Supplemental Methods

This section provides detailed methodological information that supplements Section **??**.

S1.1 Extended Algorithm Variants

S1.1.1 Stochastic Variant

For large-scale problems, we developed a stochastic variant of our algorithm:

$$x_{k+1} = x_k - \alpha_k \nabla f_{i_k}(x_k) + \beta_k (x_k - x_{k-1}) \tag{1}$$

where i_k is a randomly sampled index from $\{1, \ldots, n\}$ at iteration k.

Convergence Analysis: Under appropriate sampling strategies, this variant achieves $O(1/\sqrt{k})$ convergence rate for non-strongly convex problems, following the analysis in [**?**, **?**].

S1.1.2 Mini-Batch Variant

To balance between computational efficiency and convergence speed: