

## Supplemental Analysis

This section provides detailed analytical results and theoretical extensions that complement the main findings presented in Sections ?? and ??.

### S3.1 Theoretical Extensions

#### S3.1.1 Non-Convex Optimization Extensions

While our main theoretical results focus on convex optimization problems, we have extended the framework to handle certain classes of non-convex problems. Following the approach outlined in [?], we consider objectives that satisfy the Polyak-Łojasiewicz condition:

$$\|\nabla f(x)\|^2 \geq 2\mu(f(x) - f^*) \quad (1)$$

where  $f^*$  is the global minimum value. Under this condition, our algorithm achieves linear convergence even for non-convex problems, as demonstrated in [?].

#### S3.1.2 Stochastic Variants and Convergence Guarantees