

## Supplemental Methods

This supplemental section provides extended methodological details, including complete generative model specifications, mathematical derivations, and implementation algorithms.

### Complete Generative Model Specifications

#### Matrix A: Observation Likelihoods

The observation likelihood matrix defines the probabilistic mapping from hidden states to observations:

$$[A = \begin{pmatrix} P(o_1|s_1) & P(o_1|s_2) & \dots & P(o_1|s_n) \\ P(o_2|s_1) & P(o_2|s_2) & \dots & P(o_2|s_n) \\ \vdots & \vdots & \ddots & \vdots \\ P(o_m|s_1) & P(o_m|s_2) & \dots & P(o_m|s_n) \end{pmatrix}]$$

**Normalization:** Each column sums to 1, representing a valid probability distribution over observations for each state.

**Interpretation:** - Rows correspond to observation modalities - Columns correspond to hidden state conditions - Entry  $A[i,j]$