

## Supplemental Methods

This supplemental section provides methodological details, including generative model specifications, mathematical derivations, and implementation algorithms.

### Generative Model Specifications

#### Matrix A: Observation Likelihoods

The observation likelihood matrix defines the probabilistic mapping from hidden states to observations:

$$A = \begin{pmatrix} P(o_1 | s_1) & P(o_1 | s_2) & \dots & P(o_1 | s_n) \\ P(o_2 | s_1) & P(o_2 | s_2) & \dots & P(o_2 | s_n) \\ \vdots & \vdots & \ddots & \vdots \\ P(o_m | s_1) & P(o_m | s_2) & \dots & P(o_m | s_n) \end{pmatrix}$$

**Normalization:** Each column sums to 1, representing a valid probability distribution over observations for each state.

**Interpretation:** - Rows correspond to observation modalities - Columns correspond to hidden state conditions - Entry ( $A[i,j]$ ) represents the probability of observing ( $o_i$ ) given state ( $s_j$ )