Conclusion

Summary of Contributions

This work establishes Containment Theory as a computationally verified alternative foundation for Boolean reasoning. Our primary contributions are:

1. Rigorous Implementation

We provide a complete computational framework implementing: - **Form construction**: Void, mark, enclosure, and juxtaposition operations - **Reduction engine**: Polynomial-time reduction to canonical forms with step traces - **Theorem verification**: Automated checking of all nine Spencer-Brown consequences - **Boolean correspondence**: Verified isomorphism to Boolean algebra - **Evaluation semantics**: Sound extraction of truth values

2. Formal Verification

All theoretical claims are computationally verified: - Both axioms (Calling and Crossing) demonstrated - Nine derived consequences (C1-C9) verified by reduction - De Morgan's laws established - Boolean axiom set confirmed - Consistency (non-contradiction) proven

Key Findings

The Minimality of Distinction

The entire Boolean algebra emerges from a single cognitive primitive: **making a distinction**. This suggests that: - Logic is fundamentally spatial - Boolean reasoning requires minimal axiomatic commitment - Complexity in formal systems may be reducible

Self-Reference as Dynamics

Rather than generating paradoxes, self-referential forms in boundary logic produce **temporal oscillation**. The imaginary value $j = \langle j \rangle$ is not contradictory but dynamic—suggesting that self-reference naturally leads to process rather than paradox.

Geometric Foundations

Boundary logic's success demonstrates that geometric intuition can serve as mathematical foundation. The mark creates inside/outside; enclosure creates negation; juxtaposition creates conjunction. These spatial operations suffice for propositional completeness.

Implications

For Foundations of Mathematics

Containment Theory demonstrates that alternative foundations exist with different trade-offs: - **Set Theory**: Power and generality at cost of axiom complexity - **Boundary Logic**: Minimality and intuition for finite structures

Neither replaces the other; they serve different purposes.

For Computer Science

Digital logic gains: - Direct correspondence between forms and circuits - Reduction-based optimization potential - Geometric visualization of Boolean functions

For Cognitive Science

The calculus provides formal tools for studying [**?**, **?**, **?**]: - Distinction as primitive cognitive act - Negation as boundary crossing - Self-reference as oscillation - Attention as juxtaposition

Future Work

Immediate Extensions

1. **Variable quantification**: Extending to predicate logic
2. **Arithmetic integration**: Incorporating Bricken's iconic arithmetic
3. **Imaginary value computation**: Full treatment of self-referential dynamics

Long-term Research

1. **Category-theoretic formalization**: Forms as a category with natural transformations
2. **Quantum boundary logic**: Superposition in boundary notation
3. **Neural boundary networks**: Boundary-based machine learning architectures

Open Questions

1. **Is the consequence system complete?** Do C1-C9 generate all Boolean identities?
2. **What are tight complexity bounds?** Optimal reduction

Reproducibility

All results are reproducible: - Complete source code: `project/src/` - Test suite: `project/tests/` - Scripts: `python3 scripts/02_run_analysis.py` - Documentation: This manuscript and `AGENTS.md`

The implementation uses only standard Python libraries with no external dependencies beyond numpy and matplotlib for visualization.

Closing Remarks

G. Spencer-Brown opened *Laws of Form* with:

> *"A universe comes into being when a space is severed or taken apart."*

Our computational verification confirms that this simple act—making a distinction—suffices to generate the complete Boolean algebra. The boundary is both primitive and powerful, creating structure from void through the minimal commitment of two axioms.

Containment Theory stands as a testament to mathematical minimalism: that complexity often arises from simplicity, and that the foundations of logic may be more spatial than symbolic.

"We take as given the idea of distinction and the idea of indication, and that we cannot make an indication without drawing a distinction."

— G. Spencer-Brown, *Laws of Form* (1969)