - α	(diag. 3×4}	90º + α • 2	90º - α	90º +/- α	90º
Chamber side vs. cardinal	0º	α	α • 4	α	α	α • 2
Entrances side vs. cardinal	α	---	α • 2	α • 2	0º / α • 2	α • 2
Long. axis vs. cardinal	{α ÷ 2}	α ÷ 2	α • 2	α	α ÷ 2	{diag. 1×3}
Long. axis vs. chamber side	{α ÷ 2}	α + α ÷ 2	α • 2	0º	α ÷ 2	{---}

Prehistoric tomb surveying (7)
Dolmens along the German Baltic coasts

Dirk Kruithof, September 2019

The current paper makes part of a broader study through which the existence of a prehistoric geometry was tracked down. Different stages of tomb setups found at consecutive times and in various areas in Europe form the steppingstones towards the chronicled geometry in ancient mathematical texts. The current paper focuses on one of these areas, the German Baltic coasts. It confirms that the setup and orientation of the studied dolmens can be understood within a grid-of-squares geometry (see PTS-1 [1]). Grid lines and diagonals through adjacent squares determine the layout of the ground plan of the tombs.

The measuring method

The measurements of this study can be divided into two methods: The method used in Mecklenburg and the one in Schleswig-Holstein. During the first week of May 2007 a few selected ground plans in Mecklenburg were measured by means of a handheld sighting compass (Recta DP-10, ±0.3º), a goniometer, a water level, a tape measurer and rope. Levelled sightlines were constructed by means of water level and rope and then the orientation of those lines was taken by the sighting compass. Inside a dolmen the orientations were derived from already measured ones using the goniometer. Additionally, all sizes of and distances between sightlines were taken in order to check the geometrical forms that rolled out. Although the instruments were easy to carry, the measuring method was very time consuming. Therefore, a small total-station-like measuring tool was developed that combined an electronic angle measurer (Stabila, ±0.1º) and a laser distance measurer (Leica D5, ±0.5mm). This tool together with a laser leveller (KBW-0629, ±1mm/m) was used in

Figure 1
Measuring the angle between two levelled sightlines in Mecklenburg.

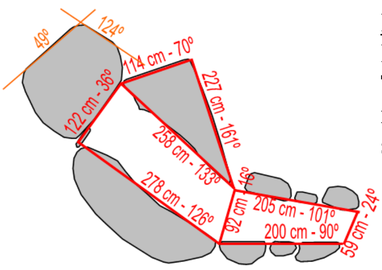


Figure 2
Dolmen 326 of Neu Gaarz.
The ground plan evolves as the result of the measured orientations, sizes and distances.

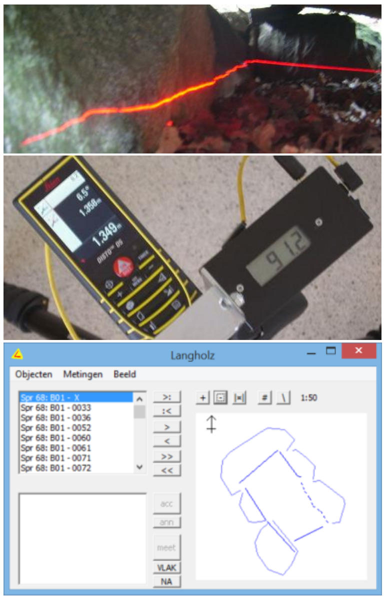


Figure 3
Plotting a 2D cross-section. A laser-leveller is used to project a cross-section on the stones. With an electronic angle and distance measurer the 2D data of the curves is taken. The data is inputted to the application SiteMsm.

[1] Reverences to the other papers of this study are abbreviated 'PTS-n', where PTS stands for the study title 'Prehistoric tomb surveying' and 'n' for the serial number of the paper.

Schleswig-Holstein in July 2011. A session started off by setting up a cross-section of a tomb directly on the stones. Small stickers were attached to the stones at the levelled laser line, and thereby the curves of the stones could be measured in a horizontal plane. The measuring tool was calibrated for true north using a sighting compass (Recta DP-10, ±0.3º) and next the distances and corresponding angles of all stickers were measured and processed. The processing was done by the for this purpose developed application SiteMsm. On the fly it showed the ground plan of the tomb so that faulty measurements could be corrected on the site. The measurement fault was 1 cm per meter at most, which is less than 1º in azimuth.

Figure 4
Urdolmen 358 of Thelkow.
The ground plan encompasses a Pythagorean triangle (3×4×5) as known from the grid-of-squares geometry.

The tomb selection

The dolmens were selected along holyday routes via the internet and the atlas of Sprockhoff. Larger tombs were avoided for two reasons: It would take to much time to measure them and the number of relationships between features grows exponentially by the number of stones and interstices. In small tombs the easier and more comprehensible setups are expected

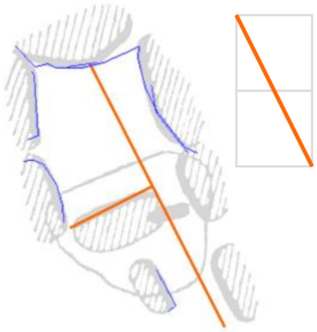


Figure 5
Dolmen 91 of Hemmelmark.
The measured data (blue) is used as an overlay over the ground plan in the Atlas of Sprockhoff. The straightforward construction of the longitudinal axis does not relate to northwestern part of the dolmen.

In Mecklenburg three dolmens have been orientated. With a fourth, the Urdolmen Thelkow 358, the orientation of the only flat stone got lost. Nevertheless, from the photographs it can be concluded that the dolmen comprises an inscribed Pythagorean triangle with sides 3×4×5 – the main characteristic of the grid-of-squares geometry (figure 4). Two dolmens were recently dismantled (Serrahn 384 and 385b [2]). Yet they are included based on the ground plans drawn by Schuldt.

In Schleswig-Holstein 11 dolmens were orientated. Furthermore, the data of Hemmelmark 91 got corrupted because of an unstable underground. Nevertheless, the orientation of the eastern wall and the portal could be established. The longitudinal orientation is 153º, which is constructed by a diagonal through two adjacent squares from due south (figure 5). Additionally, Langholz 68 was orientated, although this dolmen got restored for its major part (figure 6). The dolmen has a setup according the practitioner diagram (see PTS-1). The grid is orientated towards 167º, one of the most common orientations.

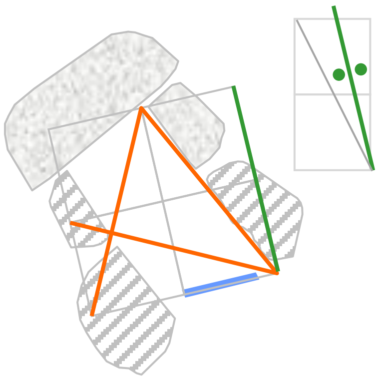


Figure 6
Dolmen 68 of Langholz.
The dashed stones lay scattered around and were restored. The door slab (blue) is virtually restored in this ground plan. It was restored with a slightly different orientation.

[2] In this paper the numbering of Sprockhoff is followed.

Overview of the processed dolmens

Rotensande 61

Overgrown by a diverse vegetation so that the status of this dolmen is unsure. The northern side stone closest to the entrance has subsided into the chamber because of a nearby oak. (In the ground plan this has been corrected.) Nevertheless, two features give hold for the inscribed Pythagorean triangle: the proportion 4:3 for the length and width, and the sightline along the undisturbed side stone of the southern wall.

Rotensande 64

A small rather well conserved dolmen. Even the capstone rests in situ. This made it hard to measure all parts of the chamber. The chamber is unique in being a rectangle in the proportion 2:3. Since the entrance faces due east, nothing more can be said about the geometrical structure.

Rotensande 65

Another well conserved dolmen. A heavy capstone rests in situ and it looks as if the left stone near the entrance has subsided under the weight. Probably this is not true since the door slab would have been set aside then [3]. Both the ground plan and the placement of the door slab confirm a straightforward setup.

Lehmberg 67

This is one of the few dolmens near the Baltic Sea that have the inscribed Pythagorean triangle with its short sides along the side stones. The eastern side stone near the entrance has subsided into the chamber a little.

Hemmelmark 81

The tomb has an elongated ground plan. Its most striking feature is a gap in the chamber side of the northern wall. It allocates for the top of the inscribed Pythagorean triangle. This special feature approves on the geometrical setup. The base of the triangle fits to the shorter southern side stone.

Panker 187 / Darry ???

In the pda software two ground plans were added to dolmen 187 accidentally. One being the real Panker 187 dolmen, while the other has not been identified properly [4]. Both dolmens display a variation on the same theme. They have side stones at about azimuth 143º, but the chamber of dolmen 187 is configured by a diagonal through two adjacent squares and that of the unknown dolmen by three adjacent squares.

Ruserberg 203n,s

These twin tombs stand about 20 meters apart in the same Langbett. The door slab of 203n stands along a grid line. The grid has an orientation of 63º, which fits the setup via a diagonal through two adjacent squares. In dolmen 203s the supports near the entrance bend into the chamber like a bottle neck. Maybe this is not the original

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[3] Because of the well-preserved state, this dolmen likely has been restored. Regrettably no restoration information could be tracked down. If the door slab has been restored it probably got its original position.

[4] It was found the surroundings of Lütjenberg, probably near Darry, but no matching references were found.

situation. Near the end stone, some capstones lie bestrewn. The grid of Ruserberg 203s has an orientation of 13º and supports forked sightlines at 167º and 193º as encountered in hunebedden D13 and D14 in the Netherlands (see PTS-5).

Wetterade 209

The dolmen has the same dimensions as the rectangular dolmen Panker 187. Since the interstices in dolmen 209 have a different position the ground plan features an inscribed Pythagorean triangle while dolmen 187 does not. The corresponding grid is rotated 3º from the cardinal points, which agrees with the difference in azimuth for the sightlines running along the northern side stones. The side stone near the end stone has a sightline at 117º and the one near the entrance at 120º.

Barkvieren Sch95

Sprockhoff did not include this Urdolmen in this atlas and thus the dolmen is numbered according the system of Schuldt. This is one of Schuldt's four prototype Urdolmens in Mecklenburg, discovered in 1945 and plundered in 1956 [5]. When Schuldt excavated the dolmen the chamber filling had been removed, but about the chamber itself he says that it "bis auf den etwas verschobenen Deckstein noch völlig intakt war" [6]. So, we may assume that the supports reside in situ nowadays.

Neu Gaarz 326

This dolmen represents the final phase of the Urdolmen type. It already has a passage and with a chamber length of 2.7 meters it is rather long. But the most spectacular feature is the northern side stone, which resides on a plinth [7]. Both outer sides have the proportion of a trysquare 1×2 and thus the inner side fits a diagonal through two adjacent squares [8]. The main grid of this tomb supplies for a setup via a diagonal through three adjacent squares. The end stone, southern side stone and the southern wall of the passage stand along diagonals in the main grid.

Figure 7
Dolmen 326 of Neu Gaarz.
The eastern side stone rests on a plinth. This construction can have served an easy manner to orientate the side stone very precisely.

Serrahn 384

Unfortunately, the dolmens of Serrahn have been dismantled by the landowners in the void of authority during the unification of Germany. Luckily, Schuldt made accurate ground plans and/or excavation plans of them already. During the excavation of this dolmen, it appeared that the door slab did not rest on the threshold any longer but was placed askew [9]. Thus, the situation of the excavation plan seems to be a secondary construction. Within the geometrical setup it emphasizes the special position of the interstice of the left wall. With or without, the setup does not change.

[5] Schuldt 1966, Second part, megalithic tomb number 29, called "Barkvieren Grab 1". In the text the years are mixed up: "erst 1956 bekannt, nachdem es 1945 durch Unberufene geöffnet und ausgeraubt worden war".
[6] Schuldt 1972, p.19
[7] Schuldt 1972, p.33 Abb.13
[8] The chamber side of the stone should match a diagonal in the sub grid but diverges +2º from it. Instead of 135º it now has an azimuth of about 137º, which aligns at the southern major standstill of the moon. Perhaps the geometrical setup intended a rough alignment that had to be fine tuned afterwards.
[9] Schuldt 1972, p.22 and p.46 Abb.24

Serrahn 385b

This is one of the dismantled dolmens of Serrahn and thus the geometrical setup is based on the field plan of Schuldt solely. This does not seem to be a problem, since dolmen 385b has a double grid (discussed further on), which would not appear in an inaccurate ground plan. Moreover, the dolmen can be compared to D22 in the Netherlands (see PTS-5).

Frauenmark 409

In this well preserved Urdolmen we find the most perfect application of a prismatic marking. It has been carved in the end stone (figure 8) and seems to highlight the geometrical configuration of the dolmen. While the northwestern side stone disturbs the idea of paralleling walls, the marking does 'repair' it. Another remarkable feature is seen near the entrance. Here the door slab does not fit the entrance completely. The gap is filled up by means of dry masonry.

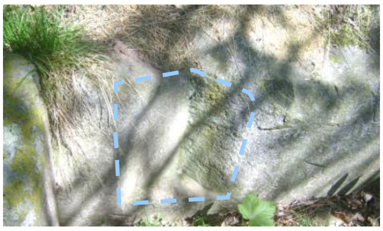


Figure 8
In the end stone of Frauenmark 409 a prismatic hole has been carved (blue dashed lines).

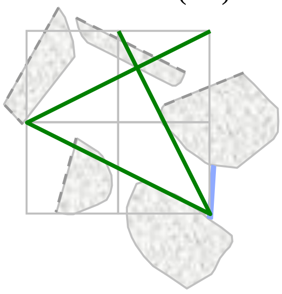
Overview of the dolmen setups

Figure 9 shows the orientated tombs. In the titles you find the nearest village and the number of the dolmen. In parenthesis the azimuth of the grid is given. Other azimuth values can be deduced from the geometrical principles (see PTS-1). In the ground plans a grey dotted line indicates where the form of a stone could not be measured anymore. Thresholds and door slabs are coloured blue. The alignment along the grid lines seems to be one of their features. In dolmen Ruserberg 203s the hatched stones indicate a possible disturbance because of the displaced capstones.

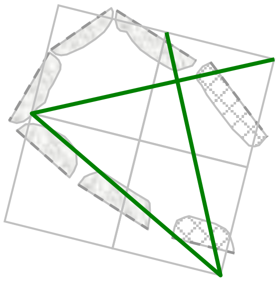
A diagonal through a field of 2×3 squares

In PTS-4 (figure 32 and 33) and in PTS-5 (figure 5) we have encountered some setups that produce sightlines at 167º within a grid at 43º. The Dutch hunebed D22 shares the grid at 43º with D21 while both hunebedden have sightlines at 167º created in the grid. Moreover, the setup of hunebed D22 is identical to that of D13 although both tombs have different shapes. The relation between the azimuth values 43º and 167º can be described as the construction of a diagonal through a field of 2×3 squares. Apart from the TRB West Group this relation is found in the regions of the TRB North Group too. A straightforward application is found in dolmen Rotensande 64, which has its ground plan formed by a field of 2×3 squares orientated towards the cardinal points. A little less straight forward comes with the setup of Serrahn 385b, which could be linked to hunebed D22 too, because of the atypical placement of an end stone (PTS-5, figure 1). The door slab of Serrahn 385b is placed along a diagonal 2×3 and thereby confirms the grid of squares having an orientation of 56º.

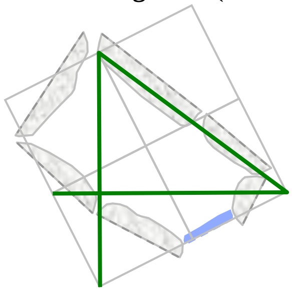
Rotensande 65 (90º)



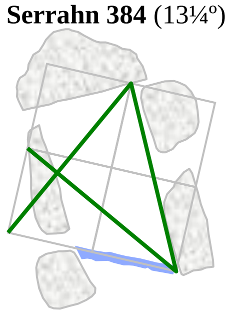
Ruserberg 203s (90º+13¼º)



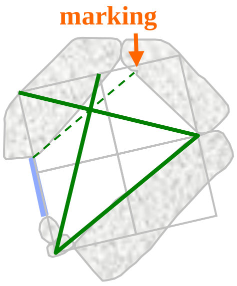
Ruserberg 203n (90º-26½º)



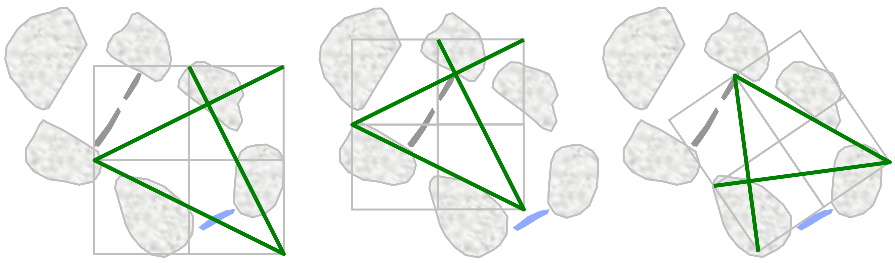
Serrahn 384 (13¼º)



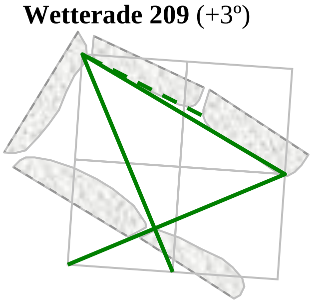
Frauenmark 409 (90º-13¼º)



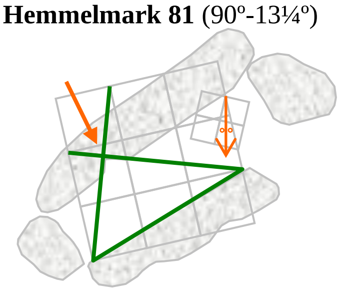
Serrahn 385b (0º / 0º / 56¼º)



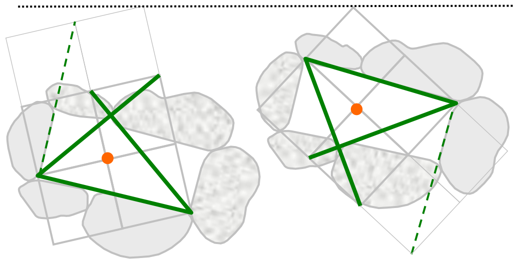
Wetterade 209 (+3º)



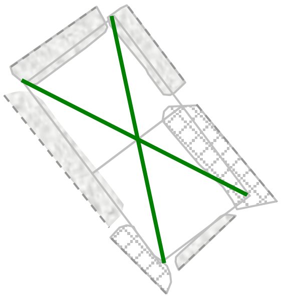
Hemmelmark 81 (90º-13¼º)



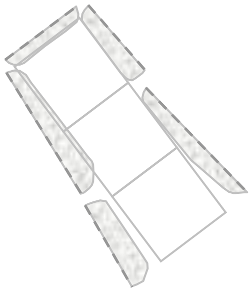
Barkvieren Sch95 (43º / 90º-13¼º)



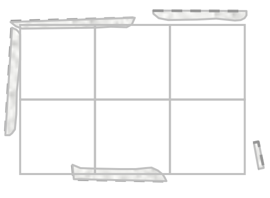
Panker 187 (53¼º)



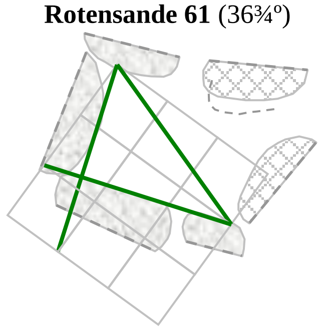
Darry ??? (53¼º)



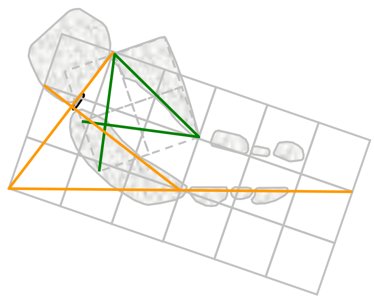
Rotensande 64 (90º)



Rotensande 61 (36¾º)



Neu Gaarz 326 (18½º)



Lehmberg 67 (13¼º)

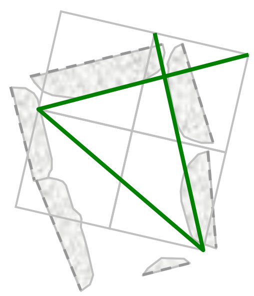


Figure 9
The ground plans and related geometrical setups of some dolmen near the Baltic Sea. The grid orientation is given in parenthesis behind the site name. Numbering according Sprockhoff (except Sch92, which is numbered according to Schuldt).

In the southern Langbett on the Ruserberg in Schleswig-Holstein we find two dolmens having their end stones aligned at the same sightline of 43º. The southern dolmen can be related to the sightline because of its grid but the northern cannot. Probably the alignment results of the orientation of the Langbett. Ruserberg 203n applies a rather straight forward setup by means of a repeated diagonal through two adjacent squares. The tomb gets its grid orientation from such a diagonal and the inscribed triangle is produced by the same diagonal.

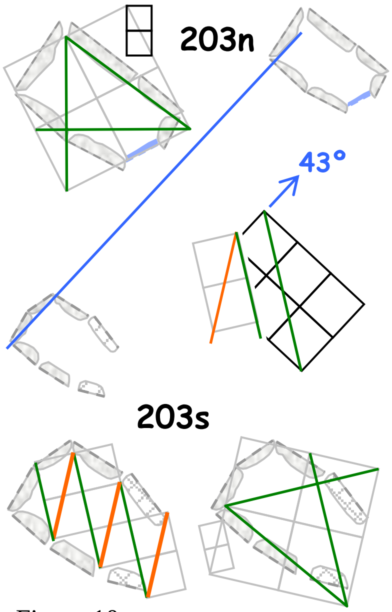


Figure 10
The geometrical setup of the dolmens of the Ruserberg. Dolmen 203n has a straightforward setup. The setup of dolmen 203s starts off with a sightline at 43º. See the text for more details.

Ruserberg 203s utilises two grids. The first grid derives its orientation from a diagonal through a field of 2×3 squares. It is used to setup forked sightlines at 167º and 193º. Furthermore, it forms the base grid of a secondary grid in which the inscribed Pythagorean triangle is constructed. The same geometrical construction of forked sightlines as in Ruserberg 203s is found in the Dutch hunebedden D13 and D22 (see PTS-5). Although the setup diverges a little, the concept itself must have belonged to the mindset of the surveyors. It demonstrates once more that in Neolithic times surveying was performed by skilled surveyors who maintained interregional contacts.

An amazing application of the relation between the grids at 43º and 167º is found in the prototype Urdolmen of Barkvieren. Dolmen Sch92 has one large and one small stone, making up each side wall. It has an almost but deliberately not exact oblong ground plan (figure 12). Because two opposing edges of the tomb do make a true square angle, both other edges not being square must be intentional. This false rectangle appears to be a feature that fits exemplarily to a setup with two inscribed Pythagorean triangles in a grid. Both grid orientations are closely related to each other via the construction of a diagonal in a field of 2×3 squares. It is applied in such a way that the orientation of one grid can be created using the other grid (figure 12c). The base of the Pythagorean triangle within the first grid runs along one side wall of the tomb and that of the other grid along the opposite wall. Both


Figure 11
Urdolmen Sch92 near Barkvieren (Mecklenburg, Germany). The photograph is taken in 2007.

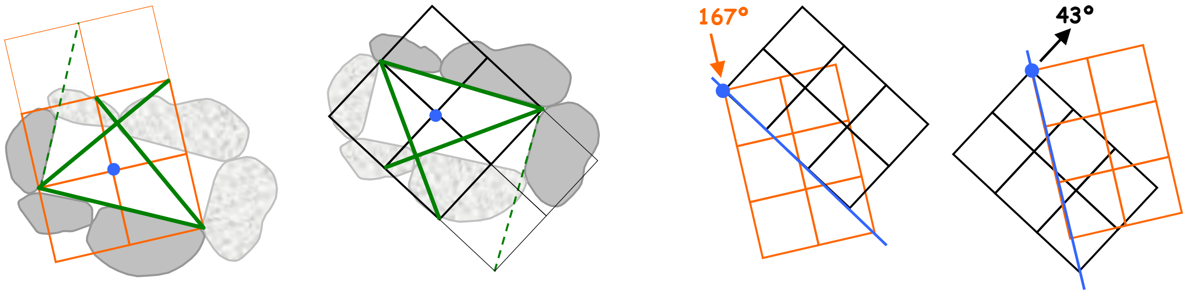


Figure 12
The ground plan of Urdolmen Sch92 is the result of two interwoven practitioner diagrams (see PTS-1).
(a) The patter in a grid at 167º fits the southern wall.
(b) The pattern in a grid at 43º fits the northern wall.
(c) The grids of both patterns can be orientated via one another.

triangle tops are positioned *between* the two stones of the side walls precisely. Therefore, the tomb length is a little more as twice the width. The grid at 77º / 167º produces walls at 103º / 193º. The orientation of the western wall and that of the grid can be described as 180º + 13º = 193º and 180º – 13º = 167º. These azimuths appear to be the most common orientations amongst megalithic tombs. On its turn, the grid at 43º / 133º produces walls at 106º / 196º. Although the grid orientation is applied quite often, the wall orientations are seen less common.

Schuldt typified dolmen Sch92 as a prototype Urdolmen so that it must be dated in the Frühneolitikum [10]. This means that it is one of the earliest megalithic monuments near the Baltic Sea. Yet the Neolithic surveyor demonstrated a creative application of the Pythagorean triangle in an orientated grid. It can be doubted if he knew on forehand how to construct such a harmonious ground plan. The blue spot of figure 12a and 12b not only forms the centre point for the rotated grids but even the centre of bisection of the inscribed triangles. Maybe the surveyor tried a lot of setups and ultimately picked the best looking. Still it proves a high understanding of the possibilities of the grid. Apparently Neolithic surveyors were familiar with the grid-of-squares geometry already when megalithic building arrived in the TRB culture. Whether the surveying moved along with megalithic building or that megalithic building got incorporated in an existing surveying, cannot be answered without further study.

[10] Schuldt 1972, p.92

Prehistoric tomb surveying (8) Small necropolises in the French Ardèche

Dirk Kruithof, September 2019

The current paper makes part of a broader study through which the existence of a prehistoric geometry was tracked down. Different stages of tomb setups found at consecutive times and in various areas in Europe form the steppingstones towards the chronicled geometry in ancient mathematical texts. The current paper focuses on one of these areas, the small causes of the southern Ardèche in France. It confirms that the setup and orientation of the studied dolmens can be understood within a grid-of-squares geometry (see PTS-1 [1]). Grid lines and diagonals through adjacent squares determine the layout of the ground plan of the tombs.

Introduction

The study of the orientations of the Ardèchoise dolmens started in 2005 and can be divided into two phases: an experimental and a systematic one. In the experimental phase the azimuth of back and side slabs were measured in order to compare them with the same measurements elsewhere - Ireland, Brittany, Rhodopi, Mecklenburg and Drenthe. The orientations were obtained using a handheld sounding compass (Recta DP10, ±0.3º) and a water leveller. Because of the slanting sides a levelled stick on a tripod was used to setup a horizontal sightline along a slab's surface. Still the working method was prone to a few degrees fault, but the main question to solve did not require exactness: *Is there a reason to believe that frequently utilized azimuth values exist independent from the latitude?* After some time, a few clusters of values could be appointed indeed. Meanwhile the study of some Dutch hunebedden (PTS-4) and German dolmens (PTS-7) had advanced some similarities in the setup of ground plans. Encouraged by these findings, the second phase was started. After an investment in photogrammetry necessities [2], three small necropolises of Caussenard type dolmens were selected. In 2012 3D models were produced of the group of La Devèse, in 2014 of Les Géandes and in 2015 of Les Granges.

The measuring method

Before taking the photographs, the cardinal points were measured off in a horizontal plane and marked by special targets. Afterwards, all photos were processed in Photomodeler Scanner (a photogrammetry application) and a point cloud was produced. Although it is possible to create a triangulated surface from a point cloud, this was not enforced. In a cross-section of such a surface even the smallest irregularities of a stone become visible. Such a high degree of detail is overdone (often we even can not be sure about the exact original placement of a stone) and it disturbs the general picture . Instead the point cloud was reduced to a small horizontal slice, which formed the basis for the ground plan. The produced ground plans reflect the dimensions at ground level.

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[1] Reverences to the other papers of this study are abbreviated 'PTS-n', where PTS stands for the study title 'Prehistoric tomb surveying' and 'n' for the serial number of the paper.
[2] Camera: Nikon 300s. Software: Photomodeler Scanner. Laser leveller: Leica Line L2P5 (±0.3 mm/mtr).

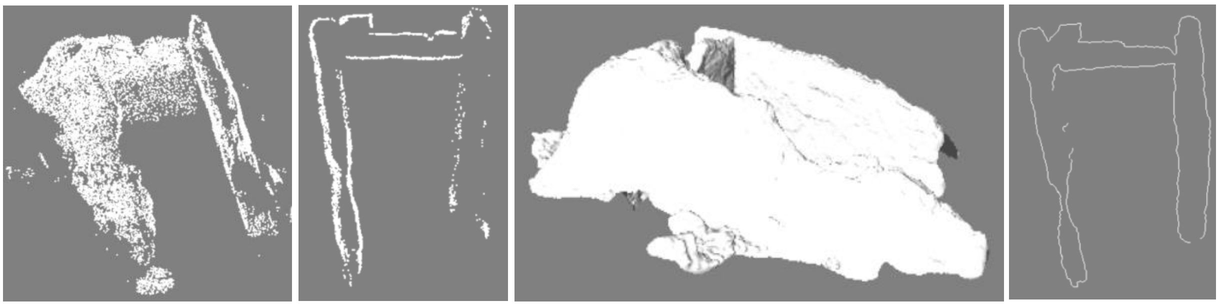

Figure 1
Photogrammetry software creates a 3D model from several carefully taken photographs. From left to right: a point cloud, a slice of the point cloud in top view, a triangulated surface, and a cross section through the surface in top view. (The dolmen in question is La Devèse A.)

Overview of the processed dolmens

La Devèse

This group is found near Orgnac-l'Aven - an area with beautiful caves. The group consists of five dolmens numbered A to E. Compared to some other dolmens around they form a close group. In this respect the entwined orientation of La Devèse A and B is compared to that of D21 and D22 (Drenthe, the Netherlands) in PTS-4. The orientations of dolmens C and E stand on their own. La Devèse D has been damaged so awfully that its orientation got lost. The other dolmens of the group all have a guard stone that shows the orientation of the grid. La Devèse B seems to have an orientated slab outside the chamber firmly fastened in the mound. A sightline along the stone runs diagonally at 167º in the orientation grid (dotted green line in figure 2). The actual placement as well as the geometrical pattern that evolves from it stand on their own. Therefore, it is unsure if the current position of the stone has a special intention.

Les Géandes

The numbering of the tombs differs per author. The current numbering is the latest one by Gély and Pape [3]. From 2002 to 2008 all dolmens of the group have been excavated. In general, the excavation plans are comparable to the cross-section through the 3D models of this study, except the plan of Les Géandes 2. Its excavation plan is less trapezoidal than the cross-section of the 3D model. How this can be is a mystery. When describing the slabs of the chamber the authors conclude that "le plan en trapèze de la chambre, plus étroit vers l'entrée, correspond donc bien à l'état initial" [4]. Yet the chamber does not constrict so much in their excavation plan. A photograph from 1896 [5] shows the dolmen in the same condition as nowadays, except that the entrance slab has been removed or destructed. Also, in the excavation plan of dolmen number 5 there is a mismatch with the 3D model. In the model the guard stones stand perpendicular to the chamber axis according to a faulty restoration in 1979 [6]. Apparently, the new excavation plan reflects the situation based on the traces in the mound. The original positions of the stones are copied into the ground plan of figure 2. Les Géandes 1 and 3 are heavily damaged.

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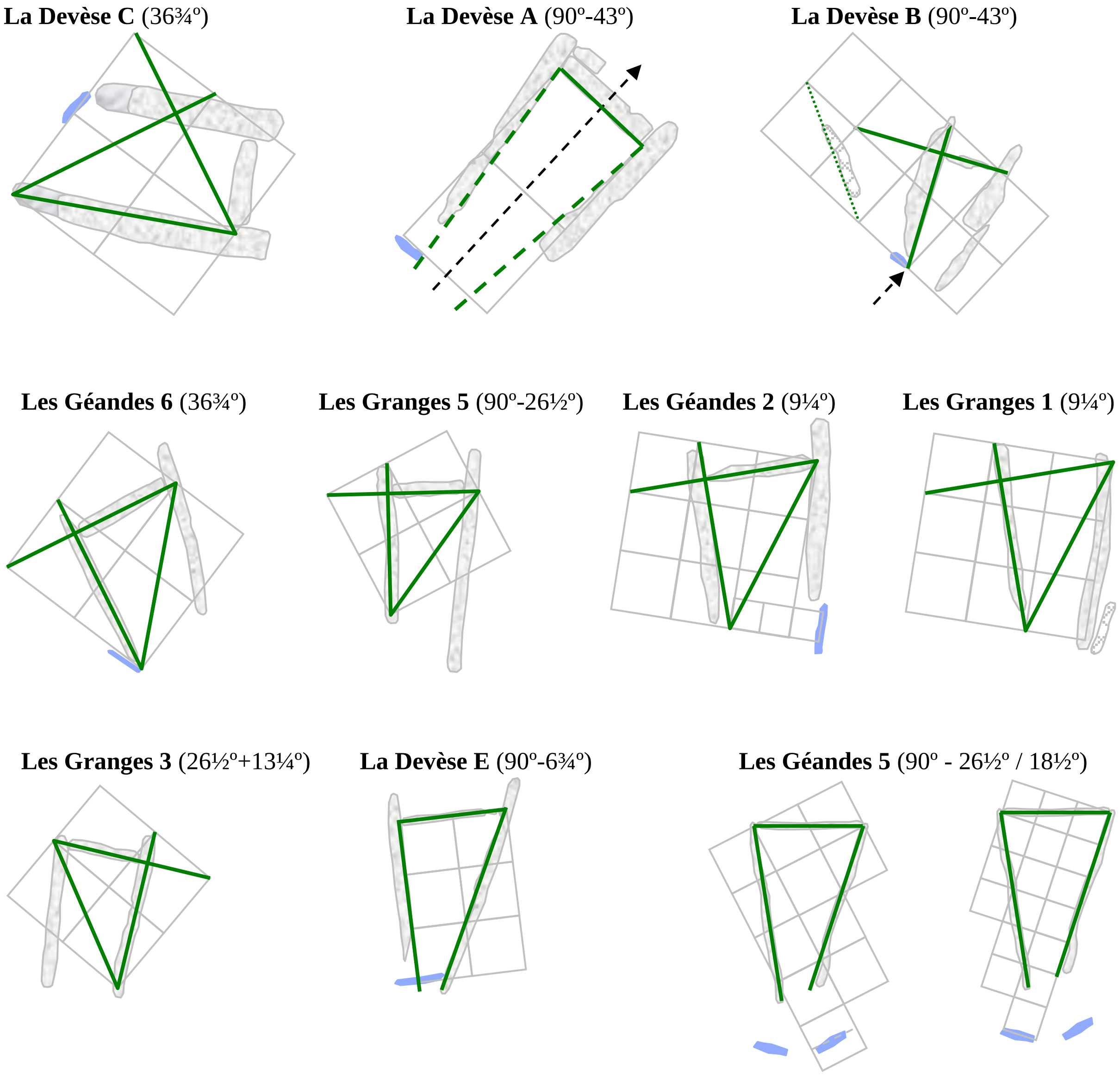
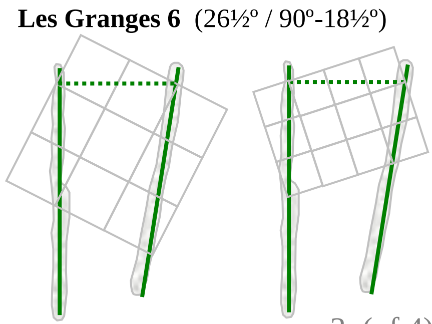
[3] Gély and Pape 2014, p.62
[4] Gély and Pape 2014, p.90
[5] Gély and Pape 2014, p.84, fig.94
[6] Gély and Pape 2014, p.107

Les Granges

Of the six visited dolmens four (numbered 1, 3, 5 and 6) were in such a state that a 3D model could be produced. Dolmen 2 lies hidden in the mound, and the eastern side slab of dolmen 4 is damaged. Additionally, the sightline along the back slab of dolmen 4 runs east-west, so there is insufficient information to get the dolmen orientated. Dolmen 6 is missing its back slab, but the setup seems to mirror the situation of Les Géandes 5.



<u>Figure 2</u>
The ground plans and related geometrical setups of some necropolises in the Ardèche. In the titles you find the local denomination followed by the azimuth of the grid in parenthesis. Other azimuth values can be deduced from the geometrical principles involved. In the ground plans a grey dotted line indicates that from there the form of a stone could not be measured anymore. Guard stones are coloured blue. Their alignment along the grid lines seems to be one of their features.



The ongoing development of tomb setups

The most eye-catching difference between the Cause type dolmens of the French Ardèche and the dolmens along the German Baltic Coast (PTS-7) is how the inscribed triangle is positioned within the tomb plan. The German dolmens tend to have the slant side of the triangle running along one of the dolmen walls, but in the Cause type dolmens it forms the diagonal of the tomb. Dolmen La Devèse C is the only exception, which could be linked to the fact that this is the only dolmen facing west.

Compared to the setups of the dolmens along the Baltic Coast, the Cause type dolmens show a lot more diversity. There is bigger share of diagonals through three adjacent squares and the share of bisected angles is less. Maybe the diversity shows a more experimental approach and reflects a deeper geometrical understanding. After all the Cause type dolmens were built almost a millennium later. It would not take long before setups applying diagonals through five adjacent squares got invented in the early Bronze Age (see PTS-9). Such setups result in just another right triangle, having sides 5×12×13. In the three groups of this paper the correct understanding of the setup by diagonals through three adjacent squares forms a prerequisite (see PTS-2).

Prehistoric tomb surveying (9)
Mycenaean tholos tombs

Dirk Kruithof, September 2019

The current paper of tholos tomb geometry must be seen as part of a more general study of prehistoric tomb surveying. By this study of Neolithic tombs in western Europe the author recognized a widespread pattern that points out a common geometrical approach of tomb layout. The geometry involved can be described at best as a 'grid of squares'-geometry. Grid lines and diagonals through adjacent squares determine the layout of the tomb plan and the orientation of the tomb. This type of geometry can be linked to the algorithms of finding integral right triangles as ascribed to Plato and Pythagoras and to a main problem type in Babylonian Seleucid and Egyptian Demotic mathematical texts [1]*. The current study of tholos tomb geometry is the onset to fill up the gap between the late Neolithic times and the genesis of these Seleucid and Demotic texts. The stomion (portal) of the tholos tomb is chosen as the subject of interest.*

The architectonics of a tholos tomb

All Mycenaean tholos tombs have a very particular arrangement in common. An open corridor (the dromos) is connected to a covered dome (the tholos) by means of a short portal (the stomion). Because the tholos has a circular shape, it cannot add much to the understanding of the geometry or orientation. Seen from the tholos, the dromos is the prolongation of the stomion and shares its orientation. It is a little wider than the stomion and the length seems to depend on the hillside more than on geometry. Especially with a large extent, the dromos adds to the dignified appearance of the stomion. Within the Treasury of Atreus, at each side of the stomion there is a base that probably supported a pilaster. The stomion forms the separation of the outer and the inner space of the tomb, what once was emphasized by a door. Enclosed between tholos and dromos the stomion has a clearly defined dimension and orientation. So, if the late Neolithic practitioner geometry is continued in the tholos tomb, then most probably it will be found in the stomion.

The orientation of the stomion

Literature about the orientation of tholos tombs seems to be restricted to the dromoi of the nine tombs in the surroundings of the Mycenaean palace. Therefore, it was decided to extend the information by supplementary measurements of tombs from other regions. Meanwhile for some Mycenaean tombs the impact of distortions was investigated, and it was checked to what degree the stomion shares its orientation with the dromos.

For the supplementary measurements the method of 3D modelling was chosen. By means of photogrammetry [2] many overlapping photos are merged into a 3D model. In the photos the cardinal points are defined by four targets on a 3D printed platform (figure 1). Its angle fault

[1] See PTS-2. PTS stands for the title of the current study *Prehistoric tomb surveying*. The study is divided in several papers. Each of them covers another subject. Other papers are referred to as PTS-n, where n stands for the serial number.
[2] Photo Modeler Scanner, version 2016

between the east-west and the north-south line appears to be 0.6°. The orientation and horizontal levelling of the platform is set via an electronic levelling compass [3] having a maximum misreading of ± 0.3° in the horizontal and a negligible ± 0.1° in the vertical plane. The magnetic deviation is calculated by the NOAA Deviation Calculator being +4.1° for the Peloponnesus (October 2016) [4].

Figure 1
Tripod with the four targets and digital levelling compass.

The targets of the cardinal points turn up in each 3D model. They are used to level the cross-sections and to have true north fixed in the model. As a result of the method, angles between lines can be regarded as geometrically correct within 0.6° and the orientation of a line within 0.9°.

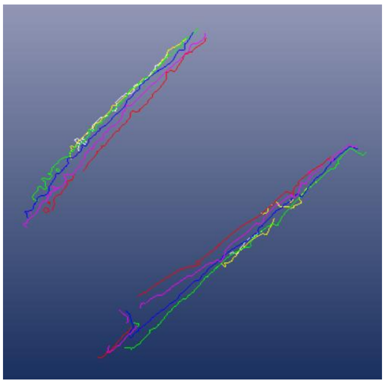


Figure 2
Cross-sections through the stomion of Ano Englianos 3. Halfway the top (red and purple) the stomion walls bend inward.

From the cross-sections it appears that even the most perfect tomb, the Treasury of Atreus, displays a deviation of 1° in the orientation between the dromos and stomion at one side of the tomb. This amounts to the same degree of deviation between the two sides of the dromos walls as reported by Maravelia [5]. It seems that a precision of 1° reflects the state of the art in Mycenaean times. For less perfect tombs, especially the ones with rubble masonry, we must reckon with larger deviations. Furthermore, many tombs have vertically curved walls because of the pressure of the surrounding hill. Near the ground and near the lintel they have kept their rectangular shape at best, but in between they bend. This effect is most noticeable close to the chamber (figure 4).

The dimensions of the stomion

In her overview of 127 tholos tombs, Dirlik [6] reports the width and length of 54 stomions. She collected these sizes from excavation reports. Appendix B lists them together with their ratio and the ratio between chamber diameter and stomion length. Many stomions have a ratio 1:2

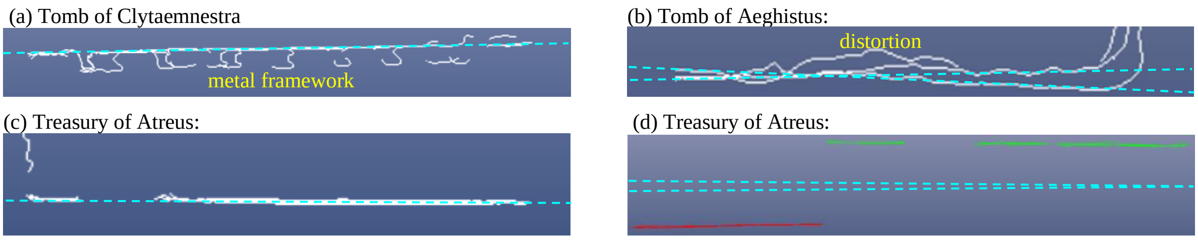


Figure 3
(a), (b), (c) Cross-sections through the left stomion wall (facing the chamber) of four tombs near Mycenae. The blue dotted lines follow the sight line along the wall.
(d) Cross-sections through the right stomion and dromos wall (facing the chamber) of the Treasury of Atreus. The dromos is green and the stomion is red. The blue dotted lines show the angle between both.

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[3] Rion HCM505b
[4] http://www.ngdc.noaa.gov/geomag-web/?useFullSite (January 2017)
[5] Maravelia 2001, pg.64
[6] Dirlik 2012, Catalogue pg. 64-99

Figure 4
Left: The dromos of the tomb of Aegistos. Already in Mycenaean times the tomb got restored.
Right: The stomion of Chalkias 1 seen from the chamber. The left wall shows a distortion at about 0.5 meter above ground level because of the pressure of the mound.

or 5:12 for the width and the length. Less frequent are the ratios 1:3, 2:3, 2:5 and 3:5. The rest (26%) does not translate into an integer ratio within 5 cm deviation. For the current paper a deviation of ±5 cm is chosen, which agrees with an orientation fault of less than 1º for stomions longer than 3 meter - the majority of the tombs in the current study. Additionally, it agrees with walls built of rubble stone, where the ending cannot be measured more precisely. Most of the stomions can be regarded to be rectangular within a fault of 5 cm, but some cannot. An obvious explanation could be that in those cases the stomion walls were placed perpendicular to the circular chamber wall and thus broaden towards the dromos, but this is not always the case. There are also stomions that narrow towards the dromos instead. Another possibility is that each stomion wall was placed by a different setup. It needs an in-depth study of all stomions involved to verify this. For now, a working explanation is chosen that the deviations are the result of imprecise surveyor's work or can be attributed to the millennia long influence of the surrounding mound. From the current study it can be concluded that there exists a rather big difference in quality amongst the tholoi setups. Already mentioned is the 1º variation in stomion and dromos orientation of the Treasury of Atreus. Apparently, the surveyors had problems to elongate parallel lines, but also a perpendicular setup could become troublesome. The lintel and the accompanying supporting walls of the tomb of Clytaemnestra deviate 2º from a real perpendicular placement compared to the stomion walls. With stomion widths of 1.45 meter and more, such a deviation will result in a measuring fault of over 5 cm.

Figure 5
The Treasury of Atreus. Sight on the relieving triangle.

The Treasury of Atreus - the Mycenaean magnum opus

The so-called Treasury of Atreus has been the largest dome of the world for about a millennium. The dome spans 14.5 meter, which was an unsurpassed achievement for so long. But the tholos tomb does not only excel in constructional sense. Above the stomion lintel one finds the so-called relieving triangle. Probably a façade with ornamental work surrounded it and filled it in - still leaving the shape of the triangle visible. Both sides of the triangle follow a diagonal through two adjacent squares [7] so that the apex forms an angle of the well-known right triangle with sides 3×4×5. The same diagonal construction forms the heart line of the tomb literally. It runs from the centre point of the chamber through the middle of the stomion and dromos creating a perfect symmetrical layout. Apparently, the architect used geometry for the layout of the tomb.

The tholos tomb has been studied by many authors, especially the construction of the dome. Como (2009), first of all interested in the thrust and the equilibrium of the stones, came to the following conclusion: "*The depth of the stomion, which is in the Treasury of Atreus equal to its height, correspond with the measure of the diagonal of the rectangle with sides M and 2M.*" [8] Como was the first to recognize the as a principle in the ground plan of the tomb. Furthermore, she fits the complete plan (including the

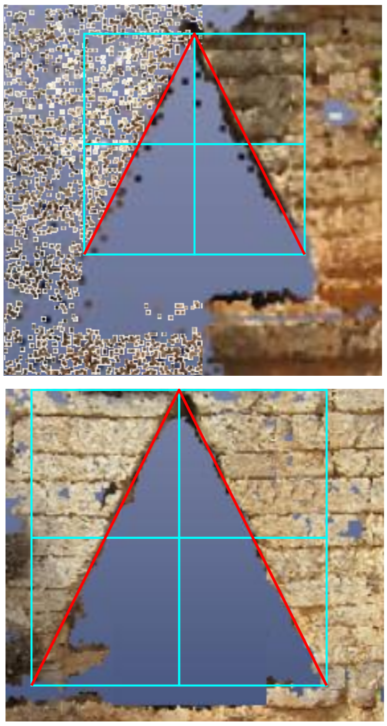

Figure 6
Top: The relieving triangle of the Treasury of Atreus. The 3D model only partly succeeded because of bad lighting conditions.
Bottom: The relieving triangle of the tomb of Clytaemnestra in the vicinity of the Treasury of Atreus.

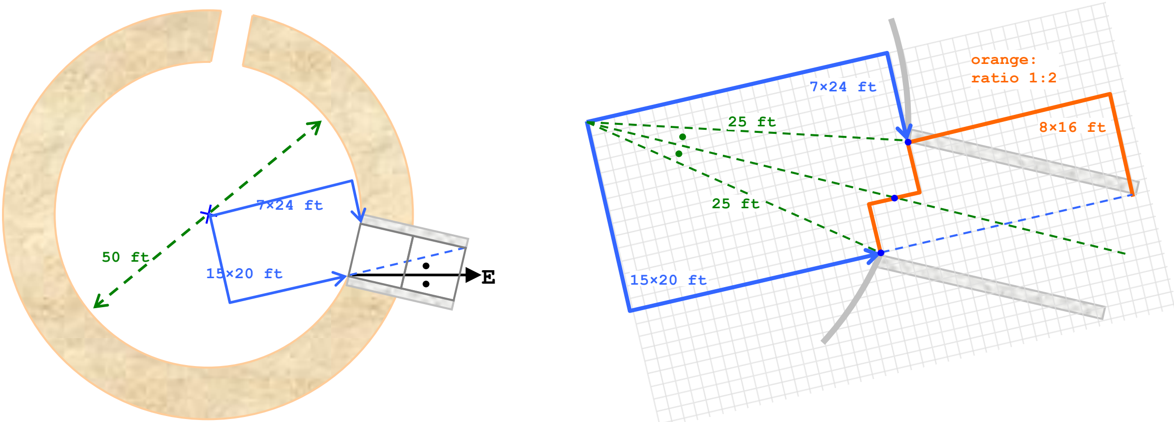

Figure 7
Left: Ground plan of the dome and stomion of the Treasury of Atreus.
Right Geometry of the connection between the dome and the stomion.
The architect used a radius of 25 feet in agreement with diagonals of trysquares with legs 7×24 and 15×20 (a five times enlargement of 3×4). These determine the joints between the stomion and chamber walls. The midpoint between both joints is found by trysquares 2:4. This midpoint defines the orientation of the stomion.

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[7] Because of the ease of reading and understanding, in this paper the wording '(a) diagonal(s) through two/three/five adjacent squares' will be replaced by the icon //.

[8] Como 2009, pg. 3

dromos) of the tomb in a square grid, which probably is a too rigid approach. A 3D analyses of the dome by Lazar (et al) shows an imperfection in the diameter at floor level of about 10 cm [9]. A 3D model of the facade of the stomion shows that the edges are close to (within ½º~1º) but not exact right angled. Over a distance of 5 meters this induces a deviation of 5 to 10 cm. When studying the geometrics of the tomb, one should not try to fit it to the actual sizes precisely but adhere to the most probable working flow. For example, in Como's ground plan the stomion length is measured from the domes circle to the line where the dromos begins, both midst the stomion. From the perspective of the working flow it is more logical to take the length from where the stomion walls connect to the chamber and facade walls. Although probably both walls differ a little in length [10], this will reflect the applied geometry better. Moreover, there is a way to clear up the position, dimensions and orientation of the stomion in terms of a coherent geometry. The next description can be followed in figure 7.

First of all, the architect adjusted the diameter of the dome in order to fit it to two trysquare diagonals of equal size: that having legs 15×20 (5 times 3×4) and 7×24. Both can be used to create a radius of 25. The midpoint of their apexes can be found by two trysquares 2×4. Additionally, the midpoint can be found right away by a trysquare 11×22 reckoned from the dome centre, which is congruent to a trysquare 1×2. This line functions as the centre line through the stomion and orientates the stomion. Both the stomion and the grid orientation are bisected by due east. This is a standard solution for tombs in the Neolithic and Bronze Age.

	Azim.	Length	Width
Ideal	103¼º	5.24 mtr	2.62 mtr
Actual	101¼º	5.4 mtr	2.7 mtr

Table 1
Measures of the Treasury of Atreus.

The above setup results in an ideal stomion orientation of 103¼º and a stomion dimension of 8×16 feet. Since the chamber diameter was taken to be 2×25 feet and actually measures 14.65 meters [11] the size of one foot can be calculated: $14.65 \div 50 = 0.293$ meters. Thereby the ideal stomion length becomes $0.293 \cdot \sqrt{(8^2+16^2)} = 5.24$ meters. This diverges 0.16 meters from the actual length. Compared to the variation in chamber diameter, it is a rather big difference. The same counts for the variation in the orientation of 2º - the actual orientation being 101º. Compared to the deviation of 1º between stomion and dromos wall it seems a rather big fault. In fact, the orientation of stomion and dromos reflect a simple elongation, so that a 1º mismatch would have been a rather big amount already. Apparently, we have to expect less precise measures. Since we do not know the exact workflow (what was measured first and how it was measured), it seems that we should esteem fitting to a well-known setup from other tombs higher than fitting to the actual measures [12].

The Mycenaean architectonics culminated in the Treasury of Atreus. No other tholos tomb fits together the elements so perfectly. No other architect implemented the trysquare 7×24 either. It seems that before the Treasury of Atreus, only trysquares 3×4 and 5×12 were known, or at least in use. We cannot know how the architect of the Treasury of Atreus found the new trysquare. Did he think of it himself? Had he heard about it? Had surveying advanced to a new way of finding Pythagorean triples? This is discussed in PTS-2.

[9] Lazar et al 2004, table 1
[10] This has not been measured but is observed with other stomions.
[11] Como 2009, pg.3
[12] If compared to tholos tomb Kakovatos A the orientation method is the same and also the way how the eastern connection of the stomion and chamber walls are found. With Kakovatos A foot of 0.303 meter was used, which - in the case of the Treasury of Atreus - would have resulted in the correct stomion dimension but a chamber diameter of 15 meter. Maybe all depends on the foot size of the surveyor at hand.

Tholos tomb Kakovatos A - Traders in Baltic amber

In a geometrical sense this tomb is less spectacular than the Treasury of Atreus and it illustrates the problem of a smaller chamber diameter. Dörpfeld excavated the tomb in 1907 and produced an accurate ground plan. The geometrical setup starts very much alike that of the Treasury of Atreus. The grid orientation comes from the bisection by due south of a (a trysquare 1×2). Within the grid the eastern wall of the stomion meets the chamber wall at coordinate 12:16, reckoned from the chamber centre, which is a four times enlargement of the trysquare 3×4. The connection of the western wall to the chamber wall cannot fit to the grid (figure 8). For Kakovatos A another geometrical technique is chosen. The stomion width is determined by the top of an inscribed right triangle 3×4×5. The triangle emerges from three (figure 9, see PTS-1). As a result of the geometrical construction the centre line of the stomion does not run through the centre point of the chamber, but half a foot eastward.

Although a little beyond the scope of this paper, tholos tomb Kakovatos A delivers some insight in the relation between Mycenaean grid-of-squares geometry and that of the northern European world. With the inscribed triangle of the stomion and the usage of bisection for the orientation, Kakovatos A resembles a setup that is widely found amongst megalithic monuments along the Baltic (figure 10). The Bronze Age people of Kakovatos were internationally orientated. The excavation of the tomb brought huge amounts of amber ornaments to the light. Merhart compared the amber spacers to those found in some Hügelgräber of south Germany and concluded: "*That the spacers of Kakovatos are imports from the North will remain a certainty even if the amber used to make them should not be Baltic as is commonly assumed.*" [13] But they appear to be Baltic indeed as the spectrum analyses of Beck, Fellows and Adams have proven. Subsequently these authors go one step further and see it as a fact "*that Mycenaean Kakovatos participated in the amber trade of the Middle Bronze Age*" [14]. Where traders are, there is not only exchange of products but also exchange of ideas. The grid-of-squares geometry seems to be an idea that survived the millennia, like that of building with huge blocks, moulding

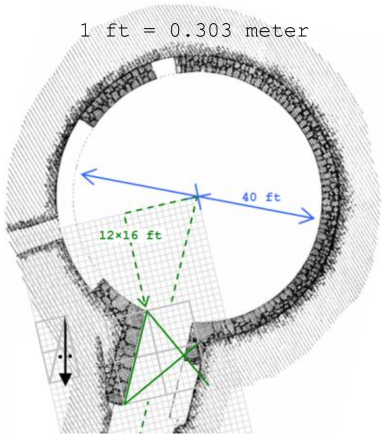


Figure 8
Geometrical setup of tomb Kakovatos A - based on the excavation plan of Dörpfeld. See the text for a description.

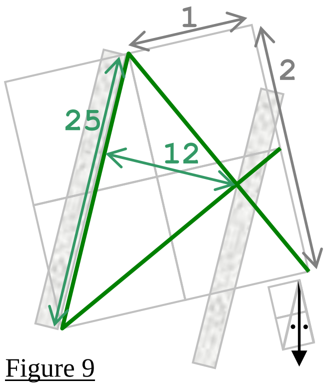


Figure 9
The geometrical interpretation of the stomion of Kakovatos A. Three produce a right triangle 3×4×5. If inscribed in a stomion, this will result in a ratio 12:25 for the width and length as is the case with Kakovatos A.

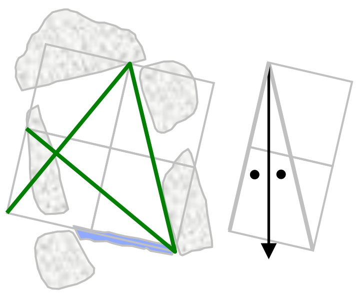


Figure 10
The geometrical interpretation of dolmen Serrahn 384 (Mecklenburg). Ground plan based on the excavation plan of Schuldt (Die mecklenburgische Megalithgräber, 1972, p.46, Abb.24)

[13] After: Beck, Fellow and Adams 1979, pg.15
[14] Beck, Fellow and Adams 1979, pg.12

pots of clay or wearing amber adornments. The grid-of-squares geometry belonged to those ideas that got passed around the world in Neolithic and Bronze Age times.

Tholos tomb Ano Englianos IV

In the southwest of the Peloponnesus king Nestor established just another realm. About hundred meters from his palace the tomb Ano Englianos IV is found. Not as wealthy and big as Kakovatos A, but of a comparable design. Seemingly the architect was aware of the displacement of the stomion centre line and thought of an ad hoc solution for it. Again, the setup starts with a grid orientation via a ◫ bisected by due south. Also comparable to the Treasury of Atreus and Kakovatos A is the connection of the eastern stomion wall to the chamber wall by an enlargement of the trysquare with legs in the ratio 3:4 (three times now). But coordinate 9:12 does not start in the centre exactly, as if the whole grid of squares got displaced half a foot. For the rest, the stomion resembles that of Kakovatos A having the orientation as the main difference. The grid-of-squares has been setup via the bisection of ◫ by a cardinal point, and then the stomion is orientated in the grid via a full ◫ [15].The orientation of Kakovatos can be calculated as *180º – ½◫ + ◫ = 193º* and that of Ano Englianos as *0º + ½◫ + ◫ = 40º* .

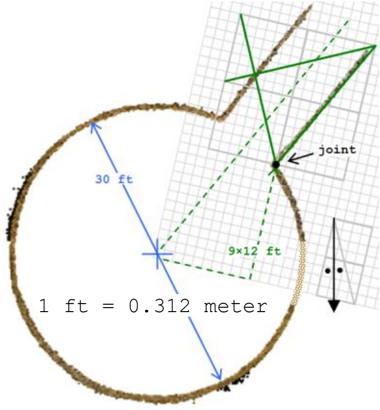


Figure 11
Tholos tomb Ano Englianos IV near the palace of Nestor. Ground plan created from a cross-section of a 3D model (see page 1).

Tholos tomb Chalkias 1

The stomion of Chalkias 1 measures 1.0×2.5 meters on average. The ratio (= 0.40 [16]) is a little less than that from a triangle by means of ◫◫◫◫◫ (= 0.42). The stomion walls bend a little and broaden towards the dromos. Based on the average orientation of the southern wall one could decide to a straightforward application of a 3×4 trysquare, which yields 37º. On the other hand, a combination of three ◫◫◫◫◫ agrees with the orientation of the northern wall (33º). Additionally, a closer look reveals that the southern wall is a little longer than the northern wall, which fits exemplarily to a construction via ◫◫◫◫◫. In fact, because of it one can follow on the construction step by step. With the grid size of about 26 cm as a (small) foot, the dome diameter measures 16 feet. From a pseudo joint of stomion and dome (the orange dot in figure 12), the wall ends near the dromos are defined by a double-◫◫◫◫◫ (the orange circles in figure 12). Because of the bending of the walls, the geometrical setup cannot be

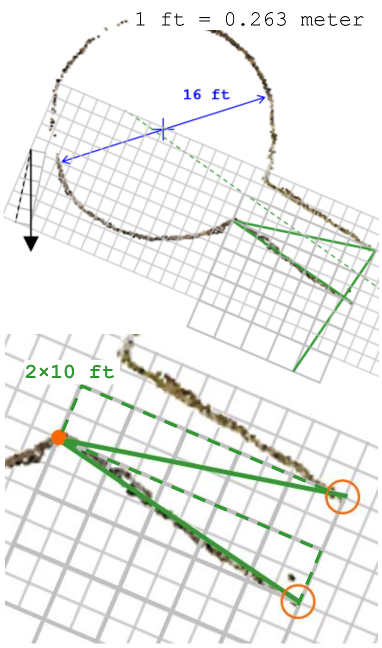
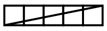


Figure 12
Tholos tomb Chalkias 1 in the middle of Messenia. Ground plan created from a cross-section of a 3D model.
Top: Grid orientation and stomion dimensions depend on ◫◫◫◫◫.
Bottom: Two such diagonals determine the length of the stomion walls (orange circles).

[15] The pattern is: {orientation} = {cardinal point} +/- {bisection 13¼º for the grid} +/- {26½º for the stomion}
[16] Based on the average width of the stomion in the 3D model. Near the dromos, the stomion width is 1.05 meters, which is the width found in Dirlik 2012, pg.85. With a width of 1.05 meters the ratio becomes 0.42, indeed.

rotated by 180º, which is normally the case. Apparently, the walls have been constructed from the dromos towards the dome. First, they follow a  in the grid but then they bend to reach the dome circle more or less perpendicular. Therefore, the stomion and the dome do not connect in a grid point.

Another reason for rejecting a 3×4 trysquare in the setup of Chalkias 1 comes from the fact that in all other investigated tombs, the diagonal of the stomion is geometrically constructed. Even the straightforward setup of the tomb of the Genii fulfils this requirement, although it does not display other features of a grid-of-squares geometry.

Overview of the investigated stomions

In six days, nineteen tombs have been visited of which seven could not be measured [17]. With three tombs the 3D model of the stomion failed partly [18] and with the tombs near the palace of Mycenae [19] it was not allowed to use the equipment. Here the orientation was re-measured tentatively using a high precision compass (± 0.3º) [20] and a 3D model was produced without having it orientated. In the end, seven stomions have been measured completely [21].

Figure 13
Areas where the studied tombs are found.

On the next pages first the stomions of the tombs in the vicinity of the Mycenaean acropolis are presented. The information is taken from Dirlik (2012) and Mickelson and Mickelson (2014). The measures of four of these tombs have been checked in the 3D model. Since the measurement targets were not available in the model, only a check on the correct proportions are made. In the case of the tomb of Clytaemnestra the given stomion dimensions appeared to be wrong [22]. The correct dimensions are calculated proportionally via the dome diameter in the 3D model. The data of the other tombs agreed with the 3D model. Subsequently, the stomions of the investigated tombs away from Mycenae are described according to the measures extracted from the 3D models. Here the equipment could be used, and all of the measures are taken from the model. Since the 3D models of the tombs of Peristeria partially failed, additional measures are taken from Dirlik (2012).

The geometrical interpretation of the ground plans is specified by means of utility lines in the orientation grid. A green arrow points to one of the cardinal points. Often the arrow is used as a bisector of a diagonal in the grid. Red dotted lines visualize the accompanying diagonal ratios. They reflect the formula for the geometrical orientation. If the grid itself has an orientation, then its construction is displayed via an inset. A red dotted line in both the setup and the main grid shows how they are interrelated. Inscribed right triangles have bright blue lines. In the second group also an image of five cross-sections through the 3D model is presented per stomion. The lowest cross-sections are taken 10 to 30 cm above ground level

[17] Ruined: Chalkias 2, Vasiliko 1 and 2. Inaccessible: Malthi 1 and 2, Dendra. Filled with earth: Koriphasio.
[18] Peristeria 1, Treasury of Atreus, Lion tomb.
[19] Treasury of Atreus, Lion tomb, Tombs of Clytaemnestra and Aeghistos.
[20] Recta DP10. By the staff of the ticket office it was not allowed to bring in the tripod with the target platform.
[21] Tiryns, Prosymna, Berbati, Ano Englianos 3 and 4, Chalkias 1 and Peristeria 2.
[22] Probably they have been copied from the Treasury of Atreus accidentally. The values are the same and this tomb succeeds that of Clytaemnestra in the catalogue.

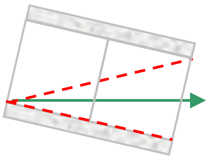
depending on the quality of the point cloud near the ground. They correspond to the intended ground plan. The highest cross-sections are taken about halfway the lintel and reveal the bending of the walls. From bottom to top the cross-sections are taken at regular distances, having the colours yellow, green, blue, purple and red.

The given 'geometrical orientation' has the following format:

{cardinal point} +/- {grid orientation} +/- {stomion diagonal in the grid}.

The orientations are given in diagonal ratios of the grid. For example, the angle of a bisected is written as ½[1:2] and is calculated by '0.5 × $\tan^{-1}$(1÷2)'.

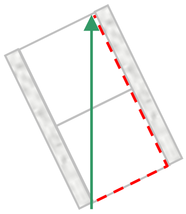
Treasury of Atreus



Geo coordinates: N:37 43 36.2 E:22 44 15.9
Dimension: 5.4 × 2.7 meter
Orientation: 101º
Masonry: sawn stone
Dating: LH IIIa/1 -LH IIIb

Proportional dimension: 1×2
Geometrical orientation: E + ½[1:2]

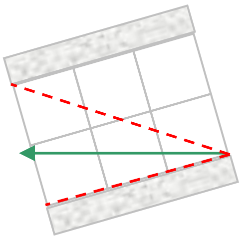
Lion tomb



Geo coordinates: N:37 43 54.6 E:22 45 19.3
Dimension: 5.15 × 2.55 meter
Orientation: 334º
Masonry: sawn stone
Dating: LH IIa

Proportional dimension: 1×2
Geometrical orientation: N - [1:2]

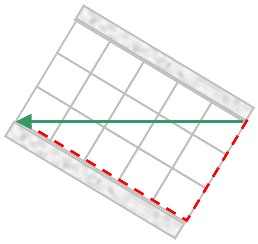
Panagia tomb



Geo coordinates: N:37 43 54.6 E:22 45 19.3
Dimension: 3.0×2.0
Orientation: 254º
Masonry: sawn stone
Dating: LH II

Proportional dimension: 2×3
Geometrical orientation: W - ½[2:3]

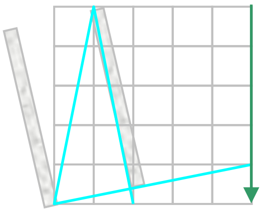
Tomb of the Genii



Geo coordinates: N:37 43 45.2 E:22 44 58.8
Dimension: 3.4 × 2.0 meter
Orientation: 301º
Masonry: sawn stone
Dating: LH IIb - LH IIIa/1

Proportional dimension: 3×5
Geometrical orientation: W + [3:5]

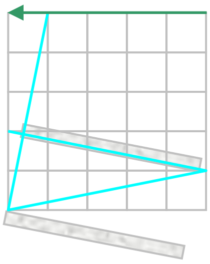
Tomb of Clytaemnestra



Geo coordinates: N:37 43 49.3 E:22 44 18.6
Dimension: 5.4 × 2.25 meter
Orientation: 168º
Masonry: sawn stone
Dating: LH IIIa/2 - LH IIIb/1

Proportional dimension: 5×12
Geometrical orientation: S - [1:5]

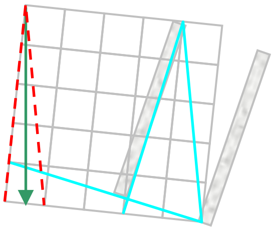
Cyclopean tomb



Geo coordinates: N:37 43 54.6 E:22 45 19.3
Dimension: 3.4 × 1.4 meter
Orientation: 280º
Masonry: hewn stone
Dating: LH IIa

Proportional dimension: 5×12
Geometrical orientation: W + [1:5]

Tomb of Aegisthus



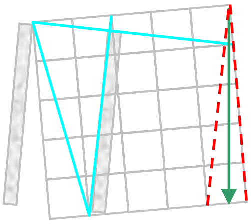
The tomb of Aegisthus is a little bit of a crackpot. For long time it stood in the centre of a heated debate concerning the now rejected Minoan supremacy over the Greek mainland during the Bronze Age. The decisive point was the single- or two-phase construction of the tomb. The ashlar facade of the stomion (sawn stone) is very long (1/3 of the total length) compared to other stomions and hence can be regarded as a second phase addition. Currently the stomion suffers from the mound pressure a lot. About 100 years ago a supporting pillar was built against the eastern wall and in 1999 the stomion was secured by heightening the floor a little and by a framework.

Geo coordinates: N:37 43 49.3 E:22 44 19.8
Dimension: 5.6 × 2.35
Orientation: 197º
Masonry: hewn and sawn
Dating: LH I / LH II

Proportional dimension: 5×12
Geometrical orientation: S + ½[1:5] + [1:5]

Figure 14
The tholos tomb of Clytaemnestra.

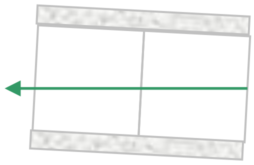
Epano Phournos tomb



Geo coordinates: N:37 43 54.6 E:22 45 19.3
Dimension: 5.0 × 2.0
Orientation: 185.5º
Masonry: hewn stone
Dating: LH IIa

Proportional dimension: 5×12
Geometrical orientation: S - ½[1:5] + [1:5]

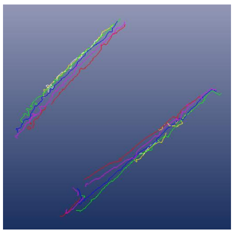
Kata Phournos tomb

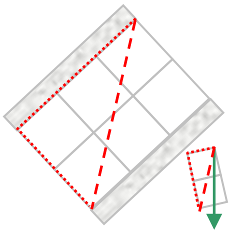


Geo coordinates: N:37 43 54.6 E:22 45 19.3
Dimension: 4.0 × 2.0
Orientation: 273º
Masonry: sawn stone
Dating: LH IIa

Proportional dimension: 1×2
Geometrical orientation: unknown

Ano Englianos 3



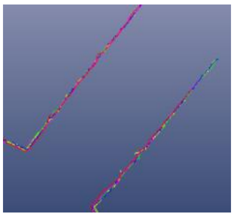


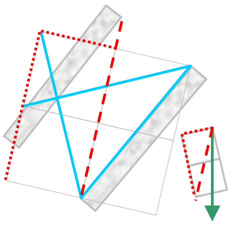
Near ground level the stomion has preserved its rectangular shape, but at the higher levels both walls tilt inward. This counts the most near the chamber.

Geo coordinates: N:37.0217024 E:21.6871232
Dimensions: 2.4 × 1.6 meter
Orientation: 46½º (±¼º) / 46½º (±½º)
Masonry: hewn stone
Dating: unknown

Proportional dimension: 2×3
Geometrical orientation: N + ½[1:2] + [2:3]

Ano Englianos 4



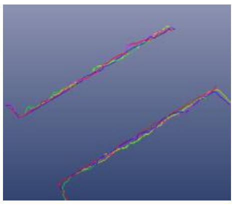


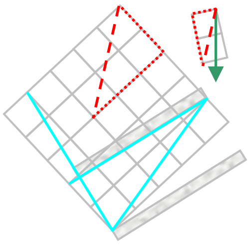
The stomion is in an almost perfect condition. Only the upper part of the eastern wall is pushed a little inward near the chamber.

Geo coordinates: N:37.0284183 E:21.696293
Dimension: 4.6 × 2.3 meter
Orientation: 38½º (±0º) / 39½º (±¼º)
Masonry: hewn stone
Dating: LH-II

Proportional dimension: 12×25
Geometrical orientation: N + ½[1:2] + [1:2]

Tiryns



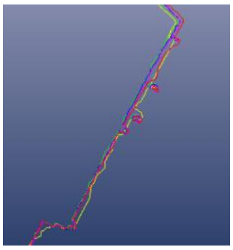


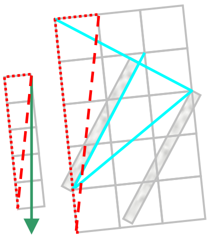
The stomion has been restored and is in a good condition still.

Geo coordinates: N:37.596520 E:22.812600
Dimension: 4.7 x 2.0 meter
Orientation: 58½º (±0º) / 58½º (±0º)
Masonry: hewn stone
Dating: LH-III

Proportional dimension: 5:12
Geometrical orientation: N + ½[1:2] + [2:3] ...
... + [1:5] = 58.3º

Peristeria 1



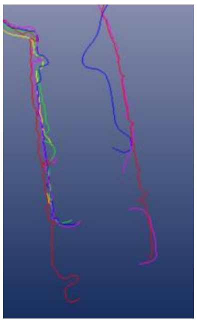


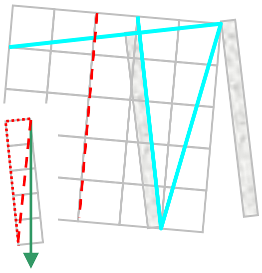
The 3D model did not succeed for the complete tholos tomb. Therefore, both walls cannot be put together in one and the same model. Only the western wall can be orientated. From the photos it appears that the lowest row of stones of this wall is set forward a little. The green cross-section displays the orientation of the stomion wall best.

Geo coordinates: N:37.276123 E:21.740118
Dimension: 5.3 × 2.3 meter [23]
Orientation: 28º (±0º)
Masonry: hewn stone
Dating: LH-I/III

Proportional dimension: 5×12
Geometrical orientation: N - ½[1:5] + [2:3]

Peristeria 2



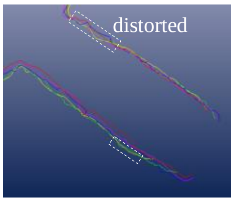


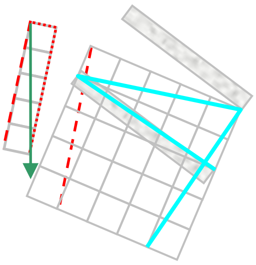
Due to its bad state the tomb is inaccessible and had to be photographed from the mound. Only the upper part of the stomion could be caught in the 3D model. The eastern wall has suffered a lot from the mound pressure. Even the modern-day traverse beams between the upper ends of both walls bend through already. Probably because of these beams the western wall tilts a little outward now. Yet this wall preserves the stomion orientation at best.

Geo coordinates: N:37.276123 E:21.740118
Dimension: 5.5 × 2.35 meter (near dromos) [24]
Orientation: 175º
Masonry: rubble stone
Dating: LH-I/III

Proportional dimension: 5×12
Geometrical orientation: S + ½[1:5] - [1:5]

Chalkias 1





Geo coordinates: N:37.361475 E:21.864388
Dimension: 2.5 × 1.0 meter
Orientation: 127º (±½º) / 127º (±½º)
Masonry: rubble stone
Dating: LH-IIa/b to LH-IIIb

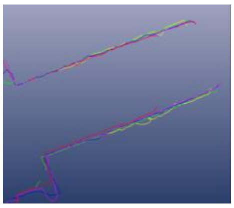
At the level of the green cross-section two heavy blocks are positioned in the edge of the stomion and the chamber. Still the northern wall is pushed inward by the pressure of the mound. There are two distortions, but they do not disturb the measurement of the orientation. The northern wall is a little shorter than the southern wall, which is taken to be the length of the stomion.

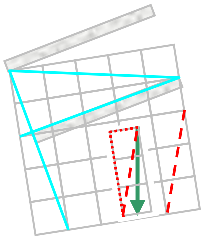
Proportional dimension: 2×5
Geometrical orientation: E + [3:4]

[23] Dirlik 2012, pg. 80

[24] Dirlik 2012, pg. 80

Prosymna



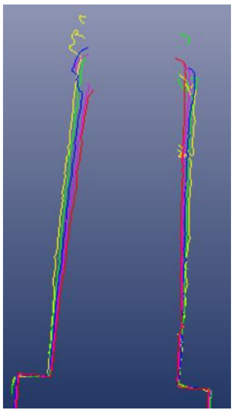


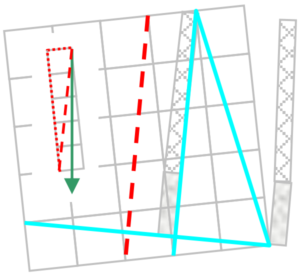
Geo coordinates: N:37.695997 E:22.769915
Dimension: 4.4 × 1.9 meter
Orientation: 69º (±0º) / 69º (±0º)
Masonry: hewn stone
Dating: LH-IIb/LH-IIIa

The stomion of the Prosymna tholos tomb is in an almost perfect condition. Only halfway both walls there is a little distortion, but this does not obscure the orientation of 69º.

Proportional dimension: 5×12
Geometrical orientation: E - ½[1:3] - [1:5]

Berbati





Geo coordinates: N:37.716615 E:22.801617
Dimension: 3.8 × 1.6 meter
Orientation: 186º (±1º) /184º (±1½)º
Masonry: hewn stone
Dating: LH-IIb/LH-IIIa

This tholos tomb is in a less well-preserved state. The orientation of the stomion walls start near the dromos with azimuth 185º but at a distance of about 1.2 meter they bend towards each other ending in orientations of 187º and 182½º near the chamber. Thus, the average orientation is about 185º. The geometrical orientation is constructed in the same way as Peristeria 2.

Proportional dimension: 5×12
Geometrical orientation: S + ½[1÷5]

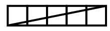
Conclusions about the stomion setups

Out of the 17 investigated stomion setups, 16 show an agreement between the geometrical orientation and the ratio of width and length. The only exception is the tomb of Kata Phournos. The stomion orientation deviates 3º from due west, which seems to be an accuracy problem. For the other tombs whose stomion ratio do match the geometrical setup, either the stomion ratio is identical to the trysquare diagonal that defines the orientation [25] or the stomion ratio arises from a triangle produced by that diagonal [26]. This can be seen as an endorsement of the hypothesis that tholos tombs got a layout according to the techniques of the grid-of-squares geometry and the practitioner diagram (see PTS-1.

It is not possible to distinguish specific geometrical setups by the masonry type of the stomion nor by a development in time. On the other hand, there does exist a regional distinction. Compared to the Messenian tombs and the other tombs in the Argolis, the ones in the vicinity of the Mycenaean acropolis stand out by their simple setup. Near Mycenae the ratio of the

[25] For example, a stomion ratio 2:3 matches the orientation of ½[1:2]+[2:3] (pattern: [grid] + [stomion]).
[26] For example, a stomion ratio 5:12 with orientation ½[1:5] arises from three .

stomion dimensions, the stomion orientation and the grid orientation can be understood within one and the same grid. Actually, in two situations the grid is orientated towards the cardinal points without any further setup. The Messenian and other Argolis tombs have the grid orientated by means of the bisection of a diagonal through adjacent squares.

Since the grid-of-squares geometry is used as early as the Neolithic, it is interesting to compare the Neolithic geometrical approach to the one applied in the tholos tombs. Because of the limited number of tombs in the comparation, it would be premature to come to real conclusions. Nevertheless, it seems as if the setups of the tholos tombs reflect a more thorough understanding of the practitioner geometry in comparison to those of the megalithic tombs. For example, amongst the Neolithic tombs no setups are found using  yet, although probably it was known in the Late Neolithic [27]. The ratio 7:24 as used in the Treasury of Atreus has neither been discovered in Neolithic nor in Bronze Age setups. Furthermore, the architect of the Treasury of Atreus merged together three different diagonal ratios (1:2, 3:4 and 7:24) in one geometrical composition of the chamber and the stomion. The layout of Neolithic tombs did not require such advanced setups.

In the list of the tholos tombs of appendix B some non-integer ratios appear rather often. Because of the repetition it may be clear that they have to do with the overall setup. Still they cannot be linked to the stomion setup without further investigation. For example, the case study of the Treasury of Atreus demonstrates how the ratio 0.37 between chamber diameter and stomion length comes to being via a setup by a .

The stomion of the Tomb of the Genii proves that not all setups follow a pure practitioner's approach with diagonals that can form integral right triangles. Here the diagonal through the stomion is based on the ratio 3:5. A practitioner diagram, which is built from such diagonals, will create a right triangle having sides 4×7½×8½ [28]. Maybe the architect had this triple in mind, although by its integral numbers 8×15×17. This triple exists in the list of the Baudhyana, verse 1.13 (around 800 BCE – see PTS 2, pg.2). Again, this can have been an advancement of the surveying methods amongst the Mycenaean architects.

[27] The rectangle by the so-called station stones of Stonehenge has a ratio 5×12.

[28] Diagonals with a ratio 3:5 produce the same Pythagorean triangle as diagonals through four adjacent squares (ratio 1:4). The practitioner diagram is created by diagonals through an odd number of adjacent squares. The sizes of the right triangle are found by the algorithm ascribed to Pythagoras ($w=n$, $h=½(n^2-1)$, $d=½(n^2+1)$, n is odd). Pythagorean triples based on an even number of adjacent squares are found by the algorithm of Plato ($w=2n$, $h=n^2-1$, $d=n^2+1$, n is even) – all according to Proclus.

Stomion	Width mtr	Length mtr	Orient. azim º	Ratio width**:**length	Orientation setup cardinal + grid + wall
Measurements taken from the 3D model					
Ano Englianos 3	1.6	2.4	46.5	2:3	N + ½[1:2] + [2:3]
Ano Englianos 4	2.25	4.6	39.0	12:25	N + ½[1:2] + [1:2]
Peristeria 1 [29]	2.35	5.5	28	5:12	N -½[1:5] + [2:3]
Peristeria 2 [30]	2.35	5.5	175	5:12	S - ½[1:5] + 0
Chalkias 1	1.0	2.5	127	2:5	E + [3:4] + 0
Tiryns	2.0	4.7	58.5	5:12	N + (½[1:2] + [2:3]) + [1:5]
Prosymna	1.7	4.1	69	5:12	E - ½[1:3] - [1:5]
Berbati	1.6	3.85	185	5:12	S + ½[1:5] + 0
Measurements taken from literature					
Treasury of Atreus	2.7	5.4	101 [31]	1:2	E + ½[1:2] + 0
Clytaemnestra	2.25 [32]	5.4	168	5:12	S + 0 - [1:5]
Aegisthus	2.35	5.6 [33]	197	5:12	S + [1:5] + [1:5]
Genii	2.0	3.4	301	3:5	W + [3:5] + 0
Lion tomb	2.55	5.15 [34]	334	1:2	N - [1:2] + 0
Cyclopean	1.4	3.4	280	5:12	W + 0 + [1:5]
Epano Phournos	2.0	5.0	185.5	5:12	S + ½[1:5] + 0
Kata Phournos	2.0	4.0	273	1:2	??
Panagia	2.0	3.0	254	2:3	W - ½[2:3] + 0
Measurements taken from the excavation plan					
Kakovatos A	2.3	4.8	193	12:25	E + ½[1:2] + 0

<u>Table 1</u>
Comparison between stomion dimension and orientation.

The tholos tomb of Berbati as an example:

The stomion ratio is 1.6 ÷ 3.85 = 0.42, which agrees with a ratio of 5:12.
A right triangle 5×12×13 is produced by three diagonals [1:5].
A diagonal ½[1:5] gives an angle of 5.7º. The orientation of 185º agrees with 180º + 5.7º.

[29] Due to bad light conditions and to time pressure the 3D models of Peristeria 1 partly failed. Therefore, the dimensions are taken from literature.
[30] Peristeria 2 is in a bad condition. Because the eastern wall got disturbed a lot near the chamber, the stomion width is taken at the dromos end.
[31] This orientation differs 2º from the orientation setup E + ½[1:2] ≈ 103¼º. See the discussion earlier in this paper.
[32] The stomion dimensions of the Tomb of Clytaemnestra in Dirlik (2012) do not agree with the ratio of 5:12 in the 3D model. See the discussion earlier in this paper.
[33] Length and width according to Kamm 2002, pg.42. Kamm gives the dimensions within a 1 cm accuracy. Since the chamber side walls are made of hewn stone, accuracy better than 5 cm is unrealistic.
[34] Length and width according to Kamm 2002, pg.61.

Appendix A, Analysis of 54 stomion dimensions

The lengths and widths of the stomions are taken from Dirlik (2012), except for the values in parenthesis. These are taken from the 3D model directly. Those of the tomb of Clytemnestra are calculated proportionally from the 3D model using the (probably) correct diameter in Dirlik.

When a ratio is close to an integer ratio this is written in brackets behind it. Close means a fault equal or less than 0.05 meter. For the column 'Stomion Ratio W/L' the faults are listed in the column 'Fault W/L'. They are calculated as the integer ratio multiplied by the stomion length compared to the stomion width. For the column 'Tomb Ratio L/D' the fault is calculated as the integer ratio multiplied by the dome diameter compared to the stomion length. This result is not displayed in a table column.

Sometimes two values are listed for the stomion width or length. If the difference is less than 0.10 meter, both values are averaged to calculate the stomion ratio. Otherwise for both values an average is given.

All measures are in meters.

Tholos tomb	Chamber Diameter	Stomion Width	Stomion Length	Tomb Ratio L/D	Stomion Ratio W/L	Fault W/L
Peristeria 1	12.10	2.33	5.50	0.45	0.42 [5:12]	0.04
Peristeria 2	10.50	2.35	5.50	0.52	0.43 [5:12]	0.06 [35]
	10.60	2.05			0.37	-
Volos	10.0	2.30	5.50	0.55	0.42 [5:12]	0.00
		1.60			0.29	-
Clytemnestra	13.40	(2.2)	(5.3)	0.40 [2:5]	0.42 [5:12]	0.01
Dendra	7.30	2.0	4.7	0.64	0.43 [5:12]	0.04
Tiryns	8.50	2.0	4.7	0.55	0.43 [5:12]	0.04
Vaphio	10.15	1.93	4.56	0.44	0.42 [5:12]	0.03
	10.35					
Prosymna	9.50	(1.7)	(4.1)	0.43 [5:12]	0.41 [5:12]	0.01
Berbati	8	(1.60)	(3.85)	0.48 [12:25]	0.42 [5:12]	0.00
Cyclopean	8	1.4	3.4	0.43 [5:12]	0.41 [5:12]	0.02
Akona 1	6.20	1.32	2.40	0.39 [2:5]	0.55	-
		1.00			0.42 [5:12]	0.00
Atreus	14.50	2.7	5.4	0.37	0.50 [1:2]	0.00
Orchomenos	14.0	2.70	5.30	0.38	0.51 [1:2]	0.05
Ano Englianos 4	9.35	2.26	~~4.62~~	0.49 [12:25]	0.49 [1:2]	0.04
		2.22	(4.57)			
Kato Phournos	10	2.0	4.0	0.40 [2:5]	0.50 [1:2]	0.00
Dimini B	8.30	1.60	3.25	0.39 [2:5]	0.49 [1:2]	0.03
Kazarma	7.20	1.7	3.0	0.42 [5:12]	0.57	-
		1.5			0.50 [1:2]	0.00
Tragana	8.50	1.45	2.80	0.33 [1:3]	0.52 [1:2]	0.05
Malthi 2	5.75	1.30	2.60	0.45	0.50 [1:2]	0.00
Voidikilia	4.80	1.25	2.35	0.50 [1:2]	0.52 [1:2]	0.05
	5.03		2.45	0.48 [12:25]		
Phytes 1	6	1.20	2.40	0.40 [2:5]	0.50 [1:2]	0.00
Akona 2	5.40	1.20	2.30	0.43 [5:12]	0.52 [1:2]	0.05
Agioi Thedoroi	3.54	0.84	0.40	0.11	2.10 [2:1]	0.04
Panagia	8	2.0	3.0	0.38	0.67 [2:3]	0.00
Ano Englianos 3		(1.6)	(2.4)		0.67 [2:3]	0.00

[35] From the geometrical interpretation it becomes clear that the ratio [5:12] can be accepted despite the 0.06 meters fault.

Routsi 1	5.50	1.5	2.3	0.42 [5:12]	0.65 [2:3]	0.03
	5.0			0.46		
Palaiochoria	3.40	1.00	1.45	0.41 [5:12]	0.66 [2:3]	0.02
	3.60	0.90				
Epano Phournos	11	2.0	5.0	0.45	0.40 [2:5]	0.00
Chalkias 1	(4.2)	(1.0)	(2.5)	0.60 [3:5]	0.40 [2:5]	0.00
Papoulias C	4.40	2.0	0.8	0.18 [1:5]	2.50 [5:2]	0.00
Katarraktis A	3.90	1.05	0.40	0.10	2.63 [5:2]	0.05
Genii	8.4	2.0	3.4	0.40 [2:5]	0.59 [3:5]	0.04
Marathon	7.0	1.60	2.60	0.37	0.61 [3:5]	0.04
Lividithi	4.62	1.09	1.74	0.38	0.63	-
		1.04			0.60 [3:5]	0.00
Kakovatos A	12.12	2.35	4.82	0.40 [2:5]	0.48 [12:25]	0.01
		2.25				
Kokla	5.40	1.25	2.60	0.48 [12:25]	0.48 [12:25]	0.00
Mouriathada	4.80	1.05	1.80	0.38	0.58	-
		0.85			0.47 [12:25]	0.01
Aeghistus	13	2.0	5.8	0.45	0.34 [1:3]	0.07 [36]
Kopanaki B	5.35	1.20	3.50	0.66	0.33 [1:3]	0.02
		1.10				
Vasiliko 1	6.50	0.92	2.75	0.42 [5:12]	0.33 [1:3]	0.00
		0.52			0.19	-
Katakalou	5.60	0.80	3.30	0.59 [3:5]	0.24 [1:4]	0.01
Chalkias Kroik.	4.0	2.30	1.75	0.44	1.31 [4:3]	0.02
Lion tomb	14	2.7	5.0	0.36	0.54	-
Analipsis	8.65	1.05	3.40	0.39 [2:5]	0.31	-
Thorikos B	9.25	1.80	3.45	0.37	0.52	-
Menidi	8.35	1.55	3.35	0.40 [2:5]	0.46	-
Kato Englianos	7.66	1.65	3.00	0.39 [2:5]	0.55	-
	7.71					
Arkines	4.70	0.78	2.80	0.60 [3:5]	0.28	-
Kambos	8.50	1.64	3.09	0.36	0.53	-
Dimini A	8.50	1.50	2.85	0.34 [1:3]	0.53	-
Osmanaga	6	1.50	1.85	0.31	0.81	-
Peristeria south	5.08	2.70	1.00	0.20 [1:5]	0.37	-
Malthi 1	6.85	1.60	1.00	0.15	0.63	-
Vlachopoulo	3.10	0.83	1.00	0.32 [1:3]	0.83	-

Number of occurrences in the table per ratio

Ratio	L/D	W/L	Ratio	L/D	W/L
[1:2]	1	11	[4:3]	0	1
[1:3]	3	3	[5:12]	7	11
[1:4]	0	1	[12:2]	4	3
[2:3]	0	4	0.37 ±0.01	9	1
[2:5]	10	4	0.45 ±0.01	7	1
[3:5]	3	3	0.54 ±0.01	2	4

[36] From the geometrical interpretation it becomes clear that the ratio [1:3] can be accepted despite the 0.07 meters fault. See text.

Prehistoric tomb surveying (10) Typification of Neolithic tomb mounds in Germany

Dirk Kruithof, September 2019

The current paper makes part of a broader study through which the existence of a prehistoric geometry was tracked down. Different stages of tomb setups found at consecutive times and in various areas in Europe form the steppingstones towards the chronicled geometry in ancient mathematical texts. Where the other papers deal with the geometrical setup of tomb chambers, this paper attends to the tomb mounds. Through the atlases of Sprockhoff many mound contours are passed down to us, of which the single tomb mounds are selected but not the Langbetten. Most of them are elliptical and the current paper tries to understand their orientation and shape by means of a grid-of-squares geometry (see PTS-1 [1]).

The mounds of D30 and D40

While studying the mutual orientations of D38, D39 and D40 (PTS-4) it occurred that the longitudinal axis of the chamber of D40 did not run in the same direction as the axis of its mound. This was a nice occasion to test if a mound could be involved in the overall orientation. The axis of the mound should match the geometrical principles of the chamber in that case. During his excavation of 1918 Van Giffen created a detailed plan of the mound including altitude lines. Near the centre, there has been quite a lot of digging, but towards the foot the altitude lines become elliptical. Only in the northwest the foot of the mound flares out a little. When one of the altitude lines is traced by a real ellipse, its central axis runs at azimuth 117º from a marking at side stone Z2' through a narrow interstice between side stone Z2 and end stone S2. This sightline is one of the lines that make up the mutual orientation between hunebedden D39 and D40. It can be set up by means of a diagonal through two adjacent squares. Diagonals through two adjacent squares form the basic principle in the geometrical setup of D40 (PTS-4). So, it appears that the setup of the mound fits the geometrical orientation of the chamber indeed.

Van Giffen delivered yet another detailed plan of a mound: that of hunebed D30, excavated in 1918 also. Although the eastern side has been disturbed heavily, again an ellipse can

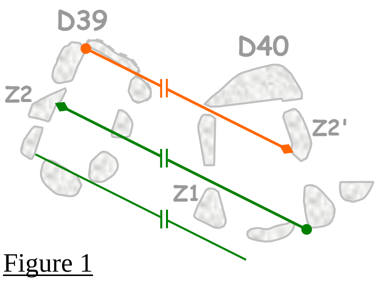


Figure 1
The mutual orientation of hunebedden D39 and D40 by means of parallel sightlines. The diamond line ending reflects a mark on a stone Z2'. The orange sightline overlaps the centre line of the mound ellipse.

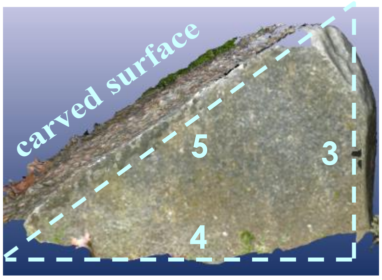


Figure 2
The left entrance stone of hunebed D30, numbered Z2' by Van Giffen. It is the only Dutch support having a carved upper surface. Since the surface runs at an inclination of 37º, a right triangle with sides 3×4×5 evolves.

[1] Reverences to the other papers of this study are abbreviated 'PTS-n', where PTS stands for the study title 'Prehistoric tomb surveying' and 'n' for the serial number of the paper.

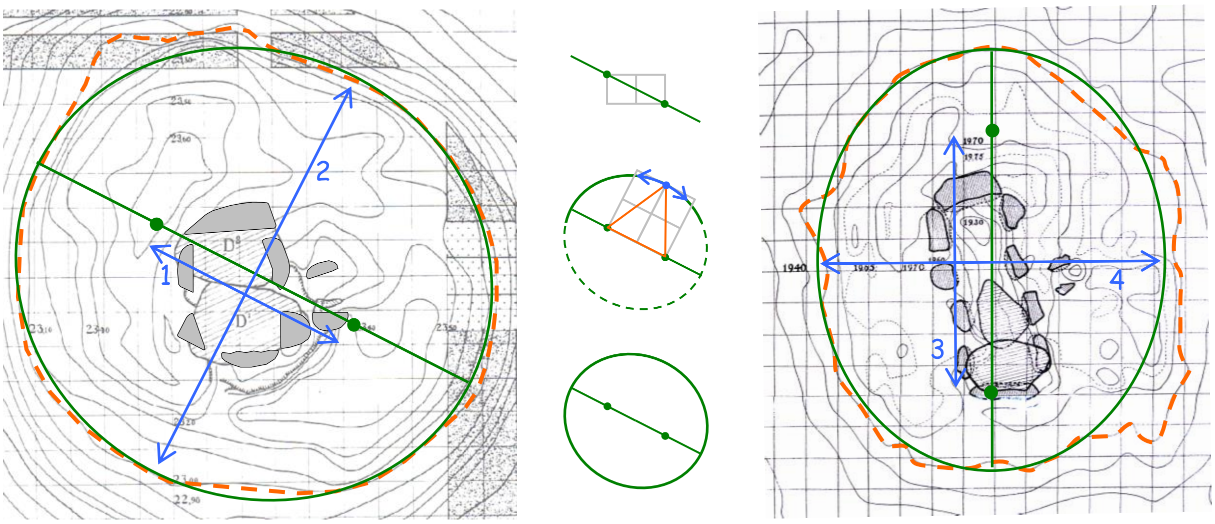


Figure 3
Hunebedden D30 and D40 have elliptical mounds based on the proportion 3:4 and 1:2 respectively.
Left: D40, *Right*: D30. Altitude lines are produced by Van Giffen (1925/27, Atlas, plate 127 and 134).
Middle: Geometrical construction of the mound ellipse of hunebed D40.

be drawn along one of the altitude lines. Here the central axis runs north-south and forms a diagonal through a field of 3×4 squares. This proportion recalls the Pythagorean triangle, which is found in the shape of the left entrance stone (figure 2). The longitudinal axis of the tomb runs at 167° more or less and thus - like D40 - the chamber and the mound axes have a different azimuth.

The mounds near the Baltic Sea

Although most of the Dutch hunebedden have been surrounded by a mound once, nowadays we find remnants mainly. By the end of the 19th century many mounds were removed, since it was believed that the mound did not belong to the original monument. Even van Giffen, who valued the mounds correctly, did not draw the remnants in his atlas. How differently is the situation in Germany. Here many mounds survived the millennia and Sprockhoff had started a similar atlas project for the German megalithic tombs, and he did record the outlines of the mounds. The current paper restricts itself to the mound drawings in the atlases of Schleswig-Holstein and Mecklenburg-Vorpommern and Brandenburg. The atlases were published in 1967. Sprockhoff rendered the situation as he encountered it, as well as his view on the original state. Figure 4 shows the contours of the mound of tomb 422. The current and 'original' situation of the mound are projected over each other. Their shapes vary a few decimetres from each other at some spots. Although this is rather much, the image of the elongated egg shape reveals itself in both contours very well.

Helmut Swieger, who worked as a draftsman for Sprockhoff, created good quality ground plans. This was revealed in 2016 when his drawing of *die Großen Sloopsteene* was compared to a modern orthographic top view. Under the auspices of the Altertumskommission für Westfalen a 3D model of the megalithic tomb was created via photogrammetry and a drone. An orthographic overlay over the drawing of Schwieger [2] demonstrates that the drawings come very close to the real situation. A few anomalies of about 10 cm should be imputed to disturbances probably rather than to imperfection.

[2] Schierhold 2016, abb.9

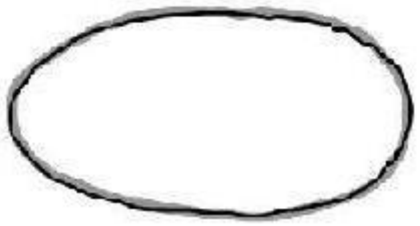


Figure 4
Contours of the mound of tomb 422 as in the atlas of Sprockhoff. The actual and idealized contours are projected over each other.

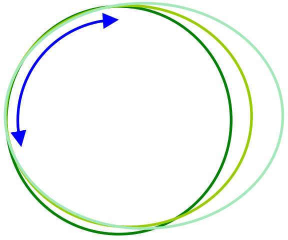


Figure 5
A circle, a 1:2-ratio ellipse and a 3:4-ratio ellipse projected over each other. In the quarter segment of the blue arrow they cannot be distinguished.

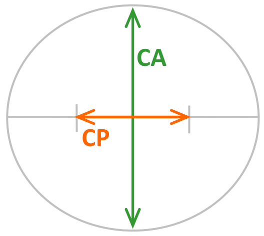


Figure 6
An ellipse is typified by the ratio between the centre points (CP) and the size of the cross-axis (CA).

Even though the accuracy of the ground plans may be very good, the mound shapes involved give more problems to typify them. Many mounds are built of segments of ellipses and for the various types they can be almost identical (figure 5). Only when a mound consists of a singular ellipse or circle, it satisfies to typify it on the basis of the shape purely. With compound forms, the segments are typified based on the position in and the orientation of the overall mound shape, which leaves room for different interpretations. Yet it fulfils the needs of this study. It will not be discussed if mound forms *should be* but instead *can be* described in terms of a grid of squares geometry. In the other papers it is demonstrated how many tomb plans fit to such a geometry and it is expected and approved that the mounds do too.

All mound shapes can be described by a circle, an ellipse or a combination. The involved ellipses distinguish themselves by the ratio of the distance between the centre points CP and the cross-axis CA (figure 6). This ratio can be CP:CA = 1:2, CP:CA = 1:3, CP:CA = 2:1 and CP:CA = 3:4. In total 85 mounds from the atlas have been typified. Only two mounds were

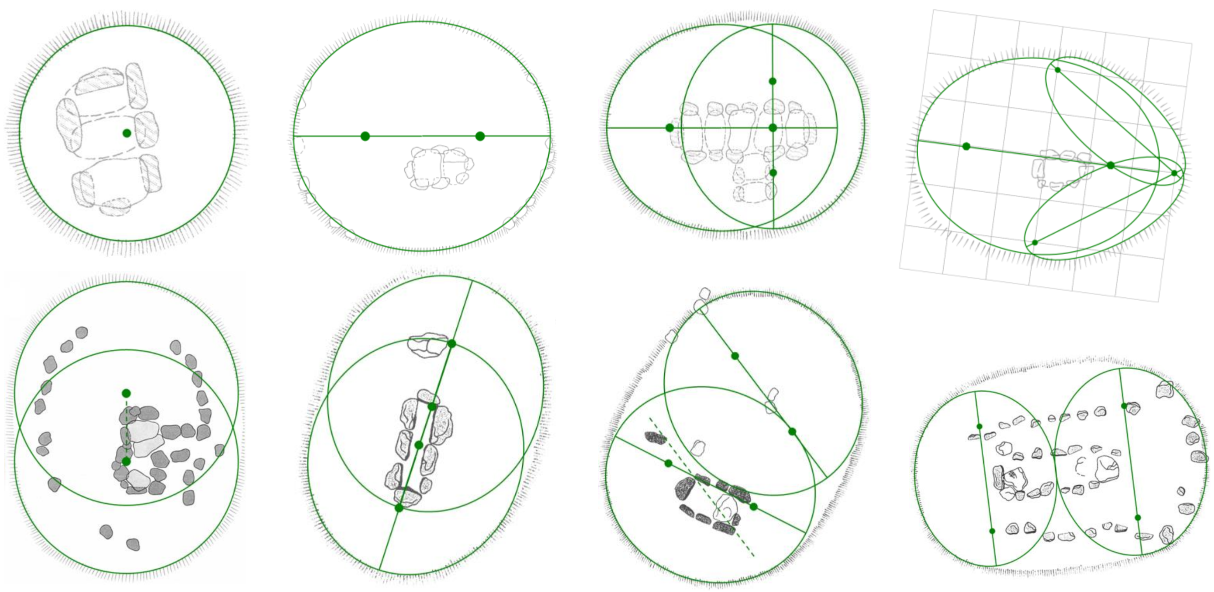


Figure 6
There exist eight main types amongst the Neolithic tomb mounds: circle, double circle, ellipse, elongated double ellipse, perpendicular double ellipse, double ellipse at an angle, parallel double ellipse and a complex type consisting of three ellipses.

disregarded because of the disturbances. Many mounds, 36 in total, show a simple setup containing just one single shape. They can be divided in 15 circles, 15 ellipses 1:2, 2 ellipses 1:3 and 4 ellipses 3:4. Six mounds have an egg shape, which is produced by one main ellipse and two additional small ellipses 2:1. Yet another three do have a comparable elongated shape, but can not be typified as egg shape. Only their main ellipses can be typified, while the elongation does not link to any composition of ellipse segments. Most other mounds can be constructed by means of two ellipses of the same type that follow on from each other, that lie perpendicular to each other or that have a specific orientation concerning the grid of squares geometry. Their angles are produced by diagonals through two (26½º) or three (18½º) adjacent squares, through a field of 3×4 squares (36½º) or by the bisection or doubling of these angles (13¼º, 6¾º, 9¼º, 53¼º). A special case is the orientation at 43º [3].

Ellipses with a ratio 1:2 and 3:4 occur in merely all groups below. Strikingly in Schleswig- Holstein all ellipses have a ratio 1:2, except for the egg-shaped ellipses. Some atypical mound setups are found in the vicinity of each other. Sprockhoff 134 and 135 have an egg shape where the axis of the main ellipses stands perpendicular to the mound axis. Both setups copy each other but have a different orientation. Near Gowens we find two mounds containing a horseshoe created of stones. The horseshoes are copies, but the mounds differ. Between Deutch Nienhof and Mielkendorf the ellipses have their axis orientated east-west. For the group of mounds near Hemmelmark the mound orientations seem to diverge +3º from the ideal orientation because of the setup. Yet two orientated dolmens do not show this deviation (PTS-7, pg.2 and pg.7). In the southern part of the island Rügen we find a lot of mound circles. Finally, there is a concentration of ellipses 3:4 in the coastal area of Wismar, where a lot of lakes are found. Apparently, there have been some preferences per region apart from the standard setups. Furthermore, the egg shapes of Sprockhoff 134 and 135 can have been the work of one and the same surveyor very well, as the horseshoes near Gowens can. These shapes must have evolved from the same geometrical setup after all, or they would never have been this identical.

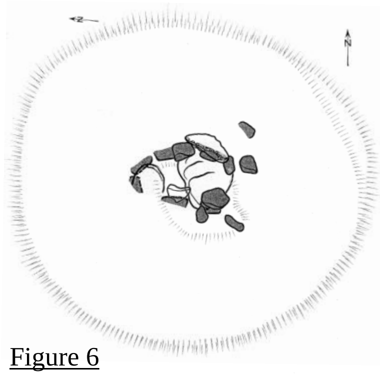


Figure 6
The plans of Sprockhoff 134 and 135 plotted over each other. There is a little aberrance at the right part of the mounds only.

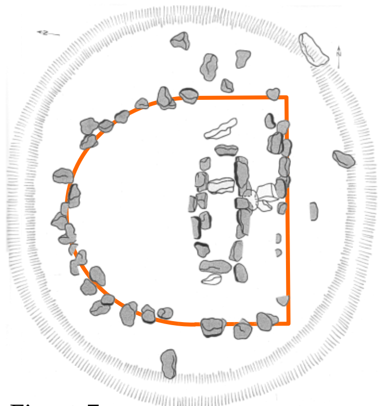


Figure 7
The horseshoe stones of the tombs near Gowens plotted over each other.

A few 'incomplete' typified mounds exist. At one side they fit to an ellipse, while they seem to be elongated at the other side. Furthermore Sprockhoff 424 stands on its own as it has a bent rectangular mound. Yet the mound can be associated with an ellipse. Except for the ’Langbetten’, this is the only rectangular mound in the atlases of Sprockhoff.

Next follows the complete list of mounds from Sprockhoff's atlases of Schleswig-Holstein and Mecklenburg-Vorpommern and Brandenburg. The mounds are listed according to their numbers in the atlases.

[3] It is found in the other papers of this study too and is created from a diagonal through a field of squares (2×3) combined with the bisection of diagonal through two adjacent squares.

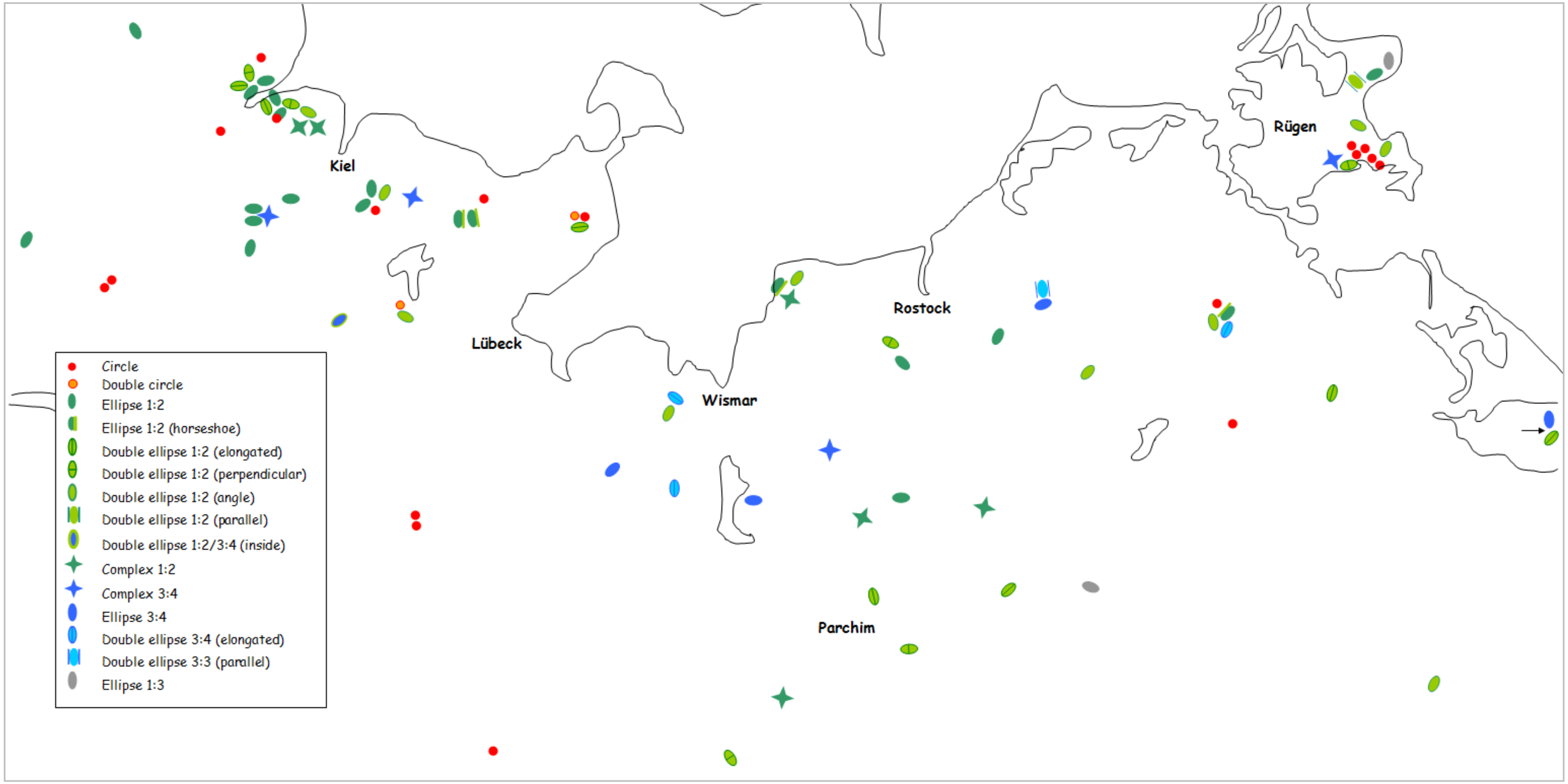
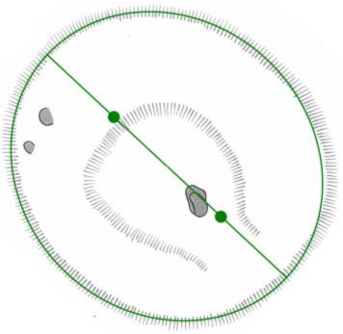


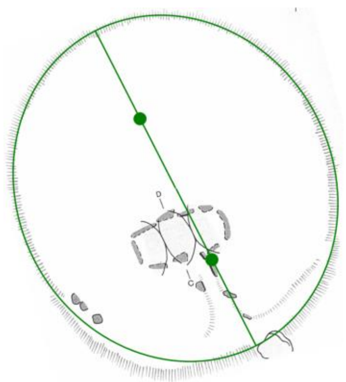
Figure 8
Map of the German Baltic coast with the Neolithic tomb mounds from Sprockhoff's atlases of Schleswig-Holstein and Mecklenburg-Vorpommern plotted in it.

List of typified mounds

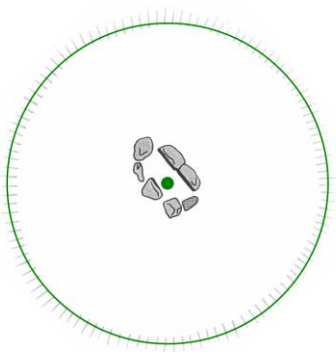
__Megalith mounds of Schleswich-Holstein__



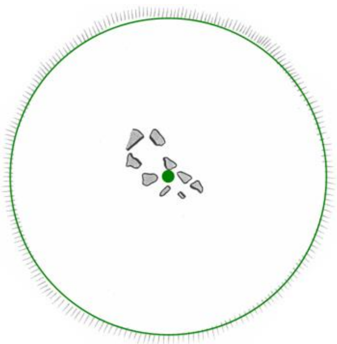
Sprockhoff 28
Munkwohlstrup
Ellipse 1:2
Orientation 90º+43º



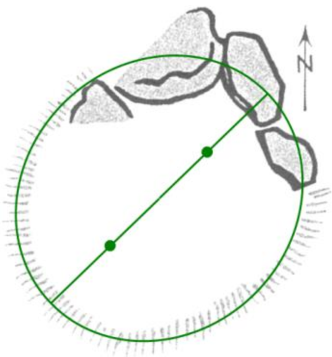
Sprockhoff 43
Idstedt
Ellipse 1:2
Orientation 180º-26½º



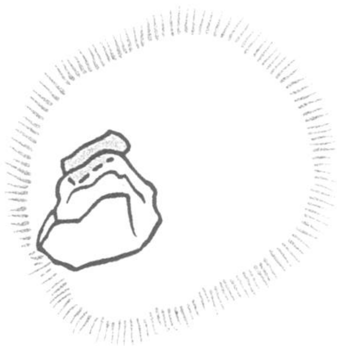
Sprockhoff 59
Holzdorf
Circle



Sprockhoff 79
Groß Wittensee
Circle

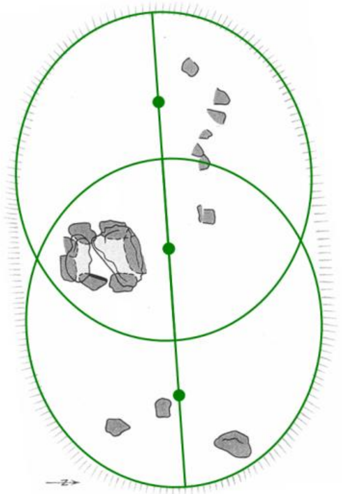


Sprockhoff 83
Hemmelmark
Ellipse 1:2
Orientation 46º
(diverges +3º)

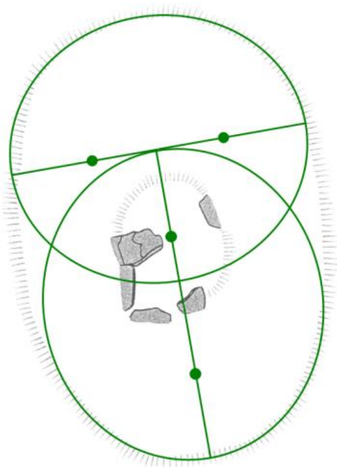


Sprockhoff 84
Hemmelmark
Disturbed

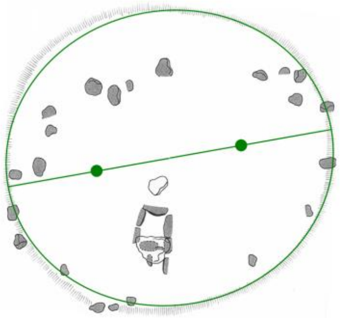
Sprockhoff number
Nearest village
Mound type
Orientation construction



Sprockhoff 85
Hemmelmark
Ellipse 1:2 / 1:2
Orientation 86º / bis
(diverges +3º)

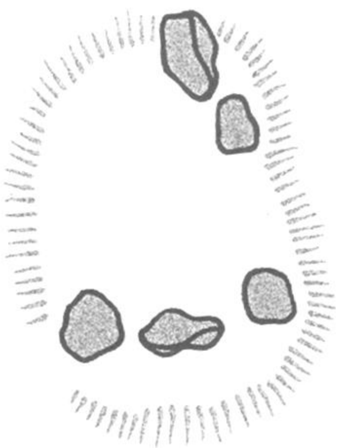


Sprockhoff 88
Hemmelmark
Ellipse 1:2 / 1:2
Orientation 80º / 90º+80º
(diverges +3º)

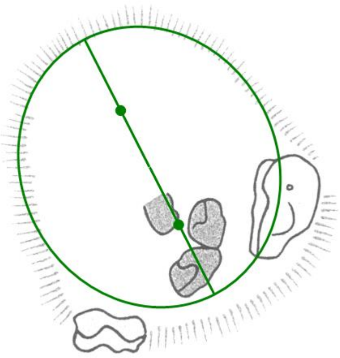


Sprockhoff 91
Hemmelmark
Ellipse 1:2
Orientation 80º
(diverges +3º)

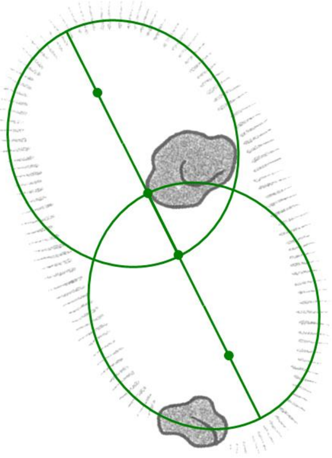
The mound orientation is problematic. Here the dolmen has a main orientation of about 185º, but measurements by the author gave about 156º for it.



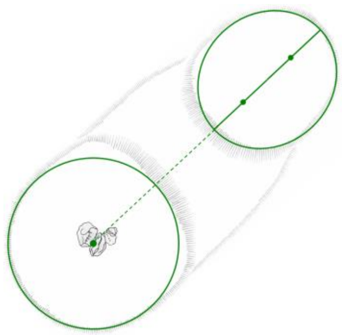
Sprockhoff 106
Lindhöft
Disturbed



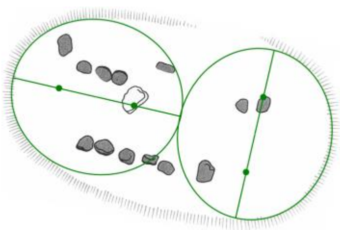
Sprockhoff 108
Lindhöft
Ellipse 1:2
Orientation 180º-26½º



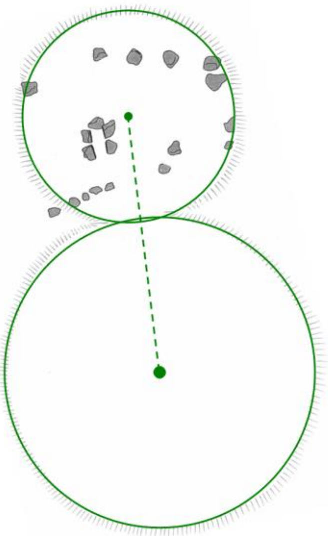
Sprockhoff 109
Lindhöft
Ellipse 1:2 / 1:2
Orient. 180º-26½º / bis



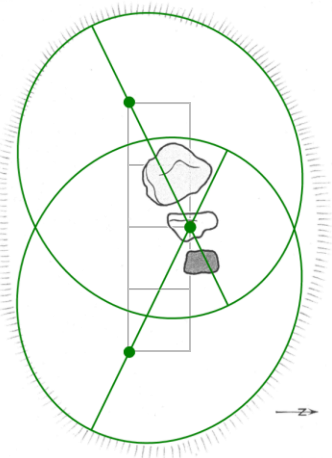
Sprockhoff 110
Lindhöft
Circle / Ellipse 1:2
Orientation 90º-43º



Sprockhoff 113
Noer
Ellipse 1:2 / 1:2
Orient. 90º+13¼º / 13¼º

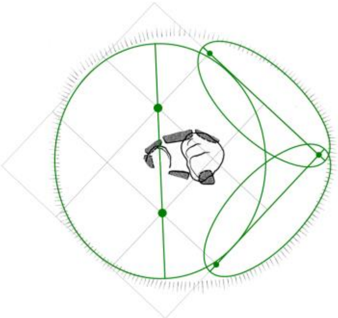


Sprockhoff 116
Grönwold
Circle / Circle
Orientation 180º-6¾º

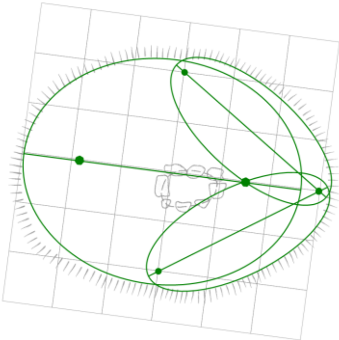


Sprockhoff 124
Birkenmoor
Ellipse 1:2 / 1:2
Orientation 90º-26½º /
90º+26½º

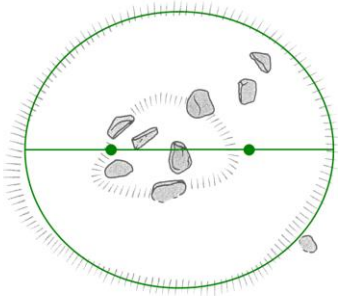
The orientation of the ellipses can easily be constructed in a grid of squares.



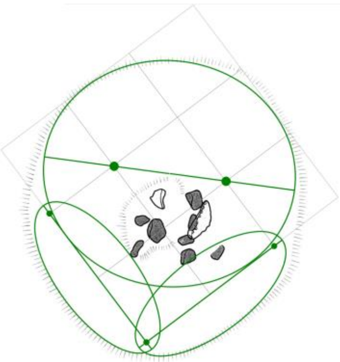
Sprockhoff 134
Kaltenhof
Ellipse 1:2 / 2:1 / 2:1
Grid orientation 43º



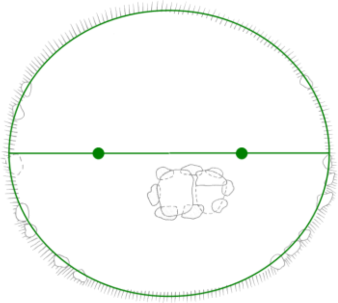
Sprockhoff 164
Deutch Nienhof
Ellipse 3:4 / 2:1 / 2:1
Grid orientation 6¾º



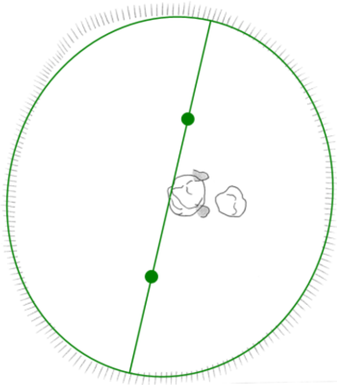
Sprockhoff 169
Mielkendorf
Ellipse 1:2
Orientation 90º



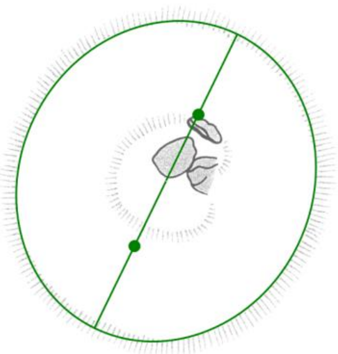
Sprockhoff 135
Borghorsterhütten
Ellipse 1:2 / 2:1 / 2:1
Grid orientation 53¼º



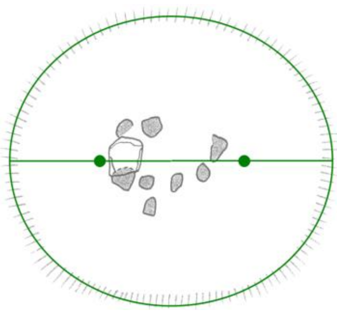
Sprockhoff 165
Deutch Nienhof
Ellipse 1:2
Orientation 90º



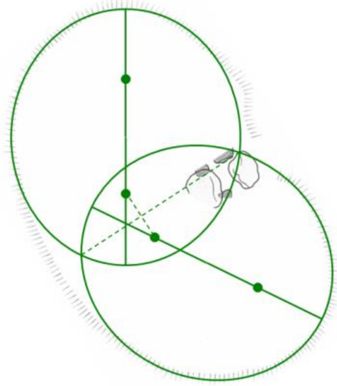
Sprockhoff 173
Eisendorf
Ellipse 1:2
Orientation 13¼º



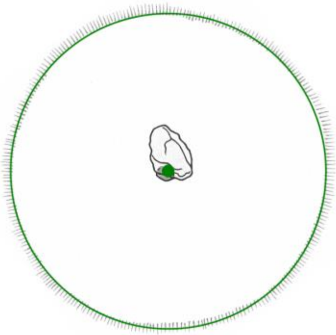
Sprockhoff 140
Süderholm
Ellipse 1:2
Orientation 26½º



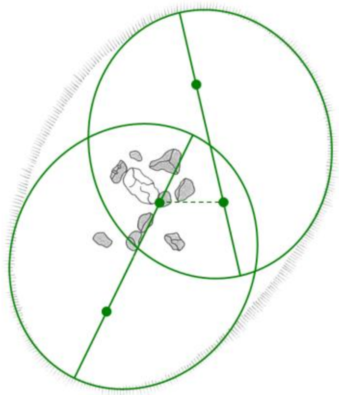
Sprockhoff 168
Deutch Nienhof
Ellipse 1:2
Orientation 90º



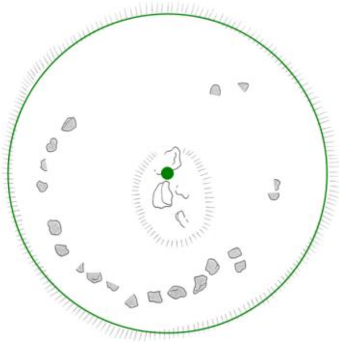
Sprockhoff 176
Seedorf
Ellipse 1:2 / 1:2
Orientation 0º / 90º+26½º



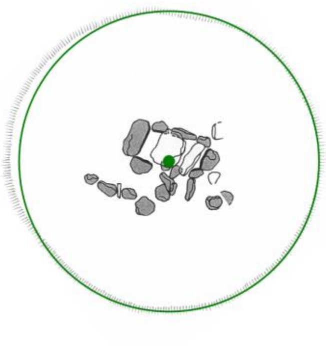
Sprockhoff 177
Thaden
Circle



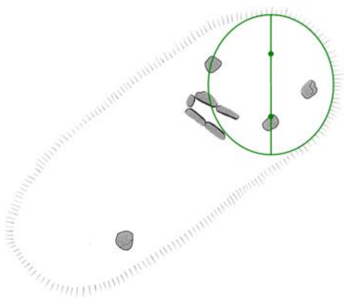
Sprockhoff 196
Hoheneichen
Ellipse 1:2 / 1:2
Orient. 180º-13¼º / 26½º



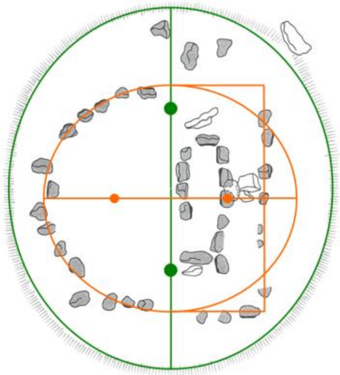
Sprockhoff 208
Blekendorf
Circle



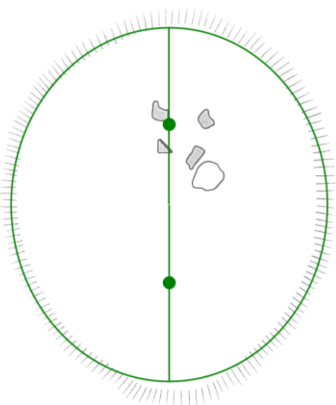
Sprockhoff 178
Thaden
Circle



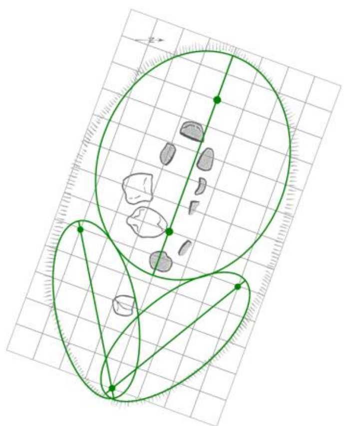
Sprockhoff 196
Rastorf
Ellipse 1:2
Orientation 0º



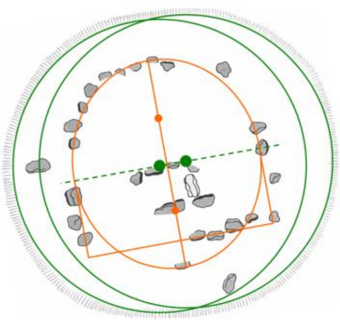
Sprockhoff 210
Gowens
Horseshoe 1:2 in
Ellipse 1:2
Orientation 90º in 0º



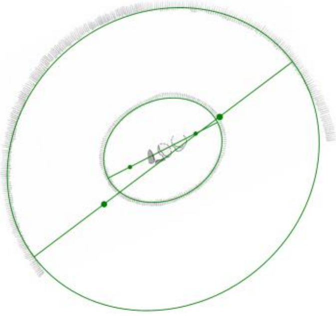
Sprockhoff 193
Liliental
Ellipse 1:2
Orientation 0º



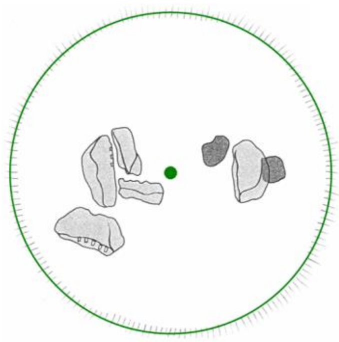
Sprockhoff 200
Lammershage
Ellipse 3:4 / 2:1 / 2:1
Grid orientation 110º



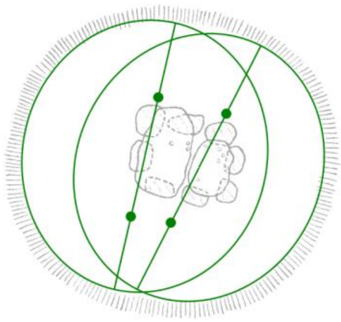
Sprockhoff 211
Gowens
Horseshoe 1:2 in
double circle
Orientation 170º in 80º



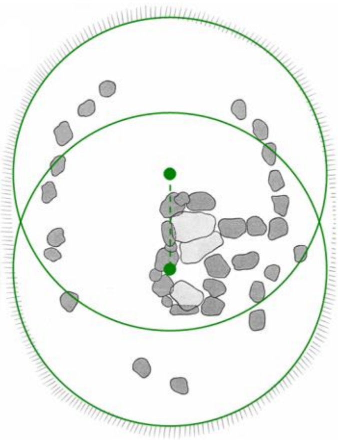
Sprockhoff 230
Gönnebek
Ellipse 3:4 in 1:2
Orient. 90º-26½º in 53¼º



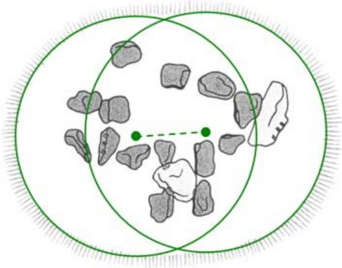
Sprockhoff 266
Lütjenbrode
Circle



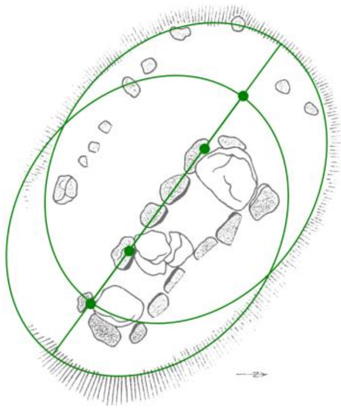
Sprockhoff 312
Everstorf
Ellipse 1:2 / 1:2
Orientation 13¼º / 26½º



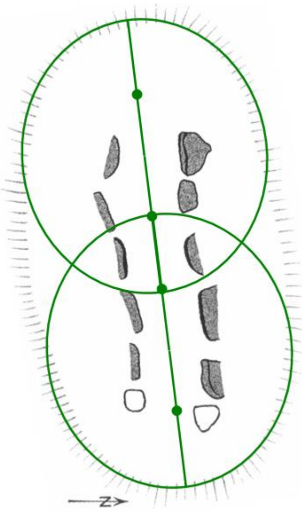
Sprockhoff 234
Damsdorf
Double circle
Orientation 0º



Sprockhoff 267
Lütjenbrode
Double circle
Orientation 86º

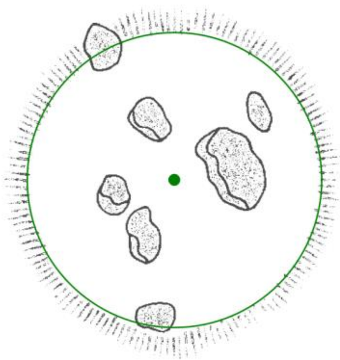


Sprockhoff 314
Everstorf
Ellipse 3:4 / 3:4
Orient. 90º+36½º / bis

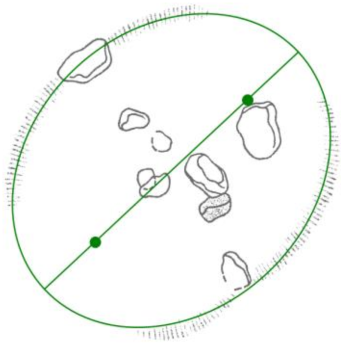


Sprockhoff 265
Lütjenbrode
Ellipse 1:2 / 1:2
Orientation 83º / bis

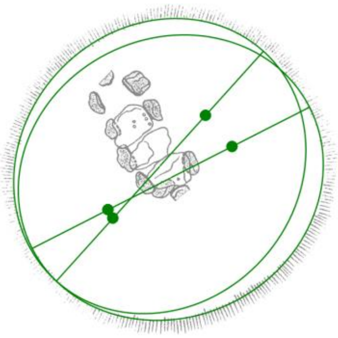
<u>Megalith mounds of Mecklenburg-Vorpommern</u>



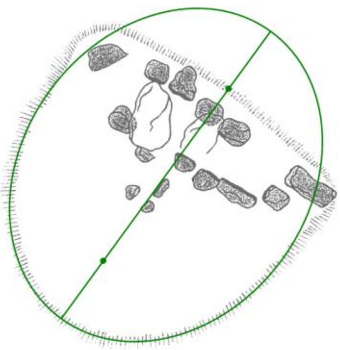
Sprockhoff 305
Barendorf
Circle



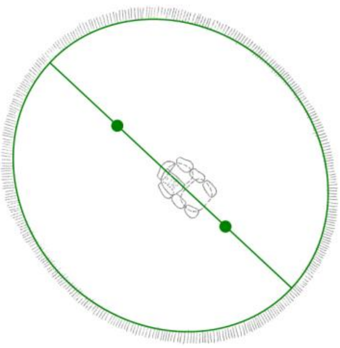
Sprockhoff 316
Klein Hundorf
Ellipse 3:4
Orientation 90º-43º



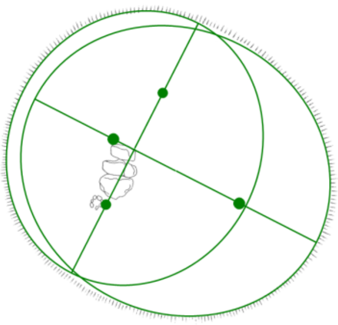
Sprockhoff 323
Mechelsdorf
Ellipse 1:2 / 1:2
Orient. 36½º / 90º-26½º



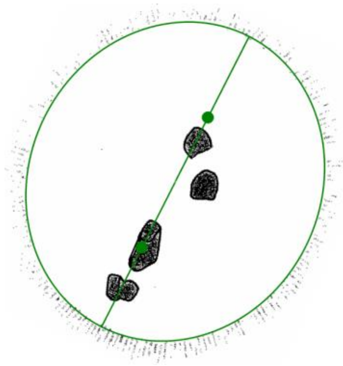
Sprockhoff 327
Gaarzerhof
Ellipse 3:4
Orientation 36½º



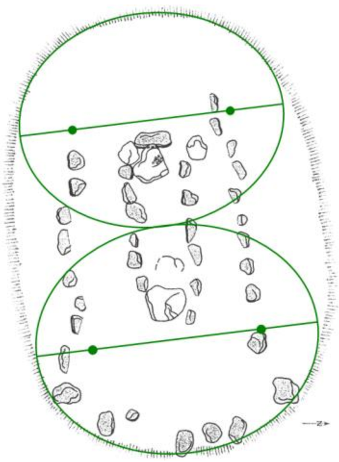
Sprockhoff 340
Hütterwohld
Ellipse 1:2
Orientation 90º+43º



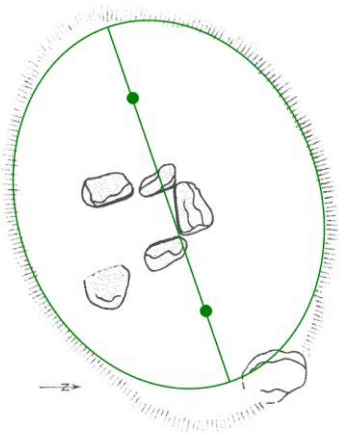
Sprockhoff 341
Ziesendorf
Ellipse 1:2 / 1:2
Orient. 26½º / 90º+26½º



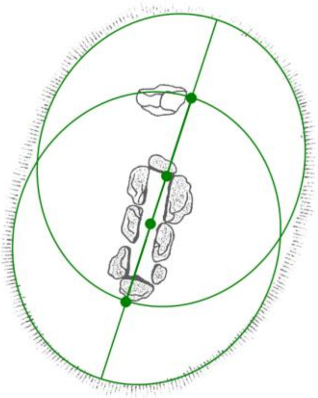
Sprockhoff 349
Cammin
Ellipse 1:2
Orientation 26½º



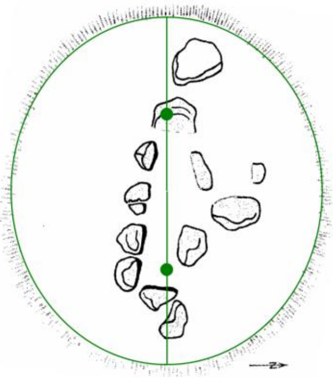
Sprockhoff 350
Gnewitz
Ellipse 3:4 / 3:4
Orientation 180º-6¾º / bis



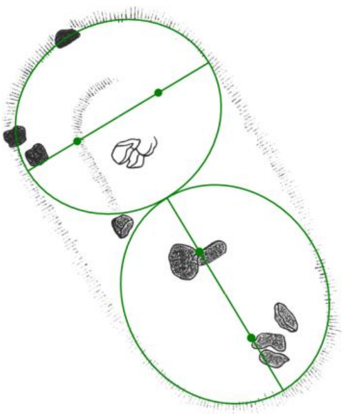
Sprockhoff 352
Gnewitz
Ellipse 3:4
Orientation 180º-18º



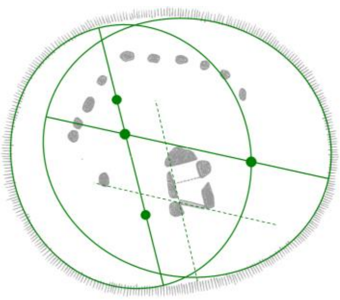
Sprockhoff 353
Liepen
Ellipse 1:2
Orientation 18º



Sprockhoff 375
Forst Tarnow
Ellipse 1:2
Orientation 90º

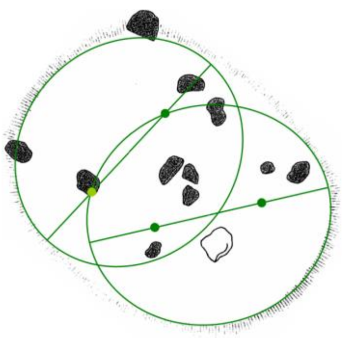


Sprockhoff 378
Lähnwitz
Ellipse 1:2 / 1:2
Orientation 59º / 149º

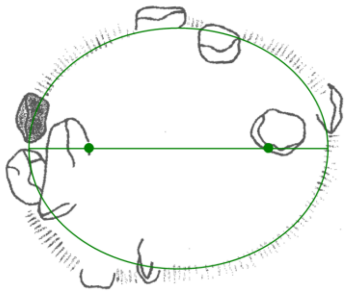


Sprockhoff 384
Serrahn
Ellipse 1:2 / 1:2
Orientation 90º+26½º /
180º-13¼º

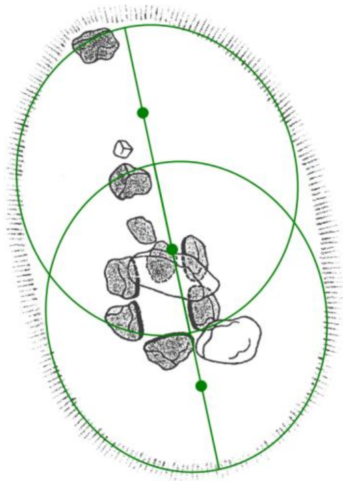
The geometrics of the dolmen is widely discussed in PTS-1 and PTS-7.



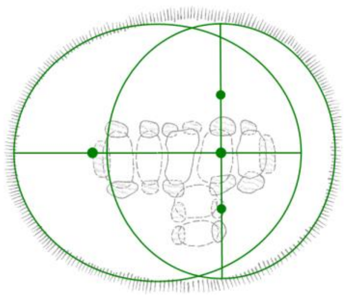
Sprockhoff 393
Remlin
Ellipse 1:2 / 1:2
Orientation 43º / 90º-13¼º



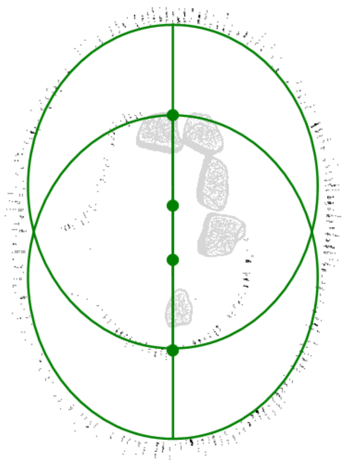
Sprockhoff 401
Kleefeld
Ellipse 3:4
Orientation 90º



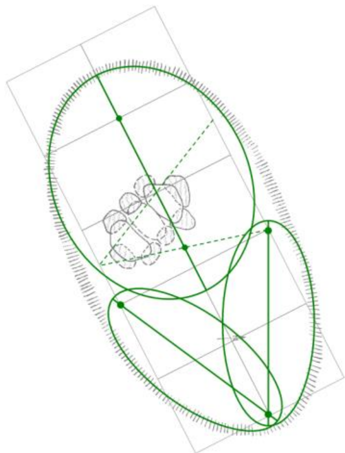
Sprockhoff 417
Forst Sandkrug
Ellipse 1:2 / 1:2
Orient. 180º-13¼º / bis



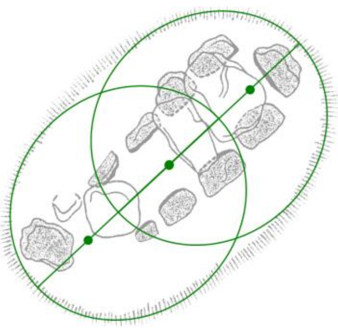
Sprockhoff 418
Wilsen bei Lübz
Ellipse 1:2 / 1:2
Orientation 90º / 180º



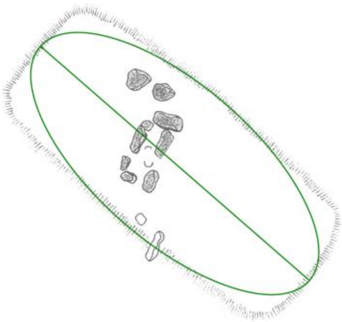
Sprockhoff 420
Camon
Ellipse 3:4 / 3:4
Orientation 0º / bis



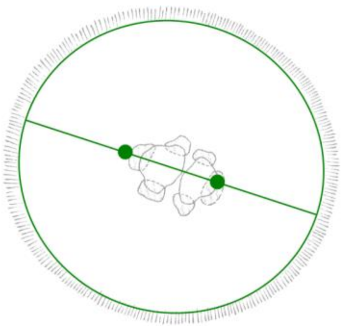
Sprockhoff 422
Moltzow
Ellipse 3:4 / 2:1 / 2:1
Grid orientation 90º-26½º



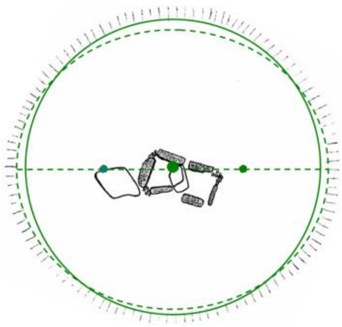
Sprockhoff 423
Sparow
Ellipse 1:2 / 1:2
Orientation 90º-43º / bis



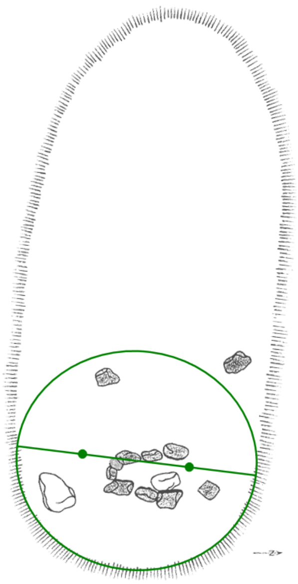
Sprockhoff 424
Loppin
Rectangle based on
ellipse 2:1
Orientation 131º



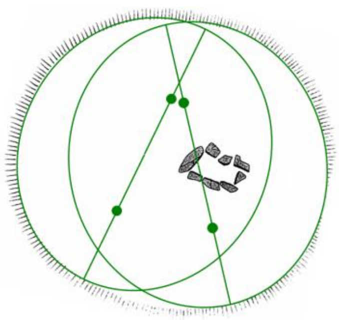
Sprockhoff 428
Waren
Ellipse 1:3
Orientation 90º+18º



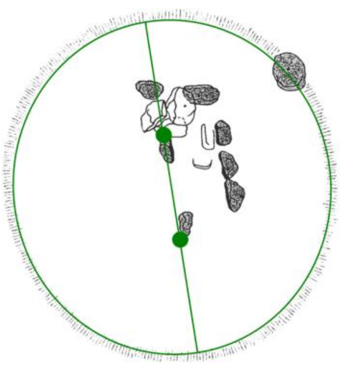
Sprockhoff 438
Freidorf
Circle (+ Ellipse 1:2)
Orientation 90º



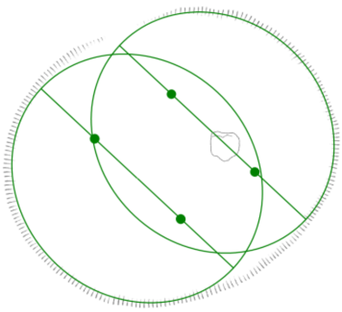
Sprockhoff 444
Dambeck
Ellipse 1:2
Orientation 6¾º



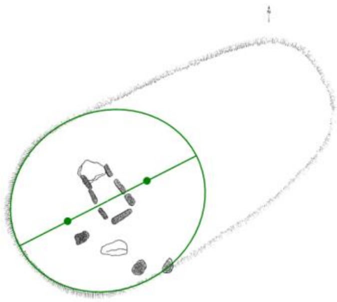
Sprockhoff 458
Dedelow
Ellipse 1:2 / 1:2
Orient. 26½º / 180º-13¼º



Sprockhoff 471
Saßnitz
Ellipse 1:3
Orientation 180º-9¼º

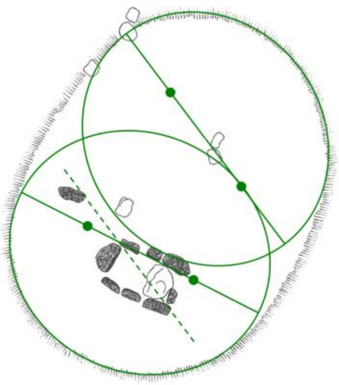


Sprockhoff 473
Dubnitz
Ellipse 1:2 / 1:2
Orientation 90º+43º / bis

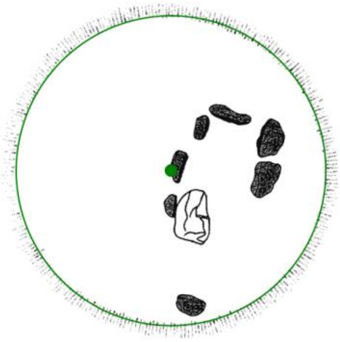


Sprockhoff 474
Mukran
Ellipse 1:2
Orientation 90º-26½º

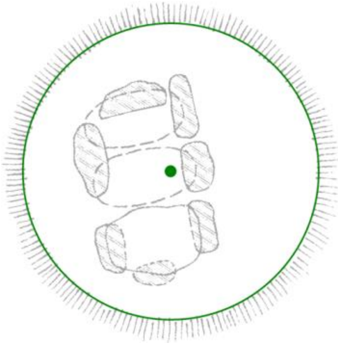
Note how the ellipse fits to the dolmen, which has a setup by means of a right triangle in a grid of squares.



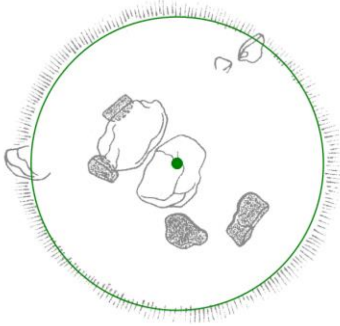
Sprockhoff 477
Hagen
Ellipse 1:2 / 1:2
Orientation 90º+26½º /
90º+53¼º



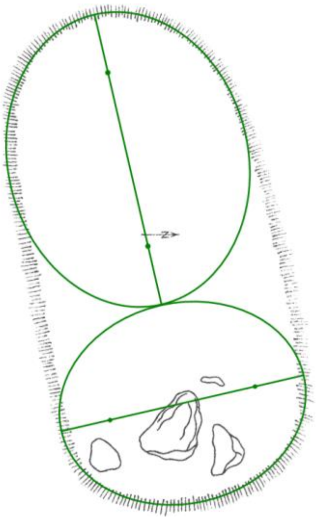
Sprockhoff 494
Seelvitz
Circle



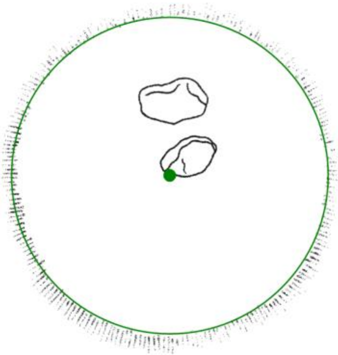
Sprockhoff 498
Nadelitz
Circle



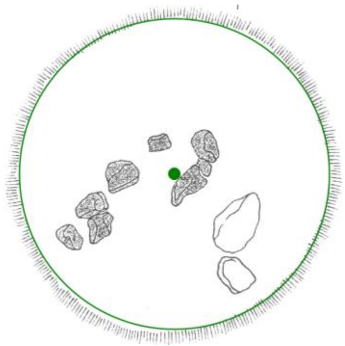
Sprockhoff 505
Lancken
Disturbed circle



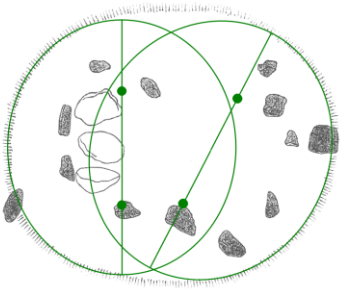
Sprockhoff 496
Nadelitz
Ellipse 3:4 / 3:4
Orientation 90º-13¼º / 180º-13¼º



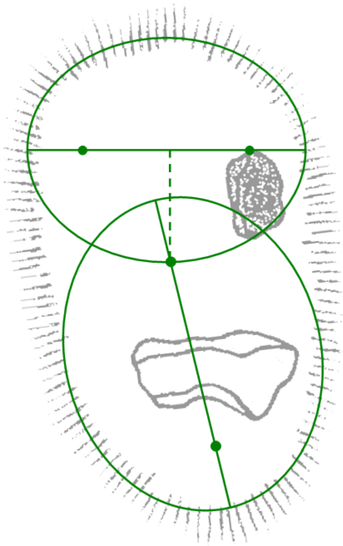
Sprockhoff 501
Lancken
Circle



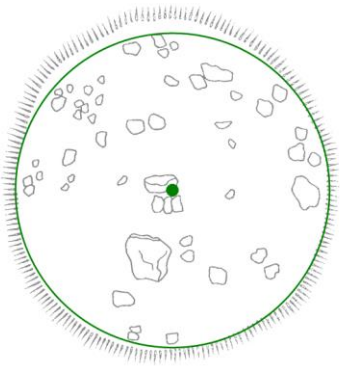
Sprockhoff 507
Preetz
Circle



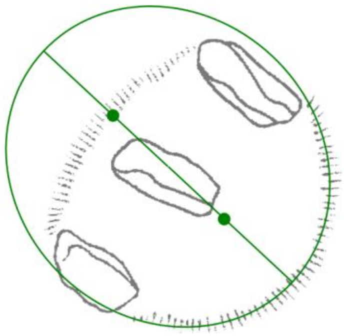
Sprockhoff 502
Lancken
Ellipse 1:2 / 1:2
Orientation 0º / 26½º



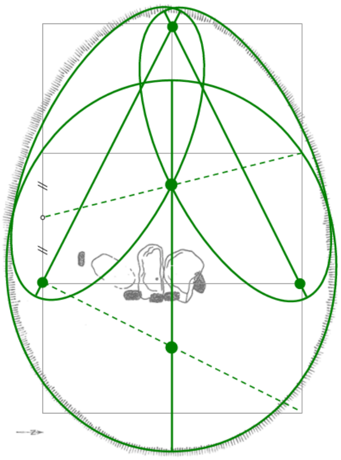
Sprockhoff 517
Nielitz
Ellipse 3:4 / 3:4
Orient. 90º / 180º-13¼º



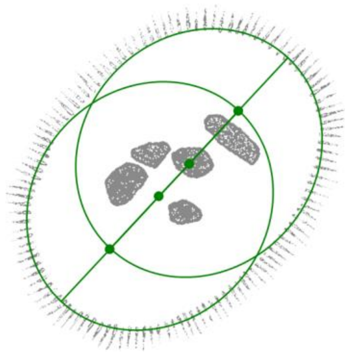
Sprockhoff 497
Nadelitz
Disturbed circle



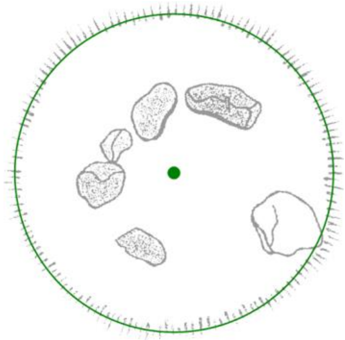
Sprockhoff 519
Nielitz
Ellipse 1:2
Orientation 90º+43º



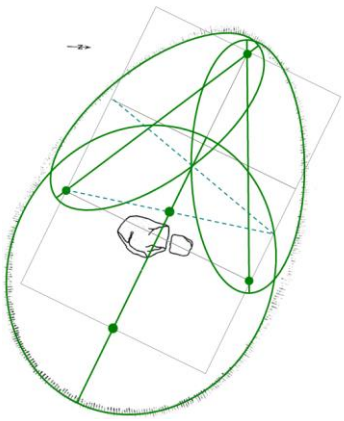
Sprockhoff 535
Groβ Labenz
Ellipse 1:2 / 2:1 / 2:1
Grid orientation 0º



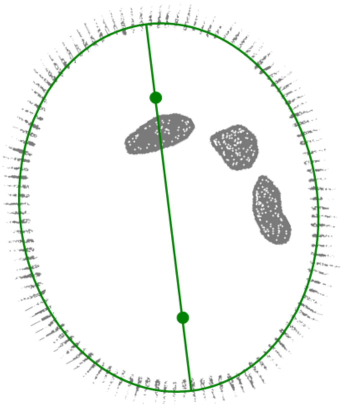
Sprockhoff 598
Schimmerwitz
Ellipse 1:2 / 1:2
Orientation 43º / bis



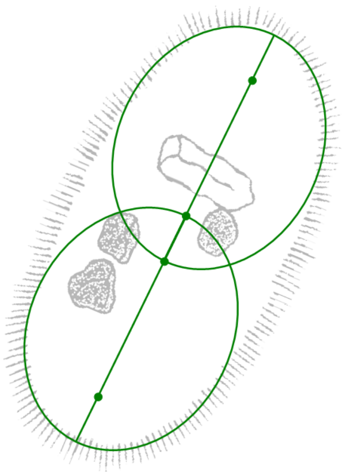
Sprockhoff 520
Nielitz
Circle



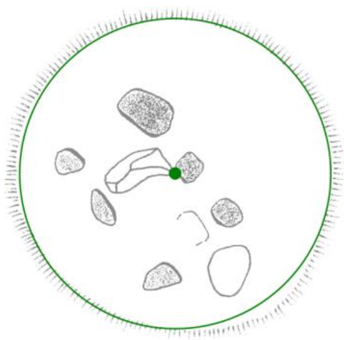
Sprockhoff 538
Dabel
Ellipse 1:2 / 2:1 / 2:1
Grid orientation 26½º



Sprockhoff 599
Schimmerwitz
Ellipse 3:4
Orientation 180º-6¾º



Sprockhoff 521
Nielitz
Ellipse 3:4 / 3:4
Orientation 26½º / bis



Sprockhoff 564
Strehlow
Circle

Prehistoric tomb surveying

Bibliography

Abdulaziz 2010, "The Plimpton 322 tablet and the Babylonian method of generating Pythagorean triples", University of Balamand archive
Allen 2000, "The origins of Greek mathematics" (lecture)
Arnadottir 2013, "Bronze working and tin trade in Bronze Age Cyprus" (thesis)
Bakker & Waterbolk 1980, "De Nederlandse Hunebedden - Rapport 1979-1980"
Bakker 1973, "De Westgroep van de Trechterbekercultuur", Part I and II
Bakker 1992, "The Dutch Hunebedden", Archaelogical Series 2 (International Monographs in Prehistory)
Bakker 1994a, "Bemerkungen zu Datierungsmethoden", Beiträge zur frühneolithischen Trichterbecherkultur im westlichen Ostseegebiet 1994, 1. Internationales Trichterbechersymposium in Schleswig (1985)
Bakker 1994b, "Eine Dolmenflasche und ein Dolmen aus Groningen", Beiträge zur frühneolithischen Trichterbecherkultur im westlichen Ostseegebiet 1994, 1. Internationales Trichterbechersymposium in Schleswig (1985)
Bakker 2010, "Megalithic Research in the Netherlands, 1547-1911"
Beck 1966, "Analysis and provenience of Minoan and Mycenaean amber, I", From: Vassar College
Beck, Southard, Adams 1967, "Analysis and provenience of Minoan and Mycenaean amber, II. Tiryns", From: Vassar College
Beck, Fellows, Adams 1979, "Analysis and provenience of Minoan and Mycenaean amber, III. Kakovatos", From: Vassar College
Beck, Southard, Adams (year unknown), "Analysis and provenience of Minoan and Mycenaean amber, IV, Mycenae", From: Vassar College
Beck, Beck 1995, "Analysis and provenience of Minoan and Mycenaean amber, V, Pylos and Messenia", From: Vassar College
Beier 1988, "Die Kugelamphorenkultur im Mittelelbe-Saale-Gebiet und in der Altmark"
Beuker 1990, "Werken met steen", Drents Museum Assen
Behrouzi and Li 2012, "Instruction Manual: Photogrammetry as a non-contact measurement system in large scale structural testing", http:/nees.illinois.edu/education/Photogrammetry Manual.pdf (August 2015)
Brindley 1986, "The typochronology of TRB West Group pottery", Palaeohistoria 28
Brindley 2003, "The use of pottery in Dutch hunebedden", Prehistoric pottery: People, pattern and purpose, BAR International Series No.4 v.4
Brindley and Lanting 1994, "A re-assessment of the hunebedden O1, D30 and D40: structure and finds", Palaeohistoria 33/34
Bruins 1949, "On Plimpton 322 - Pythagorean numbers in Babylonian mathematics", communicated by Prof. Brouwer on some meeting
Bruins 1957, "Pythagorean triads in Babylonian mathematics", The Mathematical Gazette, Vol. 41 (1957)
Burl 2005 (1983), "Prehistoric astronomy and ritual"
Castleden 2005, "Mycenaeans"
Cavanagh, Laxton 1981, "The structural mechanics of the Mycenaean tholos tomb", The Annual of the British School at Athens, Vol. 76 (1981), pp. 109-140
Chevalier 1999, "Orientations of 935 dolmens of southern France", Archeoastronomy nr.24, 1999
Como 2009, "The construction of Mycenaean tholoi" Proceedings of the Third International Congress on Construction History, Cottbus, May 2009
Cullen 1996, "Astronomy and mathematics in ancient China: the Zhou bi suan jing."

Cullen 2002, "Learning from Liu Hui - A different way to do mathematics",
Notices of the AMS, Vol. 49 (2002), No. 7
Cullen 2004, "The Suan shu shu - Writings on reckoning",
Needham Research Institute, Working papers 1 (2004)
Czebreszuk 2005, "Amber between the Baltic and the Aegean in the third and second millennia BC", Aegaeum 27, Annales dárcheologie egeennne de l'Universite de Liege et UT-PASP
Dickinson 1970, "The origins and development of early Mycenaean culture",
Doctor's thesis in Philosophy, University of Oxford
Dirlik 2012, "The tholos tombs of Mycenaean Greece"
Master's thesis in Classical Archaeology and Ancient History, Uppsala University
Czebreszuk 2007, "Amber between the Baltic and Aegean in the third and second millennia BC", Aegaeum 27
Dauben 2007, "Chinese mathematics",
In: *Katz ea 2007*, "The mathematics sourcebook", Chapter 3
Fedak, "3D Measurement Accuracy of a Consumer-Grade Digital Camera and Retro-Reflective Survey Targets", http://www.photomodeler.com/.../fedak1.pdf (August 2015)
Fritzpatrick 2008, "Euclid's elements of geometry"
Fritzpatrick 2009, "In his hands and in his head -The Amesbury Archer as a metalworker",
In: *Clark 2009*, "Bronze Age Connections Cultural Contact in Prehistoric Europe"
Galanakis 2007, "The construction of the Aegisthus tholos tomb at Mycenae and the Helladic Heresy", The Annual of the British School at Athens, Vol. 102 (2007), pp. 239-256
Gély and Pape 2014, "La nécropole des Géandes à Bourg-Saint-Andéol (Ardèche) dans le cadre du dolménisme en Bas-Vivarais", DARA 41
van Giffen 1925/27, "De Hunebedden van Nederland", Part I, II and Atlas
van Giffen, Oct. 1960 31st (letter), to the Minister of Education, Culture and Science, concerning several restorations.
Gimbutas 1956, "The Prehistory of Eastern Europe"
van Ginkel, Jager and van der Sanden 2005, "Hunebedden, Monumenten van een steentijdcultuur"
Goncalves 2008, "An alternative to the Pythagorean rule - Reevaluation problem 1 of cuneiform tablet BM34568", Historia mathematica 35 (2008)
Gonizzi *Barsanti*, *Remondino en Visintini 2012*, "Photogrammetry and Laser Scanning for Achaeological Site 3D Modelling - Some Critical Issues",
http:/ceur-ws.org/Vol-948/paper2.pdf (August 2015)
González-García, Costa-Ferrer 2003, "Possible astronomical orientation of the Dutch hunebedden", Uppsala Astronomical Observatory nr.59, 2003,
proceedings of the European Society for Astronomy in Cultures 9
González-García, Costa-Ferrer 2006, "Orientation of TRB-west megalithic monuments",
Journal for the History of Astronomy nr. xxxvii, 2006
González-García 2008, "The orientation of dolmens in Bulgaria",
Geoarchaeology and Archaeomineralogy 2008
Gupta 2011, "India's contributions to Chinese mathematics through the eighth century C.E.",
In: *Yadav & Mohan (ed) 2011*, "Ancient Indian leaps into mathematics"
Heggie 1982, "Megalithic Science"
Hiller ±1990, "The Mycenaeans and the Black Sea", ??
Hoika 1987, "Das Mittelneolithikum zur Zeit der Trichterbecherkultur in Nordostholstein",
Untersuchungen zu Archäologie und Landschaftsgeschicht 1987
Hoika 1994, "Zur Gliederung der früneolithischen Trichterbecherkultur in Holstein",
Untersuchungen und Materialien zur Steinzeit in Schleswig-Holstein 1994,
Internationale Trichterbechersymposium in Schleswig 1
Hoskin 2001, "Tombs, temples and their orientations"
Hoyrup 1998, "On a collection of geometrical riddles" (preprint),
From: Filosofi og videnskapbsteori pa Roskilde Universitetscenter, Preprints og reprints, 1998 no. 2

Hoyrup 1999, "Pythagorean 'rule' and 'theorem' - Mirror of the relation between Babylonian and Greek mathematics", From: Hoyrup's personal website, (Probably published in: International Journal of history and ethics of natural sciences, technology and medicine, Vol. 8 (2000))

Hoyrup 2002, "Lengths, widths, surfaces; A portrait of Old Babylonian algebra and its kin."

Hoyrup 2010, "Old Babylonian algebra and what it teaches us", ICM Satellite conference 'Mathematics in ancient times', Kozhikode (2010)

Hoyrup 2012, "A hypothetical history of Old Babylonian mathematics: places, passages, stages, development", Max Planckinstitute - preprint 436

Hoyrup 2016, "Seleucid, Demotic and Mediterranean mathematics versus chapters VIII and IX of the Nine Chapters" (preprint), Tsinghua Sanya International Forum (2016)

Imhausen 2006, "Ancient Egyptian mathematics - New perspectives on old sources", In: *Rowe 2006*, "Years Ago", The Mathematical Intelligencer, Vol. 28 (2006)

Imhausen 2007, "Egyptian mathematics", In: *Katz ea 2007*, "The mathematics sourcebook", Chapter 1

Kangshen ea 1999, "The nine chapters of the mathematical art, companion and commentary", Chapter 9

Katz 2007 (e.a.), "The mathematics of Egypt, Mesopotamia, China, India and Islam - A sourcebook"

Kim 2001, "Elite strategies and the spread of technological innovation", Journal of Anthropological Archaeology, Vol. 20 (2001)

Kim 2003, "Land-use conflict and the rate of the transition to agricultural economy", Journal of Archaeological Method and Theory, Vol. 10, No. 3 (2003)

Kim 2014, "From labour control to surplus appropriation", Journal of World Prehistory, Vol. 27 (2014)

Kiss 2007, "Contacts along the Danube"

Klok 1979, "Hunebedden in Nederland - Zorgen voor morgen"

Lazar ea 2004, "Corbelled domes in two and three dimensions: the Treasury of Atreus", International Statistical Review, Vol. 72, No. 2 (Aug., 2004), pp. 239-255

Langbroek 2004, "Huilen naar de maan", PIT 2

Lanting & v.d. Plicht 2002, "De ^{14}C-chronologie van de Nederlandse pre- en protohistorie, III: Neolithicum", Palaeohistoria 41/42

Lanting 1984 January 23th (letter), Request for re-opening of some trenches in the mound of hunebed D39, Archive Groninger Instute for Archaeology

Lanting 1984 November 26th-30th (document), Day report of the re-examination of some trenches in the mound of hunebed D39, Archive Groninger Instute for Archaeology

Lanting 1985 February 5th (document), Conclusive report of the re-examination of some trenches in the mound of hunebed D39, Archive Groninger Instute for Archaeology

Lanting 1986 August 22nd (document), Proposal for re- examanation of some trenches in the mound of hunebed D40, Archive Groninger Instute for Archaeology

Lanting 2008, "De NO-Nederlandse/NW-Duitse Klokbekergroep: culturele achtergrond, typologie van het aardewerk, datatering, verspreiding en grafritueel", bijlage 2, Palaeohistoria 49/50

Lemercier 2006, "The Bell Beaker phenomenon in the southeast of France", HAL archives ouvertes, Id: halshs-00087924

Maas and Hampel 2005, "Photogrammetric Techniques in Civil Engineering Material Testing and Structure Monitoring", PERS 72, January 2006

Maravelia 2002, "The orientation of nine tholos tombs at Mycenae", Archaeoatronomy, no 27 (Journal for the History of Astronomy nr. xxxiii, 2002)

Markovin 2002, "Western Caucasian dolmens", Antrhropology & Archeology of Eurasia, Vol. 41, No. 4 (2002)

Melville 2004, "Poles and walls in Mesopotamia and Egypt", Historia mathematica, Vol. 31 (2004)

Melville 2005a, "Reciprocals and reciprocal algorithms in Mesopotamian mathematics"
Melville 2005b, "The area and the side I added - Some Old Babylonian geometry", Revue d'histoire des mathématiques, Vol. 11 (2005)
Meurers-Balke and Weniger 1985, "C14-Chronologie der frühen Trichterbecherkultur im norddeutschen Tiefland und in Südskandinavien", Untersuchungen und Materialien zur Steinzeit in Schleswig-Holstein 1994, Internationale Trichterbechersymposium in Schleswig 1
Mickelson, Mickelson 2014, "Do Mycenaean tholos tombs encode astronomical alignments", Mediterranean Archaeology and Archaeometry, Vol. 14, No 3, pp. 1-14
Miranda, Belmonte, Molinero 2008, "Uncovering Seshat, New insights in the stretching of the cord ceremony", Archaeologia Baltica, Vol. 10 (2008)
Mörner, Lind 2013, "The Bronze Age in SE Sweden, Evidence for long-distance travel and advanced sun cult", Journal of Geography and Geology; Vol. 5, No. 1 (2013)
Müller 1997, "Zur absolutchronologischen Datierung der europäischen Megalithik", Tradition und Innovation, Festschrift Strahm
Muscat 2015, "Diophantus of Alexandria - Arithmetica"
Mylonas 1972, "Mycenae, A guide to its ruins and its history", third edition
Nagel 1985, "Die Erscheinungen der Kugelamphorenkultur im Norden der DDR"
Neugebauer, Sachs 1945, "Mathematical cuneiform texts" (chapter 3)
Niesiolowska 1994, "Einige Probleme der frühen Trichterbecherkultur in Polen", Untersuchungen und Materialien zur Steinzeit in Schleswig-Holstein 1994, Internationale Trichterbechersymposium in Schleswig 1
Paulson 2005, "Surveying in Egypt", WSHS 2.2 (2005)
Plofker 2007, "Mathematics in India", In: *Katz ea 2007*, "The mathematics sourcebook", Chapter 4
Plofker 2009, "Mathematics in India"
Raemaekers, Jansen 2013, "Een papieren opgraving van hunebed D12 Eexteres. Van granggraf naar dolmen.", Paleo-Aktueel 24 (2013)
Ranieri 2008, "The Pythagorean geometry of the Atreus tomb at Mycenae", Archaeologia Baltica 10
Reijs 1997, "Azimuth measurements on Dutch passage mounds", www.iol.ie/~geniet/eng (2017)
Resnikoff and Wells 2015, "Mathematics in Civilization", third edition
Robson 2001, "Neither Sherlock Holmes nor Babylon - A reassessment of Plimpton 322" Historia mathematica, Vol. 28 (2001)
Robson 2002, "Words and pictures: New light on Plimpton 322", The Mathematical Association of America, Vol. 109 (2002)
Robson 2007, "Mesopotamian mathematics", In: *Katz ea 2007*, "The mathematics sourcebook", Chapter 2
Ruggles 1984, "Megalithic Astronomy"
Ruggles 1999, "Astronomy in Prehistoric Britain and Ireland"
Ruggles & Whittle (red.), "Astronomy and Society in Britain during the period 4000-1500 BC"
van der Sanden 2015, "In het spoor van Lukis en Druyden"
Schierhold 2016, "Die Groβen Sloopsteene bei Lotte-Wersen, Kreis Steinfurt" Megalithgräber in Westfalen, Heft 1
Schuldt 1966, "Dolmen und Ganggräber an der Recknitz"
Schuldt 1972, "Die mecklenburgischen Megalithgräber"
Sprockhoff 1938, "Handbuch der Urgeschichte Deutschlands, Band 3, Die Nordische Megalithkultur"
Sprockhoff 1967, "Atlas der Megalithgräber, Teil 1, Schleswig-Holstein"
Sprockhoff 1967, "Atlas der Megalithgräber, Teil 2, Mecklenburg, Vorpommern und Brandenburg"

Sprockhoff 1967, "Atlas der Megalithgräber, Teil 3, Niedersachsen"
Sidoli ±2013, "Greek mathematics", ??
Stone 2014, "The cubit, A history and measurement commentary",
Journal of Anthropology, Vol. 2014, Article ID 489757
Stupperich 1995, "Uberlegungen zum Fussmass Mykenischer Bauten", Thetis 2, 1995, S. 21-30
Svensson 2013, "Visibility and tholos tombs in the Messenian landscape",
LUMA-GIS Thesis nr 22
van der Waerden 1954 (3de ed.), "Science awakening I"
van der Waerden 1983, "Geometry and Algebra in Ancient Civilizations", Chapter 1 and 2
Waterbolk 2009, "Getimmerd verleden"
Wiener 2014, "The interaction of climate change and agency in the collapse of civilizations ca. 2300–2000 BC"
Young Suk 2006, "Dolmens in Korea"
Zhimin 1992 "The Bronze Age in eastern parts of Central Asia",
In: *Dani, Masson 1992*, "History of civilizations of Central Asia", Vol. 1, Chapter 13