Resources

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Contents

1 Resources

This section provides comprehensive resources for learning about and working with Quadrays, synergetics, and the computational methods discussed in this manuscript.

1.1 Core Concepts and Background

1.1.1 Information Geometry and Optimization

- **Fisher information**: Fisher information (reference) — see also Eq. (**??**) in the equations appendix
- **Natural gradient**: Natural gradient (reference) — see also Eq. (**??**) in the equations appendix

1.1.2 Active Inference and Free Energy

- **Active Inference Institute**: Welcome to Active Inference Institute

- **Comprehensive review**: Active Inference — recent review (UCL Discovery, 2023)

1.1.3 Mathematical Foundations

- **Tetrahedron volume formulas**: length-based Cayley–Menger determinant and determinant-based expressions on vertex coordinates (see Tetrahedron – volume)
- **Exact determinants**: Bareiss algorithm, used in our integer tetravolume implementations
- **Optimization baseline**: the Nelder–Mead method, adapted here to the Quadray lattice

1.2 Quadrays and Synergetics (Core Starting Points)

1.2.1 Introductory Materials

- **Quadray coordinates (intro and conversions)**: Urner – Quadray intro, Urner – Quadrays and XYZ
- **Quadrays and the Philosophy of Mathematics**: Urner – Quadrays and the Philosophy of Mathematics
- **Synergetics background and IVM**: Synergetics (Fuller, overview)
- **Quadray coordinates overview**: Quadray coordinates (reference)

1.2.2 Historical and Background Materials

- **RW Gray projects — Synergetics text**: rwgrayprojects.com (synergetics)
- **Fuller FAQ**: C. J. Fearnley's Fuller FAQ
- **Synergetics resource list**: C. J. Fearnley's resource page
- **Wikieducator**: Synergetics hub
- **Quadray animation**: Quadray.gif (Wikimedia Commons)
- **Fuller Institute**: BFI — Big Ideas: Synergetics

1.3 4dsolutions Ecosystem: Comprehensive Computational Framework

The 4dsolutions organization provides the most extensive computational framework for Quadrays and synergetic geometry, spanning 29+ repositories with implementations across multiple programming languages.

1.3.1 Core Computational Modules

Primary Python Libraries

- **Math for Wisdom (m4w)**: m4w (repo)
  - **Quadray vectors and conversions**: `qrays.py` (Qvector, SymPy-aware)
  - **Synergetic tetravolumes and modules**: `tetravolume.py` with PdF-CM vs native IVM and BEAST algorithms

Cross-Language Validation

- **Rust implementation**: rusty_rays (performance-oriented)
  - Sources: Rust library implementation, Rust command-line interface
- **Clojure implementation**: synmods (functional paradigm)
  - Sources: `qrays.clj`, `ramping_up.clj`

1.3.2 Primary Hub: School_of_Tomorrow (Python + Notebooks)

Repository: School_of_Tomorrow

Core Modules

- **`qrays.py`**: Quadray implementation with normalization, conversions, and vector ops (source)
- **`quadcraft.py`**: POV-Ray scenes for CCP/IVM arrangements, animations, and tutorials (source)
- **`flextegrity.py`**: Polyhedron framework, concentric hierarchy, POV-Ray export (source)
- **Additional modules**: `polyhedra.py`, `identities.py`, `smod_play.py` (synergetic modules)

Key Notebooks

- **`Qvolume.ipynb`**: Tom Ace 5×5 determinant with random-walk demonstrations (source)
- **`VolumeTalk.ipynb`**: Comparative analysis of bridging vs native tetravolume formulations (source)
- **`QuadCraft_Project.ipynb`**: 1,255 lines of interactive CCP navigation and visualization tutorials (source)
- **Additional notebooks**: `TetraBook.ipynb`, `CascadianSynergetics.ipynb`, `Rendering_IVM.ipynb`, `SphereVolumes.ipynb` (visual and curricular materials)

1.3.3 Additional Repositories

Tetravolumes (Algorithms and Pedagogy)

- **Repository**: tetravolumes
- **Code**: `tetravolume.py`
- **Notebooks**: Atoms R Us.ipynb, Computing Volumes.ipynb

Visualization and Rendering

- **BookCovers**: VPython for interactive educational animations (repo)
  - Examples: `bookdemo.py`, `stickworks.py`, `tetravolumes.py`

1.3.4 Educational Framework and Curricula

Oregon Curriculum Network (OCN)

- **OCN portal**: OCN portal
- **Python for Everyone**: pymath page

Historical Documentation

- **Python5 notebooks**: Polyhedrons 101.ipynb
- **Historical variants**: `qrays.py` also appears in Python5 (archive)
- **Python edu-sig archives**: Python edu-sig archives tracing 25+ years of development

1.3.5 Media and Publications

- **YouTube demonstrations**: Synergetics talk 1, Synergetics talk 2, Additional
- **Academia profile**: Kirby Urner at Academia.edu

1.4 Community Discussions and Collaborative Platforms

1.4.1 Active Platforms

- **Math4Wisdom Knowledge Engineering**: Collaborative platform with various art, resources, and cross-reference materials
- **synergeo discussion archive**: Groups.io platform with ongoing community discussions and technical exchanges

1.4.2 Historical Archives

- **GeodesicHelp threads**: GeodesicHelp computations archive (Google Groups) documenting computational approaches and problem-solving techniques

1.5 Related Projects and Applications

1.5.1 Tetrahedral Voxel Engines

- **QuadCraft**: Tetrahedral voxel engine using Quadrays

1.5.2 Academic Publications

- **Flextegrity**: Generating the Flextegrity Lattice (academia.edu)

1.5.3 Context and Integration

These materials popularize the IVM/CCP/FCC framing of space, integer tetravolumes, and projective Quadray normalization. They inform the methods in this paper and complement the `src/` implementations (see `quadray.py`, `cayley_menger.py`, `linalg_utils.py`).

The ecosystem provides extensive validation, pedagogical context, and practical implementations that complement and extend the methods developed in this manuscript. Cross-language implementations serve as independent verification of algorithmic correctness while educational materials demonstrate practical applications across diverse computational environments.