

# Appendix

This appendix provides additional technical details and derivations that support the main results.

## A. Detailed Proofs

### A.1 Proof of Convergence (Theorem 1)

The convergence rate established in (??) follows from the following detailed analysis.

**Proof:** Let  $x_k$  be the iterate at step  $k$ . From the update rule (??), we have:

$$x_{k+1} = x_k - \alpha_k \nabla f(x_k) + \beta_k (x_k - x_{k-1}) \quad (1)$$

By the Lipschitz continuity of  $\nabla f$ , there exists a constant  $L > 0$  such that:

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|x - y\|, \quad \forall x, y \in \mathcal{X} \quad (2)$$

Using strong convexity with parameter  $\mu > 0$  [?, ?]:

$$f(x) - f(y) \leq \nabla f(y)^T(x - y) - \frac{\mu}{2}\|x - y\|^2 \quad (3)$$