## Resources

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## 1 Resources (References and Further Reading)

# 1.1 Quadrays and Synergetics (core starting points)

ourses (Deferences and Further Deading)

- Quadray coordinates (intro and conversions): Urner Quadray intro, Urner Quadrays and XYZ
- Synergetics background and IVM: Synergetics (Fuller, overview)

# 1.2 4dsolutions (Kirby Urner) — repositories and key artifacts

- Organization overview: 4dsolutions (GitHub org) Python-centered explorations of Quadrays and synergetic geometry.
- Math for Wisdom (m4w): m4w (repo)
  - Quadray vectors and conversions: <a href="mailto:qrays.py">qrays.py</a> (Qvector, SymPy-aware)
  - Synergetic tetravolumes and modules: tetravolume.py PdF/CM vs native IVM, BEAST
- School\_of\_Tomorrow (notebooks/code): School\_of\_Tomorrow (repo)
  - Tom Ace 5×5 determinant: Qvolume.ipynb
  - Bridging vs native tetravolumes: VolumeTalk.ipynb
- **Historical variants**: grays.py also appears in Python5 (archive).

Context: These materials popularize the IVM/CCP/FCC framing of space, integer tetravolumes, and projective Quadray normalization. They inform the methods in this paper and complement the src/ implementations (see quadray.py, cayley menger.py, linal quils.py).

## 1.3 Comprehensive index of 4dsolutions artifacts (selected)

### 1.3.1 Primary hub: School\_of\_Tomorrow (Python + notebooks)

- Repository: School of Tomorrow
- Core modules:
  - qrays.py: Quadray implementation with normalization, conversions, and vector ops (qrays.py source School\_of\_Tomorrow)
  - quadcraft.py: POV-Ray scenes for CCP/IVM arrangements, animations, and tutorials (quadcraft.py source School of Tomorrow)
  - flextegrity.py: Polyhedron framework, concentric hierarchy, POV-Ray export (flextegrity.py source School of Tomorrow)
  - Additional: polyhedra.py, identities.py, smod play.py (synergetic modules)
- · Notebooks:
  - QuadCraft\_Project.ipynb: Interactive tutorials; CCP navigation and tetra demos (QuadCraft\_Project.ipynb School of Tomorrow)
  - Qvolume.ipynb: Tom Ace 5×5 determinant; random-walk IVM volumes (Qvolume.ipynb School of Tomorrow)
  - VolumeTalk.ipynb: Bridging (CM/PdF) vs native (Ace/GdJ) tetravolumes (VolumeTalk.ipynb School of Tomorr
  - TetraBook.ipynb, CascadianSynergetics.ipynb, Rendering\_IVM.ipynb, SphereVolumes.ipynb (visual and curricular materials)

#### 1.3.2 Additional repositories

- tetravolumes: algorithms and pedagogy for tetra volumes
  - Repo: tetravolumes
  - Code: tetravolume.py
  - Notebooks: Atoms R Us.ipynb, Computing Volumes.ipynb
- rusty\_rays: Rust port highlighting cross-language consistency
  - Repo: rusty rays
  - Sources: Rust library implementation, Rust command-line interface
- synmods: Clojure/functional approach to Quadrays and synergetic modules
  - Repo: synmods
  - Sources: qrays.clj, ramping up.clj
- BookCovers: VPython for interactive educational animations
  - Repo: BookCovers
  - Examples: bookdemo.py, stickworks.py, tetravolumes.py

### 1.3.3 Additional educational resources

- Oregon Curriculum Network (OCN): OCN portal
- Python for Everyone: pymath page
- Python5 notebooks: Polyhedrons 101.ipynb

#### 1.3.4 Media and publications

- YouTube demonstrations: Synergetics talk 1, Synergetics talk 2, Additional
- Academia profile: Kirby Urner at Academia.edu
- Fuller Institute: BFI Big Ideas: Synergetics

### 1.3.5 Background and community materials

- RW Gray projects Synergetics text: rwgrayprojects.com (synergetics)
- Fuller FAQ: C. J. Fearnley's Fuller FAQ
- Synergetics resource list: C. J. Fearnley's resource page
- Wikieducator: Synergetics hub
- Quadray animation: Quadray.gif (Wikimedia Commons)

## 1.4 Geometry and volumes (Coxeter.4D context)

- Regular polytopes (Euclidean E<sup>4</sup>): H. S. M. Coxeter, Regular Polytopes (Dover ed.), p. 119 clarifies Euclidean 4D vs spacetime.
- Sphere packings and lattices: J. H. Conway & N. J. A. Sloane, Sphere Packings, Lattices and Groups (Springer)
- Cayley-Menger determinant: Cayley-Menger determinant (reference)
- Tetrahedron volume: Tetrahedron: volume (reference)
- Bareiss algorithm (exact determinants): Bareiss algorithm (reference)

## 1.5 Optimization and information geometry

- Nelder-Mead method: Nelder-Mead (reference)
- **Fisher information**: Fisher information (reference) see also Eq. (??)
- Natural gradient: Natural gradient (reference) see Eq. (??)

## 1.6 Active Inference

- Free energy principle: Free energy principle (reference)
- Comprehensive review: Active Inference recent review (UCL Discovery, 2023)

## 1.7 Community discussions and context

- Math4Wisdom: IVM→XYZ conversions (curated page)
- synergeo (groups.io): Synergetics discussion archive
- GeodesicHelp: Geodesic computations archive (Google Groups)

## 1.8 Related projects and applications

- QuadCraft: Tetrahedral voxel engine using Quadrays
- Flextegrity: Generating the Flextegrity Lattice (academia.edu)

## 1.9 Tooling

• GCC libquadmath (binary128): Official GCC libquadmath documentation

## 1.10 Cross-language and cross-platform validation

- **Rust (rusty\_rays)** and **Clojure (synmods)** mirror the Python algorithms for vector ops and tetravolumes, serving as independent checks on correctness and performance comparisons.
- POV-Ray (quadcraft.py) and VPython (BookCovers) demonstrate rendering pipelines for CCP/IVM scenes and educational animations.