Results

This section presents the theoretical results and mathematical derivations obtained through our methodological approach.

Theoretical Results

The main theoretical contribution is encapsulated in the following proposition:

Proposition 1. For any continuously differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, the gradient descent algorithm with appropriate step sizes converges to a stationary point.

Mathematical Derivations

Consider the Taylor expansion of f around point x:

$$f(x + h) = f(x) + \nabla f(x)^T h + \frac{1}{2} h^T \nabla^2 f(x) h + O(\|h\|^3)$$

For small h, the dominant term is the linear term $\nabla f(x)^T h$.

Algorithm Convergence

The convergence rate analysis yields: