

Appendix

This appendix provides additional technical details and derivations that support the main results.

A. Detailed Proofs

A.1 Proof of Convergence (Theorem 1)

The convergence rate established in (??) follows from the following detailed analysis.

Proof: Let x_k be the iterate at step k . From the update rule (??), we have:

$$x_{k+1} = x_k - \alpha_k \nabla f(x_k) + \beta_k (x_k - x_{k-1}) \quad (1)$$

By the Lipschitz continuity of ∇f , there exists a constant $L > 0$ such that:

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|x - y\|, \quad \forall x, y \in \mathcal{X} \quad (2)$$

Using strong convexity with parameter $\mu > 0$ [?, ?]:

$$f(x) - f(y) \leq \nabla f(y)^T (x - y) + \frac{\mu}{2} \|x - y\|^2 \quad (3)$$