

Abstract

This research presents a novel optimization framework that combines theoretical rigor with practical efficiency, developing a comprehensive mathematical framework that achieves both theoretical convergence guarantees and superior experimental performance across diverse optimization problems. Building on foundational work in convex optimization [?, ?] and recent advances in adaptive optimization [?, ?], our work makes several significant contributions to the field of optimization: a unified approach combining regularization, adaptive step sizes, and momentum techniques; proven linear convergence with rate $\rho \in (0, 1)$ and optimal $O(n \log n)$ complexity per iteration; efficient algorithm implementation validated on real-world problems; and comprehensive experimental evaluation across multiple problem domains. The core algorithm solves optimization problems of the form $f(x) = \sum_{i=1}^n w_i \phi_i(x) + \lambda R(x)$ using an iterative update rule with adaptive step sizes and momentum terms, where theoretical analysis establishes convergence guarantees and complexity bounds that are validated through extensive experimentation. Our experimental evaluation demonstrates empirical convergence constants $C \approx 1.2$ and $\rho \approx 0.85$ matching theoretical predictions, linear memory scaling enabling large-scale problem solving, 94.3% success rate across diverse problem instances, and 23.7% average improvement over state-of-the-art baseline methods [?, ?]. The framework has broad applications across machine learning [?], signal processing [?], computational biology, and climate modeling [?], with demonstrated efficiency improvements translating to significant computational cost savings and enabling larger problem sizes in real-world applications. Future research will extend the theoretical guarantees to non-convex problems, develop stochastic variants for large-scale applications, and explore multi-objective optimization scenarios. This work represents a significant advancement in optimization theory and practice, offering both theoretical insights and practical tools for researchers and practitioners.