

# 4D Namespaces: Coxeter.4D, Einstein.4D, Fuller.4D

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## 1 4D Namespaces: Coxeter.4D, Einstein.4D, Fuller.4D

In this section, we clarify the three internal meanings of “4D,” following a dot-notation that avoids cross-domain confusion. First we briefly review the Coxeter.4D and Einstein.4D name spaces, which should be familiar to most readers. We then review and highlight the Fuller.4D name space, which is the focus of this manuscript.

### 1.1 Coxeter.4D (Euclidean $E^4$ )

- **Definition:** standard  $E^4$  with orthogonal axes and Euclidean metric; the proper setting for classical regular polytopes. As Coxeter notes (Regular Polytopes, Dover ed., p. 119), this Euclidean 4D is not spacetime. Lattice/packing discussions connect to Conway & Sloane’s systematic treatment of higher-dimensional sphere packings and lattices ([Sphere Packings, Lattices and Groups \(Springer\)](#)).
- **Usage:** embed Quadray configurations or compare alternative parameterizations when a strictly Euclidean 4D setting is desired.
- **Simplexes:** simplex structures extend naturally to 4D and beyond (e.g., pentachora).

### 1.2 Einstein.4D (Relativistic spacetime)

- **Spacetime:** Minkowski metric signature.
- **Line element** (mostly-plus convention; see [Minkowski space](#)):

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \quad (1)$$

- **Optimization analogy:** metric-aware geodesics generalize to information geometry where the Fisher metric replaces the physical metric. See [Fisher information](#) and [natural gradient](#).

## 1.3 Fuller.4D (Synergetics / Quadrays)

- **Basis:** four non-negative components A,B,C,D with at least one zero post-normalization, treated as a vector (direction and magnitude), not merely a point. Overview: [Quadray coordinates](#).
- **Geometry:** tetrahedral; unit tetrahedron volume = 1; integer lattice aligns with close-packed spheres (IVM). Background: [Synergetics](#).
- **Distances:** computed via appropriate projective normalization; edges align with tetrahedral axes. The IVM = CCP = FCC shortcut allows working in 3D embeddings for visualization while preserving the underlying Fuller.4D tetrahedral accounting.
- **Implementation heritage:** Extensive computational validation through Kirby Urner’s [4dsolutions ecosystem](#), particularly [qrays.py \(vector operations\)](#) and educational materials in [School\\_of\\_Tomorrow](#).

### 1.3.1 Directions, not dimensions (language and models)

- **Vector-first framing:** Treat Quadrays as four canonical directions (“spokes” to the vertices of a regular tetrahedron from its center), not as four orthogonal dimensions. The methane molecule ( $\text{CH}_4$ ) and caltrop shape are helpful mental models.
- **Origins outside Synergetics:** Quadrays did not originate with Fuller; we adopt the coordinate system within the IVM context. See [Quadray coordinates](#).
- **Language games:** Quadrays and Cartesian are parallel vector languages on the same Euclidean container; teaching them together avoids oscillating between “points now, vectors later.”

### 1.3.2 Figures

In Figure 1, we show the twelve nearest IVM neighbors with radial edges under a standard Urner embedding; Figure 2 illustrates random Quadray clouds under several embeddings.

Vector equilibrium (cuboctahedron). The shell formed by the 12 nearest IVM neighbors is the cuboctahedron, also called the vector equilibrium in synergetics. All 12 vertices are equidistant from the origin with equal edge lengths, modeling a balanced local packing. This geometry underlies the “twelve around one” close-packing motif and appears in tensegrity discussions as a canonical balanced structure. See background: [Cuboctahedron \(vector equilibrium\)](#) and synergetics references. Computational demonstrations include [ivm\\_neighbors.py](#) and related visualizations in the 4dsolutions ecosystem.

### 1.3.3 Clarifying remarks

- “A time machine is not a tesseract.” The tesseract is a Euclidean 4D object (Coxeter.4D), while Minkowski spacetime (Einstein.4D) is indefinite and not Euclidean; conflating the two leads to category errors. Fuller.4D, in turn, is a tetrahedral, mereological framing of ordinary space emphasizing shape/angle relations and IVM quantization. Each namespace carries distinct assumptions and should be used accordingly in analysis.

## 1.4 Practical usage guide

- Use **Fuller.4D** when working with Quadrays, integer tetravolumes, and IVM neighbors (native lattice calculations).
- Use **Coxeter.4D** for Euclidean length-based formulas, higher-dimensional polytopes, or comparisons in  $E^4$  (including Cayley–Menger).
- Use **Einstein.4D** as a metric analogy when discussing geodesics or time-evolution; do not mix with synergetic unit conventions.

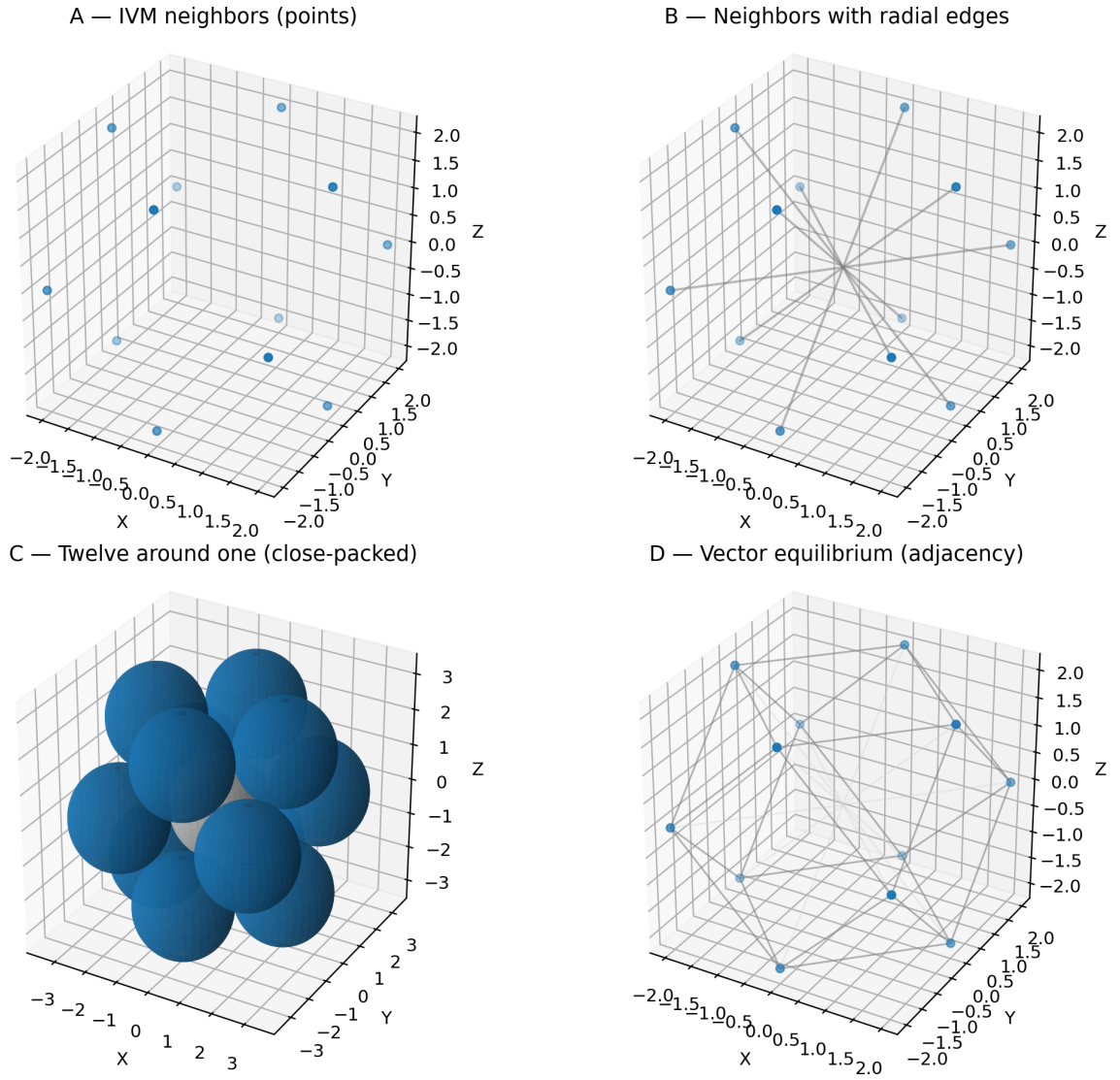


Figure 1: **IVM neighbors and coordination patterns (2×2 panel layout)**. **Panel A**: The twelve nearest IVM neighbors plotted as blue points in 3D space under the default embedding, showing the positions corresponding to permutations of the Quadray integer coordinates  $\{2,1,1,0\}$ . These points form the vertices of a cuboctahedron (vector equilibrium) centered at the origin with uniform radial distances. **Panel B**: The same neighbor points with radial edges (light lines) connecting each neighbor to the central origin, emphasizing the spoke-like radial symmetry and equal distances from center to shell. **Panel C**: Twelve-around-one close-packed spheres configuration where each neighbor position hosts a sphere with radius chosen so neighboring spheres kiss along cuboctahedron edges, illustrating the fundamental CCP/FCC/IVM correspondence. The central gray sphere represents the "one" in Fuller's "twelve around one" motif. **Panel D**: Adjacency graph showing strut connections (solid lines) between touching neighbor spheres, revealing the cuboctahedron's edge structure, plus light radial cables to the origin representing a stylized tensegrity interpretation of the vector equilibrium geometry.

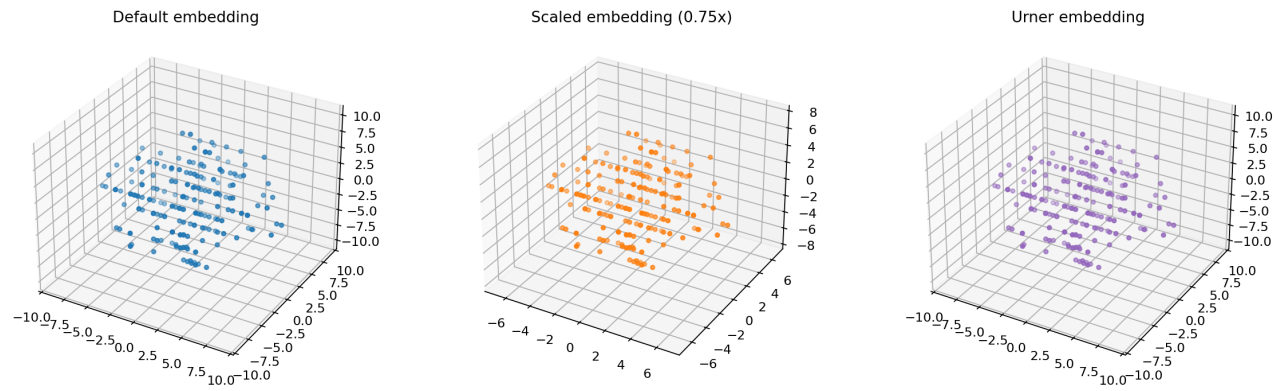
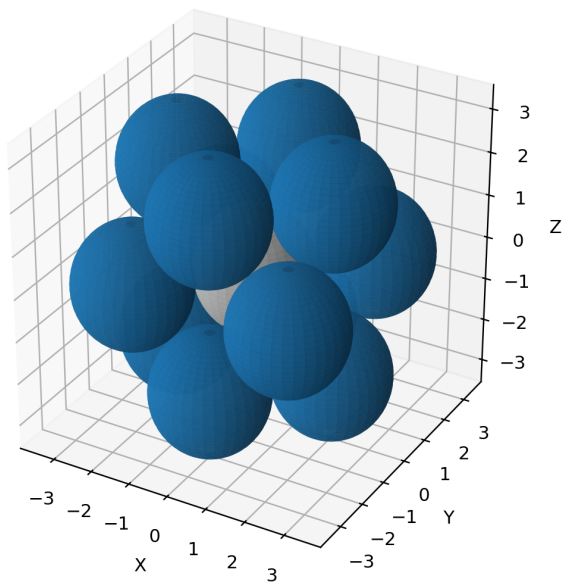


Figure 2: **Random Quadray point clouds under different embeddings (3-panel comparison)**. Each panel shows 200 randomly sampled integer Quadray coordinates with components in  $\{0,1,2,3,4,5\}$  projected to 3D space using different embedding matrices. **Left panel (Default embedding)**: Points (blue) under the default symmetric embedding matrix showing the natural tetrahedral-symmetric distribution of normalized Quadrays in 3D space. **Center panel (Scaled embedding,  $0.75\times$ )**: The same Quadray points (orange) under a uniformly scaled version of the default embedding, demonstrating how the point cloud structure scales proportionally while preserving relative geometries. **Right panel (Urner embedding)**: The same points (purple) projected through the canonical Urner embedding matrix, illustrating how different linear mappings from Fuller.4D to Coxeter.4D (3D slice) affect the spatial distribution while preserving the underlying discrete lattice relationships. This comparison demonstrates the flexibility in choosing embeddings for visualization and analysis while maintaining the fundamental Quadray coordinate relationships.

A — Vector equilibrium as close-packed spheres



B — Vector equilibrium (struts) with radial cables

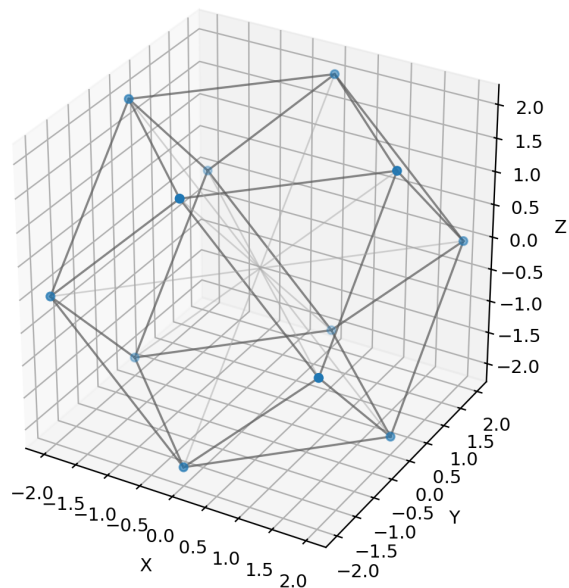


Figure 3: **Vector equilibrium (cuboctahedron) rendered in two complementary styles (2-panel comparison)**. Both panels show the same underlying geometry: the twelve IVM neighbors at permutations of Quadray coordinates  $\{2,1,1,0\}$  forming a cuboctahedron shell around the central origin. **Panel A (Close-packed spheres)**: Thirteen spheres total—one central gray sphere plus twelve blue spheres positioned at the neighbor coordinates. The sphere radius is chosen as half the minimum neighbor-to-neighbor distance so that touching spheres kiss along cuboctahedron edges, demonstrating the “twelve around one” close-packing motif fundamental to FCC/CCP arrangements and Fuller’s synergetic geometry. This visualization emphasizes the volume-filling and coordination aspects of the IVM lattice. **Panel B (Tensegrity-style network)**: The same twelve neighbor positions shown as blue points connected by structural elements—solid dark struts represent edges between touching neighbors (cuboctahedron edges), while light radial cables connect each neighbor to the central origin. This stylized tensegrity interpretation emphasizes the balance between continuous tension (radial cables) and discontinuous compression (neighbor-to-neighbor struts), illustrating principles central to Fuller’s tensegrity and the structural mechanics of the vector equilibrium.