A Physically Based Model of Ice

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1 Introduction

Animations of freezing and ice formation are becoming increasingly popular in visual effects, making prominent appearances in such recent movies as *X-Men 2, The Hulk,* and *Die Another Day*. However, the methods for modeling and animating these patterns remain fairly ad-hoc. In this sketch, we present a complete physical model for simulating ice formation.

Inspired by the three stages of solidification commonly known in statistical physics [Saito 1996], we model each stage explicitly, and design a novel algorithm that is well suited for visual simulation. Due to the fractal nature of ice, our algorithm scales naturally from the small crystal arms of a snowflake to the large frost on a car windshield. The algorithm also allows for natural, intuitive controls so that the user can guide the results toward a desired aesthetic effect.

2 Algorithm

The process of solidification is commonly broken down into three stages: chemical diffusion, surface kinetics, and heat diffusion. Depending on the physical conditions in the environment, growth can become limited by each of the three stages. In a low moisture environment, the first stage is slowest, and we get what is known as *diffusion limited* growth. In a very moist environment, the second stage is slowest and we achieve *kinetics limited* growth. In an environment with strong winds, the last stage is limiting, resulting in *heat limited* growth.

In constructing our algorithm, we make two key observations. First, each limiting case can be simulated using known algorithms. Diffusion limited growth can be simulating using *diffusion limited aggregation* (DLA) [Witten and Sander 1981]. Kinetics limited growth can be simulated using *phase fields* [Kobayashi 1993; Kim and Lin 2003]. Heat limited growth can be simulated using a fluid solver [Stam 1999]. Second, we observe that in most real-world environments, all three limiting cases exist simultaneously, but at different scales. With these observations in mind, we propose a simple yet effective algorithm for combining the above three methods. Both DLA and phase fields track the location of the advancing ice front on a regular grid. If the two algorithms are allowed to track the ice front using the same grid, then we can achieve both diffusion and kinetics limited growth simultaneously. In order maintain thermodynamic consistency, we also have DLA release heat into the phase field simulation. Heat limited growth occurs when wind forces alter the flow of heat across the surface of an ice crystal. We can introduce this limiting case by allowing a stable fluid solver to advect the heat field in phase field simulation, while treating the ice crystal as an internal obstacle. The fluid is also allowed to advect the particles present in the DLA simulation. In this way, all three limiting cases can be handled simultaneously.

Our algorithm also allows fine control over the visual aspects of both the growth animation and the final ice. The user can specify where the ice should grow, and the shape of the growing front, the 'branchiness' of the advancing front, and the degree of regularity present in the crystal arms.

3 Results

To show the flexibility of our algorithm, we ran simulations on different physical scales, as shown in Figure 1, the supplementary document and video. On the macroscale of a car, a 1024×1024 simulation completed in 3 minutes and 25 seconds. On the microscale of a snowflake, a 1024×1024 simulation completed in about 2 hours. The snowflake simulation was set to high symmetry, resulting in the increased simulation time. The window simulation in the supplementary materials and video took 4 minutes and 16 seconds on a 256×256 grid. All timings exclude rendering time.

References

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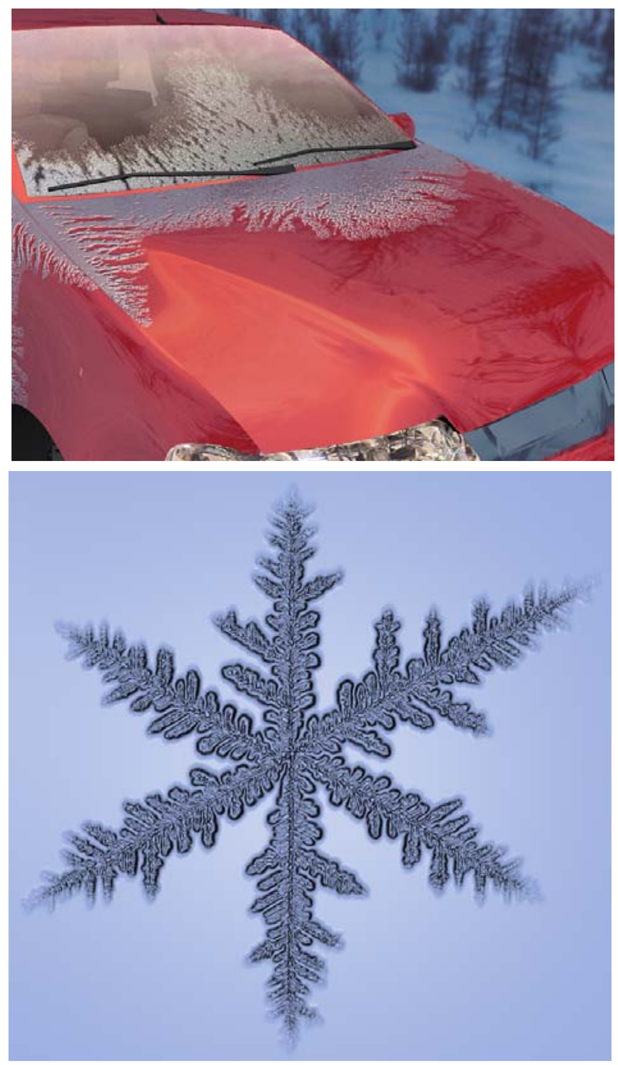

Figure 1: **Large to small growth scales handled by our algorithm.** The macroscopic scale of a pint glass and the microscopic scale of a snowflake.