

## Abstract

This research presents a novel optimization framework that combines theoretical rigor with practical efficiency, developing a comprehensive mathematical framework that achieves both theoretical convergence guarantees and superior experimental performance across diverse optimization problems. Building on foundational work in convex optimization [?, ?] and recent advances in adaptive optimization [?, ?], our work makes several significant contributions to the field of optimization: a unified approach combining regularization, adaptive step sizes, and momentum techniques; proven linear convergence with rate  $\rho \in (0, 1)$  and optimal  $O(n \log n)$  complexity per iteration; efficient algorithm implementation validated on real-world problems; and comprehensive experimental evaluation across multiple problem domains. The core algorithm solves optimization problems of the form  $f(x) = \sum_{i=1}^n w_i \phi_i(x) + \lambda R(x)$  using an iterative update rule with adaptive step sizes and momentum terms, where theoretical analysis establishes convergence guarantees and complexity bounds that are validated through