Appendix

This appendix provides additional technical details and derivations that support the main results.

A. Detailed Proofs

A.1 Proof of Convergence (Theorem 1)

The convergence rate established in (**??**) follows from the following detailed analysis.

Proof: Let x_k be the iterate at step k. From the update rule (**??**), we have:

$$x_{k+1} = x_k - \alpha_k \nabla f(x_k) + \beta_k (x_k - x_{k-1}) \tag{1}$$

By the Lipschitz continuity of ∇f, there exists a constant $L > 0$ such that:

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|x - y\|, \quad \forall x, y \in \mathcal{X} \tag{2}$$

Using strong convexity with parameter $\mu > 0$ [**?**, **?**]: