QuadMath: Front Matter and Abstract

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1 QuadMath: An Analytical Review of 4D and Quadray Coordinates

1.1 Abstract

We review a unified analytical framework for four dimensional (4D) modeling and Quadray coordinates, synthesizing geometric foundations, optimization on tetrahedral lattices, and information geometry. Building on R. Buckminster Fuller's synergetics Synergetics (Fuller) and the Quadray coordinate system, with extensive reference to Kirby Urner's computational implementations across multiple programming languages (see the comprehensive 4dsolutions ecosystem including Python, Rust, Clojure, and POV-Ray implementations), we review how integer lattice constraints yield integer volume quantization of tetrahedral simplexes, creating discrete "energy levels" that regularize optimization and enable integer-based optimization. We adapt standard methods (e.g., Nelder–Mead method) to the quadray lattice, define Fisher information in Quadray parameter space, and analyze optimization as geodesic motion on an information manifold via the natural gradient. We also integrate 4D namespaces (Quadray, Extended Euclidean, relativistic Einsteinian spaces), develop analytical tools and equations, and survey extensions and applications across AI, active inference, cognitive security, and complex systems. The result is a cohesive, interpretable approach for robust, geometry-grounded computation in 4D.

Keywords: Quadray coordinates, 4D geometry, tetrahedral lattice, integer volume quantization, information geometry, optimization, synergetics, active inference.

1.2 Manuscript structure

- Introduction: motivates Quadrays, clarifies 4D namespaces (Coxeter.4D, Einstein.4D, Fuller.4D), and summarizes contributions.
- Methods: details coordinate conventions, exact tetravolumes, conversions, and lattice-aware optimization methods (Nelder–Mead and discrete IVM descent).
- Results: empirical comparisons and demonstrations are shown inline and saved under quadmath/output/ (PNG/CSV/NPZ/MP4) for reproducibility.
- Discussion: interprets results, limitations, and implications; outlines future work.
- Appendices: equations, free-energy background, and a consolidated symbols/glossary with an auto-generated API index.

1.3 Reproducibility and data availability

- The manuscript Markdown and code to generate the PDF are available on the project repository (QuadMath on GitHub, @docxology username). See the repository home page for source, figures, and scripts: QuadMath repos-

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- The manuscript is licensed under the Apache License 2.0. See the LICENSE file for details.
- The manuscript is accompanied by a fully-tested Python codebase under `src/` with unit tests under `tests/`, complemented by extensive cross-validation against Kirby Urner's reference implementations in the 4dsolutions ecosystem.
- All figures referenced in the manuscript are generated by scripts under `quadmath/scripts/` and saved to `quadmath/output/` with lightweight CSV/NPZ alongside images.
- Tests accompany all methods under `src/` and enforce 100% coverage for `src/`; external validation includes comparisons with `qrays.py` and `tetravolume.py` algorithms.
- Symbols and notation are standardized across sections; see Appendix: Symbols and Glossary for a consolidated table of variables and constants used throughout. Equation labels (e.g., Eq. (**??**) and Eq. (**??**)) and figure labels (e.g., Figure **??**) are used consistently.