

# Supplemental Methods

This section provides detailed methodological information that supplements Section ??.

## S1.1 Extended Algorithm Variants

### S1.1.1 Stochastic Variant

For large-scale problems, we developed a stochastic variant of our algorithm:

$$x_{k+1} = x_k - \alpha_k \nabla f_{i_k}(x_k) + \beta_k(x_k - x_{k-1}) \quad (1)$$

where  $i_k$  is a randomly sampled index from  $\{1, \dots, n\}$  at iteration  $k$ .

**Convergence Analysis:** Under appropriate sampling strategies, this variant achieves  $O(1/\sqrt{k})$  convergence rate for non-strongly convex problems, following the analysis in [?, ?].

### S1.1.2 Mini-Batch Variant

To balance between computational efficiency and convergence speed: