

# Results

This section presents the theoretical results and mathematical derivations obtained through our methodological approach.

## Theoretical Results

The main theoretical contribution is encapsulated in the following proposition:

**Proposition 1.** For any continuously differentiable function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ , the gradient descent algorithm with appropriate step sizes converges to a stationary point.

## Mathematical Derivations

Consider the Taylor expansion of  $f$  around point  $x$ :

$$f(x + h) = f(x) + \nabla f(x)^T h + \frac{1}{2} h^T \nabla^2 f(x) h + O(\|h\|^3)$$

For small  $h$ , the dominant term is the linear term  $\nabla f(x)^T h$ .

## Algorithm Convergence

The convergence rate analysis yields: