

## Supplemental Methods

This supplemental section provides methodological details, including generative model specifications, mathematical derivations, and implementation algorithms. These materials support the main text by providing complete technical specifications that enable replication and extension of the quadrant structure analysis.

### Generative Model Specifications

#### Matrix A: Observation Likelihoods

The observation likelihood matrix defines the probabilistic mapping from hidden states to observations:

$$A = \begin{pmatrix} P(o_1 | s_1) & P(o_1 | s_2) & \cdots & P(o_1 | s_n) \\ P(o_2 | s_1) & P(o_2 | s_2) & \cdots & P(o_2 | s_n) \\ \vdots & \vdots & \ddots & \vdots \\ P(o_m | s_1) & P(o_m | s_2) & \cdots & P(o_m | s_n) \end{pmatrix}$$

**Normalization:** Each column sums to 1:  $(\sum_i A[i,j] = 1)$  for all  $(j)$ , representing a valid probability distribution over observations for each state. This ensures that for any hidden state  $(s_j)$ , the